Ordinary Differential Equations 2023 - Assignment Exam

Each question carries 4 marks.

1. Verify that origin is a regular singular point and calculate two independent Frobenius series solutions of $4xy'' + 2y' + y = 0$.

2. The equation $x^2y'' - 3xy' + (4x+4)y = 0$ has only one Frobenius solution. Find it.

3. Show that a function of the form $ax^3 + bx^2y + cxy^2 + dy^3$ cannot be either positive definite or negative definite.

4. If $a_1b_2 - a_2b_1 \neq 0$ show that the system

$$\begin{cases} \frac{dy}{dx} = a_1x + b_1y + c_1 \\ \frac{dz}{dx} = a_2x + b_2y + c_2 \end{cases}$$

has a single isolated critical point.