

## Verification of Kirchhoff's Voltage Law, Mesh Analysis and Equivalent Resistance

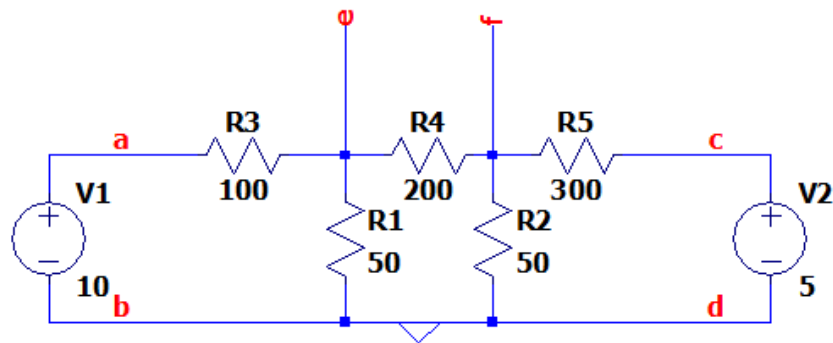
### Objectives:

1. To verify KVL and find the mesh currents and nodal voltages.
2. To verify the equivalent resistance of the circuit from different terminals

### Simulation Tool:

LTSpice – dc operating point analysis and transient analysis.

### Circuit:



### Observation:

$$I_1 = 0.06983A$$

$$I_2 = -0.00948A$$

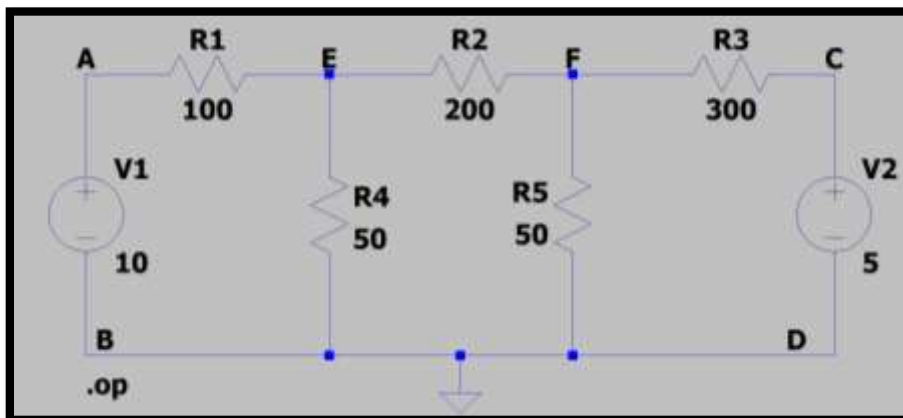
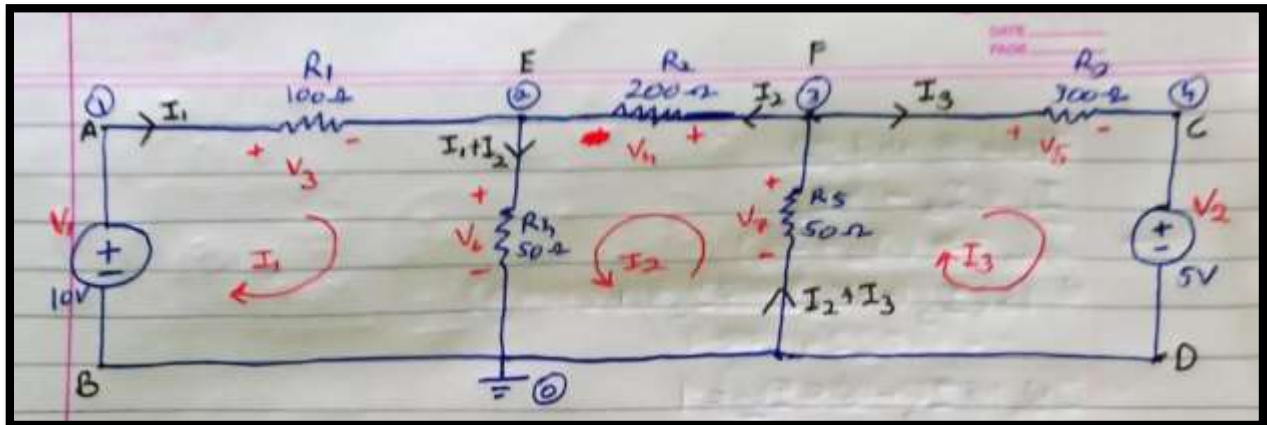
$$I_3 = -0.01293A$$

S. No	Element/Branch	Current (A)	Voltage (V)	Power (W) Dissipated
1	R1	0.06983A	6.9827 V	0.488W
2	R2	-0.00948A	-1.8965 V	0.018W
3	R3	-0.01293A	-3.8790 V	0.050W
4	R4	0.06034A	3.0172 V	0.182W
5	R5	-0.02241A	1.1205 V	0.025W
6	V1	-0.06983A	10 V	-0.698W
7	V2	-0.01293A	5 V	-0.064W

