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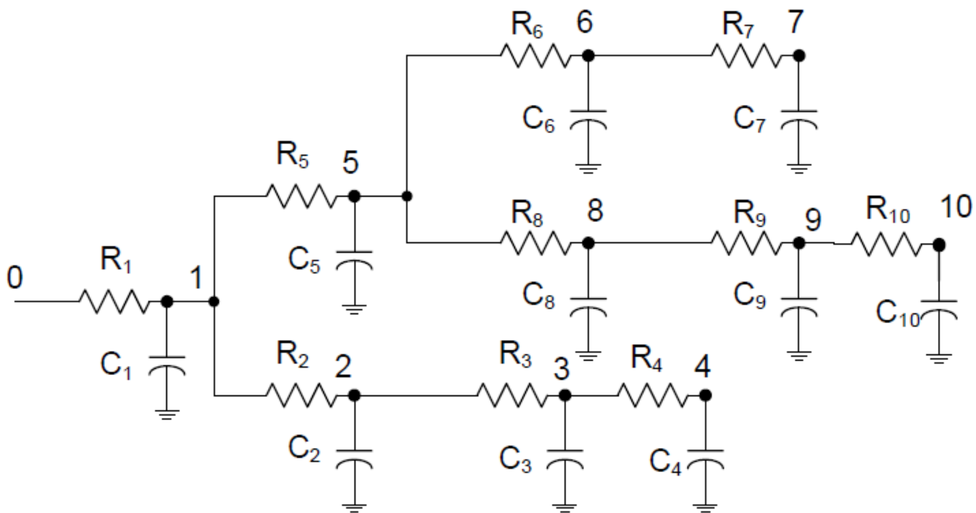
VLSI Timing Analysis
Homework 1

In the following diagram, let $R_1 = 120\Omega$, $C_1 = 2.0$, pF, $R_2 = 30\Omega$, $C_2 = 0.8$ pF, $R_3 = 95\Omega$, $C_3 = 2.5$pF, $R_4 = 125\Omega$, $C_4 = 5$pF, $R_5 = 80\Omega$, $C_5 = 1.4$pF, $R_6 = 40\ \Omega$, $C_6 = 0.5$pF, $R_7 = 120\Omega$, $C_7 = 1.4$pF, $R_8 = 250\Omega$, $C_8 = 2.9$pF, $R_9 = 380\Omega$, $C_9 = 3$ pF, $R_{10} = 580\Omega$, $C_{10} = 9$pF. Let input at node 0 be 4*s(t), where s(t) is a unit step function. You can write programs or use Matlab or other tools (such as Excel) to help you do the assignment.



First order approximation:

The input is taken as impulse function. Hence the m_0 at all the nodes is 4.

$$\mathbf{M_0^0 = M_0^1 = M_0^2 = M_0^3 = M_0^4 = M_0^5 = M_0^6 = M_0^7 = M_0^8 = M_0^9 = M_0^{10} = 4}$$

$$\mathbf{M_0^0 = M_1^0 = M_2^0 = M_3^0 = M_4^0 = M_5^0 = M_6^0 = M_7^0 = M_8^0 = M_9^0 = M_{10}^0 = 0}$$

For the first order approximation, we need moments 0 and 1.

$M_1^1 = M_1^0 - R1*(C1*M_0^1+C2*M_0^2+C3*M_0^3+C4*M_0^4+C5*M_0^5+C6*M_0^6+C7*M_0^7+C8*M_0^8+C9*M_0^9+C10*M_0^{10})$
$M_1^2=M_1^1-R2*(C2*M_0^2+C3*M_0^3+C4*M_0^4)$
$M_1^3=M_1^2-R3*(C3*M_0^3+C4*M_0^4)$
$M_1^4=M_1^3-R4*C4*M_0^4$
$M_1^5=M_1^1-R5*(C5*M_0^5+C6*M_0^6+C7*M_0^7+C8*M_0^8+C9*M_0^9+C10*M_0^{10})$
$M_1^6=M_1^5-R6*(C6*M_0^6+C7*M_0^7)$
$M_1^7=M_1^6-R7*C7*M_0^7$
$M_1^8=M_1^5-R8*(C8*M_0^8+C9*M_0^9+C10*M_0^{10})$
$M_1^9=M_1^8-R9*(C9*M_0^9+C10*M_0^{10})$
$M_1^{10}=M_1^9-R10*C10*M_0^{10}$

Node->	4	7	10
M_0	**4**	**4**	**4**
M_1	**-20.47**	**-20.57**	**-73.62**

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

Calculation: To calculate the values of B, we use the matrix method to determine B.

a. Node 4:

$$\frac{K1}{p1} = -M0 \frac{K1}{p1^2} = -M1$$

Dividing one by the other, we get

K1= 0.795 p1= -0.1987

Now, the *h (t)* is obtained by the following equation,

$$h(t) = k1 * e^{p1*t}$$

We then integrate this to obtain the step response

$$h(t) = integral\ (k1 * e^{p1*t})$$

Solving on Matlab:

$$H(t) = -\frac{7950 * \exp\left(-\frac{1987 * t}{10000}\right)}{1987} + 4$$

b. Node 7:

$$\frac{K1}{p1} = -M0 \frac{K1}{p1^2} = -M1$$

Dividing one by the other, we get

K1= 0.777 p1= -0.1944

Now, the *h(t)* is obtained by the following equation,

$$h(t) = k1 * e^{p1*t}$$

We then integrate this to obtain the step response

$$h(t) = integral\ (k1 * e^{p1*t})$$

Solving on Matlab:

$$H(t) = -\frac{1295 * \exp\left(-\frac{243 * t}{1250}\right)}{324} + 4;$$

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

c. Node 10:

$$\frac{K1}{p1} = -M0 \quad \frac{K1}{p1^2} = -M1$$

Dividing one by the other, we get

K1= 0.2173 p1= -0.054

Now, the *h(t)* is obtained by the following equation,

$$h(t) = k1 * e^{p1*t}$$

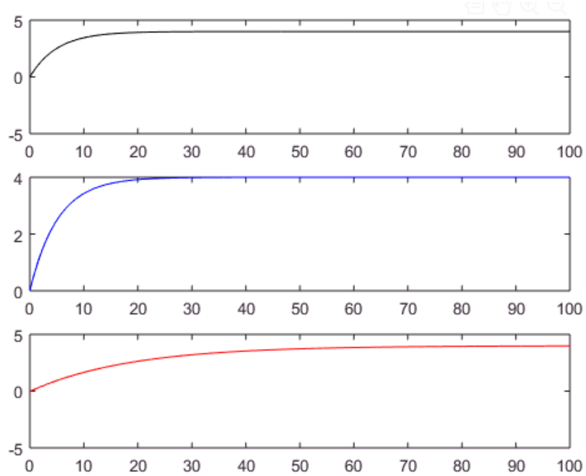
We then integrate this to obtain the step response

