Computational studies of two unconventional approximation schemes for mechanical system responses

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Overview of the presentation

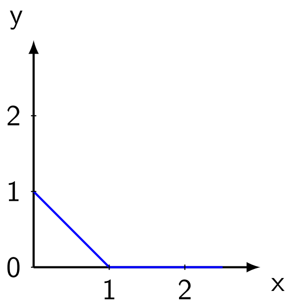
The study has two distinct parts-

- Approximation of decaying functions
    - Introduction
    - Approximation method
    - Results
    - Conclusions and future work
- Approximation of limit surfaces in frictional sliding
    - Introduction
    - Limit surfaces
    - Approximation method
    - Results
    - Conclusions and future work

Introduction

- Decaying functions
- Exponential fitting
- Approximation method
  - Construct a delayed dynamical system
  - Characteristic roots give us exponential functions
  - Exponential functions used as basis for fit

We use hockey stick function as an example to present the method. The methodology is applied to some more functions and outcomes are reported.



A dynamical model

- Linear coefficient delayed dynamical system

$$\dot{x}(t) = \int_0^t f(\tau)x(t-\tau)\,d\tau$$

- Solution for f
  - We discretize the integral and form system of simultaneous linear equation to find f

$$\dot{x} = Af$$

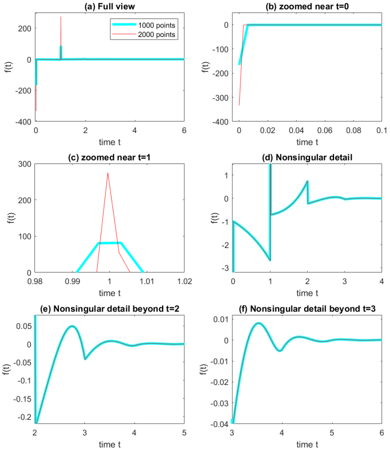
  - We used 2000 points uniformly distributed on the interval $[0, 6]$

$$\dot{\mathrm{x}} = \begin{pmatrix} -1 \\ -1 \\ \vdots \\ 0 \end{pmatrix}, \quad \mathrm{A}(:,\mathrm{k}) = conv(g_k, x)\Delta t, \quad here \;\; g_k = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \; and \; g_k(k) = 1$$

  - f is simply given by $A\backslash\dot{x}$ in MATLAB

Characterization of f

- Dirac-delta functions at $t = 0$ and $t = 1$
- Dirac-delta functions lead to discrete feedbacks
- Rest of the f lead to distributed delay
- Distributed delays are approximated by fifth order polynomials on the intervals $[0, 1], [1, 2]$ and so on
- Non-zero values of f beyond $t = 6$ are ignored



Delayed differential equation

- With discrete delayed feedback through dirac delta functions and distributed delays through polynomials our system in now defined

$$\dot{x}(t) = -x(t) + x(t-1) + \int_0^1 p_1(\tau)x(t-\tau)d\tau + ... + \int_5^6 p_6(\tau)x(t-\tau)d\tau$$

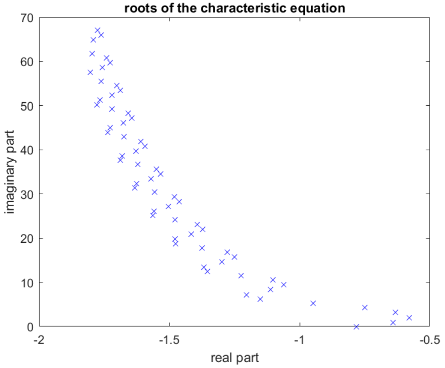
- The characteristic roots of the above equation give us exponential rates to use for approximation of $x(t)$
- Substituting $x(t) = e^{\lambda t}$ in DDE gives characteristic equation
- Transcendental equation is obtained with infinitely many roots

Characteristic roots

- Roots give us exponential functions
- Each complex root gives us two exponential functions

$$\lambda = a + ib \longrightarrow e^{at} cos(bt) \quad \& \quad e^{at} sin(bt)$$

- Real roots give only one exponential function
- These exponential functions are used as basis
- Coefficients for the functions are obtained using infinity norm solver



Least infinity-norm error

- Now with basis ready, we fit the coefficients to minimize the infinity norm error
- We use MATLAB's `linprog` to find the optimum coefficients

