Topic 2: Image Processing - Segmentation

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Segmentation

- Segmentation/classification is a process to partition a digital image into multiple regions (sets of pixels).
- Segmentation is typically used to locate objects and boundaries (lines, curves, etc.) in images.
- Medical image segmentation is used to:
  - Locate tumors and other pathologies
  - Measure tissue volumes
  - Computer-guided surgery
  - Diagnosis
  - Treatment planning
  - Study of anatomical structure

Segmentation Techniques

- There is no general solution to the image segmentation problem.

- Segmentation techniques have to be combined with domain knowledge in order to effectively solve an image segmentation problem.

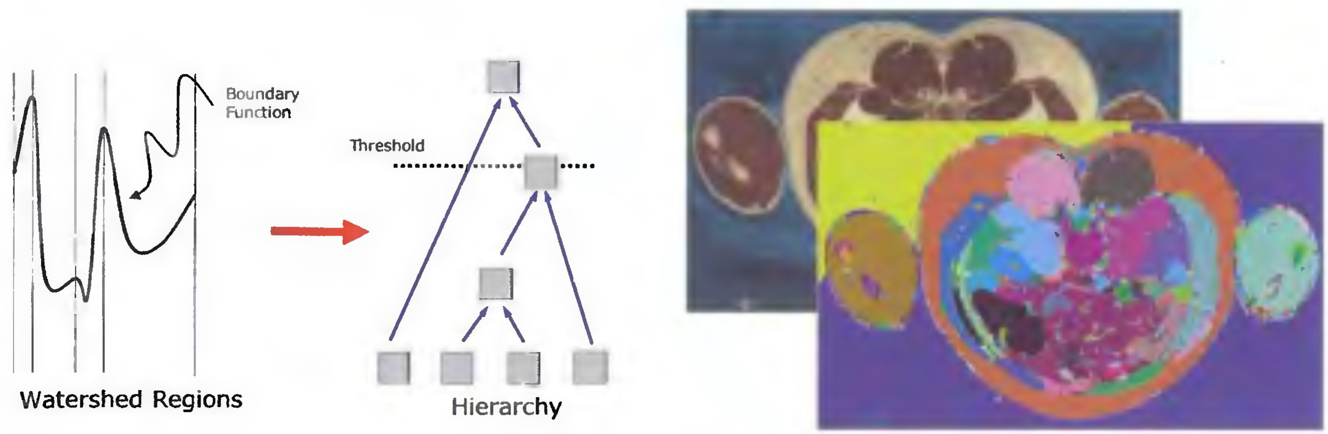
Edge Detection

- Region boundaries and edges are closely related, since there is often a sharp adjustment in intensity at the region boundaries.
- The gradient vector and its magnitude of the image intensity are used to detect boundaries.

$$\begin{bmatrix} \dfrac{\partial \phi}{\partial x} & \dfrac{\partial \phi}{\partial y} \end{bmatrix} \qquad \left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2$$

Watershed Segmentation

- The watershed technique creates a series of "catchment basins", representing local minima of image features or metrics.
- The resulting watersheds can be hierarchically organized into successively more global minima, and the resulting graph, linking all of the individual watershed regions.
- The graph hierarchy can later be used to navigate the image, interactively exploring the dataset through dynamic segmentation of the volume by selectively building 3D watershed regions from the 4D height field.



(a) (b)

Plate I. (From Figure 5.1) Watershed segmentation: (a) a 1D height field (image) and its watersheds with the derived hierarchy; (b) a 2D color data slice from the NLM Visible Human Project and a derived watershed segmentation. [Example courtesy of Ross Whitaker and Josh Cates, Univ. of Utah]

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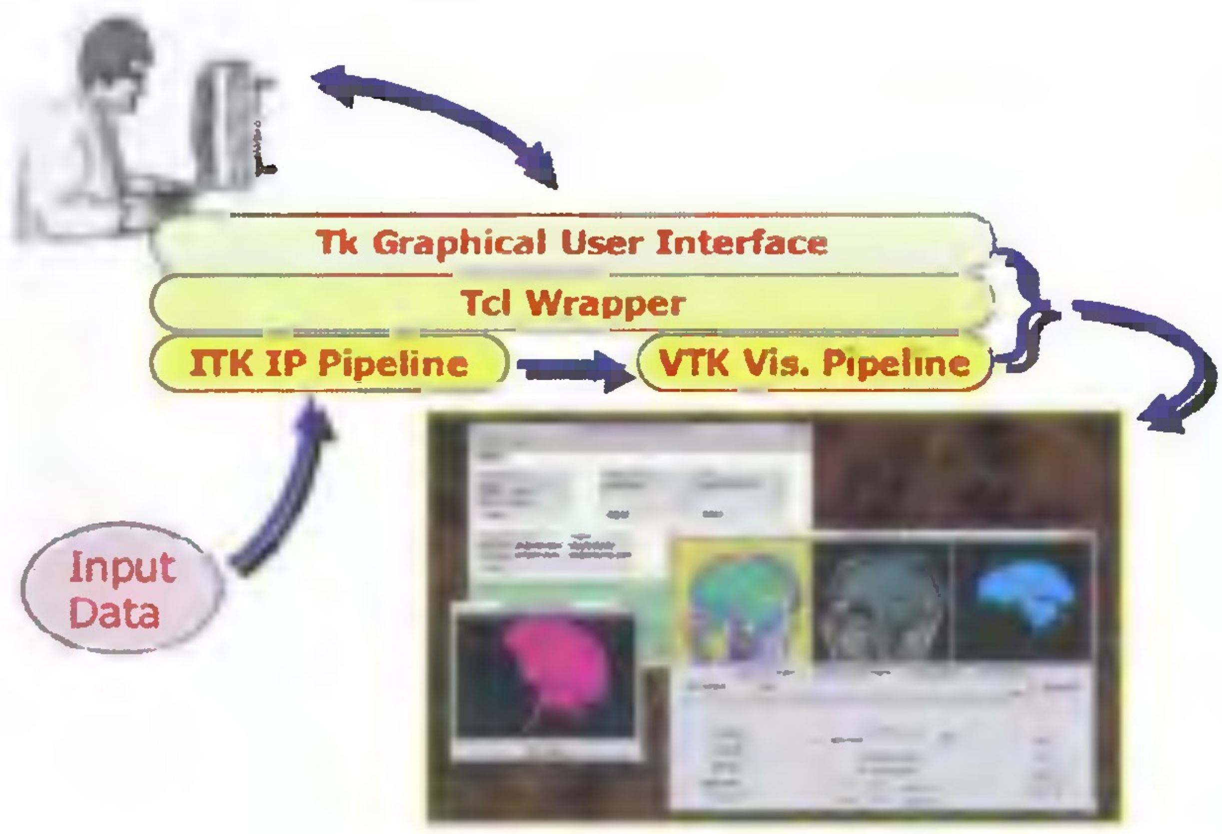
Watershed Transformation

- Watershed transformation considers the gradient magnitude of an image as a topographic surface.
- Pixels with the highest gradient magnitude intensities (GMIs) correspond to watershed lines, which represent the region boundaries.
- Water inside a region flows downhill to a common local intensity minima (LIM)
- Pixels draining to a common minimum form the watershed region.

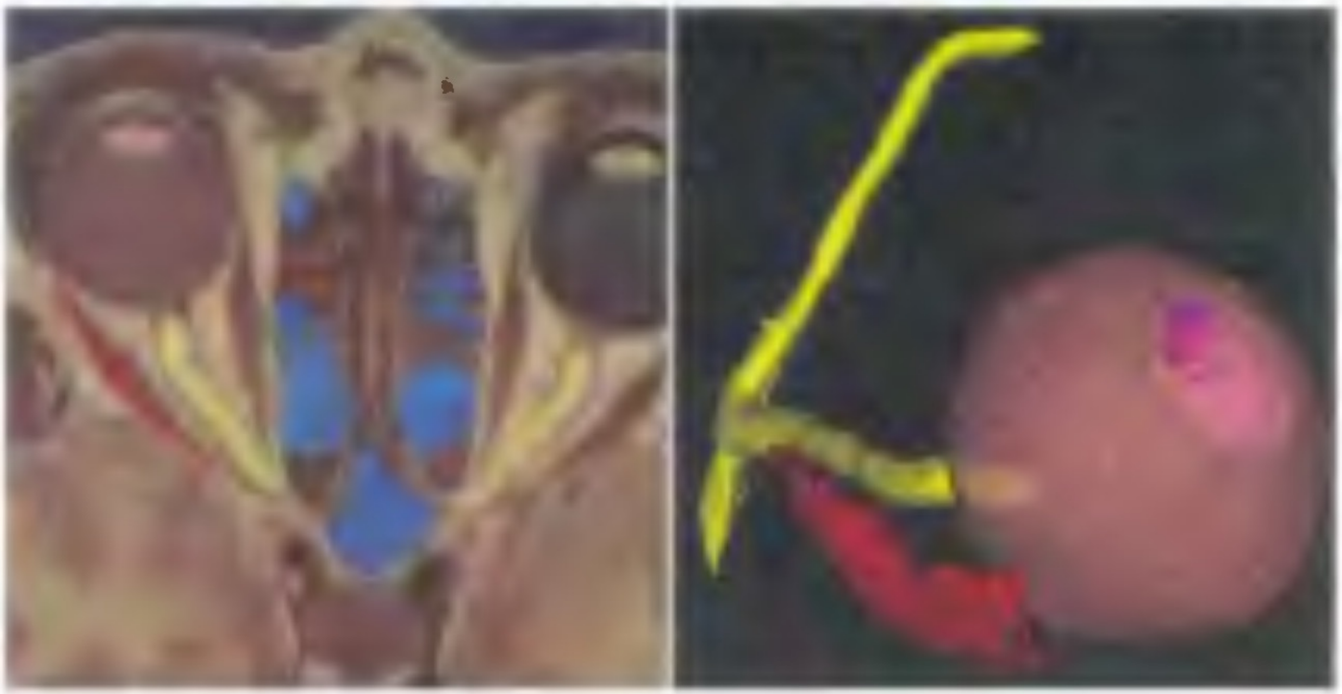
Watershed Segmentation

- An ITK watershed segmentation application with visualization provided by VTK and with the user interface provided by Tcl/TK.
- This example shows how ITK can be integrated with multiple packages in a single application.