## To Do:

1. Find the mesh currents  **$I_1$** ,  **$I_2$**  and  **$I_3$**  using the above measurements
2. Verify KVL in the independent loops
3. Compute the equivalent resistance of the circuit from terminals **a-b**, **c-d**, **e-f** and understand they are different.
4. Insert the picture of all the theoretical calculation done in your notebook

### 1. KVL and Mesh Analysis:



```
--- Operating Point ---
V(a):      10          voltage
V(e):      3.01724     voltage
V(f):      1.12069     voltage
V(c):      5          voltage
I(R5):     -0.0224138   device_current
I(R4):      0.0603448   device_current
I(R3):     -0.012931   device_current
I(R2):     -0.00948276 device_current
I(R1):      0.0698276   device_current
I(V2):     -0.012931   device_current
I(V1):     -0.0698276   device_current
```

- Calculations

KVL in loop ①:  $\sum V_{\text{Active}} = \sum V_{\text{passive}}$

$$\begin{aligned} V_1 &= V_3 + V_6 \\ 10 &= 100I_1 + 50(I_1 + I_2) \end{aligned}$$

$$\begin{aligned} \Rightarrow 150I_1 + 50I_2 &= 10 \\ \Rightarrow \boxed{15I_1 + 5I_2 = 1} &\text{--- (1)} \end{aligned}$$

$$V_3 = 100I_1$$

$$V_6 = 50(I_1 + I_2)$$

KVL in loop ②:  $\sum V_{\text{Active}} = \sum V_{\text{passive}}$

$$\begin{aligned} V_7 &= V_4 + V_6 \\ -50(I_2 + I_3) &= 200I_2 + 50(I_1 + I_2) \end{aligned}$$

$$\begin{aligned} 300I_2 + 50I_1 + 50I_3 &= 0 \\ \boxed{I_1 + 6I_2 + I_3 = 0} &\text{--- (2)} \end{aligned}$$

$$V_7 = -50(I_2 + I_3)$$

$$V_4 = 200I_2$$

KVL in loop ③:  $\sum V_{\text{Active}} = \sum V_{\text{passive}}$

$$\begin{aligned} V_7 &= V_5 + V_2 \\ -50(I_2 + I_3) &= 300I_3 + 5 \end{aligned}$$

$$\begin{aligned} 50I_2 + 350I_3 &= -5 \\ \Rightarrow \boxed{10I_2 + 70I_3 = -1} &\text{--- (3)} \end{aligned}$$

$$V_5 = 300I_3$$

From ②

$$I_1 = -I_3 - 6I_2$$

Substituting  $I_1$  in ①:

$$-15I_3 - 90I_2 + 5I_2 = 1$$

$$\Rightarrow 85I_2 + 15I_3 = -1 \quad \text{--- (5)}$$

From ③ and ④

$$850I_2 + 150I_3 = -10$$

$$\Rightarrow \frac{850I_2 + 150I_3}{50} = \frac{-10}{50}$$

$$17I_2 + 3I_3 = -2$$

$$\Rightarrow I_3 = \frac{-75}{5800} = -0.01293 \text{ A}$$

$$\boxed{I_3 = -12.93 \text{ mA}}$$

Now from ③,

$$70\left(\frac{-75}{5800}\right) + 10I_2 = -1$$

$$\Rightarrow 10I_2 = -1 + \frac{525}{580}$$

$$\Rightarrow I_2 = \frac{-55}{5800} = -0.0094827 \text{ A}$$

$$\boxed{I_2 = -9.483 \text{ mA}}$$

Now from ①

$$I_1 = -I_3 - 6I_2$$

$$= \frac{75}{5800} + \frac{55 \times 6}{5800}$$

$$I_1 = \frac{405}{5800} = 0.069827 \text{ A}$$

$$\boxed{I_1 = 69.8 \text{ mA}}$$



$$\rightarrow V_1 = 10V = V_a$$

$$\rightarrow V_2 = 5V = V_c$$

$$\rightarrow V_3 = 100I_1 \\ = 100(0.069827)$$

$$\boxed{V_3 = 6.98V}$$

$$\rightarrow V_4 = 200I_2 \\ = 200(-0.0094827)$$

$$\boxed{V_4 = -1.896V}$$

$$\rightarrow V_5 = 300I_3 \\ = 300(-0.01293)$$

$$\boxed{V_5 = -3.879V}$$

$$\rightarrow V_6 = 50(I_1 + I_2) \\ = 50(0.069827 - 0.0094827) \\ = 50(0.0603443)$$