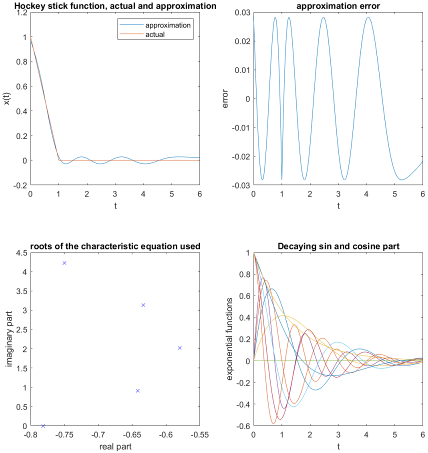
$$min \;\; f^T x \;\; such \;\; that \begin{cases} A\mathrm{x} \leq \mathrm{b} \\ A_{eq}\mathrm{x} = \mathrm{b_{eq}} \end{cases}$$

- We setup the solver such that it minimizes the infinity norm e_0

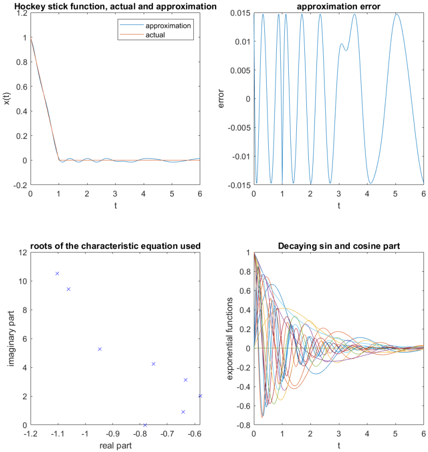
$$A\mathrm{x} - \mathrm{b} = \mathrm{e} \qquad e_0 \geq 0 \quad and \quad e_0 \geq \|e_i\| \quad \text{Here } e_i \text{ is an element of e}$$

Approximations

- Approximation using 5 roots



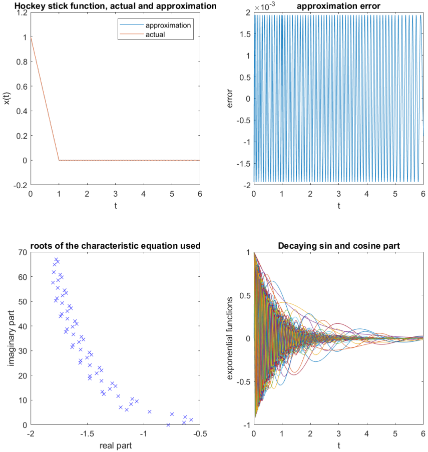
- Approximation using 8 roots



Approximations

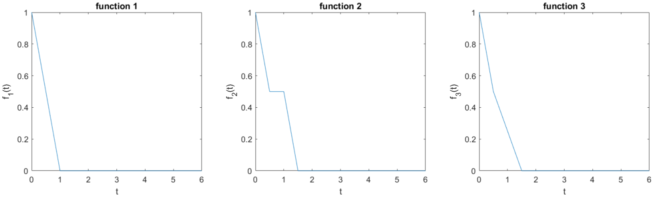
- Quality of fit improves with increase in terms
- High frequency sine and cosine term introduced
- High frequency terms contribute to sharp variations
- Equal amplitude oscillation in errors due to infinity norm fit

- Approximation using 62 roots



More functions

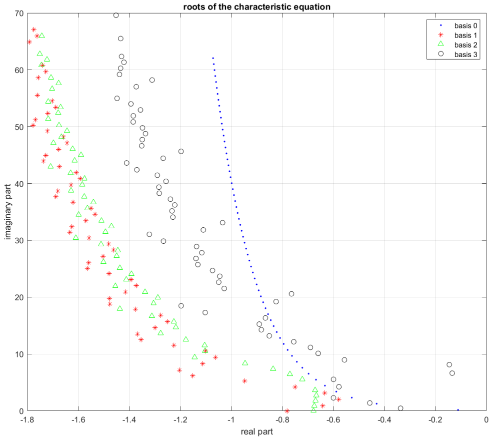
- We applied same methodology to more functions and form a collection of bases
- Let the functions be f_1, f_2, f_3 and the respective bases be basis 1, basis 2 and basis 3