An Example Using Semi-Automated Segmentation

- A user has defined the rectus muscle (highlighted on the left in red), the left eye and the optic nerve (yellow).
- These tissue types can be compared with the corresponding anatomy reflected on the right picture, revealing the difference in the highlighting color and the relative fidelity of the segmentation.



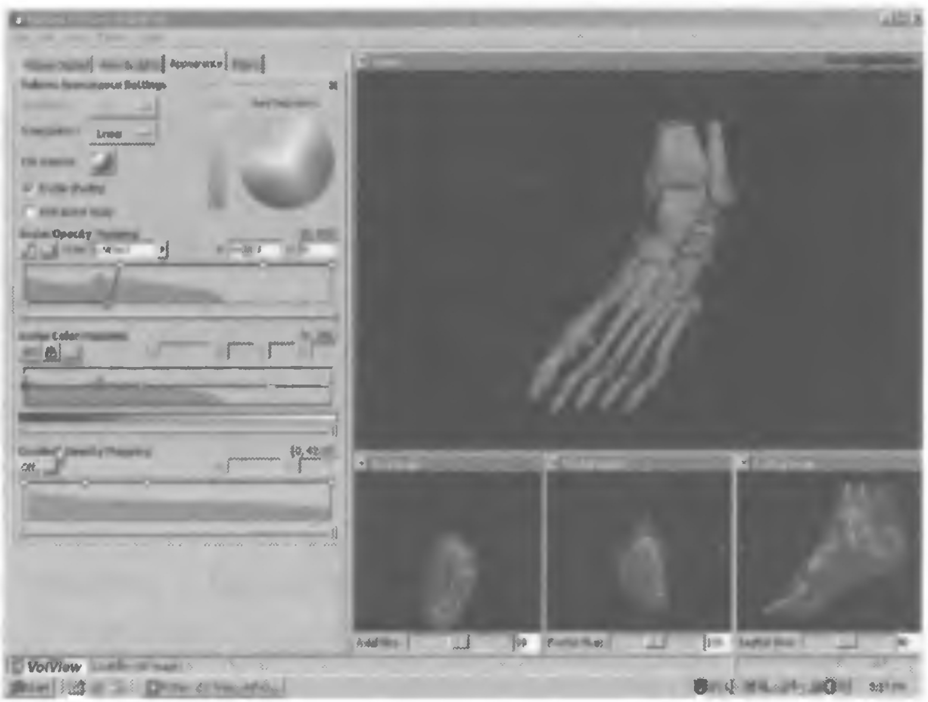
Statistical Pattern Recognition

- To treat the dataset, or a collection of datasets, as a statistical sampling of the human anatomy.

- Building upon statistics as a foundation for imaging research allows practitioners to draw upon the powerful calculus associated with probabilities and distributions of pixel or voxel measurements of the patient's internal parts.

Statistical Pattern Recognition

- A simple threshold becomes a linear discriminant on the histogram, dividing the distribution of image intensity values into two classes: “bone” and “not-bone”.
- Thresholding is particularly useful for *x*-ray CT data where intensity values have an intuitive mapping to physical density.
- More complex, multivalued data, such as MRI or color cryosections, require sophisticated statistical techniques.



Statistical Pattern Recognition

- A single, simple statistical perspective treats the entire dataset simultaneously, affecting all parts of the image at once.
- The segmentation using statistical pattern recognition can effectively partition the data into structures of interest.
- However, the objects are undersegmented. All the bones appear together as a single segment. Sometimes we need to separate and articulate the bones individually.
- Separation of the many bones of the foot can be achieved using a user interface for expert editing of the segmented system (or use other segmentation technique).

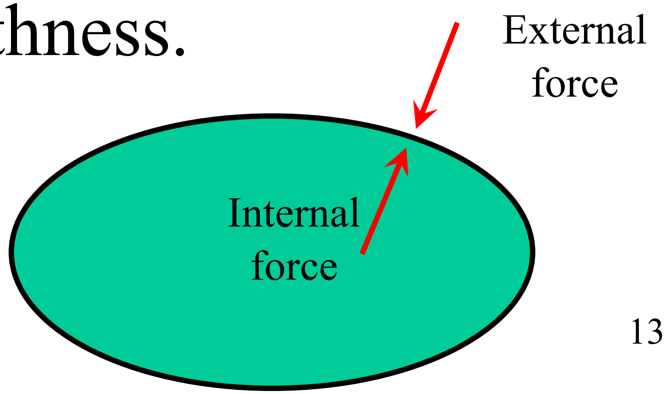
Region Merging

- Region merging is a basic means of accounting for spatial proximity and connectedness as well as alikeness of intensity values.
- A connected-threshold approach helps recursively aggregate all voxels that are connected to the marked point by an unbroken lattice of voxels that appear within a user-defined interval of intensity values.

Using a connected-threshold approach helps define a contiguous region of connected intensity values that maintain spatial as well as intensity coherence, defining specific structures of bones such as the talus or ankle bone:

Active Surfaces/Front Evolution

- Alternative approaches to region merging include the family of adaptive surface definition methods which attempt to connect the margin voxels that circumscribe the region of interest.
- These methods are built on statistical measures of similarity, or they attempt to use energy-minimizing models which balance external boundary-following metrics with internal forces that attempt to promote smoothness.



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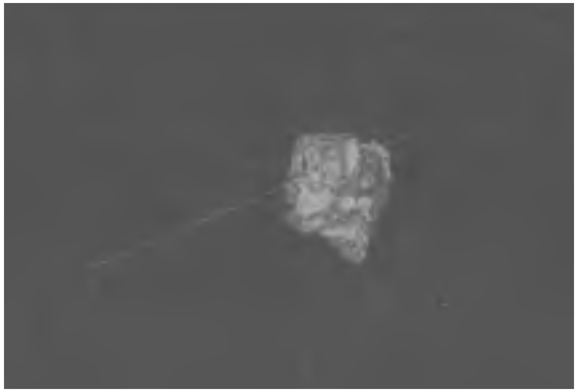
Active Surface/Front Evolution

- The principal advantage of this method is that the expected geometric properties of the segmented boundaries can be enforced. But some detailed features may be ignored.
- By using this active surface or level set method, we can apply differential geometry to the evolving front and limit the overall local curvature to recover a smooth object.

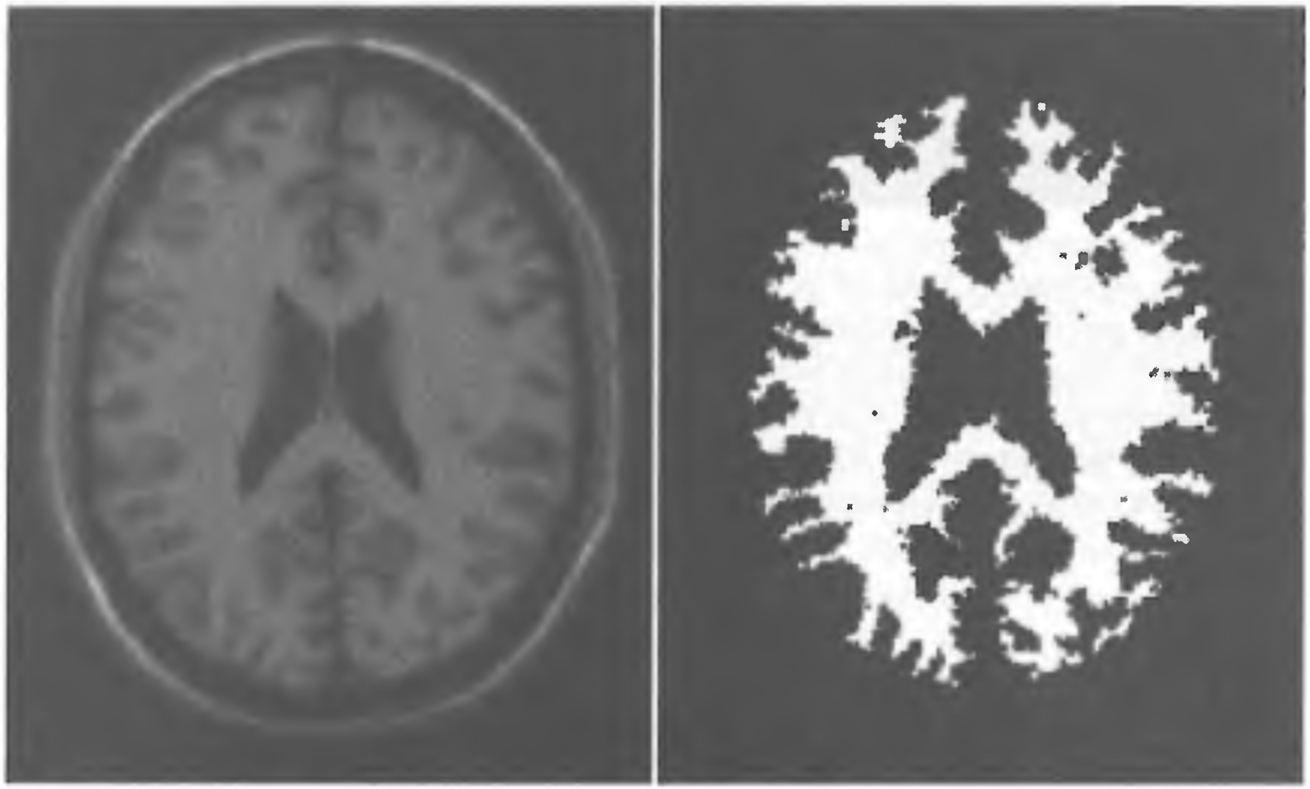


Connected-threshold method

Active surface method

Combining Segmentation Techniques

- Image statistics are powerful but often require the addition of surface geometry to generate effective results.



