# Summary

- Topics:

1. Bio-medical imaging
2. Image processing
3. Geometric modeling and computer graphics
4. Mesh generation
   - Marching Cubes/Dual Contouring
   - Tri/Tet Meshing
   - Quad/Hex Meshing
   - Quality Improvement
5. Computational mechanics
6. Bio-medical applications

# Topic 4: Mesh Generation – Unstructured Tri/Tet Meshing

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# Structured vs. Unstructured

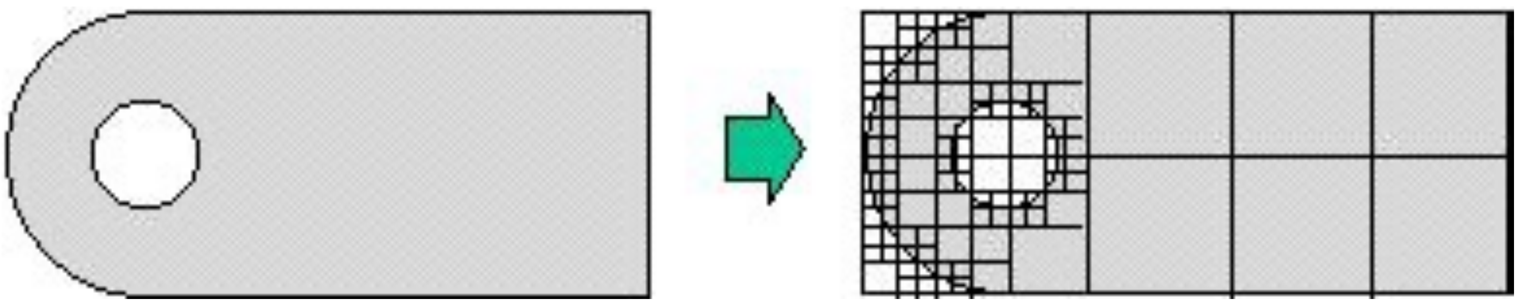
- A structured mesh can be recognized by all interior nodes of the mesh having an equal number of adjacent elements.
- Unstructured mesh generation, on the other hand, relaxes the node valence requirement, allowing any number of elements to meet at a single node.

# Triangular/Tetrahedral Meshing

- Three main methods:
  - Octree: the quadtree/octree data structure is used to generate tri/tet meshes.
  - Delaunay: generate tri/tet meshes from a given points cloud using the Delaunay criterion "empty sphere".
  - Advancing Front: starting from a given boundary tri mesh, generate tet meshes by inserting points inward. The boundary surface mesh is preserved.

# Octree

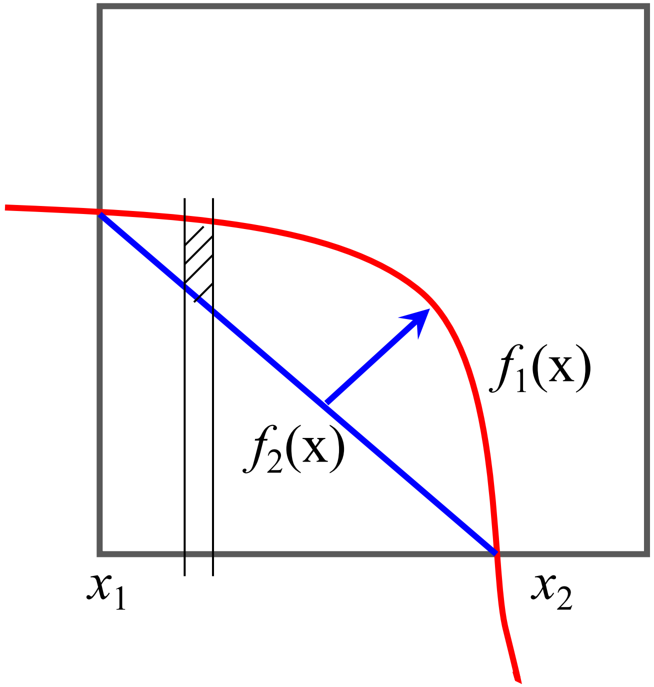
- The octree technique was primarily developed in the 1980s by Mark Shephard’s group at Rensselaer Polytechnic Institute (RPI).
- Cubes containing the geometric model are recursively subdivided until the desired resolution is reached.



Quadtree decomposition of a simple 2D object

# Surface Error

- What is the desired resolution? How to decide it?
  - Maximal distance: $\max\left|f_1(x) - f_2(x)\right|$
  - Area: $area = \int_{x_1}^{x_2} \left|f_1(x) - f_2(x)\right| \cdot \Delta x \, dx$

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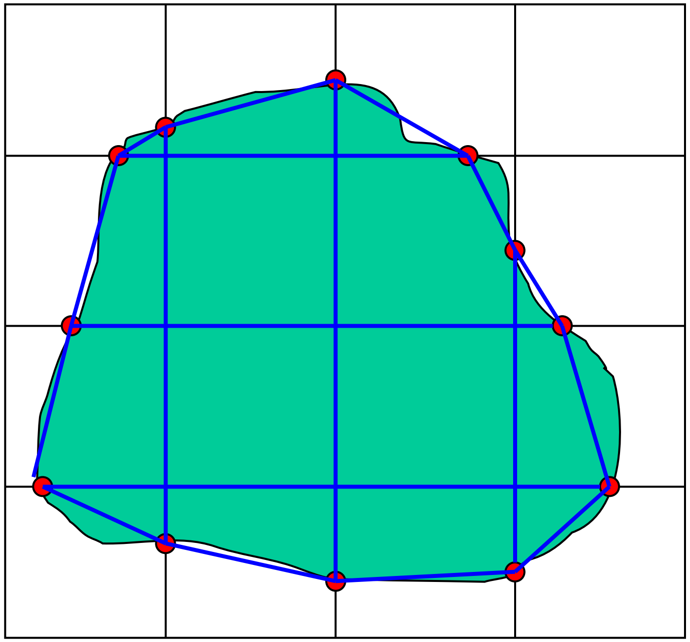
# Octree

- Irregular cells are created where cubes intersect the surface, often requiring a significant number of surface intersection calculations.
- Tetrahedra are generated from both the irregular cells on the boundary and the internal regular cells.



