

# Basic Measure Theory Problem Sheet

Jan - Jun 2026   Anoop V. P.

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- 1.** Let  $f : [0, 1] \rightarrow \mathbb{R}$  be defined as follows:

$$f(x) = \begin{cases} 0 & \text{if } x \in \mathbb{Q} \\ 1 & \text{otherwise} \end{cases}$$

Then, which of the following are correct?

- (A)  $f$  is not Riemann integrable
  - (B)  $f$  is Riemann integrable
  - (C) Limit of upper sums of  $f$  is one
  - (D) Limit of lower sums of  $f$  is zero
- 2.** Let  $A_n, n = 1, 2, 3, \dots$  be a sequence of subsets of a set  $X$ . Define  $\limsup A_n = \bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k$ . Then, which of the following are always true?
- (A)  $\limsup A_n = \{x \in X : x \in A_k \text{ for infinitely many } k\}$
  - (B)  $\limsup A_n = \{x \in X : x \in A_k \text{ only for finitely many } k\}$
  - (C)  $\limsup A_n = \{x \in X : x \in A_k \forall k\}$
  - (D)  $\limsup A_n = \{x \in X : x \in A_k \text{ for all but finitely many } k\}$
- 3.** Let  $A_n, n = 1, 2, 3, \dots$  be a sequence of subsets of a set  $X$ . Define  $\liminf A_n = \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k$ . Then, which of the following are always true?
- (A)  $\liminf A_n = \{x \in X : x \in A_k \text{ for infinitely many } k\}$
  - (B)  $\liminf A_n = \{x \in X : x \in A_k \text{ only for finitely many } k\}$
  - (C)  $\liminf A_n = \{x \in X : x \in A_k \forall k\}$
  - (D)  $\liminf A_n = \{x \in X : x \in A_k \text{ for all but finitely many } k\}$
- 4.** Let  $X$  be a set and let  $A, B$  be proper distinct subsets of  $X$ . Consider the sequence of sets
- $$A_n = \begin{cases} A & \text{if } n \text{ is even} \\ B & \text{if } n \text{ is odd} \end{cases}$$
- Which of the following are correct?
- (A)  $\limsup A_n = A \cup B$
  - (B)  $\limsup A_n = A \cap B$
  - (C)  $\liminf A_n = A \cup B$
  - (D)  $\liminf A_n = A \cap B$
- 5.** Which of the following are true?
- (A)  $\mathbb{Q} \in \mathcal{B}(\mathbb{R})$

- (B)  $\{x : 0 \leq x \leq 1 \text{ and } x \text{ is irrational}\} \in \mathcal{B}(\mathbb{R})$   
 (C) For  $a < b$ ,  $[a, b) \in \mathcal{B}(\mathbb{R})$   
 (D)  $[0, 1] \times [0, 1] \in \mathcal{B}(\mathbb{R}^2)$

**6.** Which of the following are true?

- (A)  $\{(x, 0) : x \in \mathbb{R}\} \in \mathcal{B}(\mathbb{R}^2)$   
 (B)  $\{(x, x) : x \in \mathbb{R}\} \in \mathcal{B}(\mathbb{R}^2)$   
 (C)  $\{(x, y) : y = x^2 + x, x \in \mathbb{R}\} \in \mathcal{B}(\mathbb{R}^2)$   
 (D)  $A \times \{0\} \in \mathcal{B}(\mathbb{R}^2)$  for any finite set  $A \subset \mathbb{R}$ .

**7.** Which of the following are true?

- (A)  $\mathcal{B}(\mathbb{R}^n)$  is generated by open balls of rational radii  
 (B)  $\mathcal{B}(\mathbb{R}^n)$  is generated by closed balls of rational radii  
 (C)  $\mathcal{B}(\mathbb{R}^n)$  is generated by singletons  
 (D)  $\mathcal{B}(\mathbb{R})$  is generated by  $\{[a, b) : a, b \in \mathbb{Q}\}$

**8.** Which of the following are  $\sigma$ -algebras?

- (A)  $\{A \subset \mathbb{R} : A \text{ is countable or } A^c \text{ is countable}\}$   
 (B)  $\{A \subset \mathbb{R} : A \text{ is finite or } A^c \text{ is finite}\}$   
 (C)  $\{A \subset \mathbb{R} : A \text{ is a closed interval}\}$   
 (D)  $\{A : A \subset [0, 1] \text{ and } A \in \mathcal{B}(\mathbb{R})\}$

