Analyzing a 1D Hypersonic Flow Path

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1 Introduction

This project involved analyzing the 1D Flow Path of a hypersonic jet which meant applying various thermodynamic principles that we learned over the course of this quarter in ME 132. The flow path analysis of the hypersonic jet was split into 6 sections that included a converging nozzle with a shock wave, a shock train, a combustor and finally a diverging nozzle.

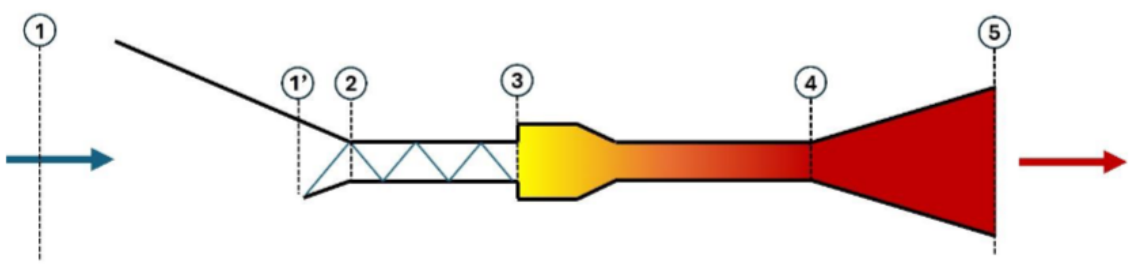

Fig 1 :1D Schematic of hypersonic flow path.

2 State 1 to 1'

Givens: From state 1 to 1', high speed air enters into the vehicle through a Busemann nozzle and is isentropically compressed with a pressure compression ratio of

$$P_{1'} = 20P_1 = 20 * (26,500) = 530,000Pa \tag{1}$$

The hypersonic vehicle is flying at a 10km altitude and traveling at Mach 7 .Based on this initial information and using the Engineering Toolbox 1, at 10km of altitude, the inlet temperature T1 was found to be 223.25K and the inlet pressure 26500Pa.

Since the process is isentropic, we want to use an isentropic relation to find T1'.

$$s_{1'} - s_1 = \int_0^T C_p(T)\frac{1}{T}\,dT - R\ln\left(\frac{P_1'}{P_1}\right) \tag{2}$$

However, in this case, the temperature change during the process is quite large and the specific heats of the air vary nonlinearly within the large temperature range. Therefore we cannot assume constant specific heats. In order to prevent laborious integrals of the Cp relations, we can use standard reference temperature entropies 2 to replace the typical Cp(T) relationship in the equation for entropy.

$$\int_0^T C_p(T)\frac{1}{T}\,dT = s_{1'}^{\circ} - s_1^{\circ} \tag{3}$$

$$s_{1'} - s_1 = s_{1'}^{\circ} - s_1^{\circ} - R\ln\left(\frac{P_1'}{P_1}\right) \tag{4}$$

In order to calculate s_1° we simply set the gas object to standard pressure(1 bar) and T1. Since this process is isentropic, we were able to set the left side of equation (2) to zero and isolate $s_{1'}^{\circ}$. So, I used this equation to find the correct temperature at point 1'.

$$s_{1'}^{\circ} = s_1^{\circ} + R\ln\left(\frac{P_1'}{P_1}\right) \tag{5}$$

Our methodology was to create an array of temperature values that ranged from the inital temperature at the inlet to 600K, with a step size of 0.5 and calculate the $s_{1'}^{\circ}$ for each temperature guess. We did this by setting a gas at the temperature at each index and standard pressure. We also calculated the value of the second half of the equation and when those two values matched, it meant that we had found the correct temperature for state 1'. We found this static temperature to be

$$T_{1static} = 432.75K \tag{6}$$

With this static temperature value and the pressure value at 1’ due to the given compression ratio of 20, we were able to calculate the stagnation temperature, stagnation pressure and Mach number. The gas composition remained the same as the inlet air which is ’O2:3, N2:11.28’.

