

BPHY101L DA1(a)

WAVE PACKETS

- A wave packet refers to the case where two (or more) waves exist simultaneously.
- It is also referred to as a "wave group" or "envelope" of localised wave action that travels as a unit.
- Principle of superposition: If any two waves are a solution to the wave equation, then the sum of the waves is also a solution. This principle holds true only for linear systems.

• Coherent and Incoherent Sources:

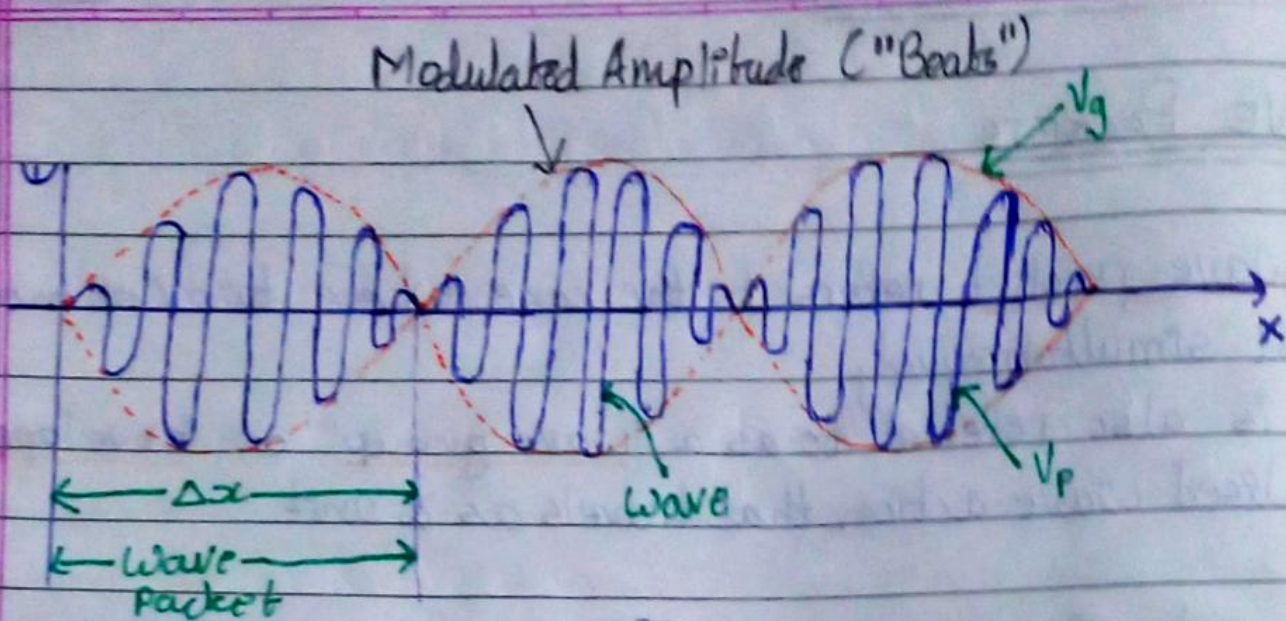
- Wave packets can be formed by the superposition of two (or more) different waves of slightly different frequencies and wavelength which means waves having phase differences.

Coherent Source: emit waves having the same frequency, wavelength and have the same phase or a constant phase difference.

Incoherent Source: emit waves that have random frequencies and phase differences.

- Therefore, wave packets cannot be formed in coherent sources of light.

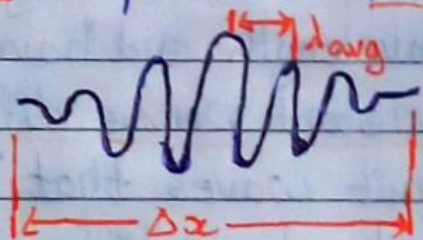
Sunlight	LED (Light Emitting Diode)	Laser Diode
→ Incoherent	→ Incoherent	→ Coherent
→ Wave packets exist	→ Wave packets exist	→ Wave packets do not exist
→ Polychromatic	→ Monochromatic	→ Monochromatic



• Localisation of wave:

→ According to de Broglie's uncertainty principle:

- A sine wave of wavelength λ implies that the momentum is precisely known but the position of particle in the wave is uncertain.
- As the number of waves increases, the wave packet becomes more localised in space but momentum is uncertain.