**9.** Which of the following definitions give a measure on  $\mathcal{B}(\mathbb{R})$ ?

- (A)  $\mu(A) = \text{number of rationals in } A$   
 (B)  $\mu(A) = 1 \text{ if } 1 \in A \text{ and } 0 \text{ otherwise}$   
 (C)  $\mu(A) = \text{number of rationals in } A \cap [0, 1]$   
 (D)  $\mu(A) = \text{number of rationals in } A^c$

**10.** Let  $n > 1$  and let  $X = \{1, 2, \dots, n\}$ . Let  $\mathcal{F}$  be the powerset of  $X$  and  $\mu$  the counting measure on  $\mathcal{F}$ . If  $f : X \rightarrow \mathbb{R}$ , then which of the following are true?

- (A)  $\int_X f d\mu = \sum_{k \in X} f(k)$   
 (B)  $\int_X f d\mu = f(1)$   
 (C)  $\int_X f d\mu = 0$   
 (D)  $\int_X f d\mu = 2f(1)$

**11.** Let  $X = \mathbb{R}$ ,  $\mathcal{F} = \{A \subset \mathbb{R} \mid A \text{ is countable or } A^c \text{ is countable}\}$ . Let  $\mu(A) = 1$  if  $A^c$  is countable and 0 if  $A$  is countable. Let  $f : (X, \mathcal{F}, \mu) \rightarrow \mathbb{R}$  be measurable. Which of the following are always true?

- (A)  $f$  is constant ae. ( $\mu$ )  
 (B)  $f$  is a non-constant continuous function

- (C)  $f$  is a non constant polynomial  
(D)  $f(x) = 0 \forall x \in \mathbb{R}$
- 12.** Let  $(X, \mathcal{F}, \mu)$  be a measure space and let  $E$  be a proper subset of  $X$ ,  $E \in \mathcal{F}$  and  $0 < \mu(E) < \mu(X)$ . Let  $f_n = \chi_E$  if  $n$  is odd and  $1 - \chi_E$  if  $n$  is even. Which of the following are true?
- (A)  $\int_X \liminf f_n d\mu < \liminf \int_X f_n d\mu$   
(B)  $\int_X \liminf f_n d\mu = \liminf \int_X f_n d\mu$   
(C)  $\int_X \limsup f_n d\mu < \limsup \int_X f_n d\mu$   
(D)  $\int_X \limsup f_n d\mu = \limsup \int_X f_n d\mu$
- 13.** Consider the space  $\mathbb{N}$  with power set sigma algebra and counting measure  $\mu$ . Let  $f : \mathbb{N} \rightarrow \mathbb{R}$  be measurable and zero ae( $\mu$ ). Which of the following are always true?
- (A)  $f(n) = 0 \forall n \in \mathbb{N}$   
(B)  $f(1) \neq 0, f(n) = 0 \forall n > 1$   
(C)  $f(n) = 0$  except for finitely many  $n \in \mathbb{N}$   
(D)  $f(n) = 0$  only when  $n$  is a prime number
- 14.** Let  $X$  be a non empty set and  $A \subset X$  be a proper subset. Consider the sigma algebra  $\mathcal{F} = \{\emptyset, X, A, A^c\}$ . Let  $f : (X, \mathcal{F}) \rightarrow \mathbb{R}$  be measurable. Which of the following are always true?
- (A)  $f = \alpha\chi_A + \beta\chi_{A^c}$  for some  $\alpha, \beta \in \mathbb{R}$   
(B)  $f = \alpha\chi_A$  for some  $\alpha \in \mathbb{R}$   
(C)  $f = \beta\chi_{A^c}$  for some  $\beta \in \mathbb{R}$   
(D)  $f \equiv 0$
- 15.** Let  $(X, \mathcal{F}, \mu)$  be a measure space. Let  $A_n \in \mathcal{F}$  such that  $A_1 \subset A_2 \subset \dots$  and  $\cup_{n=1}^{\infty} A_n = X$ . Let  $f : (X, \mathcal{F}, \mu) \rightarrow \mathbb{R}$  be a measurable function and  $f(x) \geq 0$  ae( $\mu$ ). Which of the following are always true?
- (A)  $f\chi_{A_n} \uparrow f$  ae  
(B)  $\int_{A_n} f d\mu \uparrow \int_X f d\mu$   
(C)  $\int_{A_n} f d\mu \downarrow \int_X f d\mu$   
(D)  $f\chi_{A_n} \downarrow f$
- 16.** Let  $(X, \mathcal{F}, \mu)$  be a probability measure space ( $\mu(X) = 1$ ). Let  $\{A_n\}$  be a sequence in  $\mathcal{F}$ . Which of the following are true?
- (A)  $\mu(\limsup A_n) \geq \limsup \mu(A_n)$   
(B)  $\mu(\limsup A_n) \leq \limsup \mu(A_n)$   
(C)  $\mu(\liminf A_n) \geq \liminf \mu(A_n)$   
(D)  $\mu(\liminf A_n) \leq \liminf \mu(A_n)$