Since the air is moving quickly, we can make the assumption that this is an adiabatic process and there is also no work being done to or by the system. Using this assumption and conservation of energy, we can calculate the velocity of air at 1’. We can also assume that the aircraft is not changing altitude and therefore the potential energy terms also cancel in the energy balance.

$$q_{in} + w_{in} + h_{01} + gz_1 = q_{out} + w_{out} + h_{01'} + gz_1' \tag{7}$$

$$h_{01} = h_{01'} \tag{8}$$

This simplifies to:

$$h_1 + \frac{V_1^2}{2} = h_{1'} + \frac{V_{1'}^2}{2} \tag{9}$$

$$V_{1'} = \sqrt{2\left(h_1 - h_{1'} + \frac{V_1^2}{2}\right)} \tag{10}$$

We got the values of enthalpies directly from cantera by setting the gas to the correct temperature and pressure. Using this value, we can calculate the stagnation temperature, the stagnation pressure and the Mach number.

$$Ma'_1 = \frac{V'_1}{c'_1} = 4.7943 \tag{11}$$

$$T_{o'_1} = T'_1 + \frac{V'^2_1}{2C_p} = 2377K \tag{12}$$

$$P_{o'_1} = P'_1 + \frac{V'^2_1}{2} = 2523.6kPa \tag{13}$$

Reference the appendix document for the full Matlab code on this section.

3 State 1' to 2

Once we have acquired the necessary thermodynamic properties at state 1', we can employ what we know about oblique shocks and exploit some assumptions we make to find the properties at state 2. To start we are given the shock angle, $\beta = 14$, and the deflection angle, $\theta = 0.6$, and we already know the mach number at 1'. From these values we can employ a set of 3 equations to get the Mach number at 2 which we found to be 0.5351.

$$Ma_{1',n} = Ma_{1'} sin(\beta) \tag{14}$$

$$Ma_{2,n} = \sqrt{\frac{(k-1)(Ma^2_{1',n}) + 2}{(2k)(Ma^2_{1',n}) - k + 1}} \tag{15}$$

$$Ma_2 = \frac{Ma_{2,n}}{sin(\beta - \theta)} = 0.5351 \tag{16}$$

After this step, we can start to find the thermodynamic properties at state 2. The first thermodynamic property I will concern myself with is finding the temperature at state 2. Since the air is moving very quickly, we can make the assumption that it isn't exchanging any heat with its surroundings and thus is an adiabatic process. Because of this, conservation of stagnation temperature holds from 1' to 2 — this is how we will get the temperature at 2. When inspecting the equation for stagnation temperature (see equation (11)), we see that all values only rely on T2. Cp is a function of the given T2 as well as V2 which can be found by multiplying the Mach number at 2 (Ma2) with the sound speed at 2, which is again a function of T2. Because of this, we can iterate through a list of guess temperatures at state 3, and the one that satisfies the conservation of stagnation temperature between the two states is the correct T2. This was found to be 2257 K (refer to code in appendix document). Lastly, to find the pressure at state 2, which will open the door to solve the remaining values, we must employ an equation that assumes a constant k (specific heat ratio).

$$P_2 = (P_{1'})(\frac{(2k)(Ma_{1',n}^2) - k + 1}{k+1}) = 13.551MPA \tag{17}$$

To verify if we can properly use this equation, we graphed corresponding k values to the temperature range from 1' to 2, and inspected the range of k values we received. If we determine k doesn't change that much, we can find its average value and employ that as the "constant" k value for the pressure equation. When we plotted the k values for the corresponding temperature range, we found the difference between the max k value and the minimum k value to be around 0.1.

This was sufficient enough for us to assume a constant k around the average k value within the temperature range which we found to be $k_{avg} = 1.325$. From this we can employ the pressure equation with the relevant values to obtain a $P_2 = 13.551MPa$. After this, stagnation pressure can

