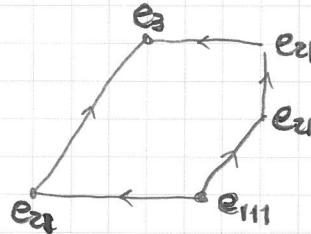


$$E_{3,3}(1+q, x) = (s_{11}+s_3) \otimes e_3 + (2s_1+s_2) \otimes e_{21} + 1 \otimes e_{111}$$



$P_{3,3}$   
(3,3)-Tamari lattice  
with labelled vertices

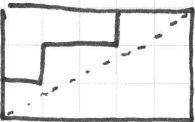
$\otimes$



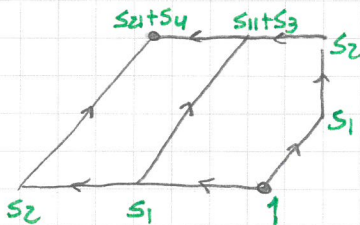
$Q_{3,3}$   
(3,3)-Tamari lattice  
with labelled vertices

Vertex label: Some symmetric function that depends only on the lattice structure, and which is closely related to the tropical geometric realization of the  $(m,n)$ -Tamari lattice.

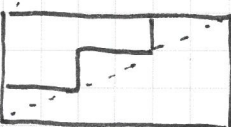
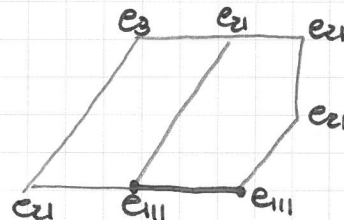
vertex label:  $e_\lambda$  where  $\lambda$  is the length partition of vertical moves of the corresponding Dyck path



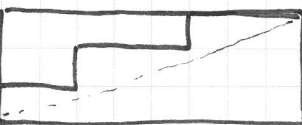
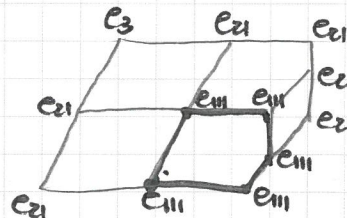
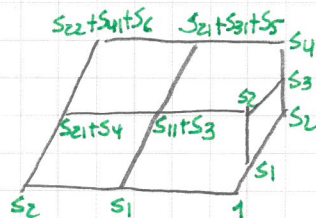
$$E_{6,3}(1+q, x) = (s_{21}+s_4) \otimes e_3 + (s_1+s_{11}+2s_2+s_3) \otimes e_{21} + (1+s_1) \otimes e_{111}$$



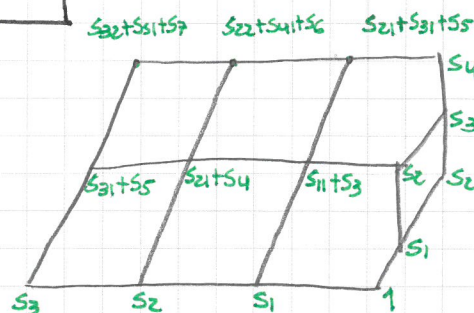
$\otimes$



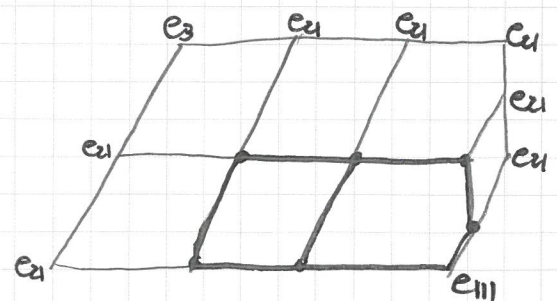
$$E_{8,3}(1+q, x) = (s_{22}+s_{41}+s_6) \otimes e_3 + (2s_2+2s_{21}+s_3+s_{31}+2s_4+s_5) \otimes e_{21} + (1+2s_1+s_{11}+2s_2+s_3+s_4) \otimes e_{111}$$



$$E_{10,3}(1+q, x) = (s_{32}+s_{51}+s_7) \otimes e_3 + (s_2+s_{21}+s_{22}+2s_3+2s_{31}+s_4+s_{41}+2s_5+s_6) \otimes e_{21} + (1+2s_1+s_{11}+2s_2+s_{21}+s_3+s_4) \otimes e_{111}$$