- A simple basis (basis 0) is also used obtained from the DDE: $\dot{x}(t) = -0.1x(t-6)$.
- Functions are fitted till $t = 6$ and projected till $t = 19$ to observe the transition

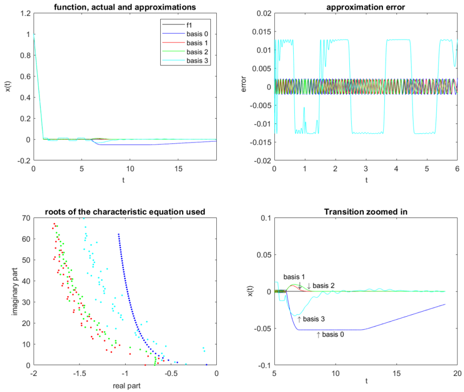
More functions

- Each of the bases is used to approximate the functions
- 62 roots are used to form the exponential functions from every basis
- Approximation from each bases are superimposed

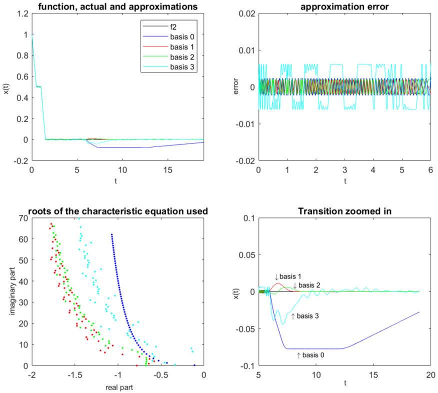


Approximations

- Approximations of f_1



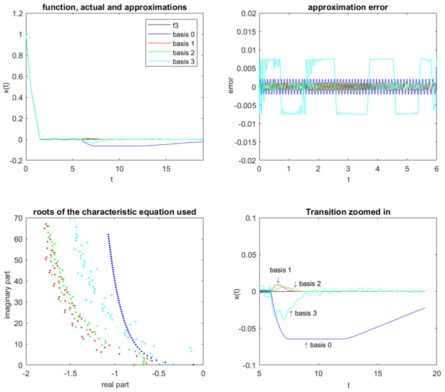
- Approximations of f_2



Approximations

- Basis 3 struggles to approximate the functions
- Slower decaying exponentials present in basis 3
- Since function are fitted only till $t = 6$, beyond it undesired responses are observed
- Large response after $t = 6$ observed which decays eventually
- Equal amplitude error with time due to infinity norm

- Approximations of f_3

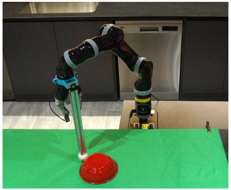


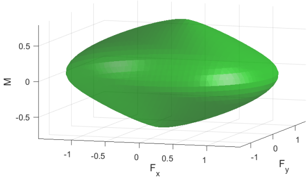
Conclusions and future work

- Conclusions
  - An approximation method for decaying function is proposed
  - We use characteristic roots of a constructed DDE to form basis for the approximation
  - With high number of terms, fit obtained is excellent
  - The method struggles to approximate when slower decaying exponents are present in the bases
  - The method provides a good fit till finite distance beyond which undesired responses are observed which eventually decay
- Future work
  - Other forms of delayed dynamical system
  - Analytical solution for f, ad hoc method for finding and approximating is used here
  - Role of slowest decaying exponential can be investigated
  - The length of the interval used, beyond which undesired responses are seen
  - Better optimization procedure to obtain superior fit

Approximations of limit surfaces in frictional sliding

- Robotic manipulation
  - Planar pushing
  - Parallel jaw grippers
- Limit surfaces
- Approximations used so far
  - Ellipsoids
  - Fourth order polynomial





Analytic model of pusher slider system

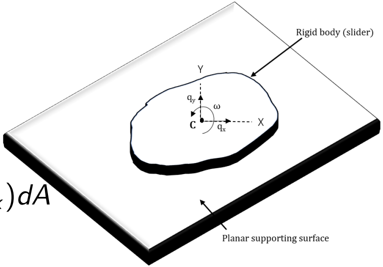
- A rigid body with reference point **C**
- Generalized load and motion vector

$$\mathrm{q} = \left(q_x, q_y, \omega\right), \mathrm{P} = \left(F_x, F_y, M\right)$$

