

For the circuit described below, perform the following two tasks. (1) Determine all gate sizes to minimize the overall circuit delay. (2) Determine all gate sizes to minimize overall area subject to the constraint that the overall circuit delay has to be less than or equal to (a) 5% (b) 10% (c) 20% above that obtained in (1).

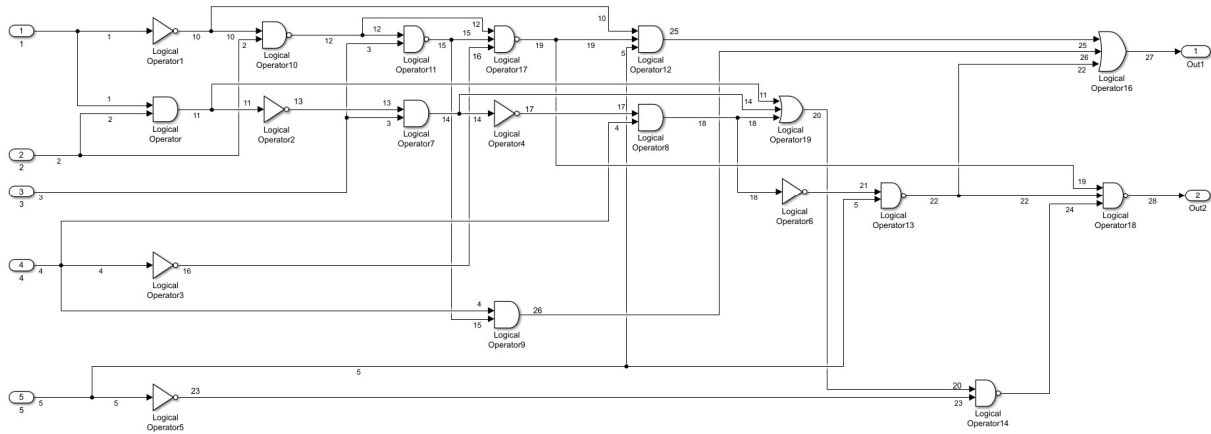
Overall area is defined as the sum of gate sizes. The size of each gate can be 1, 2, 4, or 8. The characteristic of all size-1 gates are given in the table following the circuit netlist. Assume that each primary input is driven by an $R_{drive} = 10 \text{ k}\Omega$ and each primary output is connected to a load $C_{load} = 35 \text{ pF}$. It is assumed that the arrival times for all primary inputs are zero. The delay calculation will use Elmore delay model. Assume that the wire capacitance and resistance can be ignored.

For both tasks: You need to use a table to list arrival time at all gate outputs (including primary outputs). Draw the circuit and clearly highlight critical path(s) and write down arrival times at all gate outputs.

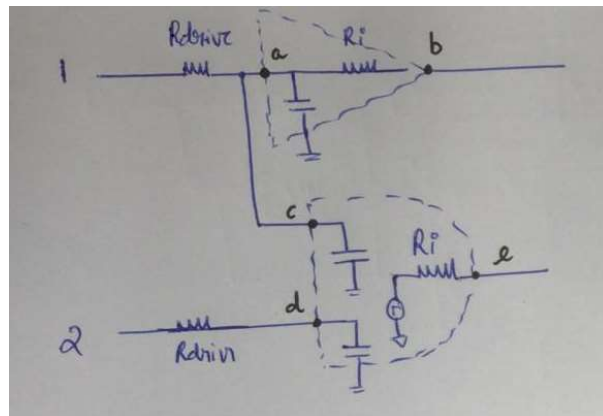
10 = NOT(1)	22 = NAND(5, 21)
11 = AND(1, 2)	23 = NOT(5)
12 = NAND(2, 10)	24 = NAND(20, 23)
13 = NOT(11)	25 = AND(5, 10, 19)
14 = AND(3, 13)	26 = AND(4, 15)
15 = NAND(3, 12)	27 = OR(22, 25, 26)
16 = NOT(4)	28 = NAND(19, 22, 24)
17 = NOT(14)	
18 = AND(4, 17)	
19 = NAND(12, 15, 16)	
20 = OR(11, 14, 18)	
21 = NOT(18)	

To solve this lab, we have to first plot the circuits based on the nodes given to us above.

