SISTEMAS CONTROLE

$\underbrace{\text{L.I.T.}}$

$$\frac{Y(s)}{X(s)} = G(s)$$

$G(s) \longrightarrow$ Ganho do sistema

Função de transferência

Exemplo:

$$G(s) = \frac{1}{s-1} \longleftarrow$$ FÍSICO ou "REAL"

SISTEMA $\longrightarrow$ PÓLOS // ZEROS

$N(s) = 1$ $\qquad$ $D(s) = s-1 = 0$

$\downarrow$

$\underbrace{P_1 = +1}_{\text{INSTÁVEL}}$ ✓

$x(t) = \delta(t) \longrightarrow y(t) = ?$

$\downarrow \mathcal{L}$

$X(s) = 1 \longrightarrow Y(s) = G(s) \cdot X(s)$

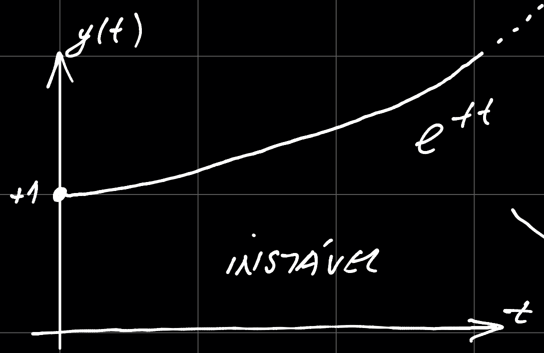
$Y(s) = G(s) \cdot 1$

CONTROLE

$$y(s) = G(s) = \frac{1}{s-1}$$

$$\mathcal{L}^{-1}$$

$$y(t) = \underbrace{e^{+t}\mu(t)}$$

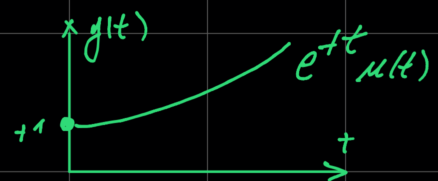


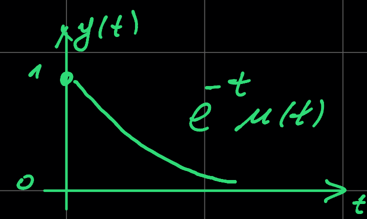
$t \to \infty$

$y(t) \to \infty$

$$\underbrace{\frac{1}{s-1}}_{\text{INSTÁVEL}} \xrightarrow{\delta(t)} \frac{1}{s+1}$$

INSTÁVEL ESTÁVEL



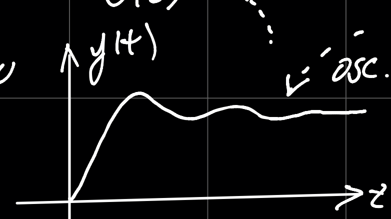


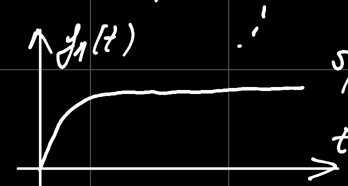
$$\frac{N(s)}{D(s)} \; G(s) \longrightarrow \frac{N_1(s)}{D_1(s)} \; G_1(s)$$

$\mu(t) = \mathcal{E}(t)$

degrau

AO DEGRAU



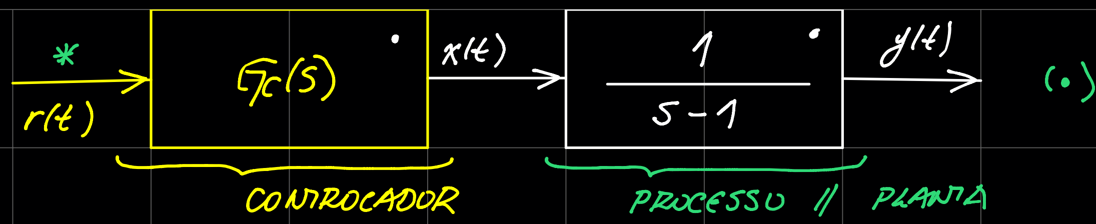


S/ OSC.

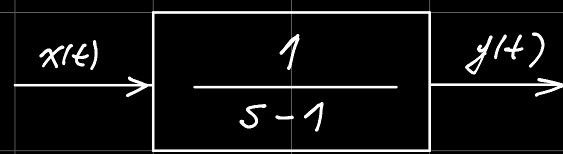
CONTROLE

- MALHA ABERTA
- MALHA FECHADA

$$\frac{1}{s+1}$$



$$\frac{Y(S)}{X(S)} = \frac{1}{S-1}$$



$$\frac{Y(S)}{R(S)} = \frac{1}{S+1} \quad \cdots\cdots \quad G_c(S) = \_\_\_\_\_\_$$

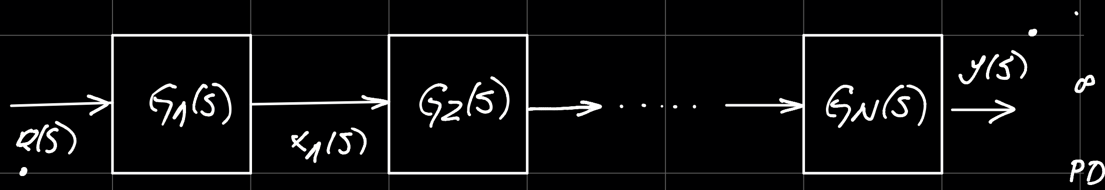
(•) CASCATA ... SÉRIE

$$Y(S)/X(S) = \frac{1}{S-1} \quad , \quad X(S)/R(S) = G_c(S) \longleftarrow$$

$$Y(S)/R(S) =$$

$$X(S) = G_c(S) \cdot R(S) \longrightarrow Y(S)/X(S) = Y(S)/G_c(S)\,R(S)$$

$$\frac{Y(S)}{R(S)} = \underbrace{G_c(S) \cdot \frac{1}{S-1}}$$



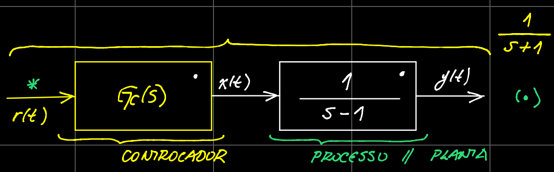
$$\frac{Y(S)}{R(S)} = \underbrace{G_1(S) \cdot G_2(S)} \cdots G_N(S) \quad !$$

$$\frac{y(s)}{R(s)} = G_c(s)\cdot\frac{1}{s-1} = \frac{1}{s+1}$$

- VIÁVEL
- RECOMENDÁVEL

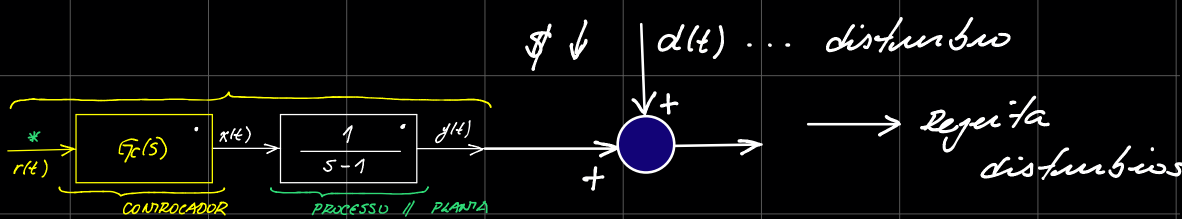
$$\therefore\ G_c(s) = \underbrace{\frac{s-1}{s+1}}_{\text{CONTROLADOR}}\ \checkmark \ \leftarrow \ldots \ \text{VIÁVEL}$$

ENG?
INCONV.?

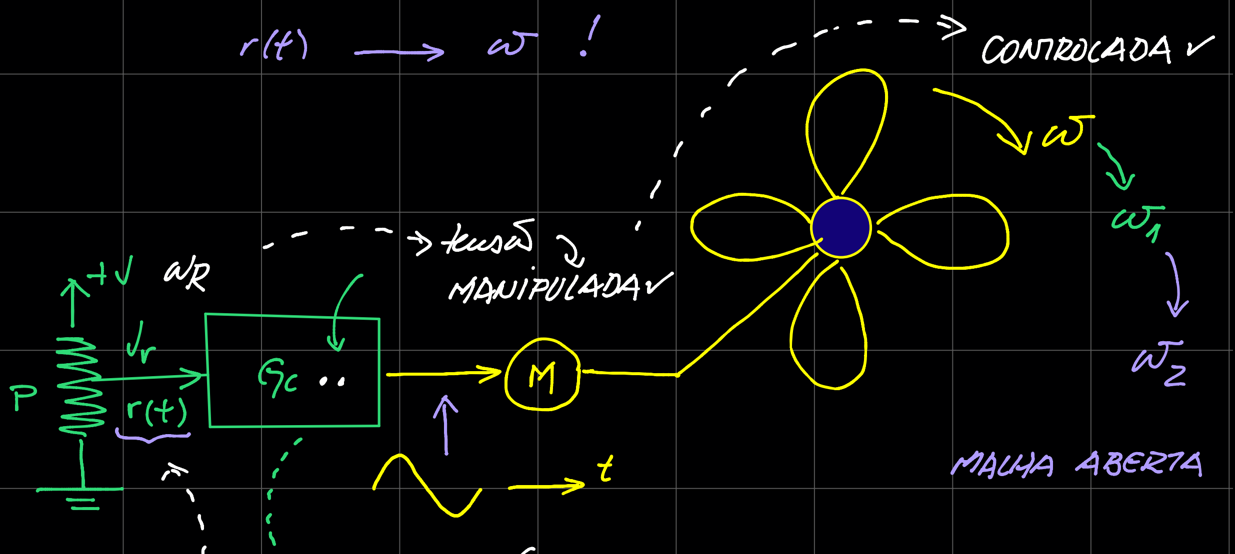


MALHA ABERTA

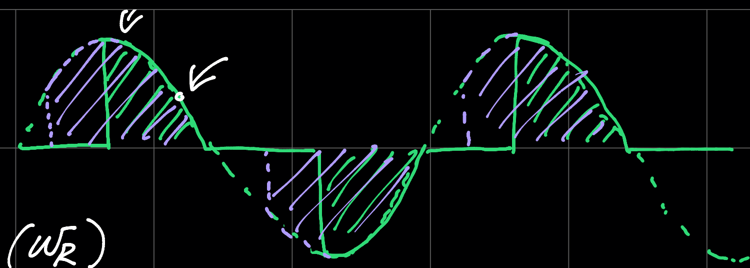
$$G_{ma}(s) = G_c(s)\cdot\frac{1}{s-1}$$