$$h(t) = integral\ (k1 * e^{p1*t})$$

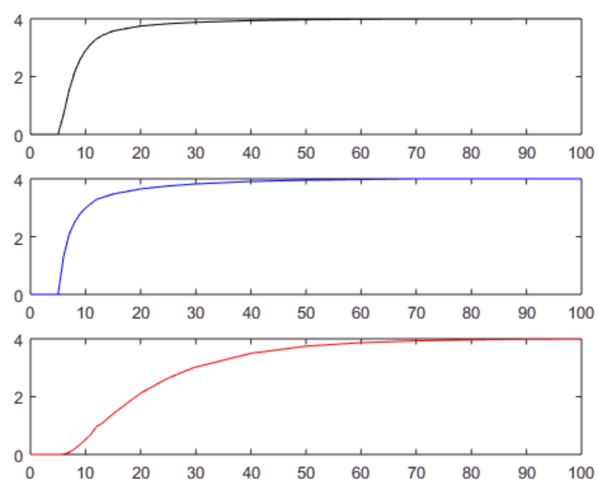
Solving on Matlab:

$$h(t) = -\frac{2173 * \exp\left(-\frac{27 * t}{500}\right)}{540} + 4$$

Plot for 1st order approximation:



Plot from HSPICE simulation:



Black: Node4; Blue: Node7; Red: Node10

From the above comparison, it is seen that the first order is not sufficient. So we move on to higher order.

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

Second order approximation:

For the second order approximation, we need M_2 and M_3

$M_2^1 = M_2^0$ - R1*(C1*M_1^1+C2*M_1^2+C3*M_1^3+C4*M_1^4+C5*M_1^5+C6*M_1^6+C7*M_1^7+C8*M_1^8+C9*M_1^9+C10*M_1^{10})
M_2^2=M_2^1-R2*(C2*M_1^2+C3*M_1^3+C4*M_1^4)
M_2^3=M_2^2-R3*(C3*M_1^3+C4*M_1^4)
M_2^4=M_2^3-R4*C4*M_1^4
M_2^5=M_2^1-R5*(C5*M_1^5+C6*M_1^6+C7*M_1^7+C8*M_1^8+C9*M_1^9+C10*M_1^{10})
M_2^6=M_2^5-R6*(C6*M_1^6+C7*M_1^7)
M_2^7=M_2^6-R7*C7*M_1^7
M_2^8=M_2^5-R8*(C8*M_1^8+C9*M_1^9+C10*M_1^{10})
M_2^9=M_2^8-R9*(C9*M_1^9+C10*M_1^{10})
M_2^{10}=M_2^9-R10*C10*M_1^{10}

$M_3^1 = M_3^0$ - R1*(C1*M_2^1+C2*M_2^2+C3*M_2^3+C4*M_2^4+C5*M_2^5+C6*M_2^6+C7*M_2^7+C8*M_2^8+C9*M_2^9+C10*M_2^{10})
M_3^2=M_3^1-R2*(C2*M_2^2+C3*M_2^3+C4*M_2^4)
M_3^3=M_3^2-R3*(C3*M_2^3+C4*M_2^4)
M_3^4=M_3^3-R4*C4*M_2^4
M_3^5=M_3^1-R5*(C5*M_2^5+C6*M_2^6+C7*M_2^7+C8*M_2^8+C9*M_2^9+C10*M_2^{10})
M_3^6=M_3^5-R6*(C6*M_2^6+C7*M_2^7)
M_3^7=M_3^6-R7*C7*M_2^7
M_3^8=M_3^5-R8*(C8*M_2^8+C9*M_2^9+C10*M_2^{10})
M_3^9=M_3^8-R9*(C9*M_2^9+C10*M_2^{10})
M_3^{10}=M_3^9-R10*C10*M_2^{10}

Node->	4	7	10
M_2	**172.54**	**225.04**	**1146.45**
M_3	**-2229**	**-3.1*10^3**	**-1.7*10^4**

First we calculate B using matrix inverse method

$$\begin{bmatrix} m_0 & m_1 \\ m_1 & m_2 \end{bmatrix} \begin{bmatrix} b_2 \\ b_1 \end{bmatrix} = -\begin{bmatrix} m_2 \\ m_3 \end{bmatrix}$$

Then we solve the quadratic equation using MATLAB to obtain P

$$b_2 \hat{p}^2 + b_1 \hat{p} + 1 = 0$$

Using these two, we solve the linear equations to obtain K

$$\frac{\hat{k}_1}{\hat{p}_1} + \frac{\hat{k}_2}{\hat{p}_2} = -m_0 \qquad \frac{\hat{k}_1}{\hat{p}_1{}^3} + \frac{\hat{k}_2}{\hat{p}_2{}^3} = -m_2$$

$$\frac{\hat{k}_1}{\hat{p}_1{}^2} + \frac{\hat{k}_2}{\hat{p}_2{}^2} = -m_1 \qquad \frac{\hat{k}_1}{\hat{p}_1{}^4} + \frac{\hat{k}_2}{\hat{p}_2{}^4} = -m_3$$

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

From this, we obtain h(t)

$$\hat{h}(t) = k_1 e^{p_1 t} + k_2 e^{p_2 t} + \cdots + k_q e^{p_q t}$$

Integrate and plot on MATLAB.

