0.0.1 Research Area

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The geometrical approach is currently becoming more and more popular and useful increasingly replacing more traditional analytic considerations in various areas of mathematics and leading to creation of new mathematical disciplines (geometric group theory, non-commutative geometry etc.). Dynamics is inherently present in this approach both in the form of the usual one-dimensional time evolution (deterministic as well as stochastic) and in the form of "non-commutative dynamics" associated with various symmetries of considered systems.

The subject area of what may be called "Geometry and Dynamics" intersects with numerous mathematical disciplines. More concretely, my active research interests include the following topics:

- Hyperbolic geometry
- Semi-simple Lie groups, their lattices and symmetric spaces
- Rigidity of groups and actions
- Geometric and combinatorial group theory
- Functional analysis on groups and groupoids, amenability
- C*-algebras and non-commutative geometry
- Spectral theory of groups, graphs and manifolds
- Foliations, laminations and equivalence relations
- Geometry of Teichmüller and outer spaces
- Dynamics on groups and manifolds; geometric flows
- Geometrical aspects of conformal dynamics
- Probabilistic methods on algebraic and geometric structures
- Asymptotic theory of Markov chains
- Stochastic Loewner evolution and conformal field theory
- Theory of cocycles; cohomology of groups and varieties
- Ergodic theory

Highlights This year my research was mostly concentrated around understanding singularity of harmonic measures arising from random walks on hyperbolic spaces.

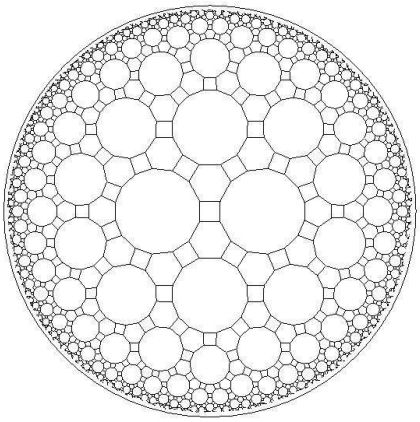

Figure 1: A typical Cayley graph of a hyperbolic group

The question whether the harmonic measure has maximal dimension has a long history in various areas (classical harmonic analysis, conformal dynamics, periodic negatively curved manifolds, continuous fractions). It is notoriously difficult, but all known answers indicate that the maximal dimension of the harmonic measure should imply that the considered system is distinguished by much higher symmetries within a given class.

One can conjecture that a finitely supported random walk on a hyperbolic group may have maximal entropy if and only if the group is a finite extension of a free group. Note that if one is allowed to consider not necessarily finitely supported measures, then re-

cent results show existence of maximal entropy random walks on an arbitrary hyperbolic group.

The "difficult" part of the above conjecture is showing that existence of a maximal entropy random walk implies that the group is a finite extension of a free group. However, in spite of a complete structure theory for finite extensions of free groups, we still do not know whether any such group carries a finitely supported maximal entropy random walk.

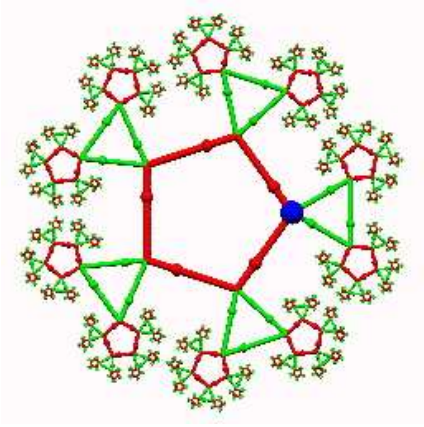

Figure 2: The free product of cyclic groups of orders 3 and 5

We look at the particular case when the group is the free product of finitely many finite groups. The main result is that, generically, in this situation no random walk has maximal entropy. The proof is based on the fact that the harmonic measure in this situation is *multiplicative Markov* (Mairesse and Matheus). Since Patterson (≡ maximal entropy) measure on the boundary is also multiplicative Markov, the standard symbolic dynamics tools (uniqueness of the maximal entropy measure for topological Markov chains) become much easier to apply.

Organization

1. Organizer of the Spring School in Probability at the Schrödinger Institute (ESI), Vienna, Austria, 2006.

2. Organizer of the special semester *Amenability* at the Schrödinger Institute (ESI), Vienna, Austria, 2007.

Collaborations

1. *State Univeristy of New York at Stony Brook, USA, University of Toronto, Canada*
   Prof. Mikhail Lyubich
   Measures in conformal dynamics

2. *Univeristy of Illinois at Urbana-Champaign, USA*
   Prof. Ilya Kapovich, Prof. Paul Schupp
   Metric properties of free group automorphisms

3. *Texas A& M University, USA*
   Prof. Rostislav Grigorchuk, Prof. Volodymir Nekrashevych
   Ergodic methods in group theory

4. *Université de Geneve, Switzerland*
   Prof. Tatiana Nagnibeda
   Ergodic properties of boundary actions

5. *ETH Zürich, Switzerland*
   Prof. Marc Burger, graduate student Theo Bühler
   Amenability of groupoids

6. *Schrödinger Institute, Vienna, Austria*
   Prof. Klaus Schmidt
   Ergodic properties of cocycles

7. *Université de Vannes, France*
   Dr. Vincent Le Prince
   Singularity of the harmonic measure

8. *University of Lisbon, Portugal*
   Dr. Pedro Freitas
   Compactifications of symmetric spaces

9. *Université Paris-7, France*
   Dr. Jean Mairesse
   Queuing systems and harmonic measures

Awards, Prizes

1. European Union, Marie Curie Research Training Network FP6: Conformal structures and dynamics (CODY), network of 10 European countries, member of the German node; total funding volume: 2.7 Mio Euro.