Conjugate Heat Transfer in Rayleigh-Benard Convection in a Square Enclosure with Sinusoidal Bottom Wall

Project report submitted to
Visvesvaraya National Institute of Technology, Nagpur
In partial fulfillment of the requirements for the award of the degree

Bachelor of Technology
In

Mechanical Engineering

by

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under the guidance of
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Declaration

I, Vivek Vijay Potdar , hereby declare that this project work titled " Conjugate Heat Transfer in Rayleigh-Benard Convection in a Square Enclosure with Sinusoidal Bottom Wall " is carried out by us in the Department of Mechanical Engineering of Visvesvaraya National of Technology, Nagpur. The work is original and has not been submitted earlier whole or in part for the award of any degree/diploma at this or any other Institution/University.

Date:

Vivek Vijay Potdar

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Certificate

This is to certify that the project titled " Conjugate Heat Transfer in Rayleigh-Benard Convection in a Square Enclosure with Sinusoidal Bottom Wall ", submitted by, Vivek Vijay Potdar in the partial fulfillment of the requirements for the award of the degree of Bachelor of Technology in Mechanical Engineering, VNIT Nagpur. The work is comprehensive, complete and fit for the final evaluation.

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Abstract

In the present work, Conjugate natural convection-conduction heat transfer in a square cavity with a finite wall thickness is studied numerically. Three walls of the cavity are taken to be of zero thickness, while the bottom wall of the cavity is thick, with a finite thermal conductivity. In conjugate heat transfer, Heat conduction in a solid is implicitly coupled with heat convection in viscous fluid flow. The main focus of study is effect of conduction in a hot thick wavy wall on a steady, laminar natural convection in square cavity. The effect of different governing parameters like Rayleigh number, thermal conductivity ratio, wall thickness ratio, Amplitude, frequency of undulation on Nusselt number is studied, while keeping the Prandtl number constant. The range within the governing parameters considered are the Rayleigh number ($5 \times 10^3 \leq Ra \leq 10^6$), the wall-to-fluid thermal conductivity ratio ($0.5 \leq Kr \leq 10$), the ratio of wall thickness to its height ($0.02 \leq D \leq 0.4$), amplitude ($0.06 \leq A \leq 0.08$), No. of undulation ($0 \leq n \leq 2$).The study is completed , analyzed and compared between two different geometries of thick bottom wall namely plane wall and wavy wall. It is observed that the fluid flow, temperature distribution pattern and heat transfer rate in the cavity can be controlled by above governing parameters.

List of Figures

Contents

List of Tables
Contents

NOMENCLATURE

d,D	Wall thickness and dimensionless wall thickness parameter
g	Gravitational acceleration
Kr	Thermal conductivity ratio
k	Thermal conductivity
ℓ	Width and height of enclosure
$\overline{Nu}_f$	Average Nusselt number
Ra	Rayleigh number
T	Temperature
u, v	Velocity components in the x and y-directions
x, y	Space coordinates
h	Convective heat transfer coefficient
A	Convective heat transfer Area
Q	Heat transfer rate
n	Direction vector
n	Number of undulation
A	Amplitude of wavy wall
Gr	Grashof Number
Pr	Prandtl Number
C_p	Specific Heat Capacity at Constant Pressure
L	Characteristic Length

Greek Symbols

α	Effective thermal diffusivity
β	Coefficient of thermal expansion of fluid
μ	Viscosity of the fluid
ν	Kinematic viscosity of the fluid
ρ	Density of the fluid
ψ	Value of the streamfunction in the the enclosure

Subscripts

w , f	Wall and Fluid
h , c	Hot and Cold
ref	Reference value
diff	Difference
p	Constant Pressure
∞	Infinity
max	Maximum Value

TABLE OF CONTENTS

1. INTRODUCTION

INTRODUCTION

1.1 Background

The convection and conduction are two main phenomenon generally observed in many the heat transfer problems. In many fields of science, including Engineering , Architecture, Medical science and also in human body, the diverse applications of interaction of these two phenomenon is commonly observed. These are main mechanisms of heat transfer within fluid and solid . Many scientists mathematically studied these two phenomenons and tried to explain them in mathematical formula. Before computers came into existence , Newton proposed less accurate method and gave formula to calculate the heat flux from a isothermal wall in convective heat transfer.

$$Q_w = h\,A\,(T_w - T_\infty)$$

As the value of heat transfer coefficient *h* mainly depends on wall temperature distribution, there is significant amount of difference is observed between theoretical and experimental results. So to do precise formulation of convective heat transfer problems which involve solid-fluid interaction, the proper coupling of the fluid and solid temperature field should be done at their interface.

Similarly for conduction, Fourier proposed theory of conduction in which he shows that amount of heat flux in a direction is directly proportional to the temperature gradient in respective direction.

$$Q_w = -KA\left(\frac{dT}{dn}\right)$$

where constant of proportionality is given as thermal conductivity. so these two formulae form the basic of heat transfer problems. In conjugate heat transfer problem, these two phenomenon coupled with each other at interface of two regions.

1.2 Rayleigh-benard convection

" Reyleigh-benard convection" is one of the types of natural convection. In 1900,the French physicist ,Henry benard conducted a simple experiment regarding natural convection in fluid heated from bottom surface and observed that a beautiful hexagonal regular pattern of convection cells is developed. These cells are called as "Benard cells" .

Later lord Rayleigh successfully analyzed convection. Rayleigh considered boundary conditions in which vertical velocity component and temperature distribution are vanishes at top and bottom boundaries. This convection is solely due to the temperature gradient which leads to formation of density gradient in the fluid from top to bottom. This creates the buoyancy force which tends to move hot fluid in upward direction. While Gravity is also trying to pull cold fluid in downward direction. But gravitational force is opposed by the viscous damping force .so the balance between these two forces is studied using non-dimensional parameter “Rayleigh Number”. After critical value of Rayleigh number, gravitational force dominates, thus instability sets and convection cells are formed. Thus buoyancy and gravity are responsible to creation of convection cells.

These cells are formed in alternate way in rotation sense from clockwise to anticlockwise. Cells are metastable in nature that a small fluctuation in governing conditions does not affect them.

1.3 Conjugate Heat Transfer

Conjugate heat transfer refers to the method of computing thermal distribution for heat conduction in solids, along with simultaneous convective heat transfer in associated fluid domain. Conjugate heat transfer mainly deals with the heat transmission between a solid and a fluid body flowing inside or over it as a result of the interaction of two objects. To do the conjugation, coupled boundary conditions are available for both the fluid and solid cell zones, where either of the zones, may contain heat sources. In conjugate heat transfer problems generally containing two or more subdomains with different phenomena describing by different differential equations. After solving the differential equation problem in each subdomain , these solution are conjugated. This coupling gave the heat flux and temperature distribution across the interface. The final outcomes of such a problems will gave the heat flux and temperature and velocity distribution in the domains.

For example, the heat transfer between a solid plate and a fluid flowing over it, is a conjugate heat transfer problem, because the heat transfer inside the solid is governed by the elliptical Laplace equation or by the parabolic differential equation, while the elliptical Navier-Stokes equation or by the parabolic boundary layer equation governs the heat transfer inside the fluid. Both domains are initialized with their corresponding properties and separate boundary conditions are provided for both the domains except at the interface. The governing partial differential equations of each subdomain is solved explicitly. A common thermal boundary condition is imposed at interface which coupled solutions of fluid and solid domains. This coupling transfer the thermal properties and characteristics across domains , which eliminates the need of explicitly providing heat transfer coefficient across the fluid-solid interface.

1.4 Bossinesq Approximation

In convective heat transfer, the fluid properties are strongly influenced by variations in temperature. In such problems , all the fluid properties are calculated using updated temperature value at current iterations . In case of natural convection, density is varies with temperature which causes buoyancy driven flow within fluid. The momentum equation and Energy equation of fluid flow is coupled and solved by considering suitable relationship of variation of density with temperature. General expression for this relationship is given by

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

The bossinesq Approximation is used to define the relationship between density and temperature. It states , for small temperature difference ,there is linear relationship between density and temperature.

$$\beta = -\frac{1}{\rho}\left(\frac{\rho - \rho_{ref}}{T - T_{ref}}\right)$$

The approximation further explains that the variation in inertia of fluid particles is insignificant but gravity is sufficiently strong enough to create appreciably difference in specific weight . Hence, when density variation is small and within limits, density can be treated as constant in unsteady and convection terms, but as a variable in body force or gravitational term in momentum equation.

$$\text{Body force term} = (\rho-\rho_{ref}) \cdot g = -\rho\, \beta\, g\, (T-T_{ref})$$

There is a constraint on the implementation of bossinesq approximation as it is valid only for those cases where value of $\rho_{eff} \ll 1$. The additional term ρ_{eff} is known as "Effective density". If value of $\rho_{eff} \gg 0.4$, then flow is considered as compressible flow.

$$\triangle \rho_{diff} = \rho - \triangle \rho = [\rho - \rho\, \beta\, (T - T_{ref})]$$

$$\rho_{eff} = \frac{\Delta \rho_{diff}}{\rho} = 1 - \beta\, (T - T_{ref})$$

1.5 Applications

There are several engineering applications of the conjugate heat transfer are present in many areas of the Science. Starting from simple examples in the 1960s, the conjugate heat transfer methods have become a more powerful tool for modeling and investigating nature phenomena and engineering systems in various areas ranging from thermal goods treatment and food processing to aerospace and nuclear reactors to, from complex procedures in medicine to atmosphere/ocean thermal interaction in meteorology, and from relatively simple units to multistage, nonlinear processes.

This convection problem in the present study has attracted a great deal of attention from researchers because of its presence in both nature and industrial applications. In nature, the convection cells formed from air rising above sunlight-warmed land or water are a major feature of all weather systems. Convection is also seen in the rising plume of hot air from fire, oceanic currents, and sea-wind formation. A very common industrial application of natural convection is free air cooling without the aid of fans: this can happen on small scales (computer chips) to large scale process equipment.

2. LITERATURE SURVEY

LITERATURE SURVEY

1. **Habibis saleh and Isak hashim** studied the conjugate heat transfer in rayleigh-benard convection in a square cavity with a finite wall thickness. In study, the horizontal bottom wall of cavity having finite thickness and thermal conductivity, while other walls having zero thickness. In problem, finite and uniform temperature difference is applied with hot bottom wall and cold top wall keeping side wall insulated. They studied the effect governing parameters like wall thickness ratio(D), Rayleigh Number(Ra), wall to fluid thermal conductivity ratio(Kr) on temperature and velocity distribution. They also studied the effect above parameters on solid-fluid interface Nusselt number. Staggered grid with Marker And Cell (MAC) method were used to solve the Problem. Grid independency test was performed on the basis of Nusselt number. The isotherms and streamlines are plotted and studied varying the one parameter while keeping other constant. Variations of average fluid Nusselt number is plotted with Rayleigh number for different wall thickness ratio and thermal conductivity ratio . Variations of Nusselt number with different wall thickness ratio at different thermal conductivity ratio is also plotted. It is observed that fluid flow development, circulation strength of flow, heat transfer through interface can be controlled by varying governing parameters. The strength of flow circulation is much higher for thin walls. Number of contra rotative cells can be controlled with parameters.

2. **D. A. KAMINSKI and C. Prakash** studied steady, laminar, natural convection flow with conduction in one of vertical wall of square cavity. One of vertical wall is heated uniformly while other wall is cold and remaining two horizontal walls are insulated. They studied the effect of vertical wall conduction on temperature variation at fluid –solid interface, fluid flow development , variations of local heat flux at interface. The SIMPLER algorithm were used during computation of problem. They also plotted isotherms and streamtraces for one and two dimensional wall conduction model. The study is done on Effect of wall thickness on flow field with help of isotherm and streamlines plot. The Governing parameters are used in mixed non-dimensional form such as the conduction ratio(K_wL/Kt) , non-dimensional form of streamfunction ($\psi_{max}L^2/\nu$) and Grashof number (Gr). The variation of local fluid-solid interface temperature and local heat flux at interface is plotted at different Grashof number with variation of conduction ratio keeping Prandtl number constant. The overall Nusselt number is calculated and tabulated for different Grashof number in range of 10^3 to 10^7 with varying conduction ratio in range of 5 to ∞ for one and two dimensional wall conduction and compared the results with lumped parameter approach and error

is calculated. The correlation was obtained between Grashof and Nusselt number. It is observed that overall Nusselt number is increases with conduction ratio and Grashof number. The temperature variations across interface increase for particular conduction ratio as Grashof number increases. In prediction of overall Nusselt number lumped parameter model gives slight under- prediction results as compared to one and two dimensional heat conduction model.