- 17.** Let  $(X, \mathcal{F}, \mu)$  be a measure space. Let  $f_n : X \rightarrow \mathbb{R}$  be measurable,  $A = \{x \in X \mid \lim f_n(x) \text{ exists}\}$ . Then,
- (A)  $A \in \mathcal{F}$
  - (B)  $A = \emptyset$
  - (C)  $A = X$
  - (D)  $A^c \in \mathcal{F}$
- 18.** Let  $(X, \mathcal{F}, \mu)$  be a measure space and  $A_n \in \mathcal{F}$ . Suppose  $\mu(X) = 1, \sum \mu(A_n) < \infty$ . Then which of the following are true?
- (A)  $\mu(\limsup A_n) = 0$
  - (B)  $\mu(\liminf A_n) = 0$
  - (C)  $\mu(\limsup A_n) = 1$
  - (D)  $\mu(\liminf A_n) = 1$
- 19.**  $(X, \mathcal{F}, \mu)$  be a probability measure space. Suppose  $f_n : X \rightarrow \mathbb{R}$  are measurable and  $|f_n| \leq 1 \text{ ae}(\mu) \forall n$ . Suppose  $f_n \rightarrow 1 \text{ ae}(\mu)$ . Then,
- (A)  $\int_X f_n d\mu \rightarrow 1$
  - (B)  $\int_X f_n d\mu \rightarrow 0$
  - (C)  $\int_X f_n d\mu \rightarrow \infty$
  - (D)  $\int_X f_n d\mu$  does not converge
- 20.** Let  $(X, \mathcal{F}, \mu)$  be a measure space and  $A_n \in \mathcal{F}$  such that  $\mu(A_n) = 0 \forall n$ . Which of the following are true?
- (A)  $\mu(\cup_{n=1}^{\infty} A_n) = 0$
  - (B)  $\mu(\cup_{n=1}^{\infty} A_n) = 1$
  - (C)  $\mu(\cup_{n=1}^{\infty} A_n) = \infty$
  - (D)  $\mu(\cup_{n=1}^{\infty} A_n) > 0$
- 21.** Let  $F$  be a countable subset of  $\mathbb{R}^n$  then  $m_*(F)$  equals:
- (A) 0
  - (B)  $\infty$
  - (C) any positive number
- 22.** Which of the following are correct?
- (A) Any countable set is measurable
  - (B) Any open set is measurable
  - (C) The set  $\{x \in \mathbb{R} : |e^x \sin x| > 1\}$  is measurable
- 23.** Which of the following sets are Borel sets?
- (A) The subset of  $[0, 1]$  whose decimal expansions starts with 2

(B) Subsets of  $\mathbb{R}$  whose complements are countable

**24.** The outer measure of the set  $\{0\} \times [-1, 1] \subset \mathbb{R}^2$  is:

- (A) 0
- (B) 1
- (C) 2

**25.** Let  $E \subset \mathbb{R}^n$  be an unbounded set.

- (A) Outer measure of  $E$  is infinity
- (B) Outer measure of  $E$  is positive, but need not be infinity always
- (C) There are unbounded sets whose outer measure is zero

**26.** Let  $E \subset \mathbb{R}^n$  be such that  $m_*(E) = 0$ . Let  $O_n$  be the open set  $O_n = \{y \in \mathbb{R}^n : d(y, E) < \frac{1}{n}\}$ .

