**Algebra: Internal Exam-1** (Total Marks - 16)

*PART A*

1. Define free abelian group. Give an example which is not a free abelian group. (2 marks)
2. Define irreducible polynomial. (1 mark)

*PART B*

1. How many polynomials are there of degree greater than or equal to 3 in $\mathbb{Z}_3[x]$? (Include 0). (1 mark)
2. Let $p$ be a prime. Show that the polynomial $x^p + a \in \mathbb{Z}_p[x]$ is not irreducible for any $a \in \mathbb{Z}_p$). (2 marks)
3. Consider the polynomial $f(x) = x^2 + 3x - 1$. Show that $f$ is irreducible in $\mathbb{Z}[\sqrt{13}]$ where $\mathbb{Z}[\sqrt{13}] = \{a + b\sqrt{13} : a, b \in \mathbb{Z}$. (2 marks)
4. Prove that $x^2 + x + 1$ is irreducible over $F$, the field of integers mod 2. (1 mark)
5. For which positive integers $n$ does $x^2 + x + 1$ divide $x^4 + 3x^3 + x^2 + 7x + 5$ in $[\mathbb{Z}/(n)][x]$?. (3 marks)

*PART C*

1. Show that the an integral domain can be embedded in a field. (2 marks)
2. Let $G \neq \{0\}$ be a free abelian group with a finite basis. Prove that all basis have same number of elements. (2 marks)