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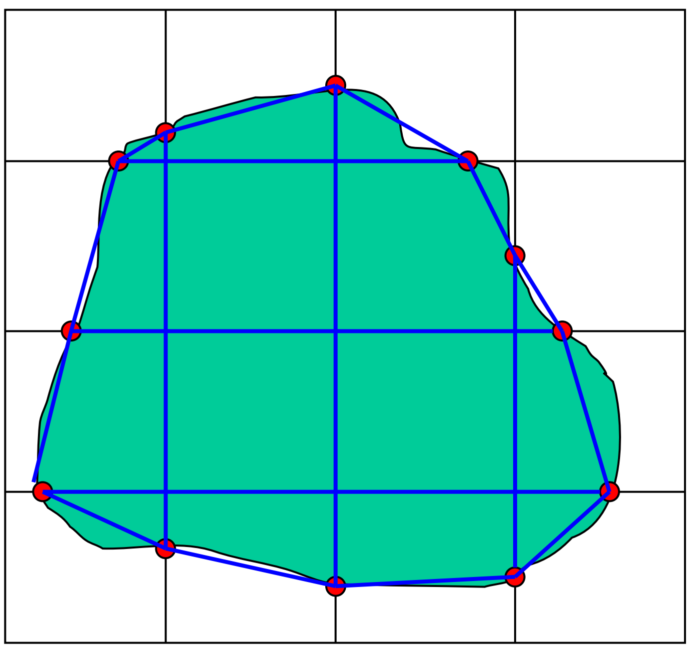
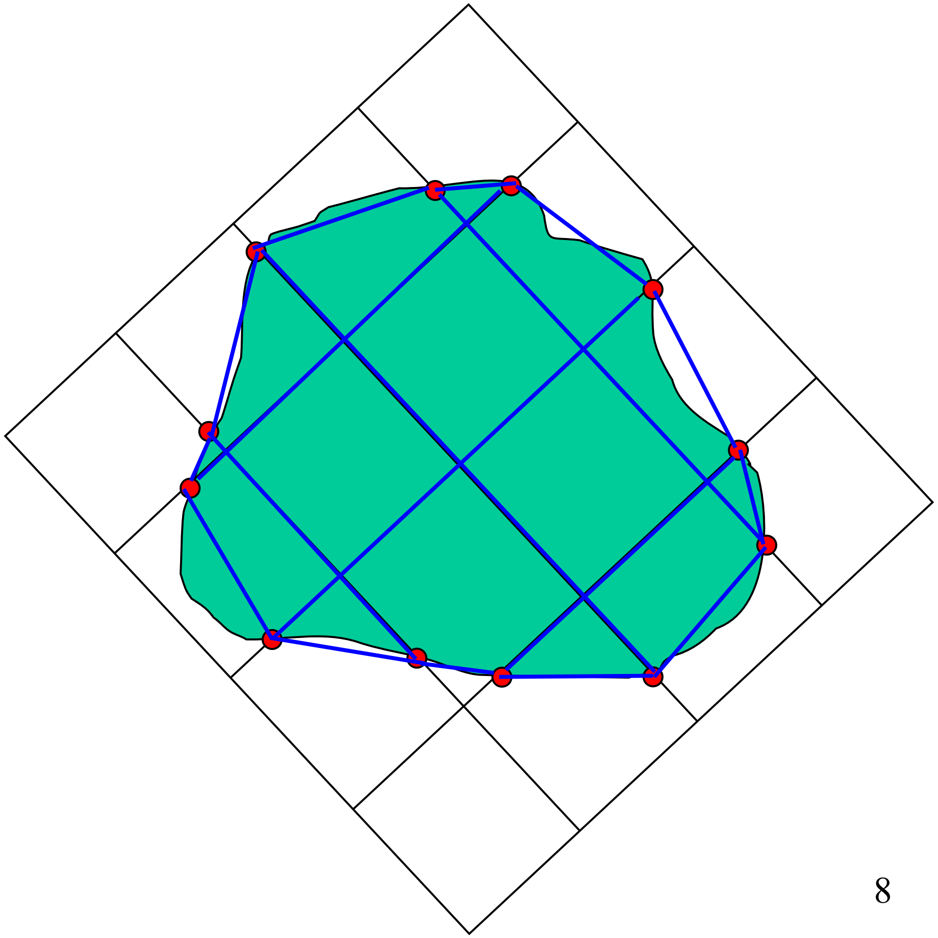
# Octree

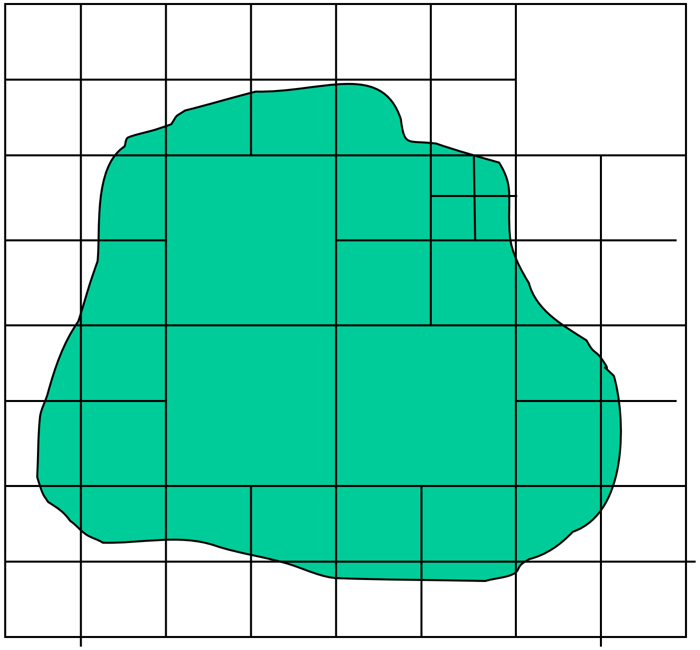
- The octree technique does not match a pre-defined surface mesh, as an advancing front or Delaunay mesh might.
- The resulting mesh will change as the orientation of the cubes in the octree structure is changed.