The plot is drawn using MATLAB Simulink as shown below



Now to determine delay at each stage, we must replace each logic gate by its cell resistance and input capacitance. The sample of the approach is shown below



To determine the delay,

$$\text{At A: } t_a = R_{\text{drive}} * (C_{\text{inv}} + C_{\text{and}})$$

$$\text{At C: } t_c = R_{\text{drive}} * (C_{\text{inv}} + C_{\text{and}})$$

$$\text{At D: } t_d = R_{\text{drive}} * (C_{\text{and}})$$

Now, for the next stages,

$$\text{At B: } t_{\text{next}} = t_a + R_{\text{inv}} * (C_{\text{next}})$$

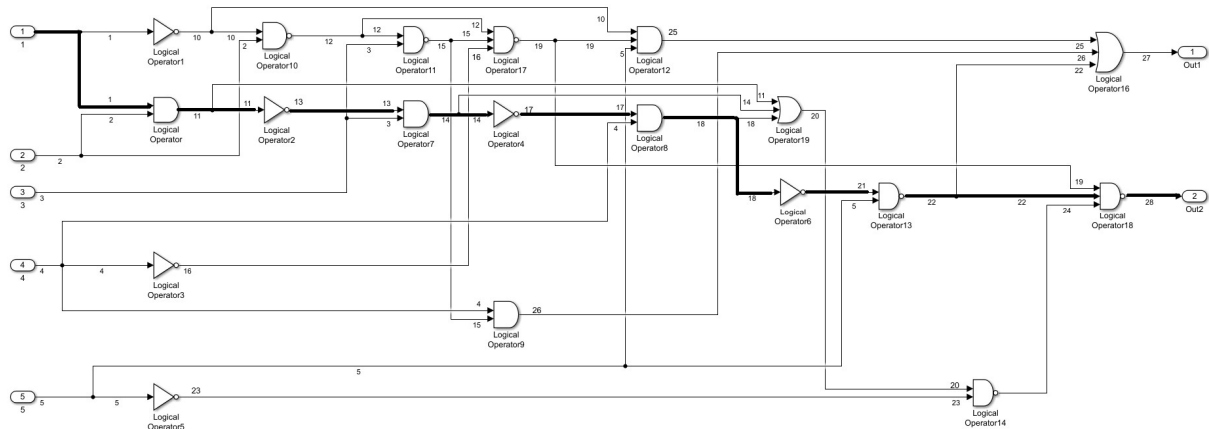
$$\text{At E: } t_{\text{next}} = \max(t_c, t_d) + R_{\text{inv}} * (C_{\text{next}})$$

When we have two or more inputs as shown, we must consider the maximum delay of the two while moving forward. This can be done in excel using the MAX function.

The tabulated values at different nodes are as shown below. (Image of EXCEL sheet used attached in appendix)

Node	Delay (ns)
1	9
2	8.5
3	8.5
4	13
5	15.5
10	16.035
11	13.32516
12	19.6393
13	16.00516
14	20.33032
15	23.063385
16	16.685
17	23.01032
18	27.33548
19	26.693245
20	28.50548
21	30.35048
22	34.855855
23	18.515
24	30.487845
25	29.213245
26	25.586395
27	125.855855
28	145.329855

As we can see in the tabulated column, the output terminals, i.e 27 and 28 have different arrival times / Elmore delay. Using this we can plot the critical path as shown below



To optimize this design, we need to now choose a sizing (1, 2, 4 or 8 as per requirement) such that the critical path delay is reduced.

After trying several combination of sizing on the gates, the optimal sizing is found as shown below,

Gate	Sizing
NOT	2
NAND3	8
AND	2
AND3	2
NAND	2
OR3	2

Using this sizing the output delays are got down to

Node	Delay(ns)
27	95.30295
28	63.6122

As we can see, of the recognized critical path has gone down to 64.61ns from 145.32ns.

Hence we can conclude that the above sizing is optimal.

