Error Analysis in Physics Laboratory

1.1 Introduction to Uncertainty

In introductory lab work, such as in Physics labs, you usually know in advance what the result is supposed to be. You can compare your actual result with the anticipated result, and calculate an actual error value. In real-world laboratory work, on the other hand, you usually *don't* know in advance what the result is supposed to be. If you *did*, you probably wouldn't be doing the experiment in the first place! When you state your final result, it's important to state also, how much you think you can trust that result, in the form of a numerical **uncertainty (or error in measurement)**. For example, you might state the volume of an object as

$$V = 43.25 \pm 0.13\,\mathrm{cm}^3 \tag{1.1}$$

When we state the uncertainty in this form, without further elaboration, it generally means that we think that the true value has about a 68% chance of being within that range. A more precise statement would include the confidence level of the uncertainty range, which might be 68% or 95% or even 99%. Usually, in an experiment we measure some number of quantities directly, and combine them mathematically to get a final result. Therefore, estimating the final uncertainty usually involves two steps. First, we must estimate the uncertainties in the individual quantities that we measure directly. Second, we must combine those uncertainties to get the overall uncertainty, in a way that corresponds to the way that we combine individual measurements to get the final result.

1.2 Estimating the Uncertainty in a Single Measurement

1.2.1 Normal analog scale

(e.g. meter stick) Estimate the final digit by interpolating between the smallest scale divisions, and make the uncertainty ±1 or ±2 in that last digit (use your judgment in deciding).

1.2.2 Analog scale with vernier

(e.g. vernier caliper or micrometer) Use the vernier scale to get the last digit, and make the uncertainty ±0.5 of that last digit.

1.2.3 Digital scale

(e.g. digital multimeter) If the reading is steady, make the uncertainty ±0.5 of the last digit; otherwise take several instantaneous readings, average them, and find the standard deviation of the mean as described below.

1.3 Estimating the Uncertainty in an Averaged Measurement

If you can make several measurements $x_1, x_2, x_3 \ldots\ldots\ldots x_N$ calculate the mean, $\bar{x}$, and use that as "the" measurement. Then calculate the standard deviation of the mean:

$$\sigma_m = \frac{\sqrt{(x_1-\bar{x})^2+(x_2-\bar{x})^2+\ldots\ldots\ldots\ldots+(x_N-\bar{x})^2}}{N} \quad (1.2)$$

and use this as the uncertainty, Δx. If your calculator has a standard deviation function, divide its result by $\sqrt{N}$ to get the standard deviation of the mean.

1.4 Combining Uncertainties in Calculated Results

In the following equations, Δx means the absolute uncertainty in x, which is the number you get from one of the methods above; it has units just like the measurement itself has. $\Delta x\%$ means the percent (or fractional) uncertainty in *x*, which is the uncertainty expressed as a percentage or fraction of the measurement; it has no units.

1.4.1 Addition and Subtraction

If $z = x + y$ or $z = x - y$

$$\Delta z = \sqrt{\Delta x^2 + \Delta y^2} \quad (1.3)$$

If you're adding and subtracting more variables, simply add more terms inside the square root.

1.4.2 Multiplication and Division

If $z = xy$ or $z = x/y$, then

$$\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2} \quad (1.4)$$

or (same thing in different notation).

$$\Delta z\% = \sqrt{(\Delta x\%)^2 + (\Delta y\%)^2} \quad (1.5)$$

If you're multiplying and dividing more variables, simply add more terms inside the square root.

1.4.3 Powers, Including Roots

If $z = x^n$, then

$$\frac{\Delta z}{z} = n\left(\frac{\Delta x}{x}\right) \quad (1.6)$$

1.4.4 More Complicated Calculations

Sometimes you can combine the three rules given above, doing the calculation one step at a time, combining uncertainties as you go along, and switching back and forth between absolute and percent uncertainties as necessary. However, you cannot do this if the same variable appears more than once in the equation or calculation, or if you have situations not covered by the rules given above, such as trig functions. In such cases you must use the general procedure given below.
The following table gives an idea about the relation between error and actual equation:

Table 1.1

Sr. No.	Relation between Z and (A, B)	Relation between Δz and (ΔA and ΔB)
1.	$Z = A + B$	$(\Delta z)^2 = (\Delta A)^2 + (\Delta B)^2$
2.	$Z = A - B$	$(\Delta z)^2 = (\Delta A)^2 + (\Delta B)^2$
3.	$Z = AB$	$\left(\frac{\Delta z}{z}\right)^2 = \left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2$
4.	$Z = A/B$	$\left(\frac{\Delta z}{z}\right)^2 = \left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2$
5.	$Z = An$	$\frac{\Delta z}{z} = n\left(\frac{\Delta A}{A}\right)$
6.	$Z = \ln A$	$\Delta Z = \frac{\Delta A}{A}$
7.	$Z = eA$	$\frac{\Delta Z}{Z} = \Delta A$

1.5 General Procedure

If $z = f(x, y)$ first calculate the differences caused by the uncertainty in each variable separately:

$$(\Delta z)_x = f((x + \Delta x), y) - f(x, y)$$
$$(\Delta z)_y = f(x, (y + \Delta y)) - f(x, y) \tag{1.7}$$

Then combine the differences to get the total uncertainty:

$$(\Delta z) = \sqrt{(\Delta z)_x^2 + (\Delta z)_y^2} \tag{1.8}$$

If there are more variables, extend these equations appropriately by adding more terms. If a variable occurs more than once in the formula for *f*(*x*,*y*), change all occurrences simultaneously when calculating the difference for that variable.

To illustrate the procedure for calculation the error in an experiment, we will work out the average (mean) value *x* and the standard deviation of the mean, $\overline{\sigma}$ and the standard deviation of an individual data point, σ, using the position measurements in the accompanying Table 1.2

Table 1.2

x_i **(m)**	$(x_i - \bar{x}_i)$ **(m)**	$(x_i - \bar{x}_i)^2$ **(m²)**
15.68	0.15	0.0225
15.42	0.11	0.0121
15.03	0.50	0.2500
15.66	0.13	0.0169
15.17	0.36	0.1296
15.89	0.36	0.1296
15.35	0.18	0.0324
15.81	0.28	0.0784
15.62	0.09	0.0081
15.39	0.14	0.0196
15.21	0.32	0.1024
15.78	0.25	0.0625

15.46	0.07	0.0049
15.12	0.41	0.1681
15.93	0.40	0.1600
15.23	0.30	0.0900
15.62	0.09	0.0081
15.88	0.35	0.1225
15.95	0.42	0.1764
15.37	0.16	0.0256
15.51	0.02	0.0004

From the above table we can make the following calculations:

$$N = 21 \sum_{i=1}^{N} x_i = 326:08m, \sum_{i}^{N} (x_i - \bar{x}_i)^2 = 1:61998m^2$$

and then evaluate the following quantities:

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{326.08}{21} = 15.53m \tag{1.9}$$

$$\bar{\sigma} = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N(N-1)}} = \sqrt{\frac{1.6201}{20.21}} = 0.062m \tag{1.10}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{(N-1)}} = \sqrt{\frac{1.6201}{20.0}} = 0.063m \tag{1.11}$$

The error or spread in individual measurements is σg 0:28 m. But for the mean $\bar{x} + \bar{\sigma} = 15.53 \pm 0.06$m. This says the average is 15.53 m which has an error of 0.06m. Or putting it another way, there is about a 68% probability that the true value of *x* falls in the range 15.47m to 15.59m. In some cases the fractional error $(\sigma/\bar{x})$, or relative error, is of more interest than the absolute value of σ. It is possible that the size of σ is large while the fractional error is small. Note that increasing the number of individual measurements on the uncertainty of the average reduces the statistical uncertainty (random errors); this improves the "precision". On the other hand, more measurements do not diminish systematic error in the mean because these are always in the same direction; the "accuracy" of the experiment is limited by systematic errors.

1.6 Suggested Experiments:

1) Measure the diameter of a wire using a screw gauge at 10 different places on the wire. Calculate the standard deviation in your measurements.
2) Measure the thickness of a tabletop at using a scale in cm. Calculate the error in your measurements.
3) Measure the period of oscillations of a pendulum using your wrist watch and record your data ten times. Estimate the standard deviation and error in your measurements.
4) Ask your partner to drop a solid object at a same height for 10 times. Measure the time of flight with your wrist watch. The same can be repeated by your other partners also. Compare the standard deviation of each of your measurements.