# Octree

- To ensure element sizes do not change too dramatically, a maximum difference in octree subdivision level between adjacent cubes can be limited to be one.



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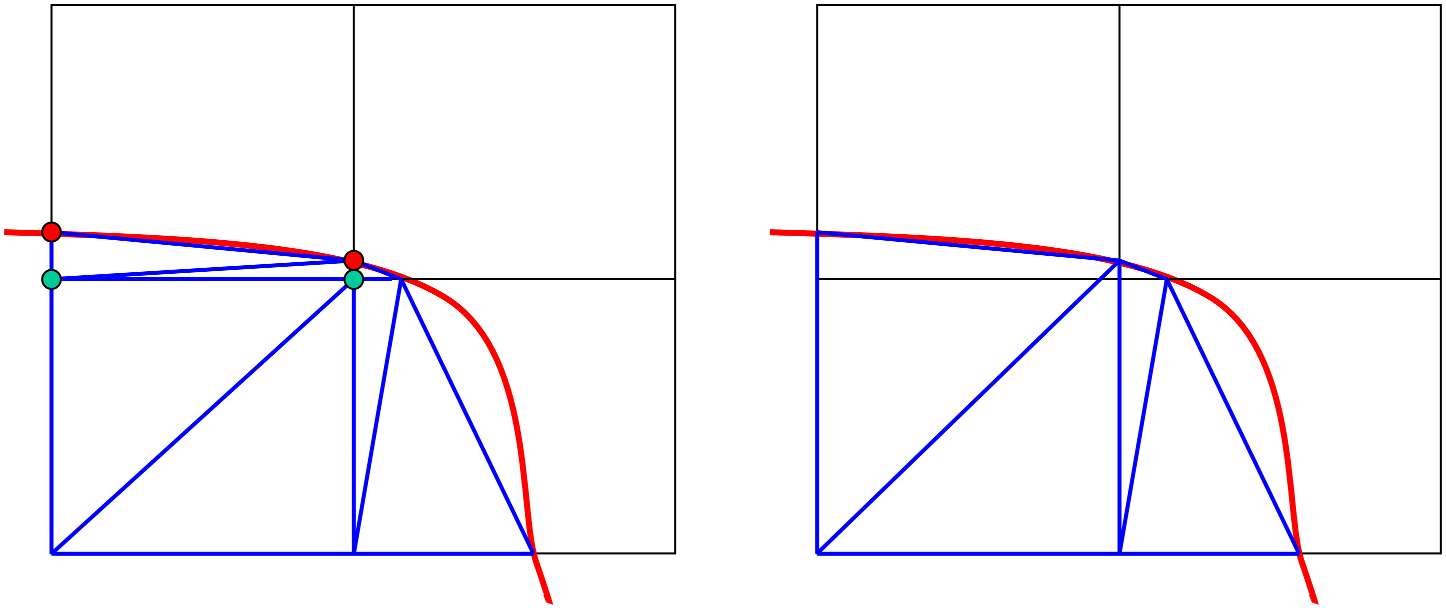
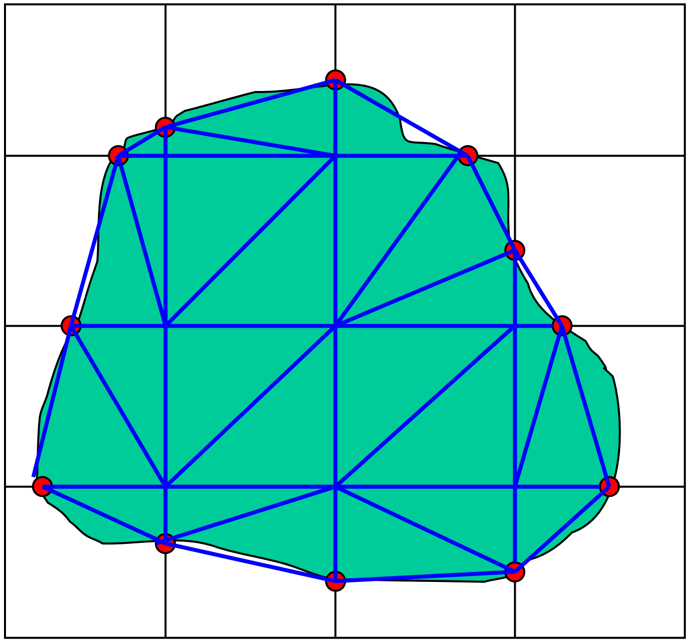
# Octree

- Smoothing and cleanup operations can also be employed to improve element shapes, e.g., merge green points to red ones.



# Marching Cubes

- Marching Cubes connects all intersection points along the boundary.
- Interior cubes are also triangulated.



