# Binomial-mixture likelihood

#### Parametrisation

#### **Details**

This likelihood is a mixture of three binomials, where two of them are simple predictors and one is general. The probability for success is

$$p = w_1 p_1 + w_2 p_2 + w_3 p_3$$

where  $w_1 + w_2 + w_3 = 1$  are the positive weights,

$$logit(p_1) = \sum_{i=1}^{m} \beta_i z_i + \beta_{2m+1} z_{2m+1}$$

$$logit(p_2) = \sum_{i=m+1}^{2m} \beta_i z_i + \beta_{2m+1} z_{2m+2}$$

and

$$logit(p_3) = \eta + \beta_{2m+1} z_{2m+3}$$

for fixed covariates  $\{z_i\}$ . The linear predictor  $\eta$  is defined in the formula.

Note:  $\beta_{2m+1}$  is the same variable in the three expressions. The allowed range for m is  $0 \le m \le 25$ .

#### **Link-function**

The link-function is given as usual, and they are all equal.

### Hyperparameters

The 2m+1 regression coefficients  $\{\beta_i\}$  are treated as hyperparameters.

## Specification

- family="binomialmix"
- Required arguments: A  $n \times 2$  matrix Y with the observations and the number of trials s, Y = (y, s), a  $n \times (2m + 3)$  matrix Z with the covariates  $Z = c(z_1, \ldots, z_{2m+3})$ , and a  $n \times 2$  matrix W with weights  $W = (w_1, w_2)$ . The inla.mdata is used as

