Harbour.Space: Mathematics and Computer Science Test

April 4, 2017

1 Mathematics

Provide explanation whenever necessary. When unable to answer fully provide as much intuition and partial work as possible.

1. What is a polynomial ?
2. Solve the equation
$$(x-1)x(x+1)(x+2) = 24$$
3. Solve the equation
$$4^{2x-2} - 4^x + |4^{x-1} - \frac{1}{3}| = -\frac{7}{3}$$
4. Solve the inequality
$$\sqrt{x^2+x+1} + 2x \leq |3x+1|$$
5. Given 3000 unique items in a set how many ways are there to select 100 non-repeating items from that ? You don't need to compute the exact number. The formula will be sufficient.
6. Draw geometric positions of points that satisfy
$$|x-y| + |y-x^2| = 2$$
7. Prove the statement: if the number $a \in N$ isn't divisible by 5, then the number $a^4 - 1$ is divisible by 5
8. Given that heights of the triangle are less than 1, prove that the radius of an inscribed circle is less than 1/3.
9. Find all values of a for which equation $ax^2-(3a^3-6a^2-1)x-3a(a-2) = 0$ satisfies condition $|x| \leq 2$?

10. Draw a chart for
$$y = \frac{x}{\sqrt{x^2 + x}}$$

11. Find all pairs of values (x, y) satisfied the system of inequalities
$$\begin{cases} x - y \leq -25, \\ x^2 - y \leq 8, \\ 4x + y \leq 1. \end{cases}$$

12. Solve the equation
$$\cos 5x - \cos 15x = \frac{3\sqrt{3}\tan 5x}{2}$$

13. Two national teams are competing in an international beach volleyball competition. Each team has 7 members, but only two can play for the team in a given match. How many ways are there to arrange a match between two teams with unique set of players ?

14. Draw a chart of $f(x) = min_{t \in [x-1, x+1]}(t^2 + 4t + 1)$.

15. Draw all solutions to $x^2 + x = y + ||x^2 + x| - y|$

16. Find the maximum value of $\cos(x + 1) + \cos(x + 3)$

17. Find all values of a for which equation $e^{x^2} = ax$ always has exactly two roots.

18. Find all pairs x, y, such that $x^3 + y$ and $y^3 + x$ are divisible by $x^2 + y^2$. Explain your solution.

19. Given value $n + 1$ from net of positive integers between 1 and $2n$. Prove that one of such numbers is divisible by another one of chosen numbers.

20. Prove that for any polynomial $P(x)$ with real coefficients, other then polynomial x, the polynomial $P(P(P(x))) - x$ is divisible by $P(x) - x$.

2 Computer Science

Provide explanation whenever necessary. Whenever writing in a programming language state what language you are using. When unable to answer fully provide as much intuition and partial work as possible.

1. What is the least number of operations necessary to sort an array of n arbitrary objects ?

2. What is the most efficient data structure to support appending to the end of the set, removing last element from the set, as well as accessing or updating ith value ? Provide explanation and complexities.

3. What is the most efficient data structure to support inserting into a set as well as selection and deletion of a value from random position ? Provide explanation and complexities.

4. What is virtual memory ? What is it used for ? Why it is necessary ?

5. Write an efficient program which given two sorted arrays $A[0..n]$ and $B[0..m]$ finds all values that are present in both arrays.

6. Describe an algorithm to find a vertex with a highest degree in an undirected graph. Describe the complexity. You do not have to write a program or a full algorithm. The description is sufficient.

7. Describe what is Object Oriented Programming ? What are the key concepts and characteristics ? Where is it used ? Write a small program demonstrating this idea.

8. Write a program that asks user's name, records it in memory, prints the number of times it saw the name since it was last started and goes back to asking user's name.

9. Describe what is a 'global variable' in computer science domain ? What is it used for ? Write a small program demonstrating this idea.

10. What is the least number of operations necessary to sort an array of n integers from $0..m$?

11. What is the most efficient data structure to support adding to a set as well as finding smallest value in the set and deleting a value added during ith iteration ? Provide explanation and complexities.

12. What is the most efficient data structure to support inserting in the beginning or the end of the set, removing element added during ith iteration and printing all elements in the right order in $O(n)$. Provide explanation and complexities.

13. Explain what is 'segmentation fault' and when it occurs ? Develop a program that demonstrates this idea.

14. Write a program that given an array $A[1...n]$ and a value S finds $0 < i < j < n$ such that $A[i] + A[j] = S$

15. Describe an algorithm that given a matrix described below determines whether matrix contains a value k. The input matrix $A[1...n, 1...n]$ has all rows and columns arranged in an non-descending order $A[i, j] < A[i, j+1]$, $A[j, i] < A[j + 1, i]$ for all $1 < i < n$ and $1 < j < n$.

16. Describe what is an 'instruction' in computer science domain ? How many instructions can be executed at a time ? Given a computer with one CPU (and one core), describe how modern systems achieve multitasking i.e. ability to execute multiple tasks at the same time.

17. Write a program without using a build in square-root operator to compute square-root of a number that is known to have an integer square root.

18. Describe what is a garbage collection and how does it work ? What are its upsides and downsides ?

19. Provide pseudo-code or a program using programming language for any of the following sorting algorithms: merge sort, quick sort, heap sort. Explain best, average and worst case time complexity.

20. What is the most efficient data structure to support all following operations: inserting into a set, finding an element by value, deleting an element by value. Provide explanation and complexities.

21. What is the most efficient data structure to support all following operations: finding minimum value, finding and deleting an element by value. Provide explanation and complexities.

22. Write a program that given a positive integer N finds all such positive integers a and b that satisfy $a2 + b2 = N$

23. Write a program that determines if a given string is a palindrome. (A palindrome is a word, phrase, number, or other sequence of characters which reads the same backward or forward)

24. Write a program using any technology that finds all files in a current directory that are older then 3 days.

25. Given a program written using a language of your choice, describe how this program is executed ?