

# Using Neighbourhoods with the Guaranteed Convergence PSO

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**Abstract**— The standard Particle Swarm Optimiser (PSO) may prematurely converge on suboptimal solutions that are not even guaranteed to be local extrema. The guaranteed convergence modifications to the PSO algorithm ensure that the PSO at least converges on a local extremum at the expense of even faster convergence. This faster convergence means that less of the search space is explored reducing the opportunity of the swarm to find better local extrema. Various neighbourhood topologies inhibit premature convergence by preserving swarm diversity during the search. This paper investigates the performance of the Guaranteed Convergence PSO (GCP SO) using different neighbourhood topologies and compares the results with their standard PSO counterparts.

## I. INTRODUCTION

The Particle Swarm Optimiser (PSO) was first introduced by Kennedy and Eberhart [1], [2]. It can be applied to virtually any problem that can be expressed in terms of an objective function for which extrema must be found.

The PSO algorithm is iterative and involves initialising a number of vectors (called particles) randomly within the search space of the objective function. These particles are collectively known as the swarm. Each particle represents a potential solution to the problem expressed by the objective function. During each time step the objective function is evaluated to establish the fitness of each particle using its position as input. Fitness values are used to determine which positions in the search space are better than others. Particles are then made to “fly” through the search space being attracted to both their personal best position as well as the best position found by the swarm so far.

The particles are “flown” through the search space by updating the position of the  $i^{\text{th}}$  particle at time step  $t$  according to the following equation:

$$\mathbf{x}_i(t+1) = \mathbf{x}_i(t) + \mathbf{v}_i(t) \quad (1)$$

where  $\mathbf{x}_i(t)$  and  $\mathbf{v}_i(t)$  are vectors representing the current position and velocity respectively. Assuming particles of dimension  $j \in 1 \dots n$ , the velocity

updates are governed by the following equation [3]:

$$\begin{aligned} v_{i,j}(t+1) = & wv_{i,j}(t) + \\ & c_1r_{1,j}(y_{i,j} - x_{i,j}(t)) + \\ & c_2r_{2,j}(\hat{y}_j - x_{i,j}(t)) \end{aligned} \quad (2)$$

where  $0 \leq w < 1$  is an inertia weight determining how much of the particle’s previous velocity is preserved,  $c_1$  and  $c_2$  are two positive acceleration constants,  $r_{1,j}$  and  $r_{2,j}$  are two uniform random sequences sampled from  $U(0,1)$ ,  $y_i$  is the personal best position found by the  $i^{\text{th}}$  particle and  $\hat{y}$  is the best position found by the entire swarm so far.

The effect of neighbourhoods on the PSO have been studied [4], [5]. It has been found that neighbourhoods can improve performance, particularly on multimodal functions. Neighbourhoods preserve swarm diversity, inhibiting early convergence to suboptimal solutions. Despite this, the standard PSO is not guaranteed to converge on a local extremum. The Guaranteed Convergence PSO (GCP SO) [6], [7] guarantees local convergence at the expense of faster convergence which reduces swarm diversity. The purpose of this paper is to determine whether the GCP SO also benefits from neighbourhoods. Two neighbourhood topologies were applied to the GCP SO and tested on a variety of benchmark functions. This paper presents the results of the experiments conducted and compares these results with the standard PSO.

The convergence behaviour of the standard PSO is discussed in Section II. Section III provides an overview of the GCP SO. Next, an overview of Neighbourhoods is provided in Section IV. Benchmark functions to measure the performance of the PSO are given in Section V. The experiment configuration can be found in Section VI. Results of the experiments are presented in Section VII. Finally, some concluding remarks are made in Section VIII.

## II. CONVERGENCE OF THE STANDARD PSO

The velocity update in Equation (2) consists of three terms. The first is the inertia term. The second is known as the cognition component because it

models the particle's personal knowledge about the search space. The third term is known as the social component because it models the interaction with the other particles.

To understand the convergence behaviour of the standard PSO, consider the scenario where the objective function is unimodal. As a particle moves towards  $\hat{\mathbf{y}}$  its personal best position must improve due to the unimodal landscape. Thus, on unimodal functions,  $\mathbf{x}_i = \mathbf{y}_i$  giving a zero cognition component. All particles are drawn by the social component toward  $\hat{\mathbf{y}}$ . When a particle reaches  $\hat{\mathbf{y}}$  then  $\mathbf{x}_i = \hat{\mathbf{y}}$  and the social component no longer plays any significant role either. Once the cognition and social components can be ignored, it is then clear that a given particle eventually stops moving. Since  $w < 1$ , the velocity is reduced by a non-zero amount during each time step.