- (A)  $m_*(O_n) = \infty$  always
- (B)  $m_*(O_n)$  is finite always
- (C)  $m_*(O_n)$  is positive always

**27.** Let  $E \subset \mathbb{R}^n$  be such that  $m_*(E) = 0$  and  $O_n$  as defined above. Then the statement  $m_*(O_n) \rightarrow 0$  is:

- (A) True always
- (B) True if  $E$  is closed
- (C) True if  $E$  is bounded

**28.** Let  $E \subset \mathbb{R}^n$  and  $x \in \mathbb{R}^n$ . Define  $E + x = \{y + x : y \in E\}$ . Suppose  $m_*(E) = 0$ . Which are correct?

- (A)  $m_*(E + x) = 0$ .
- (B)  $E + x$  is measurable
- (C)  $E + x$  need not be measurable

**29.** Let  $A_n = \{n\} \times [-n, n] \subset \mathbb{R}^2$ . If  $A = \bigcup_{n=1}^{\infty} A_n$ , then:

- (A)  $m_*(A) = 0$
- (B)  $m_*(A) = \infty$
- (C)  $0 < m_*(A) < \infty$

**30.** Let  $A = [0, 1]$ . Which of the following are correct?

- (A)  $m_*(A) = \inf\{m_*(O) : A \subset O, O \text{ open}\}$
- (B)  $m_*(A) = \sup\{m_*(K) : K \subset A, K \text{ compact}\}$

**31.** Let  $A$  be a subset of  $[0, 1]$  and  $m$  denote Lebesgue measure. Then which of the following are true?

- (A) If  $A$  is closed then  $m(A) > 0$   
 (B) If  $A$  is open then  $m(A) = m(\bar{A})$   
 (C) If  $m(int(A)) = m(\bar{A})$  then  $A$  is measurable  
 (D) If  $m(int(A)) = m(\bar{A})$  then  $A$  need not be measurable
- 32.** Define an equivalence relation in  $[1, 2]$  by  $x \sim y$  if  $x - y \in \mathbb{Q}$ . Let  $N$  contain one element from each class.
- (A)  $N$  is uncountable  
 (B)  $[1, 2] \setminus N$  is uncountable  
 (C)  $m_*(N) = 0$   
 (D)  $E \subset N$  measurable implies  $m_*(E) = 0$
- 33.** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$ . Which are necessarily true?
- (A) If  $f$  is measurable, then  $\phi \circ f$  is measurable for any continuous  $\phi$   
 (B) If  $f^2$  is measurable, then  $f$  is measurable  
 (C) If  $f$  is differentiable, then  $f'$  is measurable  
 (D) If  $g$  is measurable and  $f = g$  ae, then  $f$  is measurable
- 34.** Let  $\{f_n\}$  be real valued functions on  $[0, 1]$  converging pointwise to a continuous function  $f$ .
- (A)  $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx = \int_0^1 f(x) dx$   
 (B)  $0 \leq f_n \leq f \implies \lim_{n \rightarrow \infty} \int f_n = \int f$   
 (C)  $|f_n(x)| \leq 1/\sqrt{x} \implies \lim_{n \rightarrow \infty} \int f_n = \int f$   
 (D)  $|f_n(x)| \leq 1 \implies \lim_{n \rightarrow \infty} \int_K f_n = \int_K f$  for measurable  $K$
- 35.** Let  $\{f_n\}, \{g_n\}, f, g \in L^1(\mathbb{R}^n)$  with  $f_n \rightarrow f, g_n \rightarrow g$  ae.
- (A)  $\int(f_n + g_n) \rightarrow \int(f + g)$   
 (B)  $|f_n| \leq |f|, |g_n| \leq |g| \implies \int(f_n + g_n) \rightarrow \int(f + g)$   
 (C)  $|f_n| \leq |g| \implies \int f_n \rightarrow \int f$   
 (D)  $|f_n| \leq |g_n|$  and  $\int f_n \rightarrow \int f \implies \int g_n \rightarrow \int g$
- 36.** Let  $f_n(x) = e^{-nx^2}$  on  $[1, \infty)$ . Which are true?
- (A)  $\int_1^\infty f_n(x) dx \rightarrow 0$   
 (B)  $\sup_n \|f_n\|_1 < \infty$   
 (C)  $f_n$  converges in  $L^1[1, \infty)$   
 (D)  $f_n$  does not converge in  $L^p$  for any  $1 \leq p \leq \infty$
- 37.** Let  $m(E_n) \rightarrow 0$  and  $f \geq 0$  be measurable.
- (A)  $\int_{E_n} f \rightarrow 0$   
 (B) If  $E_{n+1} \subset E_n$  then  $\int_{E_n} f \rightarrow 0$

