Topic 2: Image Processing - Affine/Deformable Image Registration

Jessica Zhang
Department of Mechanical Engineering
Courtesy Appointment in Biomedical Engineering
Carnegie Mellon University
jessicaz@andrew.cmu.edu
http://www.andrew.cmu.edu/user/jessicaz

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Registration

- Registration is the process of finding the spatial transform that maps points from one image to the corresponding points in another image.
- Medical image registration has many applications:
  - Repeated image acquisition of a subject is often used to obtain time series information that captures disease development, treatment progress and contrast bolus propagation.
  - Correlating information obtained from different image modalities
    - MRI-CT: MRI has good soft tissue discrimination for lesion identification, while CT provides bone localization useful for surgical guidance.
    - PET/SPECT-CT/MRI: Positron emission tomography (PET) and single photon emission computed tomography (SPECT) provide functional information that can be used to locate abnormalities such as tumors, while CT/MRI provided anatomical structure, i.e., PET/MRI (study of brain tumors), and PET/CT (radiation treatment planning).

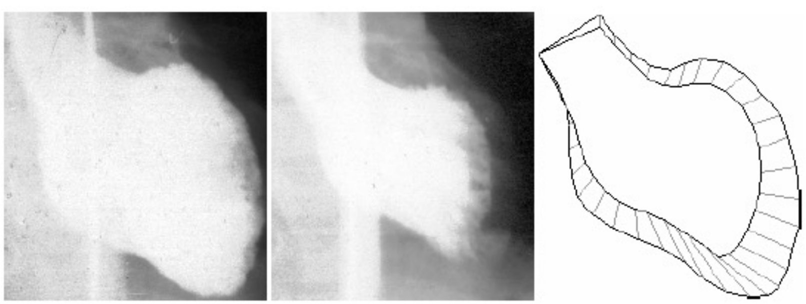
What is Image Registration?

- Image registration (sometimes called fusion) is to determine geometrical transformations to align and match two or more images obtained from different measures such as different times, different sensors, or different view points.

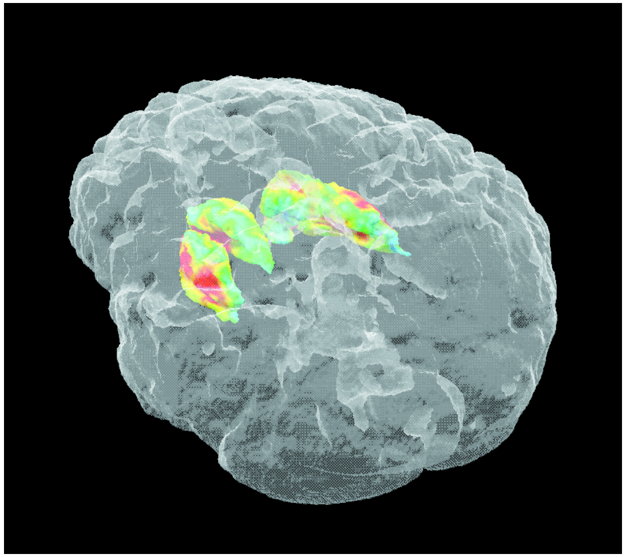
$$\textit{Determine } G \textit{ and } L \;\; s.t.$$

$$I_2(\, x,y \,) = G(\, I_1(\, L\,(x,y)\,)\,)$$

I_1 - target image, I_2 - reference image



Matching between the diastole and systole ventricles



Co-registered 3D viz of the brain and basal ganglia. The brain viz is from an MRI study, while the basal ganglia viz is depicting PET of some metabolism rate (Courtesy of PET Laboratory at Mount Sinai Medical Center, New York).

Registration Criteria

- Three criteria: landmark-, segmentation- or intensity-based criteria.
  - Landmark-based registration uses salient features selected by the user. (points, lines, corners, crossings, etc.) Minimize the distance between physical points.
  - Segmentation-based registration attempts to rigidly or deformably align the binary structures (curves, surfaces or volumes) obtained by segmentation. Minimize the distance between physical points.
  - Intensity-based registration operates directly on the image intensity by minimizing a cost function that measures the similarity between the image intensities of two images. It is very expensive because it uses full image content.

Spatial Transformation

- For 2D/2D or 3D/3D registration problems, the spatial transformation can be rigid, affine or deformable.
  - Rigid: rotation and translation.
  - Affine: rotation and translation, skew, scaling.
  - Deformable: free-form mappings with a regularization constraint to limit the allowable solution space.

Parameter Optimization

- The registration process can be viewed as an optimization problem.
- The registration criterion is the cost function to be minimized over the search space spanned by the spatial transformation parameters.

Image Interpolation

- When mapping points from one image space to another, the mapped point will generally fall on a non-grid position in the target image.
- To obtain the image intensity at the position, an image interpolation method is needed.
- Two popular methods are linear and B-spline interpolation.

A Registration Framework

- To find the spatial mapping that will bring the moving image $M(x)$ into alignment with the fixed or target image $F(x)$.
- The transform component, $T(p)$, is used to map points between two images for a given set of transform parameters p.
- The metric component, $S(\mathrm{p}|\ F, M, T)$, represents the similarity measure of how well $F(x)$ is matched with a transformed $M(x)$.

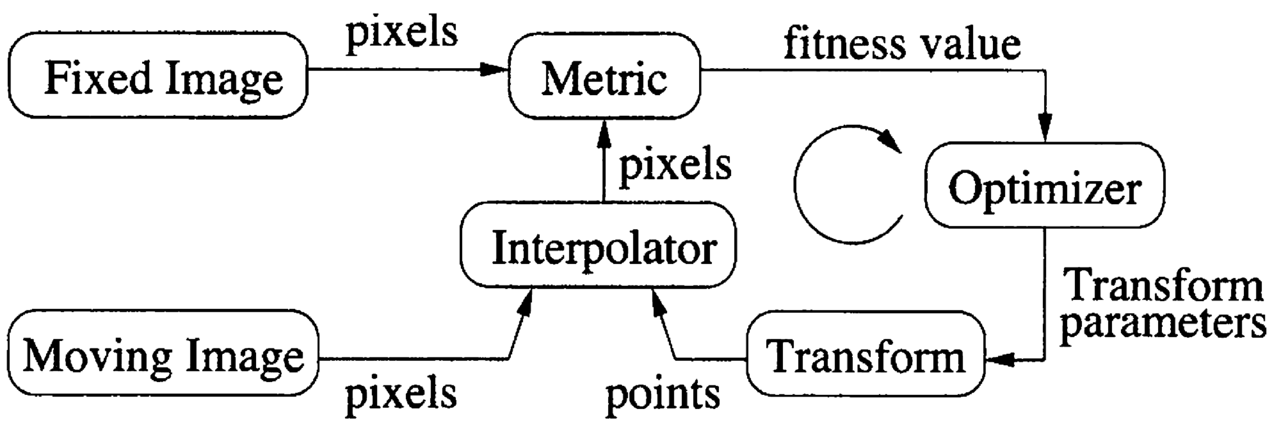


Figure 10.1. The basic components of the registration framework are two input images, a transform, a metric, an interpolator, and an optimizer.