- Net frictional load

$$F_x = \int_A f_{ax} dA \quad F_y = \int_A f_{ay} dA \quad M = \int_A (x_a f_{ay} - y_a f_{ax}) dA$$

Here $[f_a x, f_{ay}]$ are frictional forces at a contact point on the contact patch A, with coordinates $[x_a, y_a]$



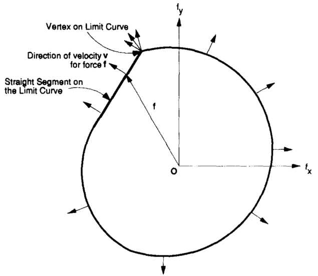
Limit surfaces

Definition

All possible combinations of frictional loads P for a given contact surface, form a convex set. The boundary of this set is called the limit surface.

Properties of Limit surfaces

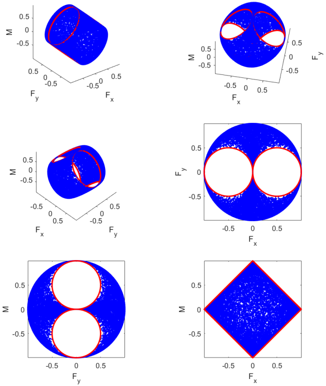
- if P is not on the limit surface $q = 0$
- q is normal to the limit surface where it is smooth
- There is a non-uniqueness in q for a given frictional load P if P is at a vertex
- There is a non-uniqueness in P for a given motion vector q if q is normal to a facet on the limit surface



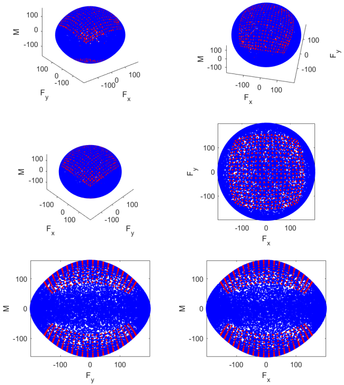
Source: S. Goyal et al, "Planar sliding with dry friction part 1.limit surface and moment function,"Wear, vol. 143, no. 2, pp. 307–330, 1991.

Limit surfaces

- Limit surface for a rigid bar with contact point at its ends



- Limit surface for square patch of edge length two



Approximation method

We define a failure surface $H(f)$ as follows-

$$H_1 = f^T Af \quad \text{1 term}$$
$$H_2 = f^T Af + (f^T Bf)^{1/2} \quad \text{2 terms}$$
$$H_3 = f^T Af + (f^T Bf)^{2/3} + (f^T Cf)^{1/3} \quad \text{3 terms}$$

The failure surface obeys the properties of the limit surface-

- $H(f) < 1$ implies f lying inside the limit surface and $v = 0$.
- $H(f) = 1$ implies f lies on the limit surface
- The velocity is given by the normal, $v = \frac{\nabla H(f)}{\|\nabla H(f)\|}$.

Here f is the frictional load vector and v is the motion vector. A, B, C are 3×3 symmetrical and positive semi-definate matrices, via the Cholesky decompostion. i.e., $A = U^T U$, where U is an upper triangular matrix.

Approximation method

We fit the failure surface with the following objective function-

$$J_{a,1} = \sum_{k=1}^{n} (f_k^T A f_k - 1)^2$$

$$J_{a,2} = \sum_{k=1}^{n} (f_k^T A f_k + (f_k^T B f_k)^{1/2} - 1)^2$$

$$J_{b,1} = \sum_{k=1}^{n} (f_k^T A f_k - 1)^2 + \|v_k - \hat{v}\|^2$$

$$J_{b,2} = \sum_{k=1}^{n} (f_k^T A f_k + (f_k^T B f_k)^{1/2} - 1)^2 + \|v_k - \hat{v}\|^2$$

Here f_k and v_k is are the load and motion vector from the fitting data, $\hat{v} = \frac{\nabla H(f_k)}{\|\nabla H(f_k)\|}$ is the approximated motion vector and A, B are matrices that are to be fitted.