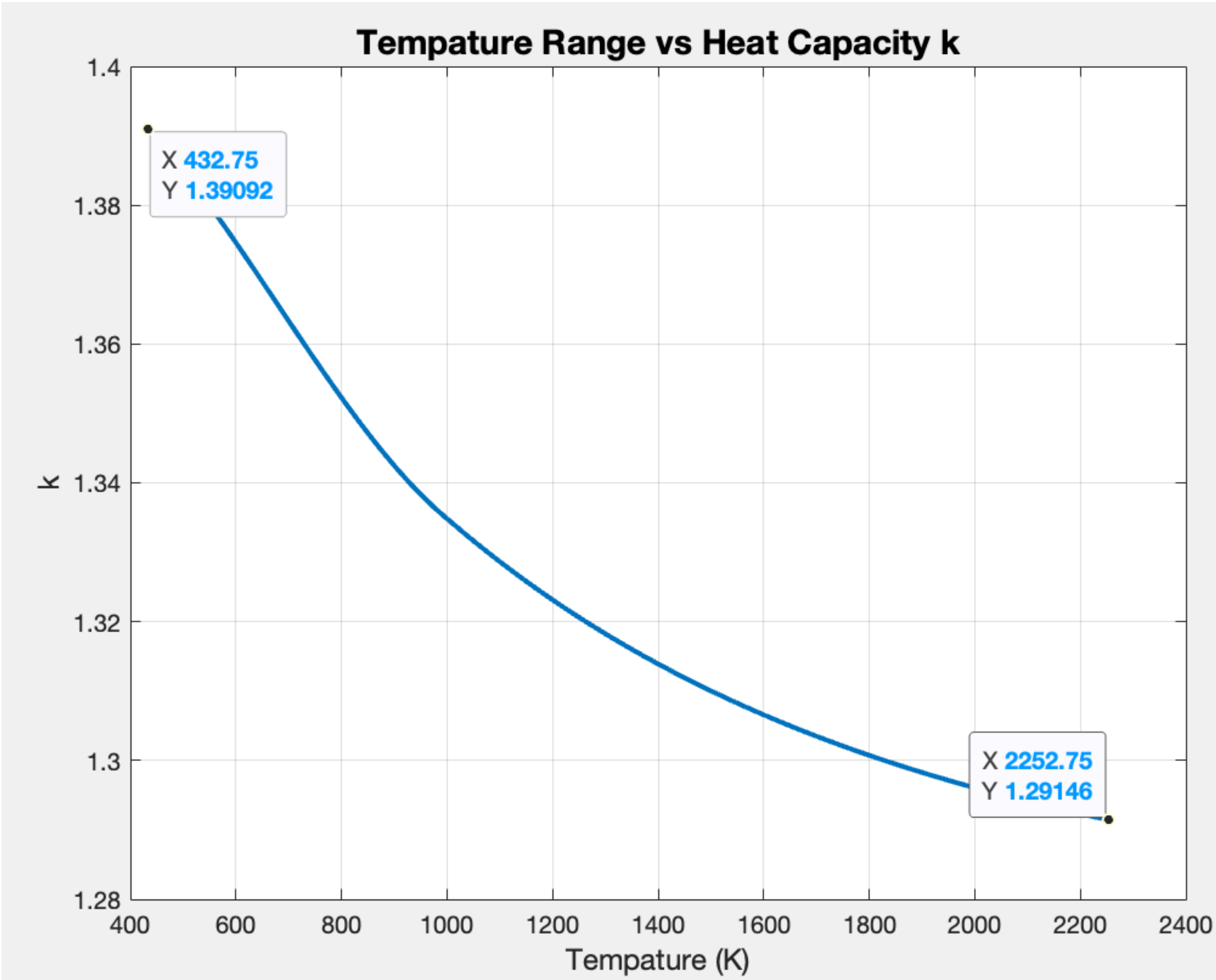

Fig 2 :K value range over relevant temperature range.

be found quite easily by including a velocity term.

$$P_{o_2} = P_2 + \frac{V_2'^2}{2} = 13.672MPa \tag{18}$$

Since stagnation temperature is constant from 1' to 2, $T_{o_2} = 2377K$, we now have all the relevant values needed for state 2. The gas composition remained the same as the inlet air which is 'O2:3, N2:11.28' since nothing related to the gas composition was altered in this section.