Transforms

- A transform T(p) is typically defined by a set of transform parameters, p. A point x of the fixed image coordinate system S_f will be mapped onto the point x' of the moving image coordinate system S_m by the expression:

$$x' = T(x \mid p)$$

$$T : R^n \mapsto R^m \quad such\ that \quad T(S_f) = S_m$$

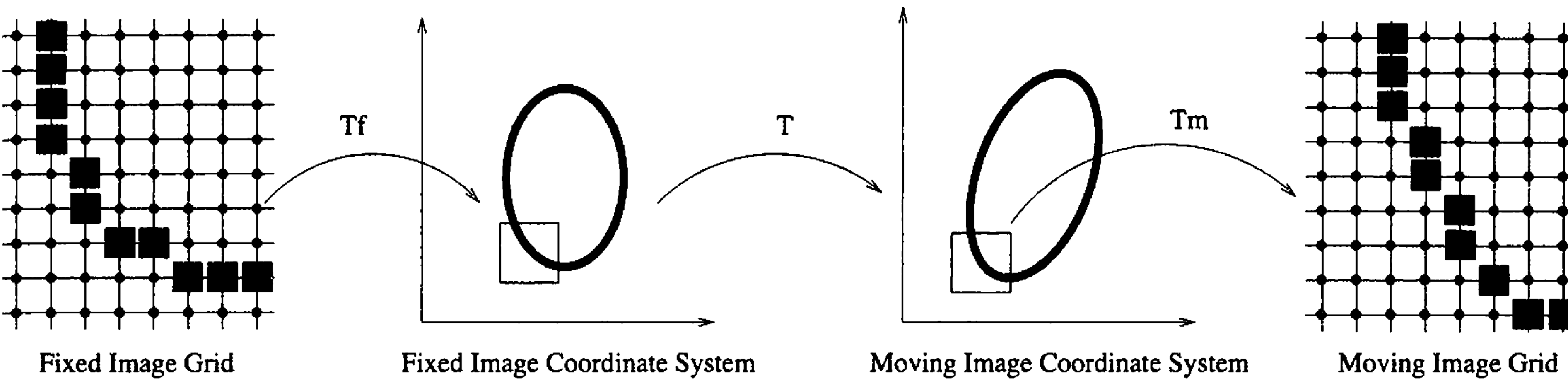
- A rigid transform in 2D space:

$$x' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \bullet \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$x' = T(x \mid p) = T(x, y \mid t_x, t_y, \theta)$$

Mapping from the Image Discrete Grid

- The mapping of pixels from the fixed image onto the moving image involves three different transforms:
  - The mapping T_f from the fixed image grid to the fixed image space.
  - The mapping T from the fixed image space to the moving image space.
  - The mapping T_m from the moving image space to the discrete grid of the moving image.



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Transforms and Metric Derivatives

- The goal of registration is to find the set of transform parameters that optimize the value of the image similarity metric.
- Some registration optimizers require function derivatives information w.r.t. the parameters of transform *p*. By using the differential chain-rule, we have:

$$\frac{\partial S(p \mid F, M, T)}{\partial p_i} = \sum_j \frac{\partial S(p \mid F, M, T)}{\partial x'_j} \bullet \frac{\partial x'_j}{\partial p_i}$$

where the matrix $\left[{\partial x'_j}/{\partial p_i} \right]$ is called the *Jacobian* of the transformation.

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Jacobian Matrix

- Given a transform T that maps a point $x = x_1, x_2, \cdots, x_n$ into another point $x' = x'_1, x'_2, \cdots, x'_n$, it is customary to call the *Jacobian* matrix

$$J = \begin{bmatrix} \frac{\partial x_1'}{\partial x_1} & \frac{\partial x_1'}{\partial x_2} & \cdots & \frac{\partial x_1'}{\partial x_n} \\ \frac{\partial x_2'}{\partial x_1} & \frac{\partial x_2'}{\partial x_2} & \cdots & \frac{\partial x_2'}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial x_n'}{\partial x_1} & \frac{\partial x_n'}{\partial x_2} & \cdots & \frac{\partial x_n'}{\partial x_n} \end{bmatrix}$$

- The determinant $|J|$ of the Jacobian matrix is often called the *Jacobian* of the transformation.

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Jacobian Matrix

- In registration, we are more interested in the following Jacobian:

$$J = \begin{bmatrix} \dfrac{\partial x_1'}{\partial p_1} & \dfrac{\partial x_1'}{\partial p_2} & \cdots & \dfrac{\partial x_1'}{\partial p_m} \\ \dfrac{\partial x_2'}{\partial p_1} & \dfrac{\partial x_2'}{\partial p_2} & \cdots & \dfrac{\partial x_2'}{\partial p_m} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial x_m'}{\partial p_1} & \dfrac{\partial x_m'}{\partial p_2} & \cdots & \dfrac{\partial x_m'}{\partial p_m} \end{bmatrix}$$

where the elements p_i are the parameters of the transform.

Jacobian Matrix

- The transformation mapping is a function of the input point coordinates and the transform parameters:

$$x'_j = T(x_1, x_2, \cdots, x_n, p_1, p_2, \cdots, p_m)$$

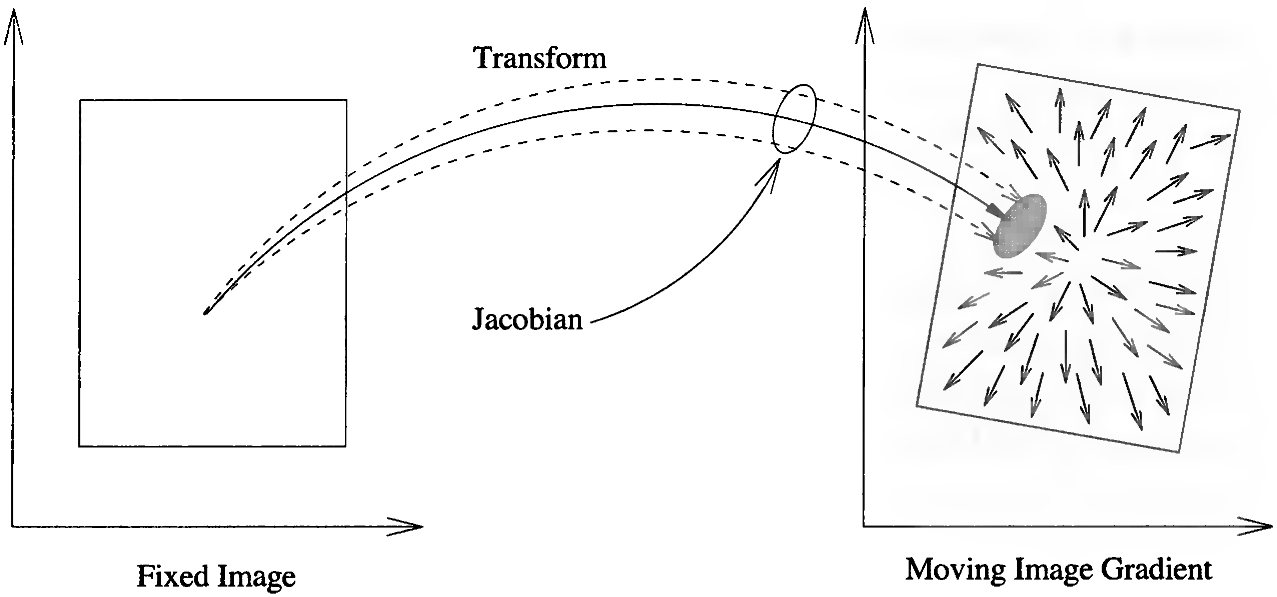
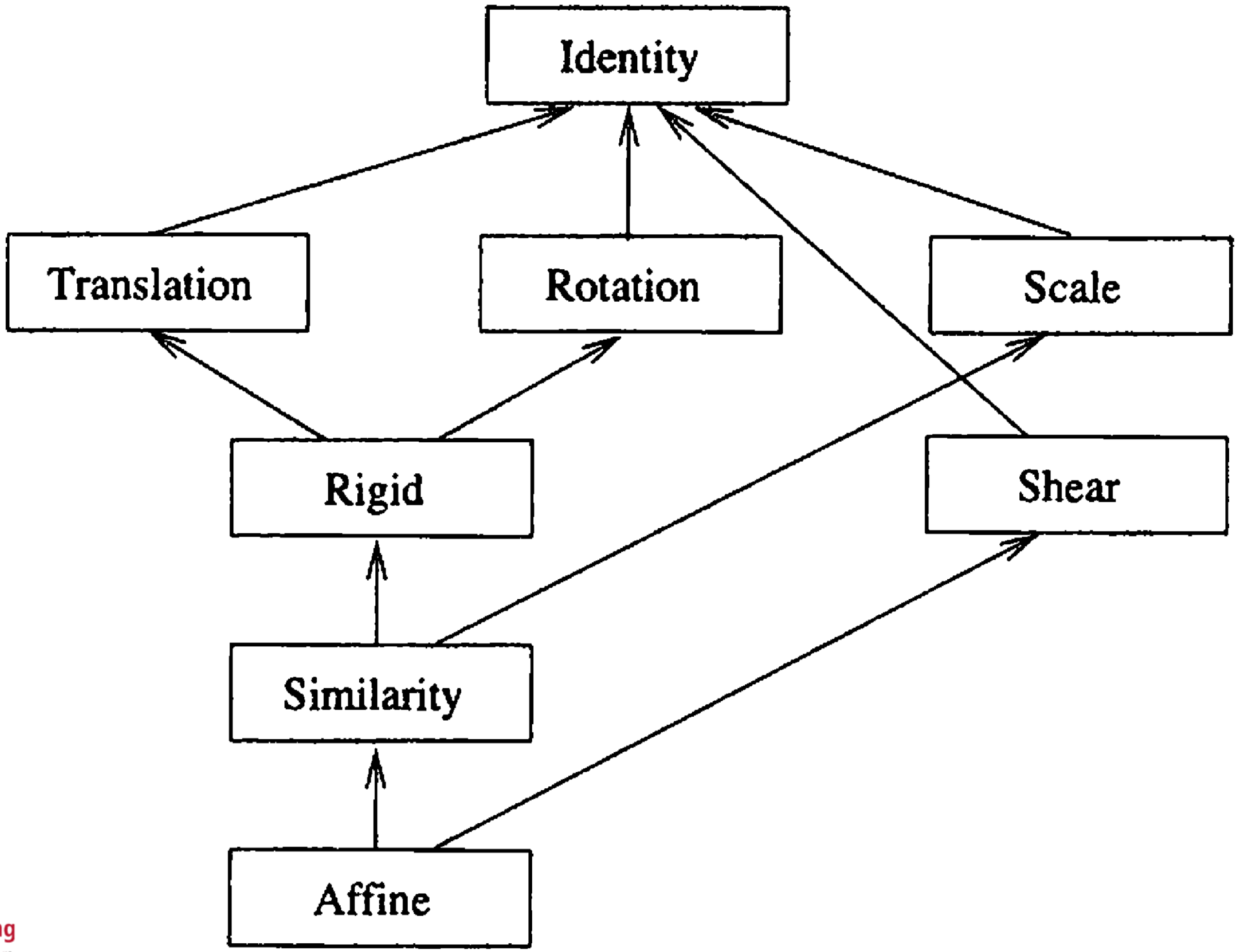


Figure 10.6. Relationship between the variations of the mapped point in the moving image space, as evaluated by the transform Jacobian, and the gradient of image intensities in the moving image. The combination of the Jacobian and the image gradient allow evaluation of the variation of one pixel contribution to the total image metric, and henceforth cumulate values for computing the image metric derivative.

Taxonomy of Spatial Transforms

- The arrows from one transform to another that can be considered to be one of its particular cases.



Transforms

- Identity transform simply maps every point to itself. It has no parameters and is only used in the context of software frameworks for validation and testing purposes.
- Translation is represented by a vector *V* indicating the displacement from the input space to the output space.
- Scaling has non-linear effects on the coordinates of mapped points (*D* is a scalar factor).

$$x' = \begin{bmatrix} x_1' \\ x_2' \\ \vdots \\ x_n' \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \qquad x' = \begin{bmatrix} x_1' \\ x_2' \\ \vdots \\ x_n' \end{bmatrix} = D \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Transforms

- Image center C is the origin of coordinates. C is a fixed point. Suppose the scaling factor is D, and the auxiliary translation is T,

$$x' = D(x - C) + C + T$$

- Rotation in 2D: suppose the rotation center is C

$$x' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \bullet \begin{bmatrix} x - C_x \\ y - C_y \end{bmatrix} + \begin{bmatrix} C_x \\ C_y \end{bmatrix}$$

- An affine transform includes translation, rotation and scaling

$$x' = \begin{bmatrix} x' \\ y' \end{bmatrix} = D\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \bullet \begin{bmatrix} x - C_x \\ y - C_y \end{bmatrix} + \begin{bmatrix} C_x \\ C_y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

