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Project Proposal

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Numerical Methods for Interface Tracking Problems

In many systems modeled with partial differential equations, interfaces play an important role in prescribing spatially- and time-varying boundary conditions. These interfaces can be fluid-fluid in the case of multiphase fluid models, fluid-solid in the case of fluid-structure interaction problems, or solid-solid in the case of contact mechanics. The modeling of these interfaces using traditional techniques requires the computationally expensive task of explicitly tracking the boundary locations of discrete bodies and searching for body penetration/interaction locations. For time-varying problems, this is often accompanied by remeshing of the governing geometry. Techniques that allow for this tracking/searching to be modeled implicitly through modifications to the governing equations or to the material properties of the bodies can greatly increase computational efficiency, making tractable many more types of interface problems.

In this project, we will investigate the numerical methods for multi-phase interface problems using different interface tracking methods. During our literature review, we will look into techniques such as the Immersed Boundary Method, Phase Field Method, Level Set Method, and Finite Cell Method. After familiarization with the different methods, we will choose a single method to implement for increased complexity problems. We will investigate this complexity in two branching approaches, one incorporating time dependence and one incorporating non-convex geometries. This project will consist of the following tasks (higher number tasks will be achieved if time permits):

Tasks

1. **Task 1:** Steady-state problem (Poisson Equation) on simple geometry (circle)
2. **Task 2:** Time-dependent conservation law on a convex, stationary geometry
3. **Task 3:** Steady-state conservation law on non-convex, stationary geometry
4. **Task 4:** Time-dependent conservation law with a time-varying, convex geometry
5. **Task 5:** Time-dependent conservation law on time-varying, non-convex geometry
6. **Task 6:** Apply solver to problem of choice.