### Hyperparameter spesification and default values

```
doc Binomial mixture
hyper
     theta1
         hyperid 56551
         name beta1
         short.name beta1
         output.name beta1 for binomialmix observations
         output.name.intern beta1 for binomialmix observations
         initial 0
         fixed FALSE
         prior normal
         param 0 100
         to.theta function(x) x
         from.theta function(x) x
     theta2
         hyperid 56552
         name beta2
         short.name beta2
         output.name beta2 for binomialmix observations
         output.name.intern beta2 for binomialmix observations
         initial 0
         fixed FALSE
         prior normal
         param 0 100
         to.theta function(x) x
         from.theta function(x) x
     theta3
         hyperid 56553
         name beta3
         short.name beta3
         output.name beta3 for binomialmix observations
         output.name.intern beta3 for binomialmix observations
         initial 0
         fixed FALSE
         prior normal
         param 0 100
         to.theta function(x) x
         from.theta function(x) x
     theta4
         hyperid 56554
         name beta4
         short.name beta4
         output.name beta4 for binomialmix observations
         output.name.intern beta4 for binomialmix observations
         initial 0
         fixed FALSE
         prior normal
```

```
param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 56555
    name beta5
    short.name beta5
    output.name beta5 for binomialmix observations
    output.name.intern beta5 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 56556
    name beta6
    short.name beta6
    output.name beta6 for binomialmix observations
    output.name.intern beta6 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 56557
    name beta7
    short.name beta7
    output.name beta7 for binomialmix observations
    output.name.intern beta7 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 56558
    name beta8
    short.name beta8
    output.name beta8 for binomialmix observations
    output.name.intern beta8 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
```

```
to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 56559
    name beta9
    short.name beta9
    output.name beta9 for binomialmix observations
    output.name.intern beta9 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 56560
    name beta10
    short.name beta10
    output.name beta10 for binomialmix observations
    output.name.intern beta10 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta11
    hyperid 56561
    name beta11
    short.name beta11
    output.name beta11 for binomialmix observations
    output.name.intern betall for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta12
    hyperid 56562
    name beta12
    short.name beta12
    output.name beta12 for binomialmix observations
    output.name.intern beta12 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
```

```
from.theta function(x) x
theta13
    hyperid 56563
    name beta13
    short.name beta13
    output.name beta13 for binomialmix observations
    output.name.intern beta13 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta14
    hyperid 56564
    name beta14
    short.name beta14
    output.name beta14 for binomialmix observations
    output.name.intern beta14 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta15
    hyperid 56565
    name beta15
    short.name beta15
    output.name beta15 for binomialmix observations
    output.name.intern beta15 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta16
    hyperid 56566
    name beta16
    short.name beta16
    output.name beta16 for binomialmix observations
    output.name.intern beta16 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
```

```
theta17
    hyperid 56567
    name beta17
    short.name beta17
    output.name beta17 for binomialmix observations
    output.name.intern beta17 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta18
    hyperid 56568
    name beta18
    short.name beta18
    output.name beta18 for binomialmix observations
    output.name.intern beta18 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta19
    hyperid 56569
    name beta19
    short.name beta19
    output.name beta19 for binomialmix observations
    output.name.intern beta19 for binomialmix observations
    initial 0
    \mathbf{fixed} FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta20
    hyperid 56570
    name beta20
    short.name beta20
    output.name beta20 for binomialmix observations
    output.name.intern beta20 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta21
```

```
hyperid 56571
    name beta21
    short.name beta21
    output.name beta21 for binomialmix observations
    output.name.intern beta21 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta 22
    hyperid 56572
    name beta22
    short.name beta22
    output.name beta22 for binomialmix observations
    output.name.intern beta22 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta23
    hyperid 56573
    name beta23
    short.name beta23
    output.name beta23 for binomialmix observations
    output.name.intern beta23 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta24
    hyperid 56574
    name beta24
    short.name beta24
    output.name beta24 for binomialmix observations
    output.name.intern beta24 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta25
    hyperid 56575
```

```
name beta25
    short.name beta25
    output.name beta25 for binomialmix observations
    output.name.intern beta25 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta26
    hyperid 56576
    name beta26
    short.name beta26
    output.name beta26 for binomialmix observations
    output.name.intern beta26 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta27
    hyperid 56577
    name beta27
    short.name beta27
    output.name beta27 for binomialmix observations
    output.name.intern beta27 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta28
    hyperid 56578
    name beta28
    short.name beta28
    output.name beta28 for binomialmix observations
    output.name.intern beta28 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta29
    hyperid 56579
    name beta29
```

```
short.name beta29
    output.name beta29 for binomialmix observations
    output.name.intern beta29 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta30
    hyperid 56580
    name beta30
    short.name beta30
    output.name beta30 for binomialmix observations
    output.name.intern beta30 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta31
    hyperid 56581
    name beta31
    short.name beta31
    output.name beta31 for binomialmix observations
    output.name.intern beta31 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta32
    hyperid 56582
    name beta32
    short.name beta32
    output.name beta32 for binomialmix observations
    output.name.intern beta32 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta33
    hyperid 56583
    name beta33
    short.name beta33
```

```
output.name beta33 for binomialmix observations
    output.name.intern beta33 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta34
    hyperid 56584
    name beta34
    short.name beta34
    output.name beta34 for binomialmix observations
    output.name.intern beta34 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta35
    hyperid 56585
    name beta35
    short.name beta35
    output.name beta35 for binomialmix observations
    output.name.intern beta35 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta36
    hyperid 56586
    name beta36
    short.name beta36
    output.name beta36 for binomialmix observations
    output.name.intern beta36 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta37
    hyperid 56587
    name beta37
    short.name beta37
    output.name beta37 for binomialmix observations
```

```
output.name.intern beta37 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta38
    hyperid 56588
    name beta38
    short.name beta38
    output.name beta38 for binomialmix observations
    output.name.intern beta38 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta39
    hyperid 56589
    name beta39
    short.name beta39
    output.name beta39 for binomialmix observations
    output.name.intern beta39 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta40
    hyperid 56590
    name beta40
    short.name beta40
    output.name beta40 for binomialmix observations
    output.name.intern beta40 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta41
    hyperid 56591
    name beta41
    short.name beta41
    output.name beta41 for binomialmix observations
    output.name.intern beta41 for binomialmix observations
```

```
initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta42
    hyperid 56592
    name beta42
    short.name beta42
    output.name beta42 for binomialmix observations
    output.name.intern beta42 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta43
    hyperid 56593
    name beta43
    short.name beta43
    output.name beta43 for binomialmix observations
    output.name.intern beta43 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta44
    hyperid 56594
    name beta44
    short.name beta44
    output.name beta44 for binomialmix observations
    output.name.intern beta44 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta45
    hyperid 56595
    name beta45
    short.name beta45
    output.name beta45 for binomialmix observations
    output.name.intern beta45 for binomialmix observations
    initial 0
```

```
fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta46
    hyperid 56596
    name beta46
    short.name beta46
    output.name beta46 for binomialmix observations
    output.name.intern beta46 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta47
    hyperid 56597
    name beta47
    short.name beta47
    output.name beta47 for binomialmix observations
    output.name.intern beta47 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta48
    hyperid 56598
    name beta48
    short.name beta48
    output.name beta48 for binomialmix observations
    output.name.intern beta48 for binomialmix observations
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta49
    hyperid 56599
    name beta49
    short.name beta49
    output.name beta49 for binomialmix observations
    output.name.intern beta49 for binomialmix observations
    initial 0
    fixed FALSE
```

```
prior normal
         param 0 100
         to.theta function(x) x
          from.theta function(x) x
     theta50
         hyperid 56600
          name beta50
         short.name beta50
         output.name beta50 for binomialmix observations
         output.name.intern beta50 for binomialmix observations
         initial 0
         fixed FALSE
          prior normal
         param 0 100
         to.theta function(x) x
         from.theta function(x) x
     theta51
         hyperid 56601
          name beta51
         short.name beta51
         output.name beta51 for binomialmix observations
         output.name.intern beta51 for binomialmix observations
         initial 0
          fixed FALSE
         prior normal
         param 0 100
          to.theta function(x) x
         from.theta function(x) x
status experimental
survival FALSE
discrete TRUE
link default logit probit
\mathbf{pdf} binomialmix
```