Assessment of purity of a given liquid by determining its refractive index

Aim

To determine the purity of a liquid in terms of refractive index using travelling microscope

Apparatus required

Travelling microscope, reading lens, 50 ml beaker, water, saw dust, etc.,

Formula

Refractive Index of the liquid $\mu = \dfrac{(C-A)}{(C-B)}$ (no unit)

where

A- Reading of the microscope when the ink dot is focused directly (cm)
B- Reading of the microscope when the ink dot is focused through water (cm)
C- Reading of the microscope when the saw dust is focused (cm)

Procedure

1. Focus ink dot marked at the bottom of the beaker directly and take the readings (MSR and VSC) in the vertical scale (A).
2. Pour ~ 20 ml of water into beaker without disturbing it.
3. Now, focus the ink dot through water by adjusting the vertical adjustment screw (Do not use the focusing knob) and take the reading (MSR and VSC) in the vertical scale. (B).
4. Sprinkle some saw dust on the surface of water gently without disturbing the set-up.
5. Focus the saw dust by adjusting the vertical adjustment screw (Do not use the focusing knob) and take the readings (MSR and VSC) in the vertical scale (C).
6. Pour out the water.
7. Repeat the experiment (step 1 and step 5) for different quantities/levels (~ 40 ml, 60 ml, 80 ml) of water and tabulate the readings.
8. Calculate the refractive index μ using the formula.

Tabulation

S. No	Reading when the ink dot is directly focused (A)			Reading when the ink dot is directly focused through water (B)			Reading when the saw dust is focused (C)			(C- A)	(C - B)	μ
	MSR	VSC	TR	MSR	VSC	TR	MSR	VSC	TR			
	cm	div	cm	cm	div	cm	cm	div	cm			
										Mean μ =		

TR= MSR + (VSC x LC)

Calculations

$$\mu = \frac{(C-A)}{(C-B)} \text{ (no unit)}$$

μ = _____________

Result

The purity of the given liquid evaluated in terms of Refractive Index is determined to be μ = _____________

Viva voce questions:

1. Why should we focus the ink dot in the beaker in this experiment?
2. Why should we use sawdust in this experiment?
2. Differentiate between actual depth and apparent depth.
3. How do you calculate the percentage of error?
4. What is the significance of this experiment is day to day life?

Determination of the number lines of a given grating using a laser source for display applications

Aim

To determine the number of lines in a given grating using a laser source of light.

Apparatus required

He-Ne laser or semiconducting laser, grating, scale, grating stand

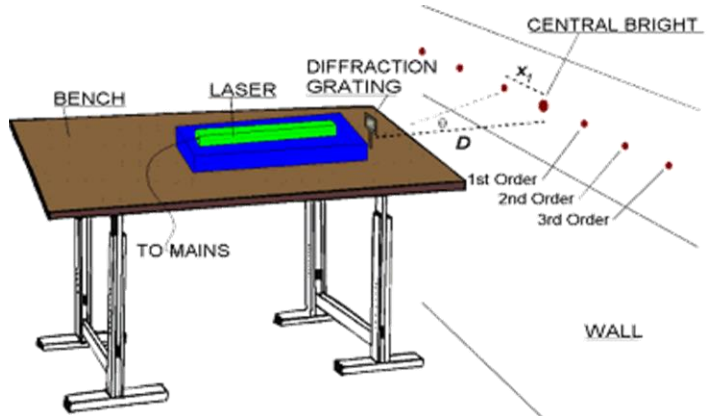


Fig. Schematic of diffraction grating setup and its diffraction pattern

Formula

$$N = \frac{\sin(\theta)}{n\lambda} \text{ lines per meter}$$

where

λ- Wavelength of the laser light used in the experiment (nm),

θ - Angle of diffraction (degree)

n - Order of diffraction,

N-The density of lines in the grating = ———— lines/meter.

Procedure

1. The grating is held normal to the laser beam at a distance D (~ 30 cm) from the screen.
2. The laser light is switched on and it is diffracted by the grating.
3. Symmetric weaker spots corresponding to different orders (n= 1,2,3,4,5,..) of diffraction will be observed around a central bright spot
4. The distances (2L) between the spots on either side of the central spot corresponding to various orders is measured and tabulated.
5. Step 4 is repeated for different values of D (~ 35 cm, 40 cm,45 cm, 50 cm).
6. The wavelength λ is calculated using the formula.