Example

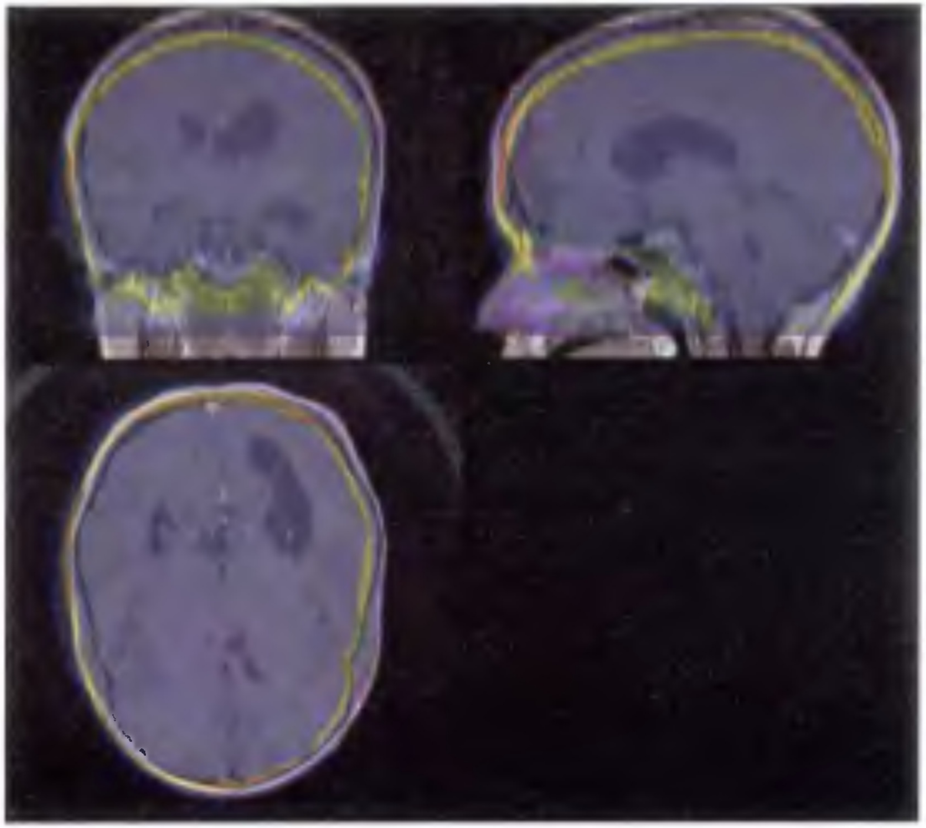


Plate IX. (From Figure 10.34) Registration start with the center of the CT image initially aligned with the center of the MR-T1 image. The CT image is shown as a color overlay on top of a gray-scale MR-T1 image. The yellow color corresponds with the bright intensity regions of the CT image.

Image Registration Techniques

Classified by different transformation models:

- Affine registration: shear, rotation, scale, translation
  - Differential affine motion
  - Fourier based matching
  - etc.

- Elastic/Deformable registration
  - Elastic registration
  - Viscous fluid registration
  - Level sets registration
  - etc.

Differential Affine Motion

Let f(x, y) and g(x, y) represent test and reference images, they are related by an affine transform:

$$f(x, y) = g(m_1x + m_2y + t_x,\ \ m_3x + m_4y + t_y)$$

Affine matrix: $A = \begin{pmatrix} m_1 & m_2 \\ m_3 & m_4 \end{pmatrix}$ displacement vector: $t = \begin{pmatrix} t_x \\ t_y \end{pmatrix}$

Symmetric translation:

$$f(x\text{-}\ t_x/2,\ y\text{-}\ t_y/2) = g(m_1x + m_2y + t_x/2,\ \ m_3x + m_4y + t_y/2)$$

Assume $m_5 = t_x/2$, $m_6 = t_y/2$, we have

$$f(x\text{-}\ m_5,\ y\text{-}\ m_6) = g(m_1x + m_2y + m_5,\ \ m_3x + m_4y + m_6).$$

The goal is to estimate 6 parameters: $m = (m_1, m_2, m_3, m_4, m_5, m_6)^T$.

Differential Affine Motion

Define a quadratic error function:

$$E(m) = \Sigma\, [f(x- m_5, y- m_6) - g(m_1 x + m_2 y + m_5,\ m_3 x + m_4 y + m_6)]^2$$

$$\approx \Sigma\, [(f(x,y) - m_5\, f_x - m_6\, f_y)$$
$$-\, (g(x,y) + (m_1 x + m_2 y + m_5 - x) g_x + (m_3 x + m_4 y + m_6 - y)\, g_y)]^2$$

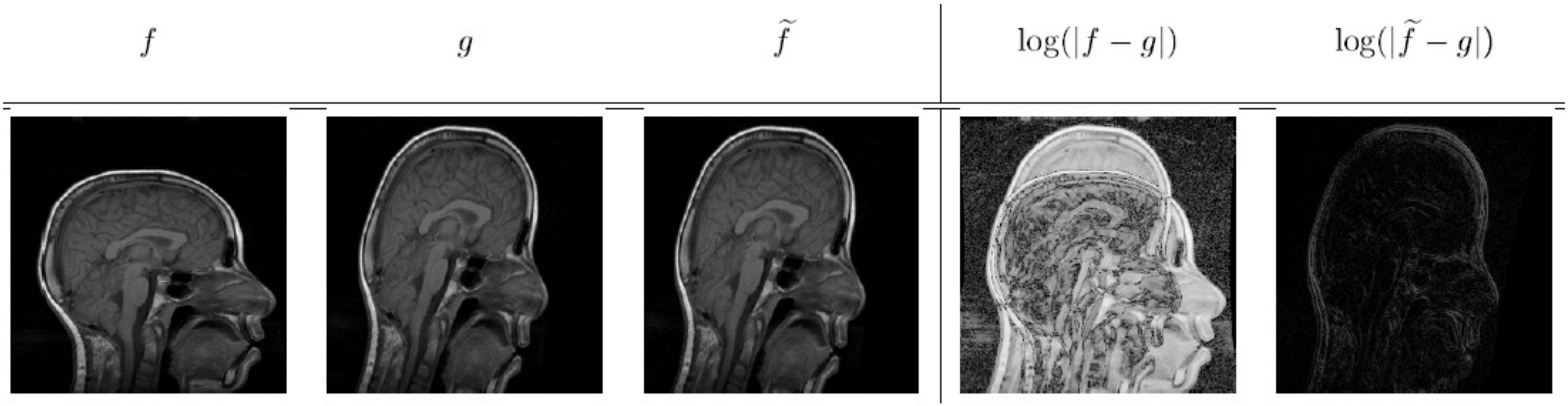
$$= \Sigma\, (\, k - c^T m\,)^2$$

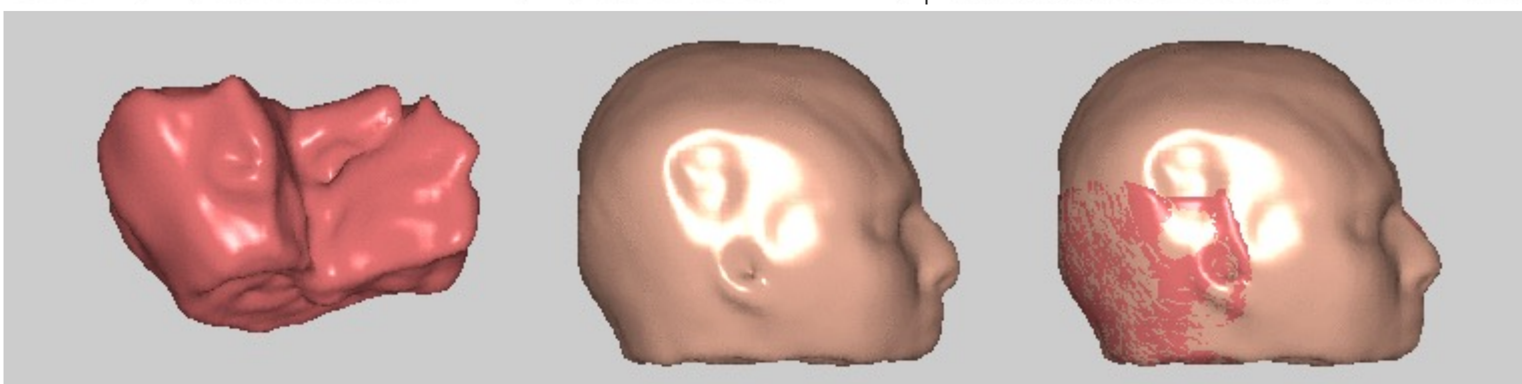
where: $c^T = (x g_x \quad y g_x \quad x g_y \quad y g_y \quad (f_x + g_x) \quad (f_y + g_y))$
$k = f - g + x\, g_x + y\, g_y$

Minimize the error function w.r.t. m:

$$\frac{dE}{dm} = \sum -2c(k - c^T m) = 0 \qquad\qquad m = \left(\sum cc^T\right)^{-1}\left(\sum ck\right)$$

Differential Affine Motion





S. Periaswamy et al. **Differential Affine Motion Estimation for Medical Image Registration**. SPIE's 45th Annual Meeting, San Diego, CA, 2000.
(Matlab code is available at http://www.cs.dartmouth.edu/~farid/research/registration.html)

Fourier-based Matching

- Suitable for images with large distortion and different contrasts.

