Summary

- Topics:

1. Bio-medical imaging
2. Image processing
3. Geometric modeling and computer graphics
4. Mesh generation
    - Marching Cubes/Dual Contouring
    - Tri/Tet Meshing
    - Quad/Hex Meshing
    - Quality Improvement
5. Computational mechanics
6. Bio-medical applications

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Topic 4: Mesh Generation – Unstructured Quad/Hex Meshing

Jessica Zhang
Department of Mechanical Engineering
Courtesy Appointment in Biomedical Engineering
Carnegie Mellon University
jessicaz@andrew.cmu.edu
http://www.andrew.cmu.edu/user/jessicaz

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Tri/Tet vs. Quad/Hex

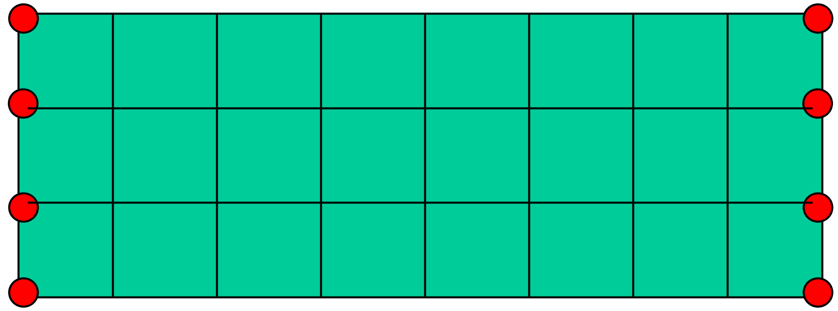
- Triangular/Tetrahedral mesh generation is easier because all polygons or polyhedra can be decomposed into triangles or tetrahedra.

- Not all polygons or polyhedra can be decomposed into quadrilaterals or hexahedra directly.

- Extension from 2D quadrilateral algorithm to a 3D hexahedral method is not generally straightforward.

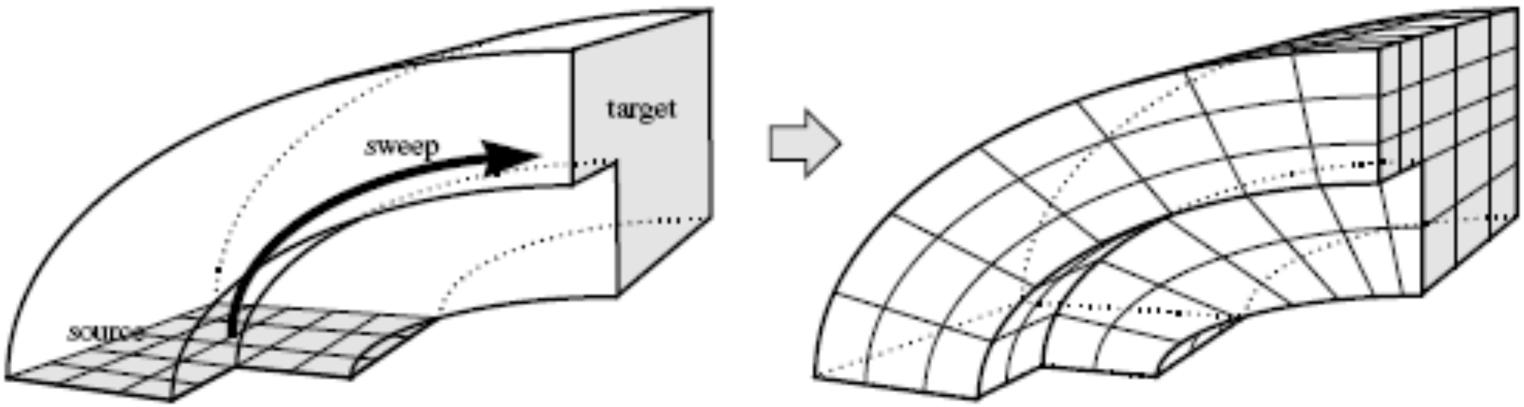
Mapped Meshing

- Mapped meshing generates structured quad/hex meshes.
- For mapped meshing to be applicable, opposite edges of the area to be meshed must have equal numbers of divisions.



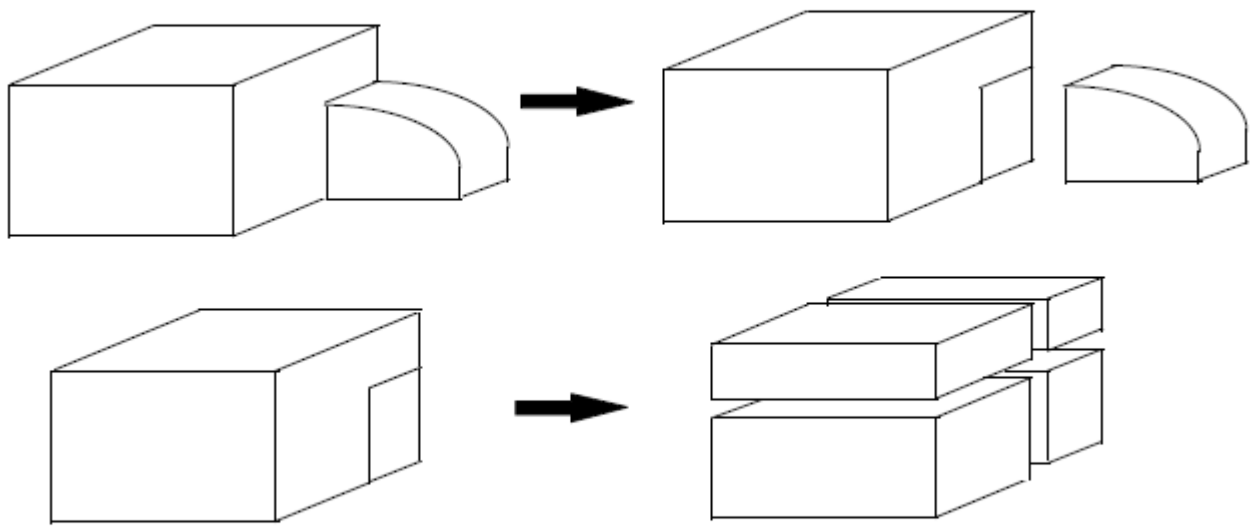
Mapped Meshing

- In 3D, each opposing face of a topological cube must have the same surface mesh.



Mapped Meshing

- This can often be impossible for an arbitrary geometric configuration or can involve considerable user interaction to decompose geometry into mapped meshable regions and assign boundary intervals.



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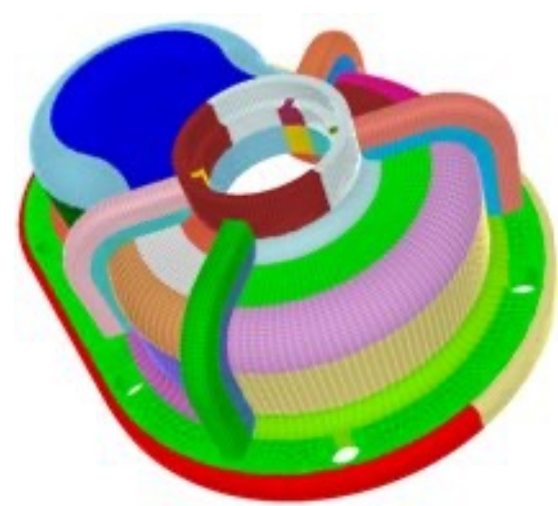
Mapped Meshing

- Research has been done through the CUBIT project at Sandia National Labs to automatically recognize features and decompose geometry into separate mapped meshable areas and volumes.

  http://cubit.sandia.gov/

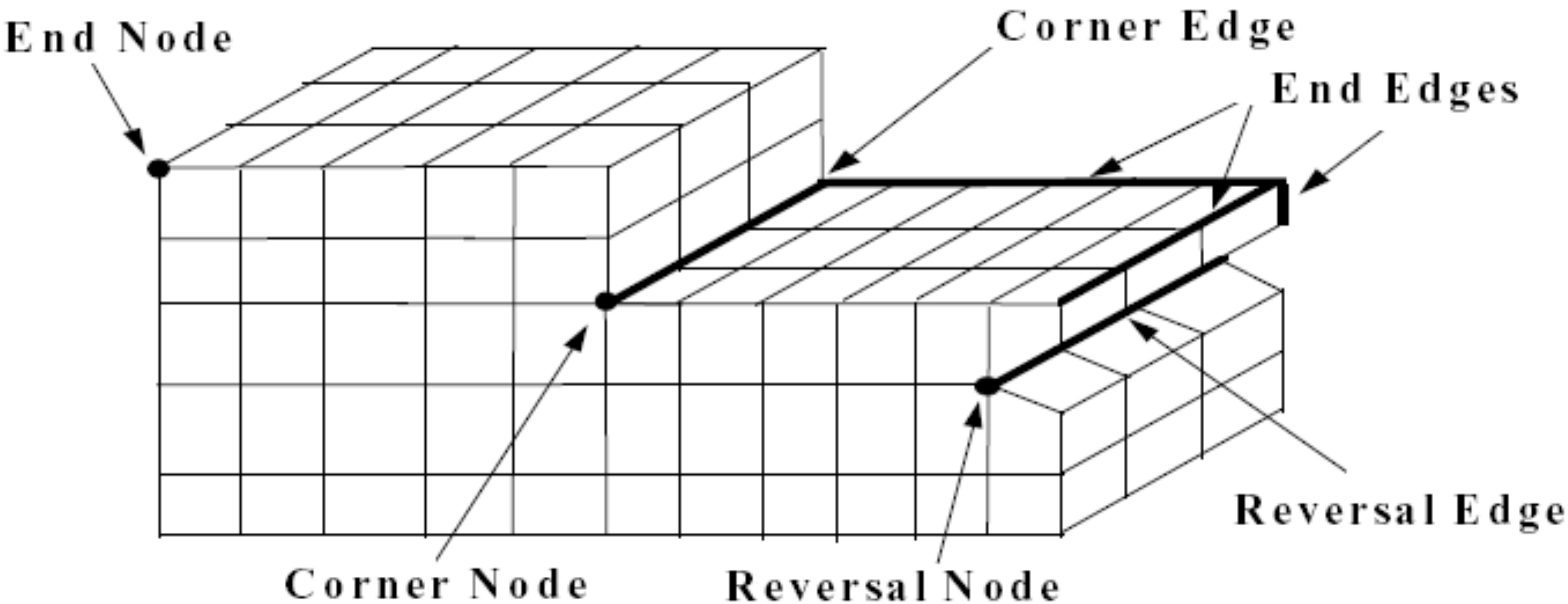
Cubit
http://cubit.sandia.gov/

- CUBIT encompasses ongoing research efforts to improve and discover new mesh generation algorithms, develop new tools for geometry cleanup and simplification and handle ever more complex preprocessing tasks for computational simulations.
  - CUBIT Geometry and Mesh Generation Toolkit
  - Common Geometry Module (CGM)
  - Mesh Verification with VERDICT
  - Cubit Adaptive Mesh Generation Library (CAMGL)
  - Graphical User Interfaces with CLARO
  - A Mesh Oriented Database



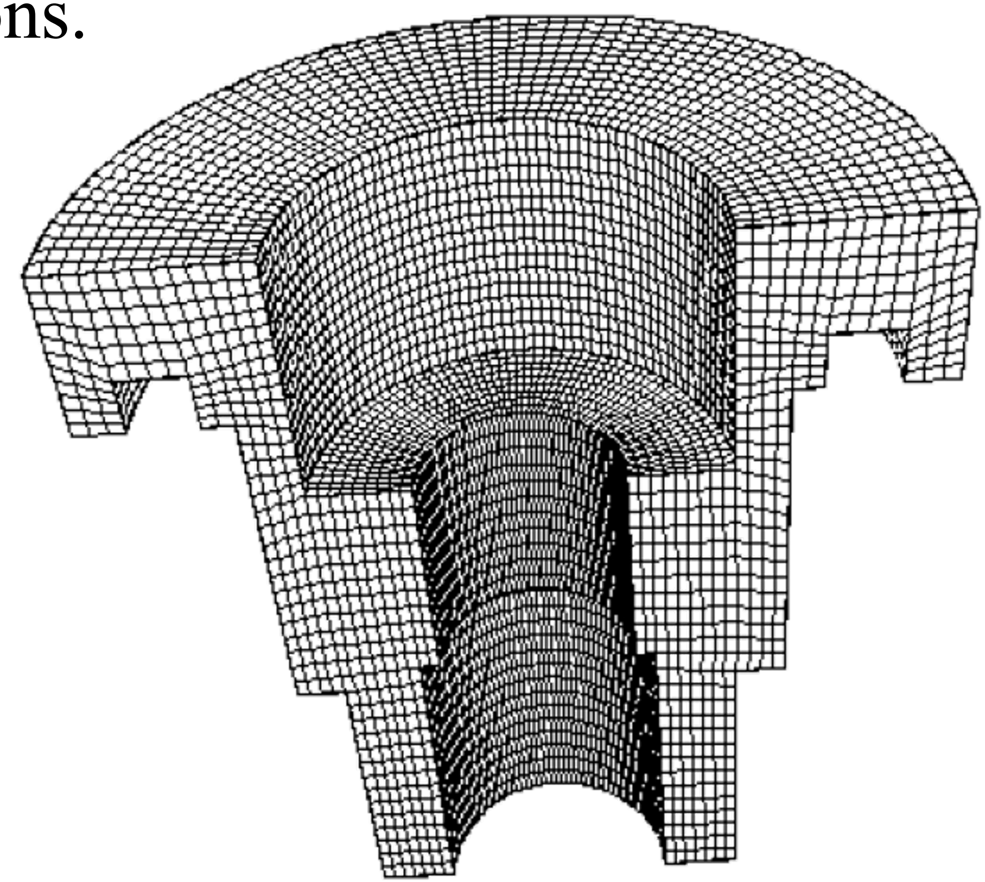
Submapping

- Rather than decomposing the geometry directly, it is important to determine an appropriate *virtual* decomposition based on corner angles and edge directions.