Tabulation

Diffraction Order n	D	2L	L	tan θ (L/D)	θ $\tan^{-1}$ (L/D)	Sin θ	Mean sin θ	N
	centimeters				*degrees*			*Lines/meter*
1	30							
	35							
	40							
	45							
	50							
2	30							
	35							
	40							
	45							
	50							
3	30							
	35							
	40							
	45							
	50							
4	30							
	35							
	40							
	45							
	50							
5	30							
	35							
	40							
	45							
	50							
							Mean N =	

Observations

For n = 1; Mean sin θ = ____________________________

For n = 2; Mean sin θ = ____________________________

Result

The density of the lines in the given grating was determined to be N = _____________________

Viva voce question

1. What is the type of the laser you used in the laboratory?
2. What is the reason for serial of light spots appearing on the measuring scale?
3. Distinguish between laser source and conventional light source.
4. Define grating element.
5. What are the requisites of good grating?
6. What is meant by diffraction of light?
7. Comment on: Grating with larger number of rulings per cm is always preferable.
8. what is the significance of the experiment

Aim: To study Black body radiation and verify Wien's Law

Apparatus Required: Black body radiation kit, Desk Top Computer

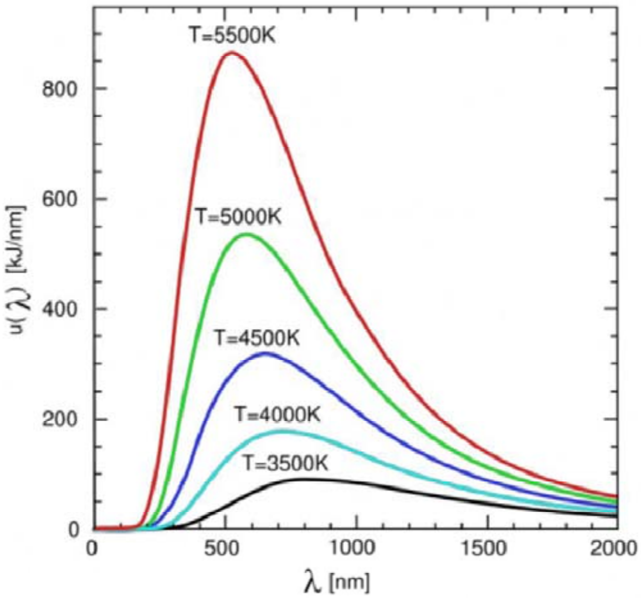
Theory of the Black Body Experiment

An incandescent light source that emits light through a small cavity is a "perfect emitter." By definition, a perfect light emitter is one that emits light rays throughout an infinite number of frequencies in the visible and invisible electromagnetic spectrum. When light from the black body is cast through a prism, the observed spectrum is continuous, and no overlapping of the spectral lines occurs.

In this experiment, parallel light rays travel through the collimating lens, which allows the light rays to remain parallel. Passing through the prism, the light rays refract and project in front of the aperture slit over the light sensor. The light sensor detects and records the light intensity as voltage.

Unlike other light sources, changes in light intensity from an incandescent black body is solely dependent on temperature. Increasing the temperature of the black body light source increases the light intensity. For any given temperature, there appears to be an optimal wavelength for reaching a maximum light intensity.

The angle of the emitted light depends upon the refraction index of the prism and the wavelength of the rays. Shorter wavelengths show more "bend" than longer wavelengths and therefore exhibit higher indices of refraction. Here is a two dimensional plot of a spectrum of a black body with different temperatures as shown below.



Important Laws :

1. **Rayleigh- Jeans law:**

$$\boxed{\rho(\nu)d\nu = \frac{8\pi\nu^2 kT}{c^3}d\nu}$$

2. **Planck's Law :**

$$\boxed{\rho(\nu)d\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{\exp(h\nu/kT)-1}d\nu}$$

Notes:

1. See the Operational Notes section of this manual for important setup reminders.
2. Before beginning the experiment, the following instructions are to be followed:
a) Set the collimating and aperture slits.
b) Check the position of the prism.
c) Ensure the cables from the rotary motion sensor are properly inserted into Science Workshop.
d) Check to see that the black body light is turned on and is emitting steady light.
The bulb can be turned on from the Signal Generator box in Data Studio. If the bulb does not turn on or emits intermittent bursts of light, see the Troubleshooting section of this manual.

Calibration

1.Calibrate the Rotary Motion Sensor

Determining the wavelength from the prism spectrophotometer requires and exact measurement of the angle. To calibrate the rotary motion sensor, determine the ratio of the disk radius to the pin radius (approximately 60:1) as follows:

a) Remove the prism mount and the light sensor bracket from the degree plate by unscrewing the two small thumbscrews. Start the Data Studio program and select a rotary motion sensor. Make a digits display of the angular position, and turn the degree plate so that the zero degree mark is exactly aligned with the index mark.

b) Start recording data. Slowly and continuously turn the degree plate clockwise for exactly one complete rotation.

c) Stop recording data. Record the maximum value of the angle. Divide this number by 360 (or 2π if it is in radians mode). This is the ratio of the radii. Record this number in the calculator in Data Studio.