4 State 2 to 3

After acquiring the thermodynamic properties at state 2, we then acquired the thermodynamic properties at state 3. We were given that the mach number at state 3 is 0.75. Since the air is moving very quickly, we can make the assumption that it isn't exchanging any heat with its surroundings and thus is an adiabatic process. Because of this, conservation of stagnation temperature holds from state 2 to state 3 which equates to 2377 Kelvin. To start calculating state properties, we iterated through a range of temperature values. We chose values that gradually increased starting at the temperature at state 2. We used the fact that this process is adiabatic along with the conservation of energy equation to find a guessed velocity at state 3

$$V_3 = \sqrt{2\left(h_2 - h_3 + \frac{V_2^2}{2}\right)} \tag{19}$$

.Then using this guessed velocity and guessed sound speed we calculated a guessed mach number using

$$Ma = \frac{V}{c} \tag{20}$$

.Once we found a static temperature that produced a mach number of 0.75 in our guess mach number array, we concluded that this was the static temperature at state 3, 2355 Kelvin. Using the found static temperature, the stagnation pressure was found using conservation of mass

$$\dot{m} = \frac{A\,\mathrm{Ma}\,P_0\sqrt{\frac{k}{RT_0}}}{\left(1 + \frac{(k-1)\,\mathrm{Ma}^2}{2}\right)^{\frac{k+1}{2(k-1)}}} \tag{21}$$

.By equating the mass flow at state 2 to the mass flow at state 3 stagnation pressure was found

$$P_{o_3} = \frac{\dot{m}_{\text{state 2}} \left(1 + \frac{(k_3-1)\,\mathrm{Ma}_3^2}{2}\right)^{\frac{k_3+1}{2\,(k_3-1)}}}{\mathrm{Ma}_3 \sqrt{\frac{k_3}{R T_{o_3}}}} \tag{22}$$

.This resulted in a stagnation pressure of 11.34 MPa . Finally, from stagnation pressure the static pressure was found using

$$P_{o_3} = P_3 + \frac{V_3^2}{2} \tag{23}$$

, this resulted in a static pressure of 11.103 MPa. We now have all the relevant values needed for state 3. The gas composition remained the same as the inlet air which is 'O2:3, N2:11.28' since nothing related to the gas composition was altered in this section.

Reference the appendix document for the full Matlab code on this section.

5 State 3 to 4

After the shock train which compresses the air even more, 3 to 4 features fuel and heat addition under constant pressure by adding and burning ethylene with the ambient air. If we assume conservation of mass flow rate, and we were told the combustion chamber has a constant cross-sectional area, we can employ the equation set up below.

$$(\rho_3)(V_3) = (\rho_4)(V_4) \tag{24}$$

We were also told to assume the combustion is thermally choked, meaning it reaches a temperature where its sound speed equals the fluid velocity (Ma4 = 1). From the given ODE solver, we can get the temperature and composition of the gas as a function of time through the combustion.

We can iterate through each time step to get a new gas object based on the given composition at that time, find the $\rho 4, then employ the equation above to get the gas' velocity, V4. At the same time step, we can also gett$ $3617K, with a composition listed below, with mole fractions to be around$: "$H20$: $0.06, CO2$: $0.09, O2 : 0.03, OH : 0.019, CO : 0.07, O : 0.01, N2 : 0.718$"