3. **M. Hribersek and G. kuhn** solved conjugate problem same as solved by Kaminski and prakash but with boundary- domain integral method. He solved Navier-stokes equation with vorticity- velocity formulation and calculate overall Nusselt number. Isotherms and streamlines are plotted with different Grashof number (Gr) and thermal conductivity ratio (k). The interface temperature variation and interface heat flux variations are plotted. He compared overall Nusselt number with FVM method for different Gr and k values.

4. **L. Adjlout , O. Imine, A. Azzi, M. Belkadi** studied the effect of a hot wavy wall of a laminar natural convection in an inclined square cavity with differential heating. All wall are of zero thickness. Temperature difference applied to a pair of opposite wall keeping other pair insulated. The effect of different governing parameters like Rayleigh number (Ra), Amplitude(A),Number of undulation(n) on temperature distribution, overall Nusselt number is studied The isotherms and streamlines are plotted with variation of rotation angle of cavity (θ) for one and three undulations or crests. It is observed Temperature pattern and fluid flow can be controlled by rotation angle (θ) and No. of undulations (n).

2.2 Outcomes of Literature Survey

- A considerable amount of attention has been given to investigate the effect of Rayleigh number, Thermal conductivity Ratio, Wall Thickness Ratio on interface average Nusselt number, fluid flow development, temperature distribution across the cavity by adopting different computational and numerical procedure and compared with experimental results.

- Many authors changed the position of conduction wall and studied the effect of it on various outcomes with varying the governing parameters. The change in geometry of conduction wall is studied with considering zero thickness. Various relationships were proposed between average Nusselt number and different parameters.

3. NUMERICAL METHODOLOGY

NUMERICAL METHODOLOGY

3.1 Problem Description

In the present problem, we considered two geometries for study of conjugate heat transfer in square cavity. In study, the problem is considered as two dimensional heat conduction- convection problem.

Case I :-

The schematic diagram of case 1 is shown in Figure 3.1. In case 1, we consider plane , solid, impermeable bottom wall having finite thickness d and thermal conductivity k_w while other wall having zero thickness. The bottom surface of the solid wall is heated at constant temperature T_h while top surface of cavity is cooled at constant temperature T_c keeping two vertical surface adiabatic. The cavity between top surface and solid wall is filled with air. Dimensions of square cavity is taken as $l \times l = 1 \times 1$ m.

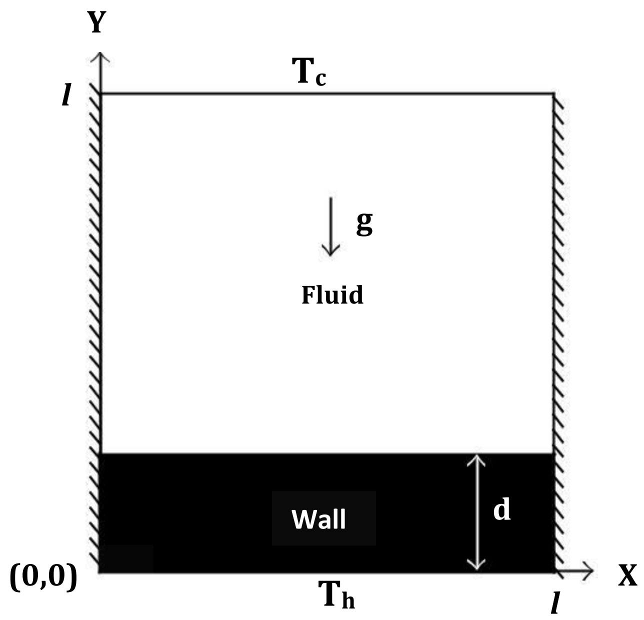


Figure 3.1. Case I - Square cavity with plane wall

Case II :-

Figure 3.2 shows the schematic diagram of case 2. Here we considered bottom wall as wavy wall having a sinusoidal nature. The sinusoidal wall is chosen as hot wall with different amplitude and two undulations. The wavy wall is solid

,impermeable in nature having thickness d and thermal conductivity k_w. Other surfaces are having same thermal parameters as in case 1.

The equation of sinusoidal wave of wall is given as

$y = f(x) = A(1 - \cos(2n\pi x))$

where, $0 \le x \le 1$

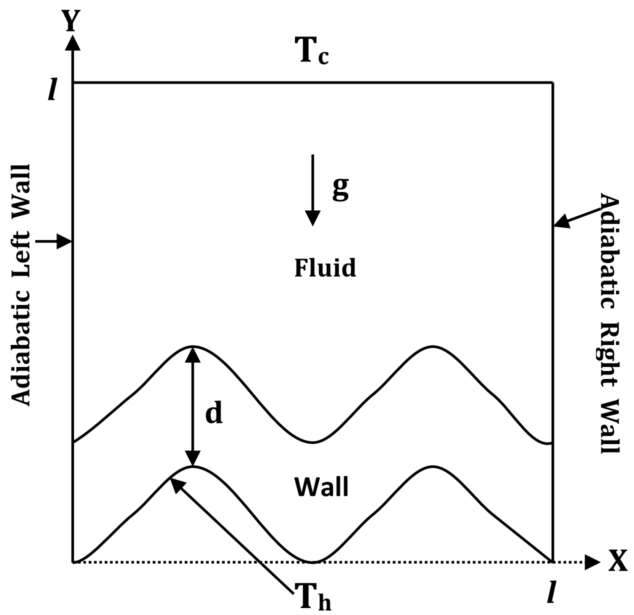


Figure 3.2. Case II - Square cavity with Sinusoidal wall

❖ The Governing Parameters:-

1. Rayleigh Number (Ra):-

   $5 \times 10^3 \le Ra \le 10^6$

   $Ra = Gr. Pr = (g .\beta. (T_h - T_c).L^3/ \nu^2) . Pr$

2. Thermal Conductivity Ratio (Kr) :-

   $0.5 \le Kr \le 10$

   $Kr = \left(\frac{K_w}{K_f}\right)$

3. Wall Thickness Ratio (D):-

   $0.02 \le D \le 0.4$

$$D = \left(\frac{d}{l}\right)$$

4. Amplitude (A) : -

   $0.06 \le A \le 0.08$

5. Number of Undulation (n) :-

   n = 1 and 2

Thermo-physical Properties of Air :-

At 300 K,

1. Density = ρ = 1.157 Kg/m^3
2. Dynamic Viscosity = μ = 1.87 × 10^{-5} N-s/m^2
3. Kinematic Viscosity = $\nu = \left(\frac{\mu}{\rho}\right)$ =1.6162 × 10^{-5} m^2/s
4. Thermal Conductivity = K_f = 0.02676 W/m-K
5. Specific Heat at constant pressure = C_p = 1011.73 J/Kg-k
6. Prandtl Number = $\left(\frac{\mu.C_p}{K_f}\right)$ = 0.707

For Case 1,

l = L = 1 m.

For Case 2,

$l \neq$ L

Table 1. The values of L for different Amplitudes

	A = 0.06	A=0.08
n=1	1.0316	1.0604
n=2	1.1273	1.2170

To simplify the analysis , following assumptions were made for computing the numerical solution while keeping the flow physics undisturbed.

1) The all physical properties of fluid is independent with respective to temperature except the density is varies with temperature which supported by bossinesq approximation.

2) Similarly, all thermo-physical properties of solid and fluid is constant everywhere in their respective domains.

3) Prandtl number of air is assumed as constant.

3.2 Ansys Fluent

There are several commercial as well as open source soft wares are available for CFD analysis. In the present study , the Ansys Fluent is used to simulate the flow in the system and quantify the Engineering parameters.

ANSYS FLUENT software contains the wide varieties physical modeling capabilities needed to model flow, heat transfer, turbulence and reactions for industrial applications ranging from air flow over an aircraft wing to combustion in a furnace, from bubble columns to oil platforms, from blood flow to semiconductor manufacturing, and from clean room design to wastewater treatment plants. Special models that give the software the ability to model in-cylinder combustion, turbo machinery, aero acoustics and multiphase systems have served to broaden its reach.

3.3 Solution Method

Solver used :-

Pressure based Type,
Absolute velocity Formulation,
Steady state ,2-Dimensional solver
With laminar viscous model.

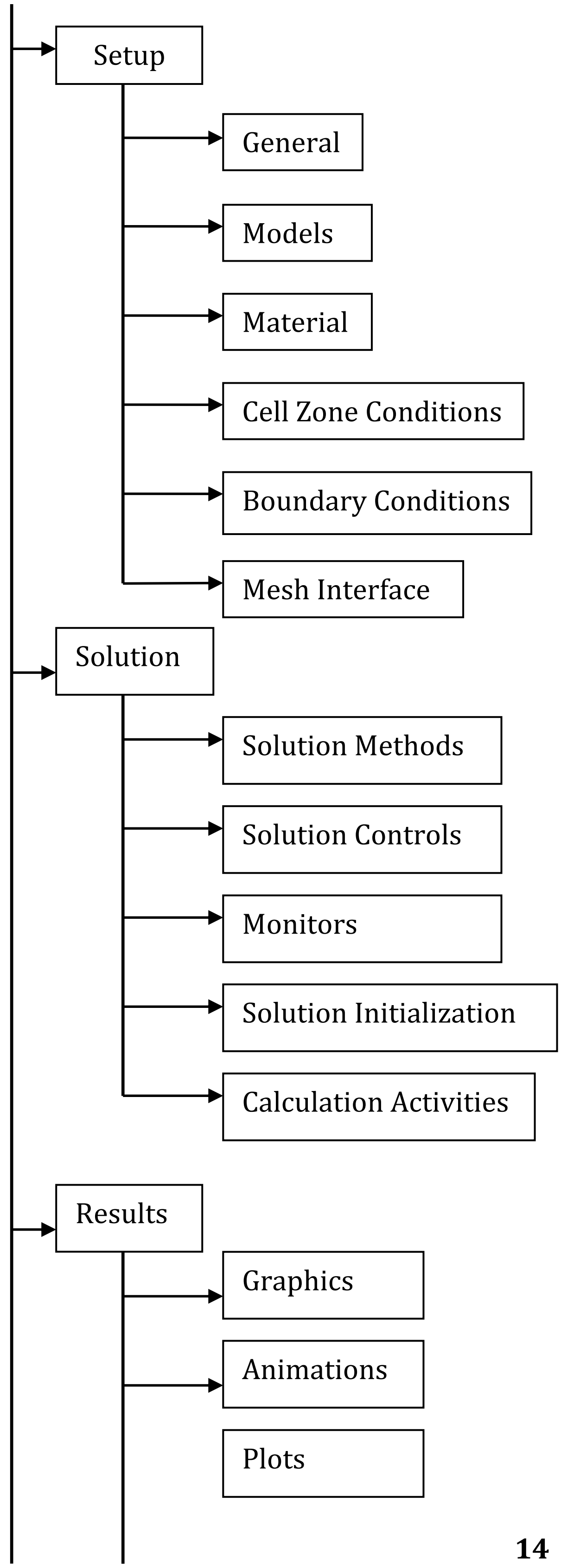
Method Used for Pressure- velocity Coupling :- SIMPLEC

Spatial Discretization Method :-

The process of descretization involved transformation of continuous functions, variables into approximate discrete parts and interpolate variable values. This methods convert Differential equations into Difference equations. The methods employed for these approximation can affect both the accuracy and stability of the numerical schemes. Hence special care must also be taken to ensure that the discretization handles discontinuous solution gracefully ,following are the discretization schemes are being used for different terms.