2. Tare the Light Sensor

Note: For best results and to avoid measurement drift, tare the light sensor before scanning the spectrum and/or before each experiment run. To turn the light source on, click the **On** button in Data Studio's Signal Generator box.

a) Rotate the light sensor arm until it hits the stop against the angle indicator on the spectrophotometer table.
b) Block the light source by placing your hands between the collimating slits and the collimating lens.
c) While the light is blocked, press the tare button on the light sensor to zero the sensor.

Procedures

A. Scanning a Spectrum

1. Tare the light sensor (as described in the section above).

2. Remove your hand to unblock the light and start recording data (Click the **Start** button in the Data Studio setup window) on the computer. Slowly rotate the light sensor arm through the spectrum.

3. To determine the initial angle from the light source, continue to rotate the arm until the light sensor has passed through the white light that passes under the prism (i.e the degree plate has rotated pass the zero degree mark).

4. Stop recording data. The initial angle (when the stop is against the angle indicator) is required to calculate the wavelength.

5. In Data Studio, make a graph of intensity vs. angular position. Measure the angle to the white light that passes directly through the spectrophotometer and under the prism. This angle is subtracted from all angles, so that all angles are referenced from the *reference line* (the parallel beams that travel in a straight line through the spectrophotometer).

6. Enter the initial angle as "Init" into the wavelength calculation in the Data Studio calculator.

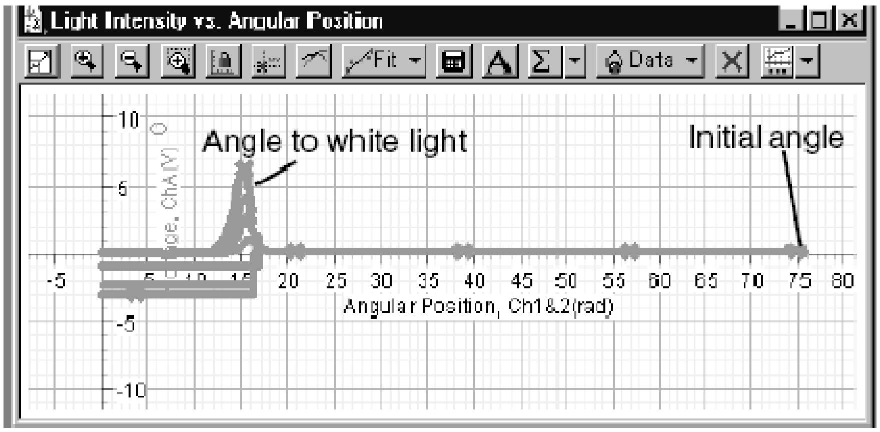


Figure 10: Measuring the Angle of the Refracted Light (Using DataStudio)

Experiment

A. Black Body Spectrum: Scanning Nested Curves from Black Bodies of Different Temperatures

Note: Before running the experiment, perform the rotary motion sensor calibration, as described in the "Calibration" section on page 9. Enter this calibration as "Ratio" into the Data Studio calculator, and click the **Accept** button.

1. Rotate the light sensor arm until it hits the stop against the angle indicator on the circular table.

Note: Before proceeding, the black body light source must be turned on and emitting light. If not, click the **On** button in Data Studio's Signal Generator dialog box.

2. Block the light source by placing your hand between the collimating slits and the collimating lens. While the light is blocked, press the tare button on the light sensor to zero the sensor.

3. Remove your hand to unblock the light and start recording data on the computer. Click the **Start** button in the Data Studio setup window, and slowly rotate the light sensor arm through the spectrum.

4. After you have scanned through the spectrum, stop rotating the arm, continue recording and push the tare button again. Place your hand between the collimating slits and the collimating lens to block the light and return the light sensor arm to its original position against the stop. Blocking the light during this time causes the return scan to be below the axis of the graph, so it does not trace back over the black body data.

