# New 0inflated models: Poisson & Binomial

## Parametrisation

This is a new implementation (Nov'22) of zero-inflated Poisson and Binomial likelihood, where we will allow for a linear predictor in both the zero-inflattion and in the mean, but one of them needs to consists of fixed effects only. This means the setup will be somewhat different that for other likelihood models.

## Details

The zero-inflated likelihood $f_0(y|\ldots)$ is defined as

$$f_0(y|\eta_1, \eta_2) = p(\eta_1)1_{[y=0]} + (1 - p(\eta_1))f(y|\eta_2)$$

where $f(y|\ldots)$ is either Poisson or Binomial. We allow for two linear predictors in the model, but one needs to be "simple" (i.e. only consists of fixed effects). The other is general and defined via the formula. Normally, the zero-inflation probability is simpler (`family="0..."`)

$$\eta_1 = \text{simple} \qquad \eta_2 = \text{formula}$$

but they can also be swapped (`family="0...S"`)

$$\eta_1 = \text{formula} \qquad \eta_2 = \text{simple}$$

## Link-function

This is similar to Poisson and Binomial.

The link-function for the 'simple'-model must be given by argument `link.simple` in the `control.family`-argument. Only link-models without covariates/parameters are currently available. The examples later on shows how this is done.

## Hyperparameters

All parameters in the simple model are treated as hyperparameters. The $j$'th element of $\eta_1$ is

$$(\eta_1)_j = \sum_{i=1}^{m} \beta_i x_{ij}$$

for covariates $x_1, \ldots,$, where $m$ is maximum 10. An intercept in this model have to be defined manually by adding a constant covariate vector.

## Specification

- `family="0poisson"`

- `family="0poissonS"`

- `family="0binomial"`

- `family="0binomialS"`

- Required arguments: As for the Poisson and Binomial (but how these arguments are given, will differ). Optional argument `link.simple`.

**Hyperparameter spesification and default values**

**0poisson**

**doc** `New 0-inflated Poisson`

**hyper**

    **theta1**

        **hyperid** `56201`
        **name** `beta1`
        **short.name** `beta1`
        **output.name** `beta1 for 0poisson observations`
        **output.name.intern** `beta1 for 0poisson observations`
        **initial** `-4`
        **fixed** `FALSE`
        **prior** `normal`
        **param** `-4 10`
        **to.theta** `function(x) x`
        **from.theta** `function(x) x`

    **theta2**

        **hyperid** `56202`
        **name** `beta2`
        **short.name** `beta2`
        **output.name** `beta2 for 0poisson observations`
        **output.name.intern** `beta2 for 0poisson observations`
        **initial** `0`
        **fixed** `FALSE`
        **prior** `normal`
        **param** `0 10`
        **to.theta** `function(x) x`
        **from.theta** `function(x) x`

    **theta3**

        **hyperid** `56203`
        **name** `beta3`
        **short.name** `beta3`
        **output.name** `beta3 for 0poisson observations`
        **output.name.intern** `beta3 for 0poisson observations`
        **initial** `0`
        **fixed** `FALSE`
        **prior** `normal`
        **param** `0 10`
        **to.theta** `function(x) x`
        **from.theta** `function(x) x`

    **theta4**

        **hyperid** `56204`
        **name** `beta4`
        **short.name** `beta4`
        **output.name** `beta4 for 0poisson observations`
        **output.name.intern** `beta4 for 0poisson observations`
        **initial** `0`
        **fixed** `FALSE`

