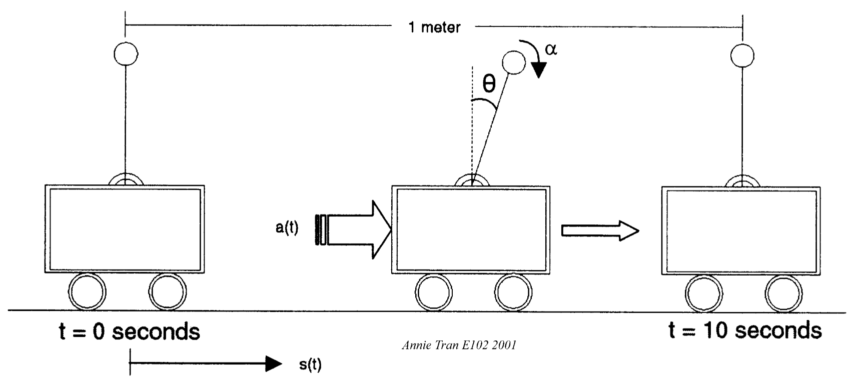
E102 Inverted Pendulum Project

Due: 5pm April 25 (Friday), 2025

You have two weeks to work on this, but start analysis and design early. You may work individually or with a partner. Only one report should be uploaded to Gradescope for each team. If you work with a partner, please include both of your names on the report.

Problem Statement



A cart with an attached inverted pendulum is to be moved from an initial rest position to a final rest position one meter away. The system is equipped with sensors for measuring the cart position $s(t)$ and the pendulum angular displacement $\theta(t)$. The pendulum is subject to an angular acceleration disturbance $\alpha(t) = 0.5\ \mathrm{rad/sec^2}$. Design and simulate a control system to carry out the movement within an overall time limit of ten seconds. The applied acceleration (control input) is limited to $|a(t)| < 0.5\ \mathrm{m/sec^2}$. The cart may not overshoot the final position.

Describing equations for the cart-pendulum system (OWN 11.56)

$$L\frac{d^2\theta(t)}{dt^2} - g\sin\theta(t) = -a(t)\cos\theta(t) + L\alpha(t)$$

$$\frac{d^2 s(t)}{dt^2} = a(t)$$

with pendulum length $L = 0.5$ m.

Suggested Approach

I Design the control system based on a linearized model of the plant

1. Formulate the linearized state space equations for the cart-pendulum plant using the small angle approximation $\sin\theta(t) \approx \theta(t); \cos\theta(t) \approx 1$.

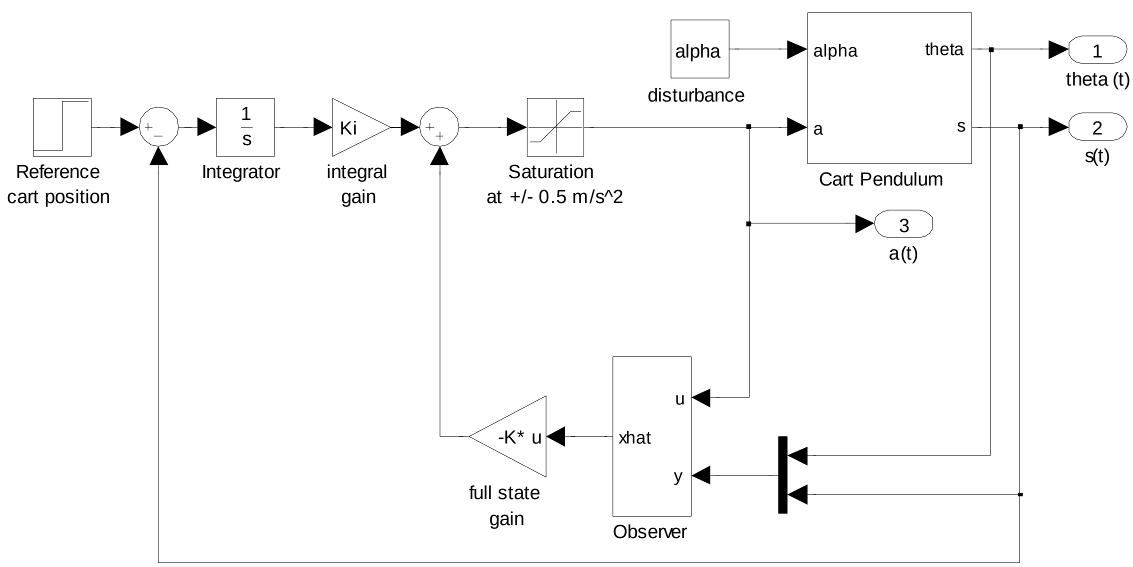
   with state vector $\mathbf{x} = \begin{bmatrix} q(t) \\ \dot{q}(t) \\ s(t) \\ \dot{s}(t) \end{bmatrix}$; control input $\mathrm{u} = a(t)$;

