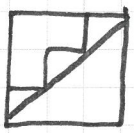
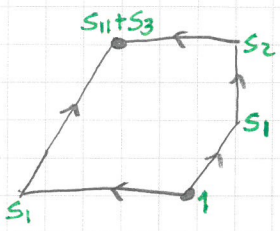
COMPUTING $\mathcal{E}_{m,n}[q+q,x] = P_{m,n} \otimes Q_{m,n}$ FOR $n=3$

A GENERAL GEOMETRIC FORMULA!

Cesar 17.10.2018

- 

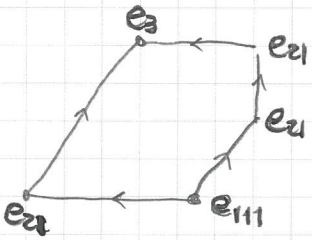
$$\mathcal{E}_{3,3}(1+q,x) = (s_{11}+s_3)\otimes e_3 + (2s_1+s_2)\otimes e_{21} + 1\otimes e_{111}$$



$P_{3,3}$

(3,3)-Tamari lattice with labelled vertices

$\otimes$



$Q_{3,3}$

(3,3)-Tamari lattice with labelled vertices

↳ vertex label: e_λ where λ is the length partition of vertical moves of the corresponding Dyck path

Vertex label: some symmetric function that depends only on the lattice structure, and which is closely related to the tropical geometric realization of the (m,n)-Tamari lattice.

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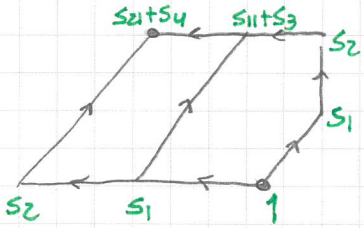
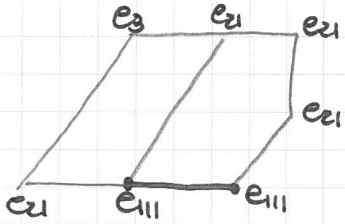
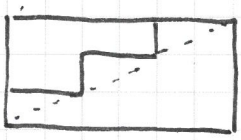
$$\mathcal{E}_{5,3}(1+q,x) = (s_{21}+s_4)\otimes e_3 + (s_1+s_{11}+2s_2+s_3)\otimes e_{21} + (1+s_1)\otimes e_{111}$$

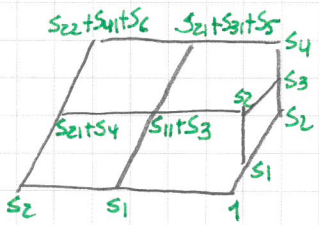


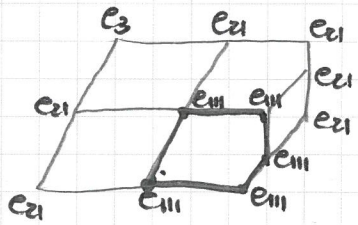
$\otimes$

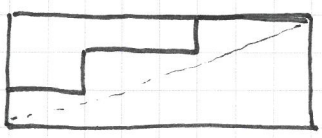


- 

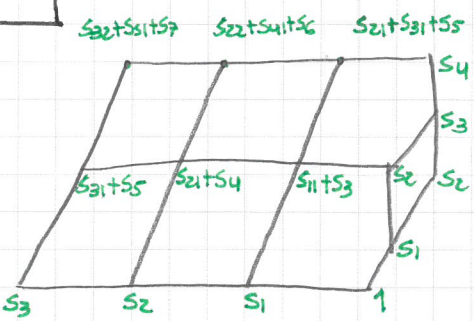
$$\mathcal{E}_{6,3}(1+q,x) = (s_{22}+s_{41}+s_5)\otimes e_3 + (2s_2+2s_{21}+s_3+s_{31}+2s_4+s_5)\otimes e_{21} + (1+2s_1+s_{11}+s_2+s_3)\otimes e_{111}$$



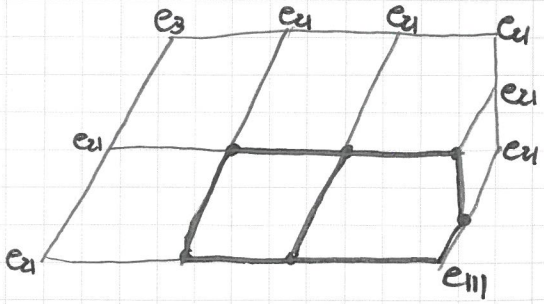


- 

$$\mathcal{E}_{8,3}(1+q,x) = (s_{32}+s_{51}+s_7)\otimes e_3 + (s_2+s_{21}+s_{22}+2s_3+2s_{31}+s_4+s_{41}+2s_5+s_6)\otimes e_{21} + (1+2s_1+s_{11}+2s_2+s_{21}+s_3+s_4)\otimes e_{111}$$