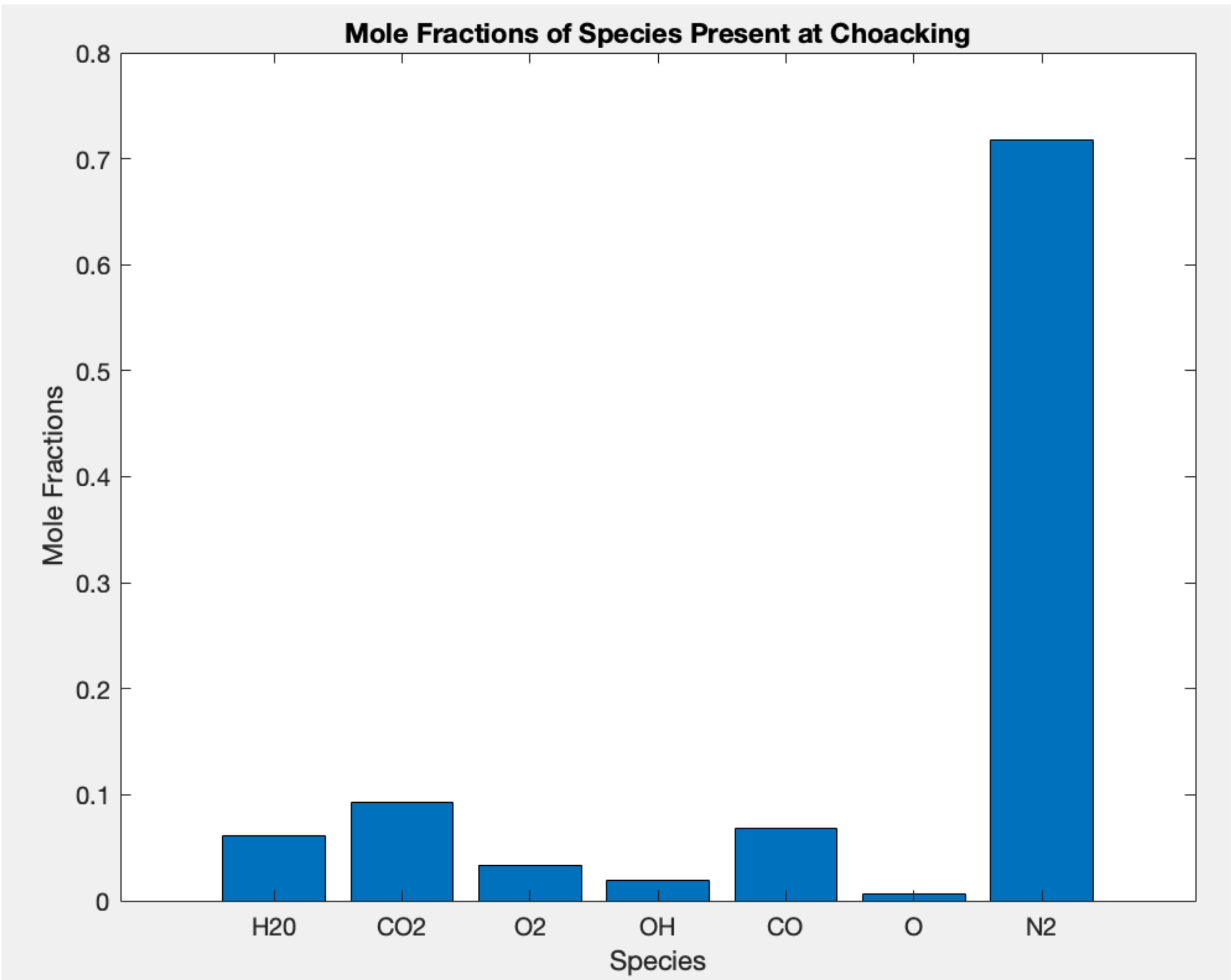

Fig 3 :Present species with a mole fraction above 0.01 when gas is chocked

Since we know combustion happened under constant pressure, we can find our P3 = P4 = 11.10 MPa. From our numerical analysis we found the gas at state 4 to have a velocity of 1146.1 m/s. From this we can employ our equations for stagnation temperature and pressure (see equations (11) and (12)). We found these values to be T04 = 3633.2 K and P04 = 11.76 MPa.

6 State 4 to 5

After finding the thermodynamic properties at state 4 when the flow is being thermally choked, we then calculated the properties at state 5 where the flow is chemically frozen (meaning the gas composition stays the same: ”H20:0.06 , CO2:0.09 , O2:0.03 , OH:0.019 , CO:0.07 , O:0.01 , N2:0.718”) and expands isentropically in the exit nozzle. This allows us to use the same chemical composition calculated at state 4 for finding properties at state 5.

Since this jet is hypersonic and thus aims to travel above Mach 1, we can assume that the flow at the nozzle exit is supersonic. According to this graph from the textbook 2, if the flow is supersonic at the nozzle exit and the process is isentropic (in the problem statement), there is no shock in the no shock in the nozzle. Since there are no shocks in the nozzle the back pressure equals the pressure at the exit. This corresponds to the PF/PG line on the figure. Based on this, we can make the assumption that P5 is equal to ambient pressure which is 26500 Pa.

To calculate the properties as state 5, we begin by defining our initial conditions: the temperature at state 4, 3617K, the velocity at state 4, 1146.1 m/s and the (back pressure) ambient pressure at 10 km above sea level, 26500 Pa.