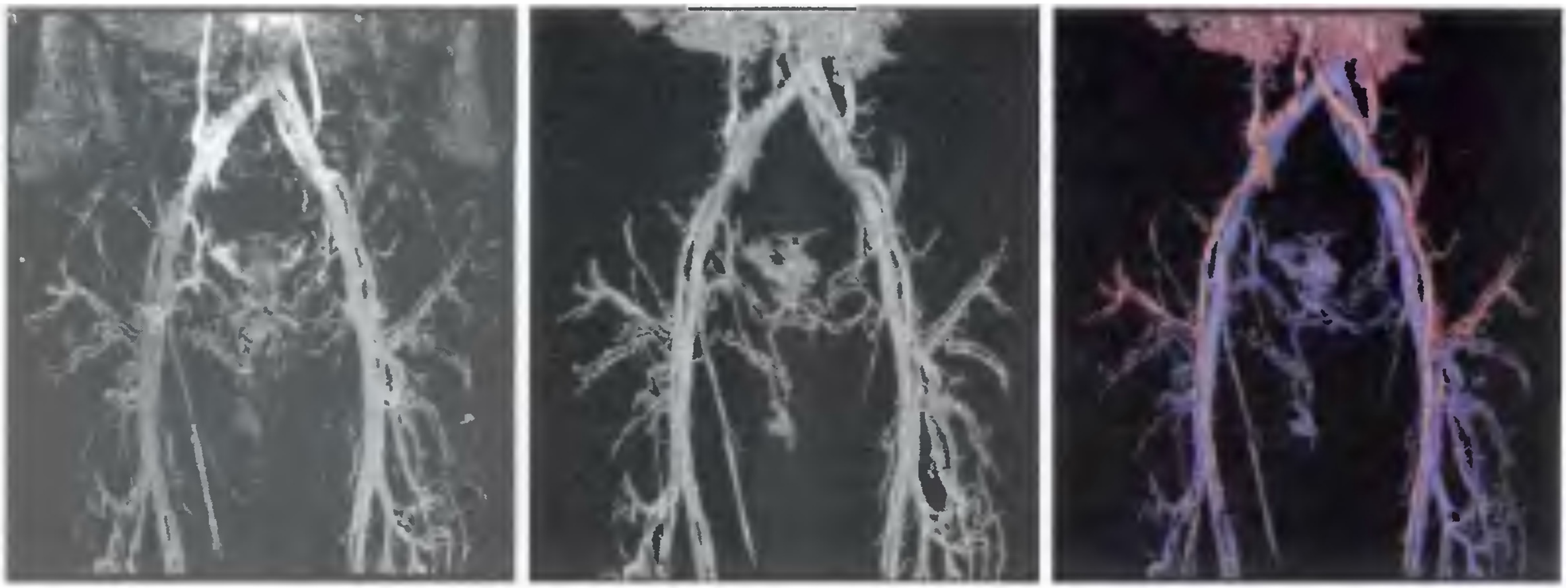
Segmentation results of a brain MRI slice using a combination of image statistics and active surface

Fuzzy Connectedness

- Images are inherently fuzzy due to spatial and temporal resolution limitations, noise and other artifacts.
- Fuzzy connectedness allows capturing the spatio-topological concept of hanging-togetherness of image elements, even in the presence of a gradation of their intensities stemming from natural material heterogeneities, blurring and other artifacts.

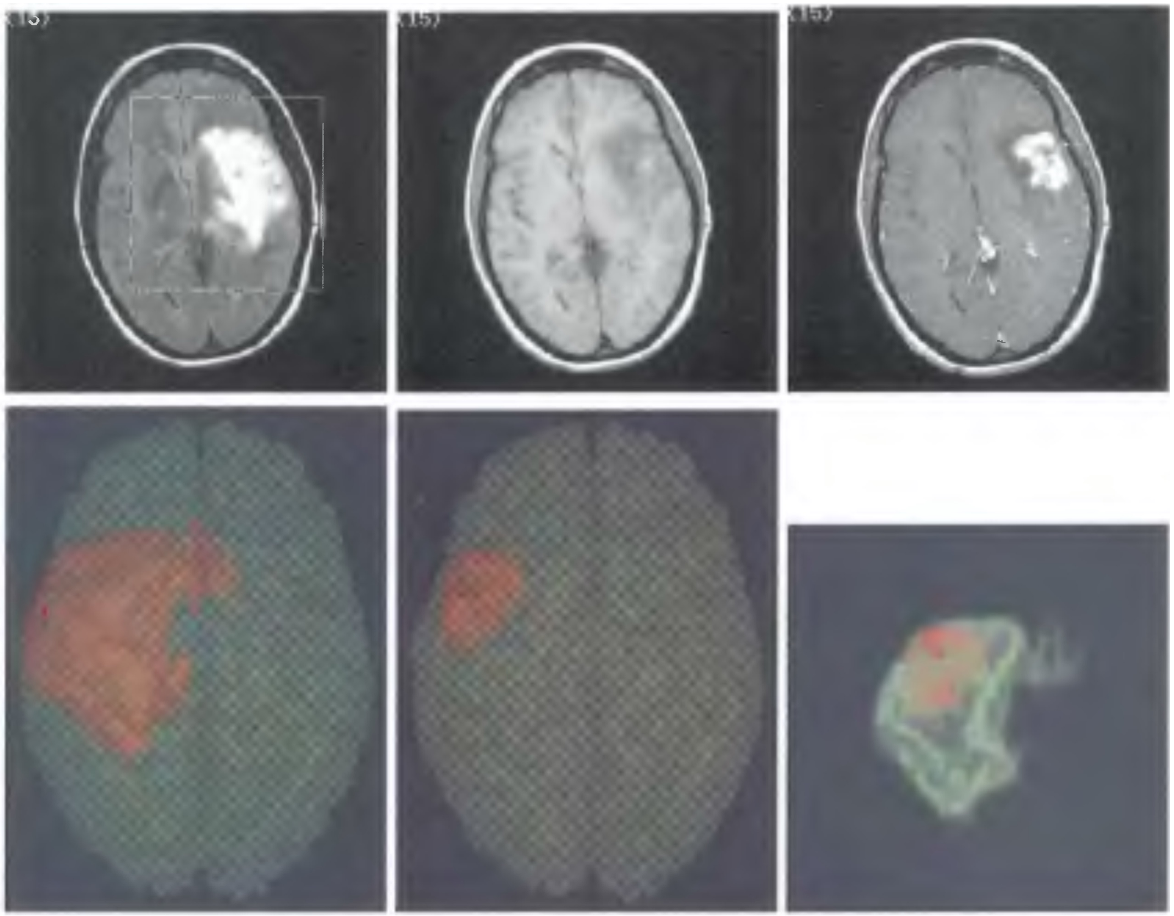
Fuzzy Connectedness

Segmentation via fuzzy connectedness



Left: CT angiography (CTA) of blood vessels acquired with the blood-pool contrast agent MS325. Middle: segmented result of the entire vascular tree via generalized fuzzy connectedness. Right: The arteries and veins are separated by using iterative relative fuzzy connectedness.

Segmentation via Fuzzy Connectedness



Brain tumor MRI data: Segmentation results of edema plus tumor.

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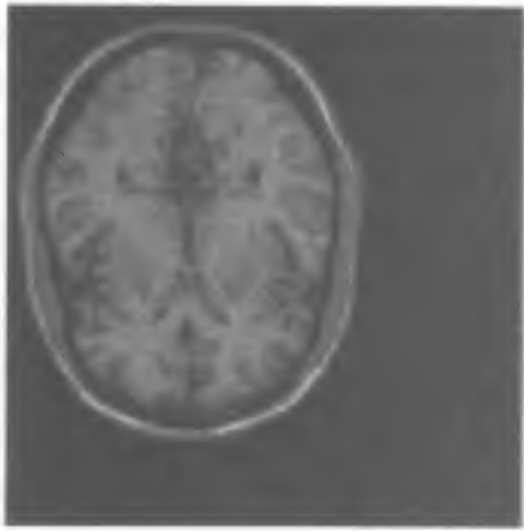
Markov Random Field (MRF) Models

- The region-based segmentation methods make use of the homogeneity of inner regions in images. However, they may fail to capture the global boundary and shape properties of the object, leading to noisy boundaries and holes inside the object.

- MRF models are probabilistic models that use the correlation between pixels in a neighborhood to decide the object region. Compared to typical region growing methods, MRF includes more information in the pixel's neighborhood.

Original MRI image



Segmented result

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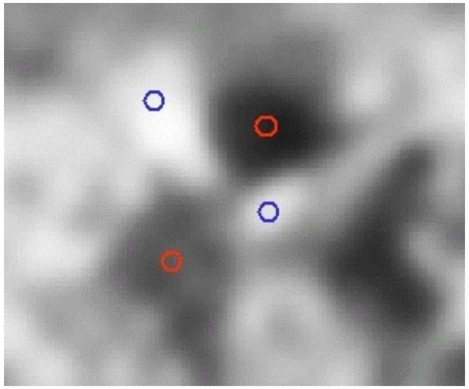
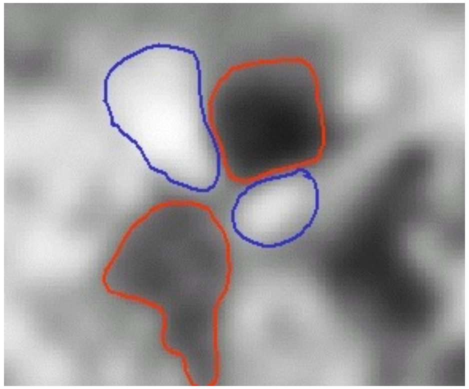
Seeded Region Growing Methods

- Takes a set of seeds at input marking each object to be segmented.
- The regions are iteratively grown by comparing all unallocated neighboring pixels to the regions, until all pixels are allocated to a region.





