

Functions of Several Variable and Differential Geometry 2024 - Viva Voce Exam

Maximum marks 16. Each question carries 2 marks

1. Can two level sets intersect each other? why or why not?
 2. Let S be a level set of function f , contained in \mathbb{R}^2 . Can S be a level set of another function g which is not equal to f .
 3. Let S be the unit circle in \mathbb{R}^2 . Give an example of a parametrized curve on S passing through the point $(1, 0)$.
 4. Write true or false with justification. Let S be an n -surface and let p be a point on S . Then the union of the tangent space and normal space at p gives the vector space \mathbb{R}_p^{n+1} .
 5. Let S be the union of S_1 and S_2 where S_1 and S_2 be two unit circles in \mathbb{R}^2 centred at $(0, 0)$ and $(3, 0)$. What are the possible smooth unit normal vector fields of S .
 6. When do you call a tangent space S_p at p of a 3-surface is right handed.
 7. Let $S = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 = 1, x_3 \geq 0\}$. Can S be n -surface? why or why not?
 8. Give an example of a geodesic in the 2-sphere in \mathbb{R}^3 .
 9. Let p be a point on an n -surface and v be a an element in the tangent space at p . Can you find a geodesic passing through p with its velocity v at p ? Give a short explanation.
 10. State true or false with justification: The velocity vector field along a parametrized curve α in S is parallel if and only if α is a geodesic.
 11. Establish this equality: $\dot{X} \cdot Y + X \cdot \dot{Y} = X' \cdot Y + X \cdot Y'$. Here X and Y are two smooth vector fields tangent to an n -surface S along a parametrized curve $\alpha : I \rightarrow S$.
 12. Explain the difference between Euclidean parallel and Levi-Civita parallel.
 13. Prove: For each smooth tangent vector field X on S , the function which sends v in to $D_v X$ is a linear map from S_p to S_p .
 14. What is a shape operator and why it is called so?
 15. Construct a tangent vector field on the plane curve.
 16. Explain circle of curvature and radius of curvature
- Extra 1. How do you reparametrize a parametrized curve defined on $(-1, 1)$ to a parametrized curve defined on $(2, 4)$.
- Extra 2. Can a Möbius band be a 2-surface? why or why not?

Hope all went well. Keep smiling and moving forward; better times are ahead.