**prior** normal

**param** 0 10

**to.theta** function(x) x

**from.theta** function(x) x

**theta5**

    **hyperid** 56205

    **name** beta5

    **short.name** beta5

    **output.name** beta5 for 0poisson observations

    **output.name.intern** beta5 for 0poisson observations

    **initial** 0

    **fixed** FALSE

    **prior** normal

    **param** 0 10

    **to.theta** function(x) x

    **from.theta** function(x) x

**theta6**

    **hyperid** 56206

    **name** beta6

    **short.name** beta6

    **output.name** beta6 for 0poisson observations

    **output.name.intern** beta6 for 0poisson observations

    **initial** 0

    **fixed** FALSE

    **prior** normal

    **param** 0 10

    **to.theta** function(x) x

    **from.theta** function(x) x

**theta7**

    **hyperid** 56207

    **name** beta7

    **short.name** beta7

    **output.name** beta7 for 0poisson observations

    **output.name.intern** beta7 for 0poisson observations

    **initial** 0

    **fixed** FALSE

    **prior** normal

    **param** 0 10

    **to.theta** function(x) x

    **from.theta** function(x) x

**theta8**

    **hyperid** 56208

    **name** beta8

    **short.name** beta8

    **output.name** beta8 for 0poisson observations

    **output.name.intern** beta8 for 0poisson observations

    **initial** 0

    **fixed** FALSE

    **prior** normal

**param** `0 10`

**to.theta** `function(x) x`

**from.theta** `function(x) x`

### theta9

**hyperid** `56209`

**name** `beta9`

**short.name** `beta9`

**output.name** `beta9 for 0poisson observations`

**output.name.intern** `beta9 for 0poisson observations`

**initial** `0`

**fixed** `FALSE`

**prior** `normal`

**param** `0 10`

**to.theta** `function(x) x`

**from.theta** `function(x) x`

### theta10

**hyperid** `56210`

**name** `beta10`

**short.name** `beta10`

**output.name** `beta10 for 0poisson observations`

**output.name.intern** `beta10 for 0poisson observations`

**initial** `0`

**fixed** `FALSE`

**prior** `normal`

**param** `0 10`

**to.theta** `function(x) x`

**from.theta** `function(x) x`

**survival** `FALSE`

**discrete** `TRUE`

**link** `default log quantile`

**link.simple** `default logit cauchit probit cloglog ccloglog`

**pdf** `0inflated`

## 0poissonS

**doc** `New 0-inflated Poisson Swap`

**hyper**

### theta1

**hyperid** `56301`

**name** `beta1`

**short.name** `beta1`

**output.name** `beta1 for 0poissonS observations`

**output.name.intern** `beta1 for 0poissonS observations`

**initial** `-4`

**fixed** `FALSE`

**prior** `normal`

**param** `-4 10`

**to.theta** `function(x) x`

**from.theta** `function(x) x`

**theta2**

    **hyperid** 56302

    **name** beta2

    **short.name** beta2

    **output.name** beta2 for 0poissonS observations

    **output.name.intern** beta2 for 0poissonS observations

    **initial** 0

    **fixed** FALSE

    **prior** normal

    **param** 0 10

    **to.theta** `function(x) x`

    **from.theta** `function(x) x`

**theta3**

    **hyperid** 56303

    **name** beta3

    **short.name** beta3

    **output.name** beta3 for 0poissonS observations

    **output.name.intern** beta3 for 0poissonS observations

    **initial** 0

    **fixed** FALSE

    **prior** normal

    **param** 0 10

    **to.theta** `function(x) x`

    **from.theta** `function(x) x`

**theta4**

    **hyperid** 56304

    **name** beta4

    **short.name** beta4

    **output.name** beta4 for 0poissonS observations

    **output.name.intern** beta4 for 0poissonS observations

    **initial** 0

    **fixed** FALSE

    **prior** normal

    **param** 0 10

    **to.theta** `function(x) x`

    **from.theta** `function(x) x`

**theta5**

    **hyperid** 56305

    **name** beta5

    **short.name** beta5

    **output.name** beta5 for 0poissonS observations

    **output.name.intern** beta5 for 0poissonS observations

    **initial** 0

    **fixed** FALSE

    **prior** normal

    **param** 0 10

    **to.theta** `function(x) x`

    **from.theta** `function(x) x`

**theta6**

    **hyperid** 56306

    **name** beta6

    **short.name** beta6

    **output.name** beta6 for 0poissonS observations

    **output.name.intern** beta6 for 0poissonS observations

    **initial** 0

    **fixed** FALSE

    **prior** normal

    **param** 0 10

    **to.theta** function(x) x

    **from.theta** function(x) x

**theta7**

    **hyperid** 56307

    **name** beta7

