

# THE $F_\tau(Q; Z) = \overline{\mathcal{E}_\tau(Q+1; Z)}$ FOR VARIOUS SHAPES AND LARGE $n$

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ABSTRACT. Work in Graz June 2023. We take a large  $n$  and compute  $\mathcal{E}_\tau(Q+1; Z)$ , then the large part  $\lambda_1$  in  $e_\lambda$  is removed  $\bar{e}_\lambda = e_{\lambda_2 \dots \lambda_k}$ . Then  $F_\tau(Q; Z)$  is independent of  $n$  and is the *stable form*.

$$F_0(Q; Z) = \mathbf{1} \otimes e_0$$

$$F_1(Q; Z) = s_1 \otimes e_0 + \mathbf{1} \otimes e_1$$

$$F_{11}(Q; Z) = s_2 \otimes e_0 + s_1 \otimes e_1 + \mathbf{1} \otimes e_2$$

$$F_2(Q; Z) = s_2 \otimes e_0 + (\mathbf{1} + s_1) \otimes e_1$$

$$F_{111}(Q; Z) = s_3 \otimes e_0 + s_2 \otimes e_1 + s_1 \otimes e_2 + \mathbf{1} \otimes e_3$$

$$F_{21}(Q; Z) = (s_3 + s_{11}) \otimes e_0 + (s_1 + s_2) \otimes e_1 + \mathbf{1} \otimes e_{11} + s_1 \otimes e_2$$

$$F_3(Q; Z) = s_3 \otimes e_0 + (\mathbf{1} + s_1 + s_2) \otimes e_1$$

$$F_{1111}(Q; Z) = s_4 \otimes e_0 + s_3 \otimes e_1 + s_2 \otimes e_2 + s_1 \otimes e_3 + \mathbf{1} \otimes e_4$$

$$F_{211}(Q; Z) = (s_4 + s_{21}) \otimes e_0 + (s_2 + s_3 + s_{11}) \otimes e_1 + s_1 \otimes e_{11} + s_2 \otimes e_2 + \mathbf{1} \otimes e_{21} + s_1 \otimes e_3$$

$$F_{31}(Q; Z) = (s_4 + s_{21}) \otimes e_0 + (s_1 + s_2 + s_3 + s_{11}) \otimes e_1 + (\mathbf{1} + s_1) \otimes e_{11} + s_2 \otimes e_2$$

$$F_4(Q; Z) = s_4 \otimes e_0 + (\mathbf{1} + s_1 + s_2 + s_3) \otimes e_1$$

$$F_{11111}(Q; Z) = s_5 \otimes e_0 + s_4 \otimes e_1 + s_3 \otimes e_2 + s_2 \otimes e_3 + s_1 \otimes e_4 + \mathbf{1} \otimes e_5$$

$$F_{2111}(Q; Z) = (s_5 + s_{31}) \otimes e_0 + (s_3 + s_4 + s_{21}) \otimes e_1 + s_2 \otimes e_{11} + (s_3 + s_{11}) \otimes e_2 + s_1 \otimes e_{21} + s_2 \otimes e_3 + \mathbf{1} \otimes e_{31} + s_1 \otimes e_4$$

$$F_{221}(Q; Z) = (s_5 + s_{31}) \otimes e_0 + (s_3 + s_4 + s_{21}) \otimes e_1 + s_2 \otimes e_{11} + (s_1 + s_3 + s_{11}) \otimes e_2 + (\mathbf{1} + s_1) \otimes e_{21} + s_2 \otimes e_3$$

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$$F_{32}(Q; Z) = (s_5 + s_{31}) \otimes e_0 + (s_2 + s_3 + s_4 + s_{21}) \otimes e_1 + (\mathbf{1} + s_1 + s_2) \otimes e_{11} + (s_1 + s_3 + s_{11}) \otimes e_2$$

$$F_{41}(Q; Z) = (s_5 + s_{31}) \otimes e_0 + (s_1 + s_2 + s_3 + s_4 + s_{11} + s_{21}) \otimes e_1 + (\mathbf{1} + s_1 + s_2) \otimes e_{11} + s_3 \otimes e_2$$

$$F_5(Q; Z) = s_5 \otimes e_0 + (\mathbf{1} + s_1 + s_2 + s_3 + s_4) \otimes e_1$$

$$F_{111111}(Q; Z) = s_6 \otimes e_0 + s_5 \otimes e_1 + s_4 \otimes e_2 + s_3 \otimes e_3 + s_2 \otimes e_4 + s_1 \otimes e_5 + \mathbf{1} \otimes e_6$$

$$F_{21111}(Q; Z) = (s_6 + s_{41}) \otimes e_0 + (s_4 + s_5 + s_{31}) \otimes e_1 + s_3 \otimes e_{11} + (s_4 + s_{21}) \otimes e_2 + s_2 \otimes e_{21} \\ + (s_3 + s_{11}) \otimes e_3 + s_1 \otimes e_{31} + s_2 \otimes e_4 + \mathbf{1} \otimes e_{41} + s_1 \otimes e_5$$

$$F_{2211}(Q; Z) = (s_6 + s_{22} + s_{41}) \otimes e_0 + (s_4 + s_5 + s_{21} + s_{31}) \otimes e_1 + (s_3 + s_{11}) \otimes e_{11} + (s_2 + s_4 - s_{11} + s_{21}) \otimes e_2 \\ + (s_1 + s_2) \otimes e_{21} + \mathbf{1} \otimes e_{22} + (s_3 + s_{11}) \otimes e_3 + s_1 \otimes e_{31} + s_2 \otimes e_4$$