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Level Set Methods

- Level set method was proposed to track moving interfaces by Osher and Sethian in 1988.
- Represent the evolving contour using a signed function, where the actual contour is its zero level.
- According to the motion equation of the contour, one can easily derive a similar flow for the implicit surface to reflect the propagation of the contour.
- Implicit, parameter free, can change the topology and is intrinsic.

Representing Surfaces with Volumes

- The level set method relies on an implicit representation of a surface, using a scalar function

$$\phi : \underset{x,y,z}{U} \mapsto R$$

where $U \in R^3$ is the domain of the volume. Thus, a surface is referred to as an isosurface of ϕ.

$$S = \{\vec{x} \mid \phi(\vec{x}) = c\}$$

The embedding ϕ is represented on a discrete grid, i.e., a volume.

Surface Normal

- The normal of an isosurface is given by the normalized gradient vector. Typically we identify a surface normal with a point in the volume domain *D*.

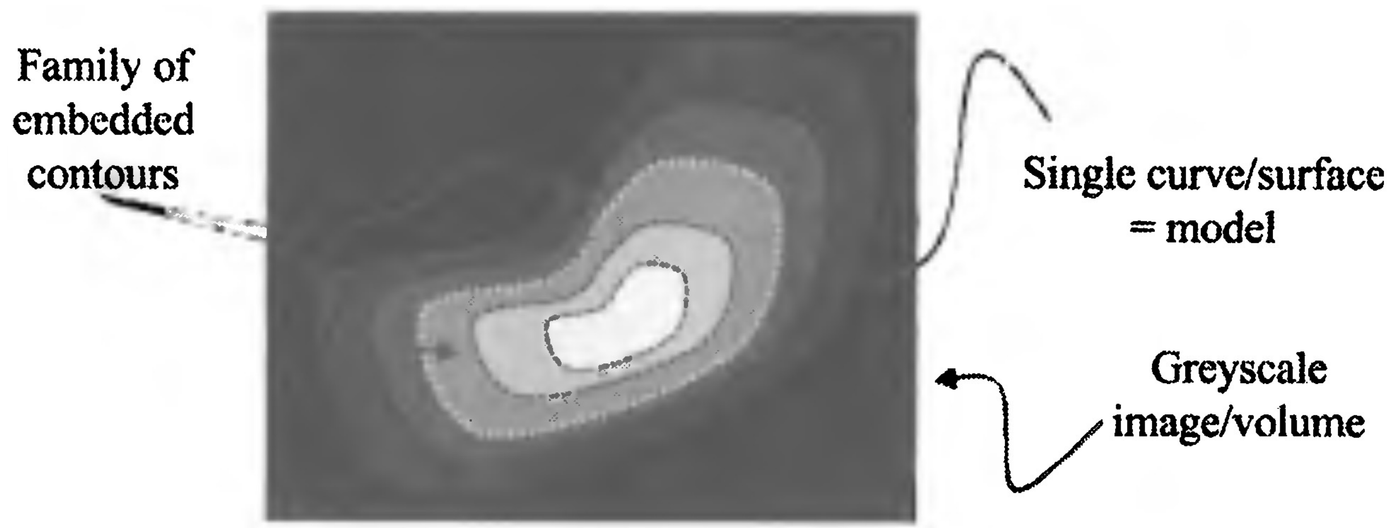
$$\vec{n}(\vec{x}) = \frac{\nabla \phi(\vec{x})}{\left|\nabla \phi(\vec{x})\right|} \quad where \quad \vec{x} \in D$$

The normal is positive if it points outward.

- On a discrete grid, the gradient vector can be approximated using central differences. In the presence of noise, it is helpful to compute first derivatives using some smoothing filters (convolution with a Gaussian).

Representing Surfaces with Volumes

- A single volume contains a family of nested isosurfaces, arranged like the layers of an onion.



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Curvature

- The curvature of the isosurface can be computed from the 1^{st} and 2^{nd} structure of the embedding ϕ.
- The normal vector is $\vec{n}(\vec{x})$, and the 3x3 matrix of derivatives of this vector

$$N = -\begin{bmatrix} \vec{n}_x & \vec{n}_y & \vec{n}_z \end{bmatrix}$$

- The projection of this derivative onto the tangent plane of the isosurface gives the shape matrix, β. Let P denote the normal projection operator

$$P = \vec{n} \otimes \vec{n} = \frac{1}{\|\nabla \phi\|^2} \begin{pmatrix} \phi_x^2 & \phi_x \phi_y & \phi_x \phi_z \\ \phi_y \phi_z & \phi_y^2 & \phi_y \phi_z \\ \phi_z \phi_x & \phi_z \phi_y & \phi_z^2 \end{pmatrix}$$

Curvature

- The tangential project operator is $T = I - P$, then the shape matrix is

$$\beta = NT = TD^2\phi T$$

where $D^2\phi$ is the Hessian of ϕ. The shape matrix β has three real eigenvalues

$$e_1 = k_1 \qquad e_2 = k_2 \qquad e_3 = 0$$

The corresponding eigenvectors are the principal directions (in the tangent plane) and the normal, respectively.

Curvature

- The mean curvature is the mean of the two principal curvatures $\frac{1}{2}(k_1 + k_2)$, which is ½ of the trace of β, which is equal to the trace of N.

- The Gaussian curvature is $k_1 k_2$.

- The total curvature, also called the deviation from flatness, D, is the root sum of squares of the two principal curvatures, which is the Euclidean norm of the matrix β.

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Deformable Surfaces

- The evolution equation for a parametric surface gives rise to an evolution equation (differential equation) on a volume, which encodes the shape of that surface as a level set.
- In geometric modeling, a surface is represented as a two-parameter object in 3D space, i.e., a surface is a local mapping $\vec{S}$:

$$\vec{S} : \underset{r}{V} \times \underset{s}{V} \mapsto \underset{x,y,z}{R^3}$$

where $V \times V \subset R^2$.

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Deformable Surfaces

- A deformable surface exhibits some motion over time *t*

$$\vec{S} = \vec{S}(r, s, t)$$

- The surface normal

$$\vec{N} = \vec{N}(r, s, t)$$

- Local deformation of the surface can be described by an evolution equation

$$\frac{\partial \vec{S}}{\partial t} = \vec{G}(\vec{S}, \vec{S}_r, \vec{S}_s, \vec{S}_{rr}, \vec{S}_{rs}, \vec{S}_{ss}, \cdots)$$

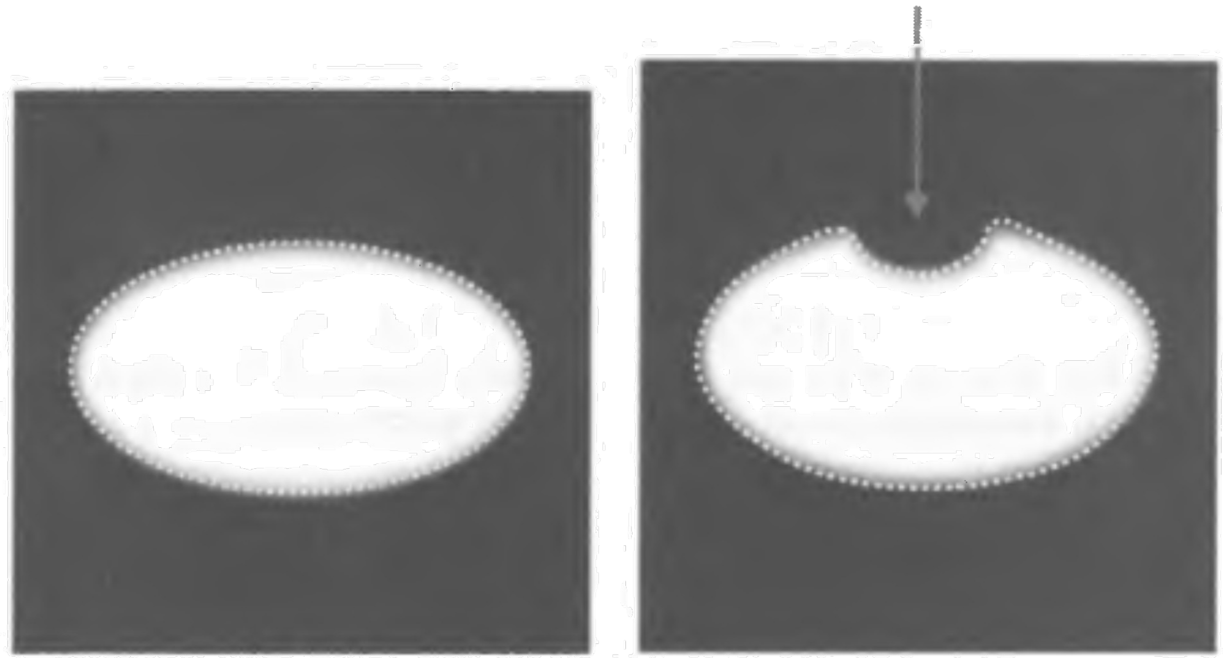
- There are various ways to define the evolution equation.

