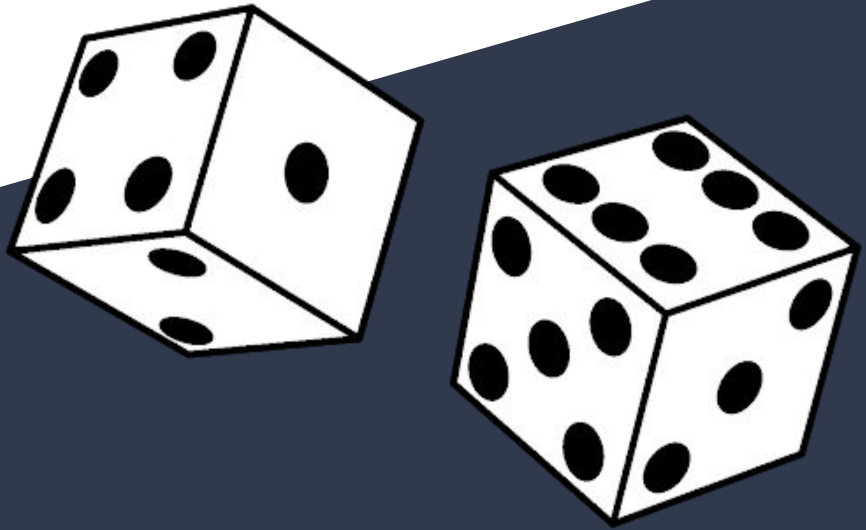
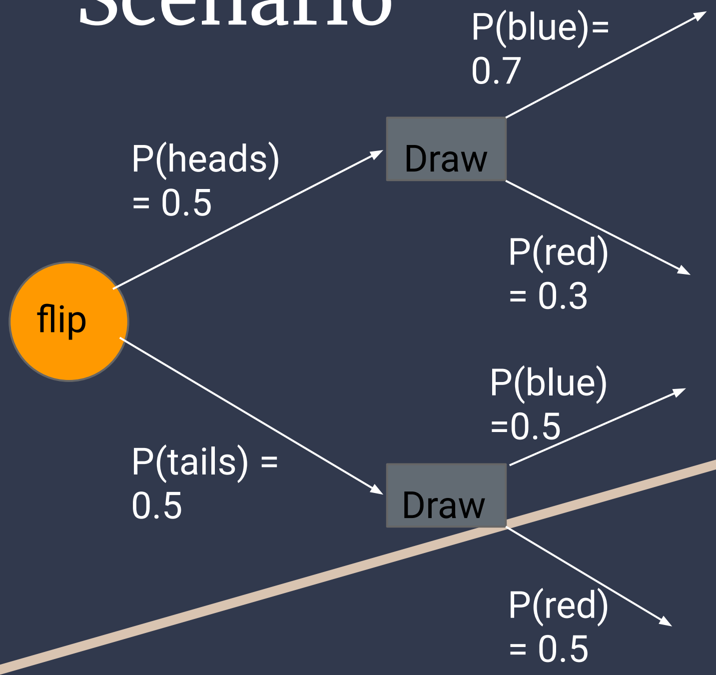
Tutorial
Basics of Probability Theory