a. <u>Node 4:</u>

From Matrix method,

B2= 52.8697 B1= 19.0845

Solving for P,

P2= -0.063 P1=-0.297

Matrix method to obtain K,

K2= 0.03 k1=1.03

The response,

$$H(t) = k1 * e^{p1*t} + k2 * e^{p2*t}$$

On integrating,

$$H(t) = -\frac{10 * \exp\left(-\frac{63 * t}{1000}\right)}{21} - \frac{1030 * \exp\left(-\frac{297 * t}{1000}\right)}{297} + 4$$

b. <u>Node 7</u>

From Matrix method,

B2= 31.20 B1= 17.01

Solving for P,

P2= -0.067 P1=-0.478

Matrix method to obtain K,

K2= 1.45 k1=0.06

The response,

$$H(t) = k1 * e^{p1*t} + k2 * e^{p2*t}$$

On integrating,

$$h(t) = -\frac{725 * \exp\left(-\frac{239 * t}{500}\right)}{239} - \frac{60 * \exp\left(-\frac{67 * t}{1000}\right)}{67} + 4$$

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

c. Node 10

From Matrix method,

B2= 41.65 B1= 17.83

Solving for P,

P2= -0.066 P1=-0.361

Matrix method to obtain K,

K2= -0.37 k1=0.33

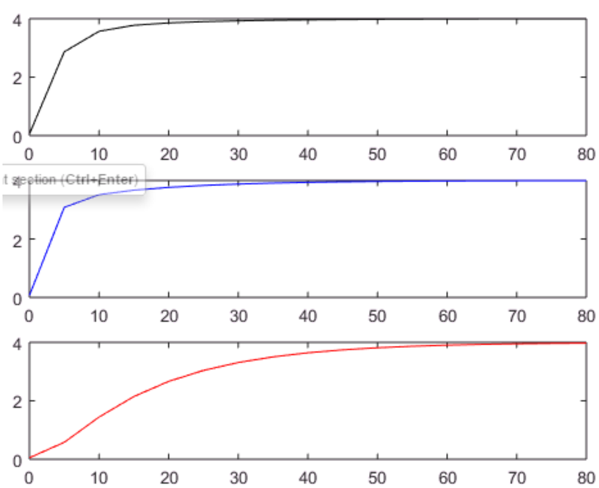
The response,

$$H(t) = k1 * e^{p1*t} + k2 * e^{p2*t}$$

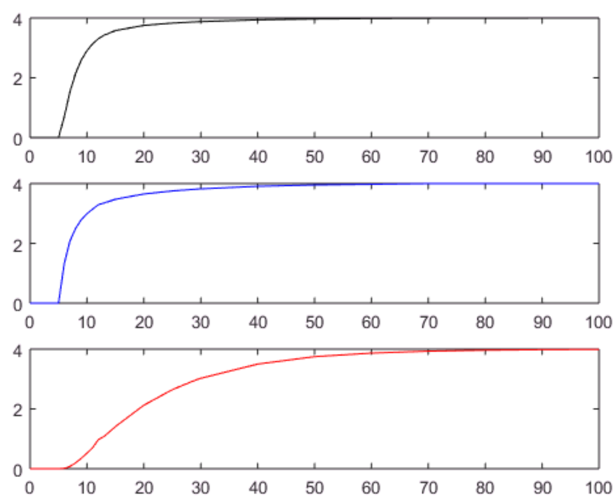
On integrating,

$$h(t) = \frac{20 * \exp\left(-\frac{361 * t}{1000}\right)}{19} - 5 * \exp\left(-\frac{33 * t}{500}\right) + 4$$

Plot for 2nd order approximation:



Plot from HSPICE simulation:



Black: Node4; Blue: Node7; Red: Node10

From the above comparison, it is seen that the second order is not sufficient. So we move on to higher order.

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

Third order approximation:

For the second order approximation, we need M_4 and M_5

$M_4^1 = M_4^0$ - R1*(C1*M_3^1+C2*M_3^2+C3*M_3^3+C4*M_3^4+C5*M_3^5+C6*M_3^6+C7*M_3^7+C8*M_3^8+C9*M_3^9+C10*M_3^{10})
M_4^2=M_4^1-R2*(C2*M_3^2+C3*M_3^3+C4*M_3^4)
M_4^3=M_4^2-R3*(C3*M_3^3+C4*M_3^4)
M_4^4=M_4^3-R4*C4*M_3^4
M_4^5=M_4^1-R5*(C5*M_3^5+C6*M_3^6+C7*M_3^7+C8*M_3^8+C9*M_3^9+C10*M_3^{10})
M_4^6=M_4^5-R6*(C6*M_3^6+C7*M_3^7)
M_4^7=M_4^6-R7*C7*M_3^7
M_4^8=M_4^5-R8*(C8*M_3^8+C9*M_3^9+C10*M_3^{10})
M_4^9=M_4^9-R9*(C9*M_3^9+C10*M_3^{10})
M_4^{10}=M_4^9-R10*C10*M_3^{10}

$M_5^1 = M_5^0$ - R1*(C1*M_4^1+C2*M_4^2+C3*M_4^3+C4*M_4^4+C5*M_4^5+C6*M_4^6+C7*M_4^7+C8*M_4^8+C9*M_4^9+C10*M_4^{10})
M_5^2=M_5^1-R2*(C2*M_4^2+C3*M_4^3+C4*M_4^4)
M_5^3=M_5^2-R3*(C3*M_4^3+C4*M_4^4)
M_5^4=M_5^3-R4*C4*M_4^4
M_5^5=M_5^1-R5*(C5*M_4^5+C6*M_4^6+C7*M_4^7+C8*M_4^8+C9*M_4^9+C10*M_4^{10})
M_5^6=M_5^5-R6*(C6*M_4^6+C7*M_4^7)
M_5^7=M_5^6-R7*C7*M_4^7
M_5^8=M_5^5-R8*(C8*M_4^8+C9*M_4^9+C10*M_4^{10})
M_5^9=M_5^8-R9*(C9*M_4^9+C10*M_4^{10})
M_5^{10}=M_5^9-R10*C10*M_4^{10}

Node->	4	7	10
M_4	**$3.2*10^4$**	**$47.5*10^3$**	**$2.62*10^5$**
M_5	**$-4.8*10^5$**	**$-1.5*10^6$**	**$-3.96*10^6$**

Note: We follow the same method as in the second order approximation. Here we will have three values of B, P and K.