Submapping

- The separate map-meshable regions are then meshed separately.
- This method is suitable for blocky shapes and volumes that have well defined corners and cube-like regions.



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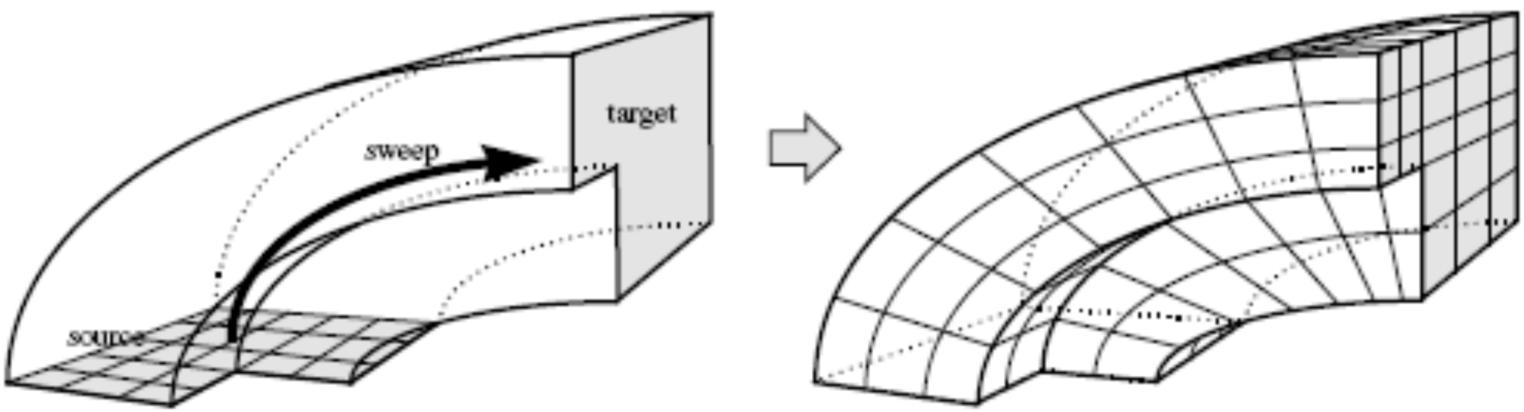
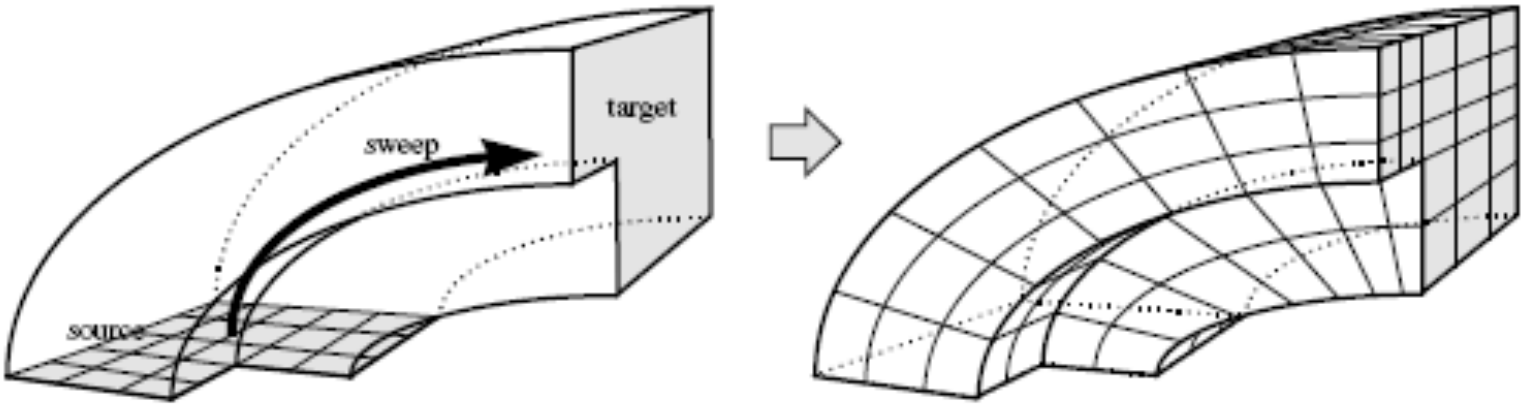
Sweeping

- Sweeping, sometimes referred to as $2\frac{1}{2}$-D meshing, is another class of mapped hex meshing.
- A quadrilateral mesh can be *swept* through space along a curve. Regular layers of hex are formed at specified intervals using the same topology as the quad mesh.



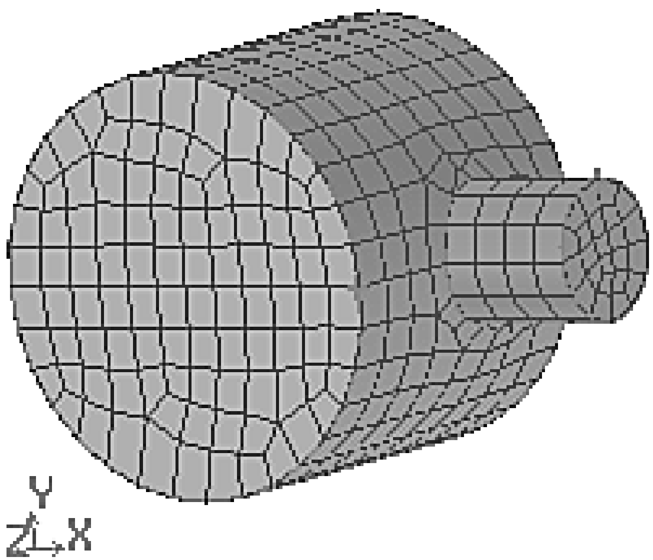
Sweeping

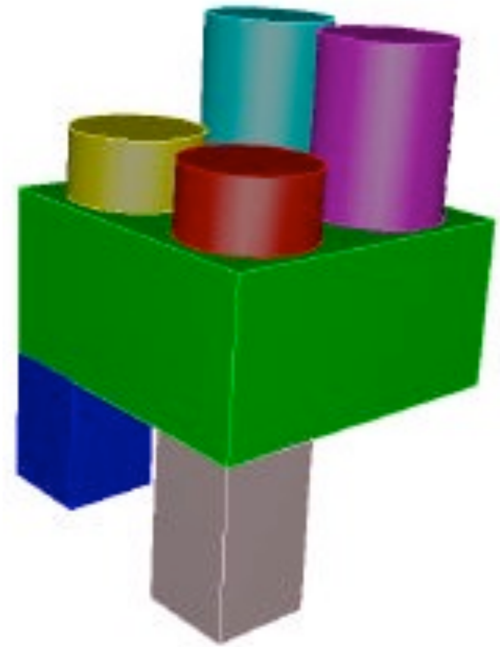
- This technique can be generalized to mesh certain classes of volumes by defining so-called *source* and *target* surfaces.
- Provided the source and target surface have similar topology and the surfaces are connected by a set of map-meshable surfaces, the quad elements of the source area can be swept through the volume to generate hexes.
- Care must be taken in locating internal nodes during the sweeping process.



Sweeping

- Ted Blacker generalizes and extends the applicability of sweeping in the *Cooper Tool*.
- The Cooper tool allows for multiple source and target surfaces while still requiring a single sweep direction.
- With this tool, the topology is allowed to branch or split along the sweep direction. In addition, the topology of source and target surfaces are not required to be similar.





Ted D. Blacker, (1996). "The Cooper Tool", Proceedings, 5th International Meshing Roundtable, pp.13-29.

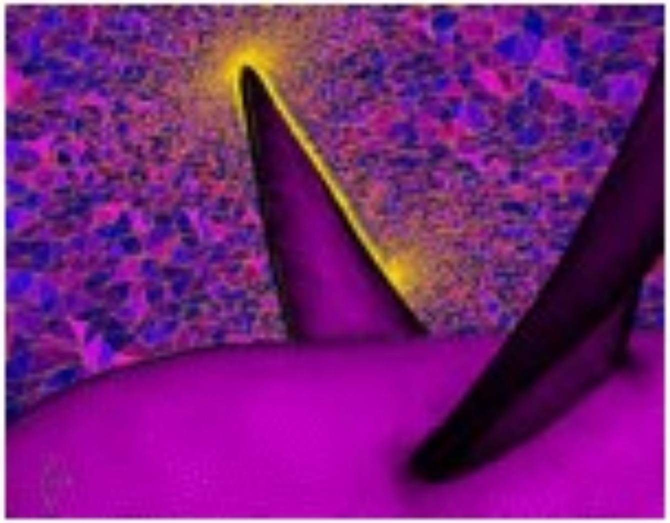
Cooper Tool

- The Cooper tool is provided as part of the Fluent pre-processor, Gambit.

  http://www.ansys.com/Products

GAMBIT: Geometry and Mesh Generation

- GAMBIT is Fluent’s geometry and mesh generation software.
- GAMBIT's combination of CAD interoperability, geometry cleanup, decomposition and meshing tools results in one of the easiest, fastest, and most straightforward preprocessing paths from CAD to quality CFD meshes.
- Highly automated meshing, the size function driven meshing, and boundary layer mesher.



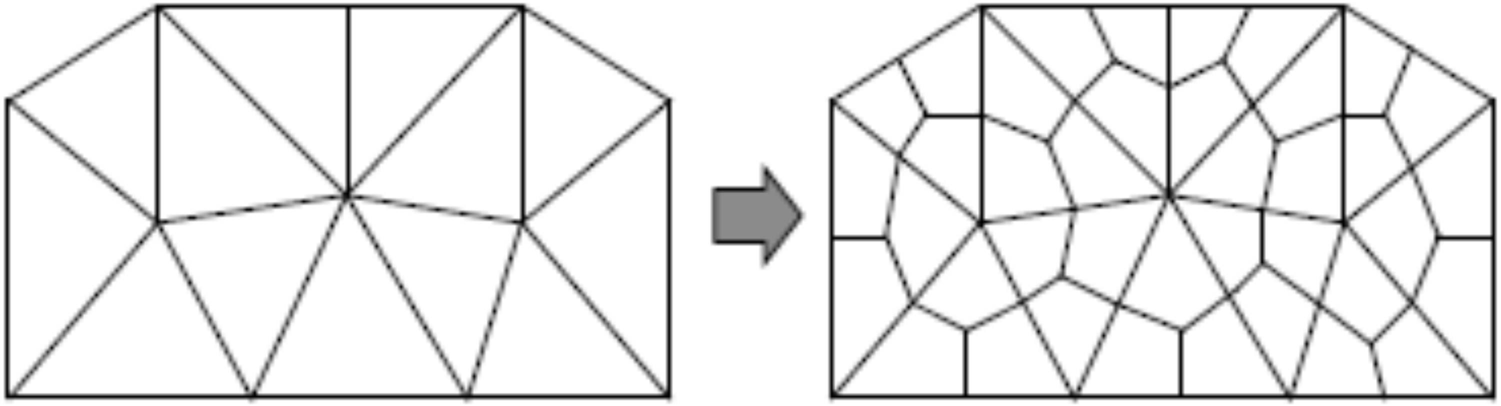
Boundary layer and tet mesh for a marine propeller

Unstructured Quad Meshing

- Two main categories:
  - Indirect approaches: the domain is first meshed with triangles, which are converted into quads later.
  - Direct approaches: Quads are placed on the surface directly, without first going through the process of triangle meshing.