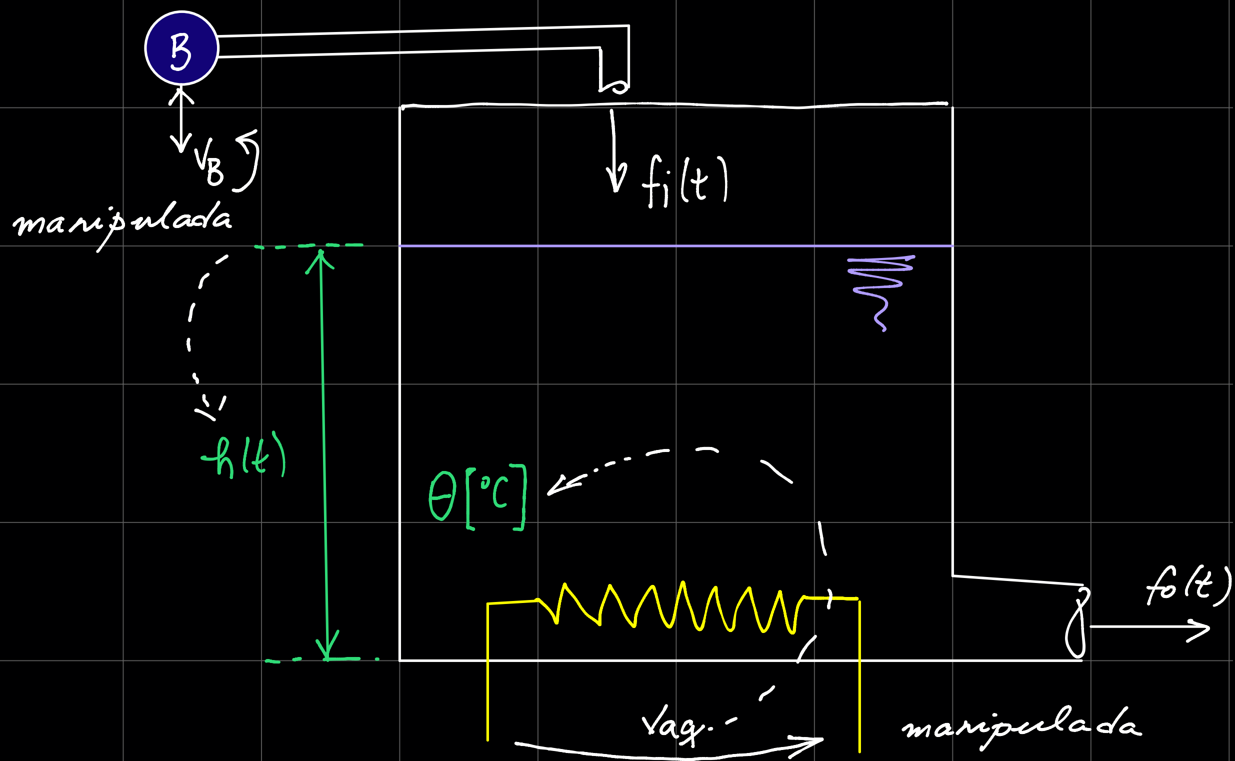


r(t) → ω !



• REFERÊNCIA ou SET-POINT (W_R)

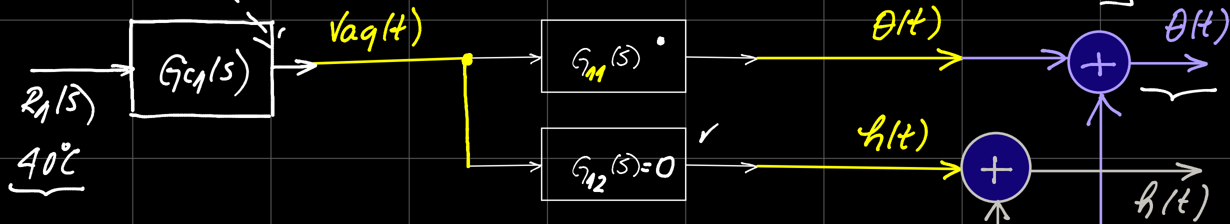


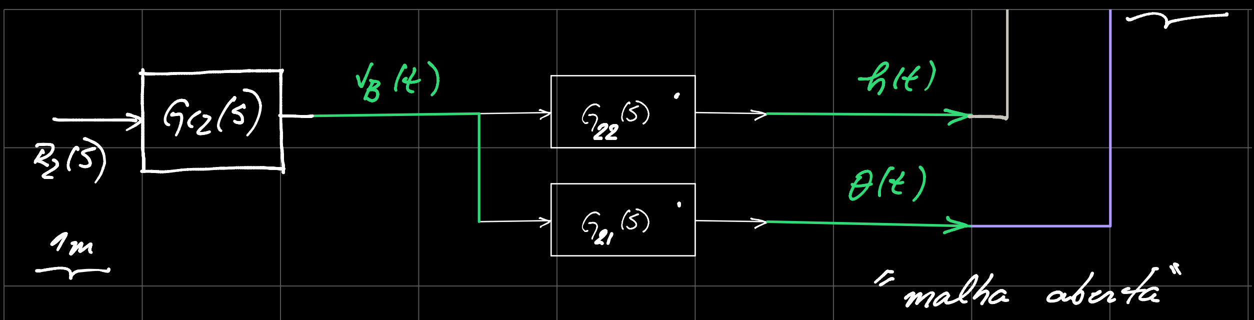


(•) CONTROLADA: Temperatura e nível

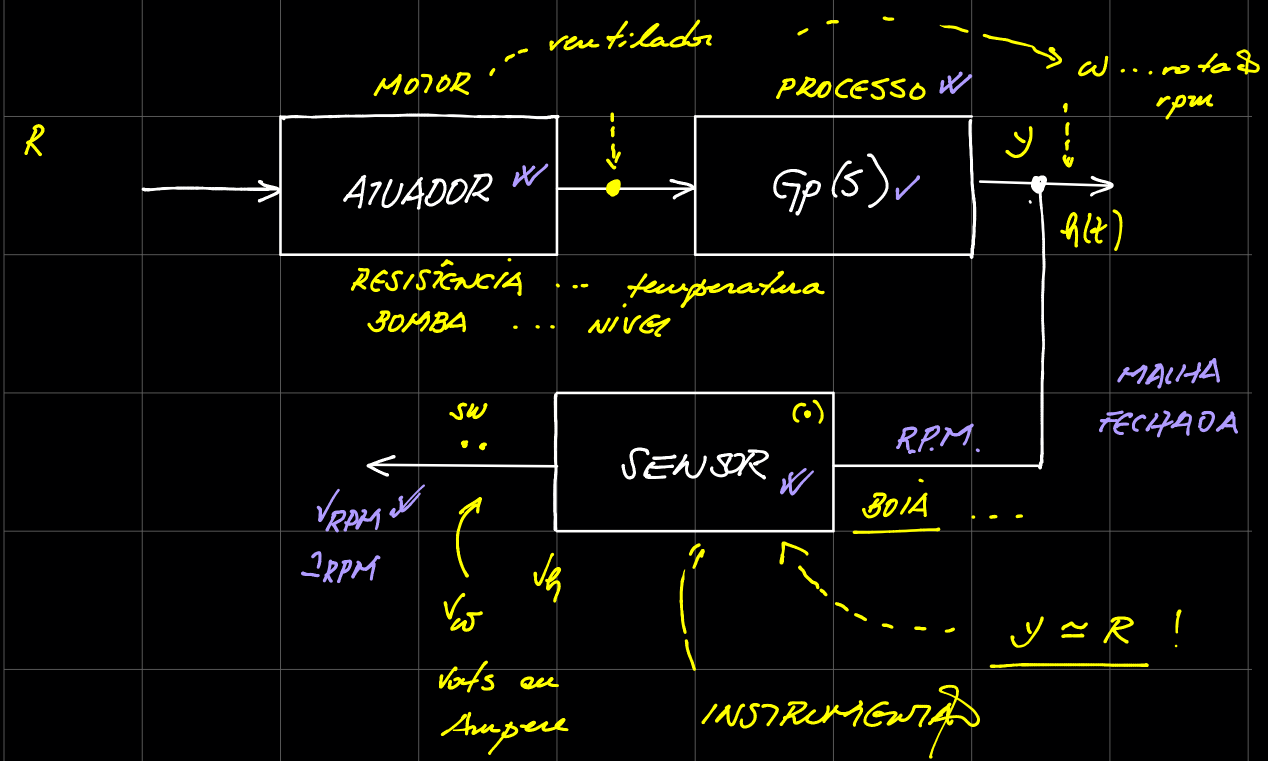
2 variáveis manipuladas MIMO

2 variáveis controladas

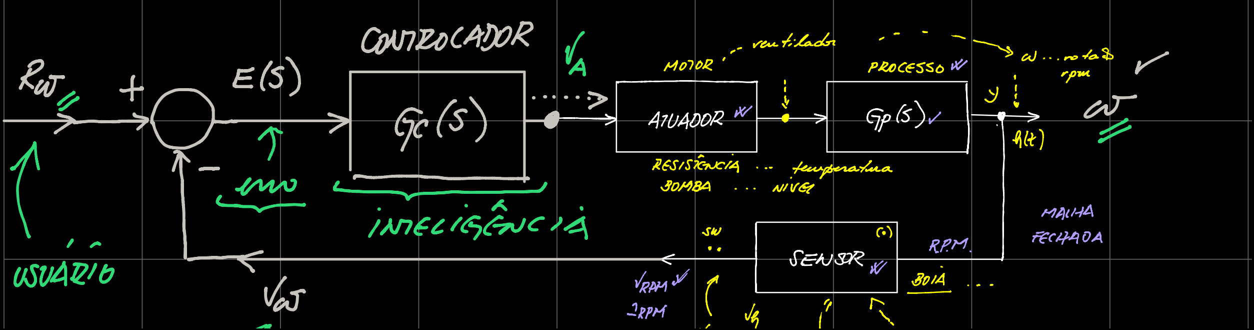




$R_2(s)$
$G_{c2}(s)$
$v_B(t)$
$G_{22}(s)$
$G_{21}(s)$
$h(t)$
$\theta(t)$
1m
"malha aberta"



MOTOR
ventilador
ω ... rotação
rpm
PROCESSO
R
ATUADOR
Gp(S)
y
h(t)
RESISTÊNCIA ... temperatura
BOMBA ... NÍVEL
MALHA
FECHADA
SW
SENSOR
(•)
R.P.M.
BOIA ...
V_{RPM}
I_{RPM}
V_ω
V_h
volts ou
Ampere
$y \simeq R$!
INSTRUMENTAÇÃO



R_ω
+
−
E(S)
CONTROLADOR
Gc(S)
V_A
erro
INTELIGÊNCIA
USUÁRIO
V_ω
ATUADOR
Gp(S)
ω
SENSOR

medida

V_w

Volts ou Ampere

INSTRUMENTAÇÃO

$y \simeq R$!

$E(s)$ ... sinal ... ERRO ATUANTE

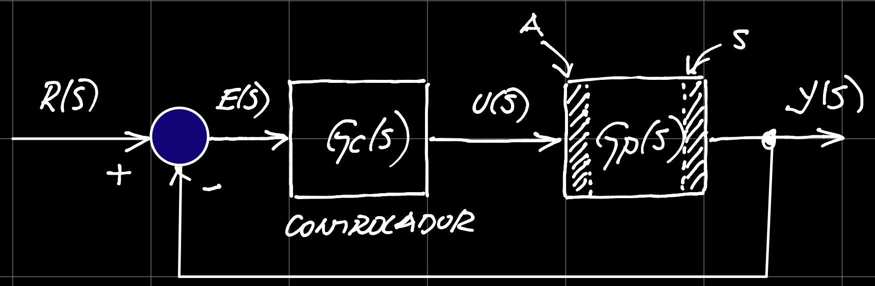
$R_w(s)$ ... sinal ... REFERÊNCIA ajustada pelo usuário

objetivos $\longrightarrow$ erro $\rightarrow 0$ ... $y(t) \rightarrow R(t)$

QUAL o $G_c(s)$ que garante ?

Existe mais de um $G_c(s)$?

Se existir, como escolher $G_c(s)$?