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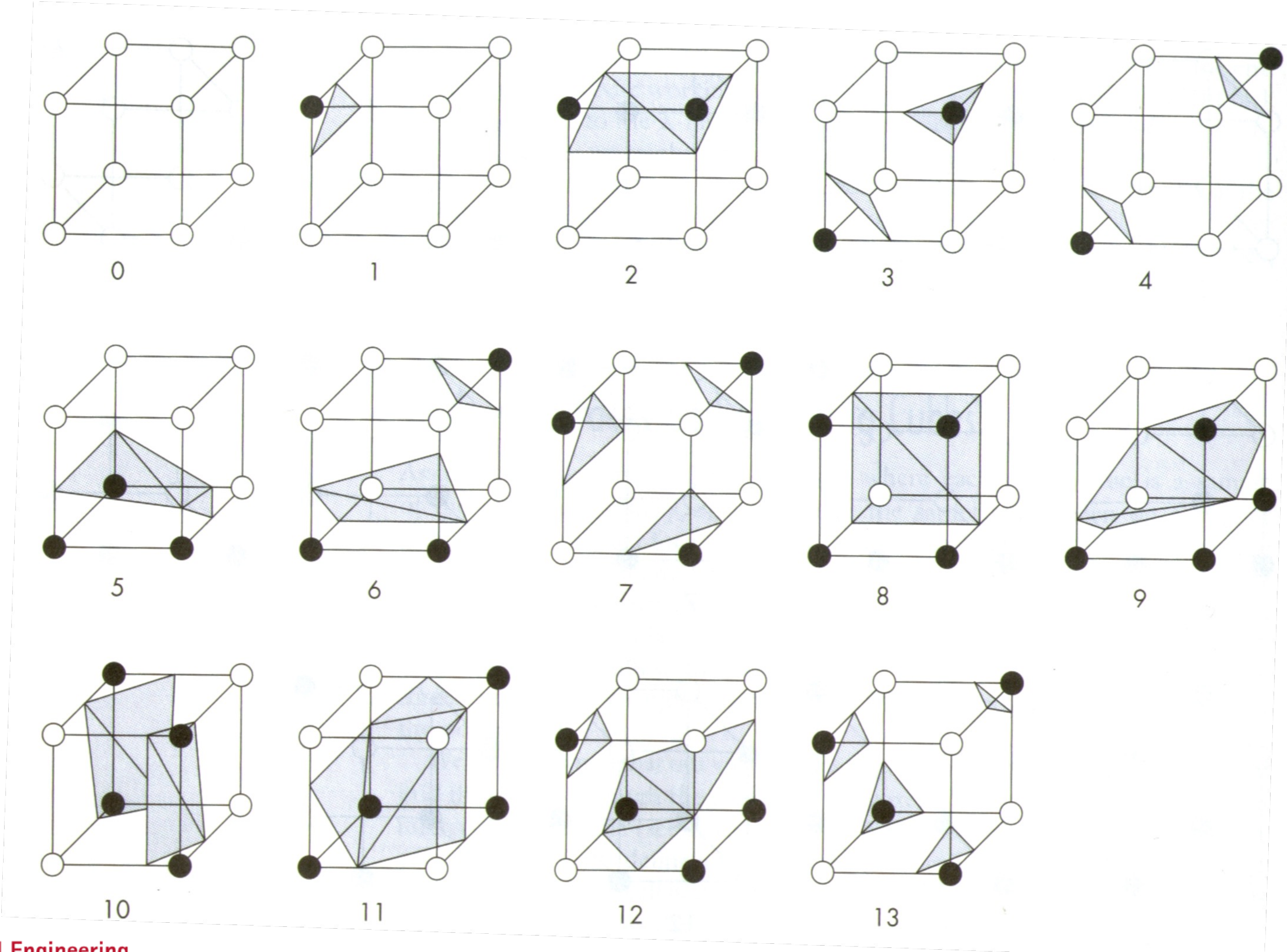
# Marching Cubes in 3D

- Marching Cubes and its variants visit each cell, and generate one or more polygons for each cube in the grid that intersects the contour.

