ME 543 Computational Fluid Dynamics presentation

Name:- Vivek Vijay Potdar

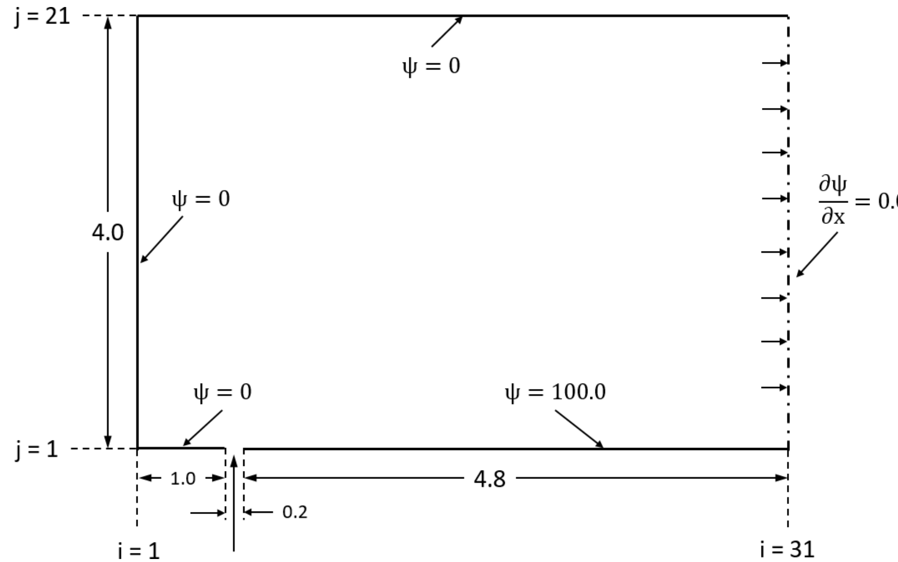
Roll No :- 224103330

Assignment 1

Governing Equation :-

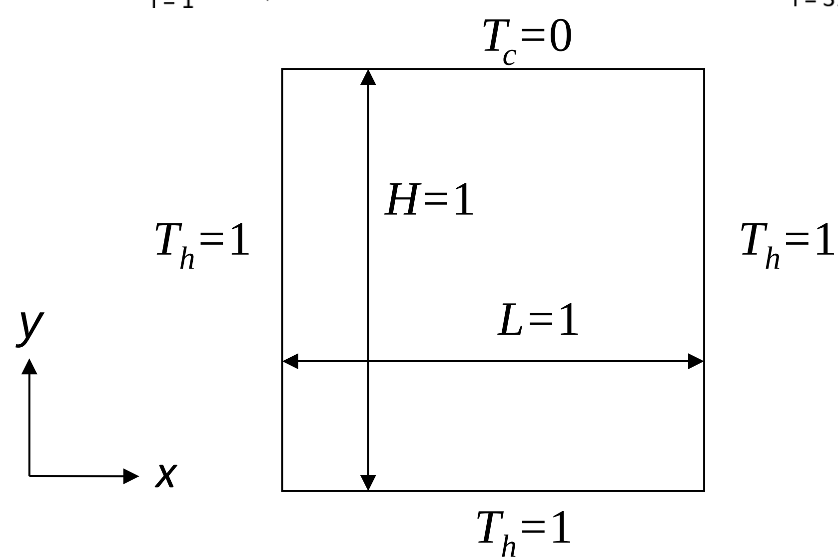
1)First Problem

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$



2)Second Problem

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$



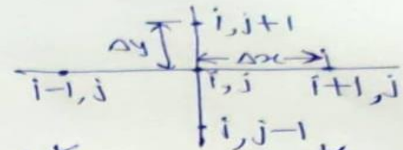
Assignment 1

- **Governing Equations** :-

Assignment-1

General PDE :-

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$



$$\beta = \frac{\Delta x}{\Delta y}$$

1. Jacobi iterative Method

$$\phi^{k+1}_{i,j} = \frac{1}{2(1+\beta^2)} \left[\beta^2 \phi^{k}_{i,j-1} + \phi^{k}_{i-1,j} + \phi^{k}_{i+1,j} + \beta^2 \phi^{k}_{i,j+1} \right]$$

2. Point gauss seidal Method

$$\phi^{k+1}_{i,j} = \frac{1}{2(1+\beta^2)} \left[\beta^2 \phi^{k+1}_{i,j-1} + \phi^{k+1}_{i-1,j} + \phi^{k}_{i+1,j} + \beta^2 \phi^{k}_{i,j+1} \right]$$

3. Point Succesive Over relaxation (PSOR) Method

$$\phi^{k+1}_{i,j} = (1-\omega)\, \phi^{k}_{i,j} + \frac{\omega}{2(1+\beta^2)} \left[\beta^2 \phi^{k+1}_{i,j-1} + \phi^{k+1}_{i-1,j} + \phi^{k}_{i+1,j} + \beta^2 \phi^{k}_{i,j+1} \right]$$

ω = over relaxation factor

4. Line gauss siedal iterative Method

$$\phi^{k+1}_{i-1,j} - 2(1+\beta^2)\, \phi^{k+1}_{i,j} + \phi^{k+1}_{i+1,j} = -\beta^2 \left(\phi^{k+1}_{i,j-1} + \phi^{k}_{i,j+1} \right)$$

↳ Row wise

$$\beta^2 \phi^{k+1}_{i,j+1} - 2(1+\beta^2)\, \phi^{k+1}_{i,j} + \beta^2 \phi^{k+1}_{i,j-1} = -\left(\phi^{k+1}_{i-1,j} + \phi^{k}_{i+1,j} \right)$$

↳ column wise

5. Alternating direction Implicit Method

Step 1:- x sweep $\Rightarrow$ $k \rightarrow k+\frac{1}{2}$

$$\phi^{k+\frac{1}{2}}_{i-1,j} + \phi^{k+\frac{1}{2}}_{i+1,j} - 2(1+\beta^2)\, \phi^{k+\frac{1}{2}}_{i,j} = -\beta^2 \left(\phi^{k+\frac{1}{2}}_{i,j-1} + \phi^{k}_{i,j+1} \right)$$

Step 2:- y sweep $\Rightarrow$ $k+\frac{1}{2} \rightarrow k$

$$\beta^2 \phi^{k+1}_{i,j+1} + \beta^2 \phi^{k+1}_{i,j-1} - 2(1+\beta^2)\, \phi^{k+1}_{i,j} = -\left(\phi^{k+1}_{i-1,j} + \phi^{k+\frac{1}{2}}_{i+1,j} \right)$$