    **short.name** beta7

    **output.name** beta7 for 0poissonS observations

    **output.name.intern** beta7 for 0poissonS observations

    **initial** 0

    **fixed** FALSE

    **prior** normal

    **param** 0 10

    **to.theta** function(x) x

    **from.theta** function(x) x

**theta8**

    **hyperid** 56308

    **name** beta8

    **short.name** beta8

    **output.name** beta8 for 0poissonS observations

    **output.name.intern** beta8 for 0poissonS observations

    **initial** 0

    **fixed** FALSE

    **prior** normal

    **param** 0 10

    **to.theta** function(x) x

    **from.theta** function(x) x

**theta9**

    **hyperid** 56309

    **name** beta9

    **short.name** beta9

    **output.name** beta9 for 0poissonS observations

    **output.name.intern** beta9 for 0poissonS observations

    **initial** 0

    **fixed** FALSE

    **prior** normal

    **param** 0 10

    **to.theta** function(x) x

    **from.theta** function(x) x

**theta10**

        **hyperid** `56310`

        **name** `beta10`

        **short.name** `beta10`

        **output.name** `beta10 for 0poissonS observations`

        **output.name.intern** `beta10 for 0poissonS observations`

        **initial** `0`

        **fixed** `FALSE`

        **prior** `normal`

        **param** `0 10`

        **to.theta** `function(x) x`

        **from.theta** `function(x) x`

**survival** `FALSE`

**discrete** `TRUE`

**link** `default logit loga cauchit probit cloglog ccloglog loglog log sslogit logitoffset quantile pquanti`

**link.simple** `default log`

**pdf** `0inflated`

## 0binomial

**doc** `New 0-inflated Binomial`

**hyper**

    **theta1**

        **hyperid** `56401`

        **name** `beta1`

        **short.name** `beta1`

        **output.name** `beta1 for 0binomial observations`

        **output.name.intern** `beta1 for 0binomial observations`

        **initial** `-4`

        **fixed** `FALSE`

        **prior** `normal`

        **param** `-4 10`

        **to.theta** `function(x) x`

        **from.theta** `function(x) x`

    **theta2**

        **hyperid** `56402`

        **name** `beta2`

        **short.name** `beta2`

        **output.name** `beta2 for 0binomial observations`

        **output.name.intern** `beta2 for 0binomial observations`

        **initial** `0`

        **fixed** `FALSE`

        **prior** `normal`

        **param** `0 10`

        **to.theta** `function(x) x`

        **from.theta** `function(x) x`

    **theta3**

        **hyperid** `56403`

        **name** `beta3`

**short.name** `beta3`

**output.name** `beta3 for 0binomial observations`

**output.name.intern** `beta3 for 0binomial observations`

**initial** `0`

**fixed** `FALSE`

**prior** `normal`

**param** `0 10`

**to.theta** `function(x) x`

**from.theta** `function(x) x`

**theta4**

**hyperid** `56404`

**name** `beta4`

**short.name** `beta4`

**output.name** `beta4 for 0binomial observations`

**output.name.intern** `beta4 for 0binomial observations`

**initial** `0`

**fixed** `FALSE`

**prior** `normal`

**param** `0 10`

**to.theta** `function(x) x`

**from.theta** `function(x) x`

**theta5**

**hyperid** `56405`

**name** `beta5`

**short.name** `beta5`

**output.name** `beta5 for 0binomial observations`

**output.name.intern** `beta5 for 0binomial observations`

**initial** `0`

**fixed** `FALSE`

**prior** `normal`

**param** `0 10`

**to.theta** `function(x) x`

**from.theta** `function(x) x`

**theta6**

**hyperid** `56406`

**name** `beta6`

**short.name** `beta6`

**output.name** `beta6 for 0binomial observations`

**output.name.intern** `beta6 for 0binomial observations`

**initial** `0`

**fixed** `FALSE`

**prior** `normal`

**param** `0 10`

**to.theta** `function(x) x`

**from.theta** `function(x) x`

**theta7**

**hyperid** `56407`

**name** `beta7`

**short.name** `beta7`

**output.name** `beta7 for 0binomial observations`
**output.name.intern** `beta7 for 0binomial observations`
**initial** `0`
**fixed** `FALSE`
**prior** `normal`
**param** `0 10`
**to.theta** `function(x) x`
**from.theta** `function(x) x`

**theta8**

**hyperid** `56408`
**name** `beta8`
**short.name** `beta8`
**output.name** `beta8 for 0binomial observations`
**output.name.intern** `beta8 for 0binomial observations`
**initial** `0`
**fixed** `FALSE`
**prior** `normal`
**param** `0 10`
**to.theta** `function(x) x`
**from.theta** `function(x) x`