a. Node 4

From Matrix method,

B3=45.47 B2= 59.93 B1= 18.85

Solving for P,

P3= -0.06 P2= -0.38 P1=-0.87

Matrix method to obtain K,

K3= 0.029 K2= -0.37 k1=0.33

The response,

$$H(t) = k1 * e^{p1*t} + k2 * e^{p2*t} + k3 * e^{p3*t}$$

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

On integrating,

$$H(t) = \frac{152 * \exp\left(-\frac{87 * t}{100}\right)}{87} - \frac{199 * \exp\left(-\frac{19 * t}{50}\right)}{38} - \frac{29 * \exp\left(-\frac{3 * t}{50}\right)}{60} + 4$$

b. Node 7

From Matrix method,

B3=4.5*10^4 B2= 205*10^4 B1= 1.47*10^3

Solving for P,

P3= -0.007 P2= -0.06 P1=-0.48

Matrix method to obtain K,

K3= 0 K2= 0.05 k1=1.51

The response,

$$H(t) = k1 * e^{p1*t} + k2 * e^{p2*t} + k3 * e^{p3*t}$$

On integrating,

$$H(t) = -\frac{151 * \exp\left(-\frac{12 * t}{25}\right)}{48} - \frac{5 * \exp\left(-\frac{3 * t}{50}\right)}{6} + 4$$

c. Node 10

From Matrix method,

B3=-395 B2= -181 B1= 4.78

Solving for P,

P3= -0.06 P2= -0.08 P1=-0.47

Matrix method to obtain K,

K3= 0.174 K2= 0.176 k1=-0.52

The response,

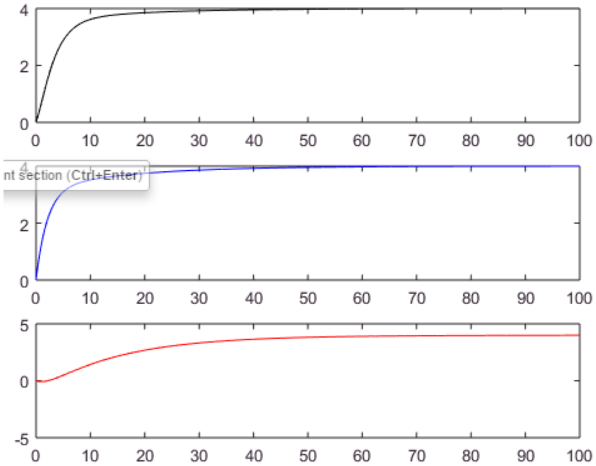
$$H(t) = k1 * e^{p1*t} + k2 * e^{p2*t} + k3 * e^{p3*t}$$

On integrating,

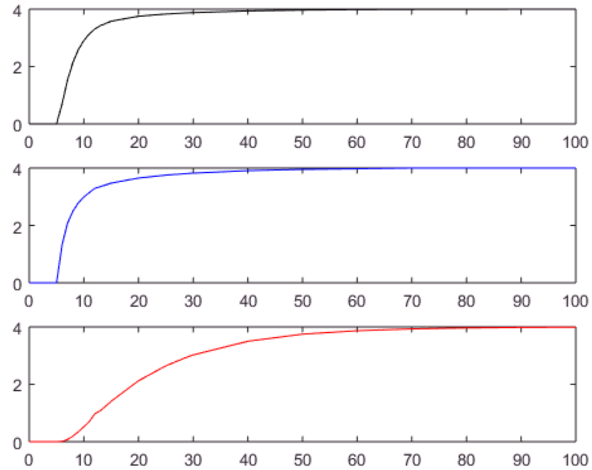
$$h(t) = \frac{52 * \exp\left(-\frac{47 * t}{100}\right)}{47} - \frac{29 * \exp\left(-\frac{3 * t}{50}\right)}{10} - \frac{11 * \exp\left(-\frac{2 * t}{25}\right)}{5} + 4$$

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

Plot for 3rd order approximation:



Plot from HSPICE simulation:



Black: Node4; Blue: Node7; Red: Node10

From the above comparison, it is seen that the third order is not sufficient. So we move on to higher order.

<u>Fourth order approximation:</u>

For the second order approximation, we need M_6 and M_7

$M_6^1 = M_6^0$ - $R1*(C1*M_5^1+C2*M_5^2+C3*M_5^3+C4*M_5^4+C5*M_5^5+C6*M_5^6+C7*M_5^7+C8*M_5^8+C9*M_5^9+C10*M_5^{10})$
$M_6^2=M_6^1-R2*(C2*M_5^2+C3*M_5^3+C4*M_5^4)$
$M_6^3=M_6^2-R3*(C3*M_5^3+C4*M_5^4)$
$M_6^4=M_6^3-R4*C4*M_5^4$
$M_6^5=M_6^1-R5*(C5*M_5^5+C6*M_5^6+C7*M_5^7+C8*M_5^8+C9*M_5^9+C10*M_5^{10})$
$M_6^6=M_6^5-R6*(C6*M_5^6+C7*M_5^7)$
$M_6^7=M_6^6-R7*C7*M_5^7$
$M_6^8=M_6^5-R8*(C8*M_5^8+C9*M_5^9+C10*M_5^{10})$
$M_6^9=M_6^9-R9*(C9*M_5^9+C10*M_5^{10})$
$M_6^{10}=M_6^9-R10*C10*M_5^{10}$

$M_7^1 = M_7^0$ - $R1*(C1*M_6^1+C2*M_6^2+C3*M_6^3+C4*M_6^4+C5*M_6^5+C6*M_6^6+C7*M_6^7+C8*M_6^8+C9*M_6^9+C10*M_6^{10})$
$M_7^2=M_7^1-R2*(C2*M_6^2+C3*M_6^3+C4*M_6^4)$
$M_7^3=M_7^2-R3*(C3*M_6^3+C4*M_6^4)$
$M_7^4=M_7^3-R4*C4*M_6^4$
$M_7^5=M_7^1-R5*(C5*M_6^5+C6*M_6^6+C7*M_6^7+C8*M_6^8+C9*M_6^9+C10*M_6^{10})$
$M_7^6=M_7^5-R6*(C6*M_6^6+C7*M_6^7)$
$M_7^7=M_7^6-R7*C7*M_6^7$
$M_7^8=M_7^5-R8*(C8*M_6^8+C9*M_6^9+C10*M_6^{10})$
$M_7^9=M_7^8-R9*(C9*M_6^9+C10*M_6^{10})$
$M_7^{10}=M_7^9-R10*C10*M_6^{10}$

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

Node->	4	7	10
M_6	**$7.36*10^6$**	**$10.6*10^6$**	**$0.5*10^8$**
M_7	**$-111.04*10^6$**	**$-1.6*10^8$**	**$-8.9*10^8$**