$\otimes$



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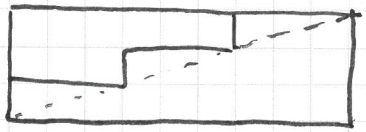
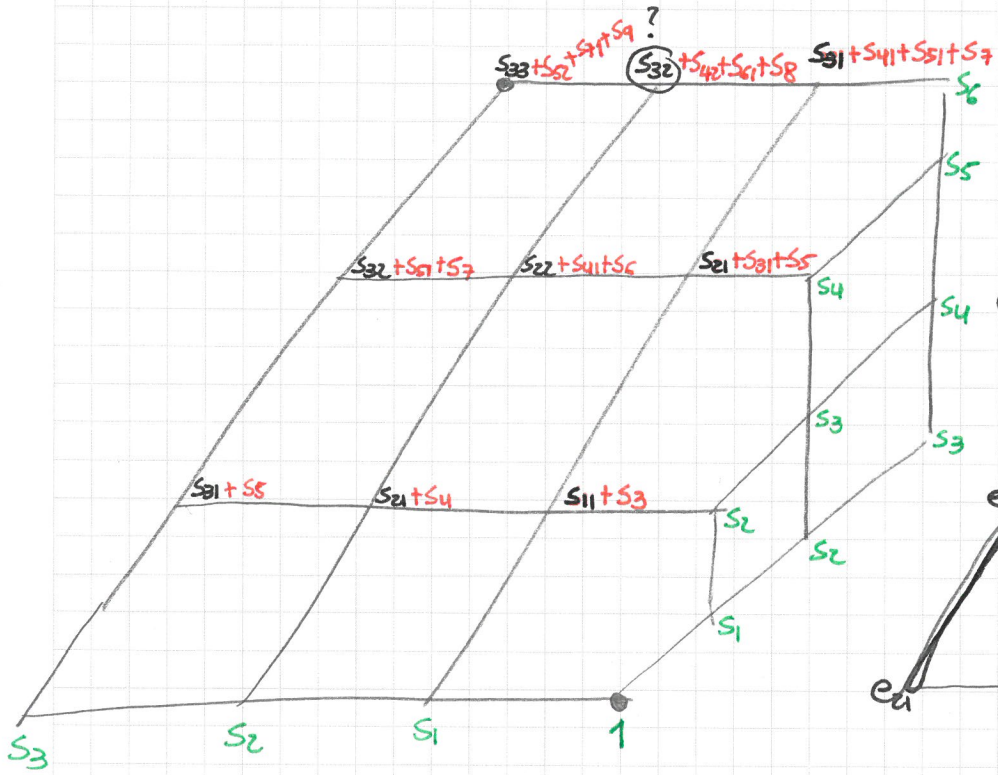
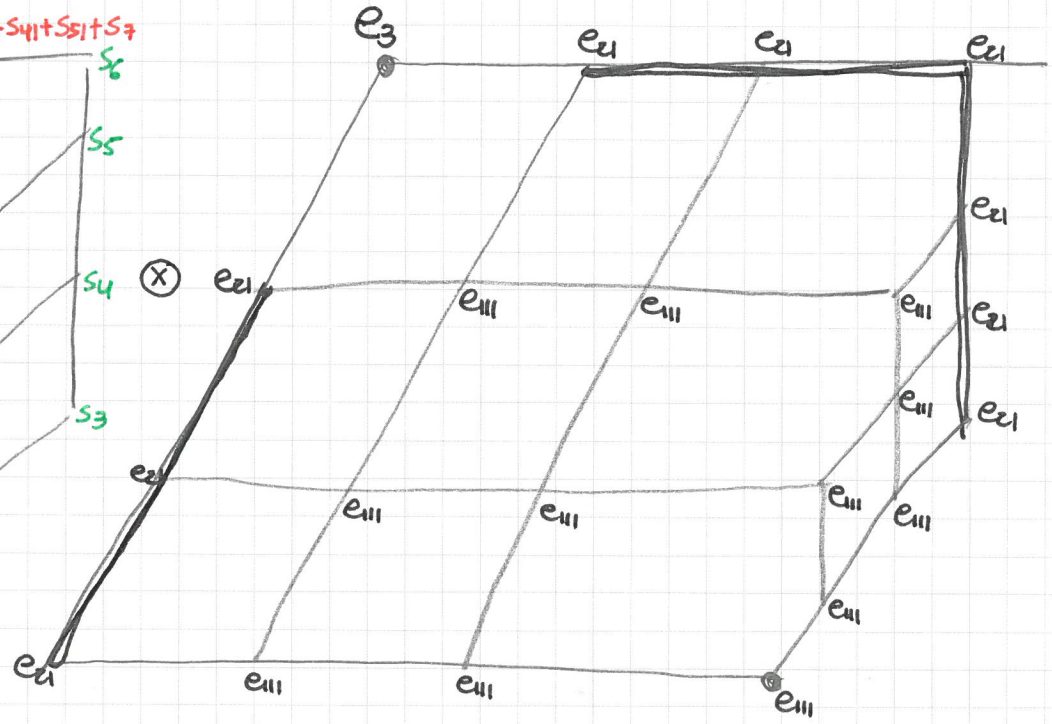
$$\mathcal{E}_{m,n}[1+q,x] = P_{m,n} \otimes Q_{m,n}$$



