An Information Theoretic Source Seeking Strategy for Plume Tracking in 3D Turbulent Fields

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***Abstract*—We present information theoretic search strategies for single and multi-robot teams to find and localize the source of a biochemical or radiological materials in turbulent flows. In our work, robots rely on sporadic and intermittent sensor readings to synthesize information maximizing exploration strategies to find and localize the position of the source. By reasoning about the spatial distribution of these sensory cues, the robots are able to construct a belief distribution over the possible positions of the source. The belief distribution is then employed to synthesize motion strategies that drives the robots to regions in the workspace that results in the largest decrease in the entropy of the belief distribution for the source position. We validate the proposed strategies in 2D and 3D environments and consider the performance of the strategies when robots have limited access to global pose information. In particular, the proposed strategies are validated using a three dimensional (3D) time-varying computational fluid model of the 2010 Deep Water Horizon oil spill.**

I. INTRODUCTION

We are interested in the development of robust sensing and tracking strategies for autonomous tracking and localization of spatio-temporal structures in dynamic environments. Specifically, we are interested in autonomous strategies that are robust to sporadic sensor measurements and can maximize the information gained throughout the process. The goal is to enable autonomous robots to effectively and efficiently localize the source of a contaminant leakage, as in an oil spill or a radioactive dispersal, and track the dispersion of biochemical and/or radiological contaminants in the presence of turbulent flows.

Existing strategies for detecting, tracking, and localizing of such structures or sources with mobile robots include the building of flow field maps [1]–[3], the estimation of concentration gradients [4]–[9], and the use of gradient-free search algorithms [6], [7]. In general, the variation in material concentrations from a source in a fluidic environment is heavily dependent on the Reynolds numbers. In lower Reynolds regimes gradient-based strategies work well since the variation in material concentrations tend to be smooth [10]. As the Reynolds number increases, as in the atmosphere or in the ocean, material transport becomes dominated by turbulent mixing [11]–[13]. The result is a highly anisotropic and unsteady sensory landscape. As such, the sporadic and intermittent nature of the sensor measurements in high Reynolds regimes renders many gradient-based strategies ineffective [14], [15].

To address some of these challenges, various bio-inspired strategies based on bacteria [5]–[7], insects [16]–[19], and crabs [20] have been proposed. However, these bio-inspired approaches are mostly ad-hoc, focused on developing novel sensor technology [16], [18], [21], or are equivalent to coverage and gradient based search strategies for single robots [6], [7], [17]–[19]. More recently, [10], [22], [23] leveraged the spatio-temporal sampling capabilities of a team of robots to better estimate the material concentrations of chemicals in the presence of turbulence. While the resulting strategies are more robust, they are fundamentally gradient-based strategies. As such, success depends on having enough robots to cover the space in order to obtain good enough estimates of the gradient. Alternatives to gradient-based strategies include reactive search strategies that can adapt to past sensory information and action [1], [24], [25]. The strategy presented in [24] is one that maximizes the information gain about the location of the source in a turbulent medium [24]. While these reactive strategies are more sophisticated, they are also computationally expensive and mostly focused on adaptive behavior in the context of single agent search strategies.

In this work, we present an information theoretic search strategy for single and multi-robot teams to find the source of a contaminant or hazardous waste plume in a turbulent medium. We build upon the strategy presented in [24] and formulate the source seeking/chemical plume localization problem as an information theoretic search strategy. The search strategy consists of making moves that maximize the change in entropy of the belief distribution of the source location. Similar to [26], [27], we rely on a particle filter representation of the posterior belief distribution to make the strategy computationally viable for large complex spaces and distributable for mobile sensing teams. While recent works have focused on information theoretic exploration strategies for simultaneous localization and mapping (SLAM) applications (see [28] and references therein), little has been done in employing similar strategies for autonomous sensing and exploration in geophysical flows. We validate the proposed strategies for single and multiple robots using data provided by a three dimensional (3D) time-varying computational fluid model of the 2010 Deep Water Horizon oil spill.

The paper is organized as follows: We formulate the problem, briefly summarize background information, and describe our assumptions in Section II. The single and multi-robot information theoretic search strategies are described in Section

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III. Section IV presents the simulation results for both the single and multi-robot strategies using both 2D and 3D models of the 2010 Deep Water Horizon Oil Spill. We conclude with a discussion of our results in Section V.