$$F_{321}(Q; Z) = (s_6 + s_{31} + s_{41} + s_{111}) \otimes e_0 + (s_3 + s_4 + s_5 + s_{11} + s_{21} + s_{31}) \otimes e_1 + (s_1 + s_2 + s_3) \otimes e_{11} \\ + \mathbf{1} \otimes e_{111} + (s_2 + s_4 + s_{11} + s_{21}) \otimes e_2 + (2s_1 + s_2) \otimes e_{21} + (s_3 + s_{11}) \otimes e_3$$

$$F_{42}(Q; Z) = (s_6 + s_{22} + s_{41}) \otimes e_0 + (s_2 + s_3 + s_4 + s_5 + s_{21} + s_{31}) \otimes e_1 \\ + (\mathbf{1} + 2s_1 + s_2 + s_3 + s_{11}) \otimes e_{11} + (s_2 + s_4 + s_{21}) \otimes e_2$$

$$F_{51}(Q; Z) = (s_6 + s_{41}) \otimes e_0 + (s_1 + s_2 + s_3 + s_4 + s_5 + s_{11} + s_{21} + s_{31}) \otimes e_1 + (\mathbf{1} + s_1 + s_2 + s_3) \otimes e_{11} + s_4 \otimes e_2$$

$$F_6(Q; Z) = s_6 \otimes e_0 + (\mathbf{1} + s_1 + s_2 + s_3 + s_4 + s_5) \otimes e_1$$

$$F_{1111111}(Q; Z) = s_7 \otimes e_0 + s_6 \otimes e_1 + s_5 \otimes e_2 + s_4 \otimes e_3 + s_3 \otimes e_4 + s_2 \otimes e_5 + s_1 \otimes e_6 + \mathbf{1} \otimes e_7$$

$$F_{211111}(Q; Z) = (s_7 + s_{51}) \otimes e_0 + (s_5 + s_6 + s_{41}) \otimes e_1 + s_4 \otimes e_{11} + (s_5 + s_{31}) \otimes e_2 + s_3 \otimes e_{21} + (s_4 + s_{21}) \otimes e_3 \\ + s_2 \otimes e_{31} + (s_3 + s_{11}) \otimes e_4 + s_1 \otimes e_{41} + s_2 \otimes e_5 + \mathbf{1} \otimes e_{51} + s_1 \otimes e_6$$

$$F_{22111}(Q; Z) = (s_7 + s_{32} + s_{51}) \otimes e_0 + (s_5 + s_6 + s_{22} + s_{31} + s_{41}) \otimes e_1 + (s_4 + s_{21}) \otimes e_{11} + (s_3 + s_5 + s_{31}) \otimes e_2 \\ + (s_2 + s_3 + s_{11}) \otimes e_{21} + s_1 \otimes e_{22} + (s_4 - s_{11} + s_{21}) \otimes e_3 + s_2 \otimes e_{31} \\ + \mathbf{1} \otimes e_{32} + (s_3 + s_{11}) \otimes e_4 + s_1 \otimes e_{41} + s_2 \otimes e_5$$

$$\begin{aligned}
F_{3211}(Q; Z) = & (s_7 + s_{32} + s_{41} + s_{51} + s_{211}) \otimes e_0 + (s_4 + s_5 + s_6 + s_{21} + s_{22} + 2s_{31} + s_{41} + s_{111}) \otimes e_1 \\
& + (s_2 + s_3 + s_4 + s_{11} + s_{21}) \otimes e_{11} + s_1 \otimes e_{111} + (s_3 + s_5 + s_{21} + s_{31}) \otimes e_2 \\
& + (2s_2 + s_3 + s_{11}) \otimes e_{21} + \mathbf{1} \otimes e_{211} + s_1 \otimes e_{22} \\
& + (s_4 + s_{21}) \otimes e_3 + (s_1 + s_2) \otimes e_{31} + (s_3 + s_{11}) \otimes e_4
\end{aligned}$$

$$\begin{aligned}
F_{421}(Q; Z) = & (s_7 + s_{32} + s_{41} + s_{51} + s_{211}) \otimes e_0 + (s_3 + s_4 + s_5 + s_6 + s_{11} + s_{21} + s_{22} + 2s_{31} + s_{41} + s_{111}) \otimes e_1 \\
& + (s_1 + 2s_2 + s_3 + s_4 + s_{11} + s_{21}) \otimes e_{11} + (\mathbf{1} + s_1) \otimes e_{111} \\
& + (s_3 + s_5 + s_{21} + s_{31}) \otimes e_2 + (s_1 + 2s_2 + s_3 + s_{11}) \otimes e_{21} + (s_4 + s_{21}) \otimes e_3
\end{aligned}$$

$$\begin{aligned}
F_{52}(Q; Z) = & (s_7 + s_{32} + s_{51}) \otimes e_0 + (s_2 + s_3 + s_4 + s_5 + s_6 + s_{21} + s_{22} + s_{31} + s_{41}) \otimes e_1 \\
& + (\mathbf{1} + 2s_1 + 2s_2 + s_3 + s_4 + s_{11} + s_{21}) \otimes e_{11} + (s_3 + s_5 + s_{31}) \otimes e_2
\end{aligned}$$

$$\begin{aligned}
F_{61}(Q; Z) = & (s_7 + s_{51}) \otimes e_0 + (s_1 + s_2 + s_3 + s_4 + s_5 + s_6 + s_{11} + s_{21} + s_{31} + s_{41}) \otimes e_1 \\
& + (\mathbf{1} + s_1 + s_2 + s_3 + s_4) \otimes e_{11} + s_5 \otimes e_2
\end{aligned}$$