**theta9**

**hyperid** `56409`
**name** `beta9`
**short.name** `beta9`
**output.name** `beta9 for 0binomial observations`
**output.name.intern** `beta9 for 0binomial observations`
**initial** `0`
**fixed** `FALSE`
**prior** `normal`
**param** `0 10`
**to.theta** `function(x) x`
**from.theta** `function(x) x`

**theta10**

**hyperid** `56410`
**name** `beta10`
**short.name** `beta10`
**output.name** `beta10 for 0binomial observations`
**output.name.intern** `beta10 for 0binomial observations`
**initial** `0`
**fixed** `FALSE`
**prior** `normal`
**param** `0 10`
**to.theta** `function(x) x`
**from.theta** `function(x) x`

**survival** `FALSE`

**discrete** `TRUE`

**link** `default logit loga cauchit probit cloglog ccloglog loglog log`

**link.simple** `default logit cauchit probit cloglog ccloglog`

**pdf** `0inflated`

**0poisson**

**doc** New 0-inflated Binomial Swap

**hyper**

    **theta1**

        **hyperid** 56501

        **name** beta1

        **short.name** beta1

        **output.name** beta1 for 0binomialS observations

        **output.name.intern** beta1 for 0binomialS observations

        **initial** -4

        **fixed** FALSE

        **prior** normal

        **param** -4 10

        **to.theta** function(x) x

        **from.theta** function(x) x

    **theta2**

        **hyperid** 56502

        **name** beta2

        **short.name** beta2

        **output.name** beta2 for 0binomialS observations

        **output.name.intern** beta2 for 0binomialS observations

        **initial** 0

        **fixed** FALSE

        **prior** normal

        **param** 0 10

        **to.theta** function(x) x

        **from.theta** function(x) x

    **theta3**

        **hyperid** 56503

        **name** beta3

        **short.name** beta3

        **output.name** beta3 for 0binomialS observations

        **output.name.intern** beta3 for 0binomialS observations

        **initial** 0

        **fixed** FALSE

        **prior** normal

        **param** 0 10

        **to.theta** function(x) x

        **from.theta** function(x) x

    **theta4**

        **hyperid** 56504

        **name** beta4

        **short.name** beta4

        **output.name** beta4 for 0binomialS observations

        **output.name.intern** beta4 for 0binomialS observations

        **initial** 0

        **fixed** FALSE

        **prior** normal

**param** 0 10
**to.theta** function(x) x
**from.theta** function(x) x
**theta5**
    **hyperid** 56505
    **name** beta5
    **short.name** beta5
    **output.name** beta5 for 0binomialS observations
    **output.name.intern** beta5 for 0binomialS observations
    **initial** 0
    **fixed** FALSE
    **prior** normal
    **param** 0 10
    **to.theta** function(x) x
    **from.theta** function(x) x
**theta6**
    **hyperid** 56506
    **name** beta6
    **short.name** beta6
    **output.name** beta6 for 0binomialS observations
    **output.name.intern** beta6 for 0binomialS observations
    **initial** 0
    **fixed** FALSE
    **prior** normal
    **param** 0 10
    **to.theta** function(x) x
    **from.theta** function(x) x
**theta7**
    **hyperid** 56507
    **name** beta7
    **short.name** beta7
    **output.name** beta7 for 0binomialS observations
    **output.name.intern** beta7 for 0binomialS observations
    **initial** 0
    **fixed** FALSE
    **prior** normal
    **param** 0 10
    **to.theta** function(x) x
    **from.theta** function(x) x
**theta8**
    **hyperid** 56508
    **name** beta8
    **short.name** beta8
    **output.name** beta8 for 0binomialS observations
    **output.name.intern** beta8 for 0binomialS observations
    **initial** 0
    **fixed** FALSE
    **prior** normal
    **param** 0 10