Indirect Methods

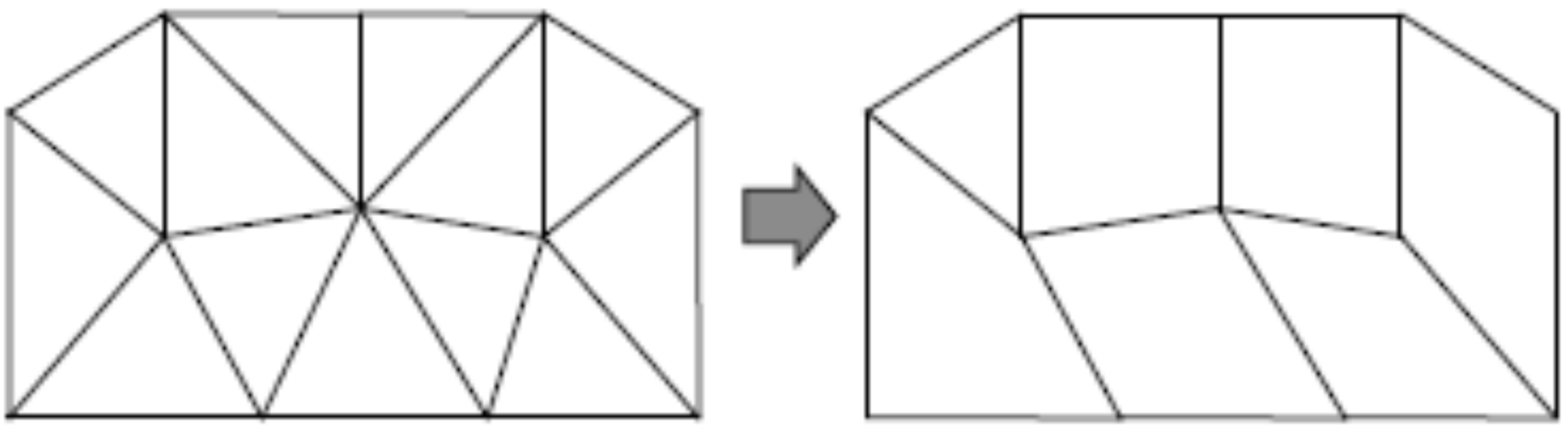
- One simple idea is to divide each triangle into three quads.
- This method guarantees an all-quad mesh, but a high number of irregular nodes are introduced into the mesh resulting in poor element quality.



Quad mesh generated by splitting each triangle into three quads.

Indirect Methods

- An alternate algorithm is to combine adjacent pairs of triangles to form a single quad.
- While the element quality increases using this method, a large number of triangles may be left.



Quad-dominant mesh generated by combining triangles.

Indirect Methods

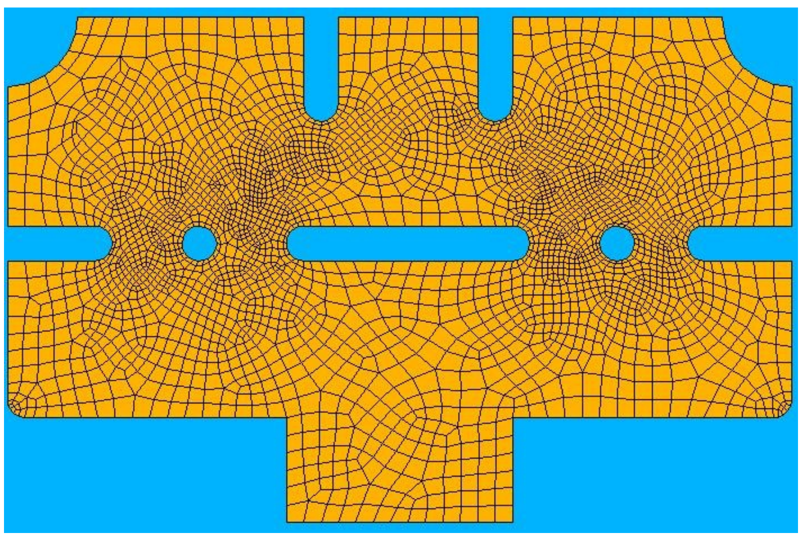
- The triangle combining method can be improved, if some care is taken. The number of quads can be maximized.

- The result is a quad-dominant mesh containing a minimal number of triangles.

- Additional local element splitting and swapping strategies were developed to increase the number and quality of quads.

Indirect Methods

- Lee proposed an advancing front approach to guarantee an all-quad mesh, provided the initial number of edges on the boundary is even.
- An initial set of front is defined consisting of the edges of triangles at the boundary of the domain.
- Triangles are systematically combined at the front, advancing towards the interior of the area.



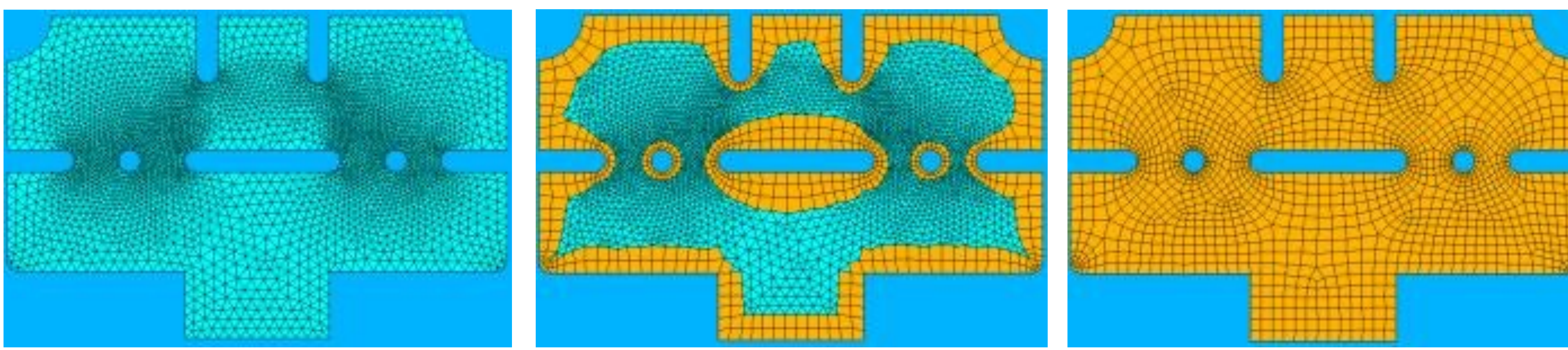
C.K Lee, and S.H. Lo (1994). "A New Scheme for the Generation of a Graded Quadrilateral Mesh," Computers and Structures, Vol.52 pp.847-857.

Indirect Methods

- Since all operations are local, indirect methods have the advantage of being very fast.
- The drawback to indirect methods is that there are typically many irregular nodes left in the mesh.
- Irregular nodes can be reduced, and hence element quality is increased by performing *topological clean-up operations*.

Indirect Methods

- Another method, known as Quad Morphing, also utilizes an advancing front approach to convert tris to quads, but is able to significantly reduce the number of irregular nodes in the mesh.
- Local edge swaps are performed and additional nodes introduced in order to ensure boundary alignment and orthogonality.
- Any number of tris may be deleted to create a single quad.



Input tri mesh | Two quad layers | Finial mesh after clean-up and smoothing

Direct Methods

- Decompose the domain into simple regions, then each region can be meshed using templates.

- Utilize a moving front method for direct placement of nodes and elements.

Quad-Tree Decomposition

- The quad-tree decomposition technique developed by Baehmann *et al*.

- After an initial decomposition of the 2D space into a quad-tree based on local feature sizes, quad elements are fitted into the quad-tree *leaves*, adjusting nodes in order to conform to the boundary.

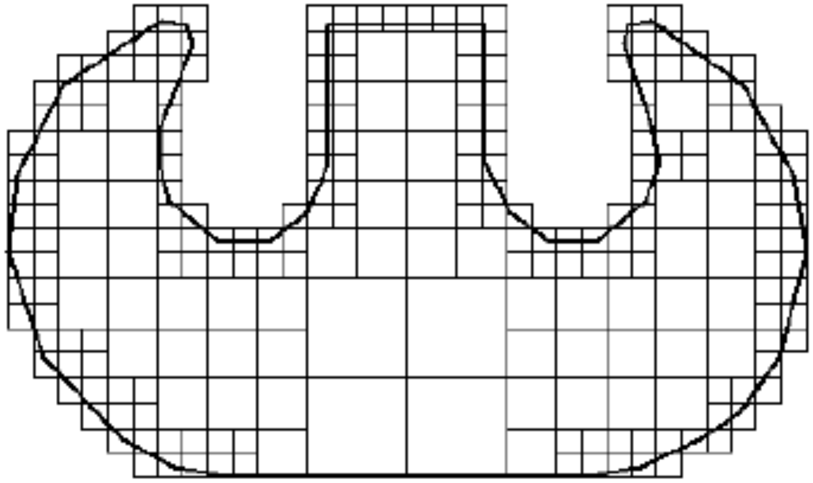


Figure 1. a region and its quadtree representation

Peggy L. Baehmann, Scott L. Wittchen, Mark S. Shephard, Kurt R. Grice and Mark A. Yerry, (1987). "Robust Geometrically-based, Automatic Two-Dimensional Mesh Generation," International Journal for Numerical Methods in Engineering, Vol.24, pp.1043-1078.

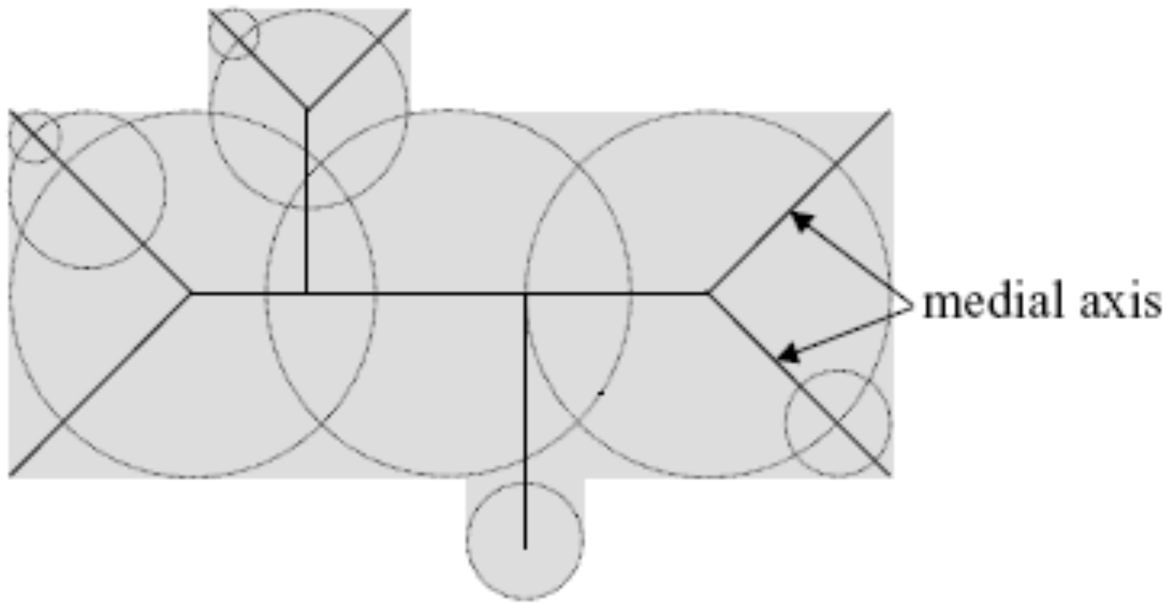
Quad Meshing by Decomposition

- The domain is recursively subdivided into simple polygonal shapes.
- The resulting polygons satisfy a limited number of templates into which quad elements are inserted.