Note that the best particle moves based only on the inertia term since  $\mathbf{x}_i = \mathbf{y}_i = \hat{\mathbf{y}}$  at the time step when it became the best. After this point, its position may improve, in which case  $\mathbf{x}_i = \mathbf{y}_i = \hat{\mathbf{y}}$  holds again. Alternatively, its position will worsen, in which case it will be drawn back to  $\mathbf{y} = \hat{\mathbf{y}}$  by the social component. This makes it likely for the best particle to stand still, particularly for values of  $w \ll 1$ .

Informally, when all the particles collapse with zero velocity on a given position in the search space, then the swarm has converged. If all the particles manage to catch up to the best particle before the optimum location is reached then the only component keeping the swarm moving is the inertia term. In this case, it is possible for the inertia weight to drive all velocities to zero before the swarm manages to reach a local extremum. Worse, the inertia is not even necessarily in a direction that leads to an extremum. Thus, it is possible for the standard PSO to converge prematurely without finding a local extremum.

For a more formal discussion and mathematical proofs about the convergence behaviour of the standard PSO see [7], [8]. The following relation should hold in order for the PSO to converge [7]:

$$\frac{c_1 + c_2}{2} - 1 < w \quad (3)$$

### III. THE GCP SO

The GCP SO was introduced by Van den Bergh [6], [7] to address the issue of premature convergence to solutions that are not guaranteed to be local extrema. The modifications to the standard PSO involve replacing the velocity update equation (see Equation (2)) of only the best particle with the fol-

lowing (based on [9]):

$$v_{i,j}(t+1) = wv_{i,j}(t) - x_{i,j}(t) + y_{i,j} + \rho(t)r_j \quad (4)$$

where  $r_j$  is a sequence of uniform random numbers sampled from  $U(-1, 1)$  and  $\rho(t)$  is a scaling factor determined using [7]:

$$\rho(0) = 1.0$$

$$\rho(t+1) = \begin{cases} 2\rho(t) & \text{if } \# \text{successes} > s_c \\ 0.5\rho(t) & \text{if } \# \text{failures} > f_c \\ \rho(t) & \text{otherwise} \end{cases} \quad (5)$$

where  $s_c$  and  $f_c$  are tunable threshold parameters. Whenever the best particle improves its personal best position, the success count is incremented and the failure count is set to 0 and *vice versa*. The success and failure counters are both set to 0 whenever the best particle changes.

These modifications cause the best particle to perform a directed random search in a non-zero volume around its best position in the search space. Recall that, in the standard PSO, the best particle can only move towards a better solution due to the inertia term (which may be 0). Therefore, the directed random search can dramatically speed up movement towards an extremum, particularly on unimodal functions. In addition, the GCP SO is guaranteed to at least converge on a local extremum (see [7] for a proof).

On multimodal functions, the GCP SO has the potential to find poorer solutions than the standard PSO. This is a consequence of faster convergence of the best particle towards a local extremum.

Consider a scenario where the best particle has rapidly found a local extremum due to the directed random search. This extremum would be a much better solution at any given point in time than it would have been in the case of the standard PSO. As a result, other particles approaching this position are not likely to be candidates for the best position. These particles miss opportunities to draw the swarm to their areas of the search space. The entire swarm may be drawn to a local solution earlier, limiting exploration.

### IV. NEIGHBOURHOODS

Neighbourhoods model the structure of social networks [4]. Implementing neighbourhoods in the standard PSO requires replacing the velocity update (see Equation (2)) with the following:

$$v_{i,j}(t+1) = wv_{i,j}(t) + c_1 r_{1,j}(y_{i,j} - x_{i,j}(t)) + c_2 r_{2,j}(\bar{y}_{i,j} - x_{i,j}(t)) \quad (6)$$

where  $\hat{y}_i$  is the best position found so far in the neighbourhood of the  $i^{\text{th}}$  particle as opposed to the best position found by the entire swarm.

The original velocity update equation models a fully connected network structure since every particle is attracted to the best solution found by the entire swarm. This topology is known as *gbest* [10]. Other topologies differ only by which particles are considered to be in the neighbourhood of the  $i^{\text{th}}$  particle and as a consequence which particles play a role in the social component of Equation (6).