$(m,n)=(q,3)$ $\mathcal{E}_{q,3} =$



$P_{q,3}$

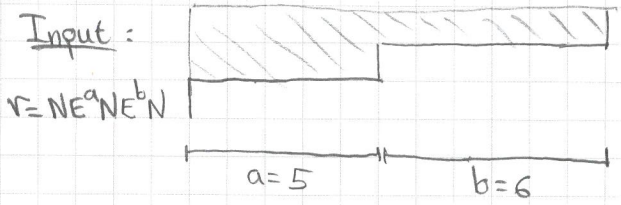


$Q_{q,3}$.

A GENERAL MASTER GEOMETRIC FORMULA.

(Works in the context of signature Catalan combinatorics that generalizes the rectangular Catalan set up)

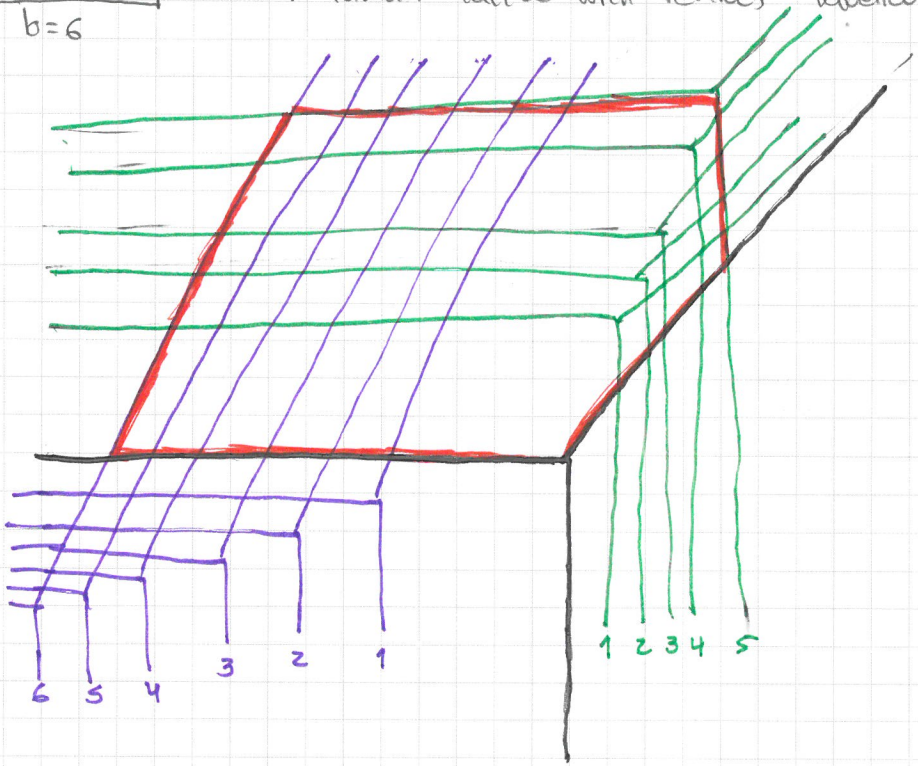
Input:

