Chi Square Test

THE CHI-SQUARE STATISTIC

The chi-square statistic measures the amount of discrepancy between observed frequencies and the frequencies that would be expected due to chance, or random sampling error. The formula for chi-square is

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

where: f_o = observed frequencies
f_e = expected frequencies

- ***Observed frequencies are the actual frequencies from our sample that fall into each category.***
- ***Expected frequencies are the frequency values that would be expected if the null hypothesis is true.***
- ***If we find no differences between our observed and expected frequencies, then our obtained $\chi 2$ value will equal 0.***
- ***But if our observed frequencies differ from those that would be expected, then $\chi 2$ will be greater than 0.***

GOODNESS OF FIT FOR KNOWN PROPORTIONS

Sample Research Question

A new professor at a midsize college wanted to see if her grade distribution after her first year of teaching was comparable to the overall college grade distribution, which has the following percentages: A – 10%; B – 22%; C – 40%; D – 21%; and F – 7%. The distribution of the new professor's grades for 323 students at the end of her first year was as follows: 38 students received As, 78 received Bs, 139 received Cs, 55 received Ds, and 13 received Fs. Does the new professor's grade distribution fit the overall college's distribution? Test, using $\alpha = .05$.

Step 1: Formulate Hypotheses

We are using the proportions/percentages from a known population distribution for our comparison. Thus, our hypotheses would be as follows:

H_0: The distribution of grades for the new professor fits the overall grade distribution of the college.
H_1: The distribution of grades for the new professor does not fit the overall grade distribution of the college.

Step 2: Indicate the Alpha Level and Determine Critical Values

After we calculate our χ^2 crit value in the next step, we will need to compare it to a critical χ^2 value. The chi-square distribution is in Table 8 near the end of this book. Various alpha levels are listed across the top and *df* are shown along the left column.

For goodness of fit, $df = \kappa - 1$, where κ refers to the number of categories.

Our problem has 5 categories (A, B, C, D, and F). Thus,

$$\alpha = .05$$

$$df = k - 1 = 5 - 1 = 4$$

$$\chi^2_{\text{crit}} = 9.488$$

Step 3: Calculate Relevant Statistics

In order to use the chi-square formula presented earlier, observed and expected frequencies are required. Observed frequencies are the actual frequencies obtained from our sample, which are given in the problem. For research questions dealing with a known population distribution, expected frequency is determined by multiplying the known proportion in the population by *n*. In other words,

$$f_e = (\text{known proportion})(n)$$

The null hypothesis specifies that the new professor's grades will not differ significantly from the proportion found in the population. We know that 10% of the college population earned As, 22% earned Bs, and so forth. If we multiply those known proportions by our sample size, then we can determine what frequencies would be expected if H_0 were true. Thus,

$$A = .10 \times 323 = 32.30$$

$$B = .22 \times 323 = 71.06$$

$$C = .40 \times 323 = 129.20$$

$$D = .21 \times 323 = 67.83$$

$$F = .07 \times 323 = 22.61$$

$$\chi^2_{\text{obt}} = \sum \frac{(f_o - f_e)^2}{f_e}$$

$$= \frac{(38-32.3)^2}{32.3} + \frac{(78-71.06)^2}{71.06} + \frac{(139-129.2)^2}{129.2} + \frac{(55-67.83)^2}{67.83} + \frac{(13-22.61)^2}{22.61}$$

$$= 1.01 + .68 + .74 + 2.43 + 4.08$$

$$= 8.94$$

Step 4: Make a Decision and Report the Results

Our obtained χ^2 value is smaller than χ^2 crit. Consequently, we will not reject H_0. We will use the following format for our conclusion. The only new element is the sample size. This is important in chi-square because *df* is based on number of categories rather than sample size. Thus, *n* cannot be determined from *df* and should therefore be reported in the results:

The new professor's grades did not differ significantly from those of the college at large. Fail to reject H_0, χ^2 (4 *df*, $n = 323$) = 8.94, $p > .05$.