- (C) If  $f$  is bounded, then  $\int_{E_n} f \rightarrow 0$   
 (D) If  $f$  is integrable and  $E_{n+1} \subset E_n$ , then  $\int_{E_n} f \rightarrow 0$

**38.** Let  $f, f_n : (X, \mathcal{F}, \mu) \rightarrow \mathbb{R}$  be measurable.

- (A)  $0 \leq f_n \rightarrow f$  uniformly  $\implies \lim \int f_n = \int f$   
 (B)  $\mu(X) < \infty, |f_n| \leq 1 \implies \lim \int g \circ f_n = \int g \circ f$  for continuous  $g$   
 (C)  $\mu(X) < \infty, |f_n| \leq 1, f_n \rightarrow f$  ae  $\implies \lim \int f_n = \int f$   
 (D)  $f_n \uparrow f \implies \lim \int f_n = \int f$

**39.** Let  $\{f_n\}$  be measurable and converge uniformly to  $f$  on  $\mathbb{R}$ .

- (A)  $\lim \int_{-\infty}^{\infty} f_n = \int_{-\infty}^{\infty} f$   
 (B)  $\lim \int_1^{\infty} f_n = \int_1^{\infty} f$   
 (C)  $\lim \int_1^2 f_n = \int_1^2 f$   
 (D)  $\lim \int_K f_n = \int_K f$  for compact  $K$

**40.** Let  $A \in \mathcal{L}(\mathbb{R}^n)$ . Which are correct?

- (A)  $\delta A \in \mathcal{L}(\mathbb{R}^n)$  for all  $\delta > 0$   
 (B)  $A + x \in \mathcal{L}(\mathbb{R}^n)$  for all  $x \in \mathbb{R}^n$

**41.** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  s.t.  $f^{-1}(\{a\})$  is a Lebesgue set  $\forall a \in \mathbb{R}$ .

- (A)  $f$  is Lebesgue measurable always  
 (B)  $f$  is Borel measurable always  
 (C)  $f : (\mathbb{R}, 2^{\mathbb{R}}) \rightarrow \mathbb{R}$  is measurable

**42.** Let  $f_n : [0, 1] \rightarrow \mathbb{R}$  be Riemann integrable and  $f_n \rightarrow f$  pointwise.

- (A)  $f$  is Riemann integrable  
 (B)  $f$  is Riemann integrable and  $\lim \int f_n = \int f$   
 (C)  $f$  is Lebesgue measurable

**43.** Continuous  $f_n \rightarrow$  continuous  $f$  pointwise on  $\mathbb{R}$ .

- (A)  $\lim \int_0^1 f_n = \int_0^1 f$   
 (B)  $0 \leq f_n \leq f \implies \lim \int_a^b f_n = \int_a^b f$   
 (C)  $|f_n| \leq e^{-x} \implies \lim \int_a^{\infty} f_n = \int_a^{\infty} f$   
 (D)  $|f_n| \leq 1 \implies \lim \int_a^{\infty} f_n = \int_a^{\infty} f$

**44.** Let  $f_n, g_n, f, g \in L^1(\mathbb{R}^k)$  with  $f_n \rightarrow f, g_n \rightarrow g$  pointwise ae.

- (A)  $\int f_n \rightarrow \int f$  and  $\int g_n \rightarrow \int g$   
 (B)  $|f_n| \leq |g_n| \implies \int f_n \rightarrow \int f$   
 (C)  $|f_n| \leq |g| \implies \int f_n \rightarrow \int f$

(D)  $|f_n| \leq |g_n|$  and  $\int f_n \rightarrow \int f \implies \int g_n \rightarrow \int g$

**45.**  $f_n(x) = e^{-nx^2}$  on  $[1, \infty)$ .

- (A)  $\int_1^\infty f_n \rightarrow 0$
- (B)  $\sup \|f_n\|_1 < \infty$
- (C)  $f_n \rightarrow 0$  in  $L^1[1, \infty)$
- (D)  $f_n$  doesn't converge in  $L^p$  for any  $p$

