Synopsis

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Two different and unconventional approximation techniques for two types of mechanical system responses are developed and presented in this study.

Many systems exhibit decaying responses, and the first problem studied involves approximating such responses. In particular, a new delayed dynamical system is constructed using convolution such that one of its responses is exactly equal to the desired response curve. The aim of approximating the curve in this way, using a dynamical system, is that the dynamical system has characteristic roots which lead to exponential curves with a prior guarantee that those exponential curves can be used in linear combination to approximate the desired decaying system response. In implementing the calculation, a combination of discrete and distributed feedbacks are found in the desired dynamical system. That leads to a delay differential equation. The characteristic equation of the DDE has infinitely many roots, and these roots give us exponential functions that can be used as a basis to fit the response of interest. The fit can be done using either 2 norm, infinity norm or any other criterion of interest to the analyst. When we increase the number of terms used in the fit, the quality of the fit improves. The primary advantage of this approach is that the underlined dynamical system and its characteristic equation give us an infinite sequence of roots, so choosing the exponential rates in the expansion is easy. In contrast, if we pose

the problem as one where both the coefficient as well as the exponent have to be fitted as design parameters, then each time we increase the number of functions involved in the fit, the whole problem including finding, the exponential roots has to be solved afresh.

The second problem is motivated by an application in robotics. One of the basic manipulation primitives that a robot uses for mechanical tasks involves pushing a manipulated object on a horizontal plane. For this task, there is a generalized load (a combination of force and moment) that needs to be applied on the body to cause motion. Given a desired motion, finding the loads constitutes a forward problem, which requires some numerical integrals over the contact patch. Given a loading direction, finding the resulting motion and the load magnitude require solution of nonlinear equations, which is more difficult. A way around this, is to solve the problem using approximated limit surfaces. A limit surface is a boundary of a convex set which consists of all possible static and sliding frictional loads, for a given contact condition. Limit surfaces are suitable for use in load-to-motion mapping. Since robots may be required to push an object in a particular direction, a simple mapping from load to incipient motion is used. This mapping uses an approximated limit load surface. Approximations for limit surfaces used have included ellipsoids in the simplest case, and multivariate polynomial expansions as a more sophisticated example. We propose a new approximation which uses a small number of symmetrical 3×3 matrices along with fractional powers of simple quadratic forms. With only one matrix, we obtain the ellipsoid approximation. However, with more matrices we obtain more accurate surfaces that are analytically tractable, e.g., for subsequent motion planning and control. Fitting these 3×3 matrices is easy with simple optimization algorithms. Several numerical examples show that with 2 or 3 such fitted matrices, the fit obtained is excellent. The new method of approximating limit load surfaces presents a significant useful generalization of the popular but somewhat inaccurate ellipsoid approach.