II. PROBLEM STATEMENT

Consider the problem of finding the source of a contaminant or hazardous waste plume in a turbulent medium. To begin with, we assume that the search area is obstacle free and can be discretized into a uniform grid of cells (2D) or voxels (3D). A cell or voxel is occupied if a robot is located within it and robots are allowed to move from their current cell or voxel to any of the neighboring cells or voxels, *i.e.*, up, down, left, right, and diagonally across the four (2D) or eight (3D) corners. In addition, every robot has the ability to localize within the workspace or domain of interest, can measure the magnitude and direction of the local flow field at its current location, and, in the case of a multi-robot team, can communicate with other members in the team. Lastly, we assume sensor measurements for detecting the presence of the plume material is binary, *i.e.*, the material is either present or not present at the robot's current position in the workspace.

In general, the expected rate of positive material plume detection in an environment depends on the spatial distance to the source, the dynamics of the surrounding flow field, the geometry of the environment, and many other factors. Since the propagation dynamics of biochemical and/or radiological material in a turbulent medium is highly complex, a common approach is to model the rate of detecting the presence of a material plume as a Poisson distribution. In this work, we employ the same statistical model for positive material plume detection in a turbulent medium as in [12], [24] and briefly summarize the approach here.

The mean rate of odor detection at position r dispersed by a source located at r_0 is assumed to follow a Poisson distribution given by:

$$\mathcal{R}(r|r_0) = \frac{R}{\ln(\frac{\lambda}{a})} e^{\frac{V}{2D}(r-r_0)\cdot \mathbf{n_3}} K_0\left(\frac{|\, r - r_0 \,|}{\lambda}\right), \tag{1}$$

$$\text{with} \lambda = \sqrt{\frac{D\tau}{1+\left(\frac{V^2\tau}{4D}\right)}},$$

where R is the emission rate of the source, τ is the mean lifetime of the chemical patch before its concentration falls off the detectable range, D is the isotropic effective diffusivity of the medium, V is the average velocity of the medium, or wind, and $K_0(\cdot)$ is the modified Bessel function of the second kind. We assume the dispersion of the plume is predominantly in the $\mathbf{n_3}$ direction where $\{\mathbf{n_1}, \mathbf{n_2}, \mathbf{n_3}\}$ denotes the orthonormal basis that spans the workspace. The probability of registering the presence of a material patch by a sensor depends on the distance of the sensor from the source and angle between $(r - r_0)$ and $\mathbf{n}_3$ where r denotes the current position of the sensor.

In the search for the source, the set of chemical detection encounters along the search trajectory carries the only available information about the relative location of the source with respect to the robot. Using this information, a robot can construct the probability distribution function for the source position using Bayes' rule $P(r_0|\mathcal{T}_t) = P(\mathcal{T}_t|r_0)P(r_0)/\left(\int P(\mathcal{T}_t|r)P(r)dr\right)$. As such $\mathcal{T}_t$ encapsulates the history of the uncorrelated plume cues it receives along this trajectory, with $P(\mathcal{T}_t|r_0)$ denoting the probability of experiencing such a history if the source of the dispersion is at r_0. Assuming that the probability of detecting a plume at each point is independent, Poisson's law is then used to estimate the probability of experiencing such a history of the chemical presence along the search trajectory as:

$$P(\mathcal{T}_t|r_0) = \exp\left(-\int_0^t \mathcal{R}(r(\bar{t})|r_0)\, d\bar{t}\right) \prod_i \mathcal{R}(r(t_i)|r_0), \tag{2}$$

where $r(t)$ is the search trajectory, and $r(t_i)$ is the position of each detection along the trajectory, and $\mathcal{R}(r(t)|r_0)$ is the expected rate of encounters at $r(t)$ for a source at r_0 [24]. It is important to mention that the assumption of independence of detections holds since the location of the plume's source is unknown.