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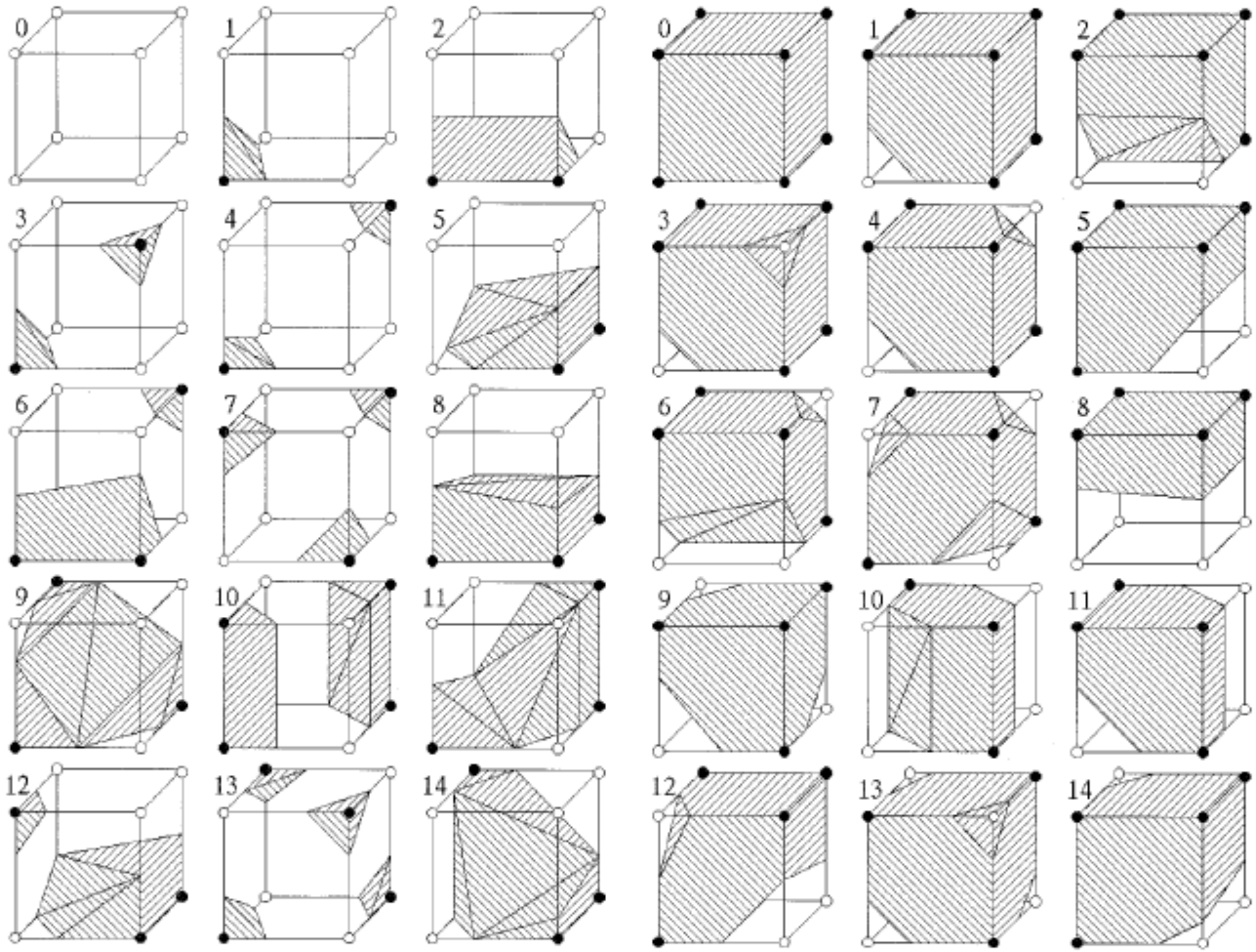
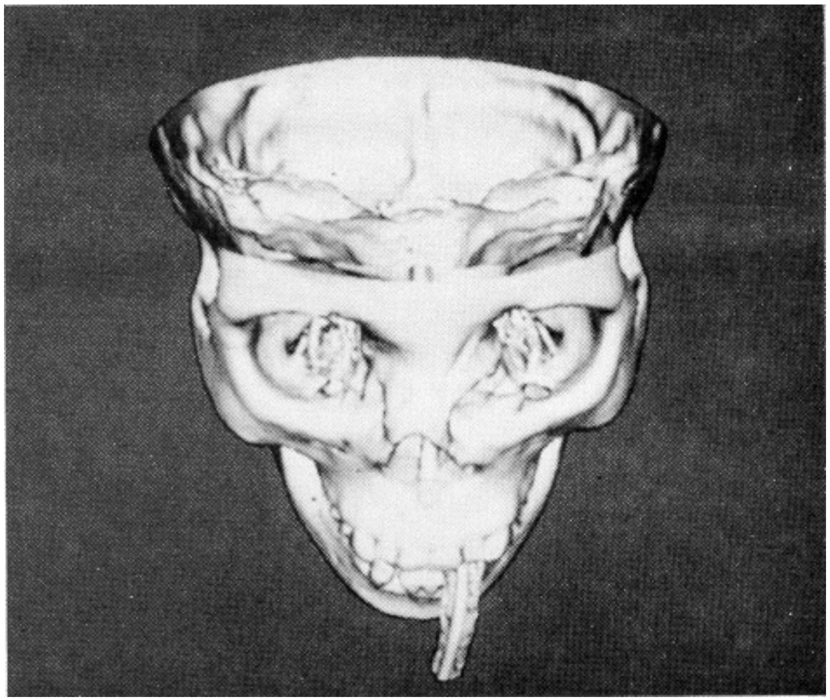
# Marching Cubes in 3D

- Marching Cubes and its variants visit each cell, and generate one or more tetrahedra for each cube in the grid that intersects the contour, and interior cubes.

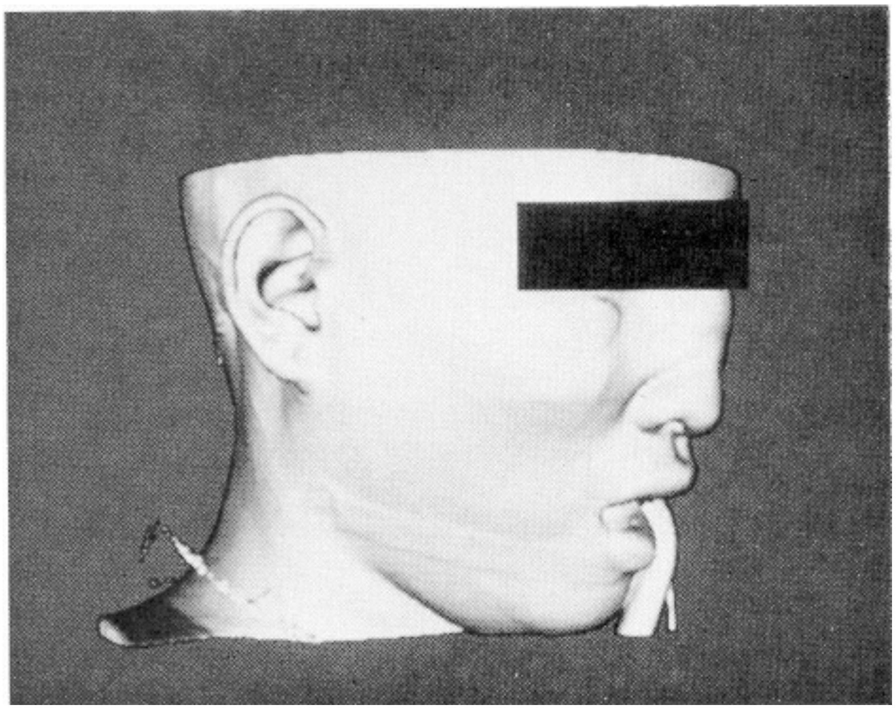


(a) (b)