Results:-

- ***Problem 1:-***

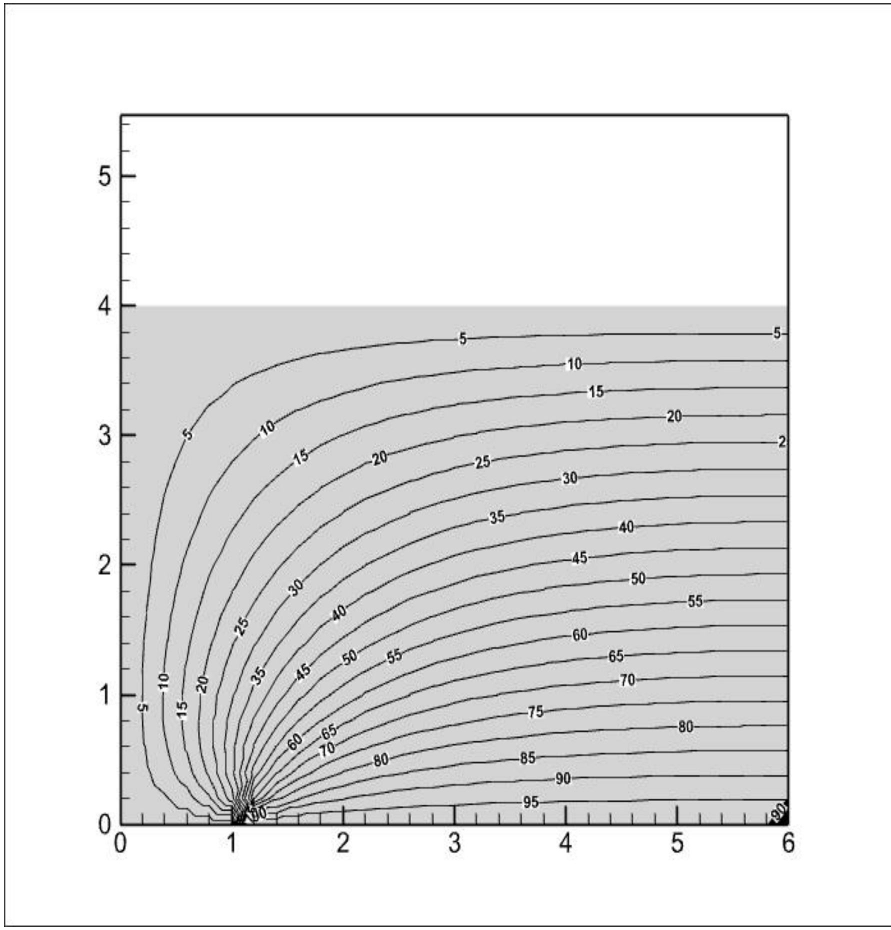


Fig 1. Stream lines for grid 31*21

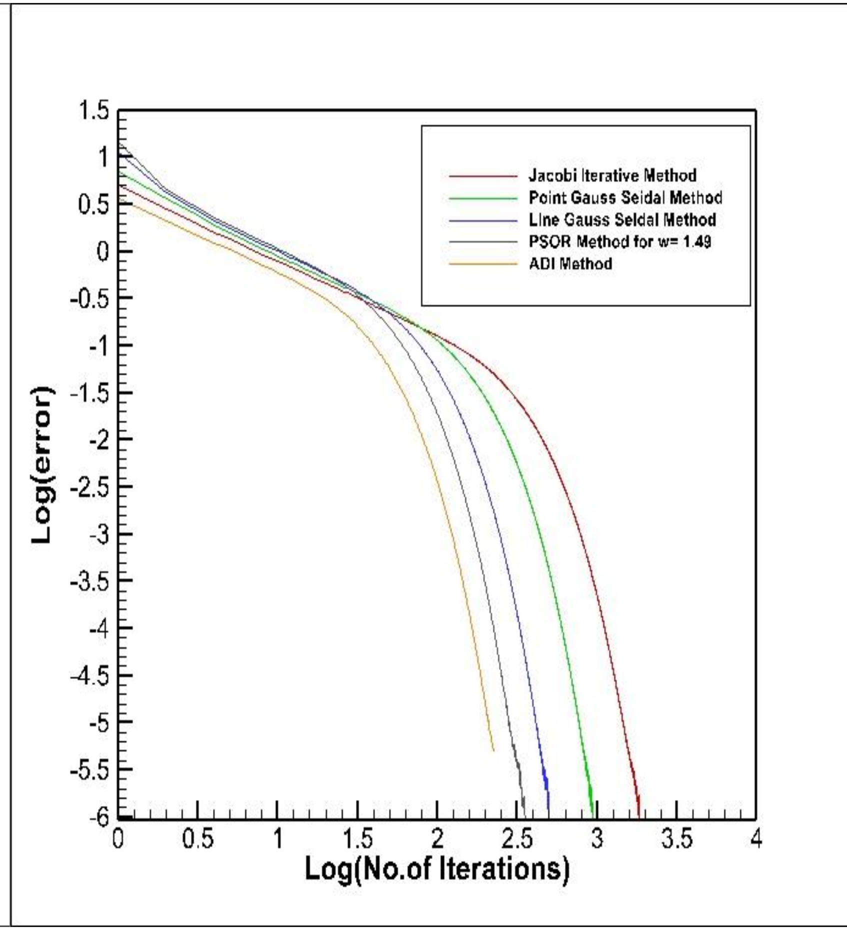


Fig 2. Comparison Study of Numbers of Iterations(grid size 31*21)