We note that (1) provides an observation model and there is no other process model beyond what we assume about the environmental dynamics and the propagation of the source in the medium. At each step, the current estimate of the belief distribution of the source location, $P_t(r) = P(\mathcal{T}_t)$, is used to determine the expected number of positive sensor measurements at the next location, *i.e.*, $\mathcal{R}(r(t)|r_0)$. Using the current belief distribution as the best estimate available for the source location, the expected number of positive sensor hits at location r can be calculated as:

$$h(r) = \int P(\mathcal{T}_t|r_0)\mathcal{R}(r|r_0)dr_0. \tag{3}$$

Let c_t denote the sensor reading at time t where $c_t = 1$ denotes the presence of the material and $c_t = 0$ otherwise. The estimated belief distribution for the source position is then updated using Poisson's law $\rho(r_j) = h^{c_t}(r_i)e(-h(r_j))$.

We assume that the robots move on a grid within the workspace and at each time step they can travel an upper bound of M moves on the grid. The maximum number of moves on the grid is determined based on the limitations and bounds on the control input and can be calculated at each time step based on the vehicle's dynamics. The objective is to localize the source of the plume in the workspace by maximizing the expected rate of information gain for the posterior distribution of the source location at every step.

III. METHODOLOGY

A. Single-Robot Search Strategy

The information gathered through the chemical encounters at each step instantly shapes the probability distribution function the robot constructs as the estimate of the location of the source, denoted by $P_t(r)$, and uses to plan its motion. A successful search strategy should drive the robot(s) to unexplored regions in the workspace, and maximizes the information gained about the source location. One strategy is to drive the robot(s) in direction that result in the steepest decrease in the entropy of the estimated distribution describing the source location. The expected rate of information gathered at each step of the such a search strategy can be computed as the expected change in the entropy of the estimated field given by:

$$\mathbf{E}[\Delta S_t(r \mapsto r_j)] = P_t(r_j)[-S_t] + [1 - P_t(r_j)][(1-\rho(r_j))\Delta S_0 + \rho(r_j)\Delta S_1], \quad (4)$$

where, $S_t = -\int P_t(r)\log(P_t(r))dr$ is the Shannon entropy of the estimated field. The first term of (4) corresponds to the change in entropy upon finding the source at the very next step. If the robot finds the source, the entropy of the probability distribution plunges to zero and the search stops. However, according to the current estimated probability distribution the chance of finding the source at r_j is just equal to $P_t(r_j)$. The second term of (4) accounts for the case when the source is not at r_j and computes the amount of information expected to be gained in case the robot does or does not receive an additional positive detection at the new position. At each new location, the robot takes one measurement with its sensor resulting in two possible outcomes: $c_t = 1$ or $c_t = 0$. To find the expected value of the change in the utility function, i.e., the entropy, (4) calculates the change in entropy for each possible move and each possible sensor outcome after the move. As such, (4) calculates the expected change in the entropy of the belief distribution based on the current estimate of the source location. Furthermore, (4) estimates the expected amount of information we acquire about the source position at the very next step.

The belief distribution for the source location $P_t(r)$ is maintained over all possible source positions. At each step, the robot chooses to move in a direction that enables it to acquire more information and decrease the uncertainty of the source position estimate. Storing and representing the belief distribution becomes computationally challenging especially when the search space spans large physical scales and/or contains complex geometry. This is especially true if robots rely on a fine grid map to calculate the log-likelihood of the belief distribution. Thus, we employ a particle filter approach to the representation of the belief distribution with limited numbers of randomly drawn particles. The use of particle filters allows us to bound the computational burden on each robot by allowing the robot to focus on more probable hypotheses while disregarding the rest.

In this work, we assume each robot stores the estimated belief distribution of the source location, $P_t(r)$, using a manageable number of weighted particles, $\{\hat{r}_i, \omega_i\}$, with $\hat{r}_i$ representing the hypothesis for the state (position) and ω_i representing the corresponding weight (probability) of hypothesis i [29], [30]. The probability mass function represented by these set of particles is mathematically equivalent to the sum of the weighted spatial impulse functions [31]:

$$\hat{P}_t(r) \approx \sum_i \omega_i \delta(\hat{r}_i - r), \quad (5)$$

Input: Current estimate of belief distribution over the possible position of the source.
Output 1: Next way point on the search trajectory.
Output 2: Updated estimate of belief distribution over the possible position of the source.