Medial Axis

- The medial axis decomposition was introduced by Tam.
- The medial axis can be thought of as a series of lines and curves generated from the midpoint of a maximal circle as it is rolled through the area.

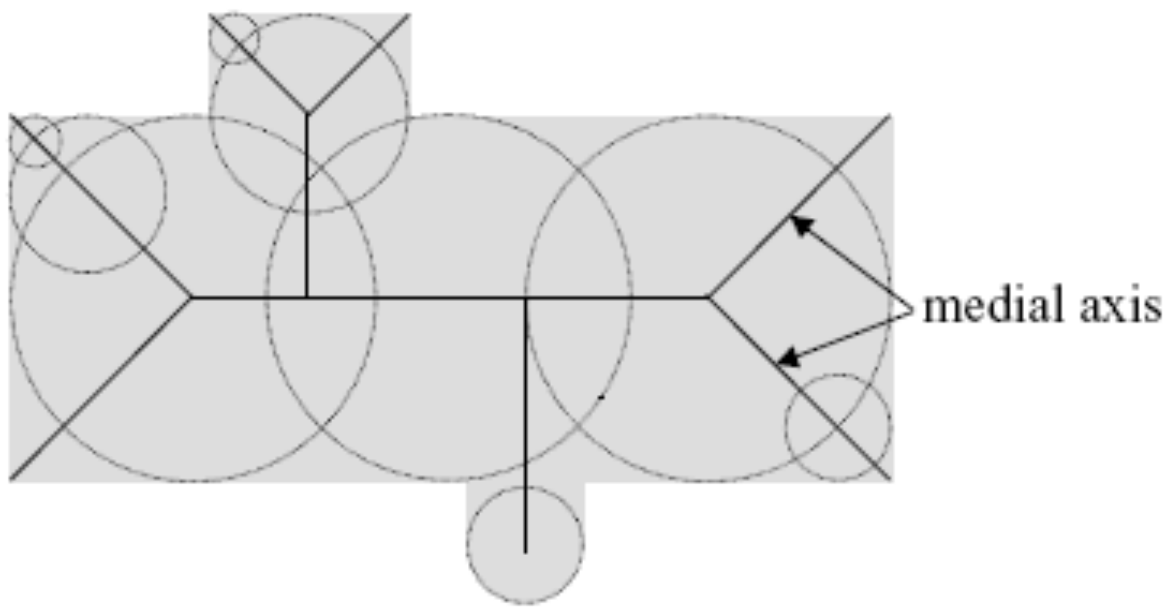
Decomposition of an area using the medial axis



T. K. H. Tam and C. G. Armstrong (1991). "2D Finite Element Mesh Generation by Medial Axis Subdivision", Advances in Engineering Software, Vol.13, pp.313-324.

Medial Axis

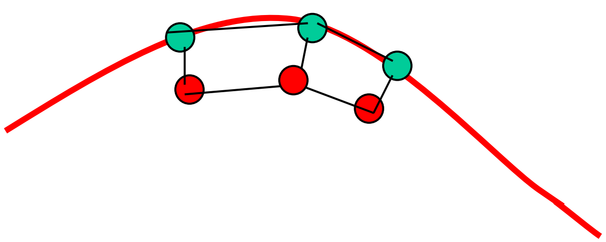
- Having decomposed the area into simpler regions, sets of templates are then employed to insert quads into the domain.
- Good programming techniques are used in order to maintain compatibility of element divisions between adjoining regions of the domain.



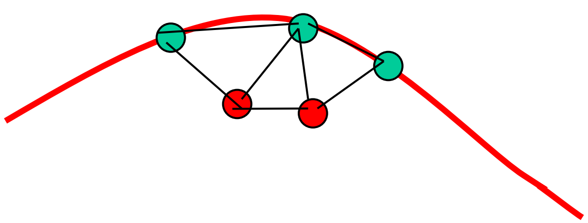
Decomposition of an area using the medial axis

Advancing Front Quad Meshing

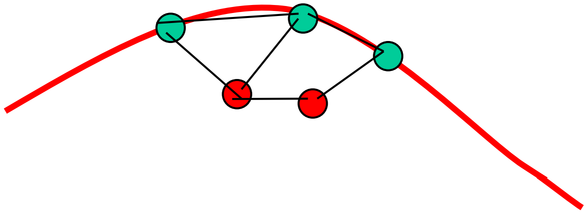
- Starting with an initial placement of nodes on the boundary, individual elements are formed by projecting edges towards the interior.
- Two triangles are formed using traditional advancing front methods and then combined to form a single quadrilateral.



Edges are projected towards the interior to form quads



Traditional advancing front method



Two triangles are combined to form a single quad.

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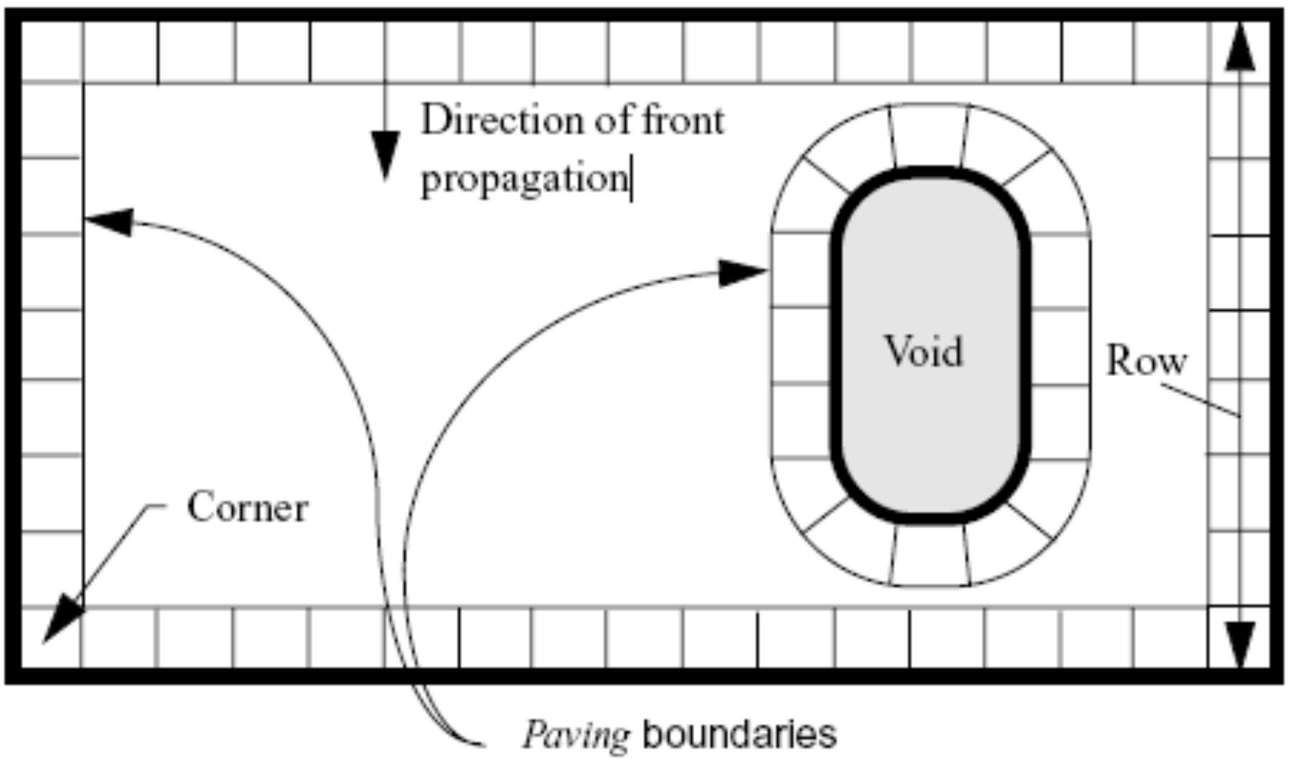
Paving

- The *paving* algorithm, introduced by Blacker and Stephenson, presents a method for forming complete rows of elements starting from the boundary and work in.



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Paving

- Methods for project of nodes, handling of special geometric situations and intersection of opposing fronts are important issues in *paving*.
- The *paving* algorithm is currently implemented as part of the CUBIT software as well as several commercial packages, including MSC Patran and Fluent's Gambit software.

http://cubit.sandia.gov/

http://www.mscsoftware.com/products/patran.cfm

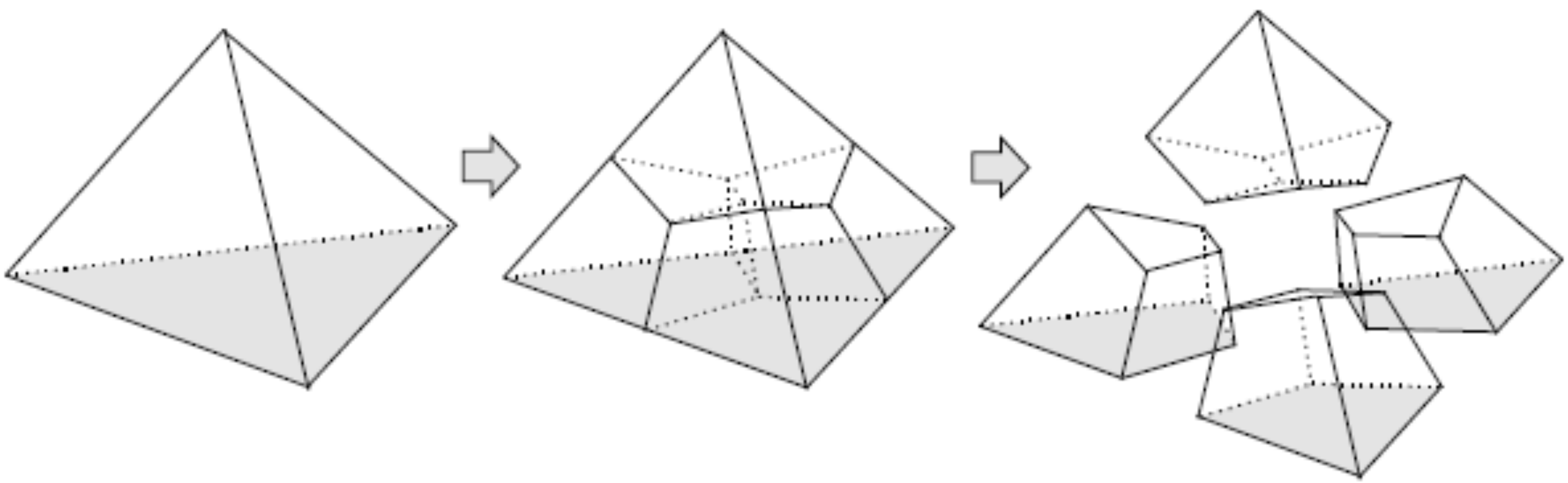
http://www.fluent.com/software/gambit/index.htm

MSC – Patran
http://www.mscsoftware.com/products/patran.cfm

- Patran is the leading pre- and post processor for CAE simulation. The program's advanced modeling and surfacing tools allow you to create a finite element model from scratch, or your existing CAD model.
- Patran imports model geometry without modifications. No translation takes place, so your CAD geometry remains intact.
- After geometry is imported, you can use Patran to define loads, boundary conditions, and material properties.

Unstructured Hex Meshing

- Indirect methods: Provided a solid can be tet meshed, each tetrahedron can be subdivided into four hexahedra. Most finite element analysts have rejected this solution due to the poor element quality of indirect methods.
- Direct methods: Generate hex meshes directly.



Decomposition of a tetrahedron into four hexahedra using **edge middle points**, **face center points**, and **the body center point**.

Indirect Methods

- An indirect method that combines tets to form hexes would therefore need to look for combinations of five or more tetrahedra to form a single hexahedra.