Approximation method

Now that A, B, C are fitted we construct the approximated limit surface. To construct the limit surface-

- Consider a generalized frictional load f
- let α be a parameter such that $H(\alpha f) = 1$-

$$\alpha^2 f_k^T A f_k = 1 \quad \text{1 term}$$

$$\alpha^2 f_k^T A f_k + (\alpha^2 f_k^T B f_k)^{1/2} = 1 \quad \text{2 terms}$$

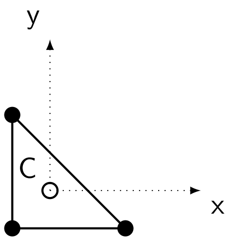
$$\alpha^2 f_k^T A f_k + (\alpha^2 f_k^T B f_k)^{2/3} + (\alpha^2 f_k^T C f_k)^{1/3} = 1 \quad \text{3 terms}$$

- The above equations are solved for α

Many such generalized load vectors uniformly distributed on the surface of the sphere are considered to plot the approximated limit surface.

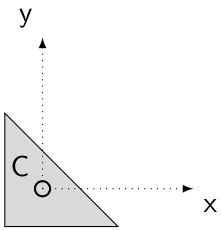
Approximated limit surfaces

- A right angled triangle with three contact points at the vertices.



- Uniform friction distribution considered

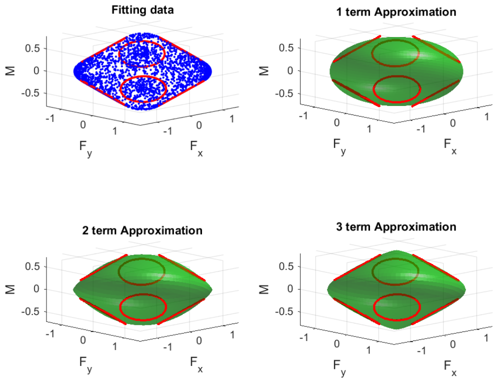
- A right angled triangle with continuous contact patch.



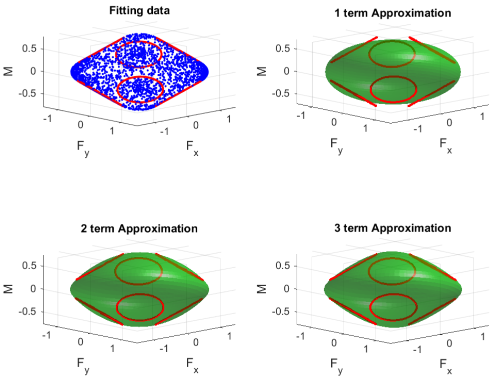
- Surface is discretized and uniform friction distribution considered

Approximated limit surface

- Approximation using J_a

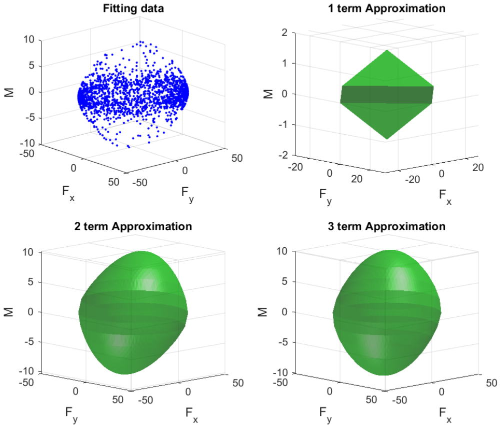


- Approximation using J_b

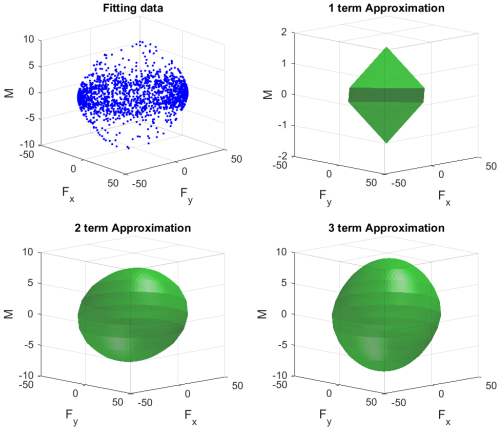


Approximated limit surface

- Approximation using J_a



- Approximation using J_b



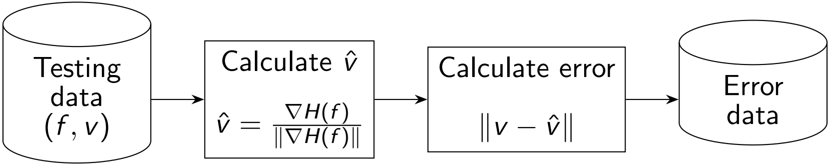
Error Measure

We look at the performance of the approximation with increasing terms, using empirical plots of errors.