repeat
- **for** *Each Accessible Way Point* **do**
  1) Calculate the expected rate of odor detection based on the current estimate of the belief distribution [Equation (3)];
  2) Calculate the belief distribution over the position of the source in case the robot moves to the new location, and
     a) does, or
     b) does not

     detect any chemical [Equation (2)];
  3) Calculate the expected variation of the entropy at the next way points [Equation (4)]
- **end**
- Move to the location of the way point expecting steepest reduction in entropy;
- Obtain sensor reading and compute the new probability distribution;

until *The Estimated Probability Distribution over the Position of the Source Meets some Confidence Measure*;

Algorithm 1: Single robot search strategy.

where $\hat{r}_i$ is a hypothesis that survived the re-sampling procedure of the previous step and the weights, ω_i, of the particles are modified as follows:

$$\omega_i(t) = \omega_i(t-1)e^{-\left(\mathcal{R}(r(t)|r_0)\right)}\left(\mathcal{R}(r(t)|r_0)\right)^{hit}. \quad (6)$$

where, *hit* is one if the sensor reading indicates presence of any chemical, and it is zero otherwise. To calculate the entropy of the particle representation of the belief distribution, we use the approach presented in [26]:

$$S \approx -\sum_{k=1}^{N} w^{(i)}_{(t-1),k} \log\left(w^{(i)}_{(t-1),k}\right). \quad (7)$$

The robot's control strategy is to move to a point where it expects the steepest decrease in the entropy of its estimate of the source position. The control decision is determined based on the expected change in the entropy of the source position estimate. At each time step, the expected information gain from any observation at the next probable robot position is calculated. Since robots are constrained to a maximum of M moves on the grid, they can determine how much information on the source's position it can expect to acquire as it moves. If no particles are within the robot's set of reachable points on the grid, then any point that is M moves away from the robot's current position is chosen as the probable next step. The single robot search strategy is summarized in Algorithm 1.

Remark 1: We note that the weight update given by (6) is different from most particle filter implementations [26], [27]. This difference is about the weights of the particles after the re-sampling procedure. The weights of the particle before the re-sampling represents the proposal distribution, and after re-sampling the samples themselves are the representative of the target probability distribution. This is because in target localization applications, robots have access to large and continuous amounts of measurement data which enables them to continuously correct or improve their estimates relatively quickly. However, when localizing the source of a plume, the algorithm relies heavily on the spatial distribution of the sporadic sensory signals along a robot's search trajectory. As such, the re-sampling serves the role of integrating past information into the current estimate of the source position. Therefore, one must take the probability of the detection history, (2), into account during the update and eliminate the less probable hypotheses when re-sampling.

Remark 2: The confidence measure to end the algorithm is somehow a delicate matter. In the environments that limit the use of other sensors or the applications in which the chemical sensor is the only mean to detect the presence of plume it is hard to verify the assumption we have. Once our confidence measure was satisfactory, we might have to send a robot directly to the estimated position of the source for direct fact check. The confidence measure can be the entropy of the estimated probability distribution, or consistency of the clues the robot receives during some period of time.

B. Multi-Robot Search Strategy

To speed up the search strategy, we extend the proposed single robot search strategy to a team of robots. This effectively increases the chances of positive encounters, and consequently expected to decrease the time needed to localize the source. To achieve this, we assume robots can communicate with one another. Each robot begins exploring the workspace following a single robot entropy minimizing strategy similar to Algorithm 1 and constructs its own belief distribution for the source position. When a robot encounters a material patch, it communicates its position to the rest of the team. Upon receiving this information, the other robots incorporate the new information into their belief distributions. While communications can be severely limited in bandwidth in a fluidic medium, *e.g.*, underwater environments, the amount of data that must be communicated is low.

C. Global vs. Body-Fixed Frames

When a robot has the ability to localize within a global coordinate frame, localization of the plume source can be absolute. This is because each particle in the filter represents a hypothesis for the source position whose positions are known in a global coordinate frame. As the robot exploits the information gained at every time step to drive its exploration strategy, the uncertainty of the position estimates will decrease resulting in a belief distribution centered around the actual position of the source.

