Automated Deduction

Laura Kovács

Outline

First-Order Theorem Proving - An Example

First-Order Logic and TPTP

Inference Systems

First-Order Theorem Proving

We will use the VAMPIRE theorem prover throughout the lecture.

Go to

`https://vprover.github.io/download.html`

and pick the route most suitable to you.

Notes:

- For Linux users, a binary is probably the easiest route
- For Mac users, you need to build from source
  - run `make vampire_rel`
- For Windows users, the easiest route is a virtual machine and then use Linux
  - only running Vampire in the browser at `https://tptp.org/cgi-bin/SystemOnTPTP`

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First-Order Theorem Proving. An Example

Group theory theorem: if a group satisfies the identity $x^2 = 1$, then it is commutative.

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More formally: in a group "assuming that $x^2 = 1$ for all x prove that $x \cdot y = y \cdot x$ holds for all x, y."

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Group theory theorem: if a group satisfies the identity $x^2 = 1$, then it is commutative.
More formally: in a group "assuming that $x^2 = 1$ for all x prove that $x \cdot y = y \cdot x$ holds for all x, y."
What is implicit: axioms of the group theory.

$$\forall x(1 \cdot x = x)$$
$$\forall x(x^{-1} \cdot x = 1)$$
$$\forall x \forall y \forall z((x \cdot y) \cdot z = x \cdot (y \cdot z))$$

Formulation in First-Order Logic

	$\forall x(1 \cdot x = x)$	A1
Axioms (of group theory):	$\forall x(x^{-1} \cdot x = 1)$	A2
	$\forall x \forall y \forall z((x \cdot y) \cdot z = x \cdot (y \cdot z))$	A3
Assumptions:	$\forall x(x \cdot x = 1)$	A4
Conjecture:	$\forall x \forall y(x \cdot y = y \cdot x)$	G

$$\boxed{A_1 \wedge A_2 \wedge A_3 \wedge A_4 \Rightarrow G}$$

In the TPTP Syntax

The TPTP library (Thousands of Problems for Theorem Provers), http://www.tptp.org contains a large collection of first-order problems. For representing these problems it uses the TPTP syntax, which is understood by all modern theorem provers, including Vampire.

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$product(e,X)=X

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In the TPTP syntax this group theory problem can be written down as follows:

```
%---- 1 * x = x
fof(left_identity,axiom,
    ! [X] : mult(e,X) = X).
%---- i(x) * x = 1
fof(left_inverse,axiom,
    ! [X] : mult(inverse(X),X) = e).
%---- (x * y) * z = x * (y * z)
fof(associativity,axiom,
    ! [X,Y,Z] : mult(mult(X,Y),Z) = mult(X,mult(Y,Z))).
%---- x * x = 1
fof(group_of_order_2,hypothesis,
    ! [X] : mult(X,X) = e).
%---- prove x * y = y * x
fof(commutativity,conjecture,
    ! [X] : mult(X,Y) = mult(Y,X)).
```

$\boxed{\forall x \mathrel{.} 1 \cdot x = x}$

Running Vampire on a TPTP file

is easy: simply use

```
vampire <filename>
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One can also run Vampire with various options, some of them will be explained later. For example, save the group theory problem in a file `group.tptp` and try

```
vampire --thanks TUWien group.tptp
```

Proof by Vampire (Slightliy Modified)

```
Refutation found.
270. $false [trivial inequality removal 269]
269. mult(sk0,sk1) != mult (sk0,sk1) [superposition 14,125]
125. mult(X2,X3) = mult(X3,X2) [superposition 21,90]
90. mult(X4,mult(X3,X4)) = X3 [forward demodulation 75,27]
75. mult(inverse(X3),e) = mult(X4,mult(X3,X4)) [superposition 22,19]
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- Proof by refutation, generating and simplifying inferences, unused formulas ...