### Example

Here is a simple example.

```
n <- 10<sup>5</sup>
size <- sample(5:10, n, replace = TRUE)</pre>
m < -5
m2 <- 2*m
beta.p1 <- rnorm(m, sd = 1/sqrt(m))</pre>
beta.p2 <- rnorm(m, sd = 1/sqrt(m))</pre>
beta.common <- rnorm(1, sd = 0.5)
beta <- c(beta.p1, beta.p2, beta.common)</pre>
Z \leftarrow matrix(NA, n, m2+3)
W <- matrix(NA, n, 2)
Y <- matrix(NA, n, 2)
x \leftarrow rnorm(n, sd = 0.5)
xx \leftarrow rnorm(n, sd = 0.5)
eta <- numeric(n)</pre>
for (i in 1:n) {
    Z[i, ] \leftarrow rnorm(m2+3)
    w <- c(rbeta(2, 1, 10), rbeta(1, 10, 1))
    w \leftarrow w/sum(w)
    W[i, ] \leftarrow w[1:2]
    p1 <- inla.link.invlogit(sum(beta.p1 * Z[i, seq_len(m)]) + beta.common * Z[i, m2+1])
    p2 <- inla.link.invlogit(sum(beta.p2 * Z[i, m + seq_len(m)]) + beta.common * Z[i, m2+2])
    eta[i] \leftarrow 1 + x[i] + xx[i] + beta.common * Z[i, m2+3]
    p3 <- inla.link.invlogit(eta[i])
    p \leftarrow w[1] * p1 + w[2] * p2 + w[3] * p3
    Y[i, ] <- c(rbinom(1, size = size[i], prob = p), size[i])
r \leftarrow inla(inla.mdata(Y, Z, W) ~ 1 + x + xx,
           family = "binomialmix",
           data = list(Y = Y, Z = Z, W = W, x = x, xx = xx),
           verbose = TRUE,
           control.inla = list(int.strategy = "eb"))
## r <- inla.rerun(r)
print(round(dig = 4, cbind(estimate = r$summary.fixed[,"mean"], true = 1)))
print(round(dig = 4, cbind(estimate = r$summary.hyperpar[,"mean"], true = beta)))
```