To find the temperature at state 5, we rely on the isentropic nature of the expansion by utilizing a similar methodology to state 1-1’. We firstly rely on the isentropic nature of the expansion but have to again consider that the temperature change during the process is quite large and the specific heats of the air may vary nonlinearly. So we cannot assume constant specific heats and thus must use equation (4) again:

$$s_5 - s_4 = s_5^\circ - s_4^\circ - R\ln\left(\frac{P_5}{P_4}\right) \tag{25}$$

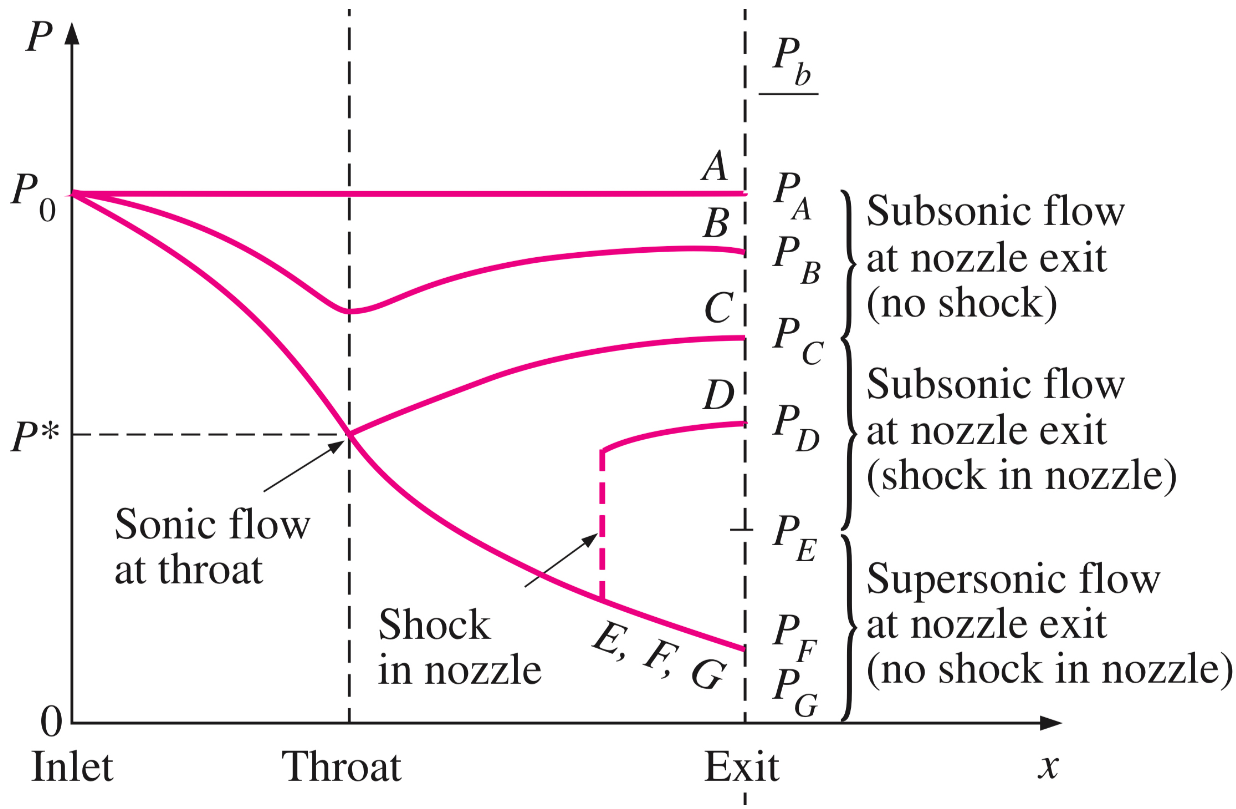

Fig 4 :The effects of back pressure on the flow through a converging-diverging nozzle

After calculating s4o, the entropy at reference pressure at state 4, we were able to rearrange the change in entropy equation in order to solve for s5o.

$$s_5^\circ = s_4^\circ + R\ln\left(\frac{P_5}{P_4}\right) \tag{26}$$

Using a range of temperatures starting from temperature at state 4 to 2730K with a step of -0.1K, we tabulated values of entropy at ambient pressure (s5) and the calculated s5o values for each temperature iteration. We were then able to find the temperature at which the second half of the above equation and s5o were equal and recorded that value as T5.