Gradient Term – Least Square Cell Based
Pressure Term - Second Order
Momentum Term – Second Order Upwind
Energy Term – Second Order Upwind

3.4 General Structure of Ansys Fluent :-



Reports

3.5 Boundary Conditions

In present study, upper surface is cold while the bottom wall is uniformly heated keeping side walls insulated. In both cases Momentum and Thermal Boundary Conditions remains same.

Table 2. Boundary Conditions

Sr. No.	Part name	Part Type	Momentum Boundary Conditions	Thermal Boundary Conditions
1.	Top_Wall	wall	Stationary wall & No Slip shear Condition	Constant Temperature= T_c = 300 K
2.	Bottom_Wall	wall	-	Constant Temperature= T_h = 301 K
3.	LHS_Fluid	wall	Stationary wall & No Slip shear Condition	Constant Heat Flux = 0 w/m^2
4.	LHS_Solid	wall	-	Constant Heat Flux = 0 w/m^2
5.	RHS_Fluid	wall	Stationary wall & No Slip shear Condition	Constant Heat Flux = 0 w/m^2
6.	RHS_Solid	wall	-	Constant Heat Flux = 0 w/m^2
7.	Fluid_Solid_ Interface	Interface	-	-
8.	Solid_Fluid_ Interface	Interface	-	-

A Mesh Interface is created by connecting Fluid_Solid_ Interface with Solid_Fluid_ Interface with coupled wall condition.

The values of the velocity are zero in the wall region and on the solid-fluid interfaces. The boundary conditions for the temperatures are the following :-

$$T_w(X,0) = T_c\,; \qquad T_f(X,1) = T_h\,;$$

$$\frac{\partial T_f(0,Y)}{\partial X} = 0\,; \quad \frac{\partial T_w(0,Y)}{\partial X} = 0\,; \quad \frac{\partial T_f(1,Y)}{\partial X} = 0\,; \quad \frac{\partial T_w(1,Y)}{\partial X} = 0$$

$$T_f(X,D)=T_w(X,D)\ ; \qquad \frac{\partial T_f(X,D)}{\partial Y}=\frac{Kr\partial T_w(X,D)}{\partial Y}.$$

3.6 Governing Equations

In present study, The thermo-physical properties of fluid is assumed to be constant except the density is varies with temperature .To couple temperature field with flow field , Bossinesq Approximation is invoked for fluid particles which relates the density with temperature linearly. With bossinesq approximation, density is assumed to constant in unsteady and convective term but variation of density is considered in body force term. Under above assumptions , the governing equations of mass ,momentum, and energy for the fluid can be written as ,

The Mass- Continuity Equation:-

$$\left(\frac{\partial u}{\partial x}\right)+\left(\frac{\partial v}{\partial y}\right)=0 \tag{1}$$

The Navier- stokes/ Momentum Equation:-

$$u.\left(\frac{\partial u}{\partial x}\right)+v.\left(\frac{\partial u}{\partial y}\right)=-\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right),$$

$$u.\left(\frac{\partial v}{\partial x}\right)+v.\left(\frac{\partial v}{\partial y}\right)=-\frac{1}{\rho}\left(\frac{\partial p}{\partial y}\right)+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+g.\beta.(T_f-T_c) \tag{2}$$

The Energy Equation:-

$$u.\left(\frac{\partial T_f}{\partial x}\right)+v.\left(\frac{\partial T_f}{\partial y}\right)=\alpha\left(\frac{\partial^2 T_f}{\partial x^2}+\frac{\partial^2 T_f}{\partial y^2}\right) \tag{3}$$

And the Energy Equation/ Laplace Equation for the impermeable wall is given as

$$\left(\frac{\partial^2 T_w}{\partial x^2}+\frac{\partial^2 T_w}{\partial y^2}\right)=0 \tag{4}$$

No- slip condition is assumed at solid-fluid interface and all surfaces. The following Non-dimensional parameters used for calculation of properties of medium :

$$\Pr = \frac{\nu}{\alpha} \ ,$$

$$Ra = \frac{g.\beta(T_h - T_c)L^3 \Pr}{\nu^2} \ .$$

Calculation of Nusselt Number :-

by Fourier's law of Heat Conduction,

$$\text{Heat Flux at Interface} = -k\frac{\partial T}{\partial y}\bigg|_{y=d} \qquad \text{(A)}$$

by Newton's law for convection ,

$$\text{Heat Flux at Interface} = h\,\Delta T \qquad \text{(B)}$$

From A and B,

For ΔT =1,

$$h = \text{Heat Flux at Interface} = -k\frac{\partial T}{\partial y}\bigg|_{y=d}$$

$$\text{Average Nusselt Number} = \text{Nu} = \frac{h.L}{k_f} = \frac{Heat\_flux\_through\_interface.L}{k_f}$$

3.7 Grid Generation

Grid generation or meshing is one of the crucial step in any simulation problem. In Grid generation , we spilt a large domain into small sub-domains and create mesh. In any numerical simulation, the discretization of governing equations is done in each small sub-domains.

In present problem of conjugate heat transfer , Two main domains as Solid and Fluid are considered. These two domains are coupled by a common interface named as Solid_Fluid_interface through which solution of one domain is coupled to solution of another domain. Two different meshes were created for each domain and a single mesh is generated by merging these two meshes. A mesh 100×100 is used for fluid domain while mesh of 20×100 for solid domain. Total Number of nodes is 12,000 in each mesh. The meshing of both domains are Structured and Non-uniform in nature. Mesh is very fine near the walls while course in middle of each domain . Meshing is done with Bi-Geometric mesh Law , having initial node spacing of 0.001 m with incremetation of 12% in spacing in both sides. All mesh generation is completed through ICEM-CFD . Figures shows some of the meshes used for simulation.

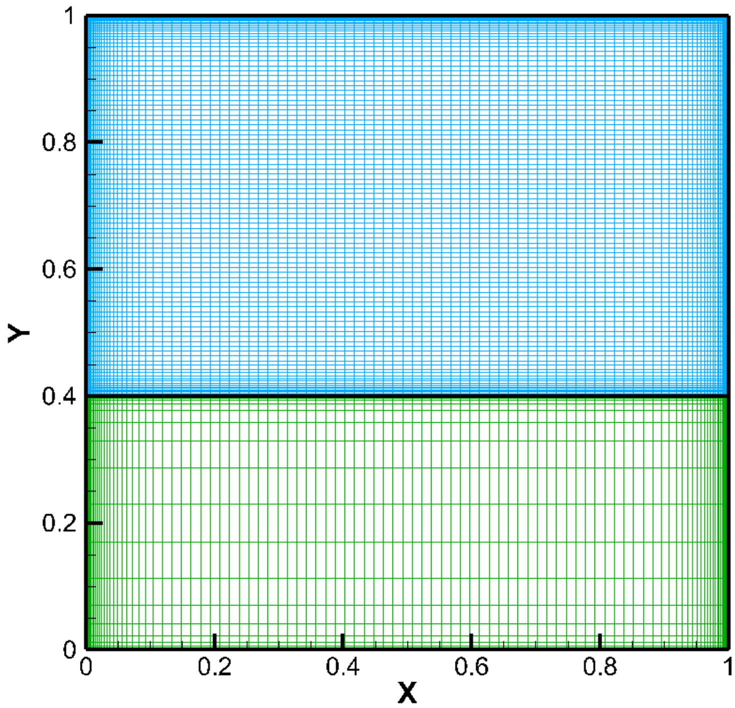


Figure 3.3. Square Enclosure with Plane Bottom Wall (D=0.4)

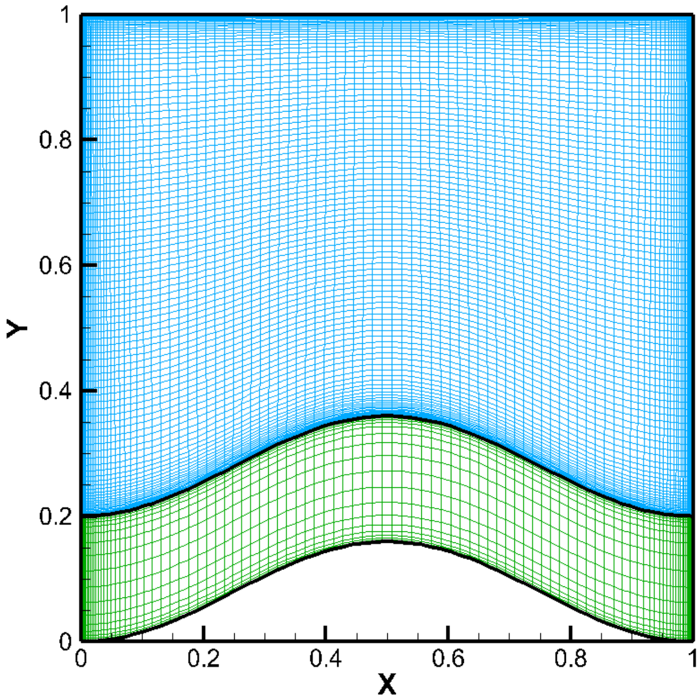


Figure 3.4. Square Enclosure with Sinusoidal Bottom Wall(D=0.2,A=0.08,n=1)

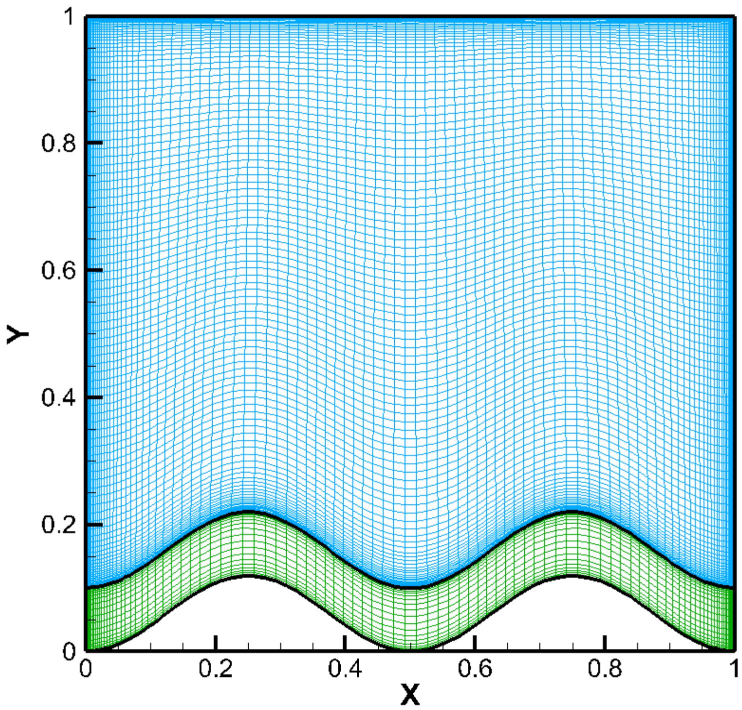


Figure 3.5. Square Enclosure with Sinusodal Bottom Wall (D=0.1,A=0.06,n=2)

Mesh Sensitivity Analysis :-

3.8 Grid Independency Study

In Numerical analysis, the solution and its convergence is largely dependent on fineness of cell and its position in the mesh. The size of each cell in the Mesh affects the outcomes of problem. So, It is necessary to check whether the solution is sensitive to change in configuration of mesh. Grid Independency Test tells us grid size suitable for numerical analysis. In Grid Independency Test , Deviation of a parameter of solution is checked by varying the size of Grid.

In present study, Average Nusselt Number $\left(\overline{Nu}_f\right)$ is calculated by variation of grid Size . In test we take grid size of 30, 60, 100, 140 for Fluid domain. The Test is done for Governing parameters D = 0.1 , Kr = 0.5 , Ra = 10^5 with plane bottom wall ,keeping all other thermal-physical boundary conditions and properties of fluid-solid constant. All the governing Equations were remain as it throughout the mesh sensitivity analysis.