$$F_7(Q; Z) = s_7 \otimes e_0 + (\mathbf{1} + s_1 + s_2 + s_3 + s_4 + s_5 + s_6) \otimes e_1$$

$$F_{11111111}(Q; Z) = s_8 \otimes e_0 + s_7 \otimes e_1 + s_6 \otimes e_2 + s_5 \otimes e_3 + s_4 \otimes e_4 + s_3 \otimes e_5 + s_2 \otimes e_6 + s_1 \otimes e_7 + \mathbf{1} \otimes e_8$$

$$\begin{aligned}
F_{2111111}(Q; Z) = & (s_8 + s_{61}) \otimes e_0 + (s_6 + s_7 + s_{51}) \otimes e_1 + s_5 \otimes e_{11} + (s_6 + s_{41}) \otimes e_2 + s_4 \otimes e_{21} + (s_5 + s_{31}) \otimes e_3 \\
& + s_3 \otimes e_{31} + (s_4 + s_{21}) \otimes e_4 + s_2 \otimes e_{41} + (s_3 + s_{11}) \otimes e_5 + s_1 \otimes e_{51} + s_2 \otimes e_6 \\
& + \mathbf{1} \otimes e_{61} + s_1 \otimes e_7
\end{aligned}$$

$$\begin{aligned}
F_{221111}(Q; Z) = & (s_8 + s_{42} + s_{61}) \otimes e_0 + (s_6 + s_7 + s_{32} + s_{41} + s_{51}) \otimes e_1 + (s_5 + s_{31}) \otimes e_{11} \\
& + (s_4 + s_6 + s_{22} + s_{41}) \otimes e_2 + (s_3 + s_4 + s_{21}) \otimes e_{21} + s_2 \otimes e_{22} + (s_5 + s_{31}) \otimes e_3 \\
& + (s_3 + s_{11}) \otimes e_{31} + s_1 \otimes e_{32} + (s_4 + s_{11} + s_{21}) \otimes e_4 + s_2 \otimes e_{41} \\
& + \mathbf{1} \otimes e_{42} + (s_3 + s_{11}) \otimes e_5 + s_1 \otimes e_{51} + s_2 \otimes e_6
\end{aligned}$$

$$\begin{aligned}
F_{22211}(Q; Z) = & (s_8 + s_{42} + s_{61}) \otimes e_0 + (s_6 + s_7 + s_{32} + s_{41} + s_{51}) \otimes e_1 + (s_5 + s_{31}) \otimes e_{11} \\
& + (s_4 + s_6 + s_{22} + s_{41}) \otimes e_2 + (s_3 + s_4 + s_{21}) \otimes e_{21} + s_2 \otimes e_{22} \\
& + (s_2 + s_5 + s_{31}) \otimes e_3 + (s_1 + s_3 + s_{11}) \otimes e_{31} + (\mathbf{1} + s_1) \otimes e_{32} \\
& + (s_4 + s_{21}) \otimes e_4 + s_2 \otimes e_{41} + s_3 \otimes e_5
\end{aligned}$$

$$\begin{aligned}
F_{3221}(Q; Z) = & (s_8 + s_{42} + s_{51} + s_{61} + s_{311}) \otimes e_0 + (s_5 + s_6 + s_7 + s_{31} + s_{32} + 2s_{41} + s_{51} + s_{211}) \otimes e_1 \\
& + (s_3 + s_4 + s_5 + s_{21} + s_{31}) \otimes e_{11} + s_2 \otimes e_{111} + (s_4 + s_6 + s_{31} + s_{11} + s_{22} + s_{41} + s_{111}) \otimes e_2 \\
& + (s_1 + 2s_3 + s_4 + s_{11} + s_{21}) \otimes e_{21} + (\mathbf{1} + s_1) \otimes e_{211} + s_2 \otimes e_{22} \\
& + (s_2 + s_5 + s_{21} + s_{31}) \otimes e_3 + (s_1 + s_2 + s_3 + s_{11}) \otimes e_{31} + (s_4 + s_{21}) \otimes e_4
\end{aligned}$$

$$\begin{aligned}
F_{431}(Q; Z) = & (s_8 + s_{42} + s_{51} + s_{61} + s_{311}) \otimes e_0 + (s_4 + s_5 + s_6 + s_7 + s_{21} + s_{31} + s_{32} + 2s_{41} + s_{51} + s_{211}) \otimes e_1 \\
& + (s_1 + s_2 + 2s_3 + s_4 + s_5 + s_{11} + s_{21} + s_{31}) \otimes e_{11} + (\mathbf{1} + s_1 + s_2) \otimes e_{111} \\
& + (s_2 + s_4 + s_6 + s_{11} + s_{21} + s_{31} + s_{22} + s_{41} + s_{111}) \otimes e_2 + (s_1 + s_2 + 2s_3 + s_4 + s_{11} + s_{21}) \otimes e_{21} + (s_5 + s_{31}) \otimes e_3
\end{aligned}$$

$$\begin{aligned}
F_{53}(Q; Z) = & (s_8 + s_{42} + s_{61}) \otimes e_0 + (s_3 + s_4 + s_5 + s_6 + s_7 + s_{31} + s_{32} + s_{41} + s_{51}) \otimes e_1 \\
& + (\mathbf{1} + 2s_1 + 2s_2 + 2s_3 + s_4 + s_5 + s_{11} + s_{21} + s_{31}) \otimes e_{11} \\
& + (s_2 + s_4 + s_6 + s_{21} + s_{22} + s_{41}) \otimes e_2
\end{aligned}$$