Statistics and Data Analysis

Tutors: Lune & Inga

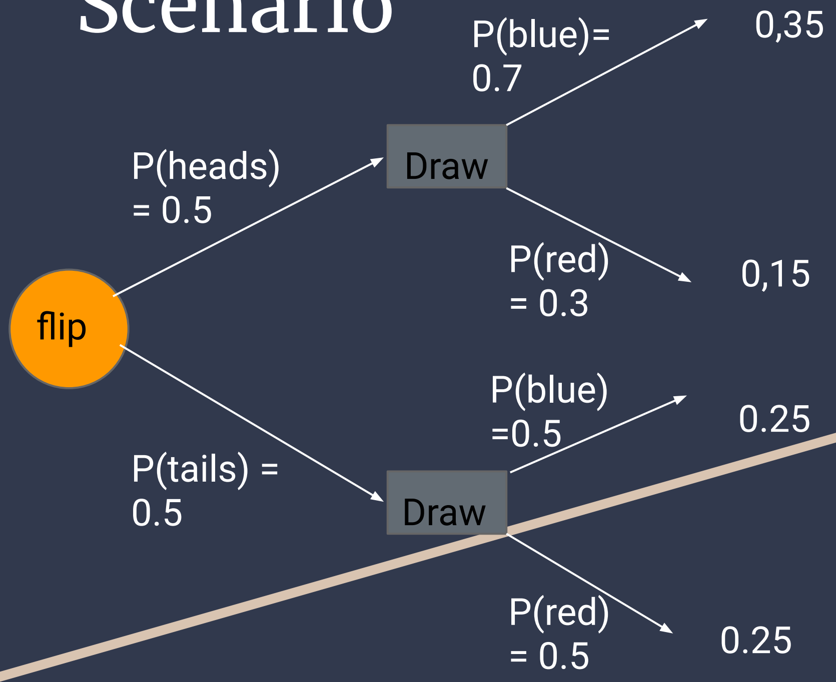


Flip-and-Draw Scenario



Calculate the full path probabilities.

Flip-and-Draw Scenario



Solution:

P(heads, blue) = 0.5*0.7=0,35

P(heads, red) = 0.5*0.3=0.15

P(tails, blue)= 0.5*0.5=0.25

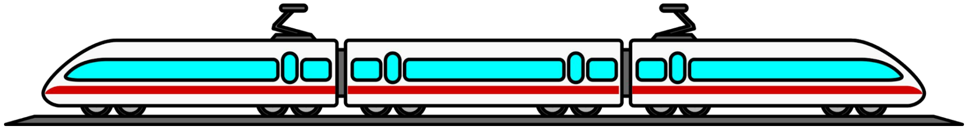
P(tails,red) =0.5*0.5= 0.25

Conditional Probability Table

John is going to work by train everyday. He wonders if the arriving time of the train is connected to the weather/season. So he started recording the numbers.

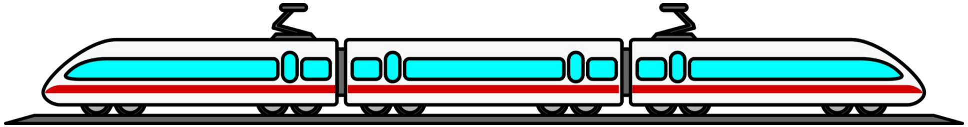
Calculate the missing values

	On Time	Delayed	$\sum$
Spring	0,2		0,25
Summer			0,25
Autumn		0,08	
Winter	0,04		0,25
$\sum$		0,45	



Conditional Probability Table

	On Time	Delayed	$\sum$
Spring	0,2	0,05	0,25
Summer	0,14	0,11	0,25
Autumn	0,17	0,08	0,25
Winter	0,04	0,21	0,25
$\sum$	0,55	0,45	1



More probability calculation

	On Time	Delayed	$\sum$
Spring	0,2	0,05	0,25
Summer	0,14	0,11	0,25
Autumn	0,17	0,08	0,25
Winter	0,04	0,21	0,25
$\sum$	0,55	0,45	1

Let's consider this table.
Calculate the following probabilities:

P(Winter | Delayed) =

P(Not summer | On Time)=

P(Delayed | Spring) =

More probability calculation

	On Time	Delayed	$\sum$
Spring	0,2	0,05	0,25
Summer	0,14	0,11	0,25
Autumn	0,17	0,08	0,25
Winter	0,04	0,21	0,25
$\sum$	0,55	0,45	1

$$P(Winter|Delayed) = \frac{0{,}21}{0{,}45} = \frac{7}{15}$$

$$P(Notsummer|OnTime) = \frac{(0{,}2+0{,}17+0{,}04)}{0{,}55} = \frac{41}{55}$$

$$P(Delayed|Spring) = \frac{0{,}05}{0{,}25} = \frac{1}{5}$$

Bayes Rule – Task

Medical Diagnostic Test

There is a cancer test, which has a true positive rate (positive, when the person has cancer) of 0.85 and a true negative rate of 0.95.The general probability to get this type of cancer is 0.02%.