- $f(x)$ is the test image, its Fourier transform is $F(u) = \int_{\Omega} f(x) e^{-i2\pi u^T x} dx$
  Let $g(Ax+t)$ be the distorted reference image with Fourier transform $G(u)$. The power spectrum is:

$$|G(u)| = \frac{1}{|\det A|} \left| F\left(\left(A^{-1} \right)^T u \right) \right|$$

- $|G(u)|$ is only related to A, not related to t (decoupled). $|G(u)|$ is normalized before estimating the linear affine parameters.

Fourier-based Matching

- For non-equispaced image data, the traditional FFT is not applicable.

- 0th and 1st order moment M_i and M_{ij} are used to determine shift. M_i is the magnitude of Fourier transform of I_i, i = 1, 2.

$$M_{ij} = \int x_j \cdot I_i(x)dx$$

- 2nd order moment (moment of inertia) M_{ijk} is used to determine rotation

$$M_{1ij} = \int M_1(\omega) \cdot \omega_i \cdot \omega_j d\omega$$

$$M_{2ij} = \int M_2(\omega) \cdot \omega_i \cdot \omega_j d\omega$$

R. Araiza et al. 3-D Image Registration Using Fast Fourier Transformation: Potential Applications to Geoinformatics and Bioinformatics.

Non-Rigid Image Registration

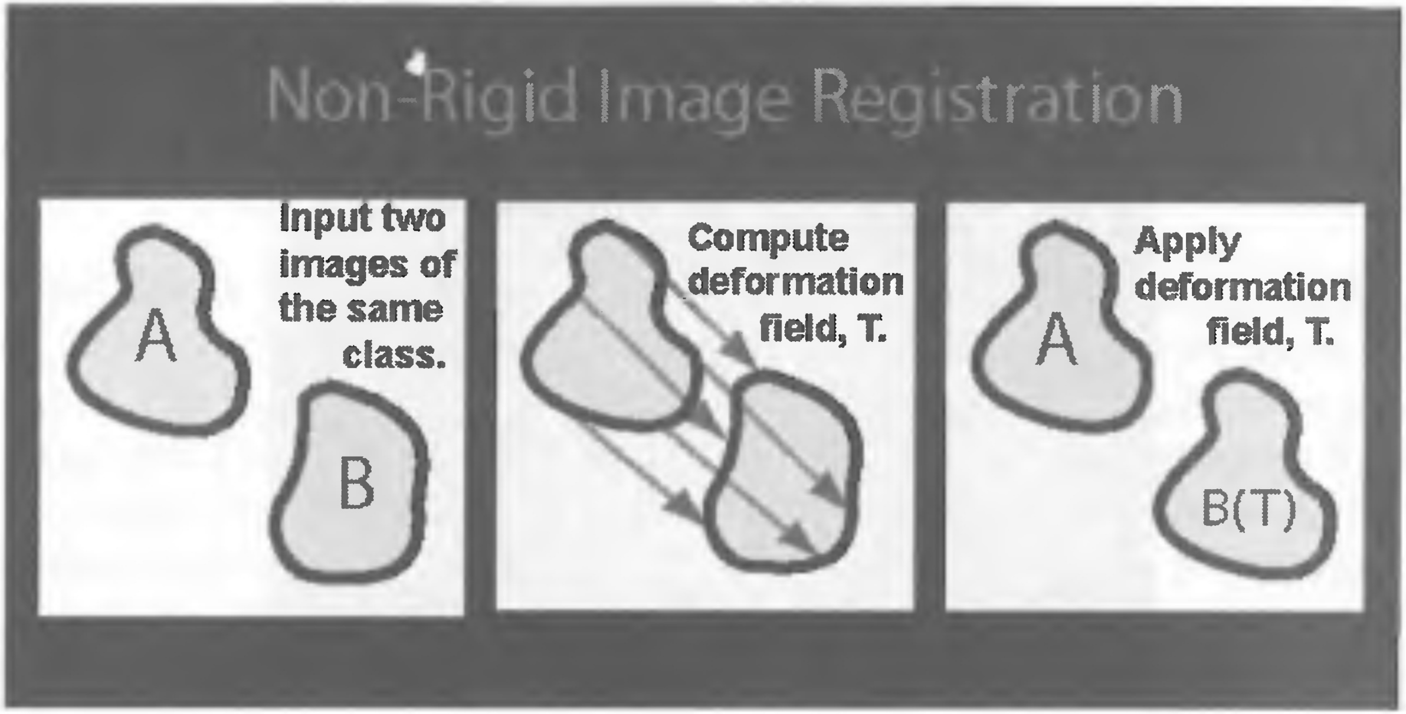


Figure 11.2. This schematic illustrates the process of non-rigid image registration. Note that the vector field is computed in the A to B direction, but B is transformed into the coordinate system of A via the inverse warp.

Deformable Registration

- To construct a minimization problem with an image matching function and a regularizing constraint on the deformation.
- Matching functions:
  - Squared intensity difference
  - Cross-correlation
  - Mutual information
- Regularizing constraints:
  - Optical flow
  - Elasticity theory
  - Fluid dynamics
  - Thin-plate spline theory

T(*x*) – reference image, *S*(*x*) – target image

The goal is to find a displacement field *u*(*x*) s.t. *T*(*x*-*u*) = *S*(*x*).

$$E_{L^2}(T,S,u) = \frac{1}{2}\int_{\Omega} \left| T(x-u) - S(x) \right|^2 dx$$

Minimize the energy function *E* to obtain $\dfrac{\partial u(x,t)}{\partial t} = -f(x,u(x,t))$

The body force $f(x,u(x,t)) = -[T(x-u) - S(x)]\nabla T\,|_{x-u}$

Optical Flow: Fast Mono-Modality Non-Rigid Registration

- Image values are conserved over time:

$$I(x,t) = I(x + v\delta t, t + \delta t)$$

where $I(x, t)$ is the image value at position x and time t, v is the velocity (or optical flow) vector, and δt is very small. The optical flow constraint, C_{of}, is gained by expanding the right hand side in Taylor series to find, ignoring 2nd order terms

$$C_{of} = I_x \bullet v + I_t = 0$$

An image point at one time instant that is translated to a new position in a later image is assumed to retain its original image value.

