

Sigmoid Function : $g(a) = \frac{1}{1 + e^{-a}}$

Simplifying $g(a)(1 - g(a))$:

$$\begin{aligned}
 g(a)(1 - g(a)) &= \left(\frac{1}{1 + e^{-a}} \right) \left(1 - \frac{1}{1 + e^{-a}} \right) \\
 &= \left(\frac{1}{1 + e^{-a}} - \frac{1}{(1 + e^{-a})(1 + e^{-a})} \right) \\
 &= \left(\frac{1 \times (1 + e^{-a})}{(1 + e^{-a}) \times (1 + e^{-a})} - \frac{1}{(1 + e^{-a})(1 + e^{-a})} \right) \\
 &= \left(\frac{1 + e^{-a} - 1}{(1 + e^{-a})^2} \right)
 \end{aligned}$$

$$g(a)(1 - g(a)) = \frac{e^{-a}}{(1 + e^{-a})^2}$$

Taking First derivative of $g(a)$, using chain rule

$$\frac{dg(a)}{da} = \frac{dg}{du} \cdot \frac{1}{u} \cdot \frac{du}{da} \quad \Big| \quad u = 1 + e^{-a}$$

$$= - \frac{1}{u^2} \frac{dg}{da} (1 + e^{-a})$$

$$= - \frac{1}{u^2} \left[\frac{dg}{da} (1) + \frac{dg}{da} (e^{-a}) \right]$$

$$= - \frac{e^{-a}}{u^2} \frac{dg}{da} (-a)$$

$$= - \frac{-1 \times e^{-a}}{u^2}$$

$$\frac{dg(a)}{da} = \frac{e^{-a}}{(1+e^{-a})^2}$$

This shows that $\frac{dg(a)}{da} = g(a)(1-g(a)) = \frac{e^{-a}}{(1+e^{-a})^2}$