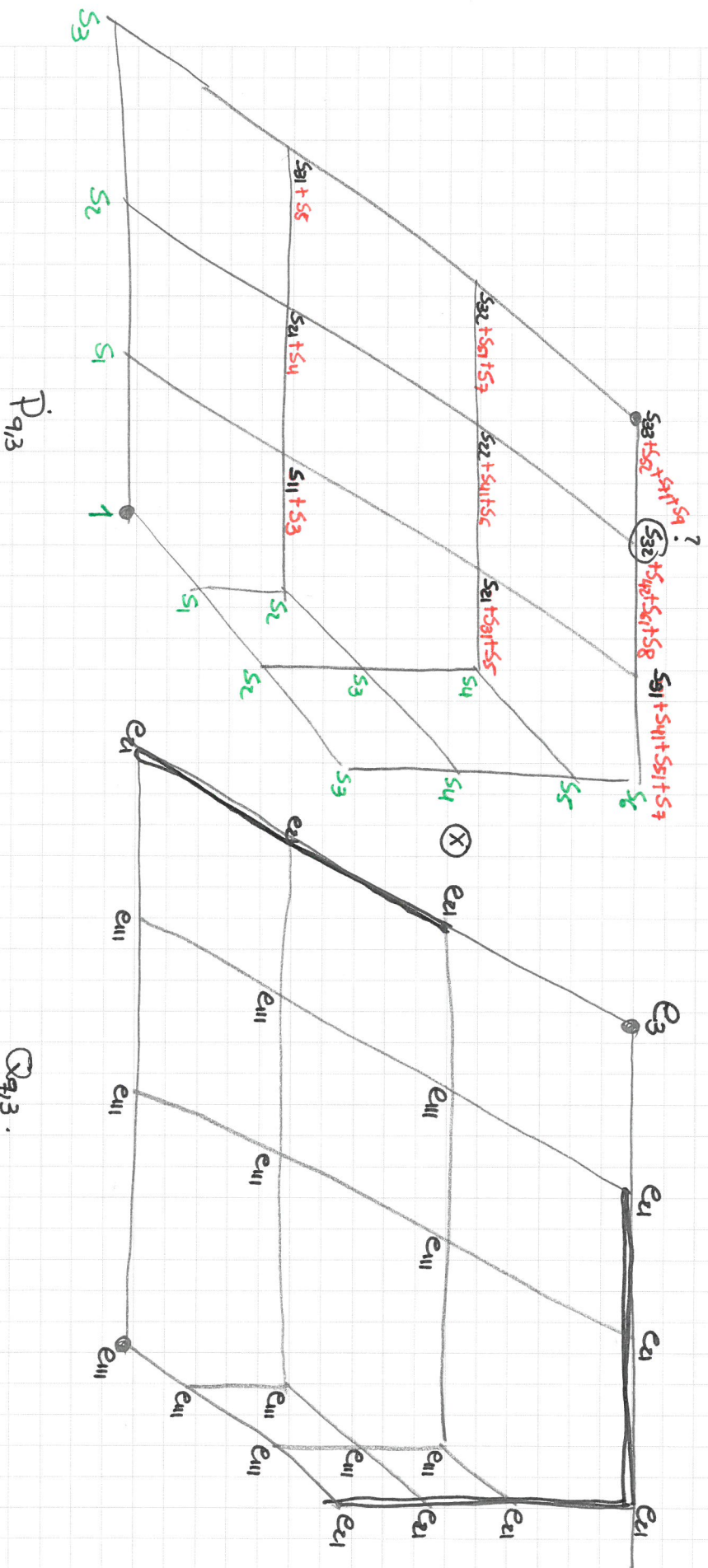
$\otimes$



$$E_{m,n} [1+q, x] = P_{m,n} \otimes Q_{m,n}$$



$$(M_{M1}) = (Q_{1,3}) \quad E_{1,3} =$$



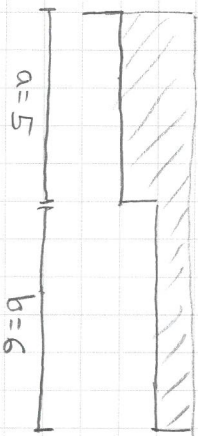


# A GENERAL MASTER GEOMETRIC FORMULA. <sup>13</sup>

(Works in the context of signature catalan combinatorics  
the generalizes the rectangular catalan set up)

Input:

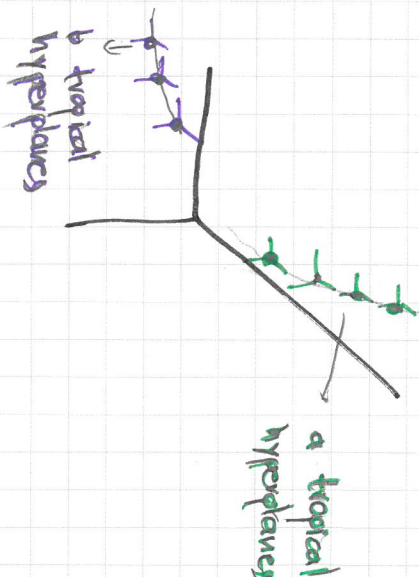
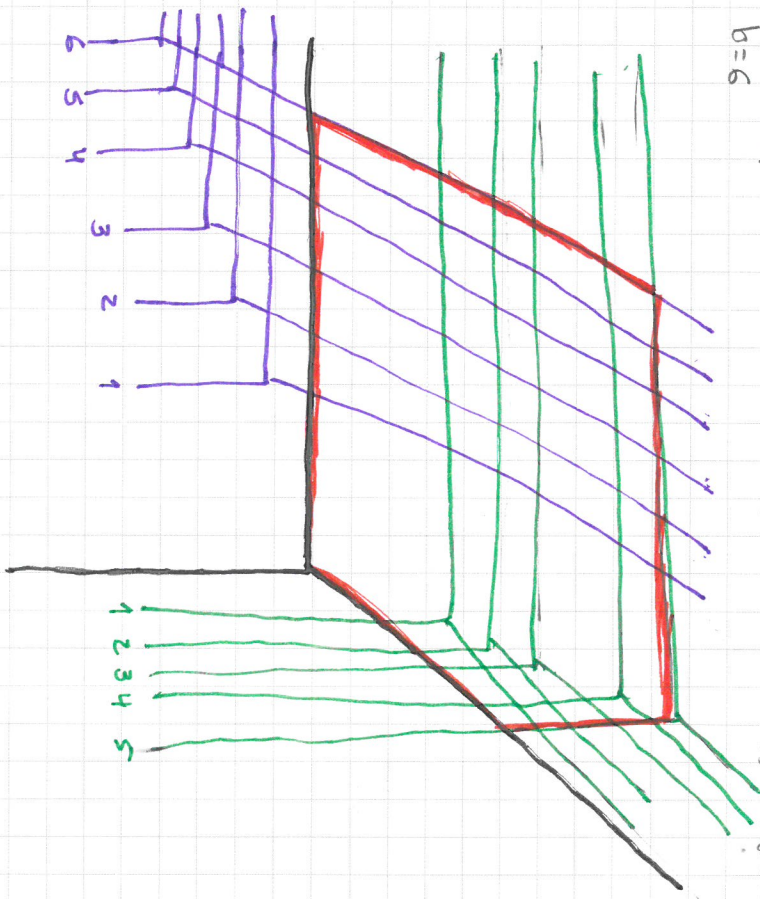
$$v = n e^a n e^b n$$



$$\mathcal{E}_r(1+q, x) = P_r \otimes Q_r$$

where  $P_r$  and  $Q_r$  are the tropical geometric realizations of the  $v$ -Tamari lattice with vertices labelled with some symmetric functions.

The  $v$ -Tamari lattice:  
tropically







## 3/3

<u>Description of Qr</u>	Qr
Qr 1	Qr 1
Qr 2	Qr 2
Qr 3	Qr 3
Qr 4	Qr 4
Qr 5	Qr 5
Qr 6	Qr 6
Qr 7	Qr 7
Qr 8	Qr 8
Qr 9	Qr 9
Qr 10	Qr 10
Qr 11	Qr 11
Qr 12	Qr 12
Qr 13	Qr 13
Qr 14	Qr 14
Qr 15	Qr 15
Qr 16	Qr 16
Qr 17	Qr 17
Qr 18	Qr 18
Qr 19	Qr 19
Qr 20	Qr 20
Qr 21	Qr 21
Qr 22	Qr 22
Qr 23	Qr 23
Qr 24	Qr 24
Qr 25	Qr 25
Qr 26	Qr 26
Qr 27	Qr 27
Qr 28	Qr 28
Qr 29	Qr 29
Qr 30	Qr 30
Qr 31	Qr 31
Qr 32	Qr 32
Qr 33	Qr 33
Qr 34	Qr 34
Qr 35	Qr 35
Qr 36	Qr 36
Qr 37	Qr 37
Qr 38	Qr 38
Qr 39	Qr 39
Qr 40	Qr 40
Qr 41	Qr 41
Qr 42	Qr 42
Qr 43	Qr 43
Qr 44	Qr 44
Qr 45	Qr 45
Qr 46	Qr 46
Qr 47	Qr 47
Qr 48	Qr 48
Qr 49	Qr 49
Qr 50	Qr 50
Qr 51	Qr 51
Qr 52	Qr 52
Qr 53	Qr 53
Qr 54	Qr 54
Qr 55	Qr 55
Qr 56	Qr 56
Qr 57	Qr 57
Qr 58	Qr 58
Qr 59	Qr 59
Qr 60	Qr 60
Qr 61	Qr 61
Qr 62	Qr 62
Qr 63	Qr 63
Qr 64	Qr 64
Qr 65	Qr 65
Qr 66	Qr 66
Qr 67	Qr 67
Qr 68	Qr 68
Qr 69	Qr 69
Qr 70	Qr 70
Qr 71	Qr 71
Qr 72	Qr 72
Qr 73	Qr 73
Qr 74	Qr 74
Qr 75	Qr 75
Qr 76	Qr 76
Qr 77	Qr 77
Qr 78	Qr 78
Qr 79	Qr 79
Qr 80	Qr 80
Qr 81	Qr 81
Qr 82	Qr 82
Qr 83	Qr 83
Qr 84	Qr 84
Qr 85	Qr 85
Qr 86	Qr 86
Qr 87	Qr 87
Qr 88	Qr 88
Qr 89	Qr 89
Qr 90	Qr 90
Qr 91	Qr 91
Qr 92	Qr 92
Qr 93	Qr 93
Qr 94	Qr 94
Qr 95	Qr 95
Qr 96	Qr 96
Qr 97	Qr 97
Qr 98	Qr 98
Qr 99	Qr 99
Qr 100	Qr 100