PHASE VELOCITY (V_p) AND GROUP VELOCITY (V_g)

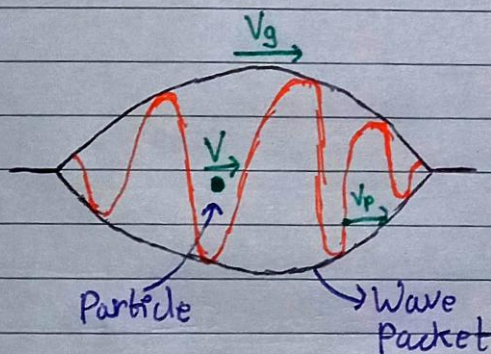
• Phase Velocity (V_p):

- The phase velocity of a wave is the rate at which the wave propagates in any medium.
- This is the velocity at which the phase of any one frequency component of the wave travels.
- $V_p = \frac{\omega}{k}$, $V_p = \nu\lambda$, $V_p = \frac{c^2}{v}$

• Group Velocity (V_g):

- The group velocity of a wave is the velocity with which the overall wave packet (envelope) propagates through space.
- Each envelope contains a group of internal waves.
- $V_g = \frac{\Delta\omega}{\Delta k}$

- In a given medium, the frequency is some function, $\omega(k)$, of the wave number. Therefore, phase velocity and group velocity depend on the frequency and the medium.



• Relation between Phase Velocity and Group Velocity:

We know that, $V_p = \frac{\omega}{k} \Rightarrow \omega = k V_p$

Therefore, $V_g = \frac{\Delta \omega}{\Delta k}$; $V_g = \frac{d}{dk} (k V_p)$

$$\Rightarrow V_g = k \frac{dV_p}{dk} + V_p$$

$$\Rightarrow V_g = V_p + \left(\frac{2\pi}{\lambda} \right) \frac{dV_p}{d\left(\frac{2\pi}{\lambda}\right)} \quad \left(\because k = \frac{2\pi}{\lambda} = \frac{\text{Time period}}{\text{wavelength}} \right)$$

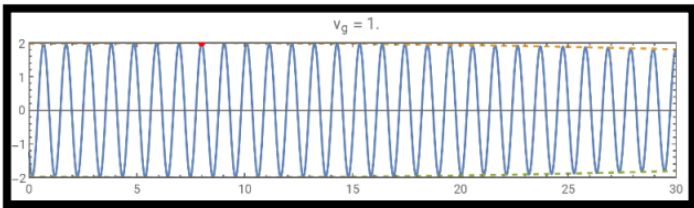
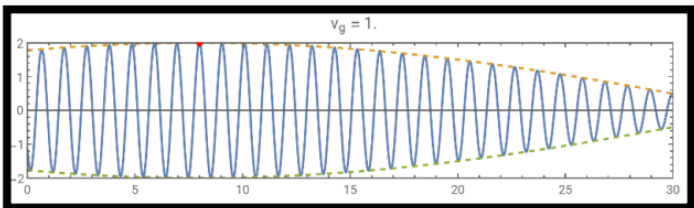
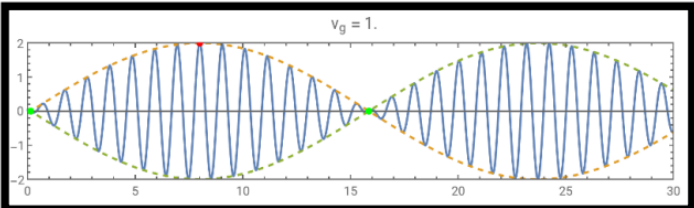
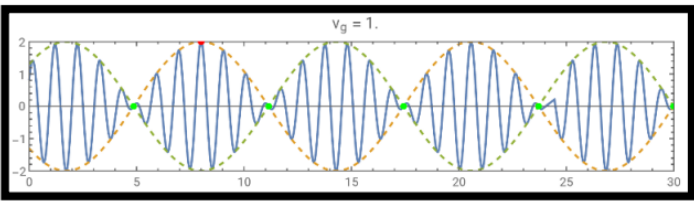
$$\Rightarrow V_g = V_p + \left(\frac{2\pi}{\lambda} \right) \frac{dV_p}{\left(-\frac{2\pi}{\lambda^2} \right) d\lambda}$$

$$\Rightarrow V_g = V_p - \lambda \frac{dV_p}{d\lambda}$$

→ Therefore, if the phase velocity does not depend on the wavelength of the propagating wave, then $V_g = V_p$.
eg:- Non-dispersive media

- Wave Patterns:

→ The wave patterns for various values of $\Delta\omega$ and Δk will not be same even if V_g is same for the different waves. The resultant wave formed by the superposition of two waves is dependent on the values of $\Delta\omega$ and Δk independently.

S. No	$\Delta\omega$	Δk	Wave pattern of the resultant waves	V_g
1	0.02	0.02		1
2	0.06	0.06		1
3	0.2	0.2		1
4	0.5	0.5		1