$$\begin{aligned}
F_{62}(Q; Z) = & (s_8 + s_{42} + s_{61}) \otimes e_0 + (s_2 + s_3 + s_4 + s_5 + s_6 + s_7 + s_{21} + s_{22} + s_{31} + s_{32} + s_{41} + s_{51}) \otimes e_1 \\
& + (\mathbf{1} + 2s_1 + 2s_2 + 2s_3 + s_4 + s_5 + s_{11} + s_{21} + s_{31}) \otimes e_{11} + (s_4 + s_6 + s_{41}) \otimes e_2
\end{aligned}$$

$$\begin{aligned}
F_{71}(Q; Z) = & (s_8 + s_{61}) \otimes e_0 + (s_1 + s_2 + s_3 + s_4 + s_5 + s_6 + s_7 + s_{11} + s_{21} + s_{31} + s_{41} + s_{51}) \otimes e_1 \\
& + (\mathbf{1} + s_1 + s_2 + s_3 + s_4 + s_5) \otimes e_{11} + s_6 \otimes e_2
\end{aligned}$$

$$F_8(Q; Z) = s_8 \otimes e_0 + (\mathbf{1} + s_1 + s_2 + s_3 + s_4 + s_5 + s_6 + s_7) \otimes e_1$$

$$F_{111111111}(Q; Z) = s_9 \otimes e_0 + s_8 \otimes e_1 + s_7 \otimes e_2 + s_6 \otimes e_3 + s_5 \otimes e_4 + s_4 \otimes e_5 + s_3 \otimes e_6 + s_2 \otimes e_7 + s_1 \otimes e_8 + \mathbf{1} \otimes e_9$$

$$\begin{aligned}
F_{211111111}(Q; Z) = & (s_9 + s_{71}) \otimes e_0 + (s_7 + s_8 + s_{61}) \otimes e_1 + s_6 \otimes e_{11} + (s_7 + s_{51}) \otimes e_2 + s_5 \otimes e_{21} + (s_6 + s_{41}) \otimes e_3 \\
& + s_4 \otimes e_{31} + (s_5 + s_{31}) \otimes e_4 + s_3 \otimes e_{41} + (s_4 + s_{21}) \otimes e_5 + s_2 \otimes e_{51} \\
& + (s_3 + s_{11}) \otimes e_6 + s_1 \otimes e_{61} + s_2 \otimes e_7 + \mathbf{1} \otimes e_{71} + s_1 \otimes e_8
\end{aligned}$$

$$\begin{aligned}
F_{221111111}(Q; Z) = & (s_9 + s_{52} + s_{71}) \otimes e_0 + (s_7 + s_8 + s_{42} + s_{51} + s_{61}) \otimes e_1 + (s_6 + s_{41}) \otimes e_{11} \\
& + (s_5 + s_7 + s_{32} + s_{51}) \otimes e_2 + (s_4 + s_5 + s_{31}) \otimes e_{21} + s_3 \otimes e_{22} \\
& + (s_6 + s_{22} + s_{41}) \otimes e_3 + (s_4 + s_{21}) \otimes e_{31} + s_2 \otimes e_{32} \\
& + (s_5 + s_{31}) \otimes e_4 + (s_3 + s_{11}) \otimes e_{41} + s_1 \otimes e_{42} \\
& + (s_4 - s_{11} + s_{21}) \otimes e_5 + s_2 \otimes e_{51} + \mathbf{1} \otimes e_{52} \\
& + (s_3 + s_{11}) \otimes e_6 + s_1 \otimes e_{61} + s_2 \otimes e_7
\end{aligned}$$

$$\begin{aligned}
F_{222111}(Q; Z) = & (s_9 + s_{33} + s_{52} + s_{71}) \otimes e_0 + (s_7 + s_8 + s_{32} + s_{42} + s_{51} + s_{61}) \otimes e_1 \\
& + (s_6 + s_{22} + s_{41}) \otimes e_{11} + (s_5 + s_7 - s_{22} + s_{31} + s_{32} + s_{51}) \otimes e_2 \\
& + (s_4 + s_5 + s_{21} + s_{31}) \otimes e_{21} + (s_3 + s_{11}) \otimes e_{22} \\
& + (s_3 + s_6 - s_{21} + s_{22} + s_{41}) \otimes e_3 + (s_2 + s_4 - s_{11} + s_{21}) \otimes e_{31} + (s_1 + s_2) \otimes e_{32} \\
& + \mathbf{1} \otimes e_{33} + (s_5 + s_{31}) \otimes e_4 + (s_3 + s_{11}) \otimes e_{41} \\
& + s_1 \otimes e_{42} + (s_4 + s_{21}) \otimes e_5 + s_2 \otimes e_{51} + s_3 \otimes e_6
\end{aligned}$$