Results:-

- ***Problem 2:-***

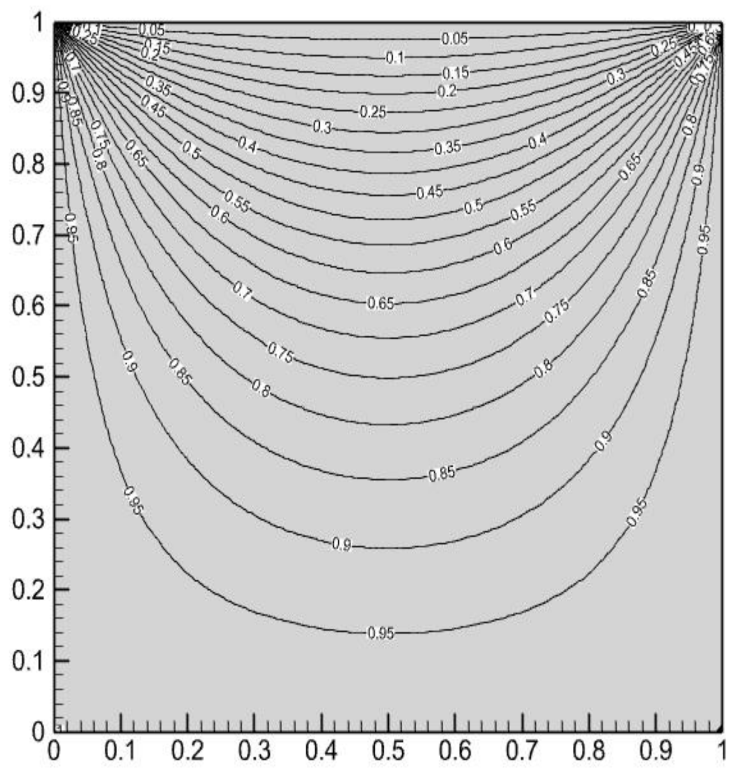


Fig 3. Isotherms for grid 100*100

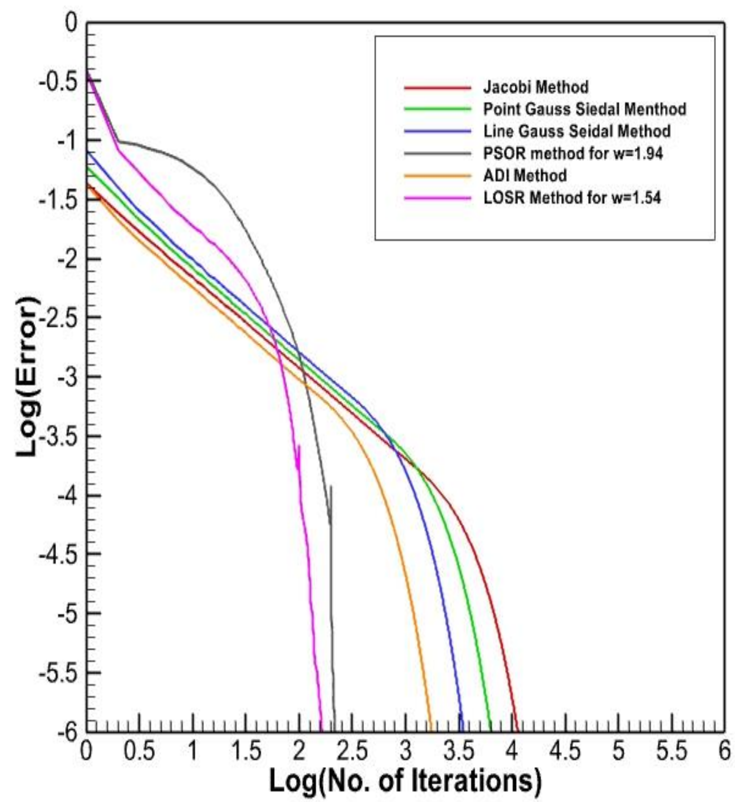


Fig 4. Comparison Study of Numbers of Iterations(grid size 100*100)

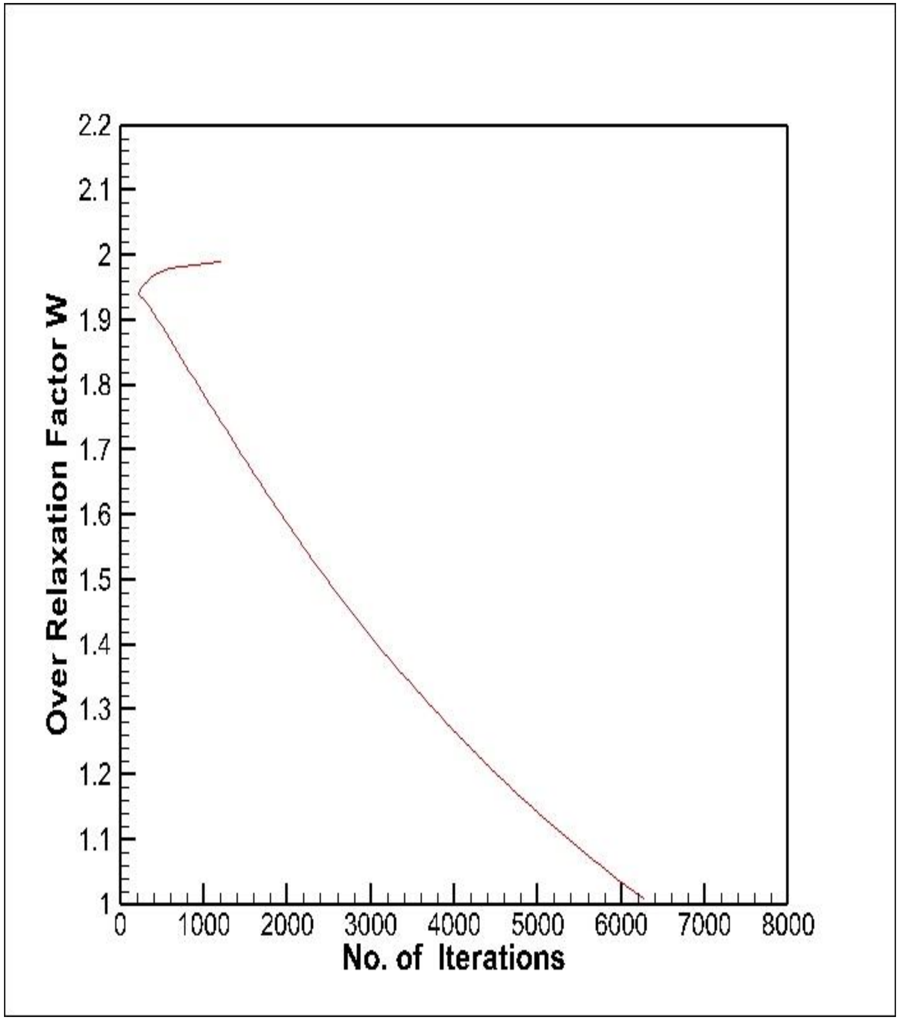


Fig 5. W vs Number of Iterations (PSOR Method) for grid 100*100

Sr. No.	*Method*	*No. of Iterations*
1	*Jacobi Method*	*1843*
2	*Point Gauss Seidel Method*	*932*
3	*Line Gauss Seidel Method*	*497*
4	*PSOR Method at w opt =1.49*	*353*
5	*ADI Method*	*225*

Problem 1 Table 1:- ***Numbers of Iterations(grid size 31*21)***

Sr. No.	Method	No. of Iterations
1	Jacobi Method	11371
2	Point Gauss Seidel Method	6365
3	Line Gauss Seidel Method	3523
4	PSOR Method at w opt =1.94	218
5	ADI Method	1765

Problem 2 Table 2:- ***Numbers of Iterations(grid size100*100)***

Assignment 2

governing Equation :-

$$\frac{\partial u}{\partial t} = \frac{1}{\mathrm{Re}_H} \cdot \frac{\partial^2 u}{\partial y^2}$$

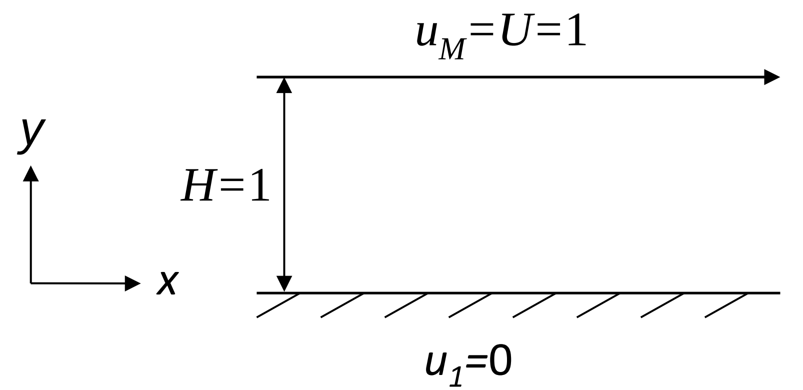


Fig.Geometry with Boundary conditions

Assignment 2

- **Governing Equations** :-

Assignment-2

PDE :-

$$\frac{\partial u}{\partial t} = \frac{1}{Re_H}\frac{\partial^2 u}{\partial y^2} \qquad \gamma_y = \frac{\Delta t}{Re_H \Delta x}$$

1. FTCS

$$u_i^{n+1} = \gamma_y u_{i-1}^n + \gamma_y u_{i+1}^n + (1-2\gamma_y)u_i^n$$

2. BTCS

$$\gamma_y u_{i-1}^{n+1} + \gamma_y u_{i+1}^{n+1} - (1+2\gamma_y)u_i^{n+1} = -u_i^n$$

3. Crank- Nicolson Scheme

Step1 :- $n \rightarrow n+\frac{1}{2}$ = FTCS scheme.