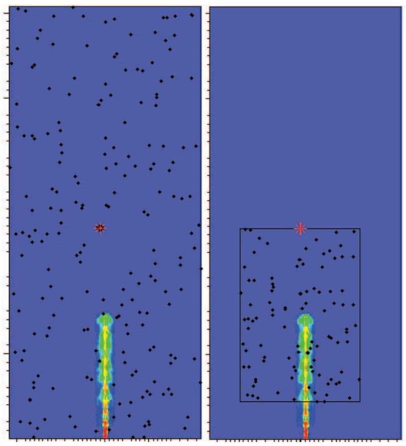
In general, it may be unrealistic to assume robots will have the ability to localize within a global frame, or that a map of the workspace is always available. In this section, we consider the extension of the described information theoretic search strategies to situations where robots must operate in an unknown environment without the ability to localize itself with respect to a reference frame. We assume robots have forward pointing sensors. During the search process, each robot generates hypotheses for the source position as particles relative to robot's current position and orientation and constructs the belief distribution for the source position in the robot's body-fixed coordinate frame. As the robot detects the presence of a material plume or after a move, the robot modifies its estimate of the belief distribution accordingly. Under these circumstances, it is possible for the source to be positioned outside the robot's range of search area internally assumed with the robot. Furthermore, although the control decisions are made according to the relative positions of the particles with respect to the robot's current pose, they do not represent actual hypotheses of the source's location in the global frame.

The main difference between having the ability to localize within a global frame and not is the information content in the *absence* of a positive detection. When the robot has the ability to localize within a global frame, a positive sensor reading provides information about the absolute position of the source. When we already have an estimate about the position of the source with respect to the robot, and since a the positive detection is in face stochastic, the absence of it provides a negligible amount information with respect to what we already know, and the lack of information does not impact the resulting estimates. Thus the particle filter representation of the belief distribution is only updated when the robot detects the presence of the material plume in the environment. However, when the robot cannot localize within a global frame, the absence of a positive detection provides a great deal of information on whether the current course the robot is moving in is a good one or not. As such, the belief distribution should be updated at each time step, regardless of the value of the sensor readings, in cases when the robots cannot localize within the workspace. These differences are shown in Fig. 1.

Lastly, to account for the robot's motion uncertainties on the particle positions and thus the entropy calculations, we represent each particle position with a Gaussian distribution and thus employ a Gaussian Mixture Model to represent the probability distribution of (5). As such, we employ the steps described in [26] to calculate the information utility function:

$$S \approx -\int_{\theta} \Big\{ \sum_{k=1}^{N} \big(w^{(i)}_{(t-1),k} p(\theta_t | \theta_{t-1} = \hat{\theta}^{(i)}_{(t-1),k})\big) \cdot \qquad (8)$$
$$\log\Big(\sum_{k=1}^{N} \big(w^{(i)}_{(t-1),k} p(\theta_t | \theta_{t-1} = \hat{\theta}^{(i)}_{(t-1),k})\big)\Big) \Big\} d\theta,$$

where $p(\theta_t|\theta_{t-1} = \hat{\theta}^{(i)}_{(t-1),k})$ is the robot's motion model and is a Gaussian probability distribution centered on the position of



(a) (b)

Fig. 1: The initialized particles representing the hypotheses for the source position for, a) estimation of the global position of the source, and b) the estimation of the position of the source in the local coordinates of the robot, The dimension of the search arena as 512×320 units.

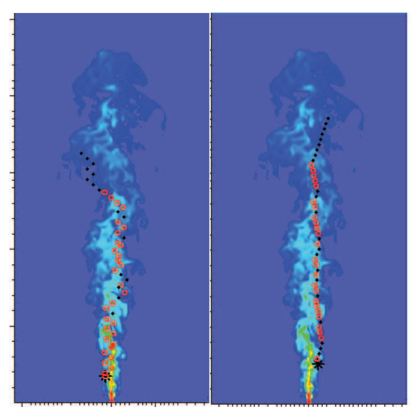
the i^{th} particle. Since there are no close form representations for this integral, numerical computation of this integral can be expensive. To reduce the computational burden, we employ the approximation strategy described in [27], [32] that has been shown to yield good approximations.