$$\boxed{V_6 = 3.0172V} = V_e$$

$$\rightarrow V_7 = -50(I_2 + I_3) \\ = -50(-0.0094827 - 0.01293)$$

$$\boxed{V_7 = 1.12V} = V_f$$

$$\rightarrow I_{R_1} = I_1 = 0.0698A$$

$$\rightarrow I_{R_2} = I_2 = -0.009483A$$

$$\rightarrow I_{R_3} = I_3 = -0.01293A$$

$$\rightarrow I_{R_4} = I_1 + I_2 = 0.069827 - 0.0094827$$

$$\boxed{I_{R_4} = 0.06034A}$$

$$\rightarrow I_{R_5} = (I_2 + I_3) \\ = -(0.0094827 + 0.01293) \\ = -0.0224127 \text{ A}$$

$$\Rightarrow I_{R_5} = -0.02241 \text{ A}$$

$$\rightarrow I_{V_1} = -I_1 = -0.06983 \text{ A}$$

$$\rightarrow I_{V_2} = I_3 = -0.01293 \text{ A}$$

~~to find~~

Power dissipated:

$$P_{R_1} = I_1 \times V_3 \\ = 0.487589 \text{ W}$$

$$\Rightarrow P_{R_1} = 0.488 \text{ W}$$

$$P_{R_5} = -I_{R_5} V_7 \\ = 0.0251189 \text{ W}$$

$$\Rightarrow P_{R_5} = 0.025 \text{ W}$$

$$P_{R_2} = I_2 \times V_4 \\ = 0.017484 \text{ W}$$

$$\Rightarrow P_{R_2} = 0.018 \text{ W}$$

$$P_{V_1} = -I_1 V_1 \\ = -0.698276 \text{ W}$$

$$\Rightarrow P_{V_1} = -0.698 \text{ W}$$

$$P_{R_3} = I_3 \times V_5 \\ = 0.050163 \text{ W}$$

$$\Rightarrow P_{R_3} = 0.05 \text{ W}$$

$$P_{V_2} = I_3 V_2 \\ = -0.064655 \text{ W}$$

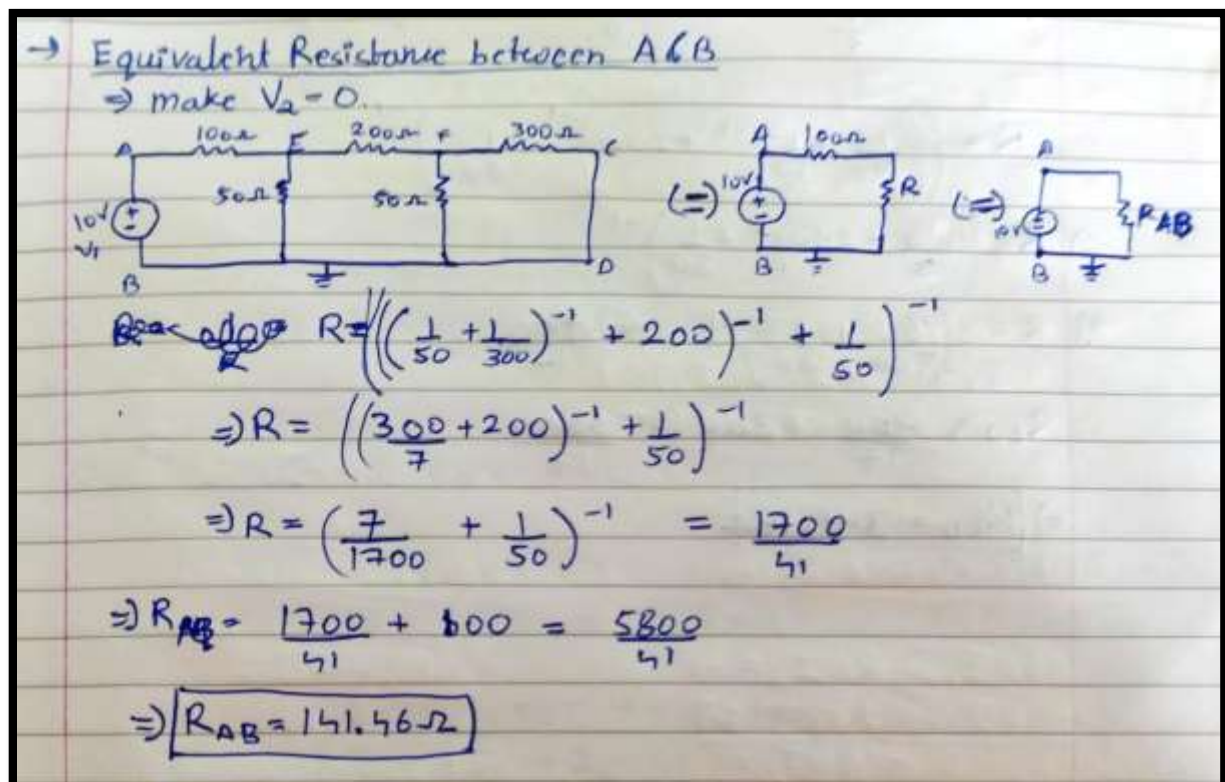
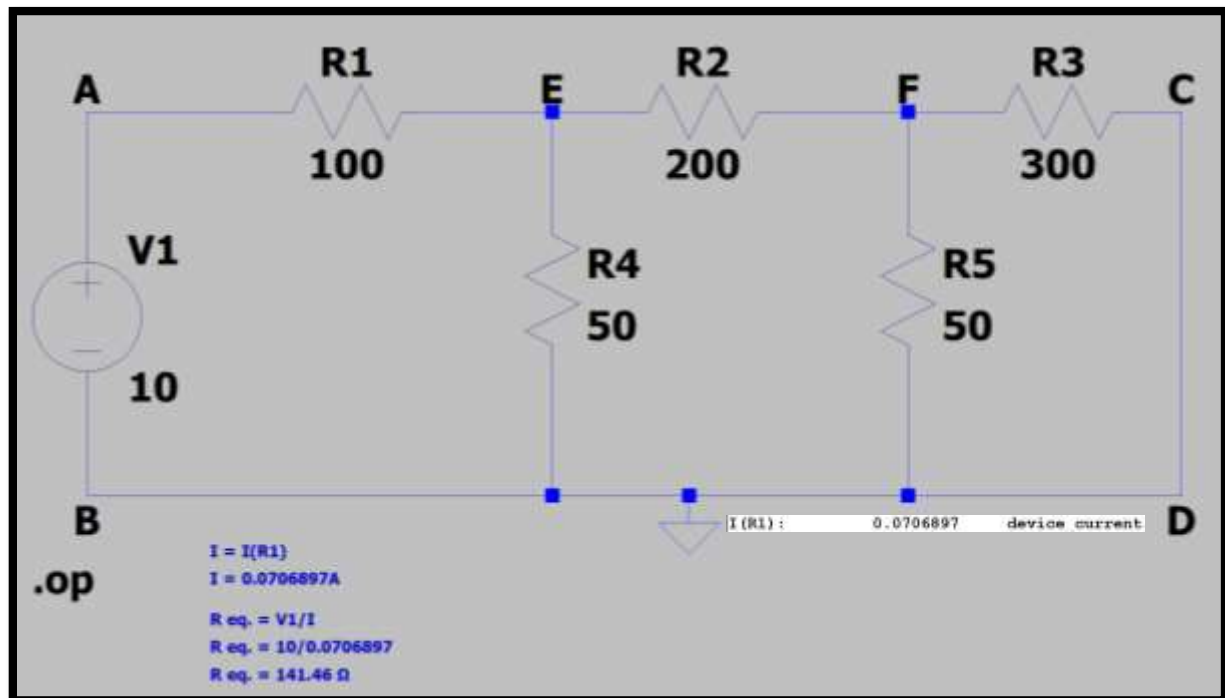
$$\Rightarrow P_{V_2} = -0.064 \text{ W}$$