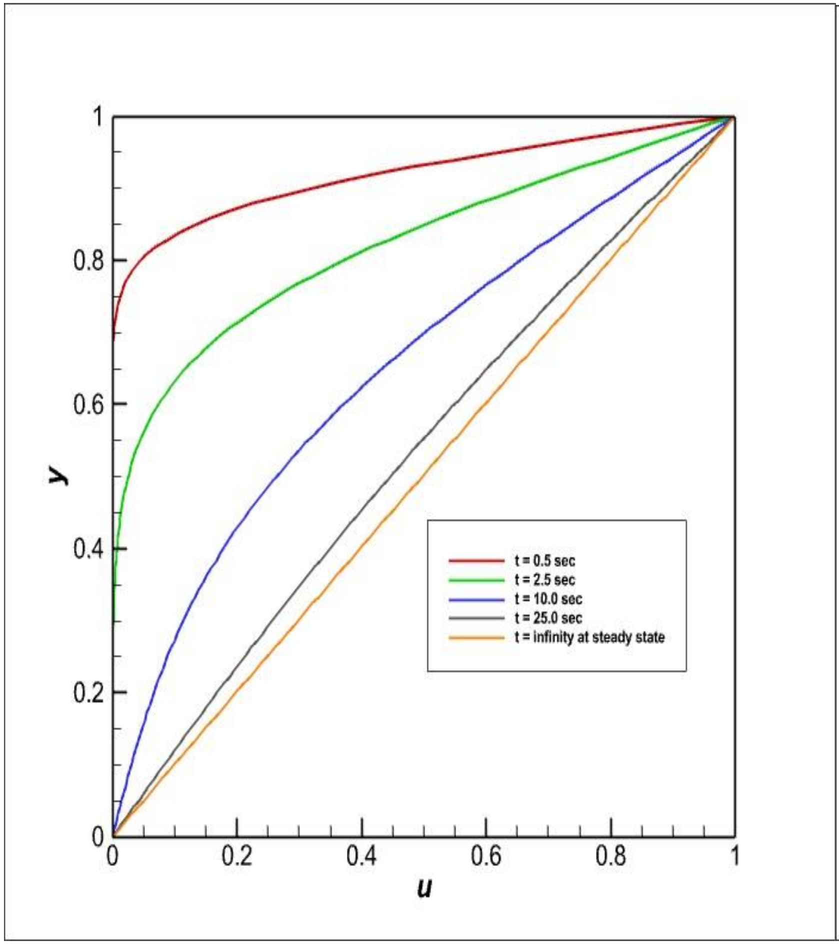
$$u_i^{n+1/2} = \gamma_y u_{i-1}^n + \gamma_y u_{i+1}^n + (1-2\gamma_y)u_i^n$$

Step2 :- $n+\frac{1}{2} \rightarrow n+1$ = BTCS scheme.

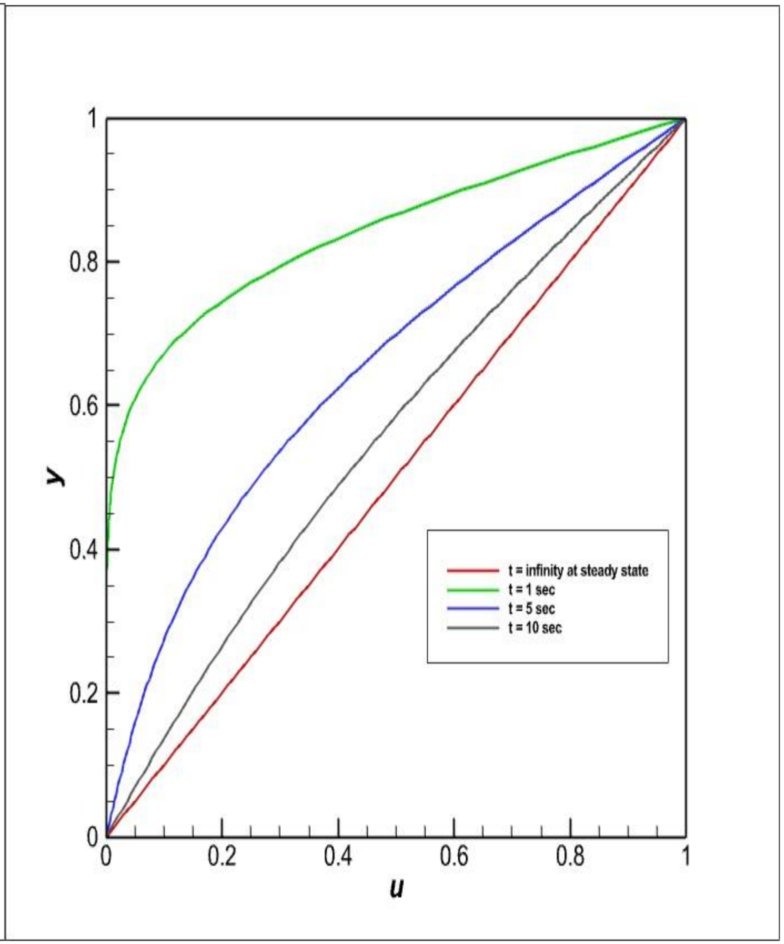
$$\gamma_y u_{i-1}^{n+1} + \gamma_y u_{i+1}^{n+1} - (1+2\gamma_y)u_i^{n+1} = -u_i^{n+\frac{1}{2}}$$