IV. SIMULATION RESULTS

In this section we present simulation results for single and multi-robot teams executing the described information theoretic strategies in 2D and 3D turbulent fields. The 2D and 3D dispersion models used in our simulations were developed to model the 2010 Deep Water Horizon Oil Spill. The dispersion database was obtained from a numerical simulation of a bubble plume in a stably stratified environment under rotation. The details and derivation of the turbulence resolving model for multiphase plumes are presented in [33]. Specifically, the plume is generated by an inlet buoyancy flux $B_i = gw_iA_i\alpha_{b,i} = 5 \times 10^{-6}m^4s^{-3}$ where $g = 9.8ms^{-2}$ is the gravity acceleration magnitude, $w_i = 4cm\,s^{-1}$ is the inlet liquid velocity, $A_i = \pi r_i^2 = 0.005m^2$ is the source cross-section area of radius r_i and $\alpha_{b,i} = 0.026$ is the inlet gas volume fraction. The initially unperturbed ambient fluid is thermally stratified with a constant slope $\zeta = 5.1K\,m^{-1}$ and the system Coriolis parameter is set to $f = 0.01s^{-1}$. The cylindrical computational domain has a height H and diameter D of $H/r_i = D/r_i \approx 67$ with Dirichlet boundary condition at the bottom, no shear and no flux at the top for the momentum and scalars respectively and open lateral boundary conditions with numerical sponges to ensure numerical stability. The domain has been spatially discretized using spectral element methods into $K = 7,540$ conforming elements in which the solution is approximatted with a 14th order polynomial expansion resulting in $\sim$ 21 million nodes. The transport equations have been integrated using the nek5000 solver [34] that has demonstrated an excellent scalability on parallel machines [35]. The results used in this work correspond to the statistically stationary solution obtained after approximately $150,000$ core-hours on a Cray XE6 using 960 2.2 GHz AMD Magny-Cours cores.

A. 2D Results

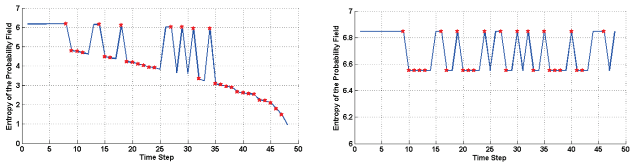
Fig. 2a and 2b respectively show the resulting single robot search trajectory in 2D when the robot has the ability to localize within a global coordinate frame versus a local body-fixed frame. The initial number of particles used for the particle filter representation of the belief distribution in the global case is 10% of the total number of cells used to represent the environment or $15,000$ particles. In contrast, we employed half the number of particles, or $7,500$, to represent the belief distribution when robots must operate in a body-fixed frame without the ability to obtain its global pose estimates. This is another advantages of using the local coordinate frame. Since it assumed basically that the robot does not know the actual dimensions of the search arena, its perception of the search field can be smaller than the actual size of the real enviroment.



(a) (b)

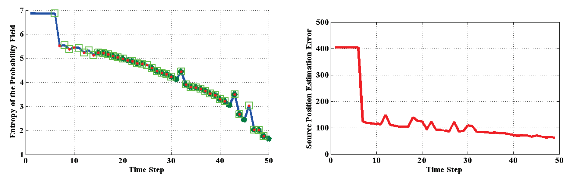
Fig. 2: Robot search trajectories when operating with (a) global pose information and (b) relative pose information. The background shows the gas volume concentration in the water due resulting from the blowout. Red corresponds to high concentration levels.

The corresponding entropy of the belief distribution for the source location over time for the single robot scenarios shown in Fig. 2a and 2b are shown in Fig. 3a and Fig. 3b. Not surprisingly, each positive detection of the material plume results in a corresponding decrease in the entropy of the belief distribution. In the case when the robot has global pose information, the entropy decreases over time. However, when the robot can only localize the source in a body-fixed frame, we do not see the same monotonically decreasing trend. This is because when robots do not have global pose information, the positive sensor readings simply steers the robot towards



(a) (b)

Fig. 3: The entropy of the belief distribution for the source position for a robot operating with (a) and (b) without global pose information. The red dots indicates positive sensor detections of the material plume.