The Graph is plotted between Average Nusselt Number $\left(\overline{Nu}_f\right)$ and Grid size. Figure shows the Graph $\overline{Nu}_f$ vs. Grid size in X and Y Direction.

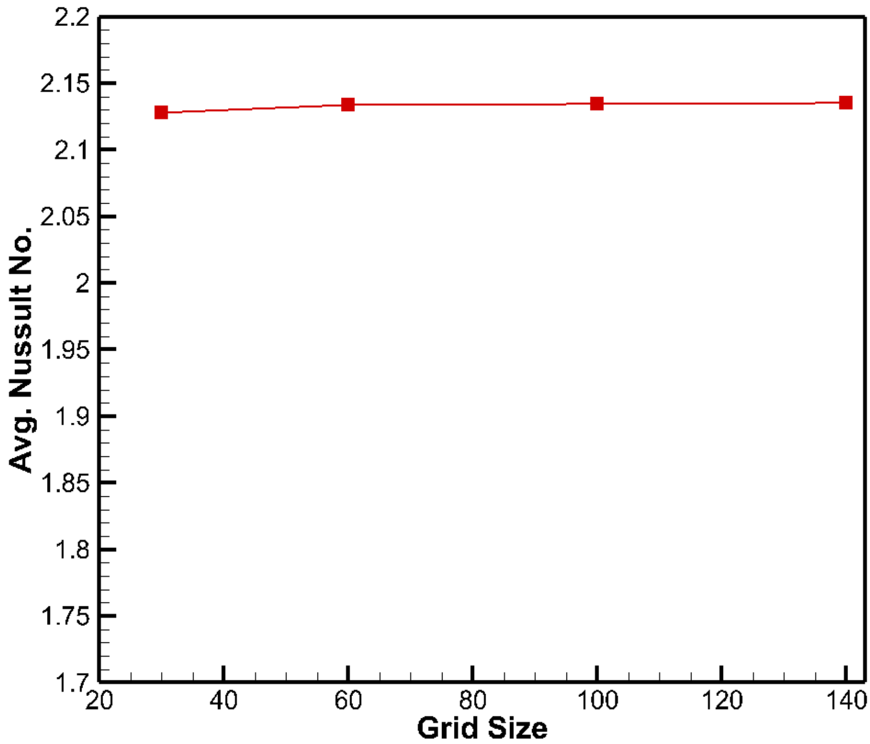


Figure 3.6. Grid Independency Study

It is observed that for mesh size 100 and above , Average Nusselt number does not show any significant variations. So ,Mesh size 100 is suitable for numerical computational procedure.

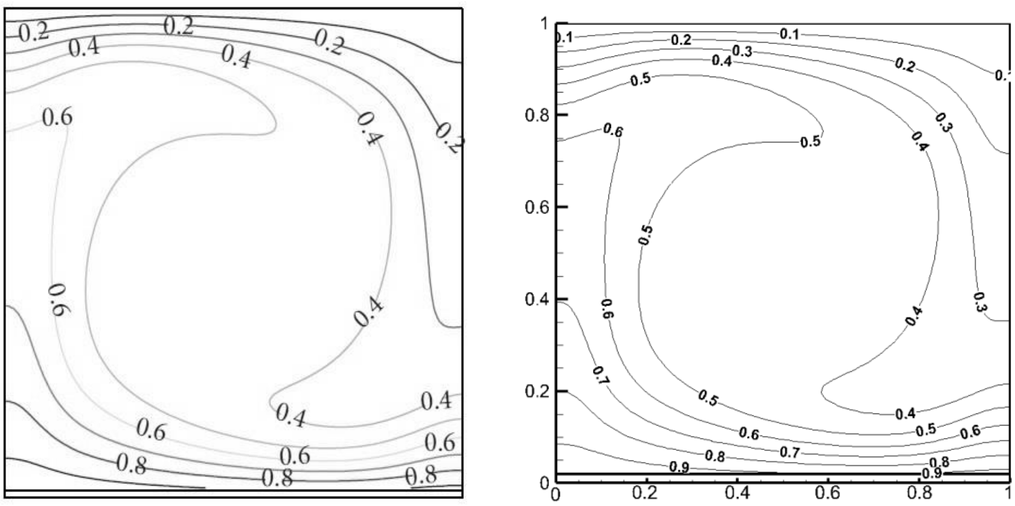
3.9 Validation Study

In order to know the accuracy and limitations of the result obtained by the our method ,we compared the obtained results with Research study done by Habibis Saleh and Isak Hashim. Comparison of average Nusselt number between the present study and other works is done for D=0.4 .

Table 3. Comparison of Nusselt Number between present study and other works

Ra →	10^5			10^6		
Kr →	1	5	10	1	5	10
Present Study	1.516	3.302	3.900	1.798	4.598	5.781
Habibis Saleh and Isak Hashim.	1.50	3.35	3.85	1.85	4.50	5.70
% Error	1.067	1.433	1.30	2.811	2.178	1.421

Comparison of Isotherms for D =0.02 at Ra $=10^5$ and Kr =1 :-



(a) (b)

Figure 3.7. (a) Isotherm plot from research paper of Habibis saleh and Isak Hashim (b) Isotherm Results obtained in present study.

4. RESULT & DISCUSSION

RESULT & DISCUSSION

The Problem of Conjugate heat transfer in a square Enclosure consists conduction through solid bottom wall parallel with convection in fluid in the cavity. In current problem, Top surface is cooled at constant temperature while bottom wall is heated at constant temperature keeping the other two side walls insulated. The conduction of solid wall affect the flow circulation and temperature distribution in the cavity. So, Heat conduction in a solid is implicitly coupled with heat convection in viscous fluid flow. The nature of flow, heat transfer rate, temperature distribution can be controlled by various governing parameters like Rayleigh number, thermal conductivity ratio, wall thickness ratio, Amplitude, frequency of undulation. The range within the governing parameters considered are the Rayleigh number ($5 \times 10^3 \leq Ra \leq 10^6$), the wall-to-fluid thermal conductivity ratio ($0.5 \leq Kr \leq 10$), the ratio of wall thickness to its height ($0.02 \leq D \leq 0.4$), amplitude ($0.06 \leq A \leq 0.08$), No. of undulation ($0 \leq n \leq 2$). Effects of above parameters on Nusselt number is calculated. Effects of two geometries of bottom wall i.e. Plane wall and Sinusoidal Wavy wall are studied , analyzed and compared the obtained results.

4.1 Case I :- Plane Bottom Wall

4.1.1 Effect of Governing Parameters on Thermal and Flow Field

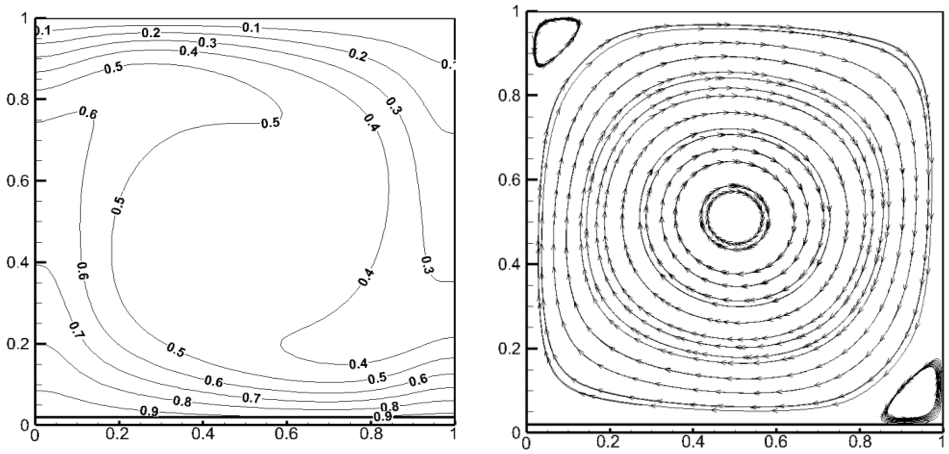
4.1.1.1 Effect of Wall Thickness Parameter D

Figure 4.1 illustrates the effects of wall thickness parameter D on the thermal field and flow fields for $Ra = 10^5$ and $Kr = 1$. Figures shows that ,the wall thickness parameter D affect the temperature distribution in solid , fluid as well as flow characteristics. From table we can see that circulation strength of fluid flow is higher for thin solid bottom wall and it goes on decreasing as D increases. This happened due to decrease in the temperature difference across the cavity. The fluid near to the hot wall has a lower density than the fluid at top cold wall. The hot fluid experience the buoyancy force so it tends to move upwards and cold fluid tries to go down . This creates the convection cell Known as main cell.

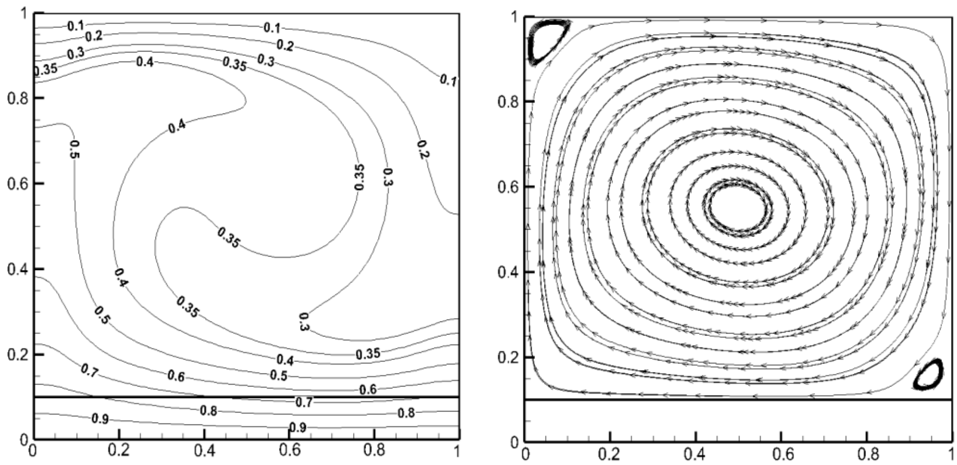
Table 4. Maximum Value of Streamfunction ψ_{max} for different wall thickness parameter D for plane wall

	Maximum Value of Streamfunction ψ_{max} ($\times 10^{-4}$)
D =0.02	6.575806
D =0.1	5.430327