The *lbest* [10] topology includes only the  $k$  nearest neighbours, in terms of particle indices, of the  $i^{\text{th}}$  particle. This forms a ring like topology and as such is also known as the circle topology.

More recently, Kennedy [5] identified other useful neighbourhood topologies including the Von Neumann topology. The Von Neumann topology connects the particles in a grid network structure where each particle is directly connected to four neighbours: above, below, to the left and to the right.

The primary purpose of neighbourhoods is to preserve diversity within the swarm by impeding the flow of information through the network. Neighbourhood topologies such as *lbest* with  $k = 2$  provide a very slow flow of information since any two randomly chosen particles communicate social information via a long indirect path. This allows particles to explore larger areas of the search space without being immediately drawn to the global best particle.

The structural layout of network connections may also have an effect on performance, particularly for multimodal functions [5].

Modifying the GCP SO to cater for neighbourhoods presents difficulties in dealing with  $\rho(t)$  since there is now the possibility that more than one best particle exists. In addition, these neighbourhood best positions may or may not be shared between multiple neighbourhoods at any given point in time. Thus, the number of best particles is constantly changing. To solve this, a value for  $\rho(t)$  is stored for each particle. Care must be taken to copy the previous value of  $\rho(t)$  to a new best particle whenever the neighbourhood best particle changes (since a particle that is no longer the best in a given neighbourhood may still be the best in other neighbourhoods).

## V. BENCHMARK FUNCTIONS

The benchmark functions in this section provide a balance of unimodal and multimodal as well as easy and difficult functions. Subsets of these functions have been used in various particle swarm studies [4], [5], [7].

For each of these functions, the goal is to find the

global minimiser. Stated formally:

Given  $f: \mathbb{R}^n \rightarrow \mathbb{R}$

find  $\mathbf{x}^* \in \mathbb{R}^n$  for which  $f(\mathbf{x}^*) \leq f(\mathbf{x}), \forall \mathbf{x} \in \mathbb{R}^n$

*Spherical*:  $\mathbf{x}^* = \mathbf{0}$ , with  $f(\mathbf{x}^*) = 0$ .

$$f(\mathbf{x}) = \sum_{i=1}^n x_i^2 \quad (7)$$

*Quadric*:  $\mathbf{x}^* = \mathbf{0}$ , with  $f(\mathbf{x}^*) = 0$ .

$$f(\mathbf{x}) = \sum_{i=1}^n \left( \sum_{j=1}^i x_j \right)^2 \quad (8)$$

*Rosenbrock*:  $\mathbf{x}^* = (1, 1, \dots, 1)$ , with  $f(\mathbf{x}^*) = 0$ .

$$f(\mathbf{x}) = \sum_{i=1}^{n/2} (100(x_{2i} - x_{2i-1}^2)^2 + (1 - x_{2i-1})^2) \quad (9)$$

*Ackley*:  $\mathbf{x}^* = \mathbf{0}$ , with  $f(\mathbf{x}^*) = 0$ .

$$f(\mathbf{x}) = -20 \exp \left( -0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2} \right) - \exp \left( \frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i) \right) + 20 + e \quad (10)$$

*Griewank*:  $\mathbf{x}^* = \mathbf{0}$ , with  $f(\mathbf{x}^*) = 0$ .

$$f(\mathbf{x}) = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos \left( \frac{x_i}{\sqrt{i}} \right) + 1 \quad (11)$$

*Rastrigin*:  $\mathbf{x}^* = \mathbf{0}$ , with  $f(\mathbf{x}^*) = 0$ .

$$f(\mathbf{x}) = \sum_{i=1}^n (x_i^2 - 10 \cos(2\pi x_i) + 10) \quad (12)$$

*Schwefel*:  $f(\mathbf{x}^*) = \mathbf{0}$ , when

$\mathbf{x}^* = (-420.9687, -420.9687, \dots, -420.9687)$ .

$$f(\mathbf{x}) = 418.9829n + \sum_{i=1}^n x_i \sin \left( \sqrt{|x_i|} \right) \quad (13)$$

Spherical, Quadric and Rosenbrock are unimodal while Ackley, Griewank, Rastrigin and Schwefel are multimodal functions. The functions have been presented roughly in order of difficulty for the PSO to find the global minimiser (see Section VII).

