Algebra: Internal Exam-2 (Total Marks - 16)

PART A

1. Define ideal. Give an example. (2 marks)
2. Define prime ideal. Give an example with justification (2 mark)

PART B (Answer any two questions out of 3, 4 and 6. Question 5 is mandatory)

3. Show that every ideal of $\mathbb{Z}$ is a principle ideal. (2 mark)
4. Show that the field $\mathbb{R}$ and $\mathbb{C}$ are not isomorphic. (2 marks)
5. Let R be a ring, and N be the set of nilpotent elements in R
   a. Prove that N is an ideal. (1.5 marks)
   b. Prove that N is contained inside every prime ideal of R. (1.5 marks)
   c. Prove thatnthe ring R/N has no nonzero nilpotent element. (1 marks)
6. Let R be a commutative ring and $a \in R$. Show that $Ia = \{x \in R : ax = 0\}$ is an ideal of R. (2 mark)

PART C (Write any two)

7. If R is a ring with unity and N is an ideal of a ring R containing a unit then show that $N = R$. (2 marks)
8. If R is a ring with unity and characteristic $n(> 1)$. Then show that R contains a subring isomorphic to $\mathbb{Z}_n$. (2 marks)
9. Let H be a subring of the ring R. Show that multiplication of additive cosets of H is well defined by the equation $(a + H)(b + H) = ab + H$ if and only if $ah \in H$ and $hb \in H$ for all $a, b \in R$, $h \in H$.