Calculate the probability of a patient having cancer given a positive test result using the Bayes Theorem.

Bayes Rule – Solution

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A \mid B) \times P(B)}{P(A)}$$

For this task:

$$P(Cancer = True|Test = Positive) = \frac{P(Test = Positive|Cancer = True) * P(Cancer = True)}{P(Test = Positive)}$$

Probabilities taking from the text:

P(Test=positive| cancer=true)=0.85
P(Cancer=True) = 0.0002
P(Test=Negative | Cancer=False) = 0.95

Calculation of P(Test=Positive):

P(Cancer=True | Test=Positive) = 0.85 * 0.0002 / P(Test=Positive)

P(Cancer=False) = 1 – P(Cancer=True) = 0.9998

P(Test=Positive|Cancer=False) = 1 – P(Test=Negative | Cancer=False) = 0.05

P(Test=Positive) = 0.85 * 0.0002 + 0.05 * 0.9998 = 0.05016

P(Cancer=True | Test=Positive) = P(Test=Positive|Cancer=True) * P(Cancer=True) / P(Test=Positive)= 0.003389154704944

More Practice! Commute problem

A person uses his car 30% of the time, walks 30% of the time and rides the bus 40% of the time as he goes to work. He is late 10% of the time when he walks; he is late 3% of the time when he drives; and he is late 7% of the time he takes the bus.

- What is the probability he took the bus if he was late?

Solution

What is the probability he took the bus if he was late?

$$P(B|L) = \frac{P(B,L)}{P(L)}$$

$$P(B|L) = \frac{0.07 * 0.4}{P(L)}$$

$$\begin{aligned} P(L) &= P(W,L) + P(B,L) + P(D,L) \\ &= 0.1 * 0.3 + 0.07 * 0.4 + 0.03 * 0.3 \\ &= 0.067 \end{aligned}$$

$$P(B|L) = 0.418$$

Commute problem

A person uses his car 30% of the time, walks 30% of the time and rides the bus 40% of the time as he goes to work. He is late 10% of the time when he walks; he is late 3% of the time when he drives; and he is late 7% of the time he takes the bus.

- What is the probability he walked if he is on time?

Solution

What is the probability he walked if he is on time?

$$P(W|T) = \frac{P(W,T)}{P(T)} \qquad = \frac{0.3 * 0.9}{P(T)}$$

$$P(T) = 1 - P(L) = 0.933 \qquad\qquad P(W|T) = 0.289$$

Coin problem

Consider 3 coins where two are fair, yielding heads with probability 0.50, while the third yields heads with probability 0.75. If one randomly selects one of the coins and tosses it 3 times, yielding 3 heads - what is the probability this is the biased coin?

Coin problem solution

Consider 3 coins where two are fair, yielding heads with probability 0.50, while the third yields heads with probability 0.75. If one randomly selects one of the coins and tosses it 3 times, yielding 3 heads - what is the probability this is the biased coin?

$$= \frac{1/3 * (0.75)^3}{2/3 * (0.5)^3 + 1/3 * (0.75)^3)} = 0.6279$$

Bayes' Theorem expression

Suppose $P(A), P(\overline{A}), P(B|A)$, and $P(B|\overline{A})$are known.

Find an expression for $P(A|B)$ in terms of these four probabilities.

Solution

$$P(A|B) = \frac{P(A,B)}{P(B)}$$

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B|A)\,P(A) + P(B|\overline{A})\,P(\overline{A})}$$

Q&A