Confidence Interval Versus Point Estimation

UNIT 3

- A ***point estimate is a single value, based on sample data, that is used to estimate the parameter of a population.***

- A ***confidence interval is a range of values that, with a specified degree of confidence, is expected to include a population parameter.***

POINT ESTIMATES

For point estimates of population parameters, the obtained sample values – with no adjustments for standard error – are reported (i.e., the numerator in our working formula for the t-statistic).

- For one-sample *t* tests, the point estimate for μ is M.
- For two-sample *t* tests, independent measures design, the point estimate for $\mu_1 - \mu_2$ is $M_1 - M_2$.
- For two-sample *t* tests, repeated measures design, the point estimate for μ_D is M_D.

CONFIDENCE INTERVALS

- LL = obtained sample statistic − (*t)(estimated standard error)*
- *UL = obtained sample statistic + (t)(*estimated standard error)

- Here, we are establishing lower and upper limits as boundaries within which population values are expected to lie.

- For one-sample t tests:

$$\text{LL} = M - t(s_M) \quad \text{and} \quad \text{UL} = M + t(s_M)$$

- For two-sample t tests, independent samples design:

$$\text{LL} = (M_1 - M_2) - t(s_{M_1 - M_2}) \quad \text{and} \quad \text{UL} = (M_1 - M_2) + t(s_{M_1 - M_2})$$

- For two-sample t tests, repeated measures design:

$$\text{LL} = M_D - t(s_{M_D}) \quad \text{and} \quad \text{UL} = M_D + t(s_{M_D})$$

ONE-SAMPLE *t* TEST

Sample Research Question

The average typing speed for the secretaries of a large company is 52 words per minute. A long-time secretary has developed finger dexterity exercises that she reports have improved her speed dramatically. The owner of the company thus hires a researcher to test the effectiveness of the exercise program. After four weeks of training in the finger exercises, the typing speed of 17 secretaries is measured. The mean number of words per minute was $M = 57$ with $SS = 3600$.

A. What would the point estimate of μ be after using the finger exercise program?
B. Establish a 90% confidence interval for the mean and write a sentence of interpretation.

A. The point estimate of μ is $M = 57$.
B. $\text{LL} = M - t(s_M)$ and $\text{UL} = M + t(s_M)$

To use the formulas for establishing confidence intervals, we need values for M, t, and s_M. The value for $M = 57$. For a 90% confidence interval with $df = 16$, the value for $t = \pm 1.746$. We now have to calculate the estimated standard error (s_M).

$$s = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{3600}{17-1}} = 15 \qquad s_M = \frac{s}{\sqrt{n}} = \frac{15}{\sqrt{17}} = 3.64$$

$$\begin{aligned} \text{LL} &= 57 - 1.746(3.64) \\ &= 57 - 6.36 \end{aligned}$$

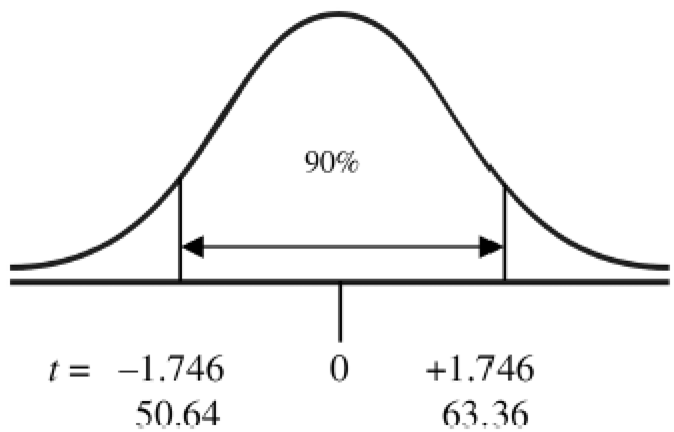
and

$$\begin{aligned} \text{UL} &= 57 + 1.746(3.64) \\ &= 57 + 6.36 \end{aligned}$$

Interpretation

We can be 90% confident that the population mean (after the finger dexterity program) would be between 50.64 and 63.36.

This is what the *t*-distribution would look like for this interval:



The *t*-values of ±1.746 form the boundaries of the middle 90% of the distribution when $df = 16$.

I. Confidence Interval for a One-Sample t Test

Research Question. A history professor was curious about the knowledge of history of graduating seniors in her state. She thus gave a 100-item history test to a random sample of 75 graduating seniors with a resulting $M = 72$ and $SS = 7400$.

A. Make a point estimate of the population mean for that state.

B. Establish a 99% confidence interval for the mean.

C. Write a sentence of interpretation.

I. Confidence Interval for a One-Sample t Test

A. $M = 72$
B. LL = 68.94 and UL = 75.06

We can be 99% confident that the population mean would be between 68.94 and 75.06.

TWO-SAMPLE *t* TEST: INDEPENDENT SAMPLES DESIGN

Sample Research Question

A high school counselor has developed a pamphlet of memory-improving techniques designed to help students in preparing for exams. One group of students is given the pamphlets and instructed to practice the techniques for three weeks, after which their memories are tested. The memories of another group of students, who did not receive the pamphlets, are also tested at this time.

Memory-pamphlet group	*No-pamphlet group*
$n_1 = 36$	$n_2 = 40$
$M_1 = 79$	$M_2 = 68$
$SS_1 = 478$	$SS_2 = 346$

A. Make a point estimate of how much improvement in memory results from following the techniques.
B. Establish a 90% confidence interval around the value for the difference between means and write a sentence of interpretation.

Memory-pamphlet group	*No-pamphlet group*
$n_1 = 36$	$n_2 = 40$
$M_1 = 79$	$M_2 = 68$
$SS_1 = 478$	$SS_2 = 346$

A. Make a point estimate of how much improvement in memory results from following the techniques.
B. Establish a 90% confidence interval around the value for the difference between means and write a sentence of interpretation.

A. The point estimate for the population difference between means $(\mu_1 - \mu_2)$ is: $M_1 - M_2 = 79 - 68 = 11$

B. $\text{LL} = (M_1 - M_2) - t(s_{M_1-M_2})$ and $\text{UL} = (M_1 - M_2) + t(s_{M_1-M_2})$

$$s_{M_1-M_2} = \sqrt{\left(\frac{SS_1 + SS_2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$= \sqrt{\left(\frac{478 + 346}{36 + 40 - 2}\right)\left(\frac{1}{36} + \frac{1}{40}\right)}$$

$$= \sqrt{(11.14)(.05)}$$

$$= .75$$

$$LL = 11 - 1.671(.75) = 11 - 1.25 = 9.75$$

and

$$UL = 11 + 1.671(.75) = 11 + 1.25 = 12.25$$

Interpretation

We can be 90% confident that the difference between population means would be between 9.75 and 12.25.

Research Question. A psychologist is investigating the effects of suggestion on conceptual problem solving. He administers the Concepts Application Test (CAT) to two groups of subjects. Prior to administering the test to one group, the researcher comments that subjects usually report that they enjoy taking the test and that it gives them

a sense of accomplishment. Another group is simply administered the test with no such remarks. The CAT scores for the two groups are presented below:

CAT with remarks	*CAT alone*
49	26
38	35
36	40
42	32
44	34
37	30
47	

a. Conduct a two-tailed t test using $\alpha = .05$ and determine the size of the effect.
b. Make a point estimate of the difference between means.
c. If the results are significant, establish a 95% confidence around the difference value and write a sentence of interpretation.