**to.theta** `function(x) x`

**from.theta** `function(x) x`

**theta9**

    **hyperid** `56509`

    **name** `beta9`

    **short.name** `beta9`

    **output.name** `beta9 for 0binomialS observations`

    **output.name.intern** `beta9 for 0binomialS observations`

    **initial** `0`

    **fixed** `FALSE`

    **prior** `normal`

    **param** `0 10`

    **to.theta** `function(x) x`

    **from.theta** `function(x) x`

**theta10**

    **hyperid** `56510`

    **name** `beta10`

    **short.name** `beta10`

    **output.name** `beta10 for 0binomialS observations`

    **output.name.intern** `beta10 for 0binomialS observations`

    **initial** `0`

    **fixed** `FALSE`

    **prior** `normal`

    **param** `0 10`

    **to.theta** `function(x) x`

    **from.theta** `function(x) x`

**survival** `FALSE`

**discrete** `TRUE`

**link** `default logit loga cauchit probit cloglog ccloglog loglog log`

**link.simple** `default logit cauchit probit cloglog ccloglog`

**pdf** `0inflated`

## Example: Poisson

```
sim.poisson <- function(prob, m)
{
    stopifnot(length(prob) == length(m) && length(prob) > 0)
    n <- length(m)
    y <- numeric(n)
    event <- (runif(n) < prob)
    idx.zero <- which(event)
    idx.non.zero <- which(!event)
    y[idx.zero] <- 0
    y[idx.non.zero] <- rpois(length(idx.non.zero), lambda = m[idx.non.zero])
    return (y)
}


## chose link-function to use for the zero-inflation probability
link.simple <- "logit"
inv.link <- inla.link.invlogit
## link.simple <- "probit"
## inv.link <- inla.link.invprobit
## link.simple <- "cloglog"
## inv.link <- inla.link.invcloglog


n <- 1000
z <- rnorm(n, sd = 0.3)
x <- rnorm(n, sd = 0.2)
xx <- rnorm(n, sd = 0.3)
zz <- rnorm(n, sd = 0.2)
E <- runif(n, min = 0.8, max = 1/0.8)

beta <- c(1, 1.1, 2.1, 0, -2, 1.2, 2.2, 0)
eta2 <- beta[1] + beta[2] * xx + beta[3] * zz + beta[4] * xx * zz
eta1 <- beta[5] + beta[6] * x + beta[7] * z + beta[8] * x * z
prob <- inv.link(eta1)
m <- E*exp(eta2)

ok <- FALSE
while(!ok) {
    y <- sim.poisson(prob, m)
    ok <- !all(y == 0)
}

## head(data.frame(y, E, x, z, xx, zz))

r <- inla(
    inla.mdata(cbind(y, E), cbind(1, x, z, x*z)) ~ 1 + xx + zz + xx*zz,
    family = "0poisson",
    data = data.frame(y, E, x, z, xx, zz),
    control.fixed = list(prec = 1, prec.intercept = 1),
    control.compute = list(cpo = TRUE),
    control.family = list(link.simple = link.simple,
                          hyper = list(beta1 = list(param = c(0, 1)),
                                       beta2 = list(param = c(0, 1)),
                                       beta3 = list(param = c(0, 1)),
                                       beta4 = list(param = c(0, 1)),
                                       beta5 = list(param = c(0, 1))))))

rr <- inla(
```

```
        inla.mdata(cbind(y, E), cbind(1, xx, zz, xx*zz)) ~ 1 + x + z + x*z,
        family = "0poissonS",
        data = data.frame(y, E, x, z, xx, zz),
        control.fixed = list(prec = 1, prec.intercept = 1),
        control.compute = list(cpo = TRUE),
        ## in this case we need to define link.simple as the main link
        control.family = list(control.link = list(model = link.simple),
                              hyper = list(beta1 = list(param = c(0, 1)),
                                           beta2 = list(param = c(0, 1)),
                                           beta3 = list(param = c(0, 1)),
                                           beta4 = list(param = c(0, 1)),
                                           beta5 = list(param = c(0, 1)))))
summary(r)
summary(rr)

res <- cbind("beta" = beta,
             "0poisson" = c(r$summary.fixed$mean, r$summary.hyperpar$mean),
             "0poissonS" = c(rr$summary.hyperpar$mean, rr$summary.fixed$mean))
res <- cbind(res,
             diff = (res[, 2]-beta),
             diffS = (res[, 3]-beta),
             "diff/sd" = (res[, 2]-beta) / c(r$summary.fixed$sd, r$summary.hyperpar$sd),
             "diffS/sd" = (res[, 3]-beta) / c(rr$summary.hyperpar$sd, rr$summary.fixed$sd))
mm <- nrow(res) %/% 2
rownames(res) <- c(paste0("beta", 1:mm, ".poisson"), paste0("beta", 1:mm, ".prob"))
print(round(dig = 2, res))
```