## VI. EXPERIMENT PARAMETERS

The PSO parameters that were used are listed in Table I. The acceleration constants and inertia weight satisfy Equation (3) and are expected to provide convergent behaviour. The GCP SO specific parameters were empirically determined. The *lbest* parameter was chosen to provide a high impedance to

TABLE I  
PSO PARAMETERS

Parameter	Value
#particles	20
$c_1 = c_2$	1.49618
$w$	0.729844
$s_c = f_c$	5
$k$ ( <i>lbest</i> )	2

communication within the swarm, since it was hypothesised that this would aid in finding the best minimiser for the multimodal functions. The benchmark function parameters that were used are listed in Table II, where  $d$  is the domain around the origin in which the PSO was initialised.

TABLE II  
FUNCTION PARAMETERS

Function	n	d
Spherical	30	100
Quadric	30	100
Rosenbrock	30	2.048
Ackley	30	30
Griewank	30	600
Rastrigin	30	5.12
Schwefel	30	500

The cross product of the *gbest*, *lbest* and Von Neumann neighbourhoods with the standard PSO and GCP SO were tested on each function resulting in six experiments per function. For each experiment the simulation was run 100 times and the mean, standard deviation, median and range of error values over all the runs were recorded. Each simulation was allowed to run for 200 000 evaluations of the objective function.

## VII. RESULTS

A table of results is provided for each of the benchmark functions in Section V. Each table consists of rows of experiments. The first column lists the names of the experiments where the prefixes G and P refer to the type of swarm used: GCP SO and standard PSO respectively and the subscripts  $g$ ,  $l$  and  $v$  refer to the neighbourhood topology used: *gbest*, *lbest* and Von Neumann respectively. The second column lists the sample mean and standard deviation of the error. Since all the benchmark functions have  $f(\mathbf{x}^*) = 0$ , the error is defined to be equal to the fitness value obtained. The notation  $\bar{x} \pm s$  where  $\bar{x}$  is the mean and  $s$  is the standard deviation is used. The third column lists the median error values and the last gives

the range of minimum and maximum error values.

For each experiment, the median error was plotted against the number of evaluations of the objective function performed. All the plots for a given benchmark function have been provided on the same system of axes. To improve the resolution, the  $\log_{10}$  values for the error have been plotted instead of the actual values. These plots can be used to compare the convergence behaviour of the various PSO configurations given a particular function. The plots have a self explanatory key.

Under the assumption of normally distributed samples, statistical hypothesis testing can be used to reason about the population mean. The assumption of a normal distribution may be relaxed in the case of large samples (typically 100 samples or more). Whenever a statistically significant difference in the population mean is mentioned, it is assumed that the simulation data is normally distributed. Be aware that this is not necessarily guaranteed, even given the relatively large samples used here. All statistical tests were performed at a 1% level of significance using the standard t-test.

TABLE III  
ERROR VALUES FOR SPHERICAL

	$\bar{x} \pm s$	Median	$R$
$G_g$	$3e-161 \pm 1e-160$	4e-164	[8e-169:1e-159]
$G_l$	$3e-93 \pm 2e-92$	4e-102	[2e-105:1e-91]
$G_v$	$6e-119 \pm 3e-118$	3e-122	[1e-126:2e-117]
$P_g$	$3e-96 \pm 2e-95$	7e-109	[4e-124:2e-94]
$P_l$	$2e-91 \pm 7e-91$	1e-92	[1e-95:5e-90]
$P_v$	$4e-112 \pm 3e-111$	6e-116	[5e-121:2e-110]

The results for the Spherical function are provided in Table III. Generally, for unimodal functions, the more connected the neighbourhood structure the better the results. The GCP SO did not benefit from neighbourhoods. There is little point in attempting to preserve swarm diversity for unimodal functions, except when using the standard PSO (from now, referred to as the PSO) since neighbourhoods may help prevent premature convergence. This explains why the  $P_v$  outperformed the  $P_g$  configuration. The Von Neumann architecture may slow down the rate of communication between the particles just enough to prevent all the particles collapsing on the same region in search space too soon. The social component of Equation (2) has more impact in diverse swarms. However, the  $P_v$  configuration did not perform significantly better than the  $P_g$  configuration. All of the configurations perform significantly better than the  $P_l$  configuration.

The *lbest* configurations were relatively unstable

having higher standard deviations. The high deviation for the  $P_g$  configuration may be explained by premature convergence. The  $G_g$  configuration was the most stable having the lowest deviation. Note that the mean and deviation of the two *lbest* configurations are comparable while the median for the GCP SO version is much better.