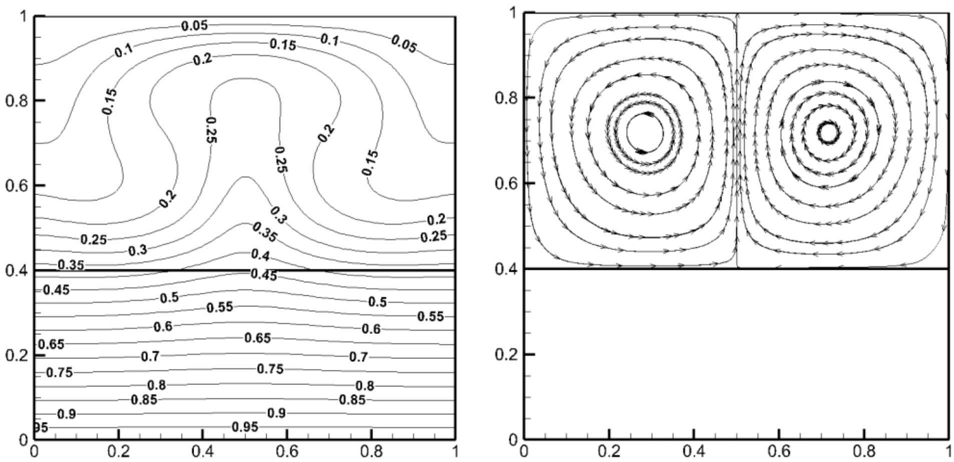
D =0.4	3.329330



D=0.02



D=0.1



D=0.4

Figure 4.1. Isotherms (left) and Streamlines(right) at Ra =10^5 and Kr =1

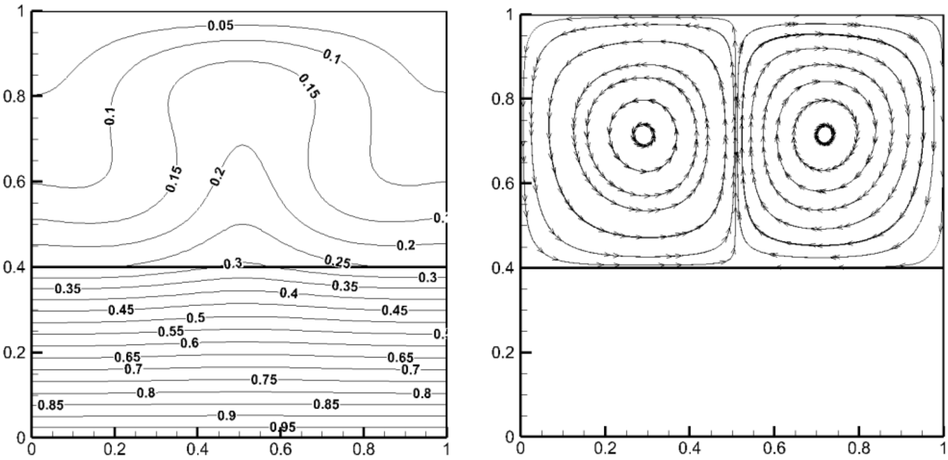
From figure we can be seen that, For Small thickness of bottom wall, a main circular cell is formed. As we increases thickness parameter D, the circular main cell becomes elliptical in nature and finally breaks into two contra –rotative cells. This two cells called as Benard cells. When fluid goes up , it gets cooled and flows in right and left planes of the enclosure. This create two successive benard cells. The cell in clockwise direction (-ve sign) formed due to natural circulation , called "main circulation cell" and the second corresponding counterclockwise (+ve sign) cell is called as "Secondary circulation cell".

4.1.1.2 Effect of Thermal Conductivity Ratio Kr

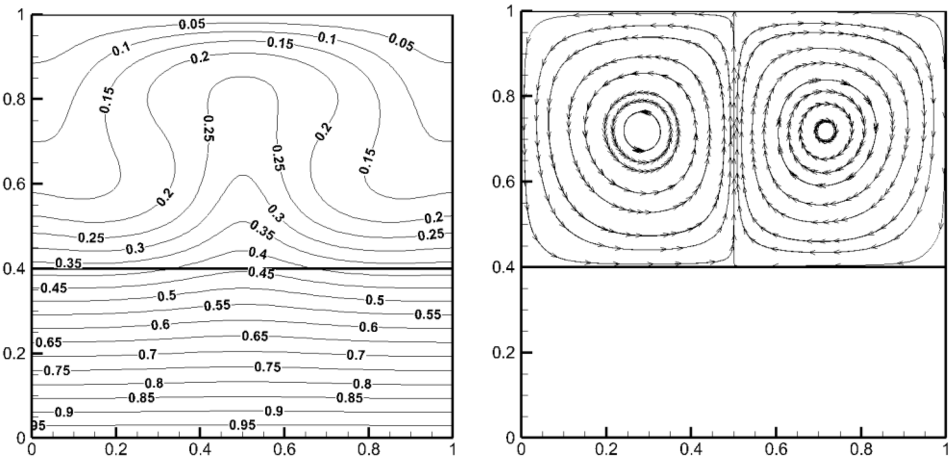
Figure 4.2 illustrates the effects of wall Thermal Conductivity Ratio Kr on the thermal field and flow fields for Ra =10^5 and D=0.4 . To show the effect we chose three different values for Kr i.e. Kr =0.5, Kr =1 ,Kr =10. From Figures we can see that two Stable contra-rotative cells i.e. clockwise main circulation cell and anticlockwise secondary cells are formed due to considerable high value of wall thickness parameter D. As we increases value of Kr , Circulation pattern is remain unaffected ,but strength of circulation goes on increases. This phenomenon occurs due to increase in wall conduction which results into increase in temperature gradient near the wall ,further leads to increase in convective heat flux. So it is observed that on increase of thermal conductivity ratio Kr, the convective heat transfer within the cavity becomes stronger.

Table 5. Maximum Value of Streamfunction ψmax for different thermal conductivity Kr

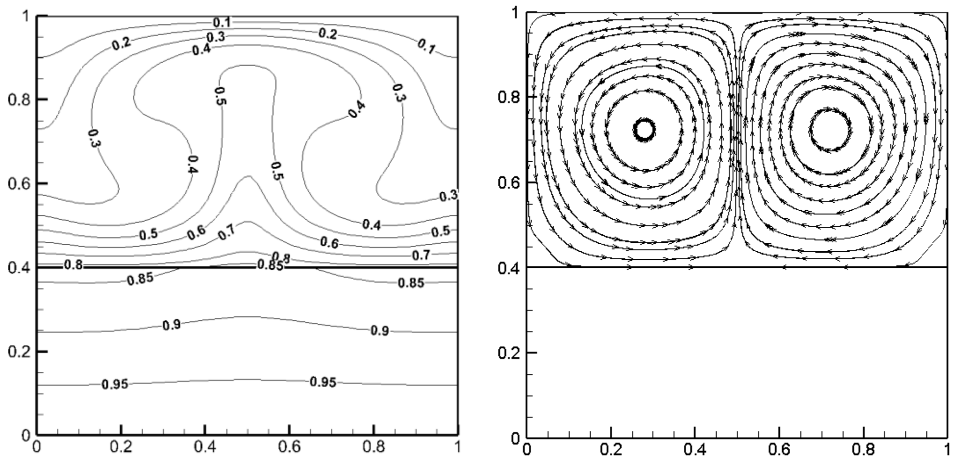
	Maximum Value of Streamfunction $\psi_{\max}$ ($\times 10^{-4}$)
Kr =0.5	2.499482
Kr =1	3.329330
Kr =10	5.380897



Kr=0.5



Kr=1



Kr=10

Figure 4.2. Isotherms (left) and Streamlines(right) at Ra =105 and D =0.4.

4.1.2 Effect of Governing Parameters on Nusselt Number

4.1.2.1 Effect of Wall Thickness Parameter D on Nusselt Number

Variations of average Nusselt number with the Rayleigh number for different values of wall thickness parameter are shown in Figure 4.3. It is observed from the graph for any value of wall thickness parameter , average Nusselt number increases as Rayleigh number increases. This is due to the increase in the Rayleigh number ,increase the convective mode of heat transfer inside the fluid. From the graph, we can conclude that as wall thickness parameter D goes on increases , the average Nusselt Number goes on decreases.

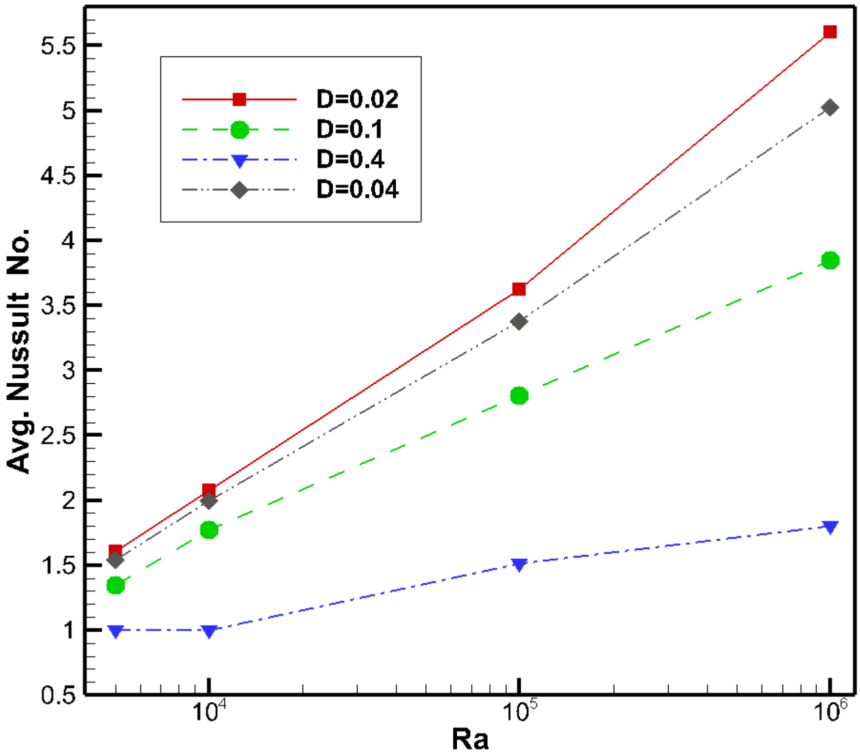


Figure 4.3. Variations of Nu with Ra for different D at Kr =1.

4.1.2.2 Effect of Thermal Conductivity Ratio Kr on Nusselt Number

Figure 4.4 show variation of the the average Nusselt number with the Rayleigh number for different value of thermal conductivity ratio Kr . The Nusselt number is increases as thermal conductivity ratio increases. This is due to the increase in the wall thermal conductivity , heat transfer rate through wall increases. This increases temperature difference across the cavity ,which motivates the convective heat transfer through the cavity .

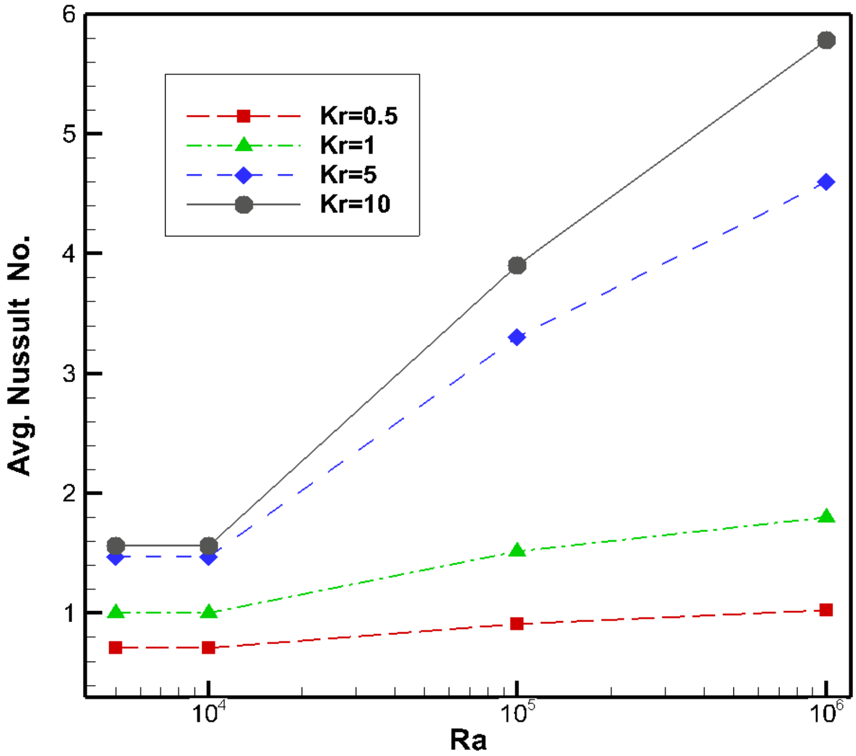


Figure 4.4. Variations of Nu with Ra for different Kr at D = 0.4.