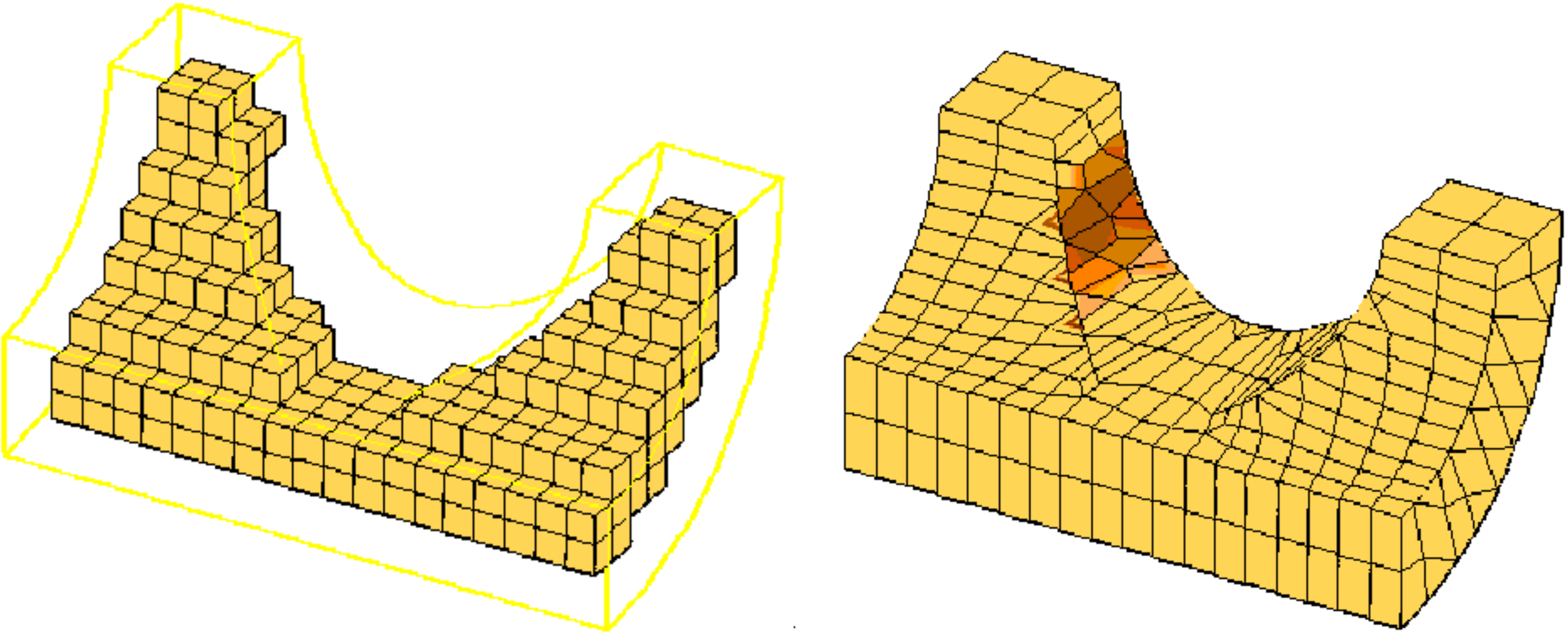
- This problem to date has not proved a reasonable nor tractable method for mesh generation.

Direct Methods

- There are currently four distinct strategies proposed for unstructured all-hex mesh generation:
  - Grid-based
  - Medial surface
  - Plastering
  - Whisker weaving

Grid-Based Methods

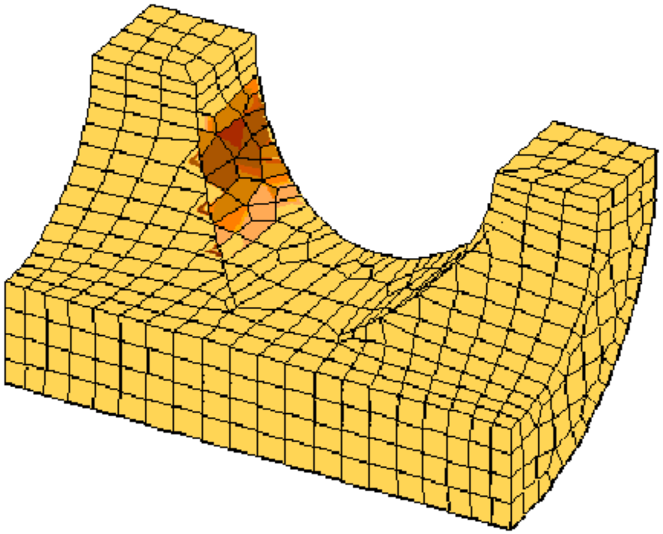
- The grid-based approach, proposed by Schneiders, generates a fitted 3D grid of hex elements on the interior of the volume.
- Hex element are added at the boundaries to fill gaps where the regular grid of hexes doesn't meet flush with the surface.



Robert Schneiders, (1996) "A Grid-Based Algorithm for the Generation of Hexahedral Element Meshes", Engineering With Computers. Vol.12 pp.168-177.

Grid-Based Methods

- Local modification and smoothing are necessary to generate topologically correct hexes.
- This method, while robust, tends to generate poor quality elements at the boundary of the volume.
- Hex elements will in general not be aligned with the boundary.
- The resulting mesh is also highly dependent upon the orientation of the interior grid of hex elements.



Available Software

- Hexar software from Cray Research

  Monika Wierse, Jean Cabello and Yoshihiko Mochizuki, (1998) “Automatic Grid Generation with HEXAR”, *Proceedings 6th International Conference on Numerical Grid Generation in Computational Field Simulations, ed. M. Cross et. al.*, University of Greenwich, UK., pp. 843-852

- MARC’s Mentat software

  http://www.mscsoftware.com/products/marc.cfm

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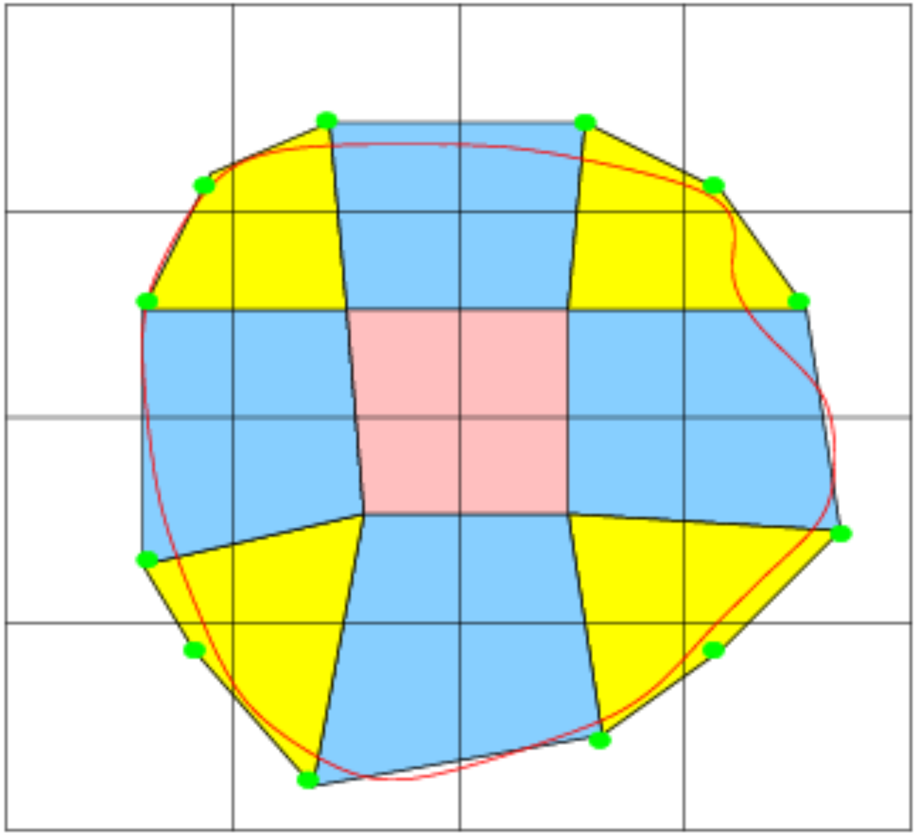
Isocontour-based Quad/Hex Meshing

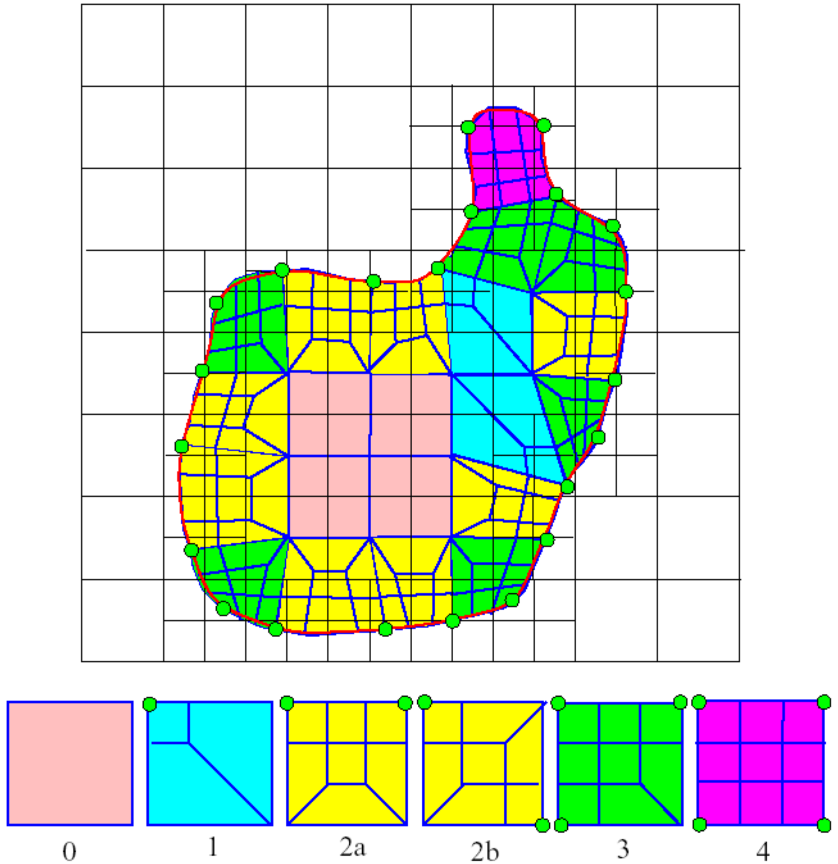
- Uniform quadrilateral meshing: analyze each interior grid point.
- Each interior grid point is shared by four cells, and we can calculate one minimizer point for each cell. Therefore we obtain four points, which construct a quad.

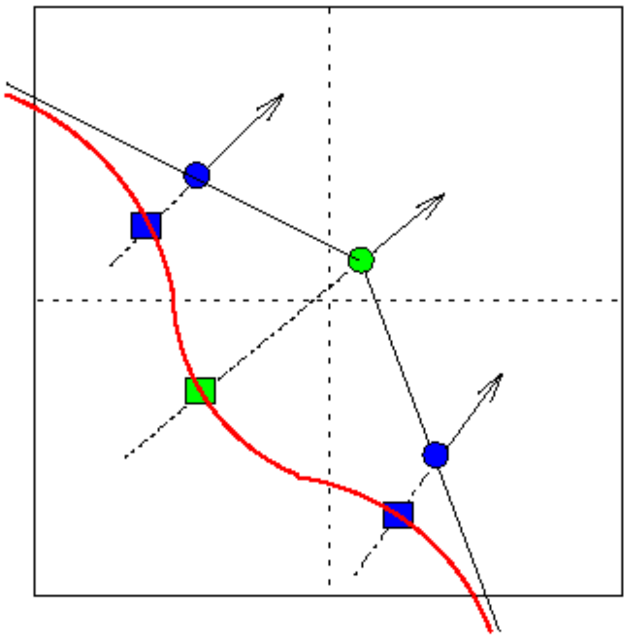


Y. Zhang, C. Bajaj. Adaptive and Quality Quadrilateral/Hexahedral Meshing from Volumetric Data. Computer Methods in Applied Mechanics and Engineering (CMAME), 195(9-12):942-960, 2006.