Optical Flow

- To minimize a global variational energy

$$E(v)=\int_\Omega\left(\Phi(C_{of})+\left|v_x\right|^2\right)d\Omega=\int_\Omega F(x,v,v_x)\,d\Omega$$

where $v_x=\nabla v$, $\Phi(C_{of})=C_{of}^2$ and $C_{of}=I_x\bullet v+I_t$.
Substitute $v'=v+\varepsilon w$ for v, then

$$\frac{dE(v')}{d\varepsilon}=\int_\Omega\frac{d}{d\varepsilon}F(x,v',v'_x)\,d\Omega$$

$$\frac{dF(x,v',v'_x)}{d\varepsilon}=\frac{\partial F}{\partial v'}\frac{dv'}{d\varepsilon}+\frac{\partial F}{\partial v'_x}\frac{dv'_x}{d\varepsilon}=\frac{\partial F}{\partial v}w+\frac{\partial F}{\partial v_x}\frac{d}{d\varepsilon}\left(\frac{dv'}{dx}\right)$$

$$=\frac{\partial F}{\partial v}w+\frac{\partial F}{\partial v_x}\frac{d}{dx}\left(\frac{dv'}{d\varepsilon}\right)=\frac{\partial F}{\partial v}w+\frac{\partial F}{\partial v_x}\frac{dw}{dx}$$

Optical Flow

$$\frac{d}{dx}\left(\frac{\partial F}{\partial v_x}w\right)=\frac{d}{dx}\left(\frac{\partial F}{\partial v_x}\right)w+\frac{\partial F}{\partial v_x}\frac{dw}{dx}$$

$$\downarrow$$

$$\frac{\partial F}{\partial v_x}\frac{dw}{dx}=\frac{d}{dx}\left(\frac{\partial F}{\partial v_x}w\right)-\frac{d}{dx}\left(\frac{\partial F}{\partial v_x}\right)w$$

Applying boundary condition $\left.\frac{\partial F}{\partial v_x}\right|_{\partial\Omega}=0$:

$$\frac{dE(v)}{d\varepsilon}=\int_\Omega\left(\frac{\partial F}{\partial v}-\frac{d}{dx}\frac{\partial F}{\partial v_x}\right)w\,d\Omega=0$$

$$\frac{\partial\Phi(C_{of})}{\partial v}-\frac{d}{dx}\frac{\partial\left|v_x\right|^2}{\partial v_x}=0 \quad \text{or} \quad I_x(I_x\bullet v+I_t)-v_{xx}=0$$

PET-CT Registration

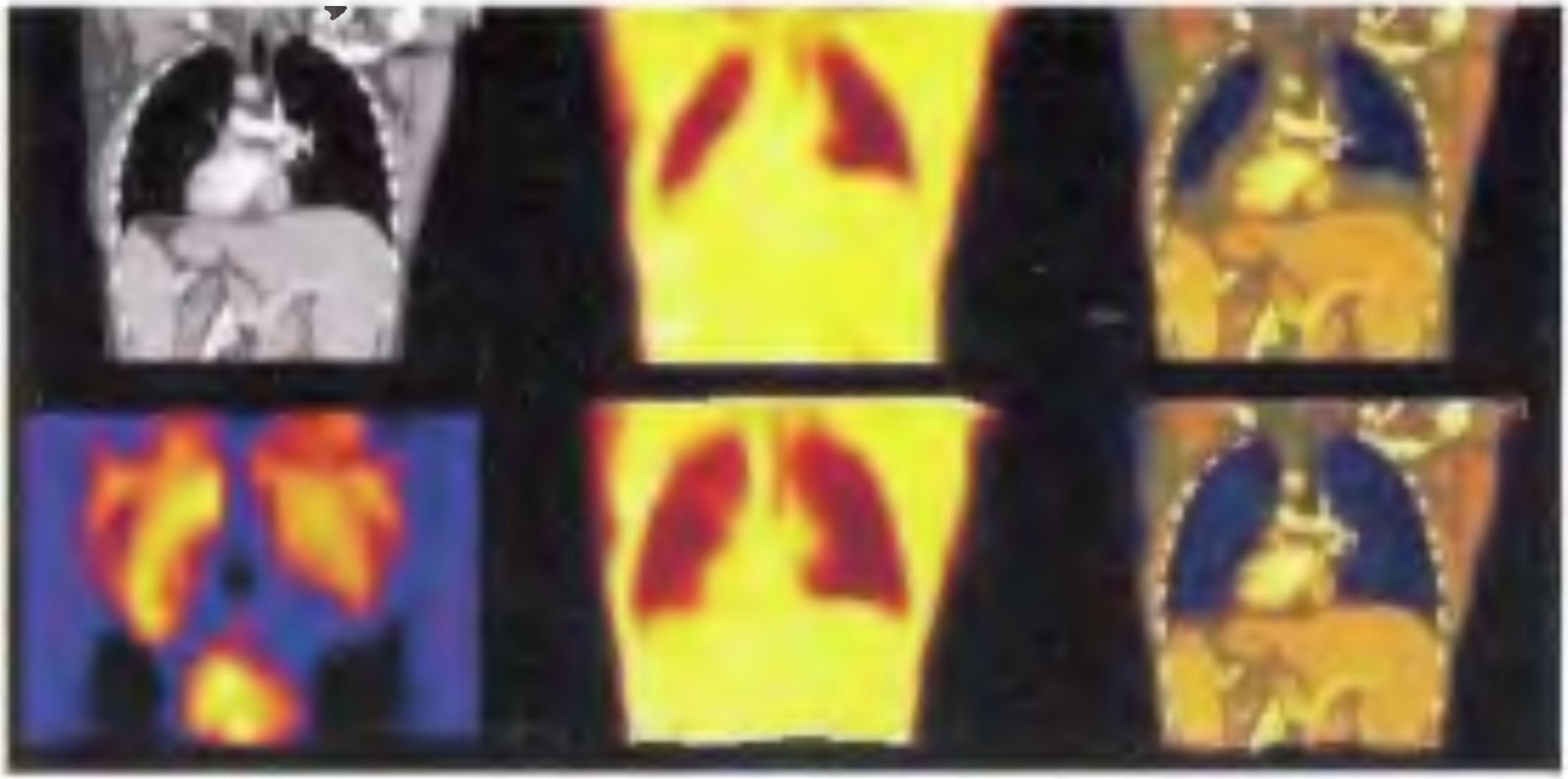


Plate XII. (From Figure 11.8) PET-CT image fusion. The original CT fixed image is at top left. The Jacobian of the transformation is at bottom left. The original PET transmission image is at top center. The warped PET transmission is at bottom center. The overlay of original images is at top right. The warped PET is fused with the CT at bottom right.

Registration using Mean Curvature

Plate IV. (From Figure 11.1) The gray-white interfaces of a human (left) and a chimp (far right) are registered by their mean curvature. The result is shown in the center column.