# Results



Bone structure

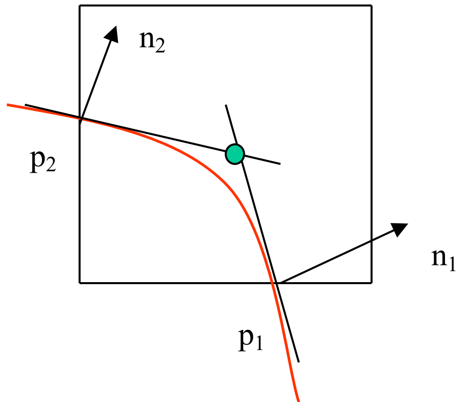


Soft tissue

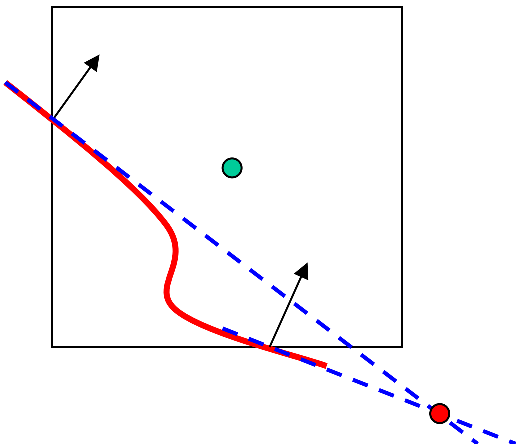
# Dual Contouring

- Dual Contouring calculates minimizer points.

$$QEF[x] = \sum_i (n_i \cdot (x - p_i))^2$$

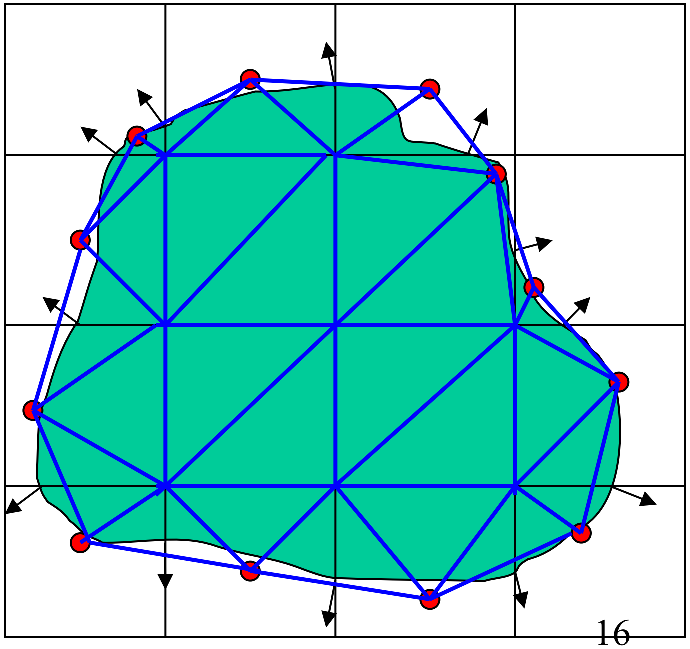



- If the intersection point of the two tangent lines are outside this octree cell, then choose the center of the cell as the minimizer point.



# Dual Contouring

- Sign change edge is an edge whose two end points lie on different sides of the isocontour.
- Method 1:
  - Sign change edge: two minimizer points and the interior end point of this sign change edge construct a triangle.
  - Interior edge in the boundary cell: the minimizer point in the boundary cell and the two end points of this edge construct a triangle.
  - Interior face: split into two triangles.





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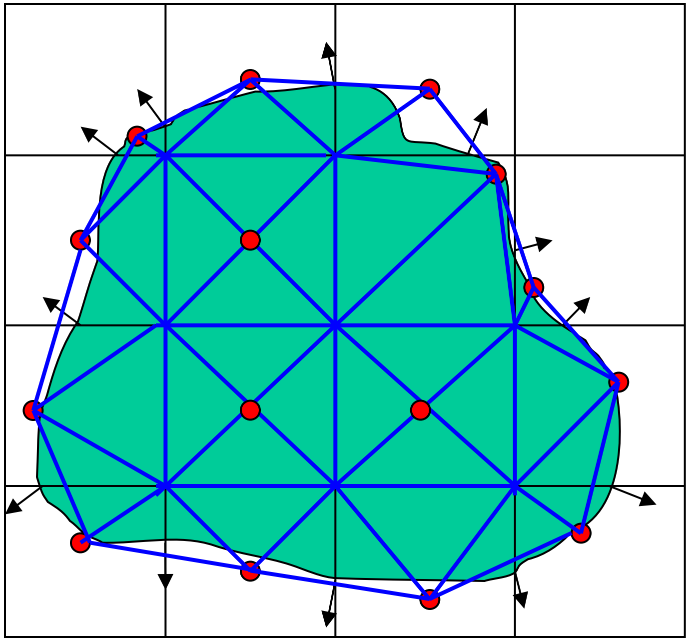
# Dual Contouring

- Method 2:
  - Signed change edge: two minimizer points and the interior end point of this sign change edge construct a triangle.
  - Interior edge: the minimizer point in the boundary cell and the two end points of this edge construct a triangle.



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# Available Software of The Octree Technique

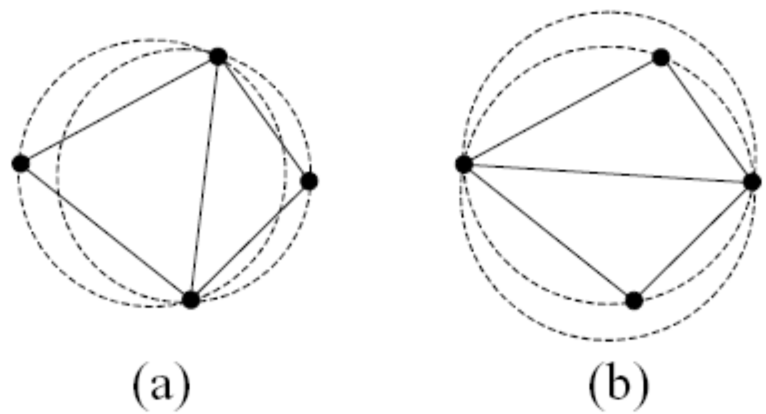
- SCOREC – Rensselaer, FMDB, Trellis, and PHASTA
  Scientific Computation Research Center (SCOREC), Rensselaer Polytechnic Institute, http://www.scorec.rpi.edu

- QMG – Steve Vavasis at University of Waterloo
  https://uwaterloo.ca/scholar/vavasis

- LBIE-Mesher – Our Computational Biomodeling Lab at CMU
  http://www.andrew.cmu.edu/user/jessicaz/

# Delaunay Triangulation

- Delaunay criterion: “empty sphere”, says that any node must not be contained within the circumsphere of any tetrahedra within the mesh.




