Ordinary Differential Equations 2023 - Seminar Exam - Series A

Write true or false. Correct and incorrect answers carry 1 and -1 mark respectively.

1. Let y_1 and y_2 be two linearly independent solutions of $y'' + P(x)y' + Q(x)y = 0$. Then the Wronskian $W(y_1, y_2)$ vanishes atleast once between two consecutive solutions of y_1 and y_2.

2. Let y_1 be a solution of $y'' + 4y = 0$ and y_2 be a solution of $y'' + 2y = 0$. Then y_1 and y_2 has same number of solutions in any interval.

3. Power series of a function is continuously differentiable within the radius of convergence.

4. Let p be a negative integer. Then $\frac{1}{\Gamma(p)} = \frac{1}{p!}$.

5. Let f be the function

$$f(x) = \begin{cases} 1 & \text{if } x \in [0, 1/2) \\ 3/2 & \text{if } x \in [1/2, 1]. \end{cases}$$

   The corresponding Bessel series diverges at $x = 1/2$.

6. The equation $\det\begin{pmatrix} a_1 - m & b_1 \\ a_2 & b_2 - m \end{pmatrix} = 0$ has a non trivial solution if the matrix has rank 2.

7. Let P_n is the n-th Legendre polynomial, then $\int_{-1}^{1} P_n^2(x)dx = 1$.

8. Let f be a real valued function on $\mathbb{R}^2$. If f satisfies the Lipschitz condition on each variable, then then partial derivatives of f exist and are continuous.

9. Continuous function on an interval $I \subset \mathbb{R}$ is always bounded.

10. Let P and Q be analytic at x_0. The differential equation $y'' + P(x)y' + Q(x)y = 0$ with the initial conditions $y(x_0) = a = y'(x_0)$ has a solution which is analytic at x_0.