The directed random search performed by the GCP SO provided an advantage both in terms of solution quality and stability of the algorithm. The GCP SO always outperformed its PSO counterparts.

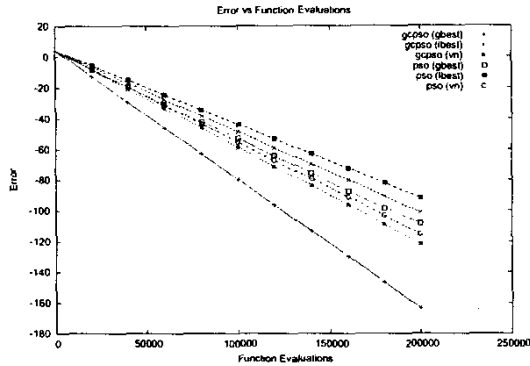


Fig. 1. Log Median Error for Spherical

Consider the plot in Fig. 1. Here the more rapid convergence of the GCP SO algorithm was evident, particularly in the case of the *gbest* topology. The GCP SO showed less of an advantage over the PSO for the neighbourhood configurations.

The directed random search performed by the GCP SO is sensitive to the value of  $\rho(t)$  in Equation (4). Recall that the success and failure counts are reset each time the best position switches from one particle to the next. In addition, if the switch takes place between particles in different areas of the search space then the value of  $\rho(t)$  may not be suited to different function landscapes. The GCP SO algorithm incurs a penalty each time the best particle changes since it takes time for  $\rho(t)$  to re-adjust and stabilise. It is possible that the neighbourhood configurations incurred more switches than the *gbest* topology resulting in less optimal values for  $\rho(t)$ .

Table IV demonstrates trends in Quadric that are similar to Spherical (both are unimodal). Once again the GCP SO configurations consistently outperformed their PSO counterparts. Again, the more connected neighbourhood topologies exhibited superior performance. The  $P_g$  configuration once again performed more poorly than  $G_g$ .

The mean error for the  $G_g$  configuration was somewhat higher than the same configuration on Spher-

TABLE IV  
ERROR VALUES FOR QUADRIC

	$\bar{x} \pm s$	Median	Range
$G_g$	$7e-139 \pm 5e-138$	$3e-143$	[1e-149:5e-137]
$G_l$	$7e-90 \pm 3e-89$	$6e-92$	[1e-94:2e-88]
$G_v$	$2e-110 \pm 1e-109$	$8e-114$	[2e-116:8e-109]
$P_g$	$2e-90 \pm 2e-89$	$3e-108$	[1e-123:2e-88]
$P_l$	$2e-89 \pm 6e-89$	$2e-90$	[5e-93:3e-88]
$P_v$	$5e-111 \pm 3e-110$	$9e-114$	[1e-117:2e-109]

ical, suggesting that Quadric is a slightly harder function to optimise. However, the neighbourhood configurations provided performance more consistent with that of Spherical.

The difference between the two Von Neumann configurations is negligible. To lesser degree, the difference between the *lbest* configurations is also very small. This suggests that the GCP SO benefited much less from neighbourhoods than the PSO on Quadric.

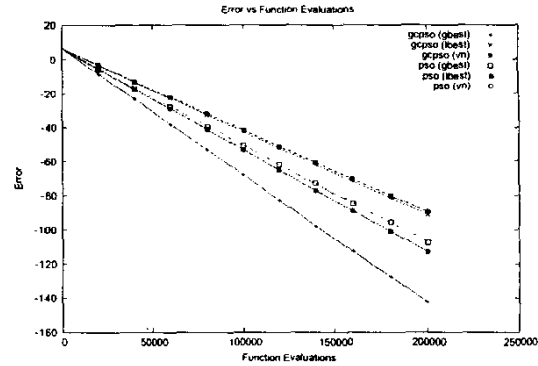


Fig. 2. Log Median Error for Quadric

Fig. 2 shows the plot for Quadric. This plot is virtually identical to Fig. 1 except for the smaller differences between the neighbourhood architectures mentioned before.