Outline

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p(X) v q(X)

![X]: (p(X) v q(X))

FOL	TPTP
$\bot, \top$	`$false, $true`
$\neg a$	`~a`
$a_1 \wedge \ldots \wedge a_n$	`a1 & ... & an`
$a_1 \vee \ldots \vee a_n$	`a1 | ... | an`
$a_1 \rightarrow a_2$	`a1 => a2`
$(\forall x_1) \ldots (\forall x_n) a$	`! [X1,...,Xn] : a`
$(\exists x_1) \ldots (\exists x_n) a$	`? [X1,...,Xn] : a`

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More on the TPTP Syntax

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%---- (x * y) * z = x * (y * z)
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5. ![X0,X1]: mult(X0,X1) = mult(X1,X0) [input]
4. ![X0]: e = mult(X0,X0)[input]
3. ![X0,X1,X2]: mult(X0,mult(X1,X2)) = mult(mult(X0,X1),X2) [input]
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```

- Each inference derives a formula from zero or more other formulas;
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Proof by Vampire (Slightly Modified)

```
Refutation found.
270. $false [trivial inequality removal 269]
269. mult(sk0,sk1) != mult (sk0,sk1) [superposition 14,125]
125. mult(X2,X3) = mult(X3,X2) [superposition 21,90]
90. mult(X4,mult(X3,X4)) = X3 [forward demodulation 75,27]
75. mult(inverse(X3),e) = mult(X4,mult(X3,X4)) [superposition 22,19]
27. mult(inverse(X2),e) = X2 [superposition 21,11]
22. mult(inverse(X4),mult(X4,X5)) = X5 [forward demodulation 17,10]
21. mult(X0,mult(X0,X1)) = X1 [forward demodulation 15,10]
19. e = mult(X0,mult(X1,mult(X0,X1))) [superposition 12,13]
17. mult(e,X5) = mult(inverse(X4),mult(X4,X5)) [superposition 12,11]
15. mult(e,X1) = mult(X0,mult(X0,X1)) [superposition 12,13]
14. mult(sK0,sK1) != mult(sK1,sK0) [cnf transformation 9]
13. e = mult(X0,X0) [cnf transformation 4]
12. mult(X0,mult(X1,X2)) = mult(mult(X0,X1),X2) [cnf transformation 3]
11. e = mult(inverse(X0),X0) [cnf transformation 2]
10. mult(e,X0) = X0 [cnf transformation 1]
9. mult(sK0,sK1) != mult(sK1,sK0) [skolemisation 7,8]
8. ?[X0,X1]: mult(X0,X1) != mult(X1,X0) <=> mult(sK0,sK1) != mult(sK1,sK0)
                                                              [choice axiom]
7. ?[X0,X1]: mult(X0,X1) != mult(X1,X0) [ennf transformation 6]
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Vampire

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Vampire

- Completely automatic: once you started a proof attempt, it can only be interrupted by terminating the process.
- Champion of the CASC world-cup in first-order theorem proving: won CASC $>$ 50 times.

What an Automatic Theorem Prover is Expected to Do

Input:

- a set of axioms (first order formulas) or clauses;
- a conjecture (first-order formula or set of clauses).

Output:

- proof (hopefully).

Proof by Refutation

Given a problem with axioms and assumptions $F_1, \ldots, F_n$ and conjecture G,

1. negate the conjecture;
2. establish unsatisfiability of the set of formulas $F_1, \ldots, F_n, \neg G$.

$$\overbrace{F_1 \wedge \ldots \wedge F_n}^{\text{axioms}} \Rightarrow \overset{\text{conjecture}}{G}$$

$$\Downarrow$$

$$\underbrace{F_1 \wedge \ldots \wedge F_n \wedge \neg G}_{\text{axioms}} \Rightarrow \bot$$

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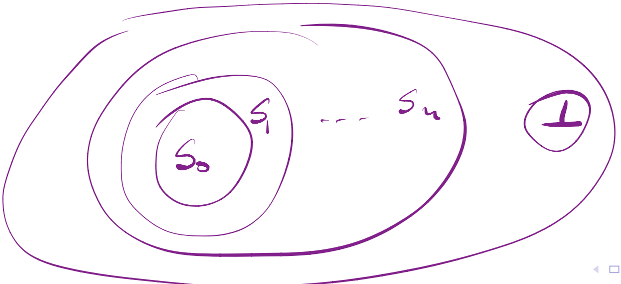
In this formulation the negation of the conjecture $\neg G$ is treated like any other formula. In fact, Vampire (and other provers) internally treat conjectures differently, to make proof search more goal-oriented.

General Scheme (simplified)

preprocessing

- Read a problem;
- Determine proof-search options to be used for this problem;
- Preprocess the problem;
- Convert it into CNF;
- Run a saturation algorithm on it, try to derive *false*. proving
- If *false* is derived, report the result, maybe including a refutation.



$S_0 \Rightarrow S_1 \Rightarrow S_2 \Rightarrow \dots \Rightarrow S_{n+1}$