## Example: Binomial

```
sim.binomial <- function(prob, p, size)
{
    ## - prob=zero-inflation-prob
    ## - binomial(size, p)
    stopifnot(length(prob) == length(p) && length(prob) == length(size)
                && length(prob) > 0)
    n <- length(prob)
    y <- numeric(n)
    event <- (runif(n) < prob)
    idx.zero <- which(event)
    idx.non.zero <- which(!event)
    y[idx.zero] <- 0
    y[idx.non.zero] <- rbinom(length(idx.non.zero),
                              size = size[idx.non.zero],
                              prob = p[idx.non.zero])
    return (y)
}


n <- 1000
z <- rnorm(n, sd = 0.3)
x <- rnorm(n, sd = 0.2)
xx <- rnorm(n, sd = 0.3)
zz <- rnorm(n, sd = 0.2)
Ntrials <- sample(1:10, n, replace = TRUE)

## chose link-function to use for the zero-inflation probability
link.simple <- "logit"
inv.link <- inla.link.invlogit
## link.simple <- "probit"
## inv.link <- inla.link.invprobit
## link.simple <- "cloglog"
## inv.link <- inla.link.invcloglog

beta <- c(1, 1.1, 2.1, 0, -2, 1.2, 2.2, 0)
eta2 <- beta[1] + beta[2] * xx + beta[3] * zz + beta[4] * xx * zz
eta1 <- beta[5] + beta[6] * x + beta[7] * z + beta[8] * x * z
prob <- inv.link(eta1)
p <- 1/(1 + exp(-eta2))

ok <- FALSE
while(!ok) {
    y <- sim.binomial(prob, p, Ntrials)
    ok <- !all(y == 0)
}

## head(data.frame(y, Ntrials, x, z, xx, zz))

r <- inla(
    inla.mdata(cbind(y, Ntrials), cbind(1, x, z, x*z)) ~ 1 + xx + zz + xx*zz,
    family = "0binomial",
    data = data.frame(y, Ntrials, x, z, xx, zz),
    control.fixed = list(prec = 1, prec.intercept = 1),
    control.compute = list(cpo = TRUE),
    control.family = list(link.simple = link.simple,
                          hyper = list(beta1 = list(param = c(0, 1)),
                                       beta2 = list(param = c(0, 1)),
```

```
                                          beta3 = list(param = c(0, 1)),
                                          beta4 = list(param = c(0, 1)),
                                          beta5 = list(param = c(0, 1)))))

rr <- inla(
    inla.mdata(cbind(y, Ntrials), cbind(1, xx, zz, xx*zz)) ~ 1 + x + z + x*z,
    family = "0binomialS",
    data = data.frame(y, Ntrials, x, z, xx, zz),
    control.fixed = list(prec = 1, prec.intercept = 1),
    control.compute = list(cpo = TRUE),
    ## in this case we need to define link.simple as the main link
    control.family = list(control.link = list(model = link.simple),
                          hyper = list(beta1 = list(param = c(0, 1)),
                                       beta2 = list(param = c(0, 1)),
                                       beta3 = list(param = c(0, 1)),
                                       beta4 = list(param = c(0, 1)),
                                       beta5 = list(param = c(0, 1)))))

summary(r)
summary(rr)

res <- cbind("beta" = beta,
             "0binomial" = c(r$summary.fixed$mean, r$summary.hyperpar$mean),
             "0binomialS" = c(rr$summary.hyperpar$mean, rr$summary.fixed$mean))
res <- cbind(res,
             diff = (res[, 2]-beta),
             diffS = (res[, 3]-beta),
             "diff/sd" = (res[, 2]-beta) / c(r$summary.fixed$sd, r$summary.hyperpar$sd),
             "diffS/sd" = (res[, 3]-beta) / c(rr$summary.hyperpar$sd, rr$summary.fixed$sd))
mm <- nrow(res) %/% 2
rownames(res) <- c(paste0("beta", 1:mm, ".binomial"), paste0("beta", 1:mm, ".prob"))
print(round(dig = 2, res))
```