Isocontour-based Quad/Hex Meshing

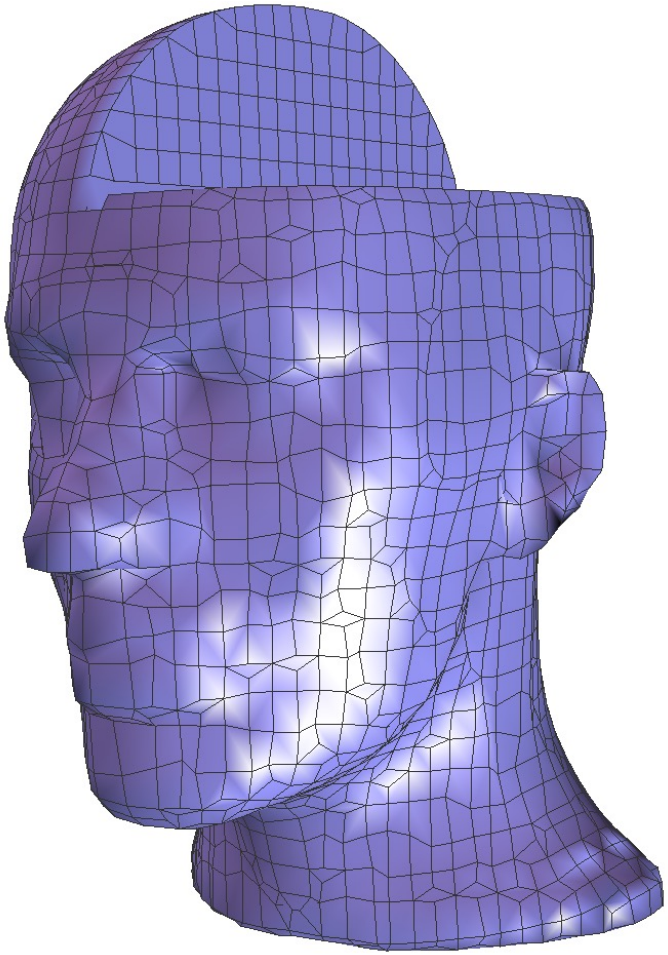
- Adaptive quadrilateral meshing: first construct uniform quad meshes, then use template to locally refine them. Finally project newly generated points onto the boundary surface.



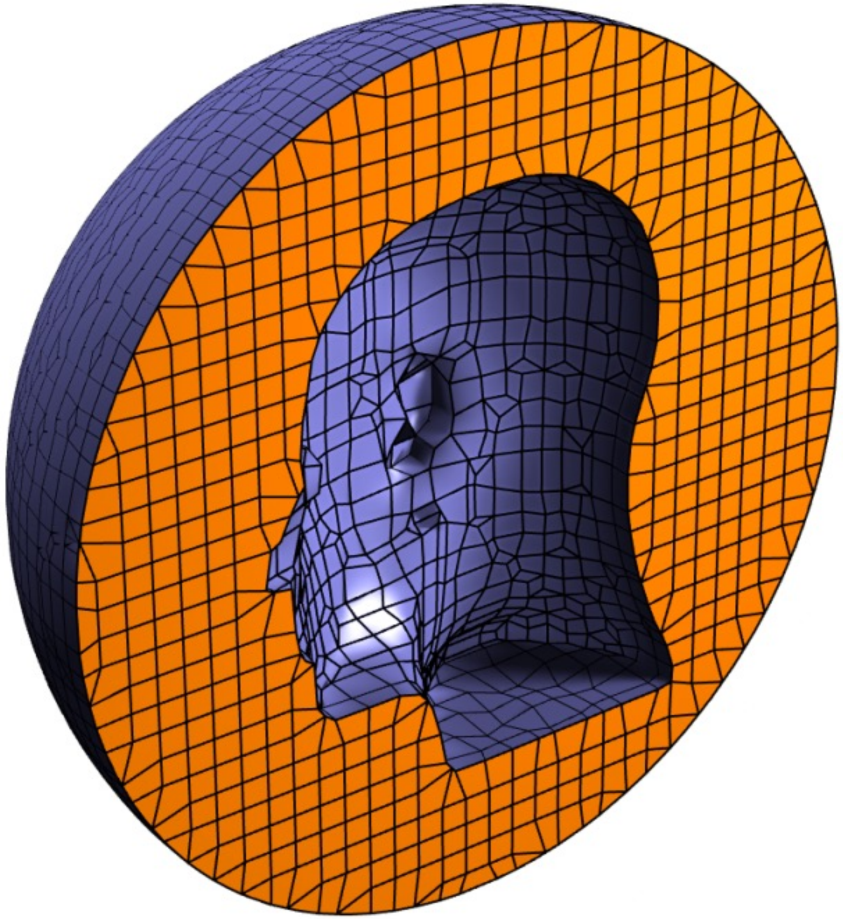


Uniform Hex Meshing

- Uniform hex meshing: analyze each interior grid point.
- Each interior grid point is shared by eight cells, and we can calculate one minimizer point for each cell. Therefore we obtain eight points, which construct a hex.



An interior mesh (23,204 hexes)

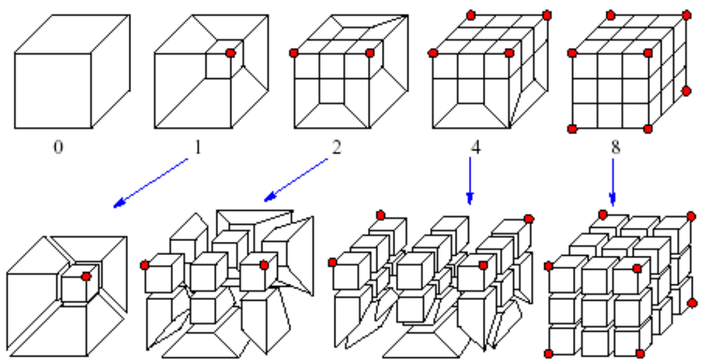


An exterior mesh (13,552 hexes)

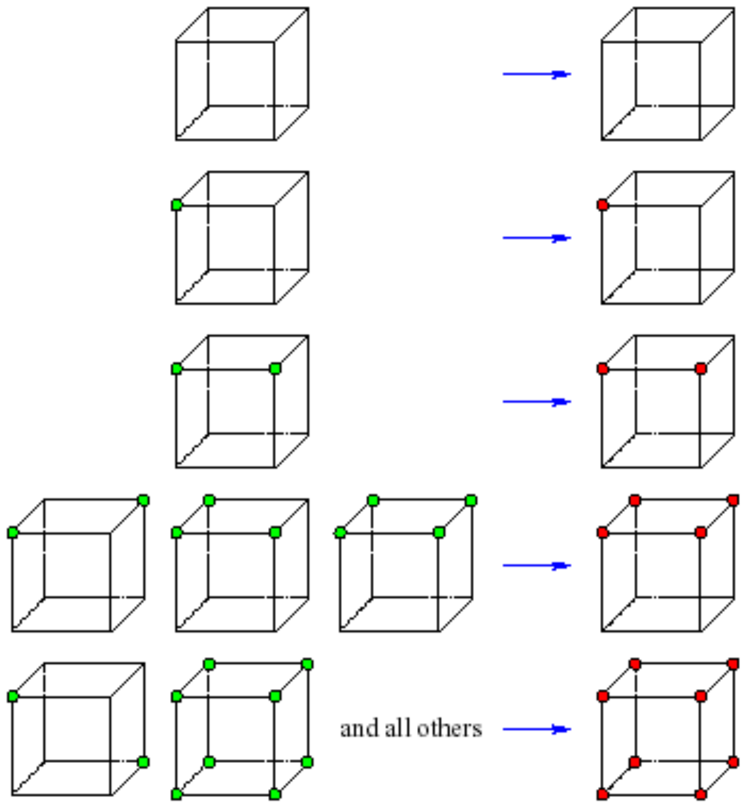
Adaptive Hex Meshing

Look-Up Table

1. Set a sign for each cell in the uniform level indicating if this cell needs to be refined or not.
2. Check each hexahedron if it belongs to one of five basic templates. If not, Keep updating the sign using the Look-Up table.
3. Construct adaptive hex meshes.



5 Basic Templates

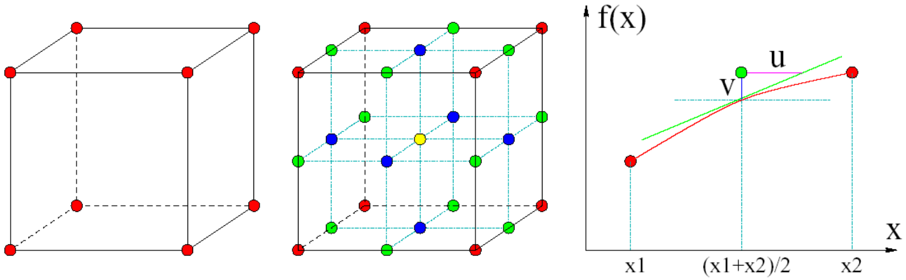


Look-Up Table

4 Mesh Adaptation

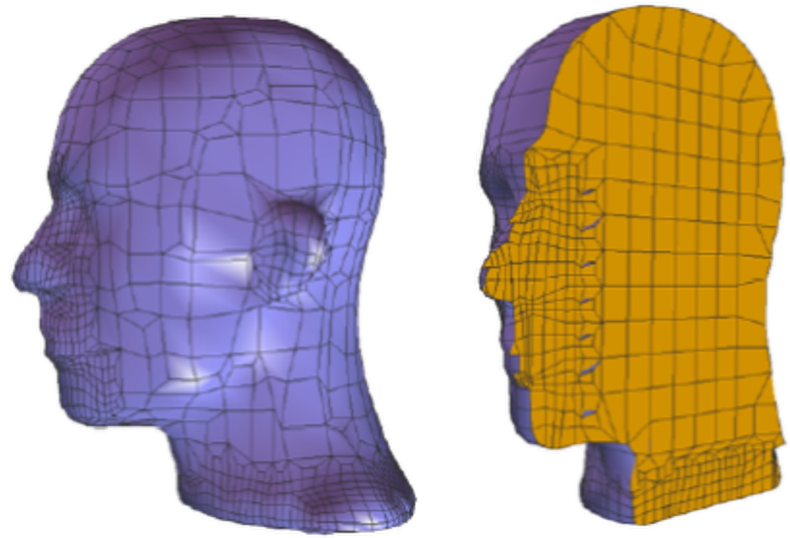
- There are three main ways to control the mesh adaptation. Users can also design an error function based on their specific requirements.
  - Feature sensitive error function
  - Various areas users are interested in
  - Finite element solutions
  - User defined

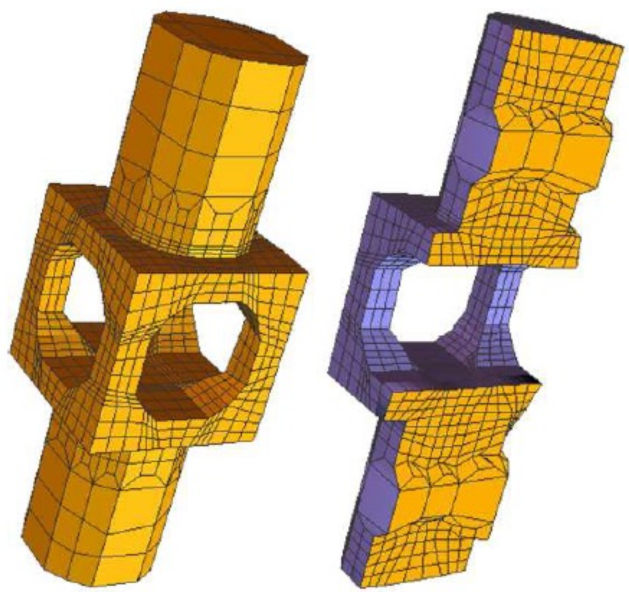
Feature Sensitive Error Function

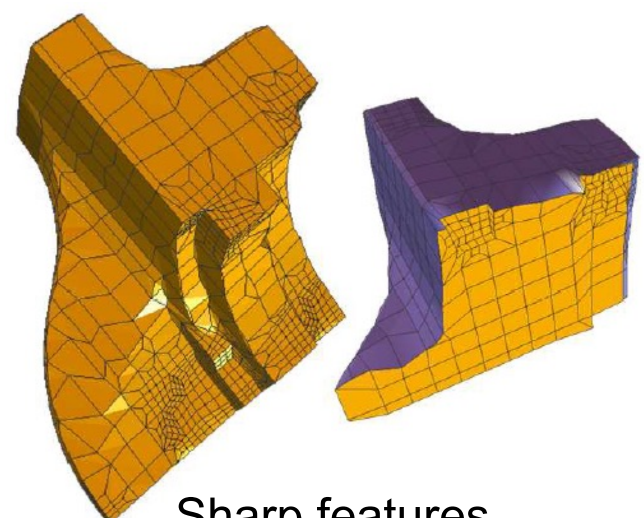


$$
\begin{aligned}
f^i(x,y,z) &= f_{000}(1-x)(1-y)(1-z) + f_{011}(1-x)yz \\
&+ f_{001}(1-x)(1-y)z + f_{101}x(1-y)z \\
&+ f_{010}(1-x)y(1-z) + f_{110}xy(1-z) \qquad (4) \\
&+ f_{100}x(1-y)(1-z) + f_{111}xyz
\end{aligned}
$$