Example of Delaunay criterion. (a) maintains the criterion while (b) does not.

# Delaunay Triangulation

- Charles Lawson and Dave Watson utilized the Delaunay criterion for developing algorithms to triangulate a set of vertices (points cloud).

- Qhull – a public domain 3D Delaunay program was developed in Univ. of Minneapolis.

  http://www.qhull.org/

- Following work: Timothy Baker at Princeton, Nigel Weatherill at Swansea, Paul-Louis George at INRIA.

  http://www.inria.fr/

## Delaunay Triangulation

- Delaunay Criterion provides the criterion for which to connect a set of existing points in space.
- A typical approach is to first mesh the boundary of the geometry from a given initial set of nodes using the Delaunay criterion.
- Nodes are then inserted incrementally into the existing mesh, redefining the triangles or tetrahedra locally to maintain the Delaunay Criterion.

# Point Insertion

- Define nodes from a regular grid of points. In order to provide for varying element sizes, a user specified sizing function can be defined to control the node insertion.
- Define nodes at triangle or tetrahedral centroids (Weatherill and Hassan).
- Define nodes at element circumcircle/sphere centers (Chew and Ruppert). This method can “guarantee quality”. Jonathan Shewchuk at CMU developed a 2D free code using this method.

  http://www.cs.cmu.edu/~quake/triangle.html

# Point Insertion

- David Marcum, Engineering Research Center (ERC) at Mississippi State Univ., developed an advancing front method. Nodes are inserted incrementally, but added from the boundary towards the interior.

  http://www.me.msstate.edu/people/faculty/dave-marcum/

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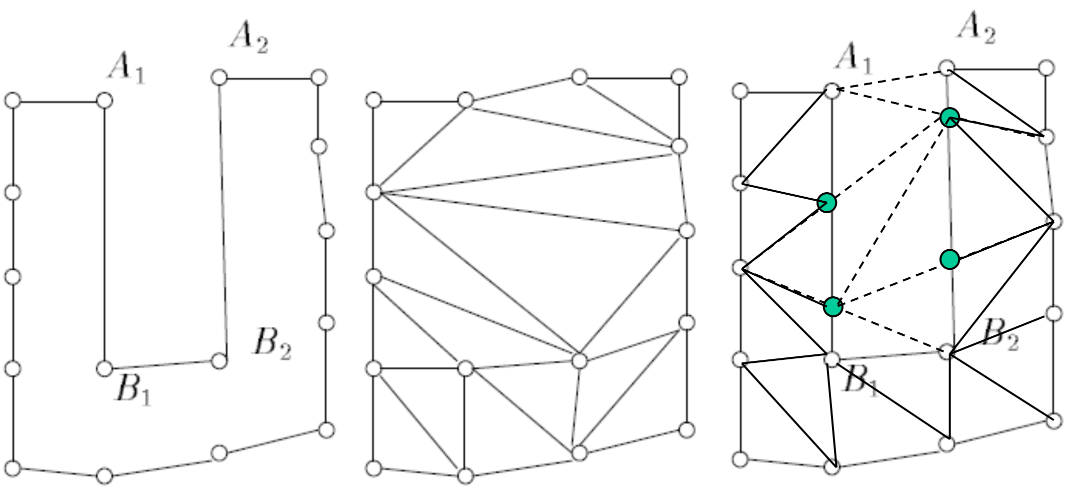
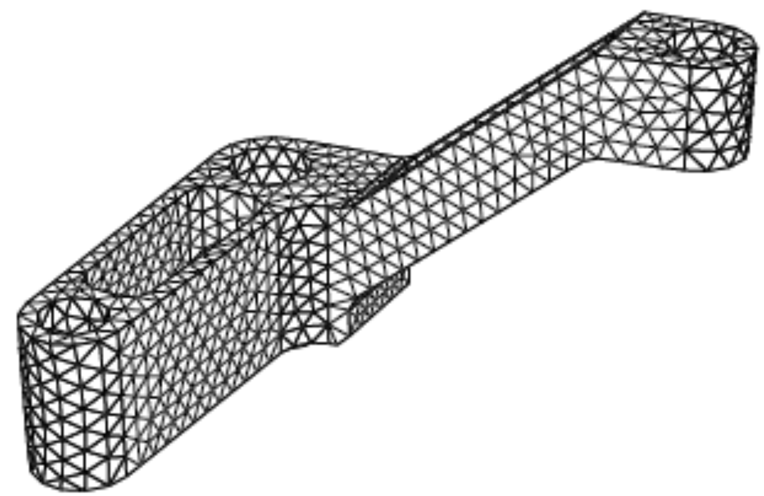
# Point Insertion

- INRIA – GSH3D inserts points along edges. Simulog Technologies




Edges are missing while their endpoints exist.