The second part of the lab requires us to reduce the area under three constraints. The delay is allowed 5%, 10%, and 20% above and we have to determine minimum area to achieve this. **The area is also said to be the sum of the sizes gates used.**

$$\text{i.e Area} = \text{sum (num of gates * size)}$$

For the optimal delay obtained, the area is calculated using the excel sheet as shown,

Gate	Total gates	Sizing of 1 * 1 gate area	Total area
NOT	6	2	12
NAND3	2	24	48
AND	4	4	16
AND3	1	6	6
NAND	5	4	20
OR3	2	6	12
		Overall Area	114

Now, let us determine if we can minimize the area under the given constraints.

a. **5%**

$$\text{Allowed delay} = \text{Optimal Delay} * 1.05$$

i.e

Node	Delay(ns)
27	100.068
28	66.7928

The optimal solution is as shown,

Gate	Sizing
NOT	2
NAND3	8
AND	2
AND3	1
NAND	2
OR3	2

The overall area and time is under condition is now shown this is now,

Gate	Total gates	Sizing of 1 * 1 gate area	Total area
NOT	6	2	12
NAND3	2	24	48
AND	4	4	16
AND3	1	3	3
NAND	5	4	20
OR3	2	6	12
		Overall Area	111

Node	Delay(ns)
27	95.303
28	63.6122

b. 10%

Allowed delay = Optimal Delay * 1.10

i.e

Node	Delay(ns)
27	104.833
28	69.9734

The optimal solution is as shown,

Gate	Sizing
NOT	1
NAND3	8
AND	1
AND3	1
NAND	2
OR3	2

The overall area and time is under condition is now shown this is now,

Gate	Total gates	Sizing of 1 * 1 gate area	Total area
NOT	6	1	6
NAND3	2	24	48
AND	4	2	8
AND3	1	3	3
NAND	5	4	20
OR3	2	6	12
		Overall Area	97

Node	Delay(ns)
27	100.887
28	69.1962

c. **20%**

Allowed delay = Optimal Delay * 1.20

i.e

Node	Delay(ns)
27	114.36
28	76.3346

The optimal solution is as shown,

Gate	Sizing
NOT	1
NAND3	8
AND	1
AND3	1
NAND	1
OR3	2

The overall area and time is under condition is now shown this is now,

Gate	Total gates	Sizing of 1 * 1 gate area	Total area
NOT	6	1	6
NAND3	2	24	48
AND	4	2	8
AND3	1	3	3
NAND	5	2	10
OR3	2	6	12
		Overall Area	87

Node	Delay(ns)
27	104.324
28	72.637

APPENDIX

1. Excel for initial sizing

Gate	Resistance	Capacitance	Sizing	R (sized)	C (sized)	Rdrive	Cload
NOT	6.7	0.5	1	6.7	0.5	10	35
NAND3	3.1564	0.55	1	3.1564	0.55		
AND	3.6043	0.4	1	3.6043	0.4		
AND3	3.6	0.6	1	3.6	0.6		
NAND	3.6043	0.45	1	3.6043	0.45		
OR3	2.6	0.7	1	2.6	0.7		

Statewise calculation of the delays
The elements in red are the delays at each node

Node	Delay	Area	Product
1	9		
2	8.5	6	1
3	8.5	2	3
4	13	4	2
5	15.6	1	3
10	16.025	5	2
11	13.32516	2	3
12	19.6393		
13	16.00516		
14	20.33032		
15	23.063385		
16	16.685		
17	23.01032		
18	27.33548		
19	26.693245		
20	28.50548		
21	30.35048		
22	34.855855		
23	18.515		
24	30.487845		
25	29.213245		
26	25.586395		
27	125.855855		
28	145.329855		

Area determination

Gate	Area	Product
NOT	6	1
NAND3	2	3
AND	4	2
AND3	1	3
NAND	5	2
OR3	2	3

Overall Area: 39

2. Excel for optimal sizing

Gate	Resistance	Capacitance	Sizing	R (sized)	C (sized)	Rdrive	Cload
NOT	6.7	0.5	2	3.35	1	10	35
NAND3	3.1564	0.55	8	0.39455	4.4		
AND	3.6043	0.4	2	1.80215	0.8		
AND3	3.6	0.6	2	1.8	1.2		
NAND	3.6043	0.45	2	1.80215	0.9		
OR3	2.6	0.7	2	1.3	1.4		

Statewise calculation of the delays
The elements in red are the delays at each node

Node	Delay	Area	Product
1	18		
2	17	6	2
3	12.5	2	24
4	26	4	4
5	31	1	6
10	25.035	5	4
11	22.32516	2	6
12	34.586395		
13	25.00516		
14	29.33032		
15	43.957575		
16	40.74		
17	32.01032		
18	36.33548		
19	46.167055		
20	36.92048		
21	39.35048		
22	49.80295		
23	34.015		
24	44.84994		
25	48.687055		
26	46.480585		
27	95.30295		
28	63.6122		

