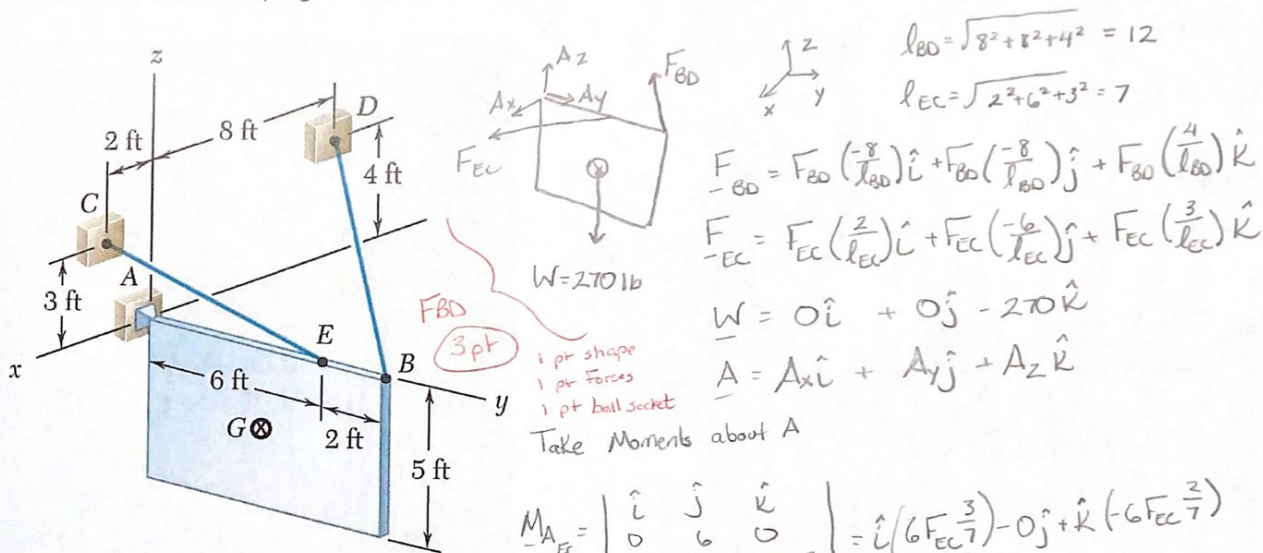Problem #1 (20 points): The sign weighs 270 lb and acts at the center of mass G of the sign (at the center of the rectangular 5 X 8 shape). The sign is supported by a ball-socket joint at A and two cables EC and BD. Determine the tension in each cable (magnitudes) and the magnitude of the support reactions at joint A.

$$l_{BD} = \sqrt{8^2+8^2+4^2} = 12$$
$$l_{EC} = \sqrt{2^2+6^2+3^2} = 7$$

$$\vec{F}_{BD} = F_{BD}\left(\frac{-8}{l_{BD}}\right)\hat{i} + F_{BD}\left(\frac{-8}{l_{BD}}\right)\hat{j} + F_{BD}\left(\frac{4}{l_{BD}}\right)\hat{k}$$

$$\vec{F}_{EC} = F_{EC}\left(\frac{2}{l_{EC}}\right)\hat{i} + F_{EC}\left(\frac{-6}{l_{EC}}\right)\hat{j} + F_{EC}\left(\frac{3}{l_{EC}}\right)\hat{k}$$

$$W = 0\hat{i} + 0\hat{j} - 270\hat{k}$$
$$A = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

W = 270 lb

FBD (3 pt)
1 pt shape
1 pt forces
1 pt ball socket

Take Moments about A

$$M_{A_{EC}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 6 & 0 \\ F_{EC}\frac{2}{7} & -F_{EC}\frac{6}{7} & F_{EC}\frac{3}{7} \end{vmatrix} = \hat{i}\left(6F_{EC}\frac{3}{7}\right) - 0\hat{j} + \hat{k}\left(-6F_{EC}\frac{2}{7}\right)$$

$$\sum M_x = 0 \quad \text{(4 pt)}$$
$$\frac{18}{7}F_{EC} + \frac{32}{12}F_{BD} - 1080 = 0$$

$$F_{EC} = 315$$

$$M_{A_{BD}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 8 & 0 \\ -F_{BD}\frac{8}{12} & -F_{BD}\frac{8}{12} & F_{BD}\frac{4}{12} \end{vmatrix} = \hat{i}\left(8F_{BD}\frac{4}{12}\right) - 0\hat{j} + \hat{k}\left(8F_{BD}\frac{8}{12}\right)$$

$$\sum M_y = 0 \quad 0 = 0$$

$$F_{BD} = 101.25 \quad M_W = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & -2.5 \\ 0 & 0 & -270 \end{vmatrix} = -1080\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\sum M_z = 0 \quad \text{(4 pt)}$$
$$-\frac{12}{7}F_{EC} + \frac{64}{12}F_{BD} = 0 \quad F_{BD} = F_{EC}(0.32143)$$

$$\sum F_x = 0 \quad \text{(2 pt)}$$
$$A_x - F_{BD}\left(\frac{8}{12}\right) + F_{EC}\left(\frac{2}{7}\right) = 0 \quad A_x = -22.7$$

$$\sum F_y = 0 \quad \text{(2 pt)}$$
$$A_y - F_{BD}\left(\frac{8}{12}\right) - F_{EC}\left(\frac{6}{7}\right) = 0 \quad A_y = 337.5$$

$$\sum F_z = 0 \quad \text{(2 pt)}$$
$$A_z + F_{BD}\left(\frac{4}{12}\right) + F_{EC}\left(\frac{3}{7}\right) - 270 = 0 \quad A_z = 101.25$$