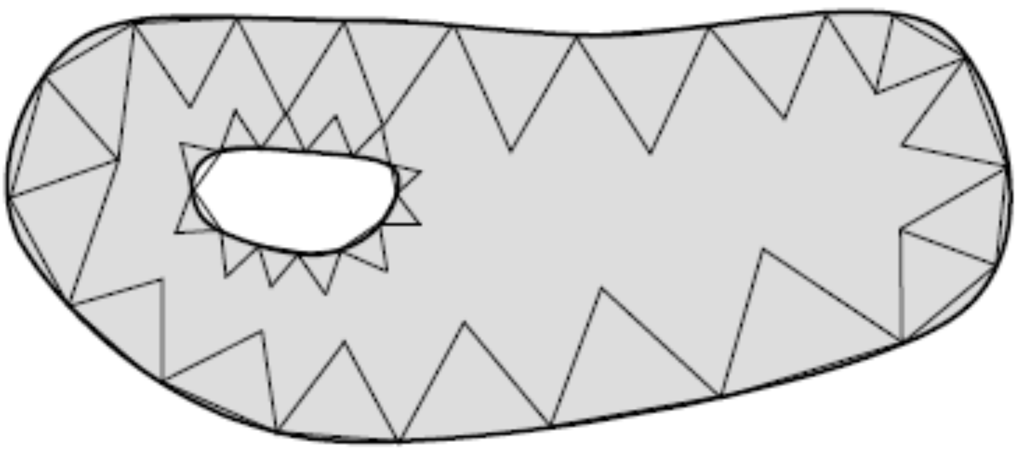
Thin structures

# Boundary Constrained Triangulation

- In many finite element applications, there is a requirement that an existing surface triangulation be maintained.
- In most Delaunay approaches, before internal nodes are generated, a 3D tessellation of the nodes on the geometry surface is produced.
- No guarantee that the given surface triangulation will be maintained.

# Advancing Front Method

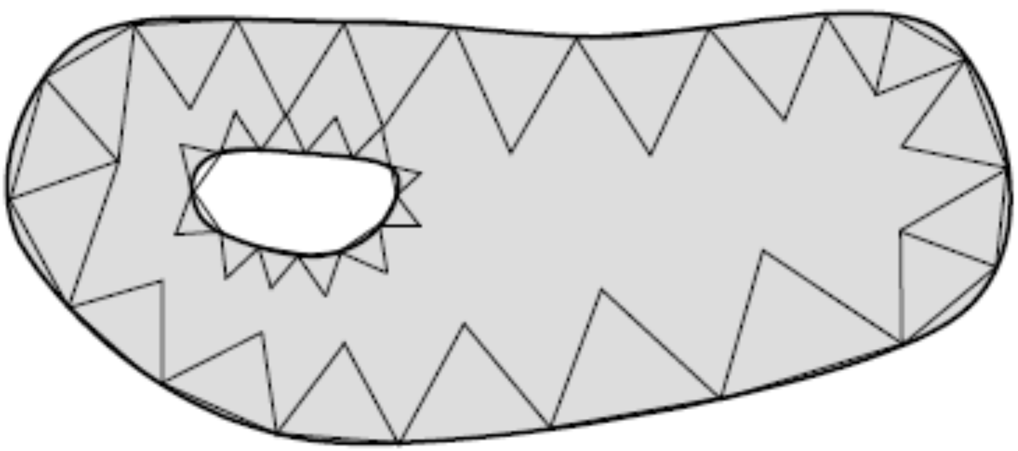
- Two main contributors: Rainald Lohner at George Mason Univ. and S. H. Lo at Univ. of Hong Kong.
- Tetrahedra are built progressively inward from the triangulated surface. An active front is maintained where new tetrahedra are formed.



Example of advancing front where one layer of triangles have been placed.

# Advancing Front Method

- As the algorithm progresses, the front will advance to fill the remainder of the area with triangles.
- In 3D, for each triangular facet on the front, an ideal location for a new fourth node is computed.



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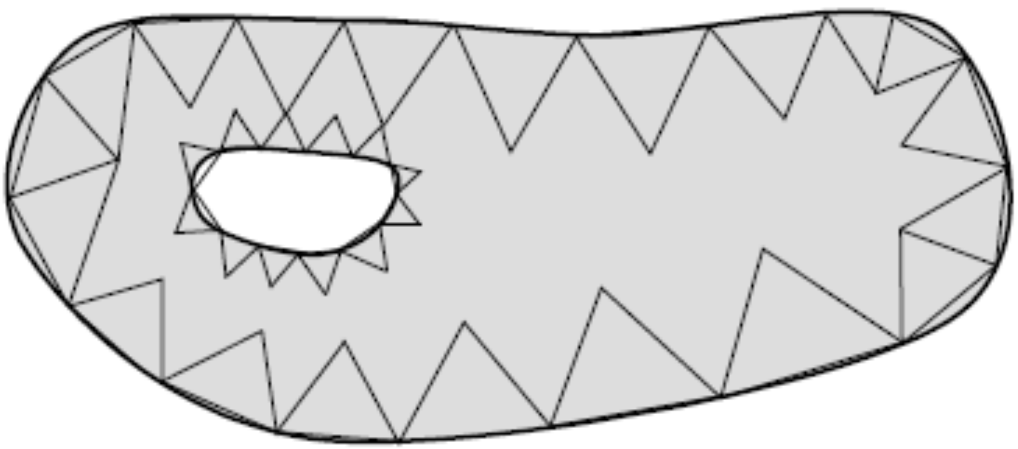
# Advancing Front Method

- The algorithm selects either the new fourth node or an existing node to form the new tetrahedron based on which will form the best tetrahedron (e.g., Delaunay criterion), as well as intersection checking.
- A sizing function can also be defined to control mesh sizes.



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# Available Codes

- A version of S. H. Lo's advancing front mesh generator is available with the ANSYS suite of mesh generation tools.

  http://www.ansys.com

- Advancing layers are also used for generating boundary layers for CFD (Computational Fluid Dynamics) applications, which is suitable for anisotropic meshing.
- Pirzadeh's code, VGRID, is available from TetraUSS in NASA, Langley.

  http://tetruss.larc.nasa.gov/

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- Nina Amenta's link: http://www.geom.uiuc.edu/software/cglist/welcome.html