$$
error = \sum \frac{|f^{i+1} - f^i|}{|\nabla f^i|}
$$







Sharp features

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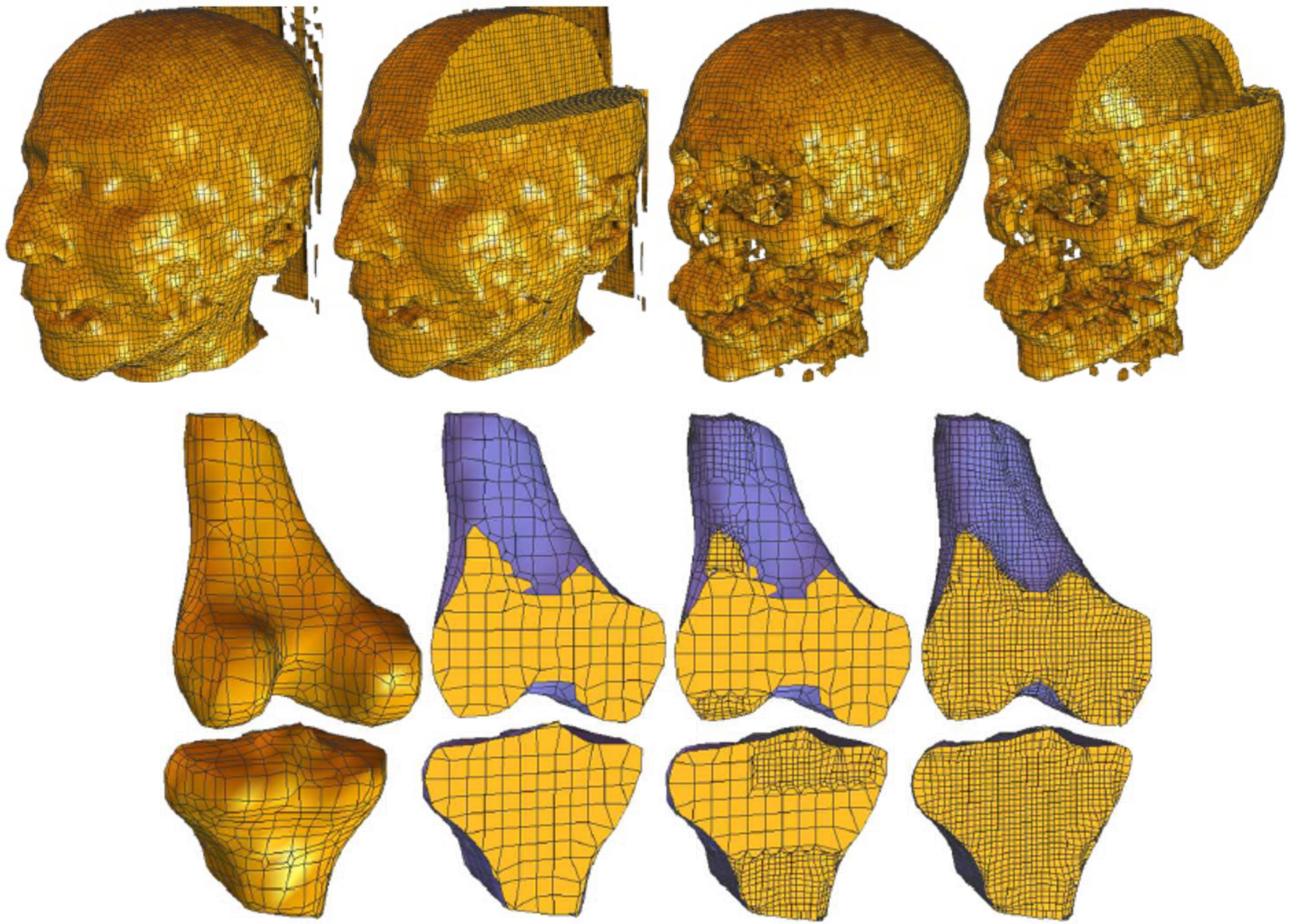
Hex Meshing Results



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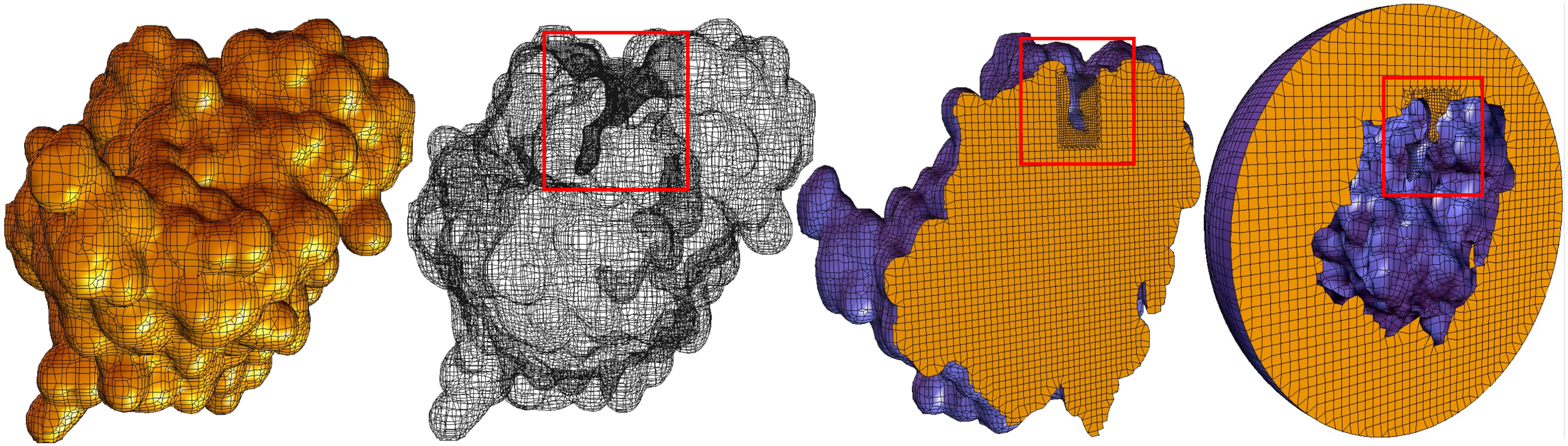
Adaptive Hex Meshes of Various Areas

- The mesh adaptation can also be controlled by choosing various areas which users are interested in. For example, the active site of the molecule, mAChE (mouse acetylcholinesterase).



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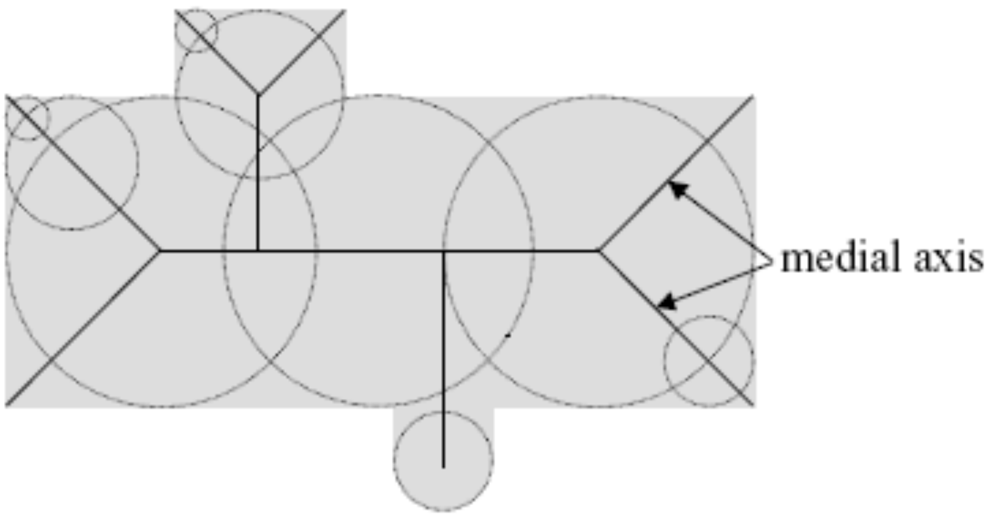
Mesh Adaptation

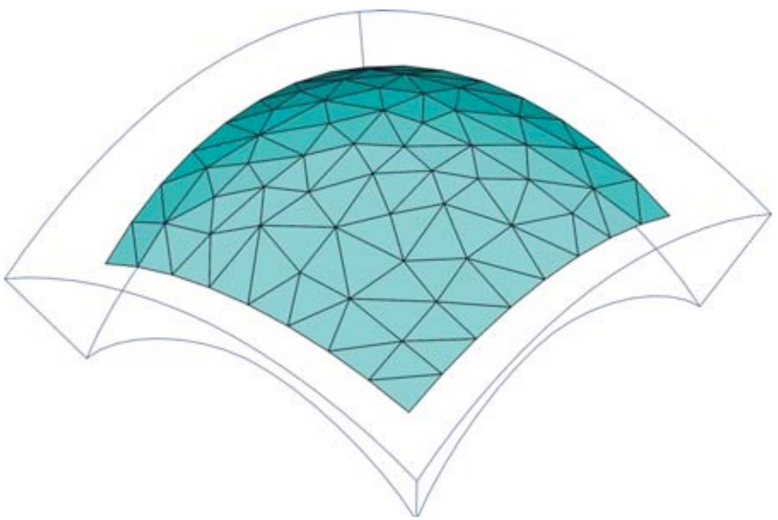
- User can also define an error function according to various application projects, then this user-defined function is plugged into the mesh generation code to control the mesh adaptation.

- Mesh adaptation can be controlled flexibly.

Medial Surface

- Medial surfaces are surfaces generated from the midpoint of a maximal sphere as it is rolled through the volume.

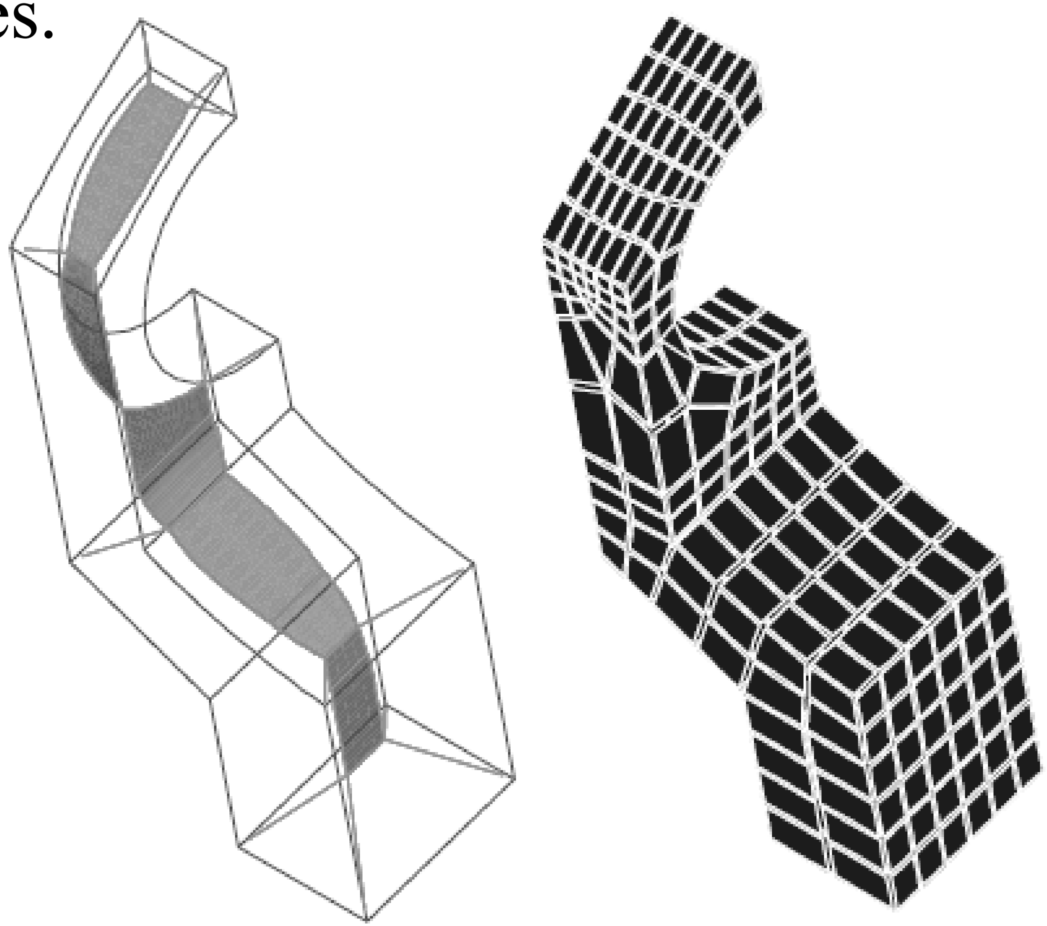




Curved surface

Medial Surface and Hex Mesh

- The domain is subdivided by a set of medial surfaces to generate map meshable regions.
- A series of templates are utilized to fill the volume with hexes.