5. a) Change the temperature of the bulb by changing the voltage applied to the bulb. You get temperature from current and voltage, as shown by

$$T = 300\,K + \frac{\dfrac{V}{0.84\,I} - 1}{4.5 \times 10^{-3}}$$

b) For each temperature, determine the maximum peak (λ_{max}) of the wavelength. Use the Wien displacement law, where $\lambda_{max}T$ =2898 μm. K, to calculate the λ_{max}.

6. Repeat steps 1 through 5 to get nested curves. On the last scan, continue the scan to the zero degree mark so that you can obtain an exact determination of the initial angle. When you have finished all 5 runs, click the **Stop** button in Data Studio.

Note: To avoid measurement drift, you must tare the light sensor (steps 1 and 2) before each subsequent scan through the spectrum.

7. Repeat the experiment 5 times to record the intensity vs wave length data at 5 different temperatures. For that 5 different voltages are to be applied to the bulb. The maximum voltage should not exceed 10V.

9. Save intensity vs wavelength data for five different temperatures in the computer. Later they can be plotted by any plotting software and print out are to be taken.

10. Find the λ_{max} for each applied voltage and at corresponding temperature. Complete the following table-1 in order to verify Wien's displacement law.

Table-1

Voltage (V)	Current (Amp)	Temperature (K)	λ_{max} (nm)	λ_{max} (nm) x T

Data Collection and Analysis

When you are ready to scan the spectrum, click the **Start** button in the Data Studio setup window. Data Studio records the results and automatically performs the calculations for you. When you have finished collecting your data, click the **Stop** button and view your results in either a graph or a table. If necessary, you can view voltage, wavelength, angle and temperature data points all in the same run. To view individual data points in a table, double click to open a table icon, click on a colored data run icon, and drag and drop the colored data run into the table. (For more information about data analysis using Data Studio, refer to the Data Studio online help guide.)

IMPORTANT OPERATIONAL NOTES

The following notes are critical to maintaining the accuracy of the calibrations and experiment:

Setting the Collimating Slits - For best results, match the slit width size on the collimating lens to the same slit width size on the aperture. For example, if you select the number 4 width from the collimating slits, select the number 4 width on the aperture. (For more information about adjusting the collimating slits, see the Instructional Manual for the Model OS-8539 Educational Spectrophotometer.)

Ensure the slit opening on the aperture slides directly over the hole on the back of the aperture bracket; otherwise light will not reach the light sensor.

Adjusting the Voltage in the Black Body Light Source - In Data Studio, set the voltage in the Signal Generator box. The recommended voltage setting is 7 volts. The voltage can be varied from zero to 10V, but continuous operation of the bulb at 10V will result in a shorter bulb life.

In Data Studio, display the temperature graph and ensure the voltage increase corresponds to a temperature increase. The sampling rate is set to 50 Hz. Also, in Data Studio, ensure that you have selected DC voltage in the Signal Generator box; do not select sine wave or another function. The black body experiment requires direct and continuous current, not alternating or pulsating current.

To improve the voltage signal display on the graphs, increase the gain on the light sensor to 10 or 100. In the Signal Generator window, use the arrow keys to adjust the gain.

Checking the position of the prism - The prism mount and prism must remain fixed at all times. If not, your data will be in error. If the prism mount rotates at any time during the scanning, discard the data, recalibrate, and take another reading.

Checking the angular display against the reading on the degree plate – Data Studio allows you to adjust the angle units to either degree or radians. If you set the units to rads in Data Studio, remember that the number on the degree plate will not correspond to the number on the display. You will need to mathematically convert degrees to radians.

If you have negative angle readings, you may have reversed the colored cables for the rotary motion sensor, rotated the degree plate in the wrong direction, or improperly mounted the light sensor arm and lenses. While taking a reading, the degree plate must rotate clockwise. For more information about the rotary motion sensor or the light sensor arm, see the instruction manual for the Educational Spectrophotometer.

Questions/Exercise

1) How does changing the temperature of the bulb affect the wavelength or light intensity? Do you notice a pattern with increasing temperature?
2) From what you remember from the lesson on the grating spectrophotometer, what differences have you observed between using a prism and a grating?
3) On a piece of paper, draw a diagram showing the position of the reference angle, measured angle and spectral lines. Do the spectral lines converge or diverge? Do the light rays overlap?
4) What would happen if you removed the collimating lens?
5) What is the relationship between the light's angle, wavelength and intensity?
6) What differences do you notice between the black body and other types of light sources you have used?
7) Replace the infrared light sensor with the high sensitivity light sensor. What differences do you notice in the graph displays?