   disturbance input $\mathrm{w} = \alpha(t)$; output vector $\mathbf{y} = \begin{bmatrix} \theta(t) \\ s(t) \end{bmatrix}$

2. Establish the stability, controllability and observability of the linearized plant.
3. Design a state feedback control system with integral action and an observer for the linearized plant. Use pole placement design.

II Simulation based on the nonlinear model of the plant:

4. Build a Simulink model of the nonlinear cart-pendulum plant together with the control system.



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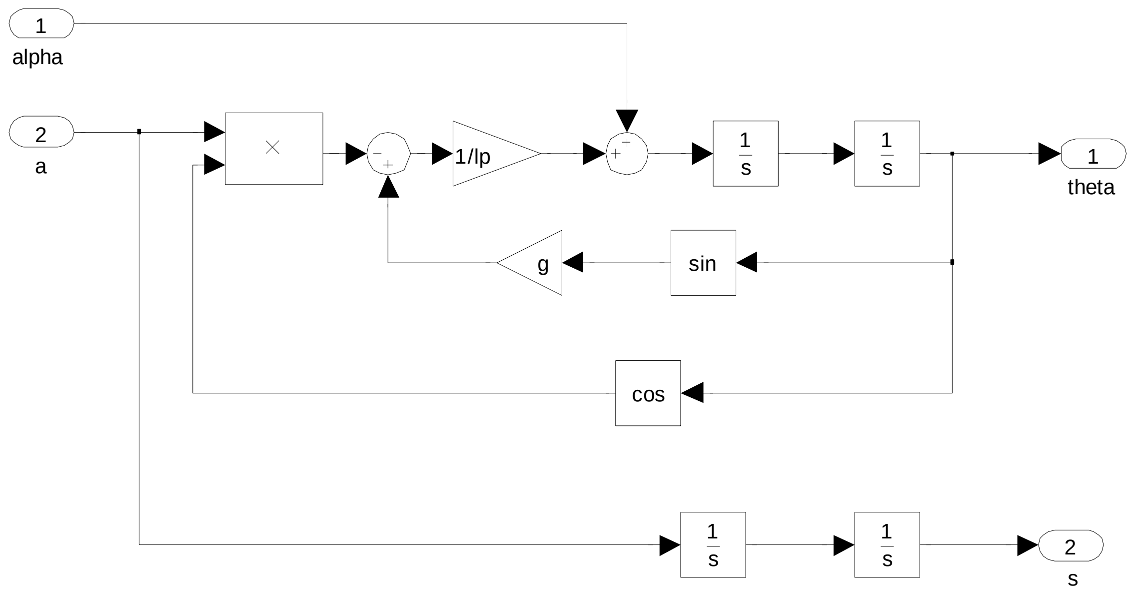

CartPendulum

5. Use the control design parameters from the designs in Part I and run the simulation.

6. Fine-tune the design to achieve the desired specifications.

7. Increment the angular acceleration disturbance $\mathrm{w} = \alpha(t)$ to find the largest allowable disturbance that maintains closed loop stability.

Project Instructions

1. This project is done in a team of two students. You may also choose to work individually.

2. A project report (no more than 4 pages, 12pt font) is due by the specified due date. Use the following format for the report

   I. Introduction
   II. State Space Design of a Linear Controller for a Linearized Plant
   III. Simulation of Control of Nonlinear Plant with the Linear Controller
   IV. Discussion and Conclusions

3. MATLAB code and Simulink block diagrams should be included in an Appendix to the report; the Appendix does not count towards the 4-page limit of the report. Response plots of $s(t)$, $\theta(t)$, $a(t)$ should be labeled, referenced and included in the body of the report.

4. The organization and writing of the report is also graded in addition to the technical results.