F_{EC} =	315 lb
F_{BD} =	101 lb
A =	353 lb

(3 pts)
1 pt each correct

$$|A| = 353$$

Problem #3 (20 points): The space truss is supported by ball-socket joints at C and D and a short link at E. A force of 6 kN is applied to joint A in the negative z direction. Determine the forces in members AB, AC, and AD. Circle if the member is in tension, T or compression, C.

FBD (5 pt)
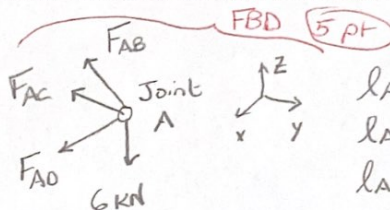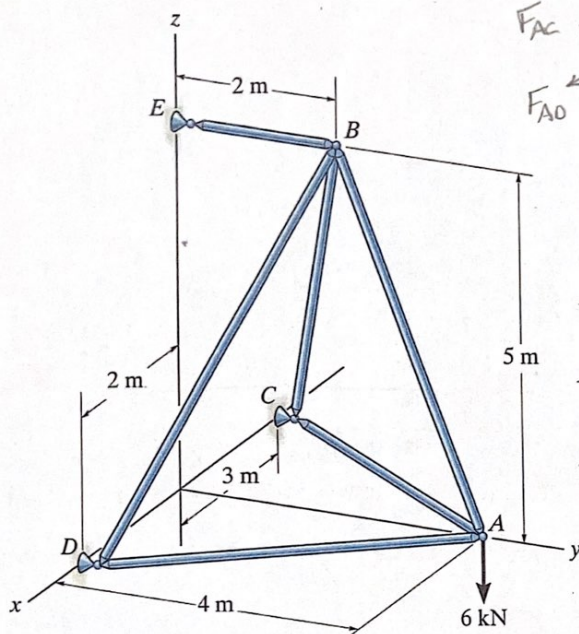
$$\ell_{AB} = \sqrt{2^2 + 5^2} = \sqrt{29} = 5.385$$
$$\ell_{AC} = \sqrt{3^2 + 4^2} = 5$$
$$\ell_{AD} = \sqrt{2^2 + 4^2} = \sqrt{20} = 4.4721$$

$$\vec{F}_{AB} = F_{AB}(0)\hat{i} + F_{AB}\left(\frac{-2}{\ell_{AB}}\right)\hat{j} + F_{AB}\left(\frac{5}{\ell_{AB}}\right)\hat{k}$$

$$\vec{F}_{AC} = F_{AC}\left(\frac{-3}{\ell_{AC}}\right)\hat{i} + F_{AC}\left(\frac{-4}{\ell_{AC}}\right)\hat{j} + F_{AC}(0)\hat{k}$$

$$\vec{F}_{AD} = F_{AD}\left(\frac{+2}{\ell_{AD}}\right)\hat{i} + F_{AD}\left(\frac{-4}{\ell_{AD}}\right)\hat{j} + F_{AD}(0)\hat{k}$$

$$\vec{F} = 0\hat{i} + 0\hat{j} - 6\hat{k}$$

$$\Sigma F_z = 0 \quad \text{(3 pt)}$$

$$F_{AB}\underbrace{\left(\frac{5}{\sqrt{29}}\right)}_{.9285} + F_{AC}(0) + F_{AD}(0) - 6 = 0$$

$$F_{AB} = 6.462$$

(1 pt) state at least one governing eqn.

$$\Sigma F_y = 0 \quad \text{(3 pt)}$$

$$\underbrace{F_{AB}}_{6.462}\underbrace{\left(\frac{-2}{\sqrt{29}}\right)}_{\substack{0.3714 \\ -2.4}} + \underbrace{F_{AC}\left(\frac{-4}{5}\right)}_{-0.8} + \underbrace{F_{AD}\left(\frac{-4}{\sqrt{20}}\right)}_{-0.8944} = 0$$

$$F_{AD} = -1.610$$

so $F_{AC} = -1.2$

$$\Sigma F_x = 0 \quad \text{(3 pt)}$$

$$F_{AB}(0) + \underbrace{F_{AC}\left(\frac{-3}{5}\right)}_{-0.6} + \underbrace{F_{AD}\left(\frac{2}{\sqrt{20}}\right)}_{0.4472} = 0$$

$$F_{AC} = F_{AD}\left(\frac{10}{3\sqrt{20}}\right)$$

(1 pt) units

(1 pt) all T or C correct

$F_{AB} =$	__6.46__ kN ⓣ or C
$F_{AC} =$	__1.2__ kN T or Ⓒ
$F_{AD} =$	__1.61__ kN T or Ⓒ

(3 pt) 1 pt each correct answer