**46.** Which are correct?

- (A)  $\chi_E$  is measurable iff  $E$  is a Lebesgue set
- (B)  $\chi_E$  is Borel measurable iff  $E$  is a Borel set

**47.** Integrability properties:

- (A)  $f$  Riemann integrable,  $g = f$  except at finite points  $\implies g$  is R-integrable
- (B)  $f$  continuous,  $g = f$  except on countable set  $\implies g$  is R-integrable
- (C)  $f$  continuous,  $g = f$  except on countable set  $\implies g$  is L-integrable

**48.** Which are  $\sigma$ -finite?

- (A) Counting measure on  $([0, 1], 2^{[0,1]})$
- (B) Counting measure on  $([0, 1], \mathcal{B}[0, 1])$
- (C)  $\mu(A) = \#(A \cap \mathbb{Q})$  on Borel sets
- (D)  $\mu(A) = \#(A \cap \mathbb{Z})$  on Borel sets

**49.** Positive linear functionals on  $C_c$  or  $C[0, 1]$ ?

- (A)  $T(f) = \sum f(1/n)$  on  $C[0, 1]$
- (B)  $T(f) = \sum f(n)$  on  $C_c(\mathbb{R})$
- (C)  $T(f) = \sum f(1/n)$  on  $C_c(0, \infty)$

**50.** On  $X = \mathbb{R} \setminus \{0\}$ , which are positive linear functionals on  $C_c(X)$ ?

- (A)  $T(f) = \sum f(1/k) + \sum f(-1/k)$
- (B)  $T(f) = \sum f(-k) + \sum f(k)$
- (C)  $T(f) = \sum f(k + 1/k)$

**51.** Lebesgue measure zero sets?

- (A) Any countable subset of  $\mathbb{R}$
- (B) Cantor set
- (C) Irrationals in Cantor set
- (D)  $k$ -dim subspace of  $\mathbb{R}^n$  ( $k < n$ )

**52.** Invertible linear map  $T$  on  $\mathbb{R}^n$ :

(A)  $E$  Lebesgue  $\implies T(E)$  Lebesgue

(B)  $E$  Borel  $\implies T(E)$  Borel

**53.** Invariance properties:

(A) Outer measure  $m^*$  is translation invariant

(B) Lebesgue measure  $m$  is translation invariant

(C) Borel measure  $\mu$  with finite compact values is a multiple of Lebesgue

**54.** Borel measure  $\mu$  where  $\mu(A + n) = \mu(A) \forall n \in \mathbb{Z}$ :

(A)  $\mu$  is zero measure

(B)  $\mu$  is constant multiple of Lebesgue

(C) Counting measure satisfies this

**55.** Linear  $A : \mathbb{R}^n \rightarrow \mathbb{R}^n$ :

(A)  $A$  singular  $\implies A(E)$  is Lebesgue for all  $E \subset \mathbb{R}^n$

(B)  $A$  invertible  $\implies A(E)$  is Lebesgue for all  $E \subset \mathbb{R}^n$

(C)  $A$  invertible  $\implies A(E)$  is Lebesgue for all Lebesgue  $E$

**56.** Positive measurable  $f$  on  $\mathbb{R}^n$ :

(A)  $\int_{E+x_0} f = \int_E f$

(B)  $\int f(x + x_0) = \int f$

(C)  $\int f(Ax) = \int f$  for any linear  $A$

**57.** Open/Closed sets in  $[0, 1]$ :

(A) Dense open  $O \implies m(O) = 1$

(B) Open  $O \implies m(O) > 0$

(C) Closed  $F$  with no interior  $\implies m(F) = 0$

**58.** Measure zero in  $\mathbb{R}^2$ ?

(A)  $C \times \mathbb{R}$  ( $C$  is Cantor)

(B)  $\mathbb{Q} \times \mathbb{R}$

(C) Countable union of lines through origin

**59.** Positive Lebesgue measure?

(A) Any unbounded set in  $\mathbb{R}^2$

(B) Any unbounded closed set

(C)  $(\mathbb{R} \setminus \mathbb{Q}) \times \mathbb{R}$

**60.** Cantor set:

(A) Closed

- (B) Perfect
- (C) Complement in  $[0, 1]$  has measure 1
- (D) Uncountable

**61.**  $L^p$  Space inclusions/products:

- (A)  $f, g \in L^1 \implies fg \in L^1$
- (B)  $f, g \in L^2 \implies fg \in L^2$
- (C)  $f \in L^1 \cap L^\infty \implies f \in L^2$