Velocity Profiles at Different time including Steady State

1. FTCS Scheme:-

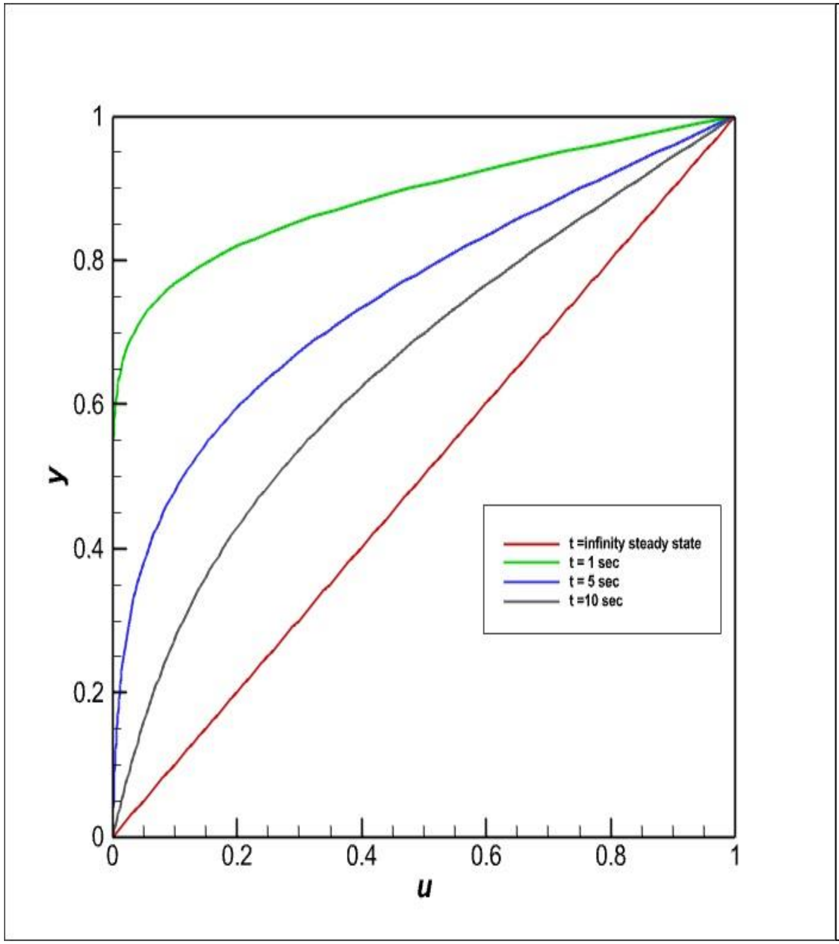


2. Crank Nicolson Scheme:-

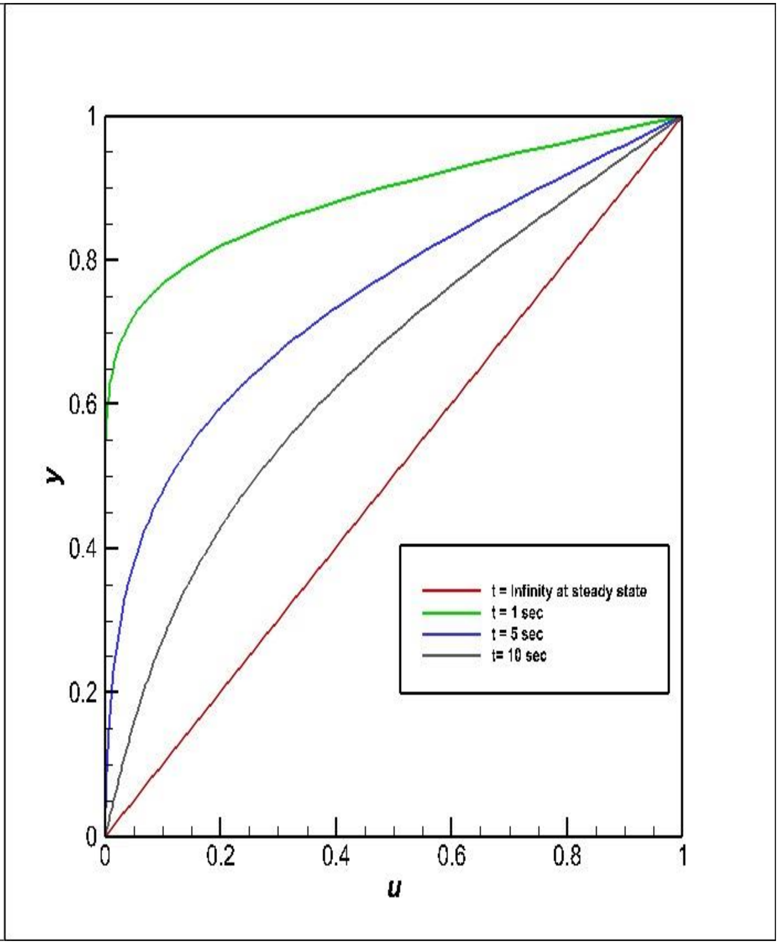


Velocity Profiles at Different time including Steady State

3. BTCS Scheme(Linear Solver Method):-

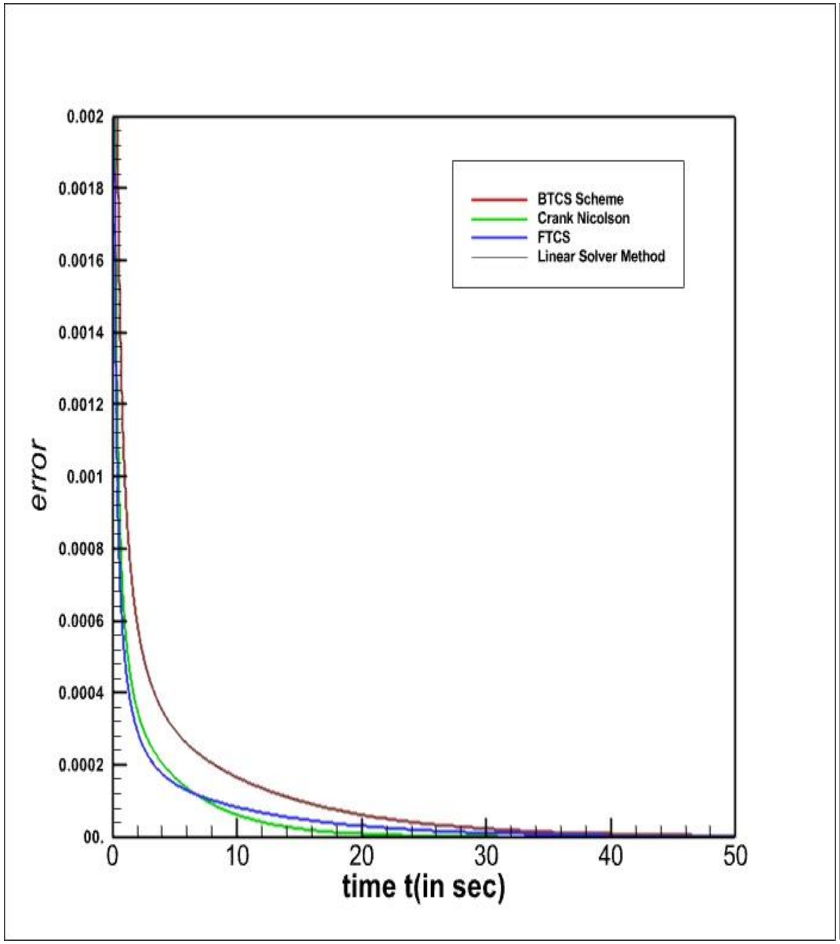


4. BTCS Scheme(TDMA):-

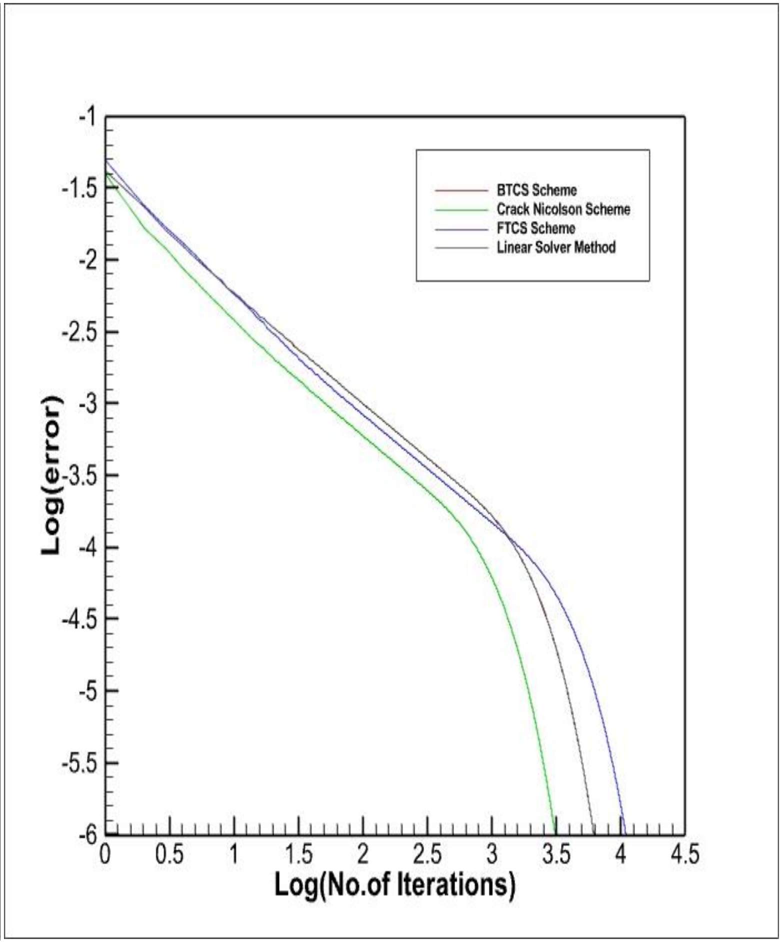


Velocity Profiles at Different time including Steady State

Convergence History (Ɛ vs t):-



Comparison Study of Number of Iterations:-



No.	*Scheme Name*	*No. of Iterations*	*Time(sec)*