4.2 Case II :- Sinusoidal Bottom Wall

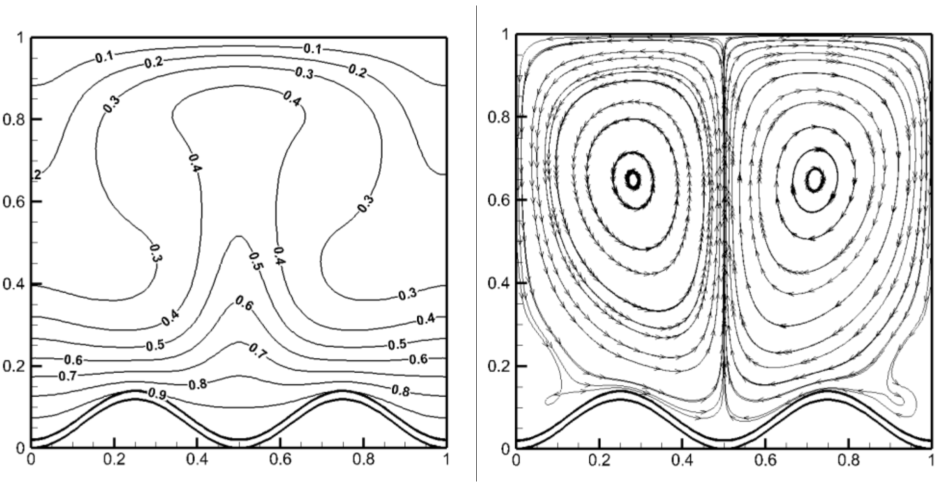
4.2.1 Effect of Governing Parameters on Thermal and Flow Field

4.2.1.1 Effect of Wall Thickness Parameter D

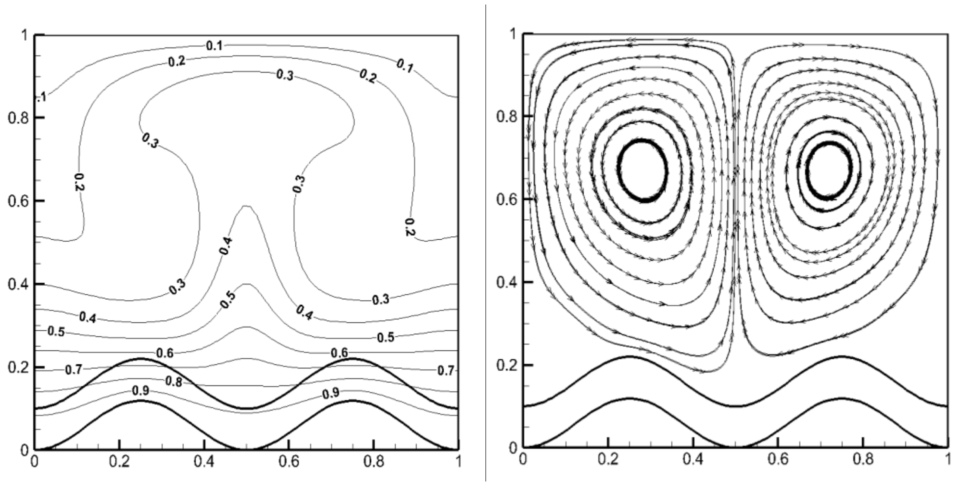
Figure 4.5 shows the effect of the wall thickness parameter D on thermal field and flow field for a sinusoidal bottom wall with two undulations, amplitude A = 0.06 for different Wall thickness parameter D at Ra = 10^5 , Kr =1 . As we can see that, the parameter D affect the fluid and solid temperature and as well as the flow characteristics . From the plots we can observed that ,due to wavy nature of wall the two contra-rotative cells are formed at thin wavy bottom wall instead of one main circulation cell as in case of plane bottom wall. As D increases , these cell becomes elliptical and main circulation cell starts to dominate. The Circulation strength of wall of D = 0.02 is much higher than the wall of D = 0.4. As D increases , the circulation strength of main cell decreases. Due to increase in D, the temperature drop across the solid increases , leads to decrease in interface wall flux. So heat transfer though the convection mode decrease. This results in decrease in circulation strength.

Table 6. Maximum Value of Streamfunction ψ_{max} for different wall thickness parameter D for Sinusoidal wall

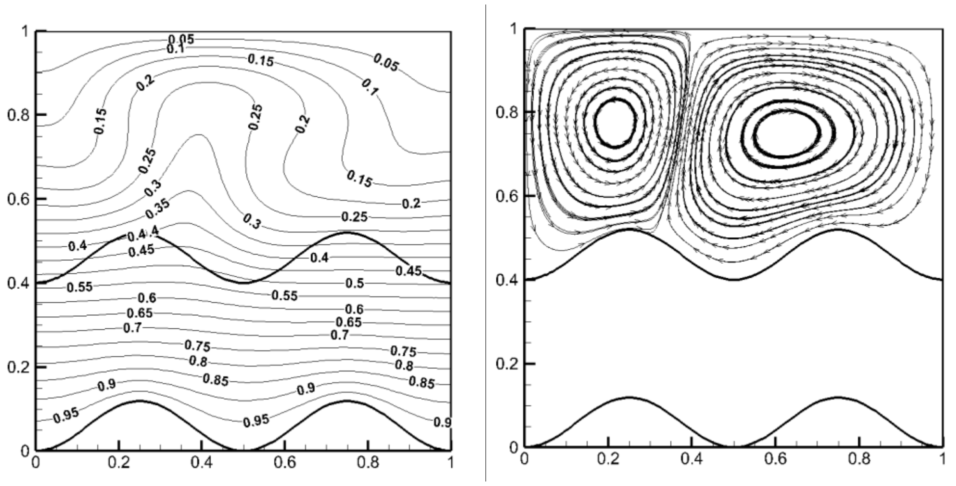
	Maximum Value of Streamfunction ψ_{max} ($\times 10^{-4}$)
D=0.02	5.128163
D=0.1	4.465025
D=0.4	1.788775



D=0.02



D=0.1



D=0.4

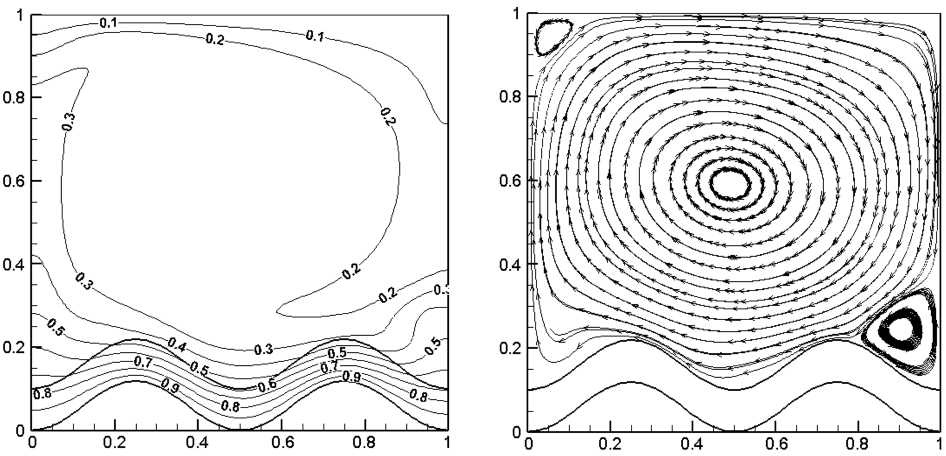
Figure 4.5. Isotherms (left) and Streamlines(right) at Ra =105 , Kr =1, A=0.06, n=2.

4.2.1.2 Effect of Thermal Conductivity Ratio Kr

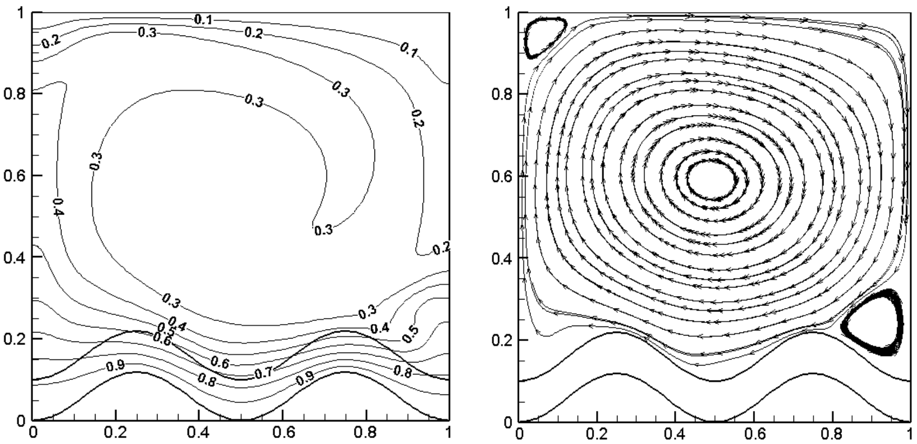
The effect of the Thermal Conductivity Ratio Kr on thermal field and flow field for a sinusoidal bottom wall is shown by figure 4.6 for Ra = 10^6, D = 0.1, A = 0.06 and n = 2. Four different thermal conductivity ratio were selected : Kr =0.5 , Kr =1, Kr =5, Kr =10 . A small circulation cell is formed in the cavity , because of high Rayleigh number (Ra =10^6) which increases convective heat transfer through fluid and remains stable as increase in Kr . Due to increase in Kr result into increase in heat transfer through wall . so , heat flux rate across the interface goes on increases ,which make convective heat transfer stronger. The table shows that , circulation strength of the main cell goes on increases as Kr increases.

Table 7. Maximum Value of Streamfunction ψ_{max} for different thermal conductivity ratio Kr for Sinusoidal wall

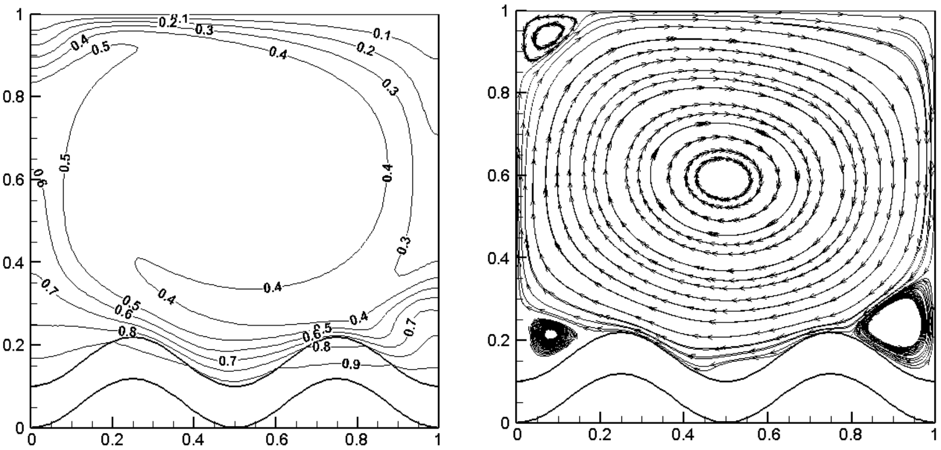
	Maximum Value of Streamfunction ψ_{max} ($\times 10^{-3}$)
Kr =0.5	1.118388
Kr =1	1.275891
Kr =5	1.503197
Kr =10	1.548791



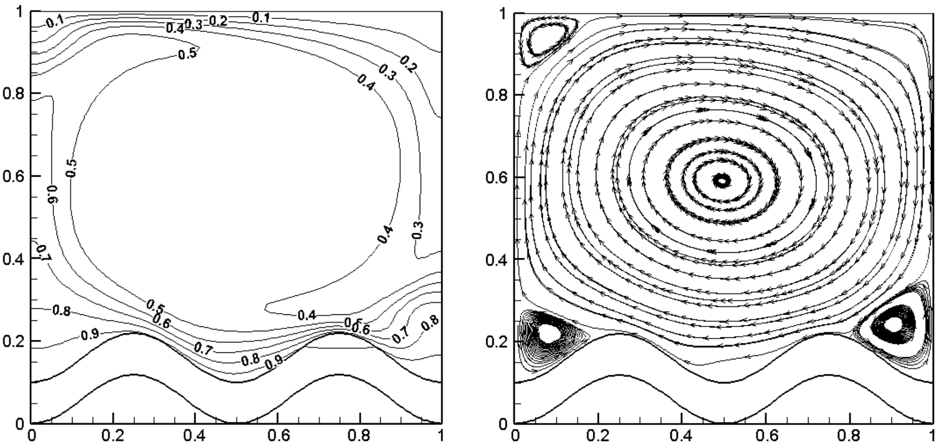
Kr=0.5



Kr=1



Kr=5



Kr=10

Figure 4.6. Isotherms(left) and Streamlines(right) at Ra=10^6,D=0.1,A=0.06.