**62.**  $L^p$  Comparisons:

- (A)  $L^1[0, 1] \subset L^2[0, 1]$
- (B)  $L^1[0, \infty) \subset L^2[0, \infty)$
- (C)  $L^2[0, 1] \subset L^1[0, 1]$
- (D)  $L^2[0, \infty) \subset L^1[0, \infty)$

**63.** Let  $f = \chi_{[0, 1/2]}$ :

- (A)  $f$  is continuous ae
- (B)  $f$  can be approximated by continuous functions in  $L^\infty$  norm
- (C)  $f = g$  ae for some continuous  $g$

**64.** Radial function integrability:

- (A)  $\chi_{|x| \leq 1} |x|^a \in L^1(\mathbb{R}^n)$  iff  $a > -n$
- (B)  $\chi_{|x| \leq 1} |x|^a \in L^1(\mathbb{R}^n)$  iff  $a < -n$
- (C)  $\chi_{|x| \geq 1} |x|^a \in L^1(\mathbb{R}^n)$  iff  $a > -n$
- (D)  $\chi_{|x| \geq 1} |x|^a \in L^1(\mathbb{R}^n)$  iff  $a < -n$

**65.** More radial functions:

- (A)  $\chi_{|x| \leq 1} |x|^a (1 - |x|)^b \in L^1$  iff  $a > -n, b > -1$
- (B)  $\chi_{|x| \leq 1} |x|^a (1 - |x|)^b \in L^1$  iff  $a > -n, b > 0$
- (C)  $\chi_{|x| \geq 1} |x|^a (1 - |x|)^b \in L^1$  iff  $a < -n, b > -1$

**66.**  $f_n(x) = x^n$  on  $[0, 1]$ :

- (A)  $f_n \rightarrow 0$  uniformly
- (B)  $f_n \rightarrow 0$  in  $L^1$
- (C)  $f_n \rightarrow 0$  in  $L^p$  for  $1 \leq p < \infty$

**67.**  $f_n(x) = x^{-n}$  on  $[1, \infty)$ :

- (A)  $f_n \rightarrow 0$  uniformly
- (B)  $f_n \rightarrow 0$  in  $L^1$
- (C)  $f_n \rightarrow 0$  in  $L^p$  for  $1 \leq p < \infty$

**68.**  $f_n(x) = x^{-n}$  on  $[2, \infty)$ :

- (A)  $f_n \rightarrow 0$  uniformly
- (B)  $f_n \rightarrow 0$  in  $L^1$
- (C)  $f_n \rightarrow 0$  in  $L^p$  for  $1 \leq p \leq \infty$

**69.**  $L^p$  Norm properties:

- (A)  $p < r < s \implies L^p \cap L^s \subset L^r$
- (B)  $\mu(X) < \infty, r < p \implies L^p \subset L^r$
- (C)  $\mu(X) < \infty, f \in L^\infty \implies \|f\|_p \rightarrow \|f\|_\infty$

**70.** Positive measurable  $f, g$  with  $fg \geq a > 0$ :

- (A)  $\mu(X) = 1 \implies (\int f)(\int g) \geq a$
- (B)  $\mu(X) < 1 \implies (\int f)(\int g) \geq a$

**71.** Measurable  $f : \mathbb{R} \rightarrow \mathbb{R}$ :

- (A)  $f \in L^1 \implies f$  is bounded
- (B)  $f \in L^1$  and continuous  $\implies f$  bounded
- (C)  $f \in L^1$  and continuous  $\implies \lim_{|x| \rightarrow \infty} f(x) = 0$
- (D)  $f \in L^1$  and uniformly continuous  $\implies f$  bounded

**72.** Product sigma algebras:

- (A)  $\mathcal{B}(\mathbb{R}) \times \mathcal{B}(\mathbb{R}) = \mathcal{B}(\mathbb{R}^2)$
- (B)  $\mathcal{L}(\mathbb{R}) \times \mathcal{L}(\mathbb{R}) = \mathcal{L}(\mathbb{R}^2)$

