Approximation of Force Fields

We chose the following objective function to get a fit. The following are the objective functions with increase in terms of the approximation. We will consider upto 3 terms in our analysis.

$$E = \sum_{k=0}^{n} ({f'}_k A f_k - 1)^2$$

$$E = \sum_{k=0}^{n} ({f'}_k A {f'}_k + ({f'}_k B f_k)^m - 1)^2, \qquad m\ is\ chosen\ as\ 0.5$$

$$E = \sum_{i=0}^{n} ({f'}_k A {f'}_k + ({f'}_k B f_k)^{m1} + ({f'}_k C f_k)^{m2} - 1)^2, \qquad m1 = \frac{1}{3}, m2 = \frac{2}{3}$$

Here f is a force vector from the Force field data and A,B,C are matrices that are found by minimizing the objective function E.(using fminunc in MATLAB)

Now that A,B,C are known, let α be some number such that αf causes slip, i.e:- αf resides on the limit surface. Therefore αf must follow the following condition-

$$\alpha^2 f' A f = 1, \qquad for\ 1\ term\ approximation$$

$$\alpha^2 f' A f + +(\alpha^2 f' B f)^m = 1, \qquad for\ 2\ term\ approximation$$

$$\alpha^2 f' A f + (\alpha^2 f' B f)^{m1} + (\alpha^2 f' C f)^{m2} = 1, \qquad for\ 3\ term\ approximation$$

Values for α can be found from the above equations.

Approximations found are plotted below-

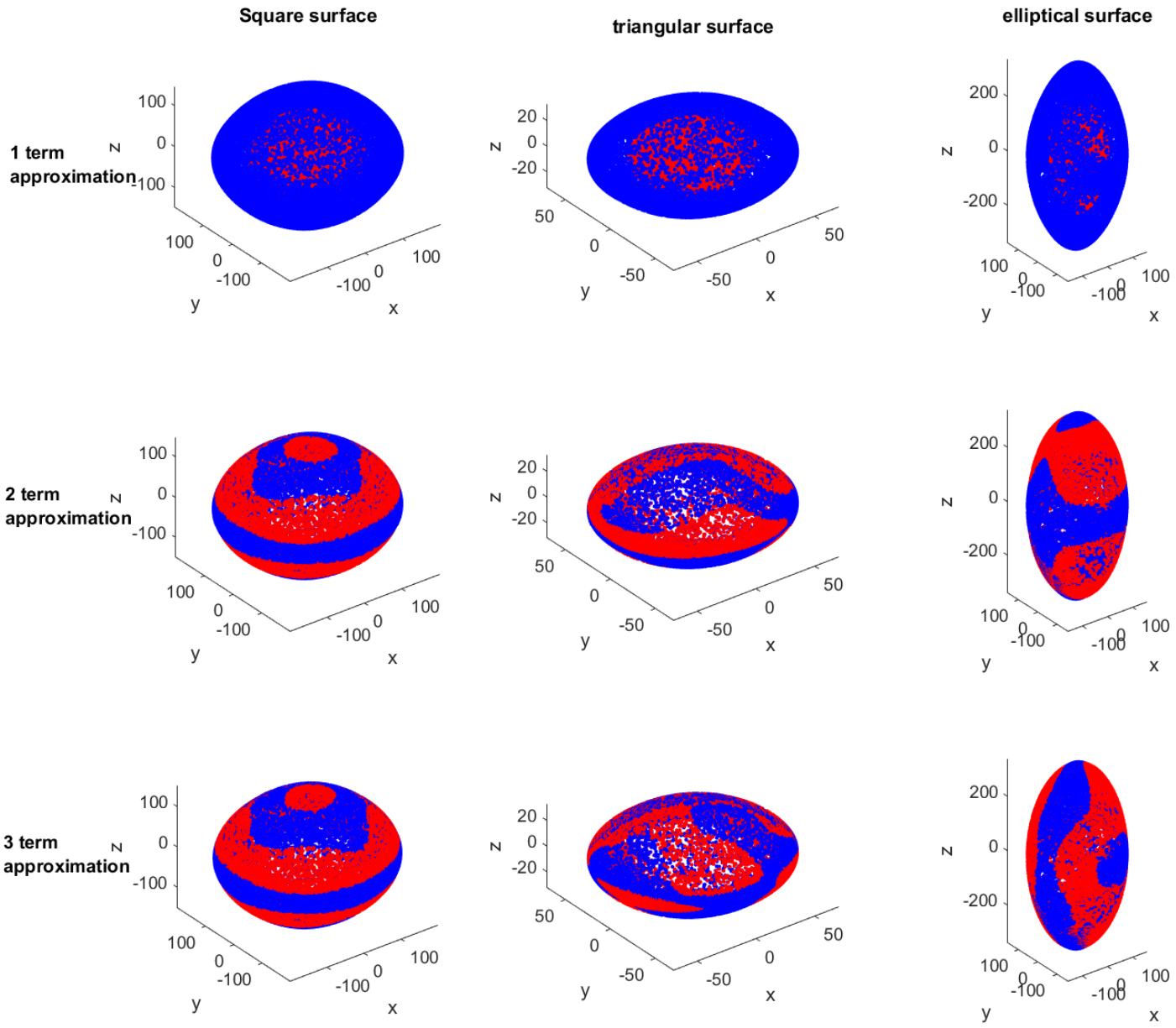

Fig1. Different approximations for different cases (Blue-original, Red-approximation)

The above figure shows upto three term approximation of limit surfaces for three different continuous surfaces, square, triangular and elliptical. The blue surface is the original data and the red surface is the approximated surface.

It can be observed the approximation gets better with increase in the number of terms in the approximation and can also be verified with the error data from the data below.

The following were the error values :-

	Square	Triangle	Ellipse
1 term	25.24	17.15	35.85
2 terms	0.86	2.05	12.36
3 terms	0.44	0.35	4.59