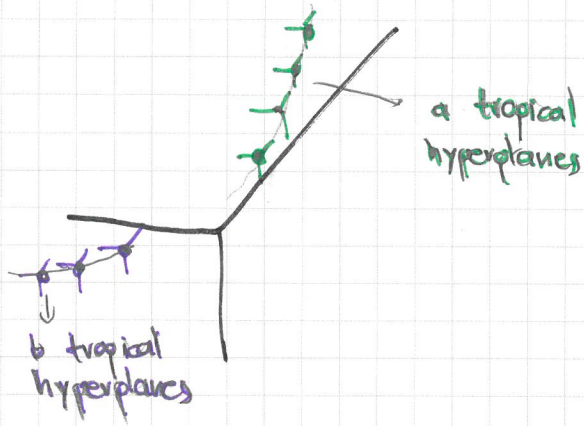


$$\mathcal{E}_r(1+q, x) = P_r \otimes Q_r$$

where P_r and Q_r are the tropical geometric realizations of the r-Tamari lattice with vertices labelled with some symmetric functions.

The r-Tamari lattice: tropically



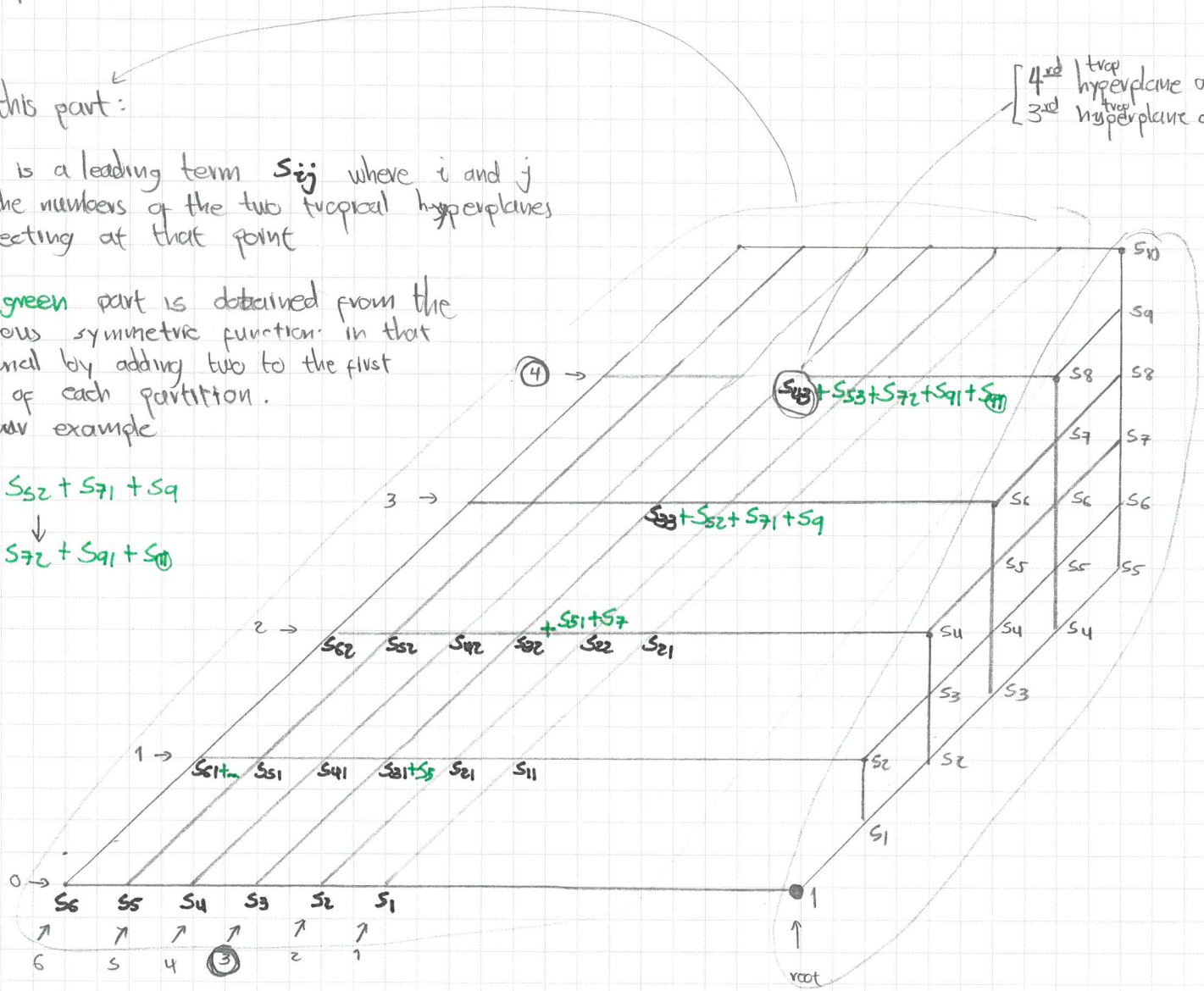


Description of P_ν :