We are able to calculate the velocity at 5 using the principle of conservation of energy (equation 6). From states 4 to 5, we have already established that there is no heat being transferred and no work being done, which allows us to reduce our equation to

$$h_4 + \frac{V_4^2}{2} = h_5 + \frac{V_5^2}{2} \tag{27}$$

Since state 5 is fully defined, we can find the enthalpy at state 5, and rearrange the equation to solve for v5. Then, after finding the speed of sound at state 5 we divided our velocity at state 5 by the speed of sound in order to calculate the Mach number. Finally, though using equations 12 and 13, we found the stagnation pressures and temperatures.

$$Ma_5 = 4.76 \tag{28}$$

$$P_{5static} = 26500Pa \tag{29}$$

$$P_{5stagnation} = 4448.186KPa \tag{30}$$

$$T_{5static} = 962.10K \tag{31}$$

$$T_{5stagnation} = 4462.63K \tag{32}$$

The composition of the gas remains the same from state 4 (post combustion)

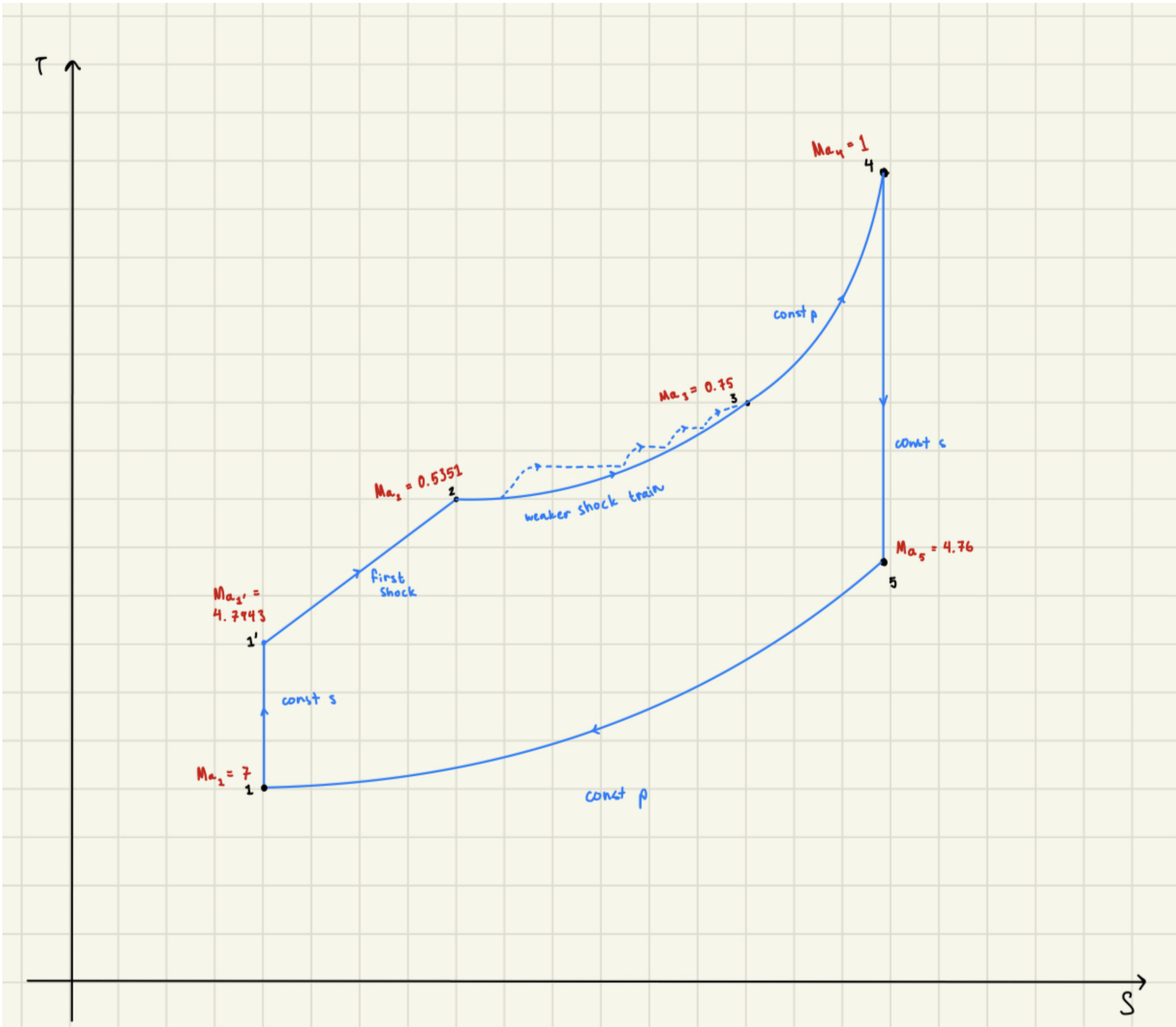

Fig 5 :Hand drawn T-S diagram with mach numbers labeled for entire system

7 Part 2a

In this section, we determine the thrust created by our engine, based on our cantera calculations, the propulsive power and the efficiency.

$$F_{thrust} = \dot{m}(V_5 - V_1) = 10.024kN \tag{33}$$

$$\dot{m} = \rho A V_2 = 11.48\frac{kg}{s} \tag{34}$$

$$\dot{P}_{propulsive} = \dot{m}(V_5 - V_1)(V_{aircraft}) = 21.057MW \tag{35}$$

The velocity of the aircraft is the same as V1 since V1 was calculated using the fact that the aircraft was traveling at Mach 7 and at 10km at altitude. Our mass flow rate was calculate using the values at state (insert state).

In order to calculate propulsive efficiency, we used the Lower Heating Value of Ethylene as our heat value because the lower heating value more accurately represents the fact that the energy used to turn water into water vapor (which would correspond to a HHV) isn't "useful" energy when it comes to propulsive power. This makes sense because if the LHV is the most energy you could get out of a fuel (and we know not all of this energy will actually be used), then using a HHV which gives us a higher number would be even more unrealistic given that most of the energy from the LHV wont be used.