1	*FTCS Scheme*	*10935*	*54.675*
2	*BTCS Scheme(Linear Solver Method)*	*6173*	*61.73*
3	*BTCS Scheme(TDMA Algorithm)*	*6178*	*61.779*
4	*Crank Nicolson Scheme*	*3095*	*30.945*

Table 3:- ***Numbers of Iterations(grid size 100)***

Assignment 3

Non dimensional governing Equations :-

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega$$

$$u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right)$$

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

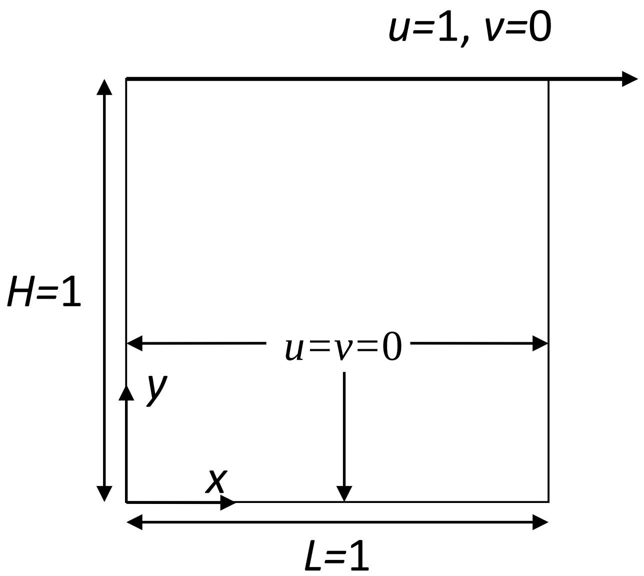
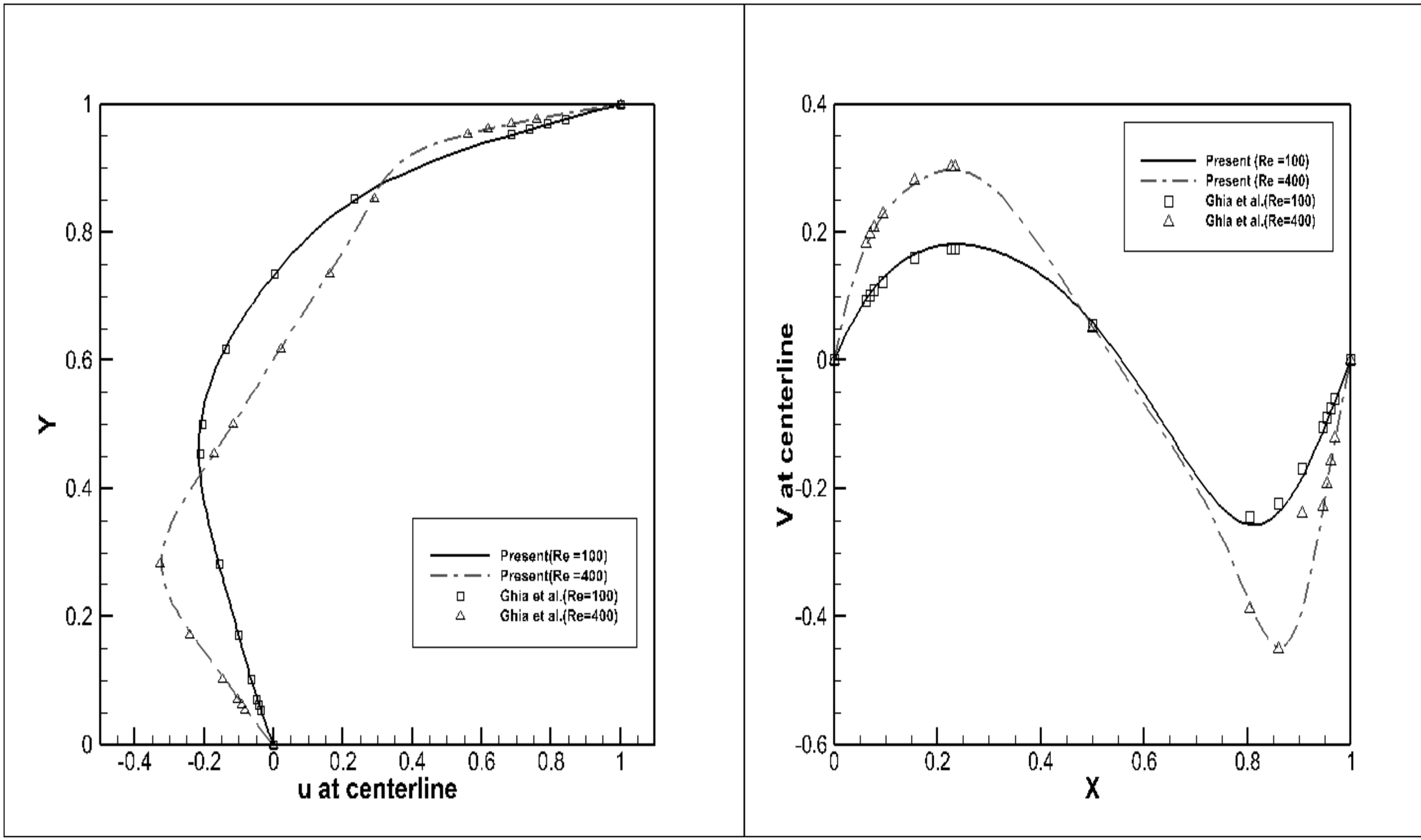