$$P_{R_4} = I_{R_4} V_6 \\ = 0.182074 \text{ W}$$

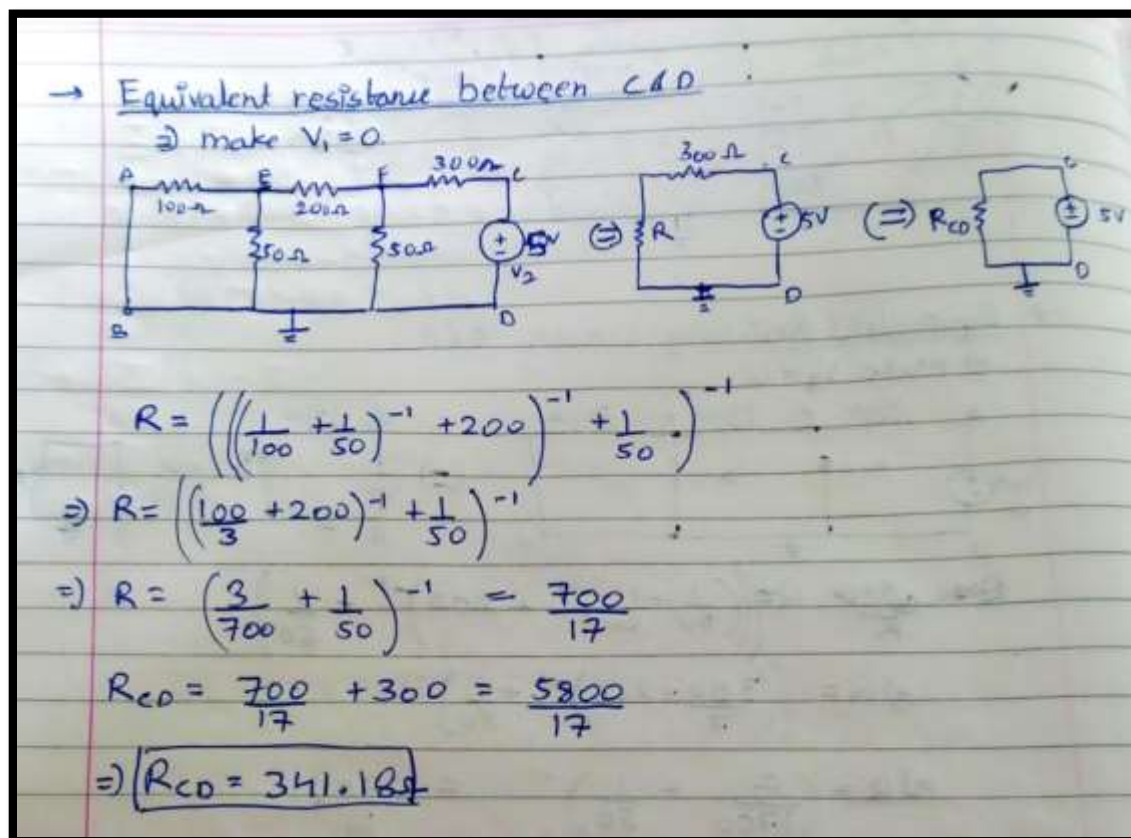
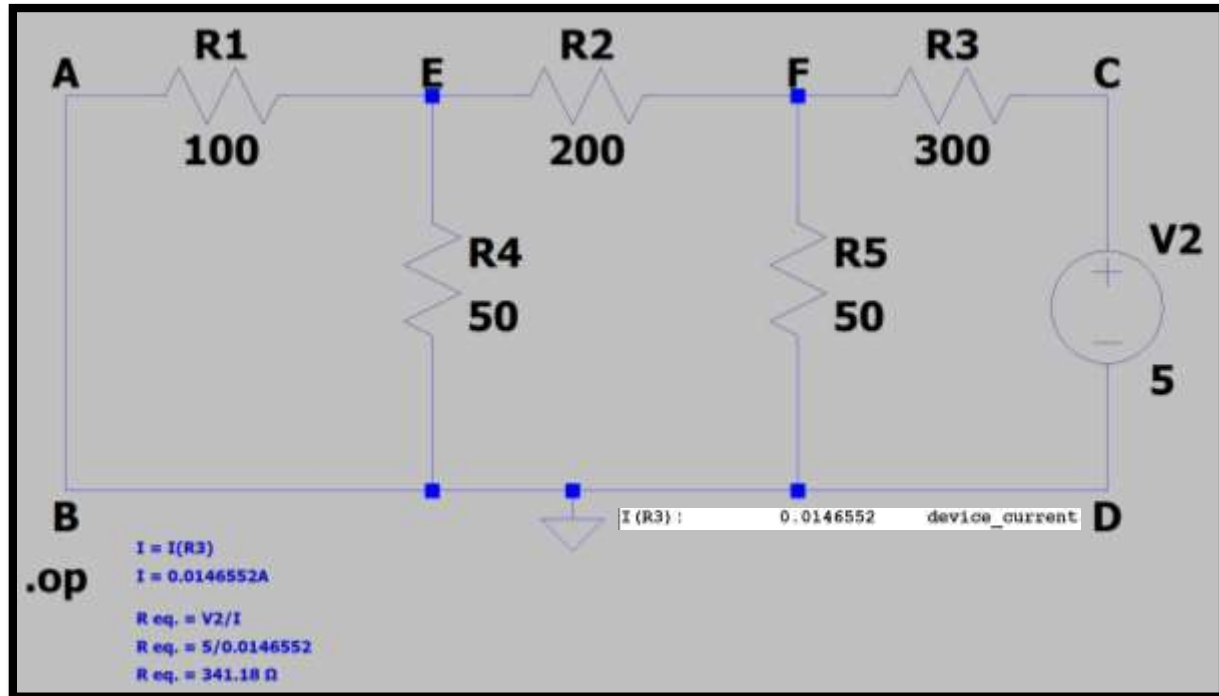
$$\Rightarrow P_{R_4} = 0.182 \text{ W}$$