Surface Deformation

- There are a variety of differential expressions that can be combined for different applications.
  - The model could move in response to some directional "forcing" function.
  - The surface could expand and contract with a spatially-varying speed.
  - The surface motion could also depend on the surface shape, e.g. curvature, or depend on both the differential geometry of the surface and outside forces or functions to control the evolution of a surface.

Level Set Method

- The level set method, proposed by Osher and Sethian, provides the mathematical and numerical mechanisms for computing surface deformations as time-varying iso-values of ϕ by solving a PDE on the 3D grid.



(a) (b)

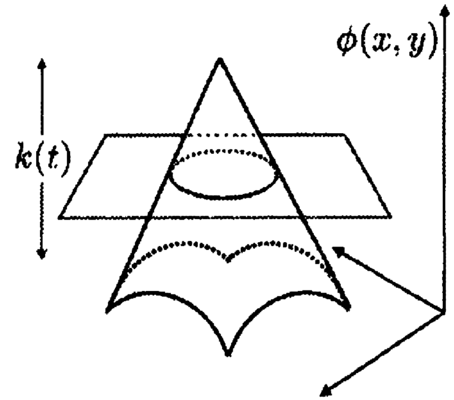
Figure 8.2. Level-set models represent curves and surfaces implicitly using greyscale images: (a) an ellipse is represented as the level set of an image; (b) to change the shape we modify the greyscale values of the image.

Level Set Method

- We denote the movement of a point $\vec{x}$ on the deforming surface as $\vec{v}$. There are two options for representing such surface movements implicitly: static and dynamic.
  - Static: A single static $\phi(\vec{x})$ contains a family of level sets corresponding to surfaces at different time *t*.

$$\phi(\vec{x}(t)) = k(t) \quad \Rightarrow \quad \nabla\phi(\vec{x}) \bullet \vec{v} = \frac{dk(t)}{dt}$$

This is a boundary value problem. A surface can't pass back over itself over time, therefore motions must be strictly monotonic: inward or outward.



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Level Set Method

– Dynamic: The approach is to use a one-parameter family of embedding $\phi(\vec{x},t)$ changes over time. k remains constant.

$$\phi(\vec{x}(t),t) = k \quad \Rightarrow \quad \frac{\partial \phi}{\partial t} = -\nabla\phi \bullet \vec{v}$$

This approach can accommodate models that move forward and backward and cross back their own path (over time).

This is an initial value problem, a potentially large computational burden.

The movements of all the level-set surfaces can be calculated from a general form of the PDE on ϕ :

$$\frac{\partial \phi}{\partial t} = -\nabla\phi \bullet \vec{v} = -\nabla\phi \bullet \vec{F}(\vec{x}, D\phi, D\phi^2, \cdots)$$

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Level Set Method

- Level-set models are topologically flexible, they can easily represent complicated surface shapes that can, in turn, form holes, split to form multiple objects, or merge with other objects to form a single structure.
- Level-set models can also incorporate many of degrees of freedom, therefore they can accommodate complex objects.

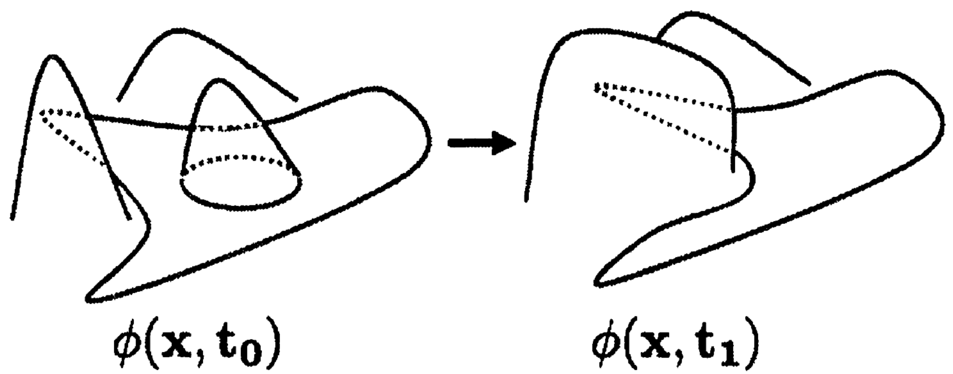

Figure 8.4. If the model is a level-set curve (represented as the shoreline of a lake) it can change as the embedding (terrain surrounding the lake) changes.

Segmentation using Level Sets

- Level-set models can lock onto edges using 2nd-derivative information from the input data, therefore they can be used to find edges, while also introducing a smoothing term.
- Suppose the model moves uphill on the gradient magnitude of the input, then the PDE is

$$\phi_t = \alpha \nabla |\nabla I| \bullet \nabla \phi + \beta |\nabla \phi| H$$

where I is the input datum, H is an energy function defined over the image.

- This model requires the initial model to be near edges in the input image in order for the 2nd derivatives of those edges to pull the surface model in the correct direction.

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Segmentation using Level Sets

- We add another speed term which encourages the model to flow into regions with greyscale values that lie within a specific intensity range.

$$\phi_t = \alpha \nabla |\nabla I| \bullet \nabla \phi + \beta |\nabla \phi| H + \gamma |\nabla \phi| (\varepsilon - |I - T|)$$

where *T* controls the brightness of the region to be segmented and ε controls the range of greyscale values around *T* that could be considered inside the object. When the model lies on a voxel with a greyscale level between T-ε and T+ε , the model expands, otherwise it contracts.

- With this combined edge- and region-based scheme, a user must specify *T*, ε , α, β, γ, as well as an initialization.

Segmentation using Level Sets

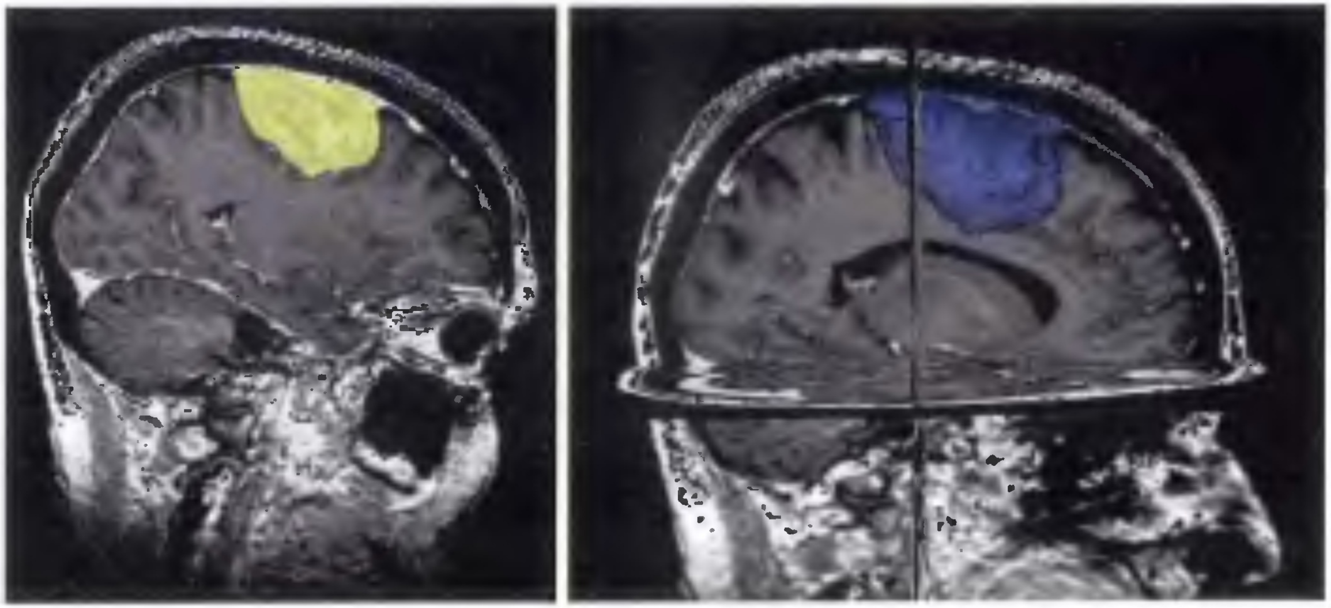


Plate VIII. (From Figure 8.12) Segmentation of MRI data using level sets: (a) a 2D slice from an MRI image; (b) 3D rendering of the level-set tumor segmentation.

Classification

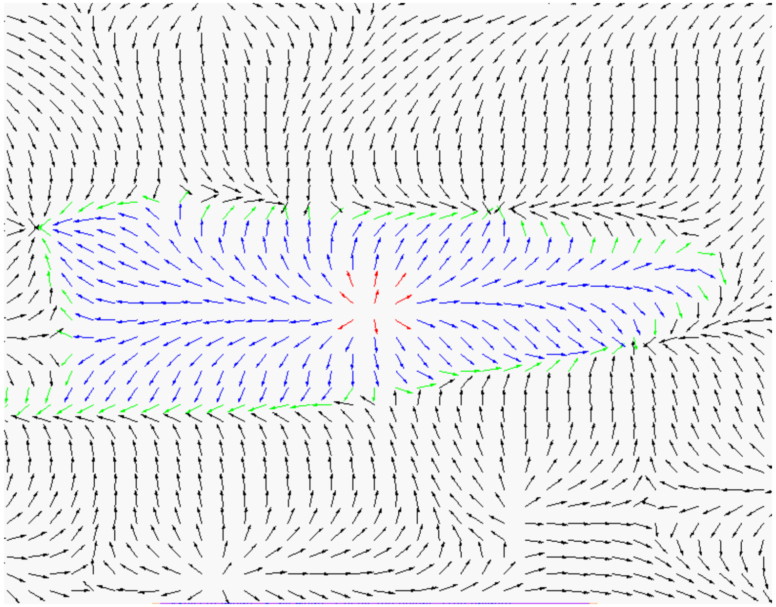
- Generates an intensity profile.
- Depending on the number of materials, groups the voxels into different intensity clusters.