$$\begin{aligned}
F_{32211}(Q; Z) = & (s_9 + s_{42} + s_{52} + s_{61} + s_{71} + s_{221} + s_{411}) \otimes e_0 \\
& + (s_6 + s_7 + s_8 + s_{22} + s_{32} + 2s_{41} + s_{42} + 2s_{51} + s_{61} + s_{211} + s_{311}) \otimes e_1 \\
& + (s_4 + s_5 + s_6 + s_{21} + 2s_{31} + s_{41} + s_{111}) \otimes e_{11} + (s_3 + s_{11}) \otimes e_{111} \\
& + (s_5 + s_7 + s_{41} + s_{22} + s_{32} + s_{51} - s_{111} + s_{211}) \otimes e_2 + (s_2 + 2s_4 + s_5 - s_{11} + 2s_{21} + s_{31}) \otimes e_{21} + (s_1 + s_2) \otimes e_{211} \\
& + s_3 \otimes e_{22} + \mathbf{1} \otimes e_{221} + (s_3 + s_6 + s_{11} + s_{31} + s_{22} + s_{41} + s_{111}) \otimes e_3 \\
& + (s_2 + s_3 + s_4 + 2s_{11} + s_{21}) \otimes e_{31} + s_1 \otimes e_{311} + (s_1 + s_2) \otimes e_{32} \\
& + (s_5 - s_{11} + s_{21} + s_{31}) \otimes e_4 + (s_2 + s_3 + s_{11}) \otimes e_{41} + (s_4 + s_{21}) \otimes e_5
\end{aligned}$$

$$\begin{aligned}
F_{3321}(Q; Z) = & (s_9 + s_{33} + s_{52} + s_{61} + s_{71} + s_{411}) \otimes e_0 + (s_6 + s_7 + s_8 + s_{32} + s_{41} + s_{42} + 2s_{51} + s_{61} + s_{311}) \otimes e_1 \\
& + (s_4 + s_5 + s_6 + s_{22} + s_{31} + s_{41}) \otimes e_{11} + s_3 \otimes e_{111} + (s_5 + s_3 + s_7 - s_{22} + s_{31} + s_{41} + s_{21} + s_{32} + s_{51} + s_{211}) \otimes e_2 \\
& + (s_1 + 2s_2 + 2s_4 + s_5 + s_{11} + 2s_{21} + s_{31}) \otimes e_{21} + (\mathbf{1} + s_1 + s_2) \otimes e_{211} + (s_1 + s_3 + s_{11}) \otimes e_{22} \\
& + (s_3 + s_6 + s_{11} + s_{31} + s_{22} + s_{41} + s_{111}) \otimes e_3 + (s_2 + s_3 + s_4 + s_{21}) \otimes e_{31} + (s_5 + s_{31}) \otimes e_4
\end{aligned}$$

$$\begin{aligned}
F_{432}(Q; Z) = & (s_9 + s_{33} + s_{52} + s_{61} + s_{71} + s_{411}) \otimes e_0 \\
& + (s_5 + s_6 + s_7 + s_8 + s_{31} + s_{32} + s_{41} + s_{42} + 2s_{51} + s_{61} + s_{311}) \otimes e_1 \\
& + (s_2 + s_3 + 2s_4 + s_5 + s_6 + s_{21} + s_{22} + s_{31} + s_{41}) \otimes e_{11} + (\mathbf{1} + s_1 + s_2 + s_3) \otimes e_{111} \\
& + (s_3 + s_5 + s_7 + s_{21} + s_{31} + s_{41} + s_{32} + s_{51} + s_{211}) \otimes e_2 + (2s_1 + 2s_2 + s_3 + 2s_4 + s_5 + 2s_{11} + 2s_{21} + s_{31}) \otimes e_{21} \\
& + (s_3 + s_6 + s_{11} + s_{31} + s_{41} + s_{111}) \otimes e_3
\end{aligned}$$

$$\begin{aligned}
F_{531}(Q; Z) = & (s_9 + s_{42} + s_{52} + s_{61} + s_{71} + s_{221} + s_{411}) \otimes e_0 \\
& + (s_4 + s_5 + s_6 + s_7 + s_8 + s_{21} + s_{22} + s_{31} + s_{32} + 2s_{41} + s_{42} + 2s_{51} + s_{61} + s_{211} + s_{311}) \otimes e_1 \\
& + (s_1 + 2s_2 + 2s_3 + 2s_4 + s_5 + s_6 + 2s_{11} + 2s_{21} + 2s_{31} + s_{41} + s_{111}) \otimes e_{11} + (\mathbf{1} + 2s_1 + s_2 + s_3 + s_{11}) \otimes e_{111} \\
& + (s_3 + s_5 + s_7 + s_{21} + s_{31} + s_{41} + s_{22} + s_{32} + s_{51} + s_{211}) \otimes e_2 + (2s_2 + s_3 + 2s_4 + s_5 + 2s_{21} + s_{31}) \otimes e_{21} \\
& + (s_6 + s_{22} + s_{41}) \otimes e_3
\end{aligned}$$

$$\begin{aligned}
F_{63}(Q; Z) = & (s_9 + s_{33} + s_{52} + s_{71}) \otimes e_0 + (s_3 + s_4 + s_5 + s_6 + s_7 + s_8 + s_{31} + s_{32} + s_{41} + s_{42} + s_{51} + s_{61}) \otimes e_1 \\
& + (\mathbf{1} + 2s_1 + 3s_2 + 2s_3 + 2s_4 + s_5 + s_6 + s_{11} + 2s_{21} + s_{22} + s_{31} + s_{41}) \otimes e_{11} \\
& + (s_3 + s_5 + s_7 + s_{31} + s_{32} + s_{51}) \otimes e_2
\end{aligned}$$