## 2. Compute Equivalent Resistance between different nodes

a-b

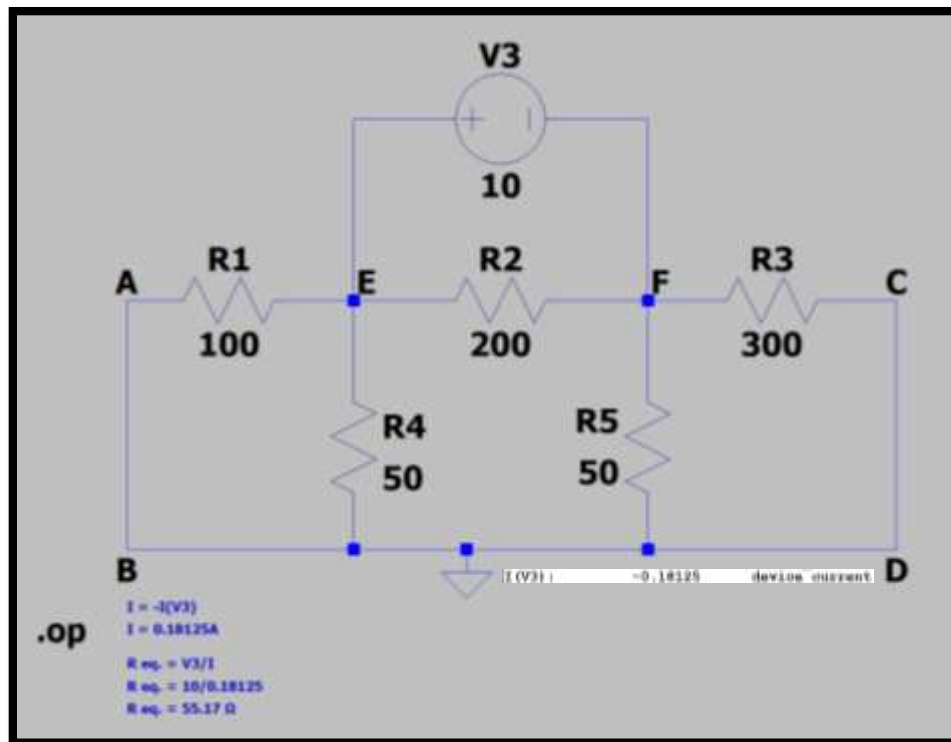


c-d





e-f



→ Equivalent resistance between E & F

⇒ make  $V_1=0$ ,  $V_2=0$  and put independent voltage source  $V_3$  between E & F

$$R = \left( \left( \frac{1}{100} + \frac{1}{50} \right)^{-1} + \left( \frac{1}{300} + \frac{1}{50} \right)^{-1} \right)^{-1}$$

$$\Rightarrow R = \frac{100}{3} + \frac{300}{7} = \frac{1600}{21}$$

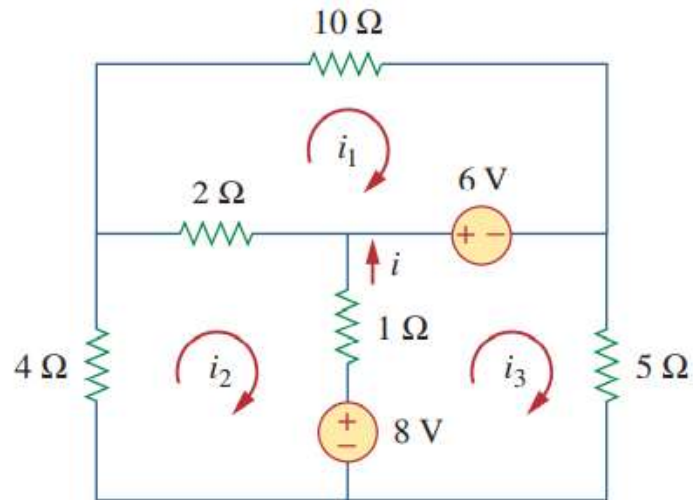
$$\Rightarrow R_{EF} = \frac{21}{1600} + \frac{1}{200}$$

$$\Rightarrow R_{EF} = \frac{1600}{29}$$

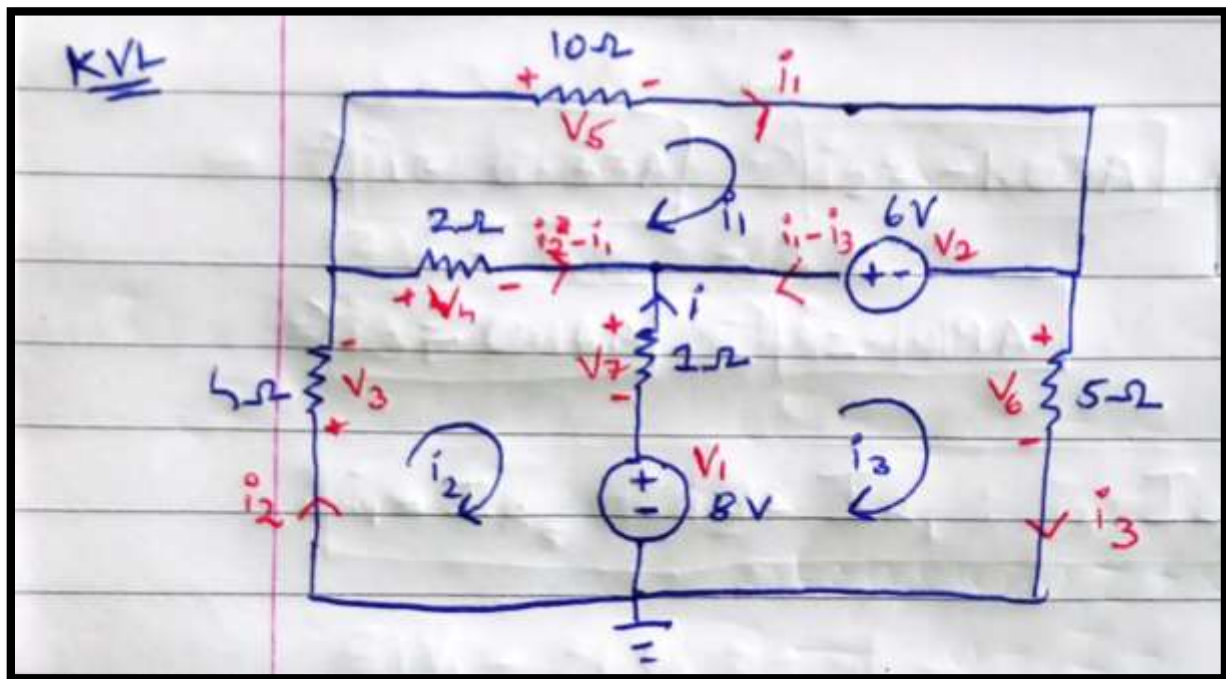
$$\Rightarrow \boxed{R_{EF} = 55.17 \Omega}$$