TABLE V  
ERROR VALUES FOR ROSEN BROCK

	$\bar{x} \pm s$	Median	Range
$G_g$	$0.0129 \pm 0.0246$	0.0052	[0.0003:0.1679]
$G_l$	$0.654 \pm 0.53$	0.5394	[0.168:4.3173]
$G_v$	$0.1801 \pm 0.097$	0.1629	[0.0291:0.552]
$P_g$	$0.058 \pm 0.43$	0.0057	[0.0004:4.3011]
$P_l$	$0.4588 \pm 0.305$	0.4212	[0.1613:2.9975]
$P_v$	$0.1617 \pm 0.0936$	0.1365	[0.0341:0.515]

Given Table V, it can be concluded that Rosenbrock is a significantly harder function than Spherical or Quadric. Rosenbrock breaks the trend of the GCP SO always performing better than the PSO on unimodal functions. Note that, if the  $s_c$  and  $s_f$  parameters are carefully tuned for Rosenbrock, then it is possible to get the  $G_g$  configuration to outperform the  $P_g$  configuration slightly [7]. This indicates that the sensitivity of the GCP SO to values of  $\rho(t)$  is dependent on the function landscape.

For Rosenbrock the directed random search of the GCP SO did not appear to provide any advantage over the PSO. In fact, the  $P_l$  configuration was significantly better than  $G_l$ . However, the trend of more connected neighbourhoods performing better on unimodal functions still holds for both GCP SO and PSO configurations. The *gbest* configurations performed significantly better than the Von Neumann configurations and these were in turn significantly better than the *lbest* configurations.

It has already been noted that the GCP SO appears not to benefit from neighbourhoods to the same extent that the PSO does. However, the GCP SO also does not appear to be any more adversely affected by neighbourhoods than the PSO on unimodal functions.

Once again the two *lbest* configurations as well as the  $P_g$  configuration demonstrated instability as evidenced by their high standard deviations. The  $P_l$  appeared to be more stable than  $G_l$ . Once again the  $G_g$  configuration was the most stable indicating that even on Rosenbrock the directed random search is not without some advantage.

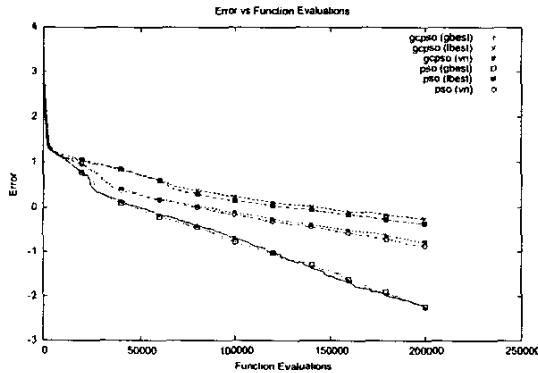


Fig. 3. Log Median Error for Rosenbrock

The plot in Fig. 3 illustrates an interesting convergence pattern for Rosenbrock. All six configurations demonstrated a similar pattern. There was a rapid decrease in error early in the search. After this, the improvement flattened out for a short while before

making another smaller dip. Finally, the error value continued to decrease at a much slower pace. Since all six configurations display this characteristic, it is not accounted for by the neighbourhood topology or the type of swarm. What is noticeable is that the *gbest* configurations maintained a much steeper descent in error value after the other configurations had flattened out.

TABLE VI  
ERROR VALUES FOR ACKLEY

	$\bar{x} \pm s$	Median	Range
$G_g$	$2.0181 \pm 1.3102$	1.9565	[1e-14:9.3975]
$G_l$	$0.2794 \pm 0.5424$	7e-15	[7e-15:2.221]
$G_v$	$0.6824 \pm 0.8269$	7e-15	[4e-15:2.5799]
$P_g$	$3.6708 \pm 1.5625$	3.3444	[3e-14:8.9018]
$P_l$	$0.0667 \pm 0.2676$	7e-15	[7e-15:1.3404]
$P_v$	$0.7098 \pm 0.845$	1e-14	[4e-15:2.887]

Consider Table VI which contains the results for Ackley. As soon as the function was multimodal, the trend regarding neighbourhoods reversed. Now, less connected networks performed better. This was motivated in Section IV.

The  $G_g$  configuration was significantly better than the  $P_g$  configuration. This may again be accounted for by the premature convergence of the PSO. Therefore, the directed random search may still be useful in a multimodal context but clearly to a lesser degree. The GCP SO seemed to break down with the *lbest* topology. The  $P_l$  configuration was significantly better than the  $G_l$  configuration. The GCP SO managed to hold its ground with the Von Neumann topology but was not significantly better than the PSO.