Note: We follow the same method as in the third order approximation. Here we will have four values of B, P and K.

a. Node 4

From Matrix method,

B4= -1.92*103 B3=-2.24*103 B2= -628.81 B1=17.33

Solving for P,

P4=-0.7452 P3= -0.382 P2= -0.066 P1=0.0275

Matrix method to obtain K,

K4=-1.385 K3= 2.036 K2= 0.032 k1=-0.0001

The response,

$$H(t) = k1 * e^{p1*t} + k2 * e^{p2*t} + k3 * e^{p3*t} + k4 * e^{p4*t}$$

On integrating,

$$h(t) = \frac{\frac{\frac{277 * \exp\left(-\frac{3*t}{4}\right)}{150} - \exp\left(-\frac{3*t}{50}\right)}{2} - \frac{203 * \exp\left(-\frac{19*t}{50}\right)}{38} - \exp\left(\frac{27*t}{1000}\right)}{135} + 4$$

b. Node 7

From Matrix method,

B4= $5.7*10^3$ B3=$18.5*10^3$ B2= 260.3 B1=-14.14

Solving for P,

P4=-136.4 P3= -1.668 P2= -0.3602 P1=-0.066

Matrix method to obtain K,

K4=-108.6 K3= 3.134 K2= 0.7206 k1=0.0609

The response,

$$H(t) = k1 * e^{p1*t} + k2 * e^{p2*t} + k3 * e^{p3*t} + k4 * e^{p4*t}$$

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

On integrating,

$$h(t) = \frac{10861 * \exp\left(-\frac{682 * t}{5}\right)}{13640} - \frac{627 * \exp\left(-\frac{167 * t}{100}\right)}{334} - \frac{305 * \exp\left(-\frac{83 * t}{1250}\right)}{332} - \frac{3603 * \exp\left(-\frac{1801 * t}{5000}\right)}{1801} + 4;$$

c. <u>Node 10</u>

From Matrix method,

B4= $2.17*10^5$ B3=$87.9*10^3$ B2= $4.6*10^3$ B1=1.218

Solving for P,

P4=-1.572 P3= -0.856 P2= -0.375 P1=-0.066

Matrix method to obtain K,

K4=0.8975 K3= -0.7019 K2= -0.331 k1=0.3354

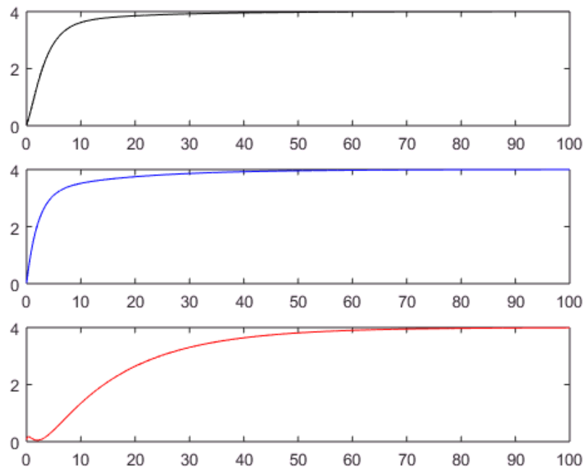
The response,

$$H(t) = k1 * e^{p1*t} + k2 * e^{p2*t} + k3 * e^{p3*t} + k4 * e^{p4*t}$$

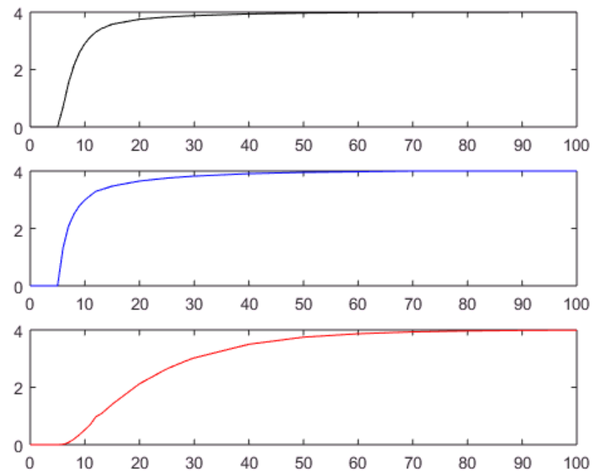
On integrating,

$$H(t) = \frac{54 * \exp\left(-\frac{47 * t}{100}\right)}{47} - \frac{26 * \exp\left(-\frac{3 * t}{50}\right)}{10} - \frac{12 * \exp\left(-\frac{2 * t}{25}\right)}{5} + 4$$

Plot for 3rd order approximation:

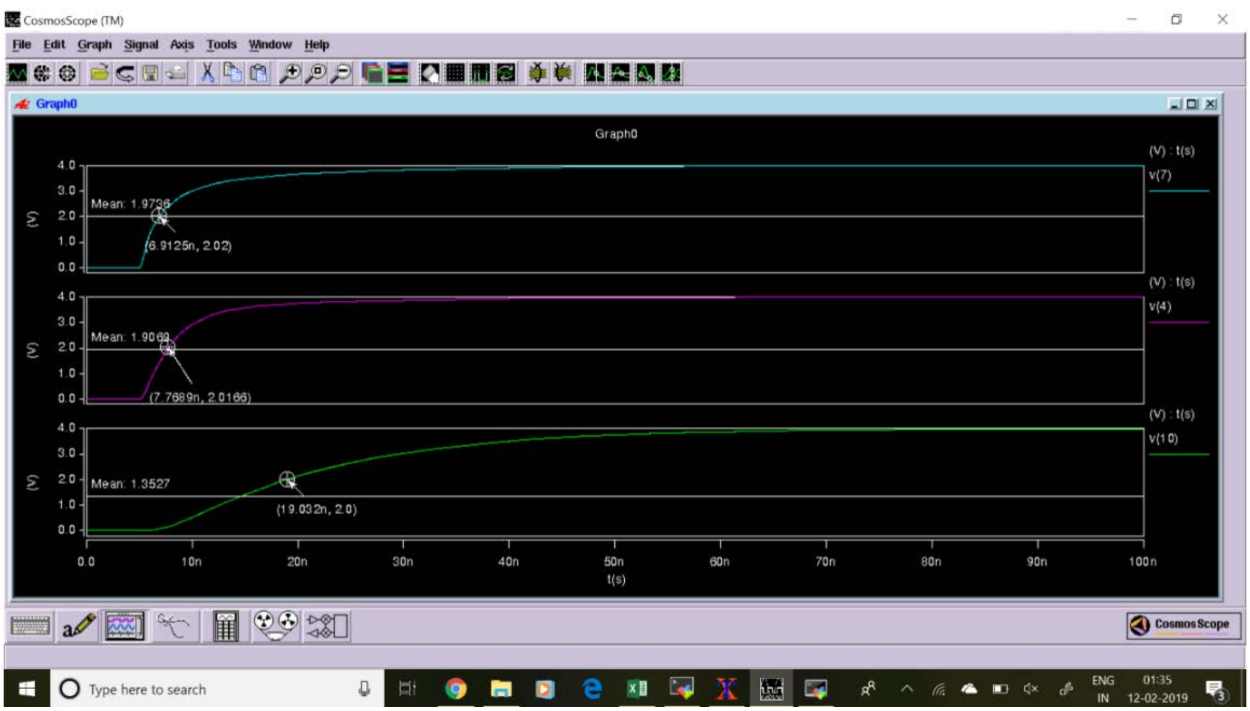


Plot from HSPICE simulation:



From the above comparison, it is seen that the fourth order is sufficient. So we move finalize this order.

The Elmore delay at these nodes are defined by First order moments at that node.



	Node 4	Node 7	Node 10
Mean (Elmore Delay)	7.5698ns	6.835ns	14.656n
$T_{50\%}$	7.768ns	6.9125ns	19.032ns
Mean	1.9069	1.9736	1.3527

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

Vinay Manjunath Prabhu
825088894

The first order moments are:

Node->	4	7	10
M_1	**-20.47**	**-20.57**	**-73.62**

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

Appendix

1. **HSPICE Code:**

```
*This file will describe the HSPICE of the given problem

*Define the voltage source that is unit step function
vs vs gnd PWL(0 0V 5n 0V 5.01n 4V 10.01n 4V 20n 4v 50n 4v)

*Define the design

R1 vs 1 120
C1 1 gnd 2pF
R2 1 2 30
C2 2 gnd 0.8pF
R3 2 3 95
C3 3 gnd 2.5pF
R4 3 4 125
C4 4 gnd 5pF
R5 1 5 80
C5 5 gnd 1.4pF
R6 5 6 40
C6 6 gnd 0.5pF
R7 6 7 120
C7 7 gnd 1.5pF
R8 5 8 250
C8 8 gnd 2.9pF
R9 8 9 380
C9 9 gnd 3pF
R10 9 10 580
C10 10 gnd 9pF

*Perform the analysis
.tran 0.01n 100n
.print tran V(vs) V(4)
.print tran V(vs) V(7)
.print tran V(vs) V(10)

.option post

.end
```

2. Calculation of B using matrix method

```
%2nd Order Approximation
%Change the values of constants at different nodes for  to obtain Variables
clc;
clear;
%Node 4
m04=4;
m14=-20.122;
m24=172.54;
m34=-2229;
a4=[m04,m14;
   m14,m24];
b4=-[m24;
    m34]';
z4=b4/a4
%Node 7
m07=4;
m17=-20.57;
m27=225.04;
m37=-3186.35;
a7=[m07,m17;
    m17,m27];
b7=-[m27;
     m37]';
z7=b7/a7
%Node 10
m010=4;
m110=-73.614;
m210=1146.4529;
m310=-17383.2;
a10=[m010,m110;
     m110,m210];
b10=-[m210;
      m310]';
z10=b10/a10
```

```
%3rd Order Approximation
%Change the values of constants for different nodes to obtain Variables
clc;
clear;
%Node4
m04=4;
m14=-20.122;
m24=172.54;
m34=-2229;
m44=32.6*10^3;
m54=-488.9*10^3;
a4=[m04,m14,m24;
    m14,m24,m34;
    m24,m34,m44];
b4=-[m34;
     m44;
     m54]';
z4=b4/a4
```

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

```
% Node 7
m07=4;
m17=-20.57;
m27=225.04;
m37=-3.18*10^3;
m47=47.5*10^3;
m57=-1.49*10^6;
a7=[m07,m17,m27;
    m17,m27,m37;
    m27,m37,m47];
b7=-[m37;
     m47;
     m57]';
z7=b7/a7
%Node 10
m010=4;
m110=-73.614;
m210=1146.45;
m310=-17.3*10^3;
m410=262.3*10^3;
m510=-3.95*10^6;
a10=[m010,m110,m210;
    m110,m210,m310;
    m210,m310,m410];
b10=-[m310;
     m410;
     m510]';
z10=b10/a10

%4th order approximation
%Change the values of constants at different nodes for  to obtain Variables
clc;
clear;
%Node 4
m04=4;
m14=-20.122;
m24=172.54;
m34=-2229;
m44=32.6*10^3;
m54=-488.9*10^3;
m64=7.36*10^6;
m74=-111.04*10^6;
a4=[m04, m14, m24, m34;
    m14, m24, m34, m44;
    m24, m34, m44, m54;
    m34, m44, m54,m64];
b4=-[m44;
     m54;
     m64;
     m74]';
z4=b4/a4
%Node 7
m07=4;
m17=-20.57;
m27=225.04;
```

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

```
m37=-3.1*10^3;
m47=47.5*10^3;
m57=-1.5*10^6;
m67=10.6*10^6;
m77=-1.6*10^8;
a7=[m07,m17,m27,m37;
    m17,m27,m37,m47;
    m27,m37,m47,m57;
    m37,m47,m57,m67];
b7=-[m47;
     m57;
     m67;
     m77]';
z7=b7/a7
%Node 10
m010=4;
m110=-73.614;
m210=1146.45;
m310=-17.3*10^3;
m410=262.3*10^3;
m510=-3.95*10^6;
m610=0.5*10^8;
m710=-8.9*10^8;
a10=[m010,m110,m210,m310;
    m110,m210,m310,m410;
    m210,m310,m410,m510;
    m310,m47,m510,m610];
b10=-[m410;
     m510;
     m610;
     m710]';
z10=b10/a10
```