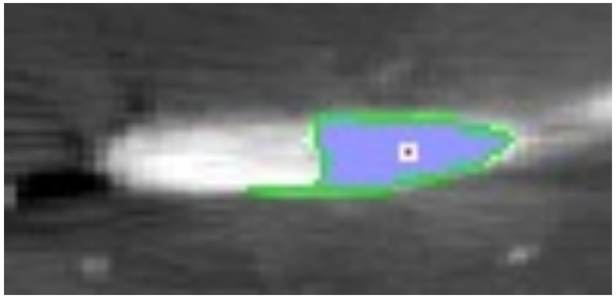
Given N intensity values we want to group them into C clusters

1. Select initial placement of the candidate clusters.
2. Compute the probability of each intensity falling into each cluster.
3. Update the position and stretch each cluster to fit better to the intensity profile.

Initial Value Picking Algorithm

- Computing
  - Gradient magnitudes
  - Vectors of the magnitudes
- Find seed points
- Follow the vectors from the seed point
- Take average of the range





Expectation Maximization (EM)

$$E\left(I,P,\theta^1\cdots\theta^c\right)=\prod_{k=1}^{N}\sum_{i=1}^{C}P_i^k g_i\left(x_k,\theta^i\right)$$

x_k : Data value

P_i^k : The probability of the k^{th} data belonging to the i^{th} cluster

θ^i : The function parameter of the i^{th} density function, $g_i(\cdot)$

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Two Steps of EM

- E-Step

$$p_i^{k(m)} = \frac{p_i^{k(m)} g_i\left(x_k, \theta^i\right)}{\sum_{i=1}^{C} p_i^{k(m)} g_i\left(x_k, \theta^i\right)}, \quad \forall i, k.$$

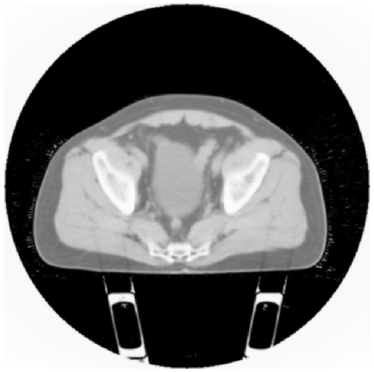
- M-Step

$$\mu_i^{(m+1)} = \left(\sum_{k=1}^{N} p_i^{k(m)}\right)^{-1} \sum_{k=1}^{N} p_i^{k(m)} x_k$$

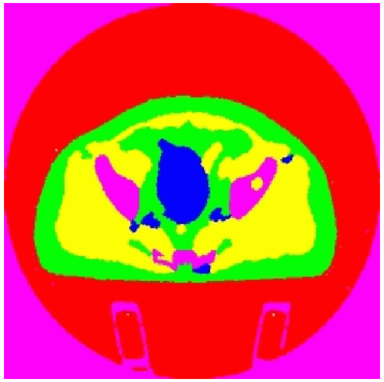
$$\sigma_i^{2(m+1)} = \left(\sum_{k=1}^{N} p_i^{k(m)}\right)^{-1} \sum_{k=1}^{N} p_i^{k(m)} \left(x_k - \mu_i^{(m+1)}\right)^2$$

Classification

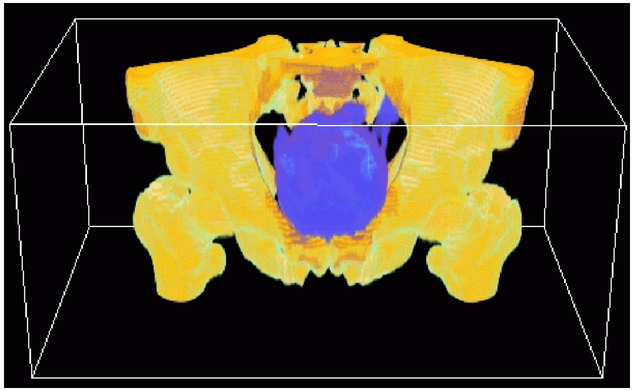
An Original Slice



After Filtering



After Classification

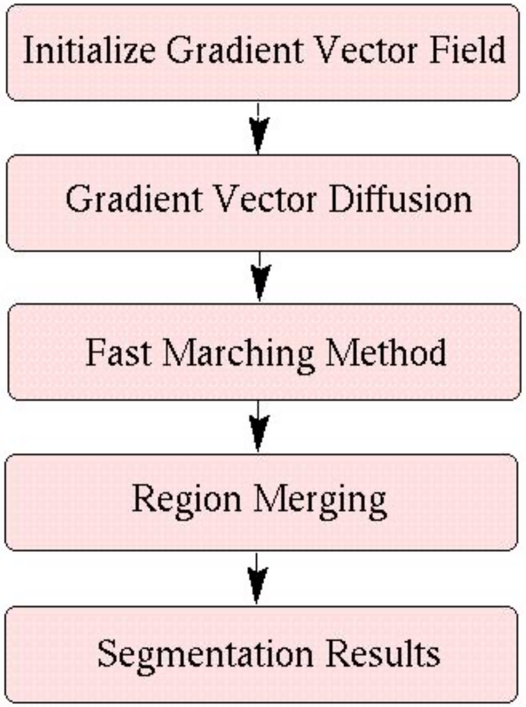


Segmented Bladder (blue)

Abdominal CT Imaging

Segmentation Using Gradient Vector and Region Merging

- Procedure



Z. Yu, C. Bajaj
Image Segmentation Using Gradient Vector Diffusion and Region Merging
Proceedings of the 16th International Conference on Pattern Recognition, vol. 2, pp. 941-944, Canada, August 11-15, 2002.

Anisotropic Gradient Vector Diffusion (AGVD)

Isotropic Diffusion (Xu *et al.*, 1998)

$$\begin{cases} \dfrac{\partial u}{\partial t} = \mu \nabla^2 u - (u - f_x)(f_x^2 + f_y^2) \\ \dfrac{\partial v}{\partial t} = \mu \nabla^2 v - (v - f_y)(f_x^2 + f_y^2) \end{cases}$$

where:

(u(t), v(t)) stands for the evolving vector field;

μ is a constant;

f is the original image to be diffused;

(f_x, f_y) = (u(0), v(0)).

Anisotropic Diffusion (Yu & Bajaj ICPR' 02)

$$\begin{cases} \dfrac{\partial u}{\partial t} = \mu \nabla (g(\alpha) \cdot \nabla u) - (u - f_x)(f_x^2 + f_y^2) \\ \dfrac{\partial v}{\partial t} = \mu \nabla (g(\alpha) \cdot \nabla v) - (v - f_y)(f_x^2 + f_y^2) \end{cases}$$

where:

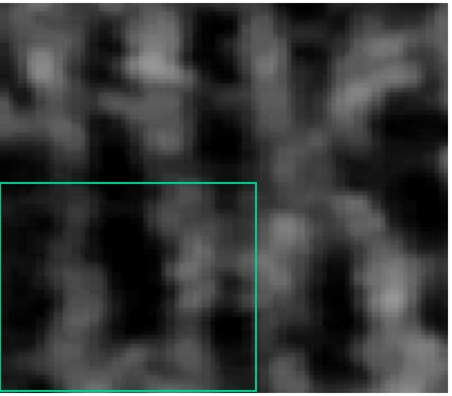
(u(t), v(t)) stands for vector field;

μ is a constant; (f_x, f_y) = (u(0), v(0)).

f is the original image to be diffused;

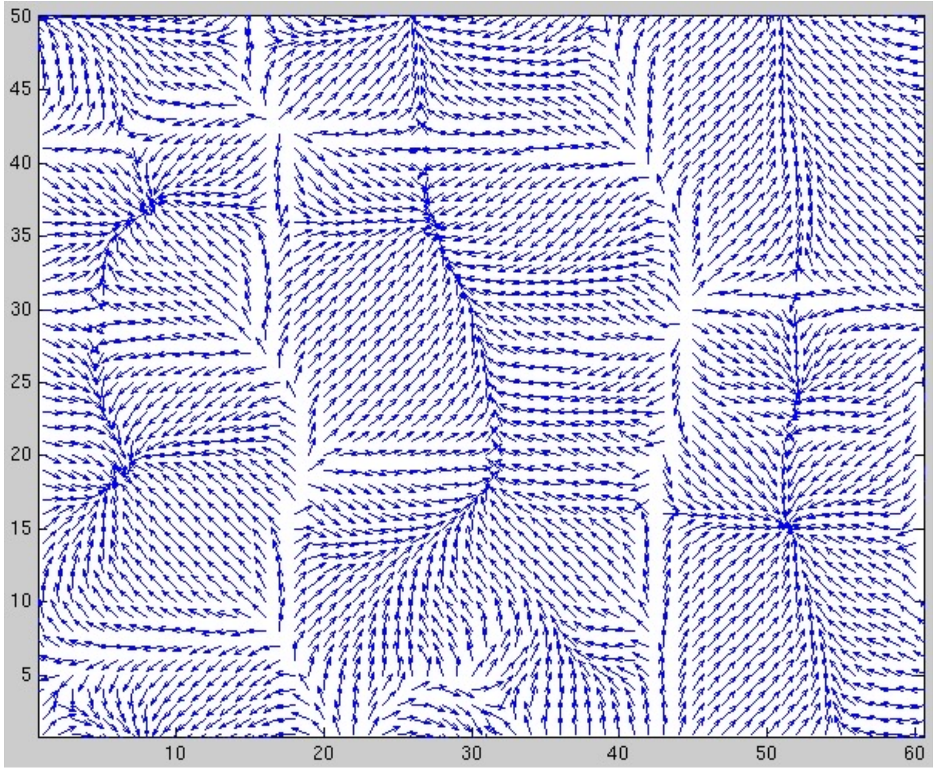
g(.) is the angle between two vectors.