The stability of the *gbest* topologies were low. Some *gbest* simulations became trapped in local minima. The neighbourhood topologies reduce the risk of this happening by maintaining swarm diversity for longer periods of time.

Fig. 4 shows the two *gbest* configurations getting trapped in a local minimum very early during the search. Only the neighbourhood configurations managed to maintain any momentum. The Von Neumann architectures converged more rapidly than their *lbest* counterparts. However, both *lbest* configurations managed to achieve significantly better solutions than either of the Von Neumann configurations. This indicates that faster convergence may not be desirable in multimodal landscapes.

The results in Table VII for Griewank display similar characteristics to Ackley. Again, it can be seen that for multimodal functions the less connected neighbourhoods performed better. The  $P_g$  configuration got trapped again. The  $G_g$  configuration was

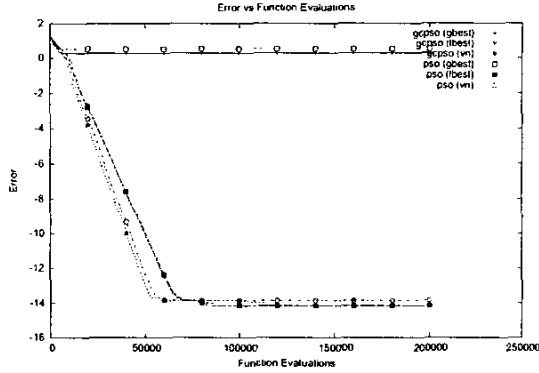


Fig. 4. Log Median Error for Ackley

TABLE VII  
ERROR VALUES FOR GRIEWANK

	$\bar{x} \pm s$	Median	Range
$G_g$	$0.0162 \pm 0.0219$	0.0074	[1e-19:0.1077]
$G_l$	$0.0039 \pm 0.0075$	1e-19	[1e-19:0.0393]
$G_v$	$0.0104 \pm 0.0145$	0.0074	[0:0.08549]
$P_g$	$0.1353 \pm 0.3154$	0.0405	[2e-19:2.1642]
$P_l$	$0.0051 \pm 0.0086$	1e-19	[0:0.0394]
$P_v$	$0.0134 \pm 0.019$	0.0074	[0:0.0903]

once again significantly better than the  $P_g$  configuration.

Although both Von Neumann configurations are significantly better than the  $P_g$  configuration they failed to keep up with the *lbest* configurations. In fact, both *lbest* architectures are significantly better than all the other topologies but neither of them is significantly better than the other. Nevertheless,  $G_l$  has a slightly better average error and stability.

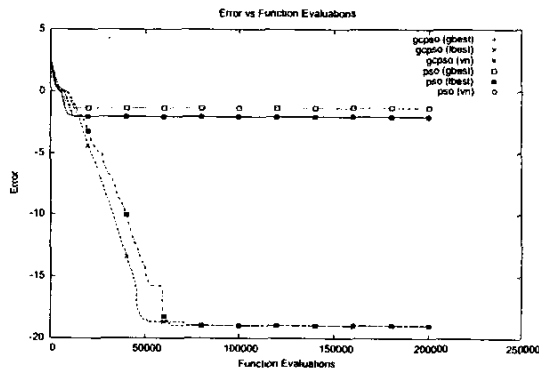


Fig. 5. Log Median Error for Griewank

Fig. 5 shows that even the Von Neumann architec-

tures got trapped early during the search. Only the two *lbest* configurations were able to maintain diversity long enough not to get trapped in a local minimum. Notice that the  $G_l$  configuration displayed a much more rapid convergence curve than the  $P_l$  configuration. This is due to the directed random search.

TABLE VIII  
ERROR VALUES FOR RASTRIGIN

	$\bar{x} \pm s$	Median	Range
$G_g$	$71.925 \pm 18.692$	71.637	[31.839:139.29]
$G_l$	$61.202 \pm 15.415$	60.692	[27.859:93.526]
$G_v$	$55.837 \pm 14.501$	54.723	[29.849:99.496]
$P_g$	$72.204 \pm 18.678$	68.652	[29.849:136.31]
$P_l$	$64.567 \pm 14.941$	63.677	[31.839:112.43]
$P_v$	$54.355 \pm 15.353$	51.738	[22.884:98.501]

The results for Rastrigin in Table VIII are interesting because the ranking of the neighbourhood architectures is different. For Rastrigin, the Von Neumann topologies were significantly better than the *lbest* topologies and they were in turn significantly better than the *gbest* topologies. This suggests that the structural layout of the neighbourhood may be significant. There was no significant difference between the GCP SO and PSO configurations. In fact, both algorithms have almost identical performance in terms of mean error as well as stability for a given neighbourhood topology. The only noticeable difference was the slightly higher worst case error for  $P_l$  which could have been caused by a single simulation.