3. Calculation of P by solving for roots of the quadratic equation

```
%2nd Order Approximation
%Change the values of constants at different nodes for to obtain Variables
clc;
clear;
%Node 4
b24=52.86;
b14=19.08;
p4=[b24 b14 1];
r4=roots(p4)
%Node 7
b27=31.2;
b17=17.01;
p7=[b27 b17 1];
r7=roots(p7)
%Node 10
b210=41.65;
b110=17.83;
p10=[b210 b110 1];
r10=roots(p10)
```

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

```
%3rd Order Approximation
%Change the values of constants at different nodes for to obtain Variables
clc;
clear;
%Node 4
b34=45.47;
b24=59.93;
b14=18.85;
p4=[b34 b24 b14 1];
r4=roots(p4)
%Node 7
b37=4.5*10^4;
b27=2.5*10^4;
b17=1.47*10^3;
p7=[b37 b27 b17 1];
r7=roots(p7)
%Node 10
b310=-395;
b210=-181;
b110=4.78;
p10=[b310 b210 b110 1];
r10=roots(p10)
```

```
%4th order approximation
%Change the values of constants at different nodes for to obtain Variables
clc;
clear;
%Node 4
b44=-1.92*10^3;
b34=-2.24*10^3;
b24=-628;
b14=-17.3;
p4=[b44 b34 b24 b14 1];
r4=roots(p4)
%Node 7
b17=5.7*10^3;
b27=1.8*10^4;
b37=260.3;
b47=-14.41;
p7=[b47 b37 b27 b17 1];
r7=roots(p7)
%Node 10
b410=1.32*10^4;
b310=4.48*10^3;
b210=12.17;
b110=-0.04;
p10=[b410 b310 b210 b110 1];
r10=roots(p10)
```

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

4. Calculation of K using the linear equations

```
%2nd Order Approximation
%Change the values of constants at different nodes for  to obtain Variables
clc;
clear;
% Node 4
m04=4;
m14=-20.122;
p24=-0.297;
p14=-0.063;
a4=[1/p14 , 1/p14^2;
   1/p24,  1/p24^2];
b4=-[m04;
     m14]';
k4=b4/a4
% Node 7
m07=4;
m17=-20.57;
p17=-0.067;
p27=-0.478;
a7=[1/p17 , 1/p17^2;
    1/p27 , 1/p27^2];
b7=-[m07;
     m17]';
k7=b7/a7
% Node 10
m010=4;
m110=-73.614;
p110=-0.066;
p210=-0.361;
a10=[1/p110 , 1/p110^2;
   1/p210,  1/p210^2];
b10=-[m010;
     m110]';
k10=b10/a10

%3rd Order Approximation
%Change the values of constants at different nodes for  to obtain Variables
clc;
clear;
%Node 4
m04=4;
m14=-20.122;
m24=172.54;
p14=-0.87;
p24=-0.38;
p34=-0.06;
a4=[1/p14, 1/p14^2, 1/p14^3;
    1/p24, 1/p24^2, 1/p24^3;
    1/p34, 1/p34^2, 1/p34^3];
b4=-[m04;
    m14;
    m24]';
k4= b4/a4
```

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

```
%Node 7
m07=4;
m17=-20.57;
m27=225.04;
p17=-0.48;
p27=-0.06;
p37=-0.0007;
a7=[1/p17, 1/p17^2, 1/p17^3;
    1/p27, 1/p27^2, 1/p27^3;
    1/p37, 1/p37^2, 1/p37^3];
b7=-[m07;
    m17;
    m27]';
k7= b7/a7
%Node 10
m010=4;
m110=-73.614;
m210=1146.4529;
p110=-0.47;
p210=-0.08;
p310=-0.06;
a10=[1/p110, 1/p110^2, 1/p110^3;
    1/p210, 1/p210^2, 1/p210^3;
    1/p310, 1/p310^2, 1/p310^3];
b10=-[m010;
    m110;
    m210]';
k10= b10/a10

%4th order approximation
%Change the values of constants at different nodes for  to obtain Variables
clc;
clear;
%Node 4
m04=4;
m14=-20.122;
m24=172.54;
m34=-2229;
m44=32.6*10^3;
p14= -0.745;
p24= -0.382;
p34= -0.06;
p44= -0.027;
a4=[1/p14 , 1/p14^2, 1/p14^3, 1/p14^4;
   1/p24 , 1/p24^2, 1/p24^3, 1/p24^4;
   1/p34 , 1/p34^2, 1/p34^3, 1/p34^4;
   1/p44 , 1/p44^2, 1/p44^3, 1/p44^4];
b4=-[m04;
     m14;
     m24;
     m34]';
y4=b4/a4
%Node 7
m07=4;
m17=-20.57;
```

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

```
m27=225.04;
m37=-3.1*10^3;
m47=47.5*10^3;
p17= 45.62;
p27= -27.24;
p37= -0.31;
p47= -0.0002;
a7=[1/p17 , 1/p17^2, 1/p17^3, 1/p17^4;
   1/p27 , 1/p27^2, 1/p27^3, 1/p27^4;
   1/p37 , 1/p37^2, 1/p37^3, 1/p37^4;
   1/p47 , 1/p47^2, 1/p47^3, 1/p47^4];
b7=-[m07;
     m17;
     m27;
     m37]';
y7=b7/a7
% Node 10
m010=4;
m110=-73.614;
m210=1146.45;
m310=-17.3*10^3;
m410=262.3*10^3;
p110= -0.066;;
p210= -0.375;
p310= -0.856;
p410= -1.572;
a10=[1/p110 , 1/p110^2, 1/p110^3, 1/p110^4;
   1/p210 , 1/p210^2, 1/p210^3, 1/p210^4;
   1/p310 , 1/p310^2, 1/p310^3, 1/p310^4;
   1/p410 , 1/p410^2, 1/p410^3, 1/p410^4];
b10=-[m010;
     m110;
     m210;
     m310]';
y10=b10/a10
```

5. MATLAB code to produce plots

```
%Integrated step response with range
clc;
clear;
% First Order Approximation
t=0:0.05:100;
z44 =-(7950*exp(-(1987*t)/10000))/1987 + 4;
z74 =-(1295*exp(-(243*t)/1250))/324 + 4;
z104= -(2173*exp(-(27*t)/500))/540 + 4;
%Plotting the function
figure(1)
subplot (3,1,1)
plot(t,z44,'Black')
subplot(3,1,2)
plot(t,z74,'Blue')
subplot(3,1,3)
plot(t,z104,'Red')
```