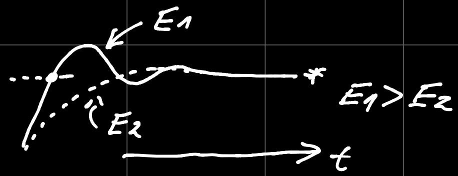
MALHA FECHADA

$R(s)$ ... REFERÊNCIA (USUÁRIO)

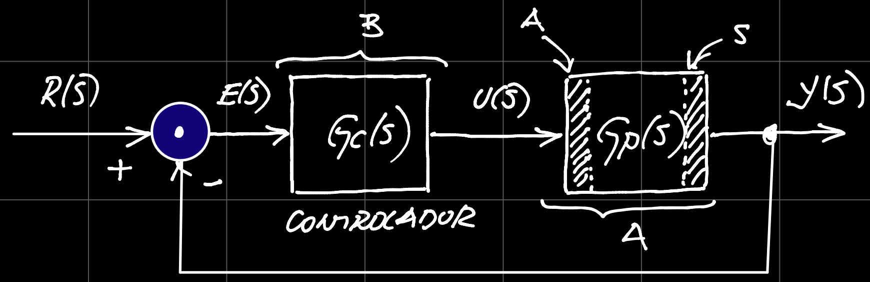
$E(s)$ ... ERRO ATUANTE

$U(s)$ ... ESFORÇO DE CONTROLE (MANIPULADA)

$y(s)$ ... VARIÁVEL CONTROLADA - SAÍDA



$\underbrace{\text{TRANSITÓRIO}}$



$$\frac{Y(s)}{R(s)} = ? \qquad \frac{E(s)}{R(s)} = ?$$

$$Y(s) = G_p(s) \cdot U(s) \quad (A)$$

$$U(s) = G_c(s) \cdot E(s) \quad (B)$$

$$E(s) = R(s) - Y(s)$$

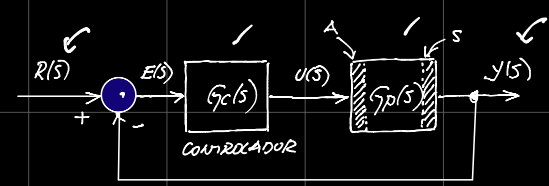
$$U(s) = G_c(s) \cdot (R(s) - Y(s))$$

$$Y(s) = G_p(s) \cdot G_c(s) \cdot [R(s) - Y(s)]$$

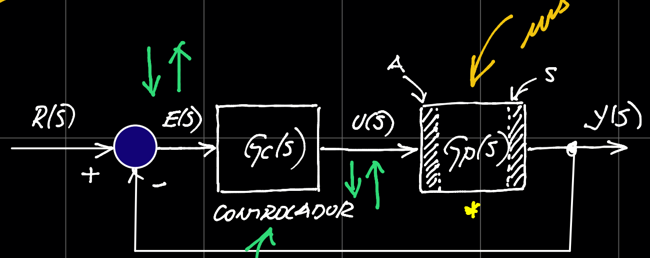
$$Y(s) = G_p(s) \cdot G_c(s) \cdot R(s) - G_p(s) G_c(s) \cdot Y(s)$$

$$Y(s)\,[1 + G_c(s)\,G_p(s)] = G_c(s)\,G_p(s) \cdot R(s)$$

$$\frac{\overset{\text{saída}}{Y(s)}}{\underset{\text{entrada}}{R(s)}} = \frac{G_c(s) \cdot G_p(s)}{1 + G_c(s) \cdot G_p(s)} \quad \ldots \quad G_{MF}(s) \; !$$



K>1 (estável)!!!



$$G_p(S) = \frac{1}{S-1}$$ *

PÓLO : $+1$

INSTÁVEL

$G_c(S) = K$ ... ganho proporcional

$$U(S) = K \cdot E(S)$$

$$G_{MF}(S) = \frac{K \cdot \frac{1}{S-1}}{1 + K \cdot \frac{1}{S-1}} =$$

$$\frac{y(S)}{R(S)} = \frac{G_c(S) \cdot G_p(S)}{1 + G_c(S) \cdot G_p(S)} = \frac{K}{S - 1 + K}$$

$y(S)$ → saída

$R(S)$ → entrada

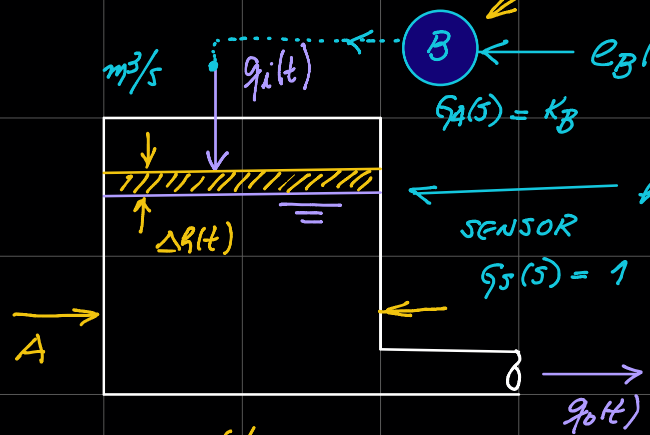
$$G_{MF}(S) = \frac{K}{S \underbrace{- 1 + K}_{>0}} = \frac{y(S)}{R(S)}$$

$$G_{MA}(S) = \frac{K}{S-1} \Rightarrow \text{PÓLO } +1$$

PÓLO : $S - 1 + K = 0$ ... $p = 1 - K$ $\underbrace{(p < 0)}$

$\underline{\underline{K > 1}}$ estável

MODELAGEM   FÍSICO $\longrightarrow$ $G_p(s)$



$e_B(t)$ ... VARIÁVEL MANIPULADA

$G_A(s) = K_B$

$h(t)$ ... VARIÁVEL CONTROLADA

SENSOR

$G_S(s) = 1$

INSTRUMENTAÇÃO

vol

$$\underbrace{\frac{\Delta h(t)\cdot A}{\Delta t}}_{Q} = q_i(t) - q_o(t)$$

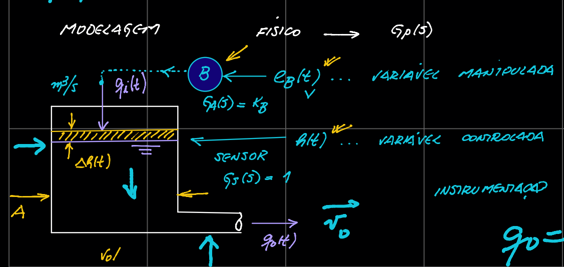
BALANÇO DE VAZÃO / FLUXO VOLUMÉTRICO

m³/s

$K_B \cdot e_B(t)$

Energia

PATM



A_0 PATM

BALANÇO DE ENERGIA

$$h(t) = \frac{v_0^2}{2g} \longrightarrow$$

$$q_o = v_0 \cdot A_0$$

m³/s   m/s   m²

$$v_0 = \sqrt{2g\,h(t)} \leftarrow$$

NÃO LINEAR

$$A \cdot \underbrace{\frac{\Delta h(t)}{\Delta t}} = \underbrace{K_B e_B(t)} - \underbrace{A_0 \sqrt{2g}\sqrt{h(t)}}$$

$\Delta t \rightarrow 0$

$$A \cdot \frac{dh(t)}{dt} = K_B \cdot e_B(t) - \underbrace{A_0 \sqrt{2g}\sqrt{h(t)}}$$

Ñ LINEAR

linearizou

$\mathcal{L}$

$$A \cdot sH(s) = K_B\, E_B(s) - \quad \sqrt{h(t)} \ldots$$

SIMULADOR

SW

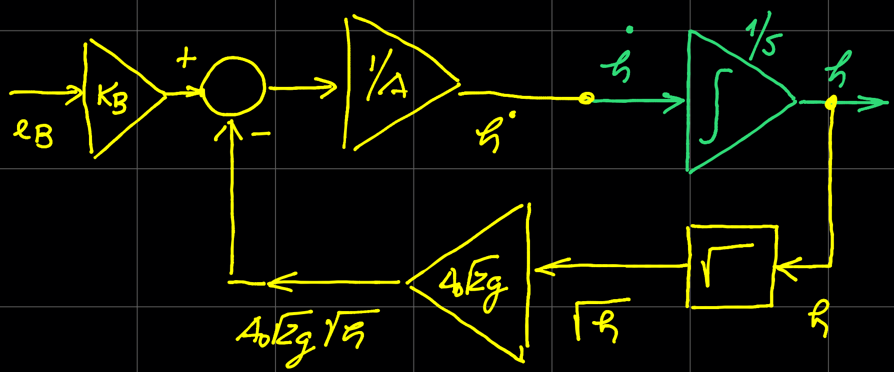
SIMULINK

$\alpha\, h(t)$

$$\underbrace{A \cdot \dot{h}}_{\text{ordem } 1} = \underbrace{K_B \cdot e_B}_{} - \underbrace{A_0\sqrt{2g}\sqrt{h}}_{}$$

$K_b = \dfrac{K_B}{A} = 1$

$K_t = \dfrac{A_0\sqrt{2g}}{A} = 1$



0

$\dot{h} = e_B - \sqrt{h}$

$h_{final} = 10$

$e_B = ?$

$e_B - \sqrt{10} = 0 \quad \therefore \quad e_B = \sqrt{10}$

$$\dot{h} = e_B - \sqrt{h} \xrightarrow{h_0} e_{B_0} = \sqrt{h_0}$$

estado estacionário

$\dot{h} = 0$

$\mathcal{L}$

$\dot{h} \Rightarrow$ linear

$\underbrace{\sqrt{h}}_{\text{"reta"}}$ ... não linear

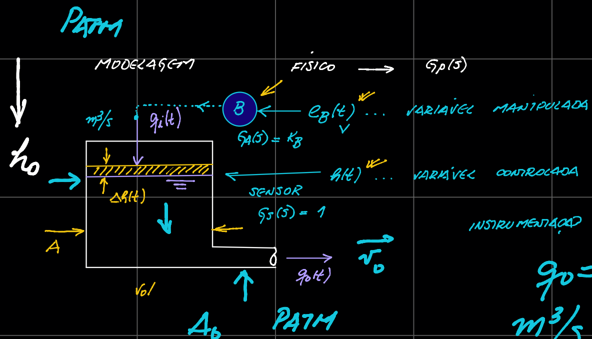
aproximação

$$\sqrt{h} \doteq \underbrace{\sqrt{h_0}}_{cte} + \frac{d\sqrt{h}}{dt}\bigg|_{h=h_0} \cdot (h - h_0)$$

série de Taylor

$$\sqrt{h} \doteq \underbrace{\sqrt{h_0}}_{cte} + \underbrace{\frac{1}{2\sqrt{h_0}}}_{cte} \cdot (h - \underbrace{h_0}_{cte})$$

$$\sqrt{h} \doteq \underbrace{\frac{1}{2\sqrt{h_0}}}\cdot h + \underbrace{\sqrt{h_0} - \frac{h_0}{2\sqrt{h_0}}}_{cte} \qquad \text{Linear } h$$



$$\dot{h} = e_B - \sqrt{h}$$

$$\dot{h} = 0$$

$$\sqrt{h_0} = e_{B0}$$

$$q_0 = v_0 \cdot A_0$$

$$m^3/s \quad m/s \quad m^2$$

$$\dot{h} = e_B - \frac{1}{2\sqrt{h_0}} \cdot h + \frac{1}{2\sqrt{h_0}} h_0 - \underbrace{\sqrt{h_0}}_{e_{B0}}$$

$$\dot{h} = \underbrace{e_B - e_{B0}}_{\Delta e_B} - \frac{1}{2\sqrt{h_0}} \underbrace{(h - h_0)}_{\Delta h}$$