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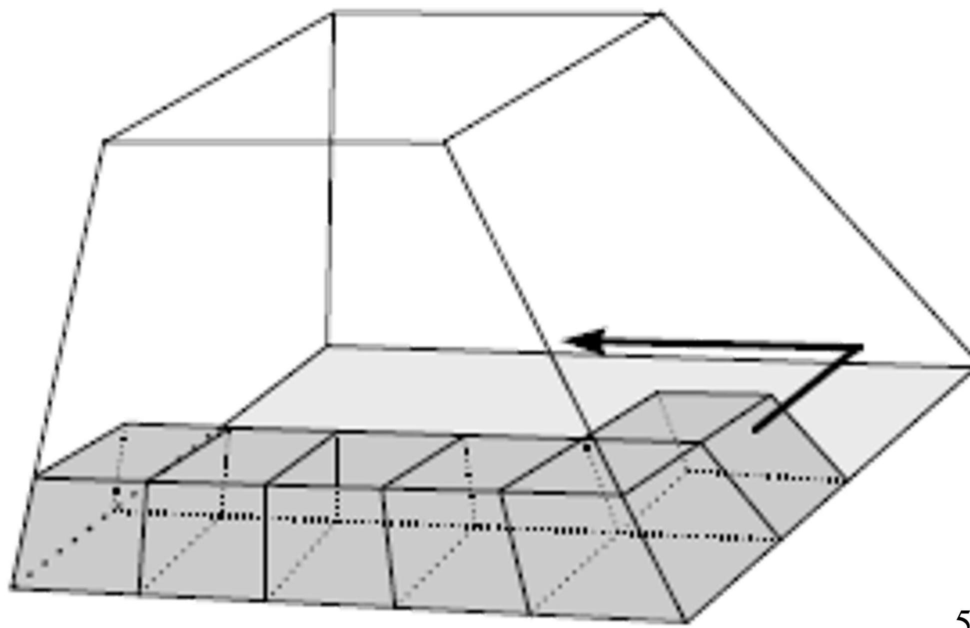
Medial Surface

- Medial surface methods are incorporated into the FEG's CADFix hexahedral mesh generator and within Solidpoint's Turbomesh software.

http://www.transcendata.com/products/cadfix/index.htm

Plastering (Advancing front hex meshing)

- Plastering is an attempt to extend the paving algorithm to three dimensions.
- Elements are first placed starting with the boundaries and advancing toward the interior of the model.



Plastering process forming elements at the boundary

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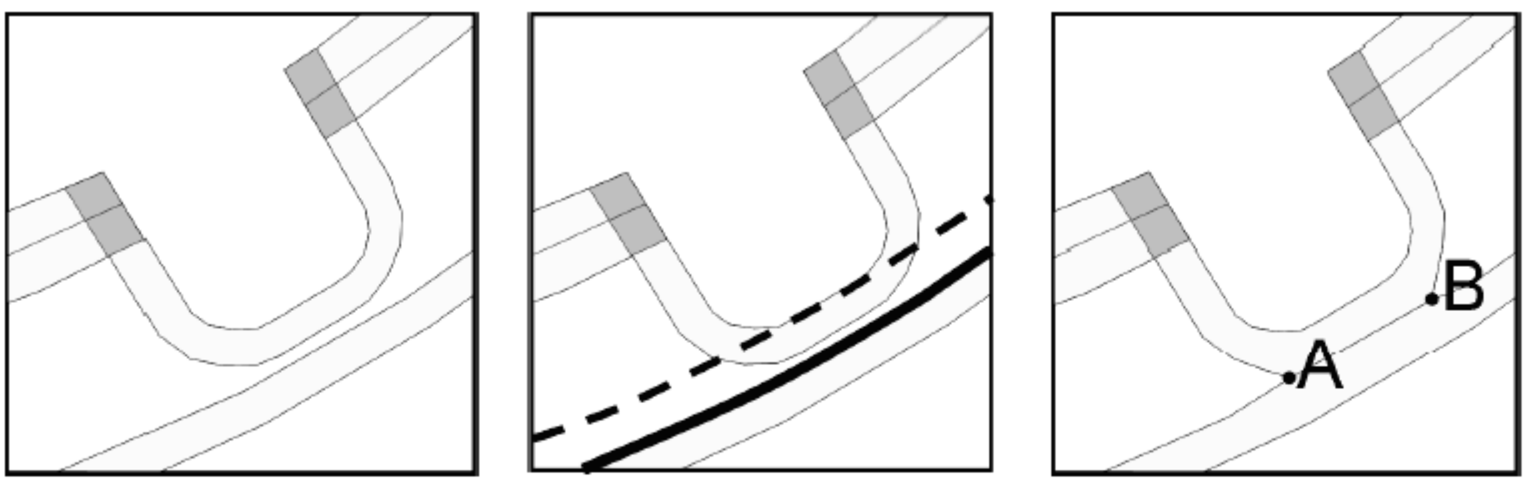
Plastering

- Plastering must detect intersecting faces and determine when and how to connect to pre-existing nodes or to seam faces.
- As the algorithm advances, complex interior voids may result, which in some cases are impossible to fill with all-hex elements.



(a) Proximity case in unmeshed void

(b) Desired front advancement

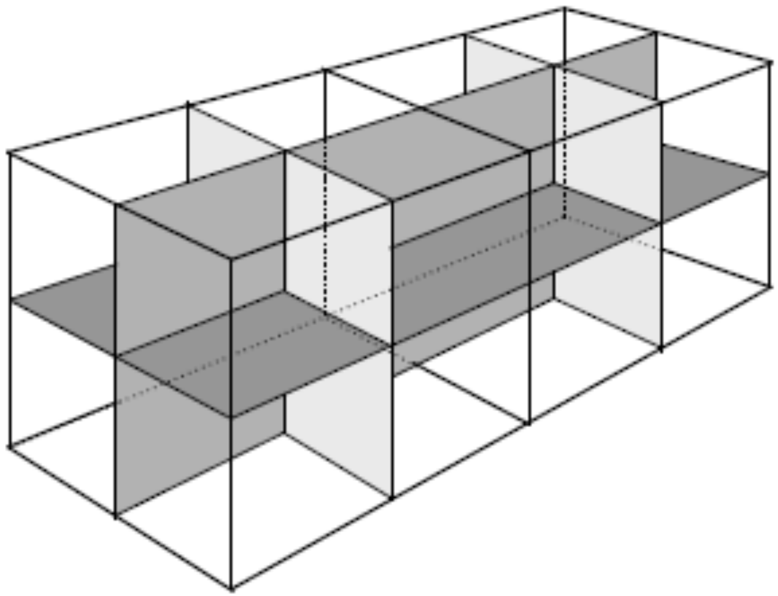
(c) Merged model, A & B are new model constraints

Plastering

- Existing elements sometimes must be modified in order to facilitate placement of hexes towards the interior.
- Currently, the plastering algorithm has not been proven to be reliable on a large class of problems.

Whisker Weaving

- Whisker weaving, introduced by Tautges and Blacker, is based on the concept of the *spatial twist continuum* (STC).
- STC is considered as a dual of the hex mesh, represented by an arrangement of intersecting surfaces which bisect hex elements in each direction.



Timothy J. Tautges, Ted Blacker and Scott Mitchell, (1996) "The Whisker-Weaving Algorithm: A Connectivity Based Method for Constructing All-Hexahedral Finite Element Meshes," International Journal for Numerical Methods in Engineering, Vol.39, pp.3327-3349.

Whisker Weaving

- The principle is to first construct the STC.

- With a complete STC, hex elements can then be fitted into the volume using the STC as a guide.

- The whisker weaving algorithm has achieved some success, but has not been proved to work for arbitrary geometry.

Hex-Dominant Methods

- All-hex meshing is difficult, people developed some hex-dominant methods, or hybrid meshing techniques.

- Whenever the tet elements interface directly with hex elements, a pyramid/prism shaped element may be formed. (ANSYS, CUBIT)

Hex-Dominant Methods

- Void sometimes remains after hex generation.
- Fill voids with tets, pyramids, or prisms.

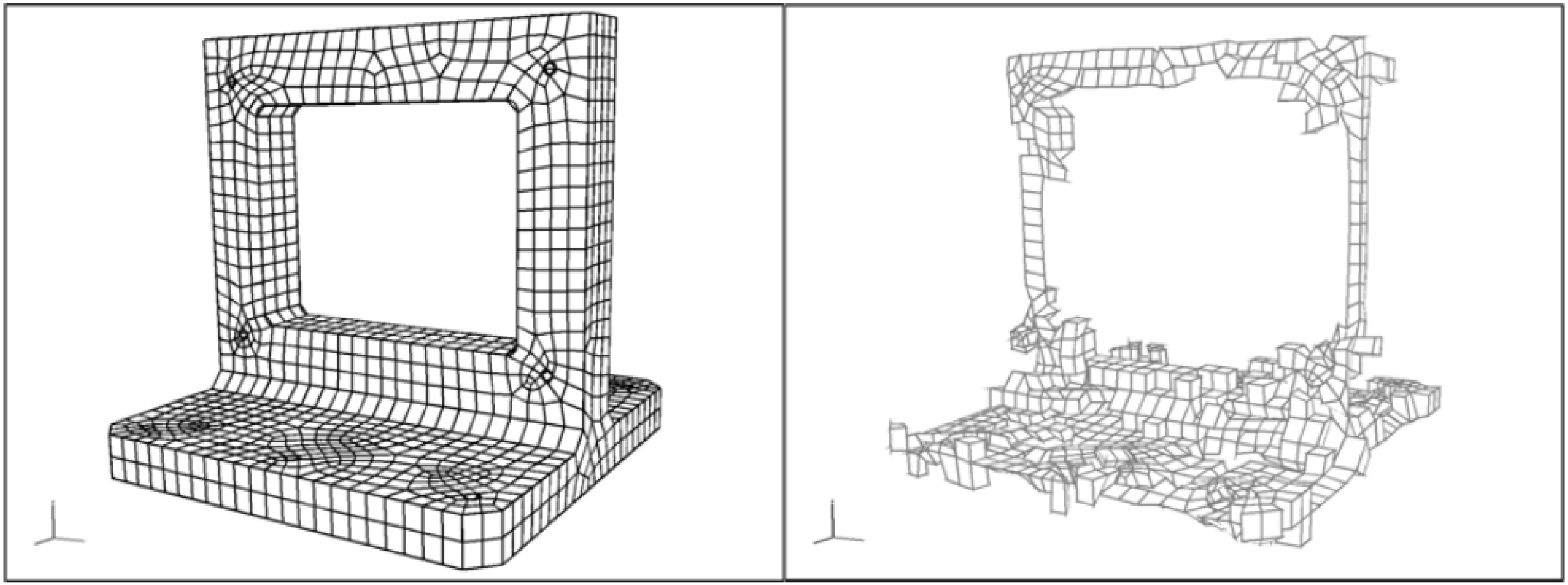


Figure 1: Winblock Model and Void Remaining after Hex Generation

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