**Do it yourself:**

For the circuit given below, Verify KVL, KCL, Nodal Analysis and Mesh Analysis using LTSpice and TinkerCAD



**1) KVL – Mesh Analysis**



KVL in loop 1:  $V_{\text{active}} = V_{\text{passive}}$

$$V_4 + V_2 = V_5$$

$$2(i_2 - i_1) + 6 = 10i_1$$

$$\Rightarrow 12i_1 - 2i_2 = 6$$

$$\Rightarrow \boxed{6i_1 - i_2 = 3} \quad \text{--- (1)}$$

$$V_4 = 2(i_2 - i_1)$$

$$V_5 = 10(i_1)$$

KVL in loop 2:  $V_{\text{active}} = V_{\text{passive}}$

$$0 = V_3 + V_4 + V_7 + V_1$$

$$0 = 4i_2 + 2(i_2 - i_1) - (i_3 - i_2) + 8$$

$$\Rightarrow \boxed{2i_1 - 7i_2 + i_3 = 8} \quad \text{--- (2)}$$

$$V_7 = -i(i_1)$$

$$i = (i_3 - i_2)$$

KVL in loop 3:  $V_{\text{active}} = V_{\text{passive}}$

$$V_1 + V_7 = V_2 + V_6$$

$$8 - 6(i_3 - i_2) = 6 + 5i_3$$

$$\Rightarrow \boxed{6i_3 - 5i_2 = 2} \quad \text{--- (3)}$$

$$V_6 = 5(i_3)$$

→ From (2)

$$i_3 = 8 - 2i_1 + 7i_2$$

→ Substituting  $i_3$  in (3)

$$\boxed{-12i_1 + 41i_2 = -46} \quad \text{--- (4)}$$

$$\Rightarrow \boxed{i_1 = 0.33A} \quad \Rightarrow \boxed{i_2 = -1.03A}$$

$$\Rightarrow \boxed{i_3 = 0.16A} \quad \Rightarrow \boxed{i = 1.19A}$$

$$\boxed{i_1 = I_{R_5} ; i_2 = I_{R_1} ; i_3 = I_{R_3}}$$

From (1) and (5)