4.2.1.3 Effect of Amplitude of Sinusoidal Wall A

Figure 4.7 shows the effect of change in Amplitude A of sinusoidal wall on the thermal field and flow field for parameter Ra =10^5 , D = 0.4 , Kr = 5 and n = 2. Two values of A is taken A=0.06, A=0.08 and results compared with Plane wall (A =0). Two circulation cells were created at plane wall ,as we increases the amplitude of wall ,the circulation cells becomes elliptical but there isn't any drastic changes in the circulation pattern of flow is observed. The increases in the amplitude of Sinusoidal wall , decreases the strength of the circulation of flow.

Table 8. Maximum Value of Streamfunction ψ_{max} for different Amplitude A for Sinusoidal wall

	Maximum Value of Streamfunction ψ_{max} ($\times10^{-4}$)

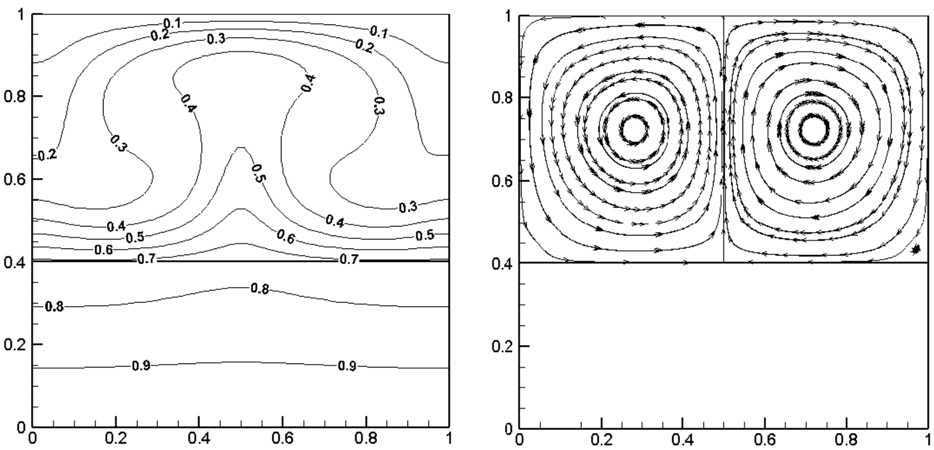
A = 0	4.963788
A = 0.06	3.050293
A = 0.08	2.078449

4.2.1.4 Effect of Number of Undulations n

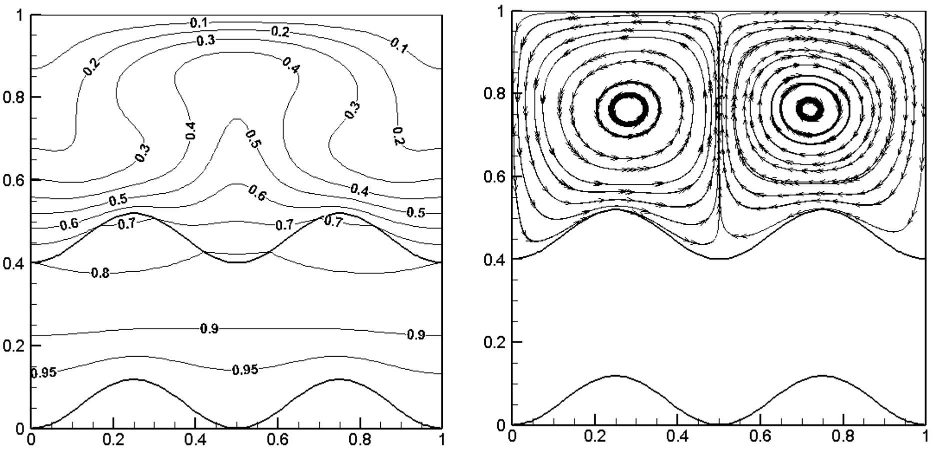
Figure 4.8 shows the effect of sinusoidal wall on the thermal field and flow field for parameter Ra $=10^6$, D = 0.2 , Kr = 5 and n = 2. The a main circulation cell is formed with plane wall . On introduction of waviness in the bottom wall ,circulation cell become elliptical and circulation strength decreases. But as we increases number of frequency of waviness , circulation strength starts on increases.

Table 9. Maximum Value of Streamfunction ψ_{max} for different Number of Undulations n for Sinusoidal wall

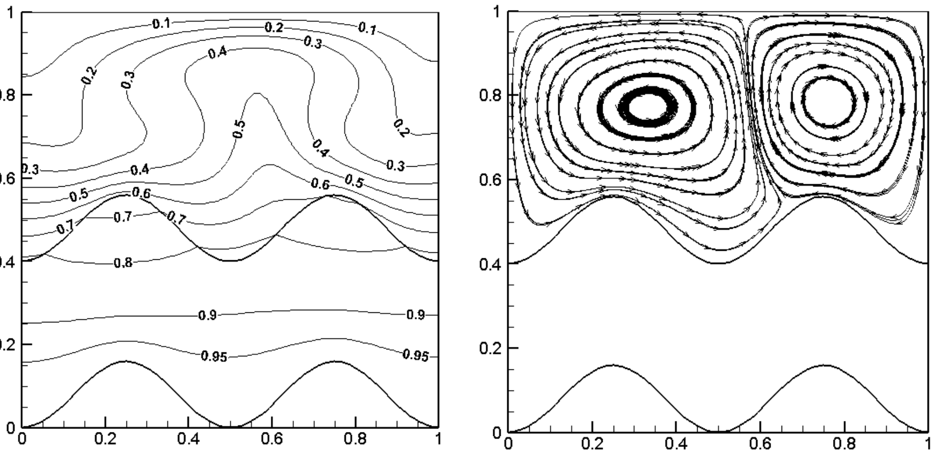
	Maximum Value of Streamfunction ψ_{max} ($\times 10^{-4}$)
n = 0	10.05457
n = 1	8.8861161
n = 2	8.994908



A = 0

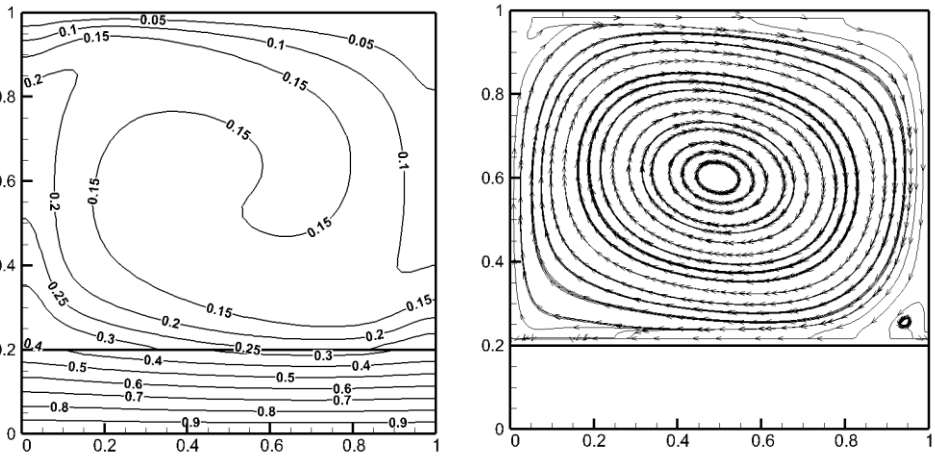


A = 0.06

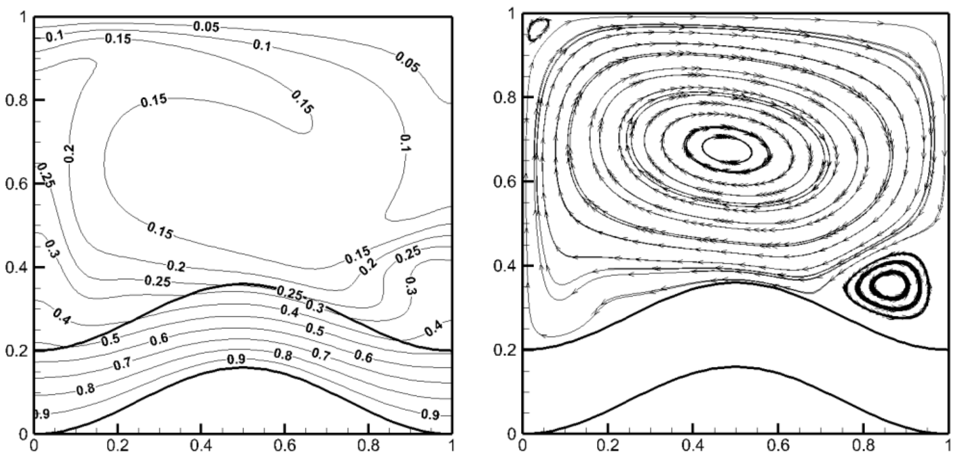


A = 0.08

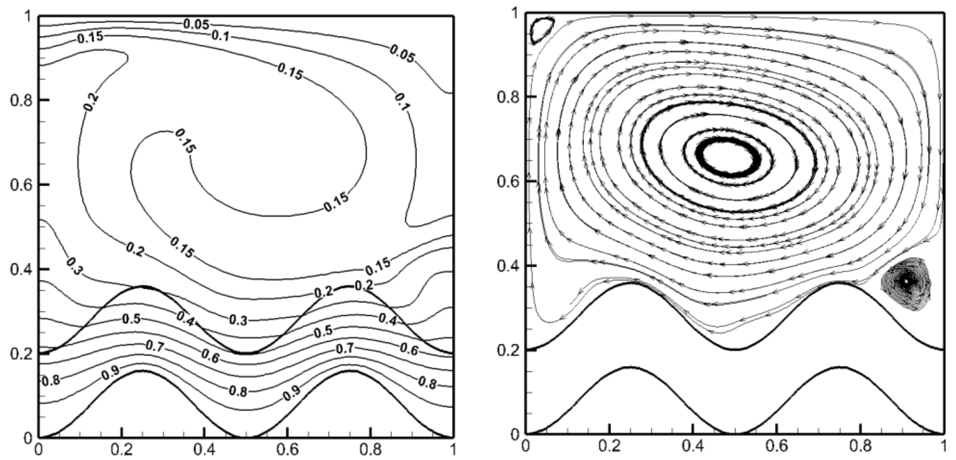
Figure 4.7. Isotherms (left) and Streamlines(right) at Ra =10^5, D=0.4,Kr =5.



n = 0



n = 1



n = 2

Fig 4.8. Isotherms (L) and Streamlines(R) at Ra=10^6, D=0.2, Kr=0.5, A =0.08.

4.2.2 Effect of Governing Parameters on Nusselt Number

4.2.2.1 Effect of Wall Thickness Parameter D

Figure 4.9 shows the variations of average Nusselt number with the Rayleigh number for different values of wall thickness parameter are shown in Figure. The graph shows that increase in Rayleigh number , increase Nusselt number across the interface for any particular value of D. Rayleigh number increases the domination of convective heat transfer i.e. increase in value of h at interface because of increase in the buoyancy forces inside the fluid. As increase in the value of wall thickness parameter D , the value of Nusselt number goes on decreases. The increment in the value of D leads to decrease in the temperature difference across the cavity. So convective heat transfer decreases i.e. decreases in value of h.