$$\Delta\dot{h} = \dot{h} - \underline{\underline{\dot{h}_0}}$$

$$\Delta\dot{h} = \dot{h}$$

$$\dot{h} = \Delta e_B - \frac{1}{2\sqrt{h_0}} \Delta h$$

$$\Delta\dot{h} = \Delta e_B - \frac{1}{2\sqrt{h_0}} \Delta h$$

modelo linear

$\mathcal{L}$

$$s \cdot \Delta H(s) = \Delta E_B(s) - \frac{1}{2\sqrt{h_0}} \, \Delta H(s)$$

$$s \cdot \Delta H(s) + \frac{1}{2\sqrt{h_0}} \, \Delta H(s) = \Delta E_B(s)$$

$$\Delta H(s) \cdot \left(s + \frac{1}{2\sqrt{h_0}} \right) = \Delta E_B(s)$$

$$\frac{\Delta H(s)}{\Delta E_B(s)} = \frac{1}{s + \underbrace{\frac{1}{2\sqrt{h_0}}}_{\text{pólo}}} \leftarrow \text{F.T.}$$

em torno do h_0

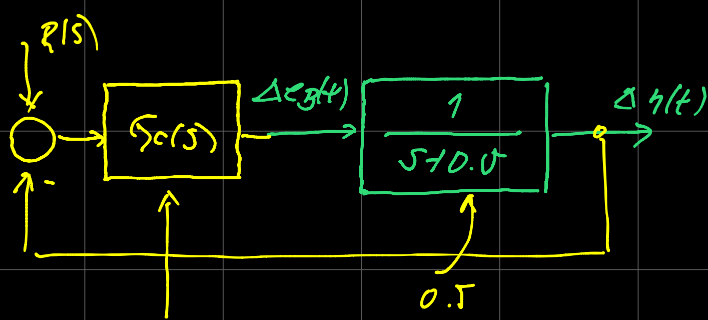
$$s + \underbrace{\frac{1}{2\sqrt{h_0}}}_{>0} = 0$$

$$p_1 = -\frac{1}{2\sqrt{h_0}} < 0 \quad \text{tempo de resposta}$$

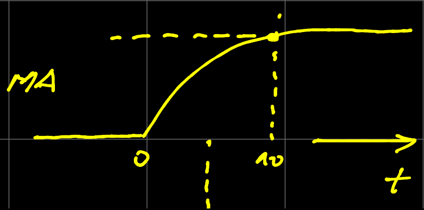
AUTOMAÇÃO | MODELO | MODELO

BOMBA + SENSOR $\longrightarrow$ $\dot{h} = e_B - \sqrt{h}$ $\longrightarrow$ $\dfrac{H(s)}{E_B(s)} = \dfrac{1}{s + \frac{1}{2\sqrt{h_0}}}$

CONTROLE $h_0 = 1m \quad \therefore \quad G_p(s) = \dfrac{1}{s + 0.5}$



DESEMPENHO $\longrightarrow$ $T_S \approx 10s$



$$\frac{1}{s+a} \xrightarrow{\mathcal{L}} e^{-at}$$

$T_s \simeq 5s$ MF

$t = 0 .. e^0 = 1$

$t = 1/a = e^{-1} \dots 0.37$

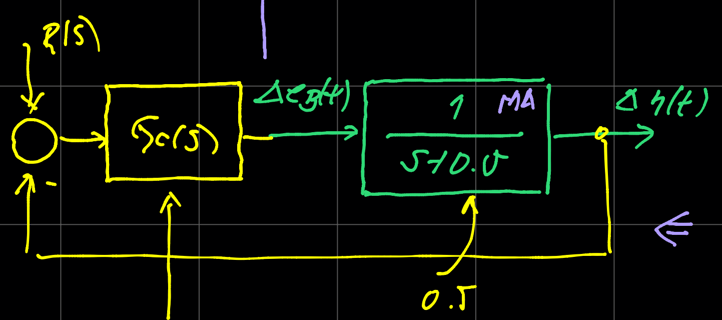
$t = 2/a = e^{-2} \dots 0.15$

$t = 3/a = e^{-3} \dots 0.05$

$t = 4/a = e^{-4} \dots 0.02$ — (98% ou 2%)

$$5s = \frac{4}{a} \quad \therefore \quad a = \frac{4}{5} = 0.8$$

$$\frac{1}{s+0.8}_{MF} \longleftarrow \frac{1}{s+0.5}_{MA} \quad \dots \text{DESEMPENHO}$$



$$\frac{y(s)}{E_B(s)} = \frac{1}{s+0.5}$$

$$\frac{H(s)}{R(s)} = \frac{1}{s+0.8}$$

$$\frac{y(s)}{R(s)} = \frac{G_{ma}(s)}{1 + G_{ma}(s)} \quad \dots \quad G_{ma}(s) = G_c(s) \cdot \frac{1}{s+0.5}$$

$$\frac{y(s)}{R(s)} = G_{MF}(s) = \frac{G_c(s)/(s+0.5)}{1 + G_c(s)/(s+0.5)} = \frac{G_c(s)}{\underbrace{s+0.5+G_c(s)}_{}}$$

$$\frac{\text{cte}}{s+0.8}$$

$$G_{MF}(s) = \frac{0.3}{s+0.8}$$

$0.5 + G_c(s) = 0.8$

$G_c(s) = 0.3$

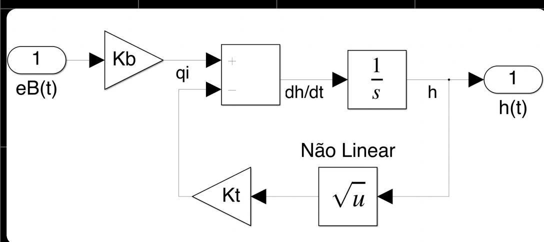
ganho

MODELO TANQUE . VARIÁVEL CONTROLADA $h(t)$

VARIÁVEL MANIPULADA $e_B(t)$

$$A \cdot \dot{h}(t) = K_B\, e_B(t) - K_t \underbrace{\sqrt{h(t)}}$$

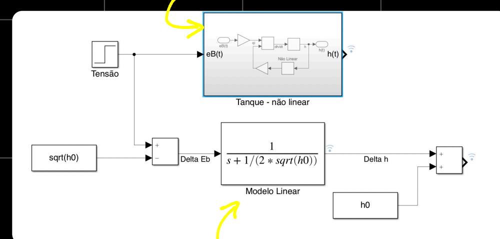
SW



linearização:

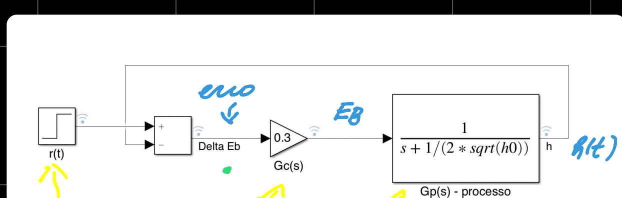
$$f(x) \doteq f(x_0) + \dot{f}(x)\Big|_{x_0} \Delta x$$

Ñ LINEAR



LINEAR

$$\frac{\Delta H(s)}{\Delta E_B(s)} = \underbrace{\frac{1}{s + \frac{1}{2\sqrt{h_0}}}}_{\text{Linear}}$$



(•) CONTROLAR o T_S

(•) Reduzir o erro

$r(t)$

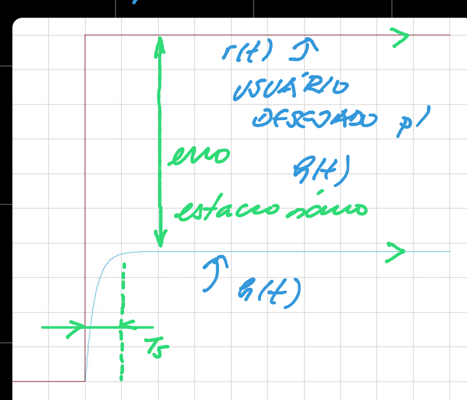
USUÁRIO

Controlador

Projeto

SW

Linear



ENGENHARIA DE CONTROLE

CAPÍTULO 03 pg 66 - 80

$$\frac{1/T}{S + 1/T}$$

$$X(S) \longrightarrow \boxed{\dfrac{1}{TS+1}} \longrightarrow Y(S)$$

PRIMEIRA ORDEM

↓

Grau máximo do $G(S) \rightarrow 1$

$$\frac{\Delta H(S)}{\Delta E_B(S)} = \frac{1}{S + \frac{1}{2\sqrt{h_0}}} = \frac{2\sqrt{h_0}}{\underbrace{2\sqrt{h_0}}_{T}\, S + 1} \longrightarrow T = 2\sqrt{h_0}$$

POLO

$P_1 = -\,1/2\sqrt{h_0}$

(estável)

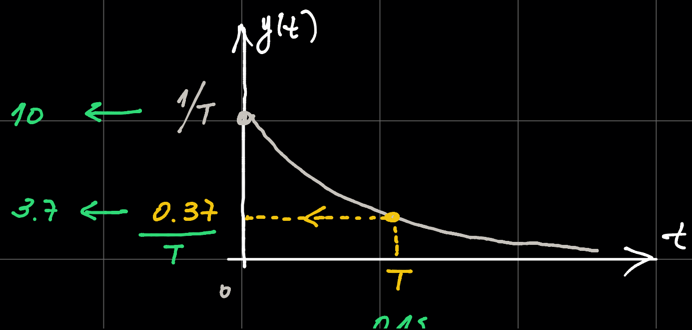
SPE

$\mathcal{L}$: 1/2

$$G(S) = \frac{1}{TS + 1} = \frac{1/T}{S + 1/T}$$

(•) $X(S) \longrightarrow$ impulso $\quad Y(S) \Rightarrow \mathcal{L}\{\delta(t)\} = 1$

$$Y(S) = G(S) = \frac{1/T}{S + 1/T} \xrightarrow{\mathcal{L}^{-1}} y(t) = \underbrace{1/T\; e^{-t/T}}$$



$$y(T) = 1/T \cdot e^{-T/T}$$

$$= \underbrace{1/T} \cdot \underbrace{e^{-1}}_{\simeq 0.37}$$

$$G(s) = \frac{10}{s+10} \rightarrow \frac{1}{0.1s+1} \quad \therefore \quad \underbrace{T = 0.1s}$$

$$X(s) \longrightarrow \text{DEGRAU} \longrightarrow \mathcal{L}\{\mu(t)\} = 1/s$$

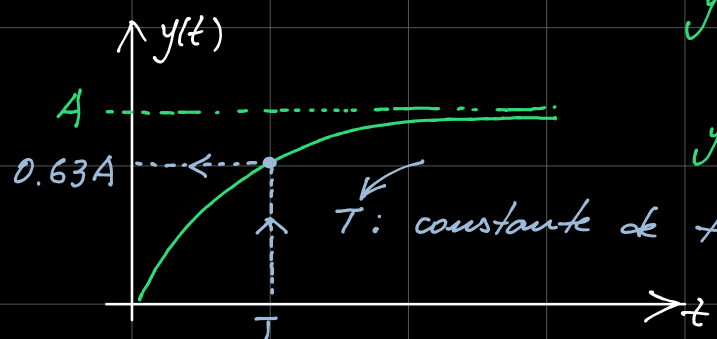
$$Y(s) = \frac{1}{Ts+1} \cdot \frac{A}{s} \longrightarrow A \ldots \text{amplitude}$$

$$= \frac{A/T}{s(s+1/T)} = \underbrace{\frac{A}{s} - \frac{A}{s+1/T}}_{\mathcal{L}^{-1}}$$

$$y(s) = A\mu(t) - \underbrace{A}e^{-t/T}\mu(t) = A(1-e^{-t/T}), \; t \geq 0$$

$$y(\infty) = \lim_{t \to \infty} y(t)$$

$$y(\infty) = A$$



$t = T$

$$y(T) = A(1-e^{-T/T}) = A(1-e^{-1}) = 0.63A$$

"TEMPO DE RESPOSTA"