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

```
%2nd order Approximation
%Define the range
clc;
clear;
t= 0:5:80;
%Plot the step response
z4 =- (10*exp(-(63*t)/1000))/21 - (1030*exp(-(297*t)/1000))/297 + 4;
z7 =- (725*exp(-(239*t)/500))/239 - (60*exp(-(67*t)/1000))/67 + 4;
z10= (20*exp(-(361*t)/1000))/19 - 5*exp(-(33*t)/500) + 4;
figure(2)
subplot(3,1,1)
plot(t,z4,'Black');
subplot(3,1,2)
plot(t,z7,'Blue');
subplot(3,1,3)
plot(t,z10,'Red');
%3rd order Approximation
%Define the range
clc;
clear;
t= 0:0.05:100;
%Plot the step response
h4 =(152*exp(-(87*t)/100))/87 - (199*exp(-(19*t)/50))/38 - (29*exp(-(3*t)/50))/60 + 4;
h7 = - (151*exp(-(12*t)/25))/48 - (5*exp(-(3*t)/50))/6 +4;
h10 = (80*exp(-(47*t)/100))/47 - (29*exp(-(3*t)/50))/10 - (12*exp(-(2*t)/25))/5 -
(600*exp(-(180*t)/50))/1801  +4 ;
figure(3)
subplot(3,1,1)
plot(t,h4,'Black');
subplot(3,1,2)
plot(t,h7,'Blue');
subplot(3,1,3)
plot(t,h10,'Red');
%4th Order Approximation
%Define the range
clc;
clear;
t= 0:0.5:100;
%Plot the step response
h4 = (277*exp(-(3*t)/4))/150 - exp(-(3*t)/50)/2 - (203*exp(-(19*t)/50))/38 -
exp((27*t)/1000)/135 + 4;
h7= (10861*exp(-(682*t)/5))/13640 - (627*exp(-(167*t)/100))/334 - (305*exp(-
(83*t)/1250))/332 - (3603*exp(-(1801*t)/5000))/1801 + 4;
h10= (60*exp(-(50*t)/100))/47 - (26*exp(-(3*t)/75))/10 - (12*exp(-(2*t)/25))/5 + 4 ;
figure(4)
subplot(3,1,1)
plot ( t, h4, 'Black');
subplot(3,1,2)
plot(t,h7,'Blue');
subplot(3,1,3)
plot(t,h10,'Red');

%Dataset from spice
clc;
dataset= xlsread('DataSet.xlsx','Sheet1','A1:F27');
```

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

```
x= dataset(:,1);
y= dataset(:,2);
subplot(3,1,1)
plot(x,y,'Black');
q= dataset(:,3);
w= dataset(:,4);
subplot(3,1,2);
plot(q,w,'Blue');
e= dataset(:,5);
r= dataset(:,6);
subplot(3,1,3);
plot(e,r,'Red');
```

6. Tabulated Moments of all nodes

	A	B	C	D	E	F	G	H	I	J	K	L
1	Node	0	1	2	3	4	5	6	7	8	9	10
2	Moments	4	4	4	4	4	4	4	4	4	4	4
3	m0	4	4	4	4	4	4	4	4	4	4	4
4	m1	0	-13.776	-14.772	-17.662	-20.47	-19.6	-19.9	-20.57	-34.5	-52.74	-73.62
5	m2	0	141.08	145.82	159.74	172.54	220.04	221.59	225.04	450.25	762.156	1146.45
6	m3	0	-1959.98	-2001.34	-2121.2	-2229	-3131.52	-3148.54	-3186.35	-6609.08	-11398	-17383.2
7	m4	0	2.91*10^4	2.96*10^6	3.1*10^4	3.26*10^4	4.67*10^4	4.69*10^4	47.5*10^3	9.91*10^4	1.71*10^5	2.62*10^5
8	m5	0	-430000	-445000	-460000	-480000	-703000	-706000	-714000	-1500000	-2500000	-3950000
9	m6	0	6.69*10^6	6.71*10^6	7.06*10^6	7.36*10^6	10.6*10^6	10.65*10^6	10.77*10^6	22.5*10^6	39*10^6	59.66*10^6
10	m7	0	-99400000	-101000000	-106000000	-111000000	-159000000	-162400000	-160000000	-580000000	-780000000	-890000000
11	Resistor		120	30	95	125	80	40	120	250	380	580
12	Capacitor		2.2	0.8	2.5	5	1.4	0.5	1.4	2.9	3	9
13												
14	Cap*moment		8.8	3.2	10	20	5.6	2	5.6	11.6	12	36
15												
16	Second term		13776	996	2850	2500	5824	304	672	14900	18240	20880
17												
18	sec*cap		0.000013776	0.000000996	0.00000285	0.0000025	0.000005824	0.000000304	0.000000672	0.0000149	0.00001824	0.00002088

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.

7. Dataset values for the HSPICE append on MATLAB

Time	Value	Time	Value	Time	Value
0	0	0	0	0	0
1	0	1	0	1	0
2	0	2	0	2	0
3	0	3	0	3	0
4	0	4	0	4	0
5	0	5	0	5	0
6	0.688211	6	1.3072	6	0.011392
7	1.524	7	2.0722	7	0.075249
8	2.1441	8	2.4995	8	0.19439
9	2.596	9	2.7891	9	0.3478
10	2.9084	10	2.993	10	0.52105
11	3.138	11	3.1464	11	0.70217
12	3.3082	12	3.2969	12	0.97379
13	3.4247	13	3.349	13	1.0765
15	3.5803	15	3.475	15	1.4079
20	3.7535	20	3.6522	20	2.1246
25	3.8309	25	3.754	25	2.646
29	3.8712	29	3.8115	29	2.9589
30	3.8793	30	3.8233	30	3.0238
40	3.9383	40	3.9096	40	3.5004
50	3.9688	50	3.9543	50	3.7487
60	3.9836	60	3.976	60	3.8674
70	3.9916	70	3.9989	70	3.9321
80	3.9957	80	3.9972	80	3.9652
90	3.9978	90	3.999	90	3.9822
100	3.9989	100	3.9995	100	3.991

All MATLAB codes, moments at other nodes and documents used are shown in the Appendix.