(a) (b)

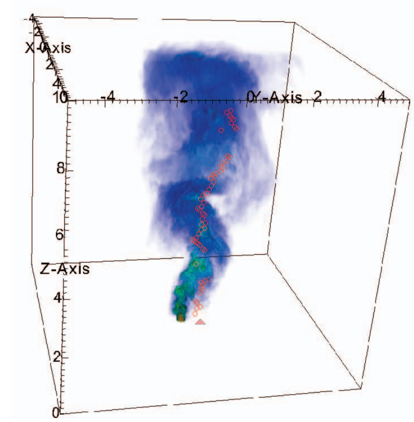
Fig. 5: (a) Entropy of the belief distribution for the source position over time for a team of three robots. Red dots, green stars, and cyan squares indicate the encounters with the material plume by different robots. (b) Error in the estimate of the source position over time for the team of three robots.

the direction of its best guess for the source position. Since the estimate for the source positive is relative to the robot's current position, the information content is limited at each step and thus do not result in an overall decrease in the entropy of the position estimate for the source.

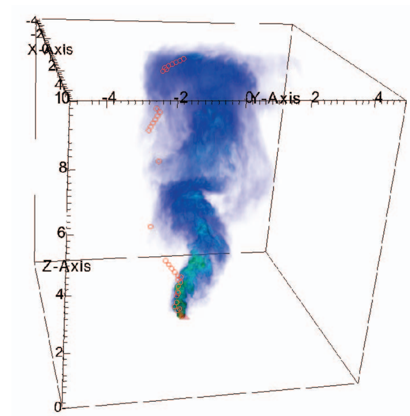
Fig. 4 shows the search trajectories for a team of three robots. In this scenario, we assume robots have the ability to localize within a common global coordinate frame. Each robots computes its own estimate of the belief distribution for the source. When a member of the team detects the presence of the material plume at its position, the information is communicated to all the other team members and each robot then updates its estimate accordingly. Fig. 5a shows the entropy of the belief distribution over time. Every positive detection of the material results in a decrease of the entropy of the source position belief distribution. Fig. 5b shows the corresponding decreases in the estimation errors of the source position over time by the team.

B. 3D Results

For the 3D simulations, the environment was discretized into a grid of $100 \times 100 \times 100$ voxels. Unfortunately, if we assume robots have the ability to localize within a global coordinate frame, this would require significant number of particles to represent the belief distribution for the source position. This was achieved using $100,000$ thousands particles or 10% of the total number of voxels used to represent the workspace and the results are shown in Fig. 6a. However, to make the approach more tractable for potentially resource



(a)

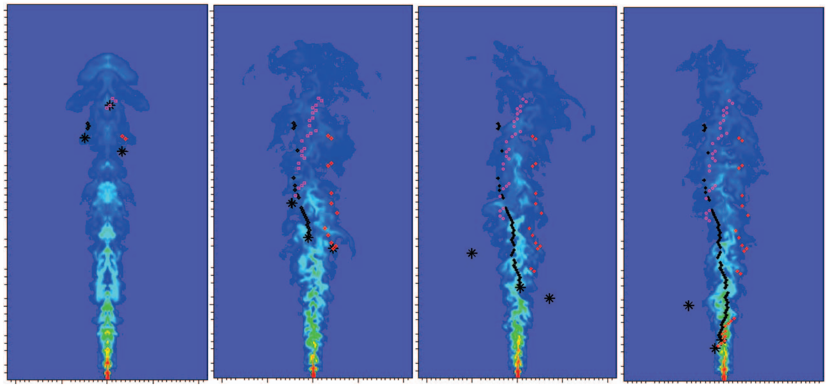


(b)

Fig. 6: Single robot search trajectory in localizing the source of a plume in a time-varying 3D turbulent flow field (a) with and (b) without global pose information. The red bubles indicate the detection events along the search trajectory.

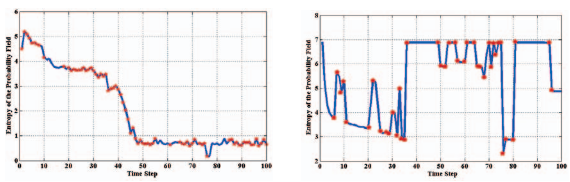
constrained mobile robots, we assumed robots localized the source position using a relative body-fixed frame, similar to the scenario where robots do not have information about its pose in a global coordinate frame. This approach significantly reduced the number of particles needed to represent the belief distribution since robots were initialized assuming its immediate environment is one eighth the volume of the actual environment. Fig. 6b shows the resulting single robot search trajectory. The corresponding entropy of the belief distribution for the source position over time for both cases are shown in Fig. 7a and 7b. We note that we see the same trends in the change of the entropy over time as in the 2D results.