$X(s) \longrightarrow$ RAMPA $\longrightarrow \mathcal{L}\{\text{rampa}\} = 1/s^2$

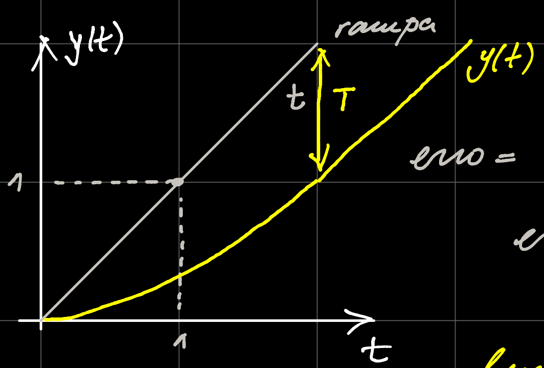
UNITÁRIA

$$y(s) = \frac{1}{s^2(Ts+1)} = \frac{1/T}{s^2(s+1/T)} =$$

$$= \underbrace{\frac{1}{s^2}}_{} - \underbrace{\frac{T}{s}}_{} + \underbrace{\frac{T}{s+1/T}}_{}$$

$\mathcal{L}^{-1}$

$$\longrightarrow y(t) = t - T + Te^{-t/T}, \quad t \geq 0$$

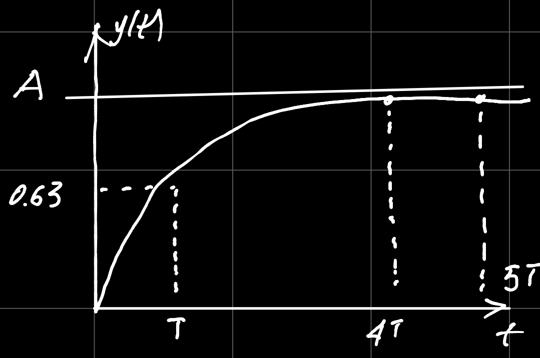


$$\text{erro} = t - t + T - Te^{-t/T}$$

$$\text{erro} = T(1 - \underbrace{e^{-t/T}}_{0})$$

$t \to \infty$

$$\lim_{t\to\infty} \text{erro} = T$$



$$y(t) = A(1 - e^{-t/T})$$

$$y(4T) = A(1 - e^{-4}) \hat{=} 0{,}98A$$

$$y(5T) = A(1 - e^{-5}) \hat{=} 0{,}99A$$

$T = 1s \longrightarrow$ 4s … 98% A

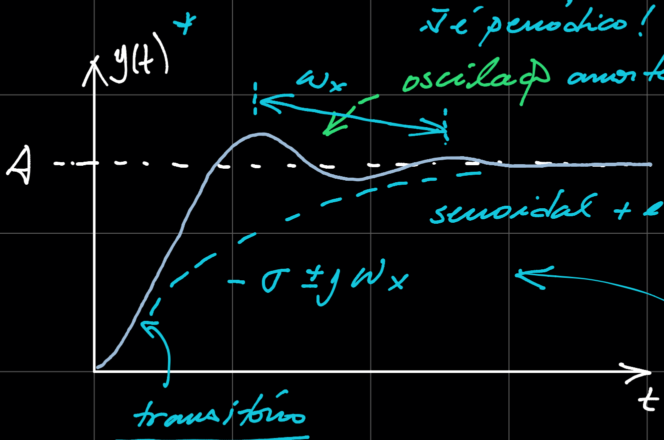
5s … 99% A

PERIÓDICO : HARMÔNICAS : FOURIER

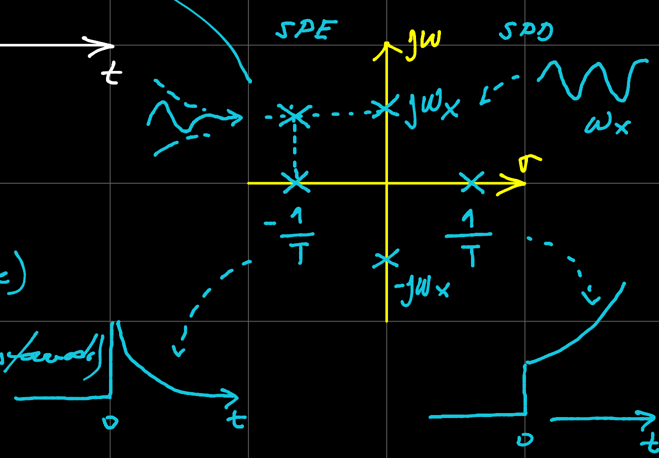
Não é periódico!

oscilação amortecida

ORDEM: Grau do denominador



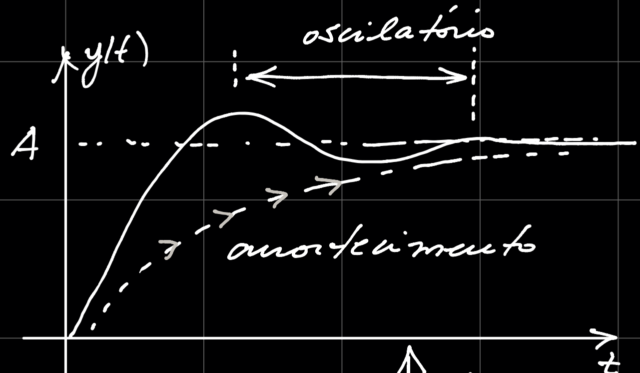
$$y(t) = \underbrace{A\left(1 - e^{-t/T}\right)}_{\text{exponencial}}$$



Laplace : transitório

Fourier : periódico (série)

Regime senoidal



ORDEM mínima : 2

$\mathcal{L}$ $\rightarrow$ $-\sigma \pm j\omega_x$

$\updownarrow \mathcal{L}$

$$X(s) \rightarrow \boxed{\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}} \rightarrow Y(s)$$

Forma canônica

(•) ω_n ... frequência natural não amortecida

(•) ξ ... grau de amortecimento

$$D(s) = s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

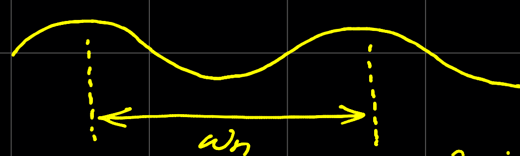
$$\Delta = 4\xi^2\omega_n^2 - 4\omega_n^2 = 4\omega_n^2 \sqrt{\xi^2 - 1}$$

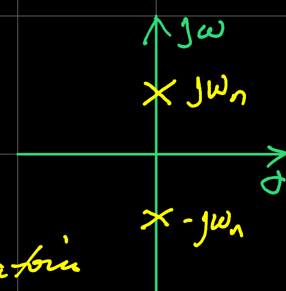
$$P_{1,2} = \frac{-2\xi\omega_n \pm 2\omega_n\sqrt{\xi^2-1}}{2}$$

$$P_{1,2} = \underbrace{-\xi\omega_n}\pm\underbrace{\omega_n\sqrt{\xi^2-1}} \quad \ldots \quad \xi^2 < 1$$

- $\xi = 0$ ... amortecimento ser nulo.

$$P_{1.2} = 0 \pm \omega_n\sqrt{-1} = \pm j\omega_n$$



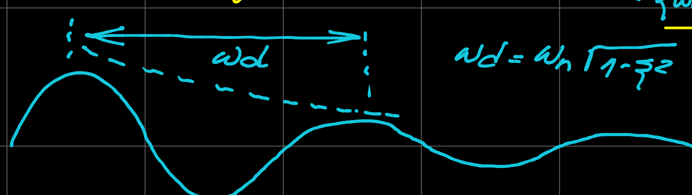


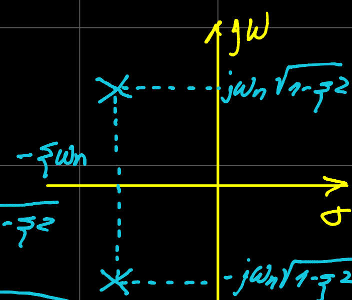
Oscilatório Puro

$P_{1,2} = \underbrace{-\xi\omega_n}\pm\underbrace{\omega_n\sqrt{\xi^2-1}}$ $\quad \xi^2 - 1 < 0$

- $0 < \xi < 1 \longrightarrow$ subamortecido

$$P_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

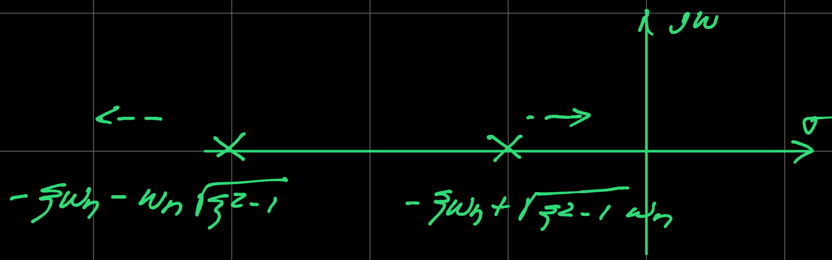




- $\underbrace{\xi = 1} \rightarrow$ amortecimento crítico

$$P_{1,2} = \underbrace{-\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}} = -\xi\omega_n$$

• $\underbrace{\xi > 1}$ $\longrightarrow$ superamortecido



$$G(s) = \frac{1}{Ts + 1}$$ ... $T \rightarrow$ constante de tempo

1ª ORDEM $T \rightarrow$ 63% valor final $\rightarrow$ degrau

$\underbrace{\begin{matrix}\delta(t)\\ e(t)\\ r(t)\end{matrix}}_{T}$

$$G(s) = \frac{1}{3600s + 1} \longrightarrow$$ $T = 3600\,s$ (1 hora)

$p = 1/3600$ (ORIGEM)