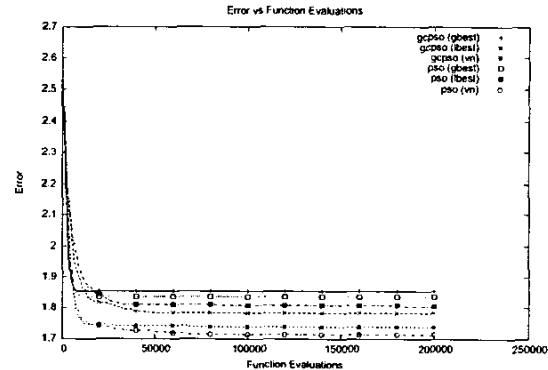


Fig. 6. Log Median Error for Rastrigin

Fig 6 shows very similar convergence characteristics between the GCP SO and PSO configurations. The medians for the PSO configurations are slightly better than the GCP SO, however, the averages for the *gbest* and *lbest* configurations show the reverse to

be true.

TABLE IX  
ERROR VALUES FOR SCHWEFEL

	$\bar{x} \pm s$	Median	Range
$G_g$	$4539 \pm 706.06$	4550.2	[2882.2:6534.1]
$G_l$	$4762.1 \pm 509.36$	4797.8	[3395.4:5863.1]
$G_v$	$4496.9 \pm 707.83$	4451.5	[2862.3:6356.5]
$P_g$	$4535.6 \pm 722.33$	4510.8	[2803.1:6179.5]
$P_l$	$4634 \pm 642.22$	4609.5	[2329.4:6219.5]
$P_v$	$4273.4 \pm 565.86$	4333.1	[2664.9:5449.1]

The results for Schwefel are presented in Table IX for completeness. Schwefel has an extremely difficult landscape with many local minima. In addition, the global minimum is located in one corner of the search space, making it even more difficult for the PSO to locate it.

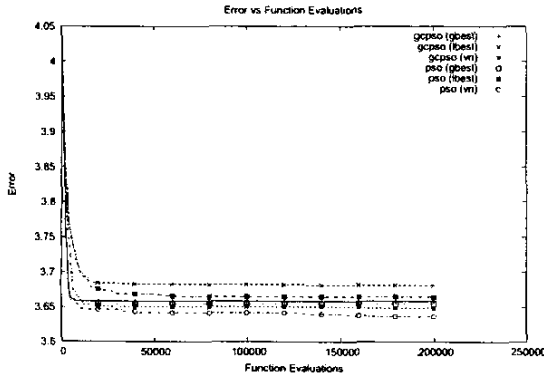


Fig. 7. Log Median Error for Schwefel

Fig. 7 shows that the *lbest* topologies performed worse than the *gbest* topologies. In fact, both *gbest* topologies performed significantly better than the  $G_l$  configuration. This behaviour is not easily explained and should be investigated further.

## VIII. CONCLUSION

The very nature of neighbourhoods impedes information flow. This is not a desirable property for unimodal optimisation as it slows convergence when there is no need to preserve diversity. The premature convergence of the standard PSO before reaching a local extremum is better solved by using the guaranteed convergence modifications. Stability of the *lbest* topologies was also poor for unimodal functions.

On the other hand, for multimodal optimisation problems, neighbourhoods are extremely desirable. In this case, slowing down convergence preserves diversity in the swarm. This results in a larger area of

the search space being explored.

Although it makes intuitive sense that sparsely connected network structures should display better performance on multimodal functions (see Section IV), it was found not to be the case in general. Rastrigin and Schwefel were both exceptions to this expectation. In both these cases, the *lbest* topology performed worse than the less sparsely connected Von Neumann topology. However, both neighbourhood topologies were generally able to exhibit superior performance to the *gbest* topology on all the multimodal functions tested.

The benefits of the directed random search in multimodal functions is greatly reduced but not eliminated. Although the GCP SO does not seem to benefit as much from neighbourhoods as the standard PSO does, it does not seem to be adversely affected by them either.

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