**73.** Rectangles and sections:

- (A)  $\sigma$ -algebra of rational rectangles is  $\mathcal{B}(\mathbb{R}^n)$
- (B)  $\sigma$ -algebra of rational rectangles is  $\mathcal{L}(\mathbb{R}^n)$
- (C)  $E \in \mathcal{L}(\mathbb{R}^2) \implies E_x \in \mathcal{L}(\mathbb{R})$

**74.** Lebesgue measurable  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ :

- (A)  $f_x$  is Lebesgue measurable  $\forall x$
- (B)  $f$  Borel  $\implies f_x$  Borel

**75.** Product measure  $m \times \mu$  ( $\mu$  counts rationals in  $[0, 1]$ ). Let  $D$  be diagonal:

- (A)  $(m \times \mu)(D) = 0$
- (B)  $(m \times \mu)(D) = \infty$
- (C)  $(m \times \mu)(D) = 1$

**76.** Product measure  $\mu \times \mu$  ( $\mu$  counts rationals). Let  $D$  be diagonal in  $[0, 1]$ :

- (A)  $(\mu \times \mu)(D) = 0$

(B)  $(\mu \times \mu)(D) = \infty$ (C)  $(\mu \times \mu)(D) = 1$ **77.**  $f(x, y) = 1$  if  $x = y$  on  $[0, 1] \times [0, 1]$ :(A)  $\int_X f(x, y) dm(x) = 0$ (B)  $\int_Y f(x, y) d\mu(y) = 1$ 

(C) Iterated integrals are same

(D)  $\mu$  not  $\sigma$ -finite  $\implies$  iterated integrals not same**78.** Subgraph  $A(f) = \{(x, y) : 0 < y < f(x)\}$ :(A)  $A(f) \in \mathcal{B}(\mathbb{R}^2)$ (B)  $m(A(f)) = \int f(x) dx$ **79.** Graph  $G(f) = \{(x, f(x))\}$ :(A)  $G(f)$  is Borel(B)  $m(G(f)) = 0$ (C)  $m(G(f)) = \infty$ **80.** Distribution function  $F_f(t) = \mu\{f > t\}$ :(A)  $F_f$  is non-increasing(B)  $\int f d\mu = \int_0^\infty F_f(t) dt$ **81.** Complete measure spaces:(A)  $(\mathbb{R}^2, \mathcal{B}(\mathbb{R}^2), m)$ (B)  $(\mathbb{R}^2, \mathcal{L} \times \mathcal{L}, m)$ (C)  $(\mathbb{R}^2, \mathcal{L}(\mathbb{R}^2), m_2)$ **82.**  $g(x) = \int_x^b \frac{f(t)}{t} dt$ :(A)  $g$  not integrable on  $[0, b]$ (B)  $g$  integrable and  $\int_0^b g = \int_0^b f$ **83.**  $f(x) = |x|^a$  for  $|x| \leq 1$ :(A)  $\int f < \infty$  if  $a > -n$ (B)  $\int f < \infty$  if  $a < -n$ (C)  $\int f < \infty$  if  $a > 0$ **84.**  $f(x) = |x|^a$  for  $|x| \geq 1$ :(A)  $\int f < \infty$  if  $a > -n$ (B)  $\int f < \infty$  if  $a < -n$ (C)  $\int f < \infty$  if  $a > 0$

85.  $f(x) = (1 - |x|^2)^a$  for  $|x| \leq 1$ :

- (A)  $\int f < \infty$  if  $a > -n$
- (B)  $\int f < \infty$  if  $a > -1$
- (C)  $\int f < \infty$  if  $a > 0$

86.  $F(x) = \int_x^{x+1} g(y)dy$  where  $g \in L^1$ :

- (A)  $F \in L^1$
- (B)  $F$  is bounded
- (C)  $\lim_{|x| \rightarrow \infty} F(x) = 0$

87.  $F(x) = \sum_{k=-\infty}^{\infty} f(x+k)$  on  $[0, 1]$ :

- (A)  $F \in L^1[0, 1]$
- (B)  $F \in L^1 \implies f = 0$  ae
- (C)  $F$  is finite ae

88. If  $\int |f_k - f| \rightarrow 0$ :

- (A)  $f_k \rightarrow f$  ae
- (B) Subsequence  $f_{k_j} \rightarrow f$  ae
- (C)  $f_k \rightarrow f$  in measure