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

ω_n ... frequência natural

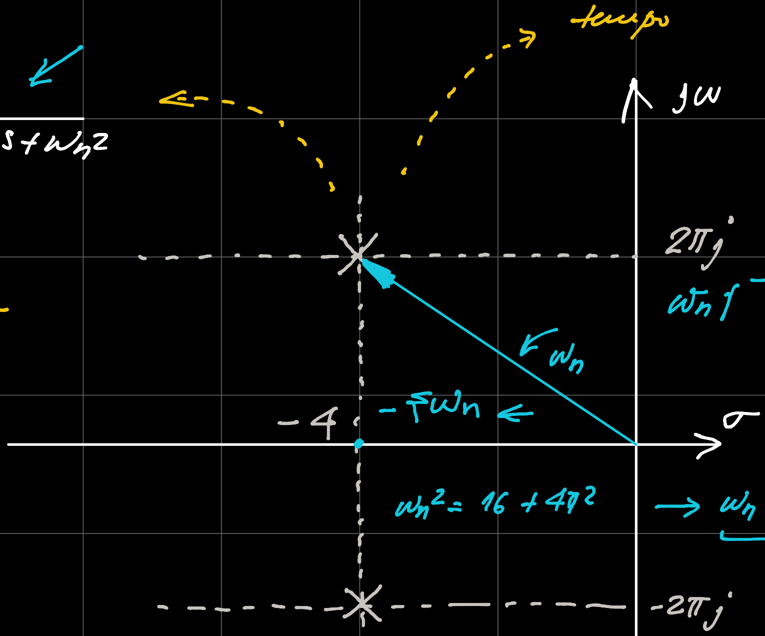
ξ ... amortecimento

$$\frac{\omega_n^2}{\underbrace{s^2+2\zeta\omega_n s}_{4}+\omega_n^2}$$

Pólos:

$$-\zeta\omega_n \pm \omega_n j\sqrt{1-\zeta^2}$$

$0<\zeta<1 \rightarrow$ SUBAMORTECIDO



$$\omega_n^2 = 16+4\pi^2 \rightarrow \underbrace{\omega_n=\sqrt{16+4\pi^2}}$$

$$-\zeta\omega_n=-4$$

$$\begin{cases}\zeta\omega_n=4 \leftarrow \\ \omega_n\sqrt{1-\zeta^2}=2\pi\end{cases}$$

$$\left[(\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)\right]^{1/2}$$

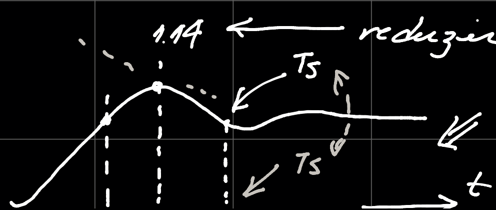
$$(\zeta\omega_n)^2+\omega_n^2-\cancel{(\zeta\omega_n)^2}=\left[\omega_n^2\right]^{1/2}=\omega_n$$

$$G(s)=\frac{16+4\pi^2}{s^2+8s+16+4\pi^2} \longrightarrow \text{(TEMPO)}$$

ω_n^2 ; ω_n^2

$$G(s)=\frac{55.48}{s^2+8s+55.48} \xrightarrow[\mathcal{L}^{-1}]{\text{tempo}}$$

(0)



t_r 0.5 0.776 $\uparrow J(\cdot)$

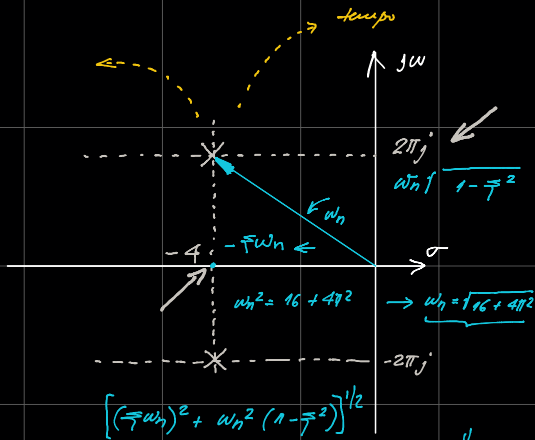
t_p $\uparrow$ manter

- MANTER O T_s $\quad \xi\omega_n \to \underline{cte}$
- REDUZIR O MÁXIMO $(\xi)\uparrow$

$$\underbrace{e^{-\xi\omega_n t}}$$

$$e^{-\xi\omega_n \cdot T_s} < 0.02$$

$$\underbrace{T_s}_{cte} \simeq \frac{4}{\xi\omega_n} \Leftarrow \underline{cte}$$



$$\left[(\xi\omega_n)^2 + \omega_n^2(1-\xi^2)\right]^{1/2}$$

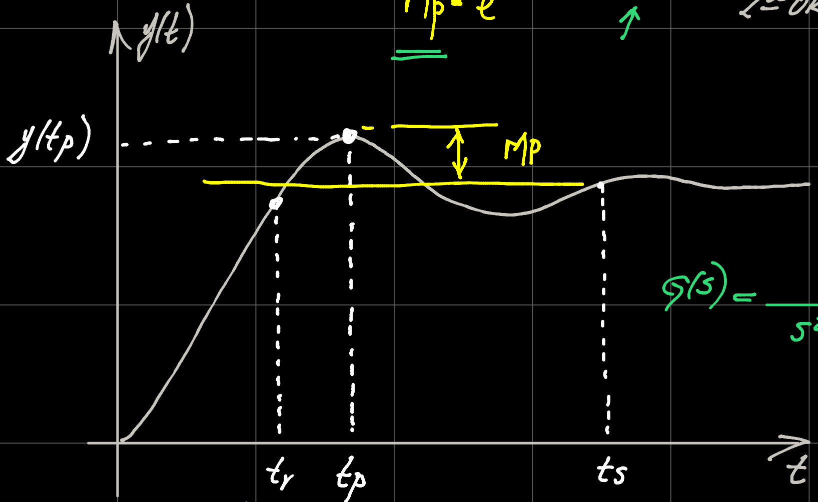
$$(\xi\omega_n)^2 + \omega_n^2 - (\xi\omega_n)^2 = \left[\omega_n^2\right]^{1/2} = \omega_n$$

PAG. 76

$$\underline{M_p} = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

2ª ORDEM

$0 < \xi < 1$



$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

t_s:

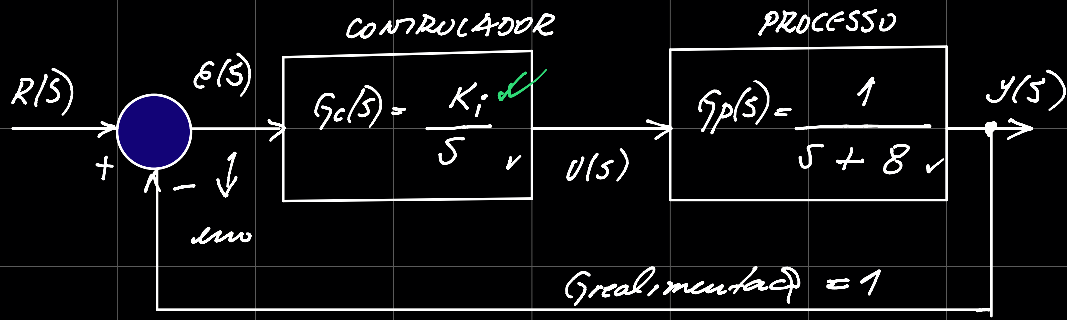
2% $\longrightarrow 4/\xi\omega_n$

5% $\longrightarrow 3/\xi\omega_n$

$$t_r = \frac{\pi - \beta}{\omega_d}$$

$\beta(\xi, \omega_n)$

$t_p \to \dfrac{\pi}{\omega_d}$, $\quad \omega_d = \omega_n\sqrt{1-\xi^2}$



$$G_{MF}(s) = \frac{G_{ma}(s)}{1 + G_{ma}(s)}$$ malha fechada

$$G_{ma}(s) = G_c(s) \cdot G_p(s) = \frac{K_i \cdot 1}{s\,(s+8)}$$

$$G_{MF}(s) = \frac{\dfrac{K_i}{s(s+8)}}{1 + \dfrac{K_i}{s(s+8)}} = \frac{K_i}{s(s+8) + K_i}$$

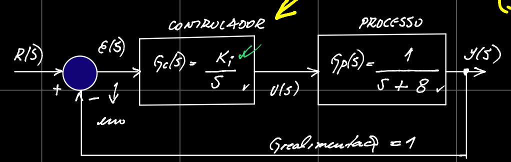
$$G_{MF}(s) = \frac{K_i}{\underbrace{s^2 + 8s + K_i}} \leftarrow \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$\xi\omega_n = 4$ $\quad$ $K_i = \omega_n^2$

ξ

Escolhendo : $K_i = 55$ (sw)

$$\Longrightarrow G_{MF}(s) = \frac{55}{s^2 + 8s + 55}$$ $\leftarrow$ 1ª curva

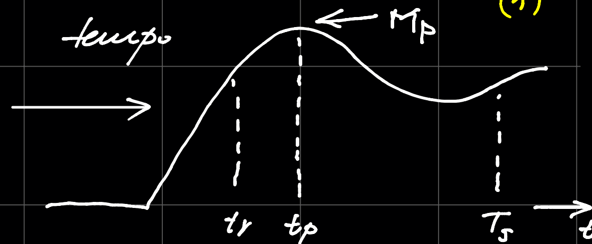


(3)

$$GMF(S) = \frac{Gma(S)}{1 + Gma(S)}$$

malha fechada

sistema em malha fechada



(1)

$\mathcal{L} \downarrow \xi, \omega_n$

PROJETO

desejado ✓ (2)

$$\underbrace{\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}}_{\text{modelo de referência}}$$

$$\frac{K_i}{s^2 + 2\xi\omega_n s + K_i}$$

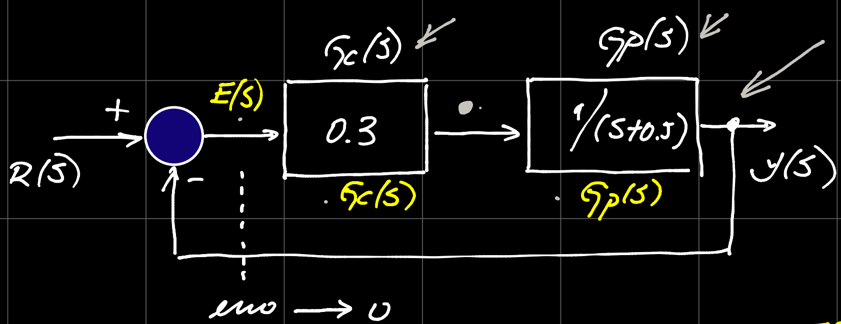
construído

(4)

Comparação

(5)

ERRO ESTACIONÁRIO   PAG. 05



$$\frac{E(S)}{R(S)} = \; ?$$

$$E(S) = R(S) - y(S)$$

$$y(S) = \underbrace{Gc(S) \cdot E(S)} \cdot Gp(S) \implies E(S)\left(1 + Gc(S)Gp(S)\right) = R(S)$$

$$E(S) = R(S) - Gc(S)\, Gp(S) \cdot E(S)$$

$$\underbrace{\frac{E(s)}{R(s)}} = \frac{1}{\underbrace{1 + G_c(s)\,G_p(s)}}$$

$$G_c(s) = 0.3$$

$$G_p(s) = \frac{1}{s+0.5}$$

$$R(s) = \frac{1}{s}$$

$$E(s) = \frac{1}{1 + 0.3 \times \frac{1}{s+0.5}} \cdot \frac{1}{s} = \frac{1}{s + \frac{0.3\,s}{s+0.5}}$$