$$\eta = \frac{\dot{P}_{propulsive}}{\dot{Q}_{in}} = \frac{(V_5 - V_1)(V_{aircraft})}{(LHV)_{ethylene}} = 3.84\% \tag{36}$$

Reference the appendix document for the full Matlab code on this section.

8 Part 2b

Our methodology to solve for the Mach number at the combustor inlet of maximum propulsion efficiency was a combination of our methodologies for states 2 to 3, 3 to 4 and 4 to 5. The way in which we calculated state 3 was dependent on the Mach number initially given at state 3 (please reference section on state 2 to 3 for exact methodology). In order to find which Mach Number at the combustor inlet lead to the maximum propulsion efficiency, we made an array of Mach numbers starting from 0.1 to 7 and looped through the code for 2 to 3 in order to have an array of pressures and temperatures that fully define state 3 for every Mach number.

We then cycled through our code and methodologies for 3 to 4 and 4 to 5 for every mach number in our array of guesses in order to have the values of propulsive efficiency that corresponded to every mach number. We then took the maximum efficiency value from that array of values and matched it to its corresponding Mach Number.

We were able to find a value, yet we do not think it is entirely correct, despite our correct methodology. In order to program part 2b, we had to create many nested for loops around our code from 2-3, 3-4, and 4-5. Somewhere along that process, we have a bug that causes our efficiency value for Mach numbers 0.75-2.955 and 4.89-7.5 to output a constant value of 1.0177%. We traced the issue to our T5 array (array of temperatures for state 5 based on each mach number) where we were getting some constant values but we unable to fix the issue.

However, even with this coding bug, there was a range of mach numbers that produced a range of efficiencies above the constant value of 1.0177%. From this range we got a peak max value of efficiency of 44.504% that corresponded to a Mach value of 4.53. Our code for this section is also included in the appendix document.

References

1 T. E. Toolbox, “Us standard atmosphere vs. altitude,” (2003).

2 Y. Cengel and M. Boles, *Thermodynamics: An Engineering Approach, 5th ed*, McGraw-Hill (2006).