Speeding up Initial Data Generation in Einstein Toolkit

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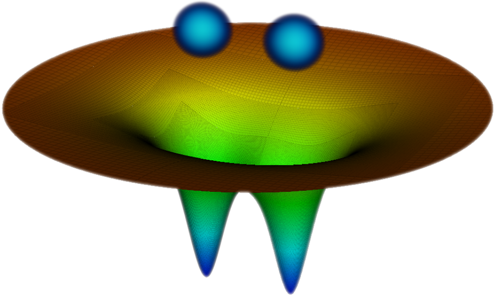
Outline

Motivation

- Modelling scenarios in astrophysics with numerical relativity simulations requires the production of suitable initial data sets.
- Initial data generated is needed for simulating black hole and neutron star mergers, cosmology and gravitational wave simulations.
- However, producing initial data is computationally intensive.

Initial Data

- Obtained by solving constraint equations of general relativity together with equilibrium equations for matter.
- Laplacian dominated elliptic boundary value problems.



Metric for binary neutron star pair.

Outline

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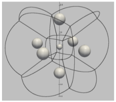
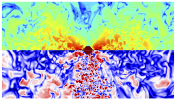
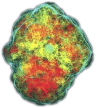
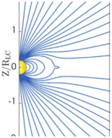
Free open source, community toolkit for astrophysics simulations.

- Cosmology
- Accretion disks after neutron star collisions
- Core collapse supernovae
- General relativistic magnetohydrodynamic simulations
- Binary black hole, neutron star mergers

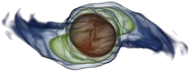

E. Bentivegna

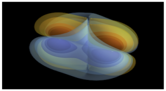

D. Siegel


C. Ott



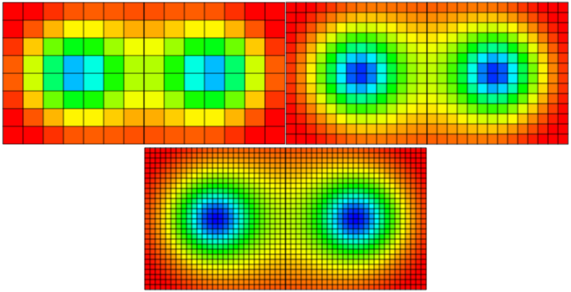
Z. Etienne


R. Haas, S. Rosofsky


B. Wardell

CT_Multilevel, a Multigrid Solver

- Existing multigrid solver in Einstein Toolkit developed by Eloisa Bentivegna. It is implemented using Cactus Computational Toolkit.
- Intended for cosmology and initial data problems.
- Multigrid solvers speed up convergence by passing the solution between a hierarchy of grids spanning the same space.



Successive Over-Relaxation (SOR)

- At every grid, CT_Multilevel solves a Dirichlet boundary value problem using SOR. It solves algebraic equation $Au = f$ iteratively:

$$u_{i+1} = u_i + \omega(Au_i - f)$$

- SOR is extremely robust. It can handle nonlinear equations without any modifications.
- But it is slow.
- We intend to modify the smoothing operation, i.e. replace SOR with a faster algorithm, while leaving the multigrid scheme intact.

Outline

Scheduled Relaxation Jacobi (SRJ)

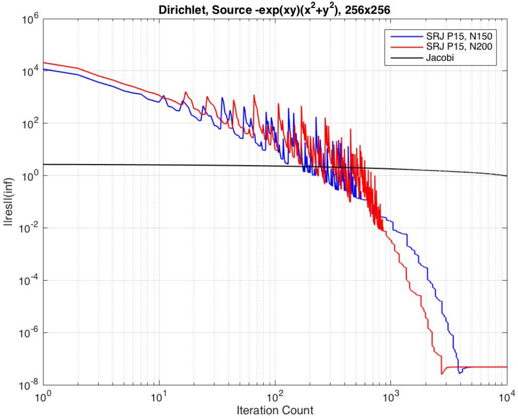
- SRJ (Adsuara et al, 2015) is an extension of Successive Over-Relaxation where a set of optimal ω, relaxation factors, are precomputed to minimise the total number of iterations.

$$u_{i+1} = u_i + \omega(Au - f)$$

- Since SRJ methodology has only been developed for linear equations, we linearize the equation using a Newton-Raphson scheme.