$$\begin{aligned}
F_{72}(Q; Z) = & (s_9 + s_{52} + s_{71}) \otimes e_0 + (s_2 + s_3 + s_4 + s_5 + s_6 + s_7 + s_8 + s_{21} + s_{22} + s_{31} + s_{32} + s_{41} + s_{42} + s_{51} + s_{61}) \otimes e_1 \\
& + (\mathbf{1} + 2s_1 + 2s_2 + 2s_3 + 2s_4 + s_5 + s_6 + s_{11} + s_{21} + s_{31} + s_{41}) \otimes e_{11} + (s_5 + s_7 + s_{51}) \otimes e_2
\end{aligned}$$

$$\begin{aligned}
F_{81}(Q; Z) = & (s_9 + s_{71}) \otimes e_0 + (s_1 + s_2 + s_3 + s_4 + s_5 + s_6 + s_7 + s_8 + s_{11} + s_{21} + s_{31} + s_{41} + s_{51} + s_{61}) \otimes e_1 \\
& + (\mathbf{1} + s_1 + s_2 + s_3 + s_4 + s_5 + s_6) \otimes e_{11} + s_7 \otimes e_2
\end{aligned}$$

$$F_9(Q; Z) = s_9 \otimes e_0 + (\mathbf{1} + s_1 + s_2 + s_3 + s_4 + s_5 + s_6 + s_7 + s_8) \otimes e_1$$

$$\begin{aligned}
F_{1111111111}(Q; Z) = & s_{10.} \otimes e_0 + s_9 \otimes e_1 + s_8 \otimes e_2 + s_7 \otimes e_3 + s_6 \otimes e_4 \\
& + s_5 \otimes e_5 + s_4 \otimes e_6 + s_3 \otimes e_7 + s_2 \otimes e_8 + s_1 \otimes e_9 + \mathbf{1} \otimes e_{10}.
\end{aligned}$$

$$\begin{aligned}
F_{211111111}(Q; Z) = & (s_{10.} + s_{81}) \otimes e_0 + (s_8 + s_9 + s_{71}) \otimes e_1 + s_7 \otimes e_{11} + (s_8 + s_{61}) \otimes e_2 + s_6 \otimes e_{21} + (s_7 + s_{51}) \otimes e_3 \\
& + s_5 \otimes e_{31} + (s_6 + s_{41}) \otimes e_4 + s_4 \otimes e_{41} + (s_5 + s_{31}) \otimes e_5 + s_3 \otimes e_{51} + (s_4 + s_{21}) \otimes e_6 \\
& + s_2 \otimes e_{61} + (s_3 + s_{11}) \otimes e_7 + s_1 \otimes e_{71} + s_2 \otimes e_8 + \mathbf{1} \otimes e_{81} + s_1 \otimes e_9
\end{aligned}$$

$$\begin{aligned}
F_{221111111}(Q; Z) = & (s_{10.} + s_{62} + s_{81}) \otimes e_0 + (s_8 + s_9 + s_{52} + s_{61} + s_{71}) \otimes e_1 + (s_7 + s_{51}) \otimes e_{11} \\
& + (s_6 + s_8 + s_{42} + s_{61}) \otimes e_2 + (s_5 + s_6 + s_{41}) \otimes e_{21} + s_4 \otimes e_{22} \\
& + (s_7 + s_{32} + s_{51}) \otimes e_3 + (s_5 + s_{31}) \otimes e_{31} + s_3 \otimes e_{32} \\
& + (s_6 + s_{22} + s_{41}) \otimes e_4 + (s_4 + s_{21}) \otimes e_{41} + s_2 \otimes e_{42} \\
& + (s_5 + s_{31}) \otimes e_5 + (s_3 + s_{11}) \otimes e_{51} + s_1 \otimes e_{52} \\
& + (s_4 + s_{11} + s_{21}) \otimes e_6 + s_2 \otimes e_{61} + \mathbf{1} \otimes e_{62} \\
& + (s_3 + s_{11}) \otimes e_7 + s_1 \otimes e_{71} + s_2 \otimes e_8
\end{aligned}$$

$$\begin{aligned}
F_{2221111}(Q; Z) = & (s_{10} + s_{43} + s_{62} + s_{81}) \otimes e_0 + (s_8 + s_9 + s_{33} + s_{42} + s_{52} + s_{61} + s_{71}) \otimes e_1 \\
& + (s_7 + s_{32} + s_{51}) \otimes e_{11} + (s_6 + s_8 + s_{41} + s_{42} + s_{61}) \otimes e_2 + (s_5 + s_6 + s_{22} + s_{31} + s_{41}) \otimes e_{21} \\
& + (s_4 + s_{21}) \otimes e_{22} + (s_4 + s_7 - s_{22} + s_{32} + s_{51}) \otimes e_3 + (s_3 + s_5 + s_{31}) \otimes e_{31} \\
& + (s_2 + s_3 + s_{11}) \otimes e_{32} + s_1 \otimes e_{33} + (s_6 - s_{21} + s_{22} + s_{41}) \otimes e_4 \\
& + (s_4 - s_{11} + s_{21}) \otimes e_{41} + s_2 \otimes e_{42} + \mathbf{1} \otimes e_{43} + (s_5 + s_{31}) \otimes e_5 \\
& + (s_3 + s_{11}) \otimes e_{51} + s_1 \otimes e_{52} + (s_4 + s_{21}) \otimes e_6 \\
& + s_2 \otimes e_{61} + s_3 \otimes e_7
\end{aligned}$$