Area determination

Gate	Area	Product
NOT	6	2
NAND3	2	24
AND	4	4
AND3	1	6
NAND	5	4
OR3	2	6

Overall Area: 114

3. Excel for 5% constraint to minimize area

Lab4 - Excel

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	Gate	Resistance	Capacitance		Sizing		R (sized)	C (sized)		Rdrive	Cload															
2	NOT	6.7	0.5		2		3.35	1																		
3	NAND3	3.1564	0.55		8		0.39455	4.4																		
4	AND	3.6043	0.4		2		1.80215	0.8																		
5	AND3	3.6	0.6		1		3.6	0.6																		
6	NAND	3.6043	0.45		2		1.80215	0.9																		
7	OR3	2.6	0.7		2		1.3	1.4																		
8																										
9	Statewise calculation of the delays																									
10	The elements in red are the delays at each node														Area determination											
11	1	15																								
12	2	17																								
13	3	12.5																								
14	4	26																								
15	5	25																								
16	10	23.025																								
17	11	22.32516																								
18	12	32.576395																								
19	13	25.00516																								
20	14	29.33032																								
21	15	41.947575																								
22	16	40.74																								
23	17	32.01032																								
24	18	36.33548																								
25	19	43.920325																								
26	20	36.90048																								
27	21	39.35048																								
28	22	49.80295																								
29	23	28.015																								
30	24	44.84994																								
31	25	48.960325																								
32	26	44.470585																								
33	27	95.30295																								
34	28	63.6122																								
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4. Excel for 10% constraint to minimize area

Lab4 - Excel

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	Gate	Resistance	Capacitance		Sizing		R (sized)	C (sized)		Rdrive	Cload															
2	NOT	6.7	0.5		1		6.7	0.5																		
3	NAND3	3.1564	0.55		8		0.39455	4.4																		
4	AND	3.6043	0.4		1		3.6043	0.4																		
5	AND3	3.6	0.6		1		3.6	0.6																		
6	NAND	3.6043	0.45		2		1.80215	0.9																		
7	OR3	2.6	0.7		2		1.3	1.4																		
8																										
9	Statewise calculation of the delays																									
10	The elements in red are the delays at each node														Area determination											
11	1	9																								
12	2	13																								
13	3	8.5																								
14	4	13																								
15	5	20																								
16	10	19.05																								
17	11	19.84817																								
18	12	28.601395																								
19	13	22.52817																								
20	14	29.37634																								
21	15	37.251715																								
22	16	42.48																								
23	17	32.05634																								
24	18	38.90451																								
25	19	44.45275																								
26	20	39.48951																								
27	21	44.93451																								
28	22	55.38698																								
29	23	76.09																								
30	24	47.41897																								
31	25	49.49275																								
32	26	42.297735																								
33	27	100.88698																								
34	28	69.19623																								
35																										

Sheet1

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5. Excel for 20% constraint to minimize area

Gate	Resistance	Capacitance	Sizing	R (sized)	C (sized)	Rdrive	Cload
NOT	6.7	0.5	1	6.7	0.5	10	35
NAND3	3.1564	0.55	8	0.39455	4.4		
AND3	3.6043	0.4	1	3.6043	0.4		
AND3	3.6	0.6	1	3.6	0.6		
NAND	3.6043	0.45	1	3.6043	0.45		
OR3	2.6	0.7	2	1.3	1.4		

Statewise calculation of the delays	Area determination
The elements in red are the delays at each node	
1 9	
2 8.5	NOT 6 1 6
3 8.5	NAND3 2 24 48
4 13	AND 4 2 8
5 15.5	AND3 1 3 3
10 16.035	NAND 5 2 10
11 15.84817	OR3 2 6 12
12 35.515855	
13 18.52817	
14 25.37634	Overall Ar 87
15 50.816495	
16 42.48	
17 28.05634	
18 34.90451	
19 52.789245	
20 35.48951	
21 37.91951	
22 58.82445	
23 18.515	
24 51.34843	
25 57.829245	
26 55.862515	
27 104.32445	
28 72.6337	