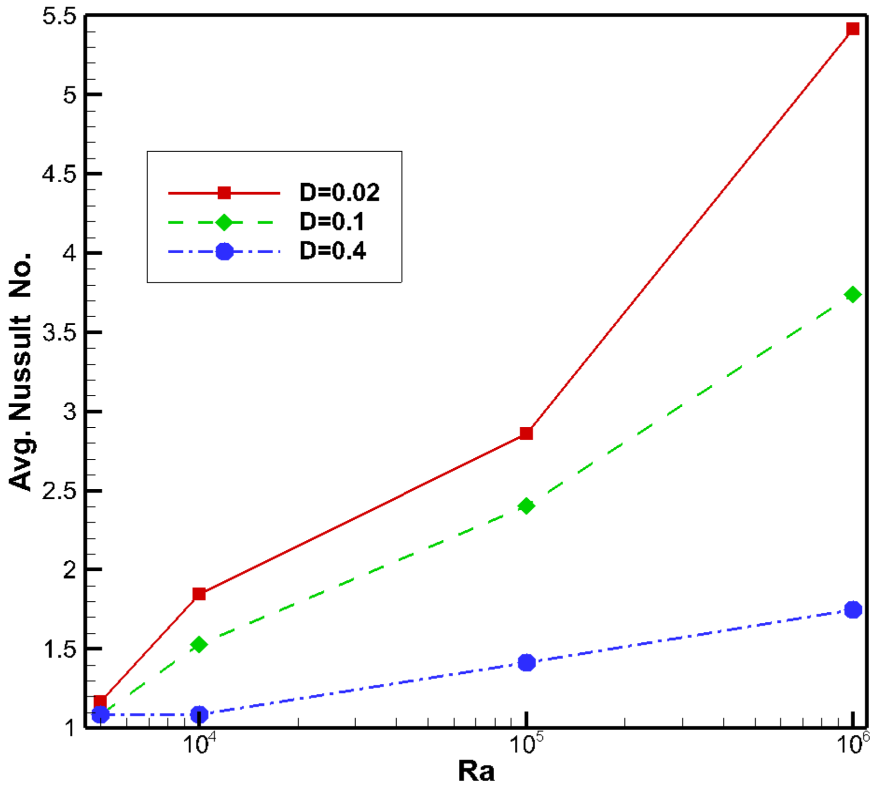


Figure 4.9. Variations of Nu with Ra for different D at Kr =1, A =0.06, n=2.

4.2.2.2 Effect of Thermal Conductivity Ratio Kr

Figure 4.10 illustrate the Effect of Thermal Conductivity Ratio Kr on Nusselt Number . The graph shows that ,The increases in the wall thermal conductivity and Rayleigh number, increase the Nusselt number . As we know on increases in Thermal conductivity ratio Kr , more heat transfer through wall takes place. Due to this temperature gradient across wall interface increase , results in increase in convective heat transfer coefficient h at the interface. So overall Nusselt number goes on increases with constant characteristic length L and constant wall thermal conductivity K_w as in Kr.

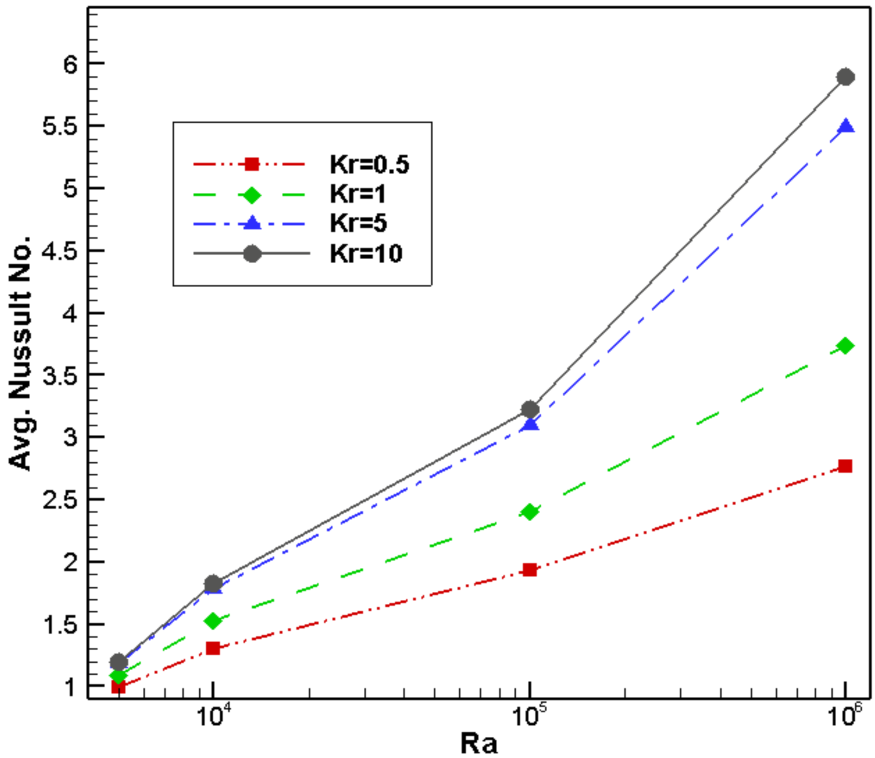


Figure 4.10. Variations of Nu with Ra for different Kr at D =0.1, A =0.06, n=2.

4.2.2.3 Effect of Amplitude of Sinusoidal Wall A and Number of Undulations n

The Effect of change in Amplitude A on Nusselt number with varying the Rayleigh number is shown by the Figure 4.11 for parameter D=0.4 , Kr =5, n=2. It is observed that the increase in Amplitude A decreases the Nusselt number. This is because as increase in Amplitude of wavy wall , there is increase in heat transfer area at the interface so heat flux rate at interface goes on decreases. Though there is increases in value of L but dominance of decrease in heat flux rate the value of Nusselt number decreases.

The figure 4.12 shows the effect of Number of undulation n on the Nusselt number at Ra = 10^5 and Ra = 10^6 . From n = 0 (plane wall case) to n = 1 , the value of Nusselt number decreases due to dominance decreases in the heat flux rate over increase in characteristics length L . From n =1 to n = 2 , the value of Nusselt number start increases. This is because of significant increase in the characteristics length L dominates over the decreases in heat flux rate at interface.

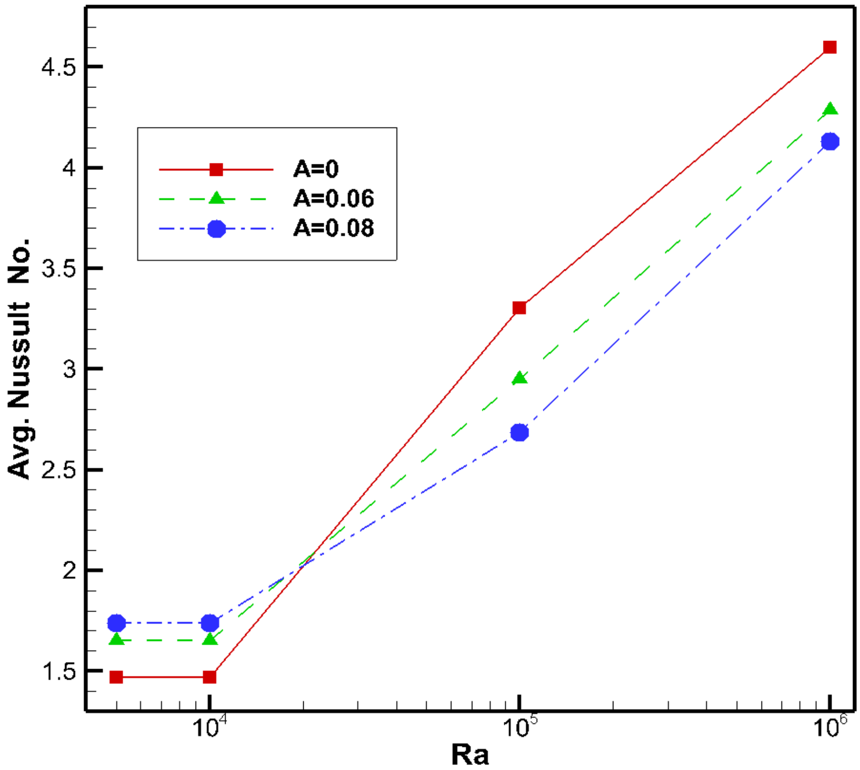


Figure 4.11. Variations of Nu with Ra for different A at D=0.4 , Kr =5, n=2.

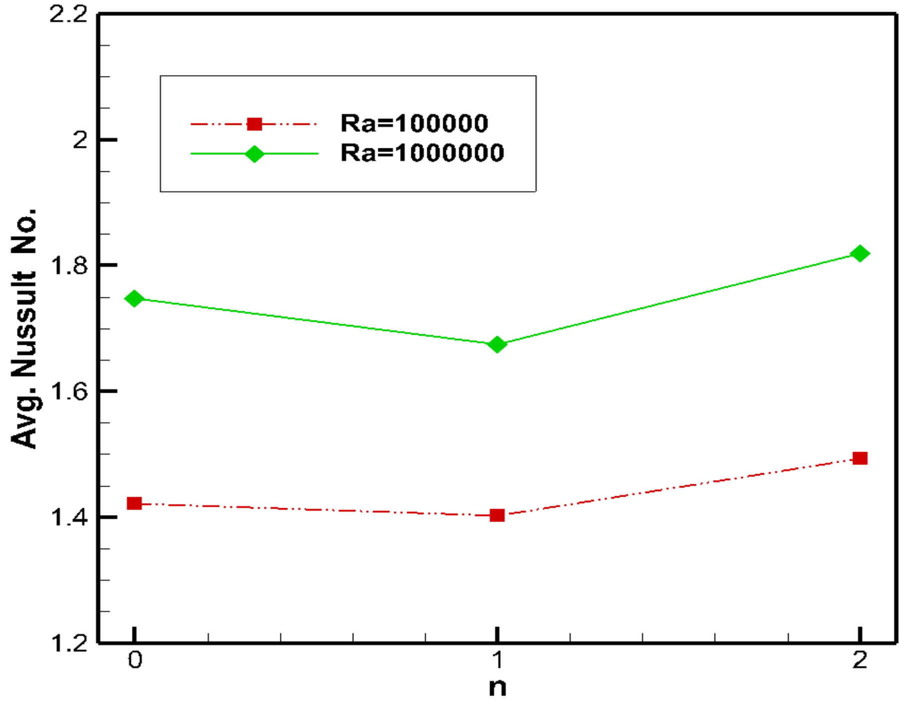


Figure 1.12. Variations of Nu with Ra for different n at D=0.2 , Kr =0.5, A = 0.08.

5. CONCLUSION

CONCLUSION

In the present simulation study , the effect of conduction heat transfer in a Sinusoidal bottom wall on Rayleigh-benard Convection in square cavity is studied. The effects of different Governing parameters i.e. Rayleigh number (Ra) , Thermal conductivity Ratio (Kr) , Wall thickness ratio (D), Amplitude (A), Number of undulations (n) are studied. The Detailed computational Results for flow and temperature field have been presented and analyzed in the form of streamlines and isotherm plots. The effects of governing parameters on Nusselt number at interface is also presented in the graphical forms. The main Conclusions of the present simulation study are as follow.

1. The strength of the flow circulation of the fluid is much higher for the thin bottom wall and for the high thermal conductivity Ratio. But strength of circulation goes on decreases as thickness of bottom wall increases or thermal conductivity ratio decreases.
2. The number of Contrarotative cells and their strength of circulation can be controlled by the thermal conductivity ratio, wall thickness ratio, Rayleigh number ,amplitude and number of undulations .
3. The strength of flow circulation is decreases on introducing wavy nature to the plain bottom bottom wall.
4. The increase in amplitude of wave decreases the circulation strength of fluid in the cavity. The strength of circulation is increases as number of undulation increases.
5. The average Nusselt number increases by increasing either the thermal conductivity ratio , the Rayleigh number or number of undulation.
6. The increase in the wall thickness ratio or the amplitude of wave, decreases the average Nusselt number.

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