$$\begin{aligned}
F_{322111}(Q; Z) = & (s_{10} + s_{43} + s_{52} + s_{62} + s_{71} + s_{81} + s_{321} + s_{511}) \otimes e_0 \\
& + (s_7 + s_8 + s_9 + s_{32} + s_{33} + 2s_{42} + 2s_{51} + s_{52} + 2s_{61} + s_{71} + s_{221} + s_{311} + s_{411}) \otimes e_1 \\
& + (s_5 + s_6 + s_7 + s_{22} + s_{31} + s_{32} + 2s_{41} + s_{51} + s_{211}) \otimes e_{11} \\
& + (s_4 + s_{21}) \otimes e_{111} + (s_6 + s_8 + s_{32} + s_{41} + s_{51} + s_{42} + s_{61} + s_{311}) \otimes e_2 \\
& + (s_3 + 2s_5 + s_6 + s_{22} + 3s_{31} + s_{41} + s_{111}) \otimes e_{21} + (s_2 + s_3 + s_{11}) \otimes e_{211} \\
& + (s_4 + s_{21}) \otimes e_{22} + s_1 \otimes e_{221} + (s_4 + s_7 + s_{41} + s_{32} + s_{51} - s_{111} + s_{211}) \otimes e_3 \\
& + (s_3 + s_4 + s_5 - s_{11} + 2s_{21} + s_{31}) \otimes e_{31} + s_2 \otimes e_{311} + (s_2 + s_3 + s_{11}) \otimes e_{32} \\
& + \mathbf{1} \otimes e_{321} + s_1 \otimes e_{33} + (s_6 + s_{11} - s_{21} + s_{31} + s_{22} + s_{41} + s_{111}) \otimes e_4 \\
& + (s_3 + s_4 + s_{11} + s_{21}) \otimes e_{41} + s_1 \otimes e_{411} + s_2 \otimes e_{42} \\
& + (s_5 - s_{11} + s_{21} + s_{31}) \otimes e_5 + (s_2 + s_3 + s_{11}) \otimes e_{51} + (s_4 + s_{21}) \otimes e_6
\end{aligned}$$

$$\begin{aligned}
F_{33211}(Q; Z) = & (s_{10} + s_{43} + s_{52} + s_{62} + s_{71} + s_{81} + s_{321} + s_{511}) \otimes e_0 \\
& + (s_7 + s_8 + s_9 + s_{32} + s_{33} + 2s_{42} + 2s_{51} + s_{52} + 2s_{61} + s_{71} + s_{221} + s_{311} + s_{411}) \otimes e_1 \\
& + (s_5 + s_6 + s_7 + s_{22} + s_{31} + s_{32} + 2s_{41} + s_{51} + s_{211}) \otimes e_{11} + (s_4 + s_{21}) \otimes e_{111} \\
& + (s_6 + s_4 + s_8 + s_{32} + s_{41} + s_{51} + s_{21} + s_{42} + s_{61} + s_{311}) \otimes e_2 \\
& + (s_2 + 2s_3 + 2s_5 + s_6 + s_{11} + s_{22} + 3s_{31} + s_{41} + s_{111}) \otimes e_{21} \\
& + (s_1 + s_2 + s_3 + s_{11}) \otimes e_{211} + (s_2 + s_4 + s_{21}) \otimes e_{22} \\
& + (\mathbf{1} + s_1) \otimes e_{221} + (s_4 + s_7 + s_{21} - s_{11} + s_{41} + s_{32} + s_{51} - s_{111} + s_{211}) \otimes e_3 \\
& + (s_3 + s_4 + s_5 + 2s_{21} + s_{31}) \otimes e_{31} + s_2 \otimes e_{311} + (s_1 + s_2 + s_3 + s_{11}) \otimes e_{32} \\
& + (s_6 + s_{11} - s_{21} + s_{31} + s_{22} + s_{41} + s_{111}) \otimes e_4 + (s_3 + s_4 + s_{21}) \otimes e_{41} \\
& + (s_5 + s_{31}) \otimes e_5
\end{aligned}$$

$$\begin{aligned}
F_{4321}(Q; Z) = & (s_{10} + s_{42} + s_{43} + s_{61} + s_{62} + s_{71} + s_{81} + s_{311} + s_{411} + s_{511} + s_{1111}) \otimes e_0 \\
& + (s_6 + s_7 + s_8 + s_9 + s_{31} + s_{32} + s_{33} + 2s_{41} + s_{42} + 2s_{51} + s_{52} + 2s_{61} + s_{71} + s_{111} + s_{211} + s_{311} + s_{411}) \otimes e_1 \\
& + (s_3 + s_4 + 2s_5 + s_6 + s_7 + s_{11} + s_{21} + 2s_{31} + s_{32} + s_{41} + s_{51}) \otimes e_{11} \\
& + (s_1 + s_2 + s_3 + s_4) \otimes e_{111} + \mathbf{1} \otimes e_{1111} \\
& + (s_4 + s_6 + s_8 + s_{21} + s_{31} + 2s_{41} + s_{51} + s_{22} + s_{42} + s_{61} + s_{111} + s_{211} + s_{311}) \otimes e_2 \\
& + (2s_2 + 2s_3 + s_4 + 2s_5 + s_6 + 2s_{11} + 3s_{21} + s_{22} + 2s_{31} + s_{41}) \otimes e_{21} \\
& + (3s_1 + 2s_2 + s_3) \otimes e_{211} + (s_2 + s_4 + s_{11} + s_{21}) \otimes e_{22} \\
& + (s_4 + s_7 + s_{21} + s_{31} + s_{41} + s_{32} + s_{51} + s_{111} + s_{211}) \otimes e_3 + (2s_3 + s_4 + s_5 + 2s_{11} + s_{21} + s_{31}) \otimes e_{31} \\
& + (s_6 + s_{31} + s_{41} + s_{111}) \otimes e_4
\end{aligned}$$