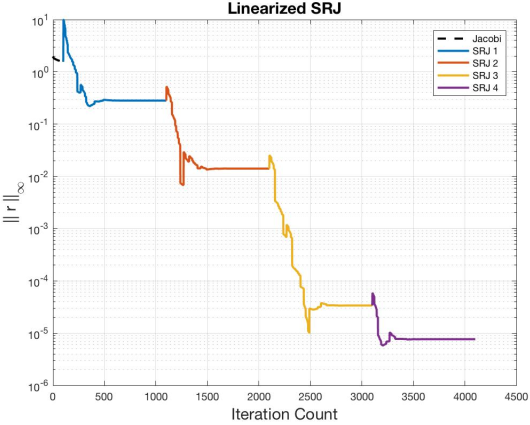
Scheduled Relaxation Jacobi Results

$$\triangle u + exp(x+y)u = (x^2 + y^2 + exp(x+y))e^{xy}, \text{ 256x256 grid.}$$



Scheduled Relaxation Jacobi Results

$$\triangle u = u^3 - \frac{3}{(x+1)^2(y+1)} - \frac{3}{(x+1)(y+1)^2},\ \text{256x256 grid.}$$



Outline

Preconditioned Krylov Subspace Method (KSM)

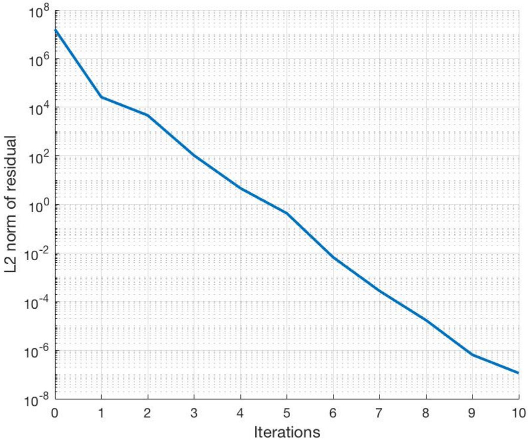
- Krylov Subspace methods are a class of iterative linear solvers.
- A suitable preconditioner improves convergence rate.
- We linearize with Newton-Raphson scheme and solve using preconditioned KSM.
- We will only show results for linear equations because all methods share the same Newton-Raphson scheme.

The Preconditioner

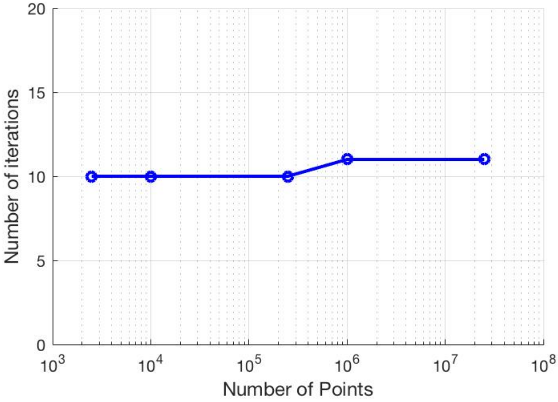
A **preconditioner** is a lower order approximation to the inverse of a matrix. Since the Laplacian is the dominant term, we construct a preconditioner to solve for it directly.

- Convert $\triangle u = f$ into an algebraic set of equations, $Mu = f$.
- Invert M matrix-free using a discrete sine transform in $\mathcal{O}(N)$ complexity.

$\triangle u + exp(x+y)u = (x^2 + y^2 + exp(x+y))e^{xy}$, 500x500 grid.



Solve$\triangle u + exp(x+y)u = (x^2 + y^2 + exp(x+y))e^{xy}$ for varying grid sizes.



Summary and Future Work

- We have two $\mathcal{O}(N)$ that reduce the number of iterations for a single grid.
- See how the schemes behaves inside a multigrid solver.
- Suitable treatment for the error equation?
- Implement in CT_Multilevel.

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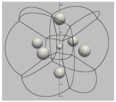

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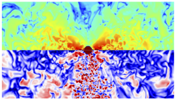

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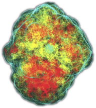

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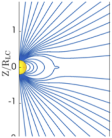

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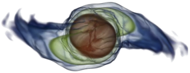

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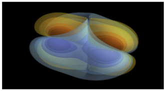

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