$\longrightarrow$ erro estacionário $t \to \infty$

$$\underbrace{\lim_{t\to\infty}} \text{erro}(t) \longrightarrow \underbrace{\lim_{s\to 0}} s \cdot E(s)$$ teorema do valor final

$$\lim_{s\to 0} \frac{s}{s + \frac{0.3\,s}{s+0.5}} = \lim_{s\to 0} \frac{1}{1 + \frac{0.3}{\underset{\uparrow 0}{s}+0.5}}$$

$$\underbrace{\text{erro}(\infty)}_{\text{estacionário}} = \frac{1}{1 + 0.3/0.5} \longrightarrow$$

$$\rightarrow G_c(s) = \frac{55.48}{s} \qquad G_p(s) = \frac{1}{s+8}$$

$$\lim_{s\to 0} \cancel{s} \cdot \frac{1}{1+\underbrace{\frac{55.48}{s(s+8)}}_{0}} \cdot \frac{1}{\cancel{s}} = \frac{1}{\infty} \rightarrow 0$$

$$\underbrace{\text{erro}(\infty)} = 0 \xrightarrow{\text{tempo}} \text{ok!} \leftarrow$$

$$G_c(s) = 0.3 \qquad G_p(s) = \frac{1}{s+0.5} \leftarrow$$

$$\text{erro}(\infty) = 0.625 \rightarrow \text{OK!}$$

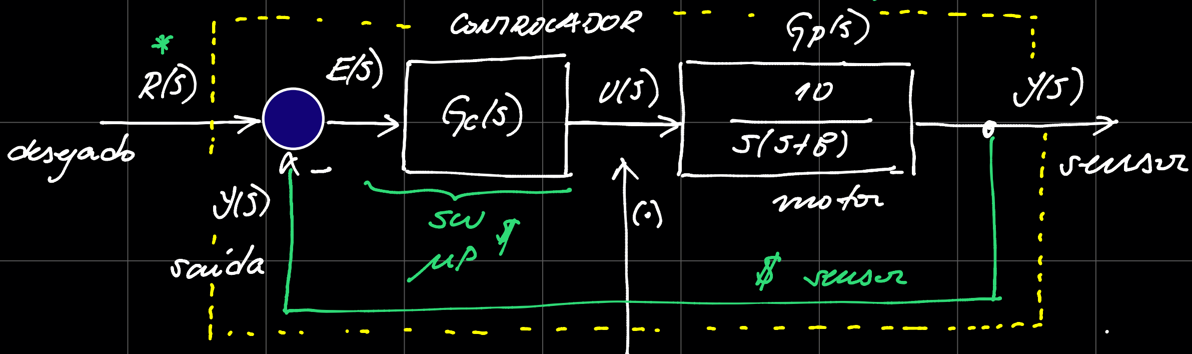
PROJETO DE CONTROLADOR

uma introdução

1º MODELO $\longrightarrow G_p(s)$

$$G_p(s) = \frac{10}{s(s+8)}$$

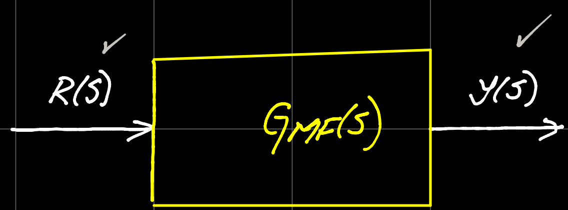
2º ESTRATÉGIA : MALHA FECHADA



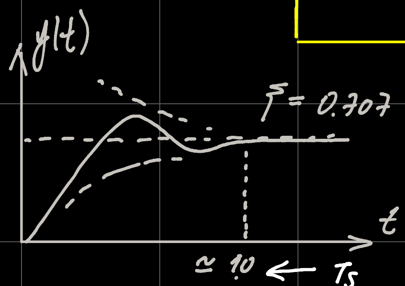
ESFORÇO DE CONTROLE

limite <u>tensão</u> (max)

3º DESEMPENHO



r(t)... degrau unitário



$\xi = 0.707$ amortecimento

erro$(\infty) = 0$

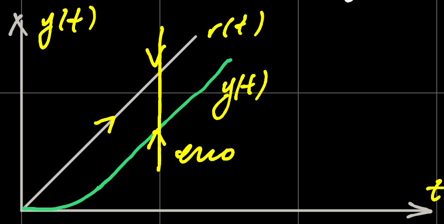
· LIMITES

· RESTRIÇÕES

↑

CONTROLADOR

r(t)... rampa unitário



$y(t) = t$

erro$(\infty) = 0.1$

⇓

PROCESSO CONTROLADOR

$$G_p(S) = \underbrace{\frac{10}{S(S+8)}}_{HW} \longrightarrow G_c(S) = \frac{K_c(S+Z_c)}{(S+P_c)}$$

SW

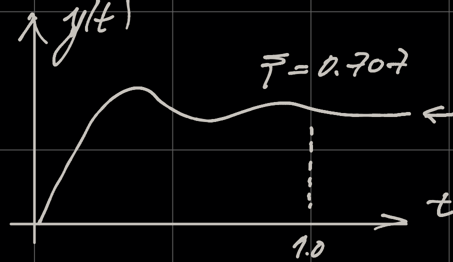
Desempenho

$G_p'(S) \leftrightarrow (?)$

Desempenho

$e(\infty) = 0$
degrau

$e(\infty) = 0.1$
rampa



← degraus

(4º) Escolha do controlador

Qual o controlador mais simples que atende ao desempenho?

$$G_c(S) = K_p$$

(5º) Sintonia do controlador : (K_p)

$E(S) = R(S) - Y(S)$ ... $G_p(S) = \dfrac{10}{S(S+8)} = \dfrac{Y(S)}{U(S)}$ $\quad \underbrace{G_c(S)} = K_p = \dfrac{U(S)}{E(S)}$

$$G_{ma}(S) = \frac{10K_p}{S^2+8S} = G_c(S) \cdot G_p(S)$$

$$G_{mf}(S) = \frac{Y(S)}{R(S)} = \frac{G_{ma}(S)}{1+G_{ma}(S)} =$$

realimentação unitária

$$\frac{Y(S)}{R(S)} = G_{MF}(S) = \frac{\frac{10K_p}{S^2+8S}}{1+\frac{10K_p}{S^2+8S}} = \underbrace{\frac{10K_p}{S^2+8S+10K_p}}_{\text{Processo + Controlador em MF}}$$

$G_{MF}(S) \equiv$ DESEMPENHO

$\mathcal{L}$ tempo

$\downarrow$

$\mathcal{L}$

$T_S = 1.0 \xrightarrow{\mathcal{L}}$ FORMA CANÔNICA

$\xi = 0.707 \xrightarrow{\mathcal{L}}$

$$\frac{\omega_n^2}{S^2+\underline{2\xi\omega_n}S+\omega_n^2}$$

$$T_S = \frac{4}{\xi\omega_n} \therefore \xi\omega_n = 4$$

2%

$$\downarrow$$

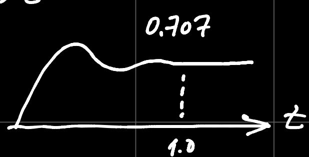
$$\frac{\omega_n^2}{S^2+8S+\omega_n^2} \quad \text{Desempenho}$$

$$\omega_n = \frac{4}{0.707} = 5.66$$

$$\omega_n^2 \simeq 32$$

$$\downarrow$$

$$\frac{32}{S^2+8S+32} = \frac{Y(S)}{R(S)} \text{ Des}$$



$\leftarrow \uparrow$ $\mathcal{L}$

$\updownarrow$! $= S$ $K_p = 3.2$

$\longrightarrow K_p = 3.2$

$$G_{MF}(S) = \frac{10K_p}{S^2+8S+10K_p} = \frac{Y(S)}{R(S)} \text{ malha}$$

$\longrightarrow K_p = 3.2 \longrightarrow$ ajusta a dinâmica do sistema.

$$G_{ma}(s) = \frac{\overset{SW}{\xrightarrow{K_p} 3.2 \cdot 10}}{s(s+8)} = \frac{32}{s(s+8)}$$

$$E(s) = \frac{1}{1+G_{ma}(s)} \cdot R(s)$$

Degrau $$E_D(s) = \frac{1}{1+\frac{32}{s(s+8)}} \cdot \frac{1}{s} = \frac{1}{s + \frac{32}{(s+8)}}$$

Rampa $$E_r(s) = \frac{1}{1+\frac{32}{s(s+8)}} \cdot \frac{1}{s^2} = \frac{1}{s^2 + \frac{32s}{(s+8)}}$$

$$\underbrace{e_D(\infty)}_{0\ (OK)} = \lim_{s\to 0} s \cdot E_D(s) = \lim_{s\to 0} \frac{\cancel{s}^{\to 0}}{\cancel{s}^{\to 0} + \frac{32}{\cancel{s+8}}} = \frac{0}{4} = 0$$

$$e_r(\infty) = \lim_{s\to 0} s\, E_r(s) = \lim_{s\to 0} \frac{s}{s^2 + \frac{32s}{s+8}} =$$

$$\lim_{s\to 0} \frac{\cancel{s}}{s^{\cancel{2}} + \frac{32\cancel{s}}{s+8}} = \lim \frac{1}{\underset{0}{s} + \frac{32}{\underset{0}{s}+8}} = \frac{1}{32/8} = \frac{1}{4} = 0.25$$

$e_r(\infty) = 0.25$ $\quad$ 25% (10%)

$>$ Ñ atende!

8 ... processo ... HW

32 ... 10 . Kp

$$\frac{1}{10K_p/8} \leq 0.1 \qquad \frac{8}{10K_p} = 0.1 \qquad 0.8 = 0.1K_p$$

$$\therefore\ K_p = \frac{0.8}{0.1} = 8 \uparrow \qquad (3.2) \qquad T_s = \frac{4}{\xi\omega_n}$$

cte

cte

$G_c(s) = K_p$

$$\boxed{e(\infty)_r = 0.1}$$

$$\omega_n^2 = 80$$

$$\omega_n = 8.9 \text{ rd/s}$$

$$G_c(s) = \frac{K_c(s+z_c)}{(s+p_c)} \qquad G_{MF}(s) = \frac{80}{s^2+8s+80} \qquad 2\omega_n\xi = 8$$

$\underbrace{K_c, z_c, p_c}$

$$0.707 \longrightarrow \xi = 4/8.9 = 0.45$$

$g_i(t)$ $g_o(t)$

$$(1)\quad \dot{h} = 2e(t) - \underbrace{4\sqrt{h(t)}}_{\bar{n}\text{ linear}} \leftarrow \text{MODELO} \rightarrow \mathcal{L}$$

$\bar{n}$ linear $\rightarrow$ lineares : $h_0 = 10m$

$$(2)\quad h_0 = 10 \rightarrow \underbrace{\dot{h} = 0}\quad \underbrace{2e_0 = 4\sqrt{10}} \rightarrow e_0 = 2\sqrt{10}$$

$h_0 = 10m$

$$\text{raiz}\quad \sqrt{h(t)} \doteq \sqrt{h_0} + \frac{1}{2\sqrt{h_0}} \cdot (h - h_0) \quad \text{reta}$$

(II)

$$f(x) \doteq \underbrace{f(x_0)} + f'(x)\Big|_{x = x_0} \cdot \Delta x \ \ldots \quad \text{Taylor}$$

$$h^{1/2} \to \tfrac{1}{2} h^{-1/2}$$

(II) em (I)

$$\dot{h} = 2e(t) - 4\sqrt{h_0} - \underbrace{\frac{4\sqrt{h_0}}{2\sqrt{h_0}}}(h - h_0)$$

$$\bar{n}\text{ linear}\quad \sqrt{h(t)} = a_0 + a_1 h(t) + \underbrace{a_2 h^2(t) + a_3 h^3(t) + \ldots}_{\ldots}$$

$\bar{n}$ linear $\quad$ $\bar{n}$ linear

$$\sqrt{h(t)} \doteq \underbrace{a_0 + a_1 h(t)}$$

$$\dot{h} = 2e(t) - \underbrace{4\sqrt{h_0}}_{2e_0} - 2(h - h_0) \qquad \Delta\dot{h} = \dot{h}$$

$$\Delta h = h - h_0$$

$$\dot{h} = 2(e(t) - e_0) - 2(h(t) - h_0) =$$

$$\underbrace{\Delta\dot{h}(t)} = 2\,\Delta e(t) - 2\underbrace{\Delta h(t)} \quad \text{em torno de } 10m$$