$$246i_1 - 41i_2 = 123$$

$$-12i_1 + 41i_2 = -46$$

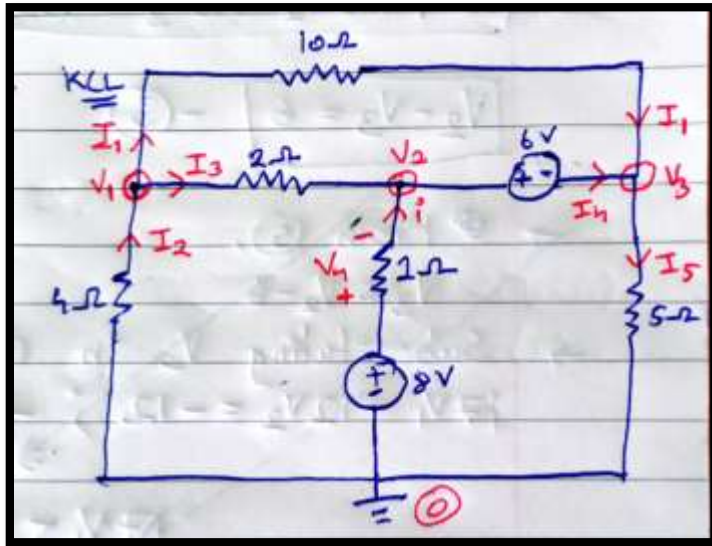
$$\hline 234i_1 = 77$$

$$\Rightarrow i_1 = \frac{77}{234}$$

$$\Rightarrow \boxed{i_1 = 0.32906A}$$



## 2) KCL – Nodal Analysis



KCL at node ①:  $\sum I_{\text{entering}} = \sum I_{\text{leaving}}$

$$I_2 = I_1 + I_3$$

$$\frac{-V_1}{4} = \frac{V_1 - V_2}{10} + \frac{V_1 - V_3}{2}$$

$$\Rightarrow -5V_1 = 2V_1 - 2V_2 + 10V_1 - 10V_2$$

$$\Rightarrow \boxed{17V_1 - 10V_2 - 2V_3 = 0} \quad \text{--- ①}$$

KCL at node ②:  $\sum I_{\text{entering}} = \sum I_{\text{leaving}}$

$$I_3 + i = I_4 \quad \text{--- ②}$$

KCL at node ③:  $\sum I_{\text{entering}} = \sum I_{\text{leaving}}$

$$I_4 + I_1 = I_5$$

$$\Rightarrow I_4 = I_5 - I_1 \quad \text{--- ③}$$

From ② and ③

$$I_3 + i = I_5 - I_1$$

$$\frac{V_1 - V_2}{2} + \frac{V_1}{1} = \frac{V_2}{5} - \left( \frac{V_1 - V_3}{10} \right)$$

$$\Rightarrow \frac{V_1 - V_2}{2} + \frac{8 - V_2}{1} = \frac{V_2}{5} - \left( \frac{V_1 - V_3}{10} \right)$$

$$V_4 = 8 - V_2$$



$$\Rightarrow 5V_1 - 5V_2 + 80 = 10V_2 = 2V_3 - V_1 + V_3$$

$$\Rightarrow \boxed{-6V_1 + 15V_2 + 3V_3 = 80} \quad - (5)$$

$$\boxed{V_2 - V_3 = 6} \quad - (5)$$

From (5),  $V_3 = V_2 = 6$

$$V_3 = V_2 = 6$$

→ Substituting  $V_3$  in (1) and (4)

$$17V_1 - 12V_2 = -12 \quad - (6)$$

$$-3V_1 + 9V_2 = 49 \quad - (7)$$

From (6) and (7),

$$51V_1 - 36V_2 = -36$$

$$-51V_1 + 153V_2 = 797$$

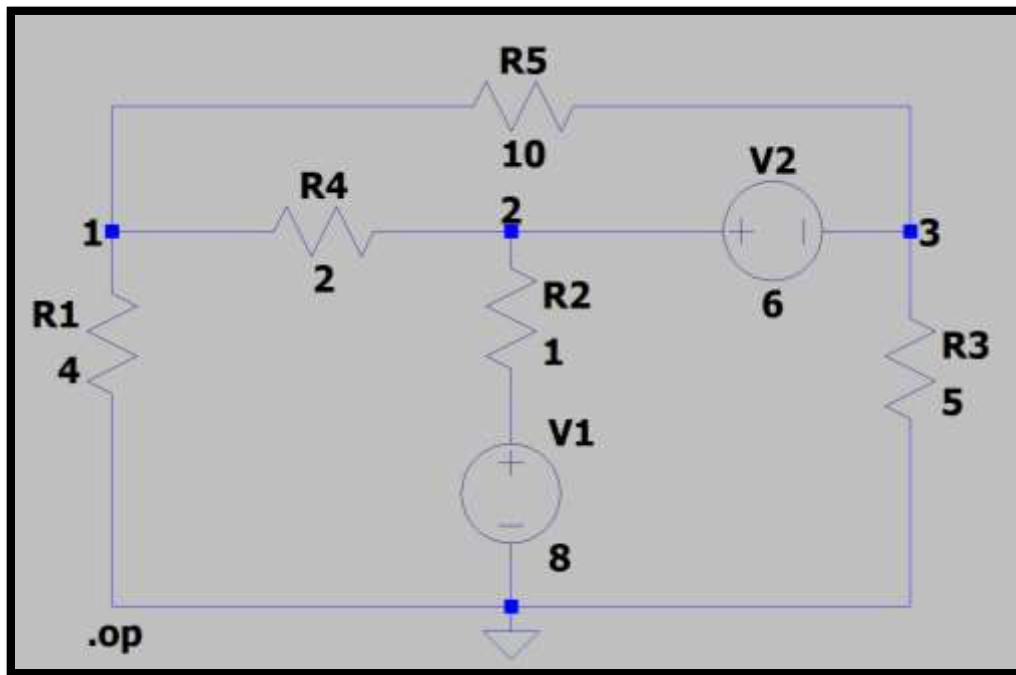
$$117V_2 = 797$$

$$\Rightarrow V_2 = \frac{797}{117}$$

$$\Rightarrow \boxed{V_2 = 6.81V} \quad \Rightarrow \boxed{V_3 = 0.81V}$$

$$\Rightarrow \boxed{V_1 = 4.10V} \quad \Rightarrow \boxed{i = 8 - V_2 = 1.19A}$$

### 3) LTSpice



#### --- Operating Point ---

V(1) :	4.10256	voltage
V(2) :	6.81197	voltage
V(4) :	8	voltage
V(3) :	0.811966	voltage
I(R5) :	0.32906	device_current
I(R4) :	1.3547	device_current
I(R3) :	0.162393	device_current
I(R2) :	-1.18803	device_current
I(R1) :	-1.02564	device_current
I(V2) :	-0.166667	device_current
I(V1) :	-1.18803	device_current

#### 4) TinkerCAD