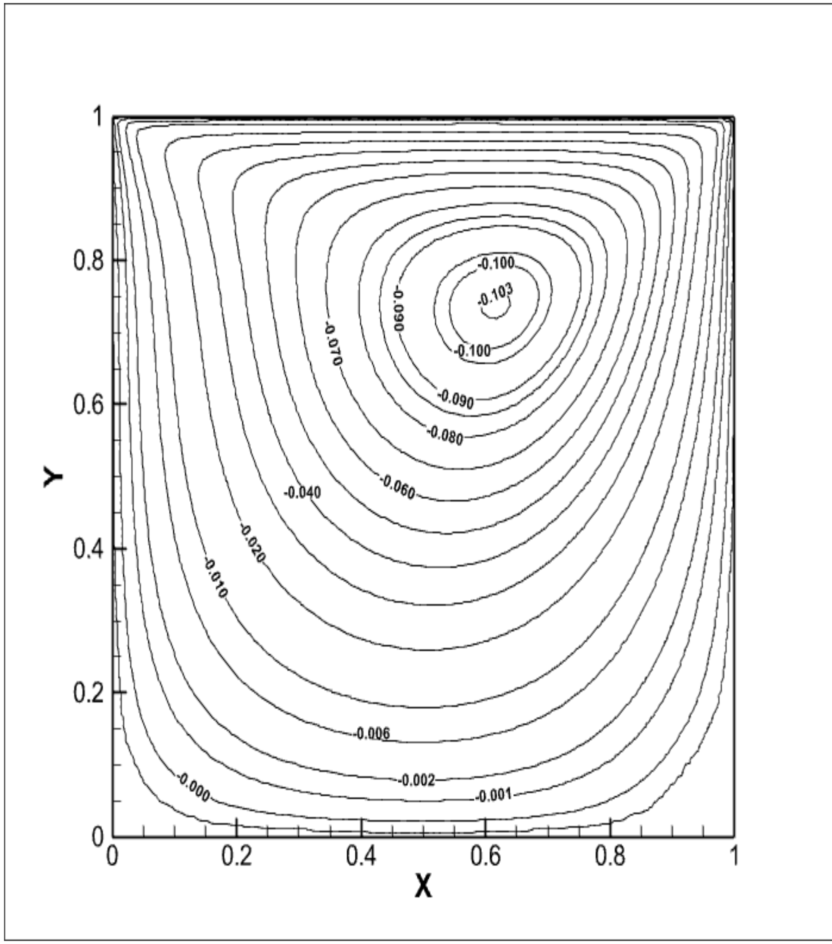
Figure: Flow inside a lid-driven cavity

Centreline u and v velocity profiles :-

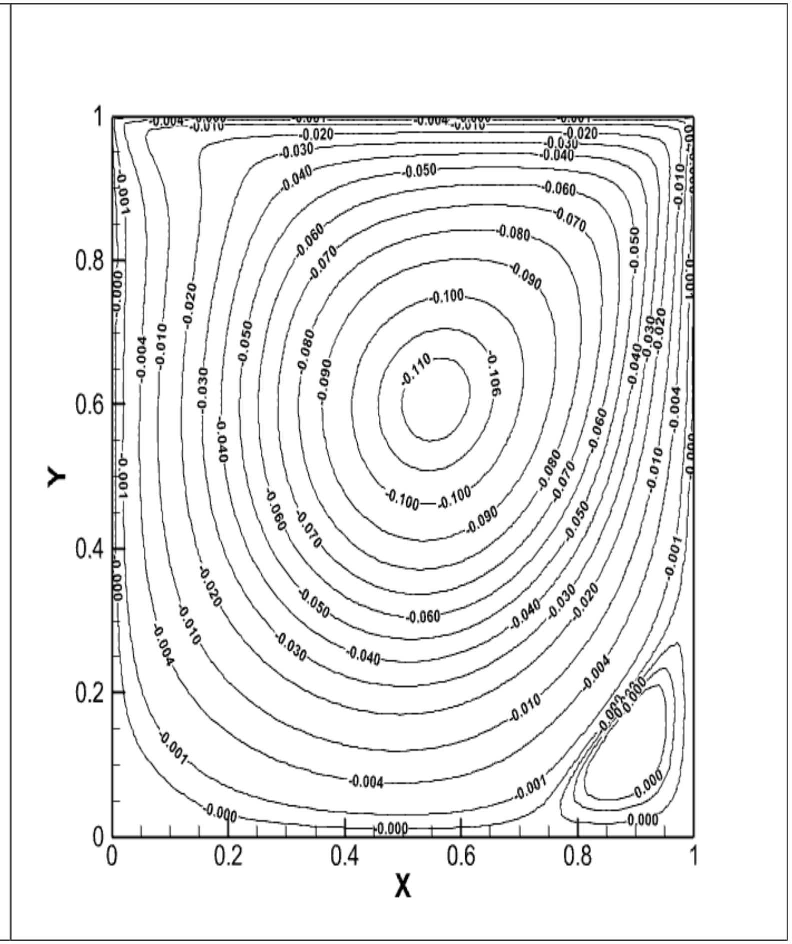


Stream lines contours :-

A) Re=100

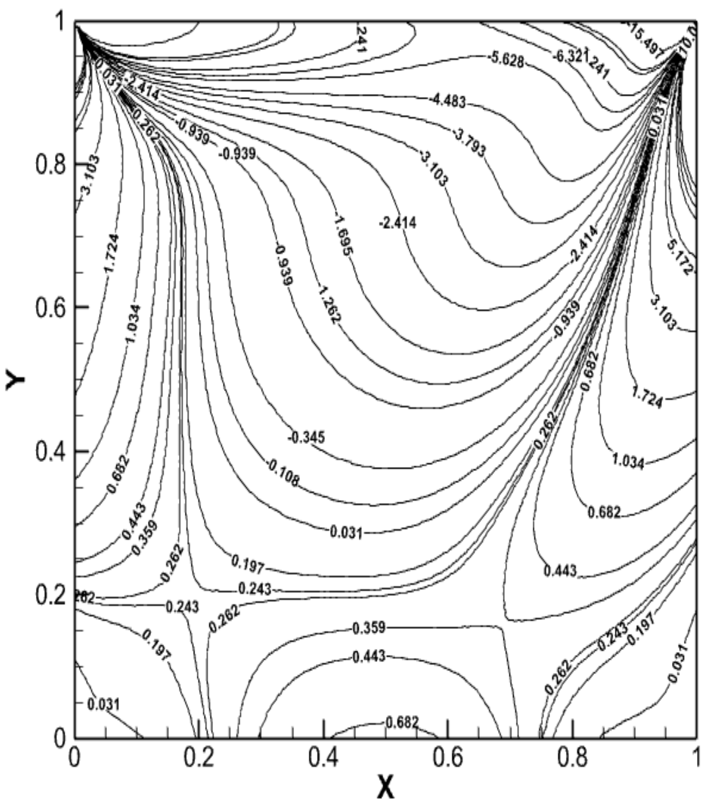