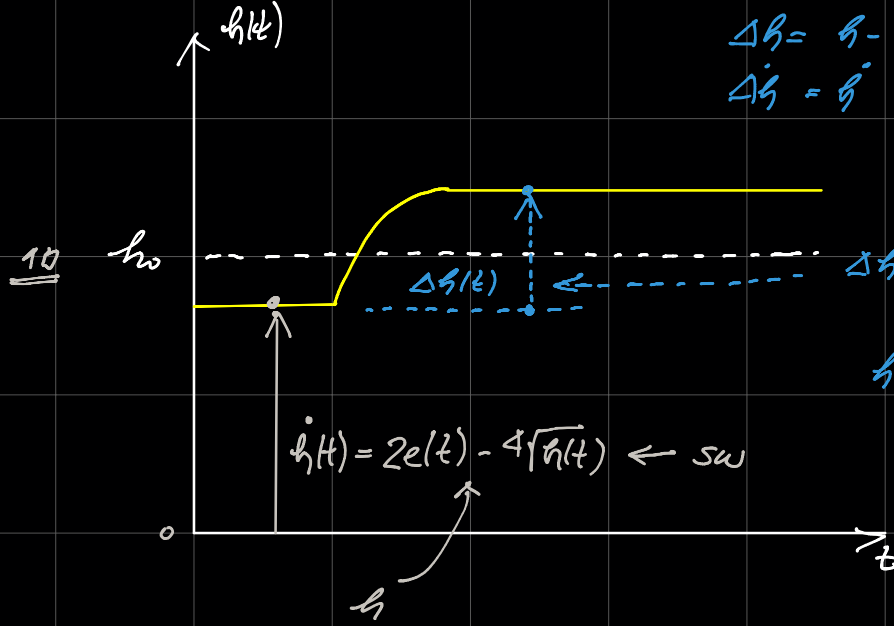
h_0

modelo de desvios

$$\dot{h}(t) = 2e(t) - 4\sqrt{h(t)} \quad \text{modelo absoluto}$$

$h(t)$

$$\mathcal{L}\quad s \cdot \Delta H(s) = 2\Delta E(s) - 2\Delta H(s)$$



$\Delta h = h - h_0$

$\Delta \dot h = \dot h$

$\Delta \dot h(t) = 2\Delta e(t) - 2\Delta h(t)$

$h(t) = \Delta h(t) + h_0$

$\Delta h = 1m$

$h = 11$

$\Delta h = -2$

$h = 8$

$$\underbrace{\Delta \dot h} = 2\Delta e(t) - 2\Delta h(t)$$

$$\mathcal{L}: s \cdot \Delta H(s) = 2\Delta E(s) - 2\Delta H(s)$$

$$\frac{\Delta H(s)}{\Delta E(s)} = \frac{2}{s+2} = G_p(s) \quad \dots \quad 1^{a} \text{ ordem}$$

$$G_p(s) = \frac{2}{2(0.5s+1)}$$

$$s\Delta H(s) + 2\Delta H(s) = 2\Delta E(s)$$

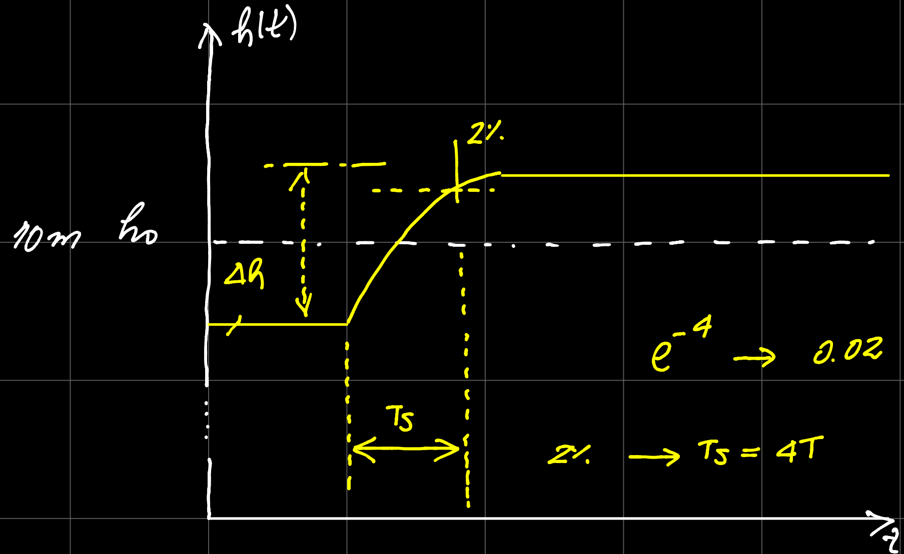
$$(s+2)\Delta H(s) = 2\Delta E(s)$$

$$G_p(s) = \frac{1}{0.5s+1}$$

T

→ $T = 0.5$ segundo

$T_s(2\%) \cong 2$ segundos



$\simeq 2seg$

$e^{-4} \rightarrow 0.02 \quad (2\%)$

$2\% \rightarrow T_S = 4T$

$$\frac{2}{s+2} \longrightarrow \frac{2}{2(0.5 \cdot s+1)} = \frac{1}{(0.5s+1)} \qquad T = 0.5s$$

$\uparrow$ T (0.63) $\qquad$ $\downarrow$ $4T \simeq 2seg.$

$\longrightarrow$ $2T \rightarrow 89\%$. $\quad$ $4T$ 98% $\quad$ T (0.63)

$3T \rightarrow 95\%$. $\quad$ $5T$ 99%

$$G_p(s) = \frac{2}{s+2} \qquad G_c(s) = \frac{K_i}{s}$$

$$G_{ma}(s) = \frac{2K_i}{s\,(s+2)}$$

$$G_{mf}(s) = \frac{G_{ma}(s)}{1 + G_{ma}(s)} = \frac{\dfrac{2K_i}{s^2+2s}}{1 + \dfrac{2K_i}{s^2+2s}}$$

realimentação unitária

$$\longrightarrow G_{mf}(s) = \frac{2K_i}{s^2+2s+2K_i} \longleftarrow \text{resposta}$$

$K_i > 0$

$$G_{mf}(s) = \frac{2K_i}{s^2 + 2s + 2K_i} \leftarrow (\cdot)$$

- T_s em 20% $\rightarrow$ $T_s = 4/\xi\omega_n$
- $e(\infty)$ rampa 0.1 ou 10% $\Leftarrow (\cdot\cdot)$
- $\xi = 1$ (crítico) ... reais!

$$G_D(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{3}{s^2 + as + c}$$

Forma canônica

$\rightarrow \xi, \omega_n$: $\xi = 1$ $\rightarrow$ $T_s = 4/\xi\omega_n$ $\rightarrow$ $\xi\omega_n = \frac{4}{1.6}$

$T_{s_{ma}} = 2s$ $\rightarrow$ $T_{s_{mf}} = 1.6s$ $\quad \underbrace{\xi\omega_n}_{\xi=1} = 1/0.4 = 10/4 = \underbrace{2.5}$

$\therefore \omega_n = 2.5$

Forma canônica

$$\frac{\omega_n^2 \swarrow 2.5^2}{\underbrace{s^2 + 2\xi\omega_n s + \omega_n^2}_{=0}}$$

$5 \qquad 2.5^2$

$\xi > 1$ •

$P_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$

$\xi = 1$ •

$P_{1,2} = -\xi\omega_n$ (DUPLO)

$\xi < 1$ •

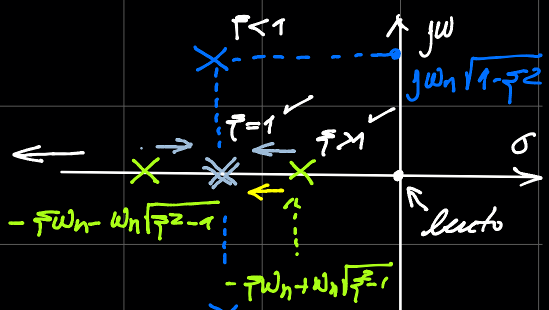
$P_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$

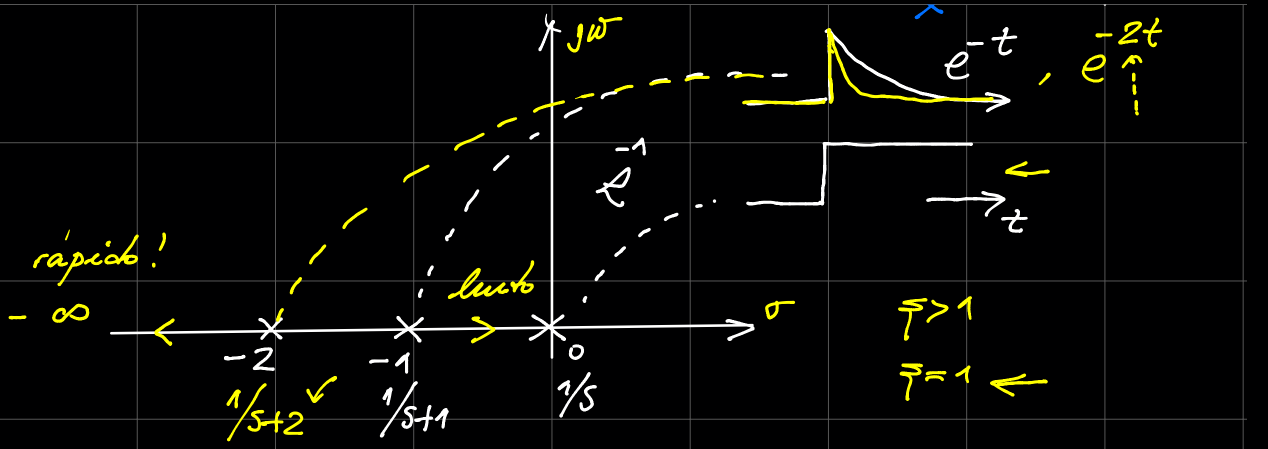
Pólos :

$$\Delta = 4(\xi\omega_n)^2 - 4\omega_n^2 = 4\omega_n^2(\xi^2 - 1)$$

$$P_{1,2} = -\frac{2\xi\omega_n \pm \sqrt{4\omega_n^2(\xi^2-1)}}{2}$$

$$P_{1,2} = -\xi\omega_n \pm \underbrace{\omega_n\sqrt{\xi^2 - 1}}$$





$$\underbrace{G_{mf}(s) = \frac{2K_i \;\; \checkmark\, \omega_n^2}{s^2 + 2s + 2K_i}}_{\text{sistema "real"}} = \underbrace{G_D(s) = \frac{2.5^2}{s^2+5s+2.5^2}}_{\text{desempeño}}$$

$$\underbrace{2K_i = 2.5^2} \longrightarrow K_i = \_\_\_\_\_\_$$

$\Downarrow$

$2\zeta\omega_n = 2$

$\zeta\omega_n = 1$

$T_s = \dfrac{4}{\zeta\omega_n} = \underline{\underline{4s}}$

"real"

$\zeta = 1 \ldots$

$\Downarrow$

$2\zeta\omega_n = 5$

$\zeta\omega_n = 2.5$

$T_s = \dfrac{4}{2.5} = \underline{\underline{1.6\,sg}}$

"deseo"

$G_c(s) = K_i/s$ ✗ $\longrightarrow$ CONTROLADOR $G_c(s) = \underline{\underline{K_p}} + \overline{K_i}/s$

CONTROLE

MODELO
$\downarrow$
$G_p(s)$
$\downarrow$
$G_c(s)$
$\downarrow$
$G_{ma}(s)$
$\downarrow$
$G_{mf}(s)_R$

Desempenho
$\downarrow$
PÓLOS
$\downarrow$
$G_{mf}(s)_D$

OK $\xrightarrow{N}$ modifica o controlador (SW $\updownarrow$)
$\downarrow$ S
$e(\infty)$

modifica o controlador
$\downarrow$
modifica o processo HW