For this part:

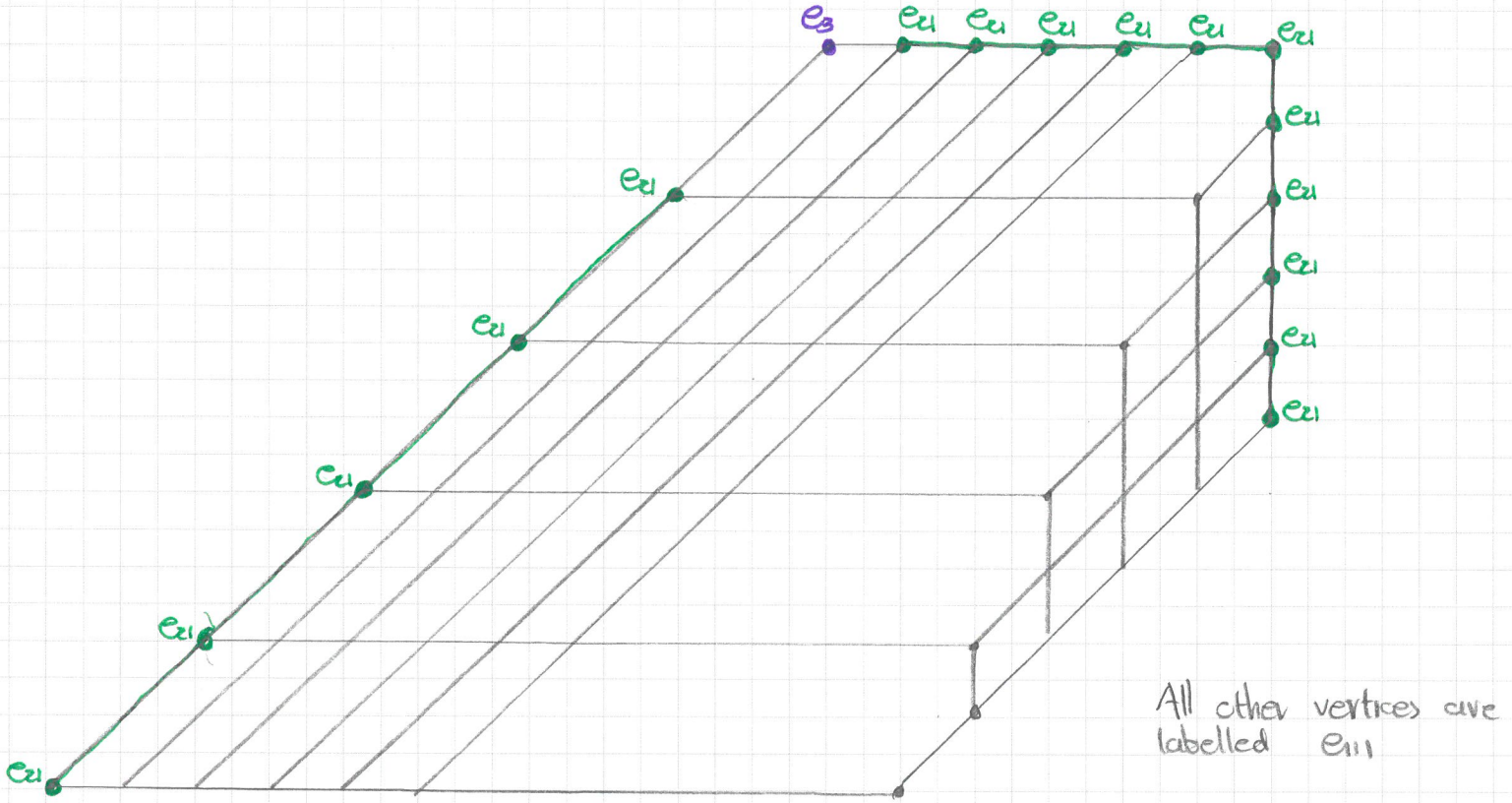
- There is a leading term s_{ij} where i and j are the numbers of the two tropical hyperplanes intersecting at that point
- The green part is obtained from the previous symmetric function in that diagonal by adding two to the first entry of each partition.
  In our example

$$s_{33} + s_{52} + s_{71} + s_9$$
$$\downarrow$$
$$s_{53} + s_{72} + s_{91} + s_{11}$$



A GENERAL MASTER GEOMETRIC FORMULA 3/3

Description of Q_r :



Q_r