



**Kannur University**  
**Department of Mathematical Sciences**  
**Real Analysis – Minor 3**  
**August – December 2025**

**Maximum Marks: 16**

**Time: 50 Minutes**

**Date: 29-09-2025**

**Instructions:** Answer all questions. Each question carries 3.5 marks. Justify each answer clearly.

1. In which of the following cases does there exist a continuous and onto function  $f : X \rightarrow Y$ ?

- a)  $X = (0, 1), Y = (0, 1]$
- b)  $X = [0, 1], Y = (0, 1)$
- c)  $X = (0, 1), Y = \mathbb{R}$
- d)  $X = (0, 2), Y = \{0, 1\}$

2. Which of the following statements is/are true?

- (a) There exists a continuous map  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that  $f(\mathbb{R}) = \mathbb{Q}$ .
- (b) There exists a continuous map  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that  $f(\mathbb{R}) = \mathbb{Z}$ .
- (c) There exists a continuous map  $f : \mathbb{R} \rightarrow \mathbb{R}^2$  such that

$$f(\mathbb{R}) = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}.$$

- (d) There exists a continuous map  $f : [0, 1] \cup [2, 3] \rightarrow \{0, 1\}$ .

3. Let  $E$  be a subset of  $\mathbb{R}$ . Then the characteristic function  $\chi_E : \mathbb{R} \rightarrow \mathbb{R}$  is continuous if and only if

- (a)  $E$  is closed
- (b)  $E$  is open
- (c)  $E$  is both open and closed
- (d)  $E$  is neither open nor closed

4. If  $f$  is defined on  $E$ , the **graph of  $f$**  is the set of points  $(x, f(x))$ , for  $x \in E$ . In particular, if  $E$  is a set of real numbers, and  $f$  is real-valued, the graph of  $f$  is a subset of the plane.

Suppose  $E$  is compact, and prove that  $f$  is continuous on  $E$  if and only if its graph is compact.

5. Discuss the continuity and uniform continuity of the function  $f : (0, 1) \rightarrow \mathbb{R}$  defined by  $f(x) = \frac{1}{x}$ .