Z. Yu, C. Bajaj
Anisotropic Vector Diffusion in Image Smoothing
Proceeding of the 9th IEEE International Conference on Image Processing, vol. 1, 2002, 828-831.

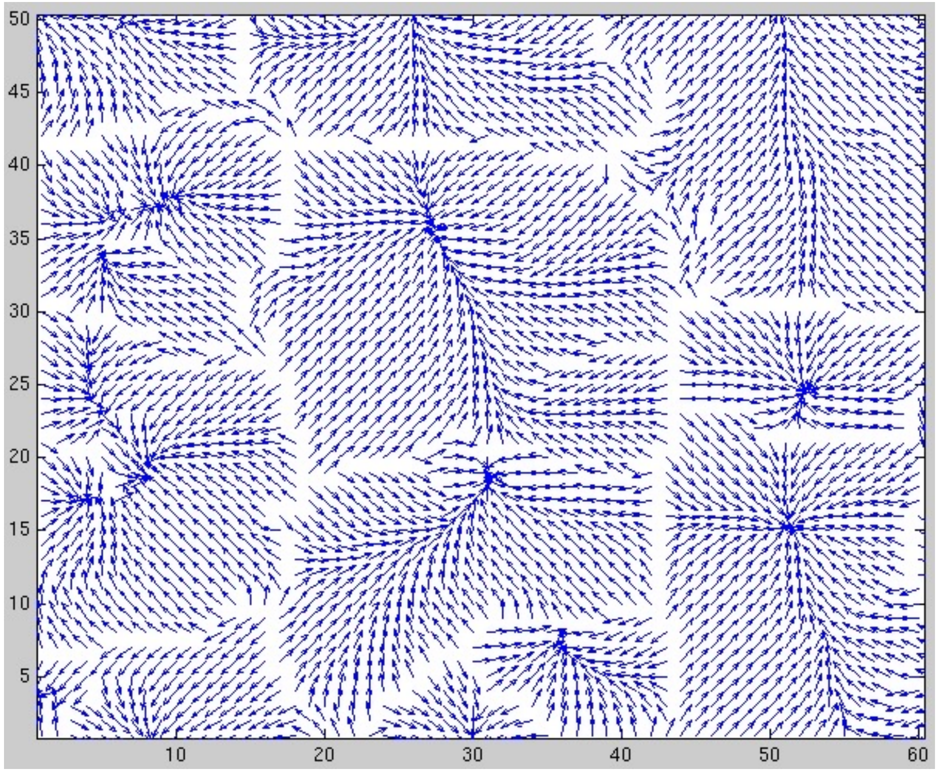


Gradient Vector Diffusion vs. Anisotropic Gradient Vector Diffusion

Isotropic diffusion



Anisotropic diffusion



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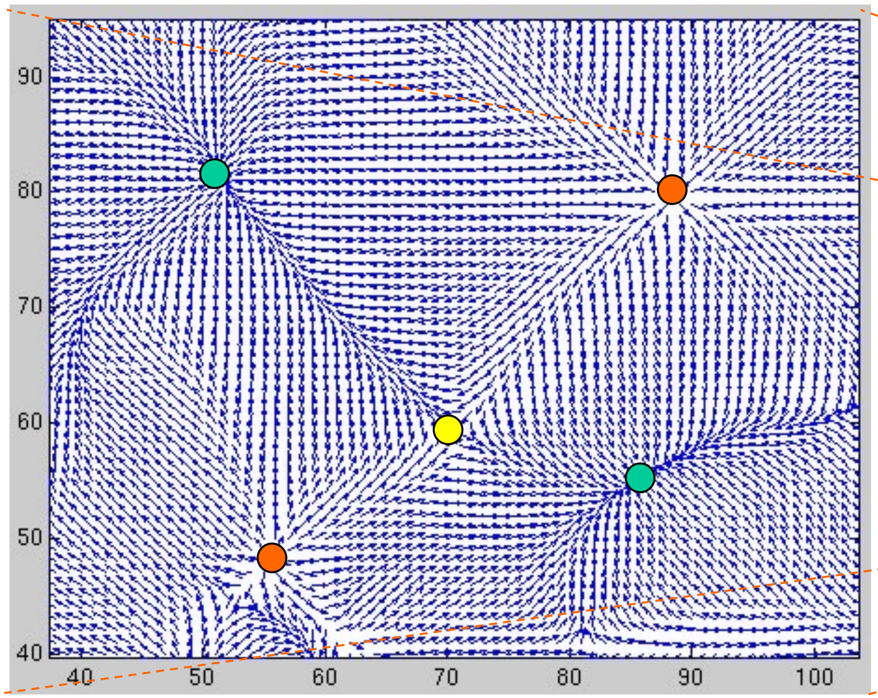
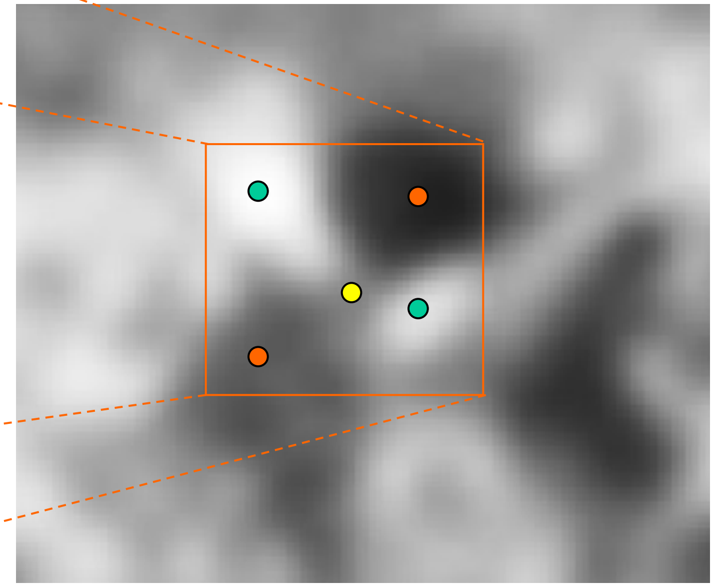
Compute Critical Points Using AGVD





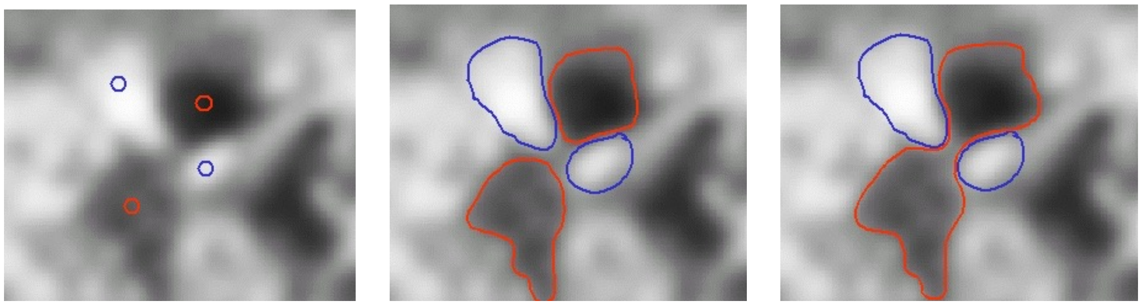
: minimum

: maximum

: saddle

Image Segmentation

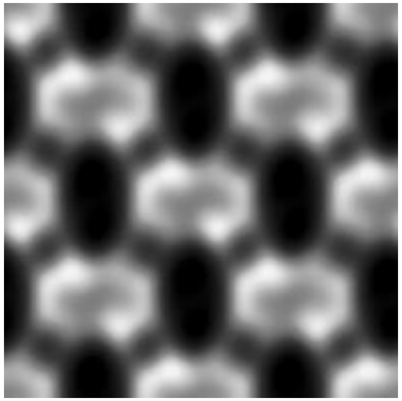
- Gradient vector domain
- Select seed points
- Fast marching method with multiple seeds
  - Marching simultaneously from each seed
  - Marching speed is controlled by the gradient
- Merging domain
  - Topology-changing for seeds in the same class
  - Topology-preserving for seeds in different class



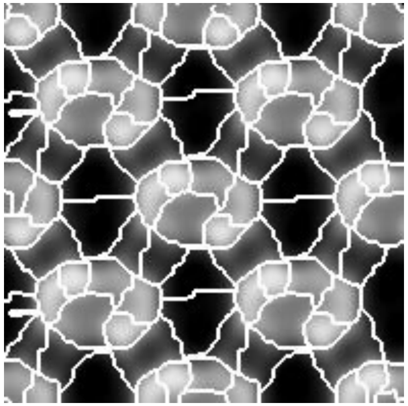
[YB2005] Z. Yu, C. Bajaj (2005) Automatic Ultrastructure Segmentation of Reconstructed CryoEM Maps of Icosahedral Viruses *IEEE Transactions on Image Processing: Special Issue on Molecular and Cellular Bioimaging*, 2005 Sep;14(9):1324-37.

[YB2003] C. Bajaj, Z. Yu, M. Auer (2003) Volumetric Feature Extraction and Visualization of Tomographic Molecular Imaging *Journal of Structural Biology*, Volume 144, Issues 1-2, October 2003, Pages 132-143.