Large Deformation Diffeomorphic Matching with Landmarks

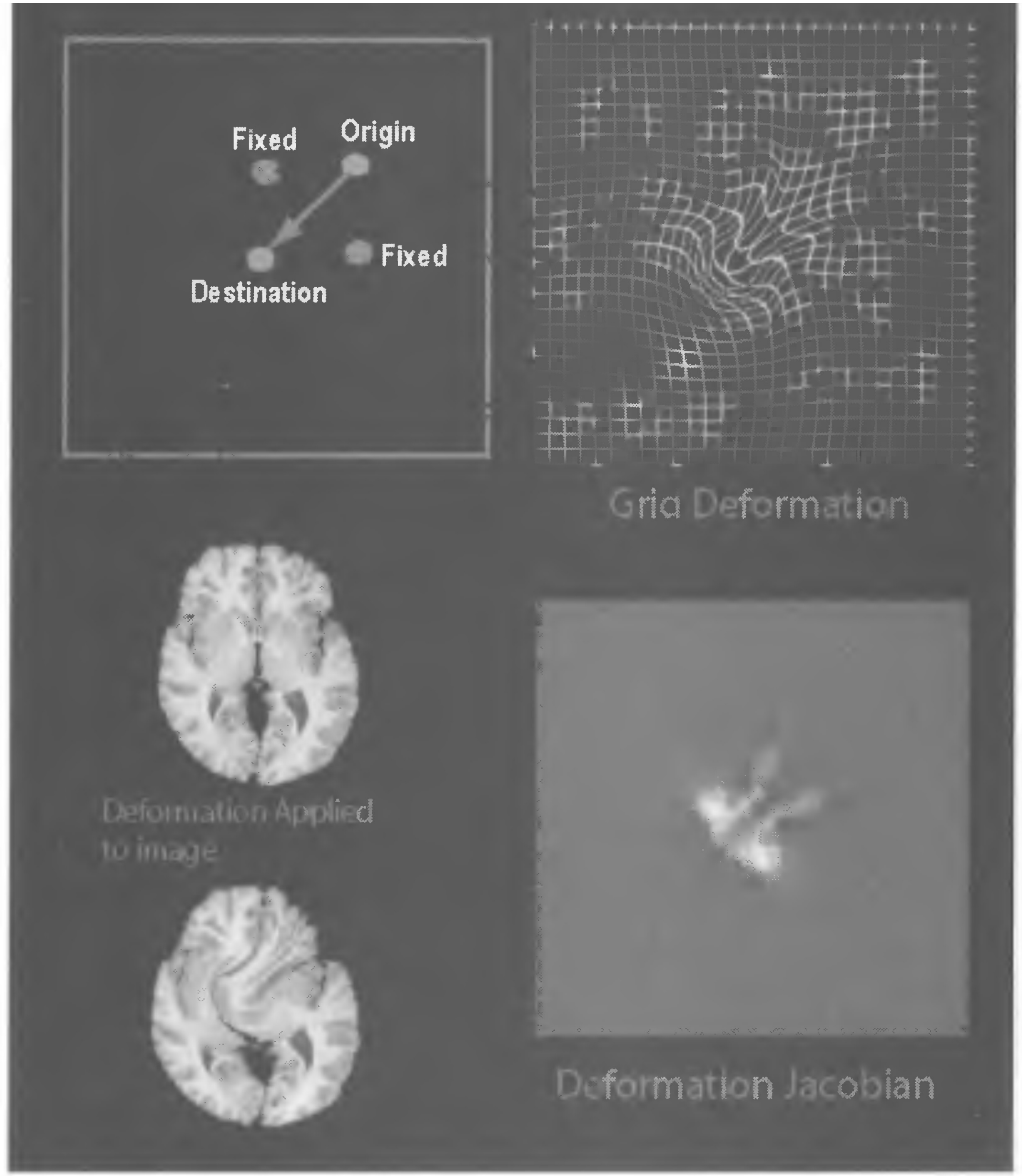


Figure 11.20. Large deformation diffeomorphic landmark matching is illustrated with an example in which one landmark is forced to pass between two others. This type of transformation would typically induce folding. The grid illustrates the smoothness of the transformation.

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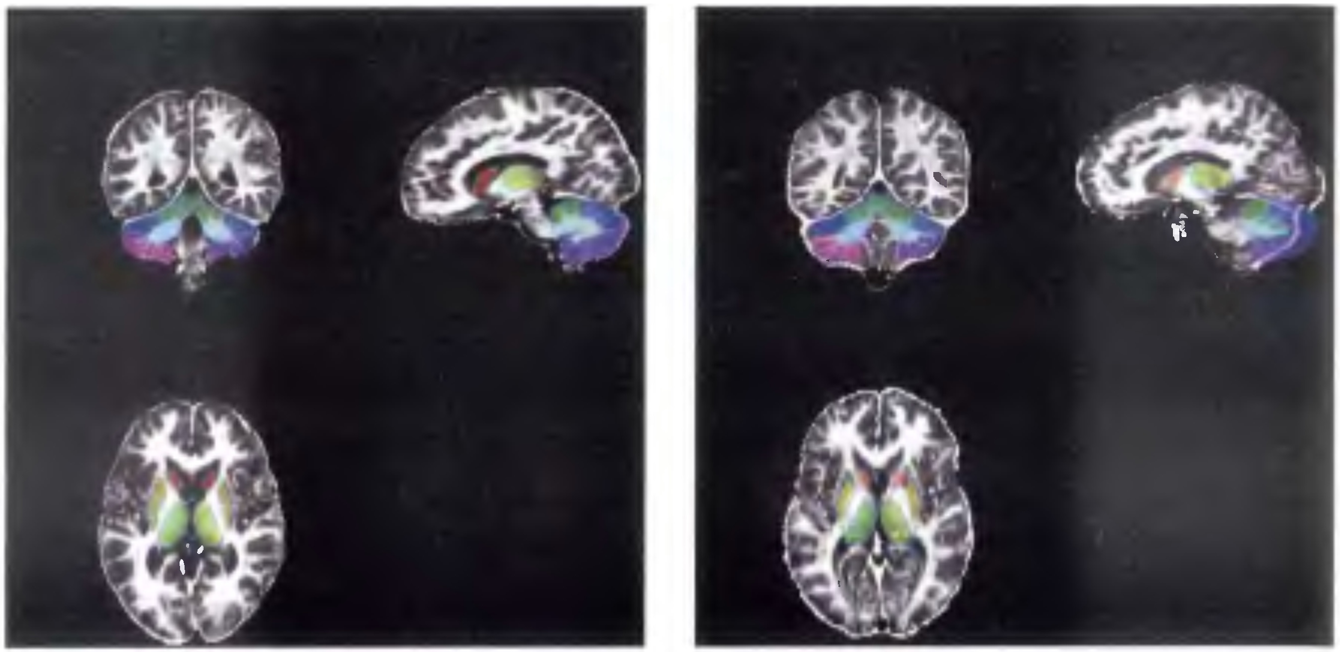
Atlas-Based Segmentation Example

- Manual segmentation is done only once on an atlas image, then non-rigid registration is used to map/align it to the subject image.



(a) (b)

Plate XIV. (From Figure 11.6) For the atlas image (a), several subcortial and cerebellar structures were hand segmented. The segmentations are shown as an overlay on top of the atlas image (left) and subject image (b).

Elastic Registration

- The image deformation is considered similar to the deformation of elastic plates:

$$\mu \Delta u + (\mu + \nu)\nabla(\nabla \cdot u) = f(x,u)$$

where μ and ν are Lame constants.

Elastic registration is derived assuming small angles of rotation and small linear transformations, it is not suitable for large nonlinear deformations.

R. Bajcsy and S. Kovacic. **Multiresolution elastic matching**. Computer Vision Graphics and Image Processing, 46:1-21, 1989.