B) Re=400

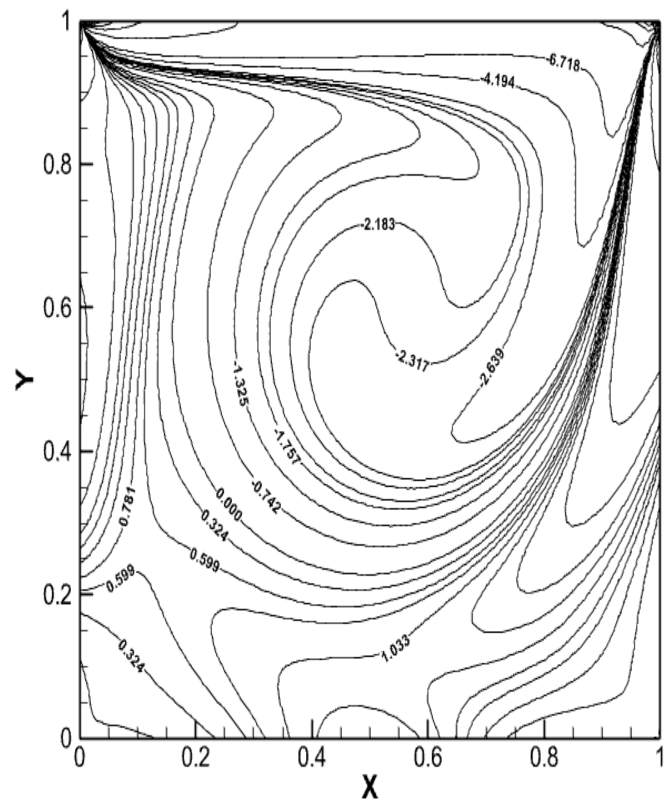


vorticity contours :-

A) Re=100

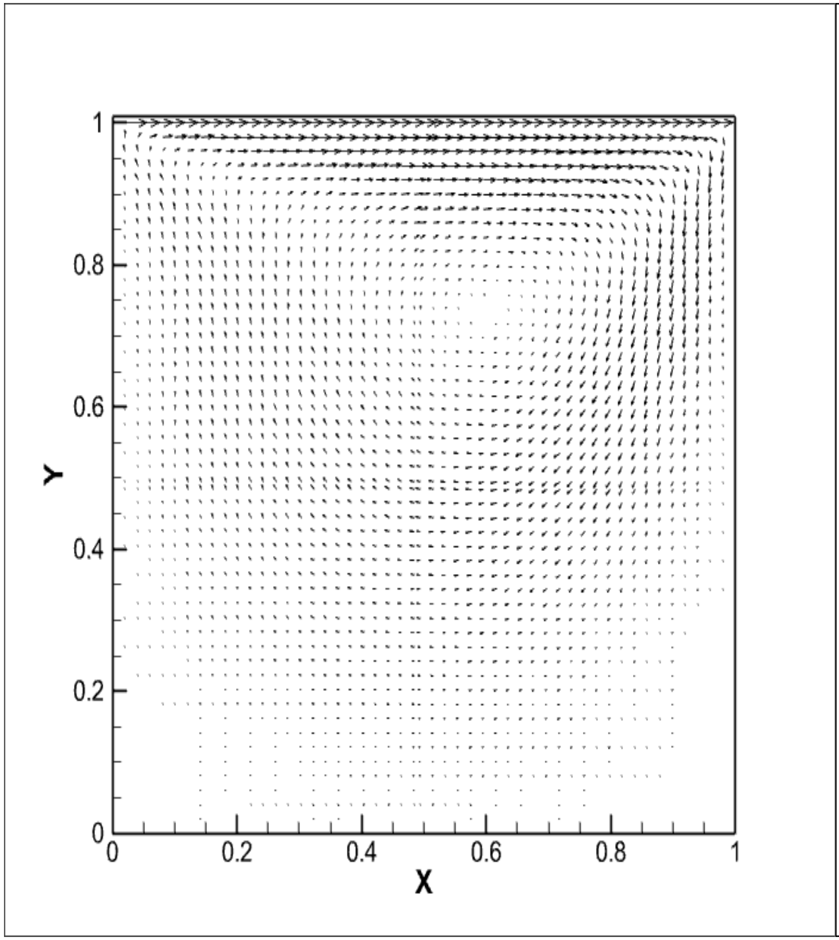


B) Re=400

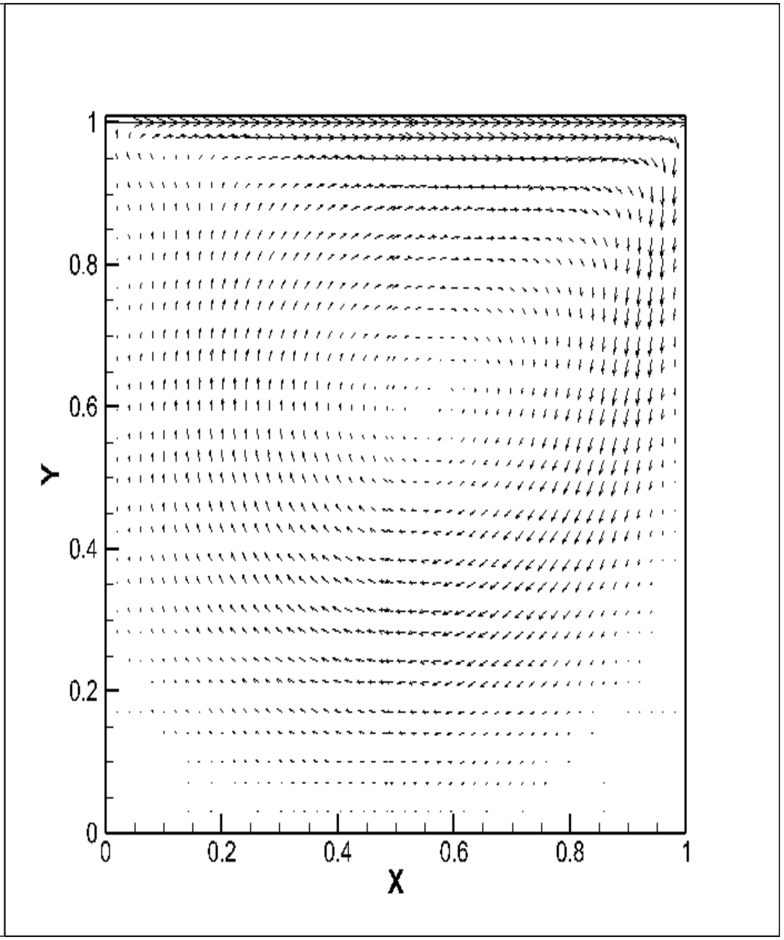


Velocity vectors :-

A) Re=100



B) Re=400



Thank you!