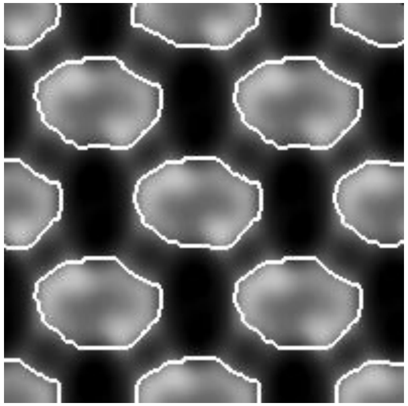
Segmentation



Original image



Initial Segmentation



Final Segmentation

Z. Yu, C. Bajaj
Image Segmentation Using Gradient Vector Diffusion and Region Merging
Proceedings of the 16th International Conference on Pattern Recognition, vol. 2, pp. 941-944, Canada, August 11-15, 2002.

Deformable Models

- Deformable models are dynamic models whose 2D or 3D shape evolves under the influence of external and internal forces.

- The process of surface evolution corresponds to the minimization of an energy function. The energy function is composed of three components:
  - The internal energy which helps keep the continuity and regularity of the contour/surface.
  - The image energy which pushes the deformable model to edge features in the original image.
  - The user-defined energy that can speed up the deformation and/or push the model out of local minima.

Deformable Models

- By using deformable models, we can use the internal energy constraints, which include global or local prior information about the object, to define the object boundary even if there is no enough gradient information in the boundary region.

- The deformable model is robust to noise and has good performance for images with a low signal-to-noise ratio (SNR).

- By using the parameterized deformable model, it is easy to achieve sub-pixel accuracy in segmentation applications.

- Deformable models are easy to interact with based on user defined forces.

Snakes

- Active contour models (snakes) developed by M. Kass *et al*. is an interactive segmentation methodology based on energy minimization.
- An active contour is a deformable contour defined by a set of points with the initial position vector $p(s)$ and the displacement vector $d(s)$, where s is the arc-length. The motion of the contour is governed by the minimization of the following energy function:

$$E = \int \left[E_{internal}(d) + E_{image}(p+d) + E_{user}(p+d) \right] ds$$

  - $E_{internal}$ controls the stretching and bending of the snake.
  - E_{image} term is a function of short-range image operators (gradient magnitude or gradient vector flow for segmentation). It attracts contour points to appropriate image features.
  - E_{user} is the energy derived from the position of the user-controlled cursor. It guides interactively the contour away from the local minimum.

Fourier Parameterized Models

- Fourier parameterized models approach the segmentation (or edge finding) as an optimization problem.

- In an image $I(x, y)$, let $v(t, p)$ be the deformable boundary template, where $t \in [0, 2\pi)$ and p is the parameter vector. The segmentation procedure is to find the parameter vector p_{opt} which will fit the model to the object of interest in an optimal way.

Fourier Parameterized Models

- To achieve the optimal parameter vector, a global objective function H(p) is defined

$$H(p) = \int_t h(t,p)\|v'(t)\|dt$$

where $h(t$, p) is a local objective function derived from image information, e.g., image gradient. To locate the edge feature of the object of interest, we set $h(t, p) = \nabla I(x,y)$

$$H(p) = \int_t \|\nabla I(v(t, p))\|\|v'(t)\|dt$$

$H(p)$ is evaluated at N discrete points on the curve to approximate the optimal solution.

Deformable Models

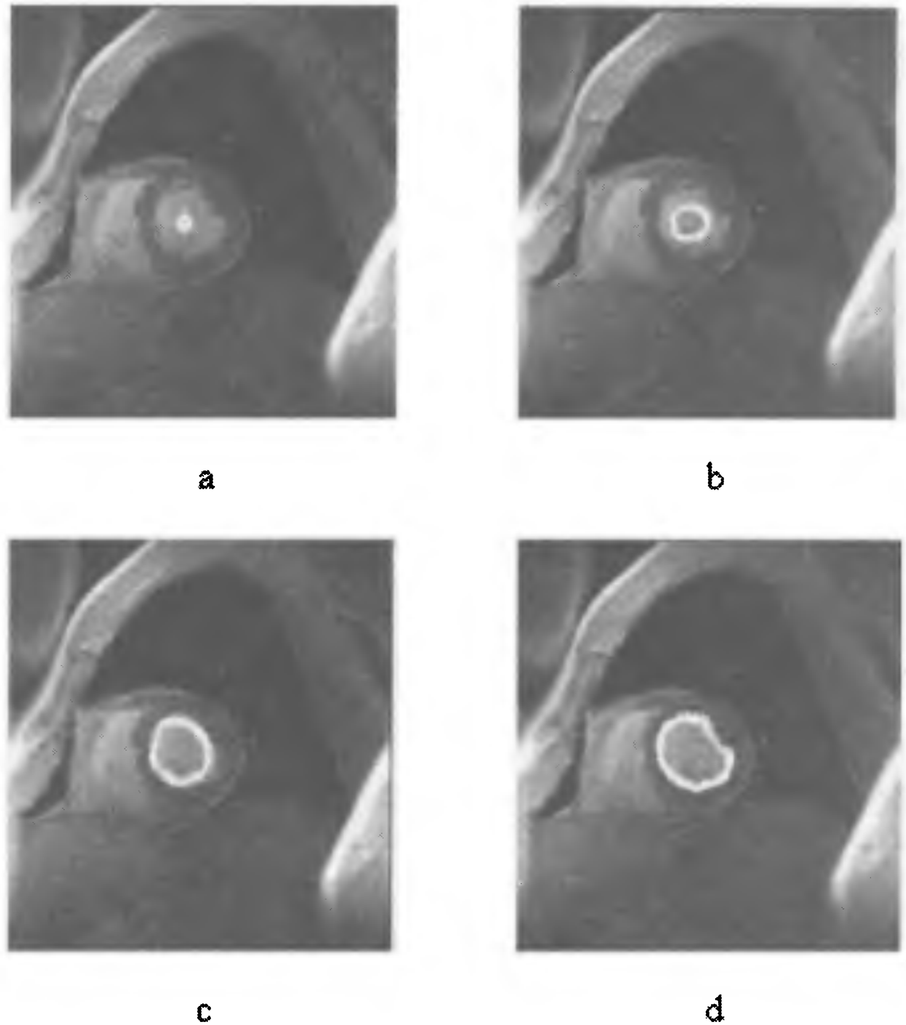


Figure 9.3. (a) A deformable model starts inside the left ventricle; (b) the deformable model expand under the effect of balloon force; (c) the deformable model begins to deform under the effect of gradient flow when close enough to the boundary; (d) final fitting result. [Data courtesy of Dr. Leon Axel, New York University]

Deformable Models

Figure 9.4. (a) One slice in the 3D volume of the jaw; (b) the segmentation result of the Voronoi diagram method and deformable model (see [6]); (c) the projection of the segmentation result onto the original image. [Data from the National Library of Medicine, Visible Human Project]

Deformable Models

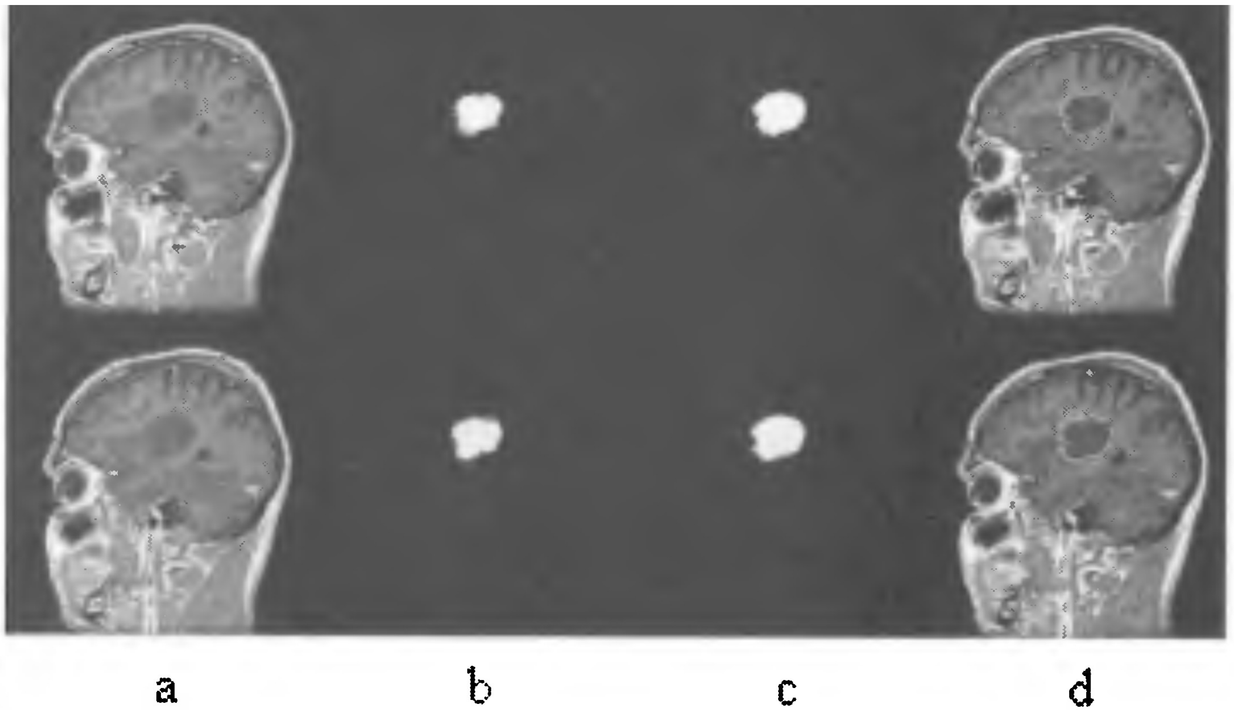


Figure 9.5. Column (a Two slices from the tumor data volume; (b The Region-based segmentation results on these Two slices; (c) the deformable model segmentation results on these slices; (d) the projection of the segmentation results onto the original image. [Data courtesy of Dr. Peter Ratiu, Harvard Brigham and Women's Hospital.]

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