Viscous Fluid Registration

- The Eulerian velocity field V is nonlinearly related to u:

$$V = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + V \cdot \nabla u$$

The deforming reference image T(x) is considered to be embedded in a viscous fluid whose motion is governed by Navier-Stokes equation, one simplification is

$$\mu \Delta V + (\mu + \nu) \nabla (\nabla \cdot V) = f(x,u)$$

$$f(x, u(x,t)) = -[T(x-u) - S(x)] \nabla T \,|_{x-u}$$

G. Christensen et. al. **Deformable templates using large deformation kinematics.** IEEE Transaction on Image Processing, 5(10):1435-1447, 1996.

Level Set Methods

- Based on viscous fluid registration, a new energy function was designed by introducing Jacobian maps [Osher 2006].

- Jacobian maps are important to study nonlinear image registration Jacobian map is defined as $|Dh|$, where h is a deformation map.

- Preservation of global density

$$\int_\Omega \left| D\vec{h}(\xi) \right|^2 d\xi = \int_\Omega d\vec{y} = 1$$

$$\int_\Omega \left| D\vec{h}^{-1}(\xi) \right|^2 d\xi = \int_\Omega d\vec{x} = 1$$

- Three probability density functions to h, h^{-1} and id (identity map)

$$P_h(\cdot) = \left| D\vec{h}(\cdot) \right|$$

$$P_{h^{-1}}(\cdot) = \left| D\vec{h}^{-1}(\cdot) \right|$$

$$P_{id}(\cdot) = 1$$

I. Yanovsky, P. Thompson, S. Osher, A. Leow. Large Deformation Unbiased Diffeomorphic Nonlinear Image Registration: Theory and Implementation. UCLA CAM Report 06-71, 2006.

Level Set Methods

- The Kullback-Leibler (KL) distance is defined as:

$$KL(X,Y) = \int_{\Omega} X \log \frac{X}{Y} \, d\vec{x} \geq 0$$

- The symmetric Kullback-Leibler (sKL) distance is chosen to quantify the magnitude of deformations.

$$sKL(P_h, P_{id}) = KL(P_{id}, P_h) + KL(P_h, P_{id})$$

Level Set Methods

$$KL(P_{id}, P_h) = \int_\Omega P_{id} \log \frac{P_{id}}{P_h} d\vec{x} = -\int_\Omega \log \left| D\vec{h}(\vec{x}) \right| d\vec{x}$$

$$KL(P_h, P_{id}) = \int_\Omega P_h \log \frac{P_h}{P_{id}} d\vec{x} = \int_\Omega \left| D\vec{h}(\vec{x}) \right| \log \left| D\vec{h}(\vec{x}) \right| d\vec{x}$$

Therefore:

$$sKL(P_h, P_{id}) = KL(P_{id}, P_h) + KL(P_h, P_{id})$$
$$= \int_\Omega (| Dh(x) | - 1) \log | Dh(x) | \, dx$$

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Properties of sKL(P_h, P_{id})

- Non-negative
- Only evaluates to zero when h is volume-preserving everywhere ($|Dh|=1$ everywhere)
- Zero-change can be recovered by minimizing sKL when we compare images differing only in noise.
- Minimizing sKL leads to unbiased deformation in the logarithmic space

Level Set Methods

- A new energy function is defined

$$E_{L^2}(T,S,u)=\frac{1}{2}\int_\Omega \left|T(x-u)-S(x)\right|^2 dx$$
$$+\lambda\int_\Omega (|Dh(x)|-1)\log|Dh(x)|\,dx$$

λ is the Lagrange multiplier. h is the deformation, and Dh is the Jacobian matrix of h. Suppose L(y) = (y-1)log$y$, then the body force becomes:

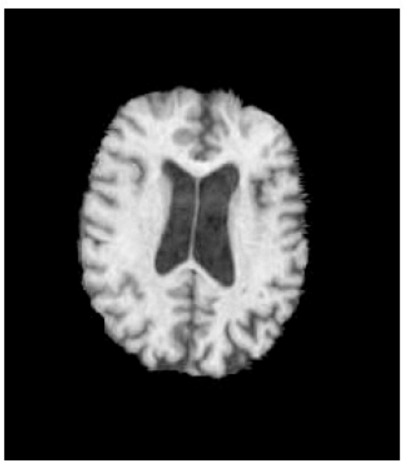
$$f(x,u(x,t))=-[T(x-u)-S(x)]\nabla T\,|_{x-u}$$
$$-\lambda\begin{bmatrix} -\dfrac{\partial}{\partial x_1}(\dfrac{\partial h_2}{\partial x_2}L')+\dfrac{\partial}{\partial x_2}(\dfrac{\partial h_2}{\partial x_1}L') \\ \dfrac{\partial}{\partial x_1}(\dfrac{\partial h_1}{\partial x_2}L')+\dfrac{\partial}{\partial x_2}(\dfrac{\partial h_1}{\partial x_1}L') \end{bmatrix}$$

$$L'=L'(|Dh|)=1+\log(|Dh|)-1/|Dh|$$

Results

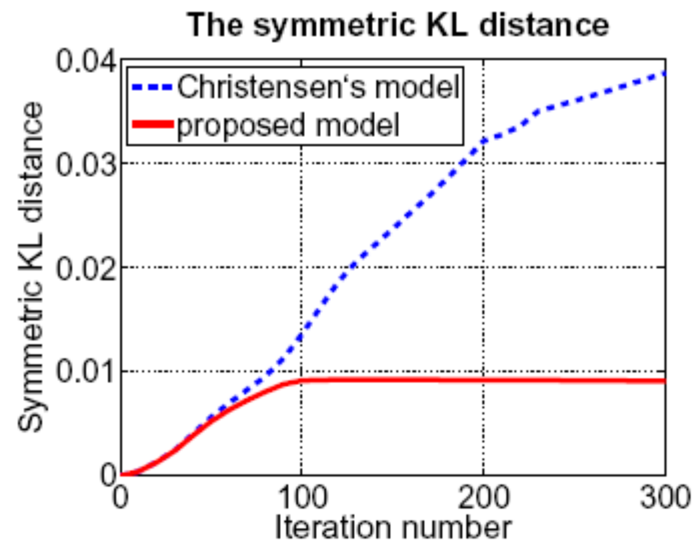
(a) T

(b) S



Christensen's Method

Osher's Method



Serial MRI example. (a) image T; (b) image S; (c) image T is deformed to image S using Christensen's model; (d) image T is deformed to image S using the proposed model.

I. Yanovsky, P. Thompson, S. Osher, A. Leow. Large Deformation Unbiased Diffeomorphic Nonlinear Image Registration: Theory and Implementation. UCLA CAM Report 06-71, 2006.

Other Level Set Methods

- A fast multiscale and multigrid method for the matching of images in 2D and 3D [Clarenz, Droske, Rumpf 2002].

$$E(\phi) = \frac{1}{2}\int_{\Omega} \left| f_1 \circ \phi - f_2 \right|^2 dx$$

f_1, f_2 are the intensity maps of the two images to be matched, and Φ is the deformation/transformation.

- A registration between two given 2D/3D images based on a matching of the edges in the images [Mumford & Shah 89]

Open Source Software

- ITK (Insight Segmentation and Registration Toolkit)
  http://www.itk.org

- AIR (Automated Image Registration)
  http://air.bmap.ucla.edu/AIR5/

- FLIRT (FMRIB's Linear Image Registration Tool)
  http://fsl.fmrib.ox.ac.uk/fsl/fslwiki/FLIRT

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