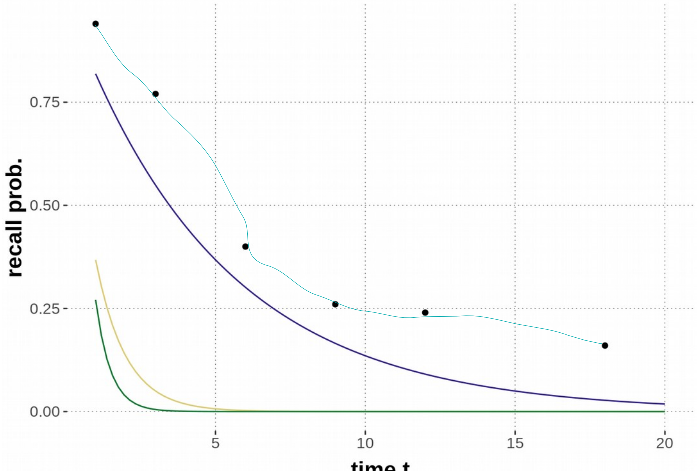
Model comparison

What's a good model?

- Fit to the data
- Simplicity



- How to combine them into one criterion? (trade-off)

AIC

$$\mathrm{AIC}(M_i, D_{\mathrm{obs}}) = 2k_i - 2\log P(D_{\mathrm{obs}} \mid \hat{\theta}_i, M_i)$$

- First term: complexity, second term: fit to data

- Akaike weights: How likely is Mi given all models in consideration? $P(M_i \mid D) \approx w_{\mathrm{AIC}}(M_i, D)$

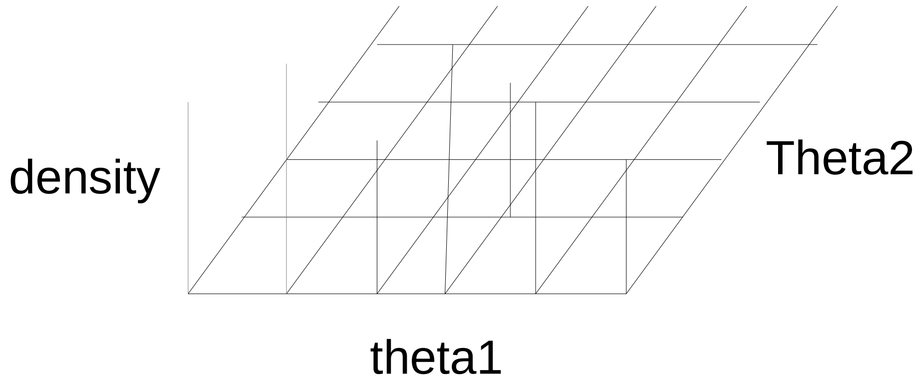
Bayes factors

$$\underbrace{\frac{P(M_1 \mid D)}{P(M_2 \mid D)}}_{\text{posterior odds}} = \underbrace{\frac{P(D \mid M_1)}{P(D \mid M_2)}}_{\text{Bayes factor}} \underbrace{\frac{P(M_1)}{P(M_2)}}_{\text{prior odds}}$$

$$\frac{P(D \mid M_1)}{P(D \mid M_2)} = \frac{\int P(\theta_1 \mid M_1)\, P(D \mid \theta_1, M_1)\, \mathrm{d}\theta_1}{\int P(\theta_2 \mid M_2)\, P(D \mid \theta_2, M_2)\, \mathrm{d}\theta_2}$$

- Model doesn’t depend on data
- Damn integrals
- Vague priors are punished implicitly

BF – grid approximation



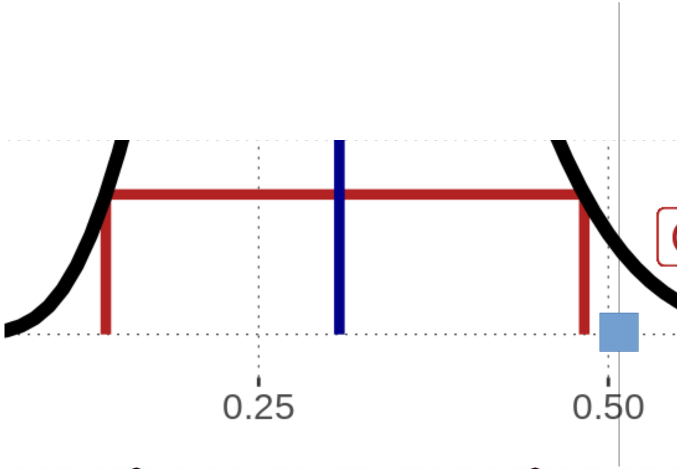
BF – naive MC

$$P(D, M_i) = \int P(D \mid \theta, M_i)\, P(\theta \mid M_i)\, \mathrm{d}\theta \approx \frac{1}{n} \sum_{\theta_j \sim P(\theta|M_i)}^{n} P(D \mid \theta_j, M_i)$$

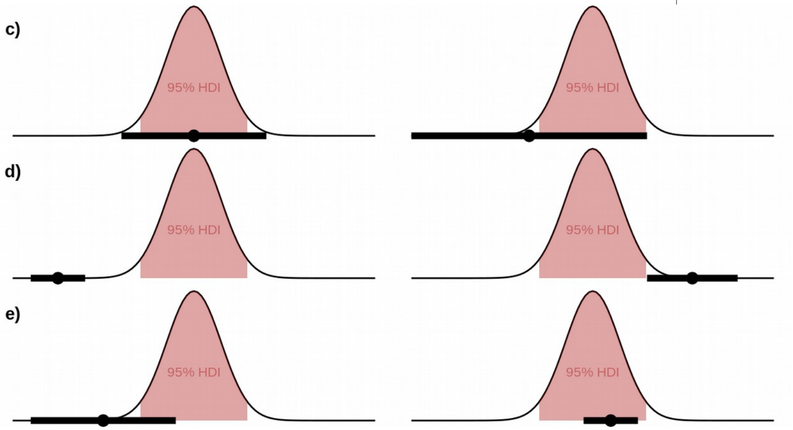
Bayesian Hypothesis testing

Estimation-based

- Both ROPE and point outside 95%HDI



- ROPE criterion: whether 95% is (fully/not at all/partially) contained in ROPE



Comparison-based point-valued Savage-Dickey

$$\mathrm{BF}_{01} = \frac{P(\theta_i = x_i, \ldots, \theta_n = x_n \mid D, M_1)}{P(\theta_i = x_i, \ldots, \theta_n = x_n \mid M_1)}$$

- BF01 = posterior / prior, really that simple!
- nesting model (1) is more general than nested model (0)
- Hypothesis can be thought as the nested model fixing some parameter values

Comparison-based interval-valued encompassing model

- $P_{M_0}(\theta,\omega) = P_{M_e}(\theta,\omega \mid \theta \in [a;b])$
  $P_{M_a}(\theta,\omega) = P_{M_e}(\theta,\omega \mid \theta \notin [a;b])$ Both models are nested under the encompassing model

- $$\mathrm{BF}_{01} = \frac{\text{posterior-odds of } H_0}{\text{prior-odds of } H_0} = \frac{P(M_0 \mid D)}{P(M_1 \mid D)} \Big/ \frac{P(M_0)}{P(M_1)}$$
  $$= \frac{P_M(\theta = \theta^* \pm \epsilon \mid D)}{P_M(\theta \neq \theta^* \pm \epsilon \mid D)} \frac{P_M(\theta \neq \theta^* \pm \epsilon)}{P_M(\theta = \theta^* \pm \epsilon)}$$