$$\begin{aligned}
F_{532}(Q; Z) = & (s_{10} + s_{43} + s_{52} + s_{62} + s_{71} + s_{81} + s_{321} + s_{511}) \otimes e_0 \\
& + (s_5 + s_6 + s_7 + s_8 + s_9 + s_{21} + s_{22} + 2s_{31} + s_{32} + s_{33} + s_{41} + 2s_{42} + 2s_{51} + s_{52} + 2s_{61} + s_{71} + s_{211} + s_{221} + 2s_{311} + s_{411}) \otimes e_1 \\
& + (s_2 + 2s_3 + 2s_4 + 2s_5 + s_6 + s_7 + s_{11} + 2s_{21} + s_{22} + 2s_{31} + s_{32} + 2s_{41} + s_{51} + s_{111} + s_{211}) \otimes e_{11} \\
& + (\mathbf{1} + 2s_1 + 2s_2 + s_3 + s_4 + s_{11} + s_{21}) \otimes e_{111} \\
& + (s_4 + s_6 + s_8 + s_{32} + s_{41} + s_{51} + s_{42} + s_{61}) \otimes e_2 \\
& + (s_1 + 2s_2 + 3s_3 + s_4 + 2s_5 + s_6 + s_{11} + 2s_{21} + s_{22} + 3s_{31} + s_{41}) \otimes e_{21} \\
& + (s_4 + s_7 + s_{41} + s_{32} + s_{51}) \otimes e_3
\end{aligned}$$

$$\begin{aligned}
F_{631}(Q; Z) = & (s_{10} + s_{43} + s_{52} + s_{62} + s_{71} + s_{81} + s_{321} + s_{511}) \otimes e_0 \\
& + (s_4 + s_5 + s_6 + s_7 + s_8 + s_9 + s_{21} + s_{22} + s_{31} + s_{32} + s_{33} + 2s_{41} \\
& + 2s_{42} + 2s_{51} + s_{52} + 2s_{61} + s_{71} + s_{211} + s_{221} + s_{311} + s_{411}) \otimes e_1 \\
& + (s_1 + 2s_2 + 3s_3 + 2s_4 + 2s_5 + s_6 + s_7 + 2s_{11} + 3s_{21} + s_{22} + 3s_{31} + s_{32} + 2s_{41} + s_{51} + s_{111} + s_{211}) \otimes e_{11} \\
& + (\mathbf{1} + 2s_1 + 2s_2 + s_3 + s_4 + s_{11} + s_{21}) \otimes e_{111} \\
& + (s_4 + s_6 + s_8 + s_{31} + s_{32} + s_{41} + s_{51} + s_{42} + s_{61} + s_{311}) \otimes e_2 \\
& + (s_2 + 2s_3 + s_4 + 2s_5 + s_6 + s_{21} + s_{22} + 2s_{31} + s_{41}) \otimes e_{21} \\
& + (s_7 + s_{32} + s_{51}) \otimes e_3
\end{aligned}$$

$$\begin{aligned}
F_{73}(Q; Z) = & (s_{10} + s_{43} + s_{62} + s_{81}) \otimes e_0 \\
& + (s_3 + s_4 + s_5 + s_6 + s_7 + s_8 + s_9 + s_{31} + s_{32} + s_{33} + s_{41} + s_{42} + s_{51} + s_{52} + s_{61} + s_{71}) \otimes e_1 \\
& + (\mathbf{1} + 2s_1 + 3s_2 + 3s_3 + 2s_4 + 2s_5 + s_6 + s_7 + s_{11} + 2s_{21} + s_{22} + 2s_{31} + s_{32} + s_{41} + s_{51}) \otimes e_{11} \\
& + (s_4 + s_6 + s_8 + s_{41} + s_{42} + s_{61}) \otimes e_2
\end{aligned}$$



$$\begin{aligned}
F_{82}(Q; Z) &= (s_{10.} + s_{62} + s_{81}) \otimes e_0 \\
&+ (s_2 + s_3 + s_4 + s_5 + s_6 + s_7 + s_8 + s_9 + s_{21} + s_{22} + s_{31} + s_{32} + s_{41} + s_{42} + s_{51} + s_{52} + s_{61} + s_{71}) \otimes e_1 \\
&\quad + (\mathbf{1} + 2s_1 + 2s_2 + 2s_3 + 2s_4 + 2s_5 + s_6 + s_7 + s_{11} + s_{21} + s_{31} + s_{41} + s_{51}) \otimes e_{11} \\
&\quad \quad + (s_6 + s_8 + s_{61}) \otimes e_2
\end{aligned}$$

$$\begin{aligned}
F_{91}(Q; Z) &= (s_{10.} + s_{81}) \otimes e_0 \\
&+ (s_1 + s_2 + s_3 + s_4 + s_5 + s_6 + s_7 + s_8 + s_9 + s_{11} + s_{21} + s_{31} + s_{41} + s_{51} + s_{61} + s_{71}) \otimes e_1 \\
&\quad + (\mathbf{1} + s_1 + s_2 + s_3 + s_4 + s_5 + s_6 + s_7) \otimes e_{11} + s_8 \otimes e_2
\end{aligned}$$

$$F_{10.}(Q; Z) = s_{10.} \otimes e_0 + (\mathbf{1} + s_1 + s_2 + s_3 + s_4 + s_5 + s_6 + s_7 + s_8 + s_9) \otimes e_1$$