

**Functions of Several Variable and Differential Geometry 2024 - Minor  
3**

Maximum Marks : 16

**Part A:** Each question carries 1 marks.

- 1 Define covariant derivative. If  $X$  is a parallel vector fields along a parametrized curve  $\alpha$  and  $f$  is a smooth function along  $\alpha$ . Show that  $(fX)' = f'X + fX'$ .
- 2 Define geodesic. Let  $\alpha$  be a parametrized curve in an  $n$ -surface with constant speed. Is it a geodesic?
3. Define parallel transport. Why this map is well defined.
4. Define Levi-Civita parallel.

**Part B:** Each question carries 4 marks.

5. Let  $C$  be unit circle centred at the origin. How many maximal geodesics can pass through  $(1, 0)$ ?. (Hint: Think about unit speed reparametrization of geodesics)
6. Let  $S$  be an  $n$ -surface and  $\alpha : I \rightarrow S$  be a parametrized curve in  $S$ . Let  $\tilde{S} = \{v \in S_{\alpha(t)} : \|v\| = 1\}$  for some  $t \in I$  and let  $\tilde{V}$  be the set of vector fields tangent to  $S$  along  $\alpha$  and has unit length at some point. Compare the cardinalities of the sets  $\tilde{S}$  and  $\tilde{V}$ .

**Part C:** Each question carries 2 marks.

7. If a an object falling down from 100 meters height in a straight line to earth. Is the path a geodesic ?
8. Prove that ,in an  $n$ -plane parallel transport is path independent.
- 9 Show that parallel transport is a vector space isomorphism which preserves dot product.