Two error measures are defined-

- Velocity based error
- Force based error

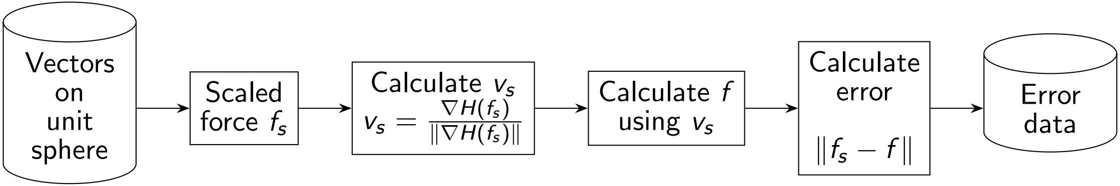
In velocity based error-



- Each data point (f_i, v_i) from the testing data is considered
- Approximated velocity $\hat{v}_i = \frac{\nabla H(f_i)}{\|\nabla H(f_i)\|}$ is calculated for each data point
- Error is calculated as norm of difference between $\hat{v}_i$ and v_i

Error measure

In force based error-



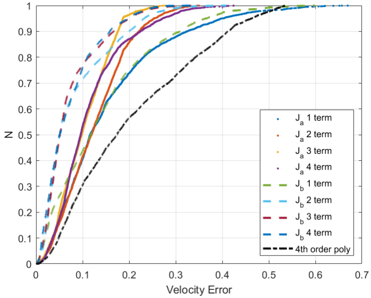
- 1000 generalized load vectors lying uniformly on the surface of a sphere are considered
- These vectors are scaled to lie on the limit surface, referred as f_s
- Approximated velocity v_s is calculated for f_s
- New loads f are calculated using v_s by intergrals or sums over all the contact points
- Error is calculated as norm of difference between f_s and f

$$Ef_i = \|f_s - f\|$$

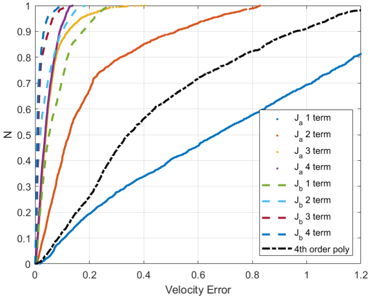
- Empirical distribution plot of these errors Ef is plotted

Velocity based error

- Three point contact



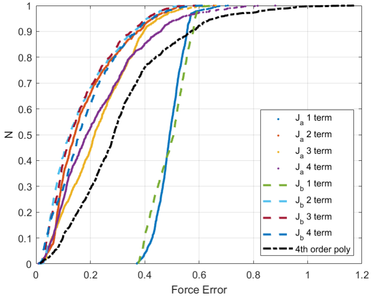
- Continuous contact patch



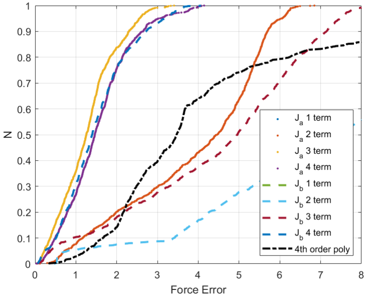
- J_b performs better than J_a as expected
- Higher term approximations perform better

Force based error

- Three point contact



- Continuous contact patch



- J_a performs better than J_b as expected
- Higher term approximations perform better

Conclusions and future work

- Conclusions
  - Approximation proposed using small number of 3×3 symmetrical matrices
  - Trade off between load and motion prediction while fitting
  - Approximation using J_a fitting approximates the limit surfaces better
  - Approximation using J_b fitting predicts the motion vectors better
  - Higher term approximations perform better than the commonly used ellipsoidal approximation
  - The higher term approximation also outperforms a more sophisticated fourth order convex polynomial approximation
- Future work
  - Testing the formulation on experimental data
  - More balanced objective function for fitting
  - More sophisticated optimization algorithms to obtain superior fits

THANKS