Fig. 8a shows the resulting search trajectories for a team of three robots. While we see lower uncertainty in the team's estimate for the source position (see Fig. 8b), the robots do not necessarily move closer to the source. This may be a potential concern in the event of false positives and as such it is important that at least one robot is tasked to the estimated position to establish a final verification. Fig. 8b shows the corresponding entropy of the belief distribution



(a) (b) (c) (d)

Fig. 4: A team of three robots executing the proposed collaborative search strategy to localize the source of an oil spill. The background shows the gas volume concentration in the water due to the spill. The robots are denoted by black $*$. The red, magenta, and black dots denote the robot trajectories. The number of the particles is the same as single agent search simulation in Fig. 2a.



(a) (b)

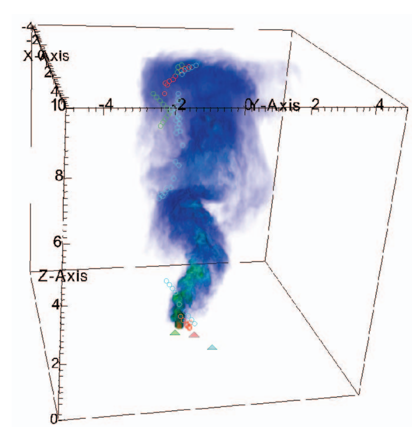
Fig. 7: Entropy for the belief distribution of the source position over time for the single robot in a 3D environment (a) with and (b) without global pose information. The red dots indicate the detection of plume by the robot.

for the source position for the team of three robots. Since the robots initialize their source position belief distributions independently, it results in differences in their initial estimates. However since the team uses the information to shape the belief distribution, every robot in the team results in a similar estimate in the absence of communication losses.

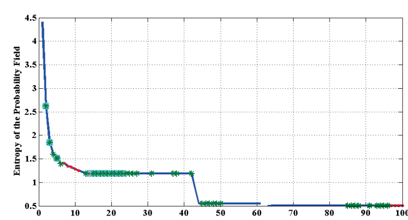
The algorithm runs surprisingly fast. With an average desktop computer equipped with Intel(R) Core(M) i7-4770 CPU @ 3.40 GHZ, the 2D simulations for the prescribe dimensions runs in less than a minutes, and the 3D simulations takes about two minuses. We refer the interested reader to our multimedia attachment for a movie of the various simulation results.

V. CONCLUSION

We presented different implementations of an information theoretic search strategy for single and multiple robots to localization the source position of a plume in turbulent mediums. In our strategies, robots search and localize the source by relying on sporadic and discontinuous sensor readings. We developed two variations of the proposed strategy where robots can rely either on global or relative pose estimates to localize the position of the emitting source. We validated the strategies



(a)



(b)

Fig. 8: (a) The last frame of the movie of the simulated dispersion source localization in 3D environment for a robot (b) The Shannon Entropy the distribution over the absolute location of the the source estimated with one of the robots in the group. Red dots, cyan squares, and green stars indicates the sensor reading reported with the robots.

in 2D and 3D using a computational model of the 2010 Deep Water Horizon Oil Spill.

In general, coordination of source seeking and cooperative localization strategies for multi-robot teams are easier to achieve when robots have the ability to localize within a common coordinate frame. Furthermore, in the absence of communication losses, the team has the ability to more efficiently localize the position of the source. We note that in our proposed implementation, the added advantage of the collaborative search strategy is that individual robots can use a smaller number of particles in their representation of the source position belief distribution. This is because the added information provided by other robots in the team enables individual robots to update their own belief distributions. As such, individually robots can obtain better estimates of the source position in a more computationally efficient manner as compared to a single robot.

ACKNOWLEDGEMENT

This work was supported by the Office of Naval Research (ONR) by ONR Awards No. N000141211019, the National Science Foundation (NSF) grant IIS- 1253917, and the Gulf of Mexico Research Initiative (GoMRI) Award No. SA 15-15.

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