A physics-based generative AI approach to model order reduction

Vedant Puri, Aviral Prakash, Yongjie Jessica Zhang, and Levent Burak Kara
Mechanical Engineering, Carnegie Mellon University

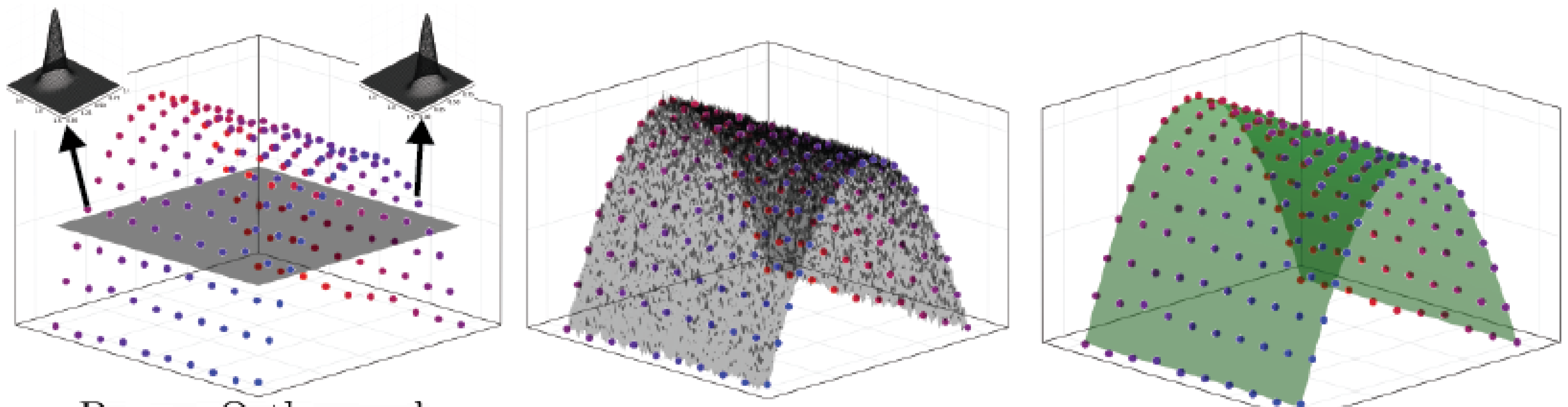
Motivation & overview

Access to high-fidelity numerical PDE solutions remains prohibitively expensive for problems that require repeated model evaluations such as design optimization. We present a generative AI-based model order reduction method that follows a separation of variable approach:

- **Spatial representation:** A low-dimensional learned manifold is fit to high-dimensional simulation data
- **Dynamics:** The governing PDE system is mapped to the latent manifold and dynamics are computed with automatic-differentiation

Method: manifold construction

Model order reduction approaches learn a low-dimensional representation of simulation data by projecting high-dimensional snapshots onto a learned lower-dimensional manifold.

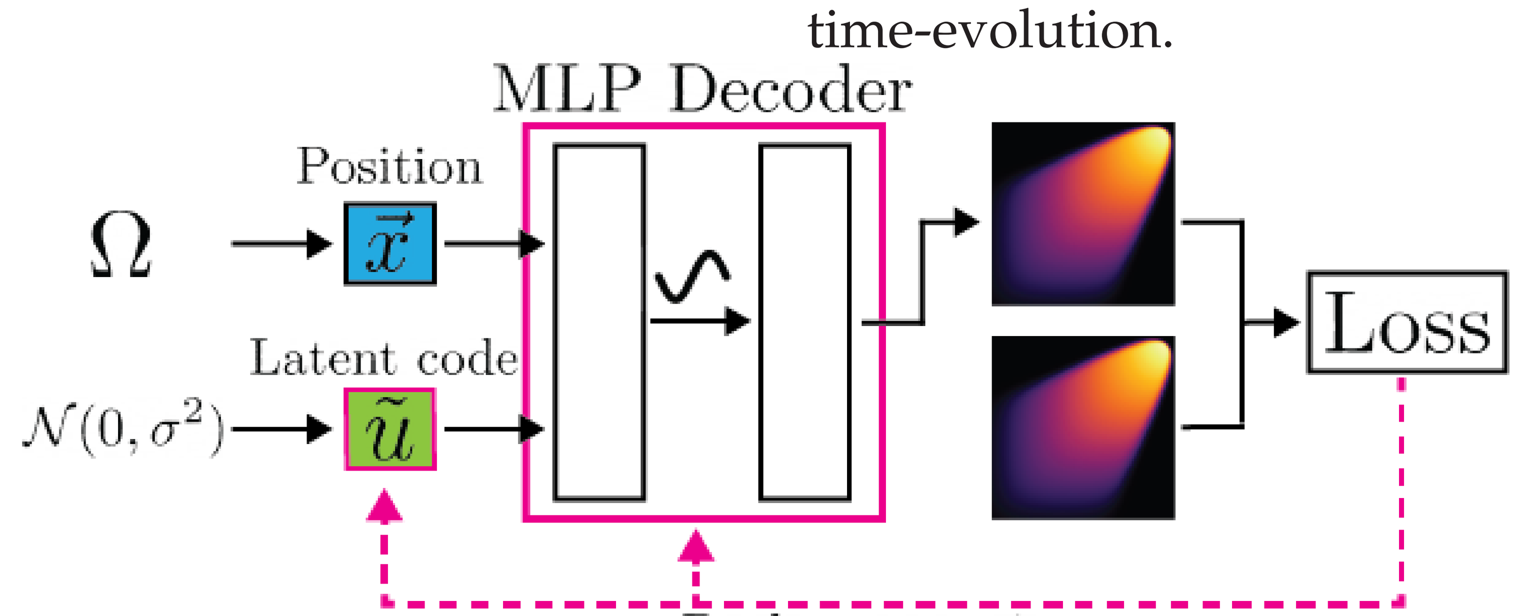


Proper Orthogonal Decomposition | Deep Auto-Encoder | Smooth Neural Field (ours)

POD attempts to fit a hyperplane to the data. This proves sub-optimal for advection-dominated problems that exhibit a slow decay in their Kolmogorov n-width.

Auto-encoders can accurately capture such data with nonlinear manifolds [2]. Leaned nonlinear manifolds, however, exhibit numerical artifacts that cause downstream problems in time-evolution.

We develop a continuous neural-field reduced modeling formulation that is trained in an **encoder-free** manner. That is, we only learn the decoder MLP and latent codes $\tilde{u}$ during training. Our training pipeline is **discretization invariant** allowing the model to learn from adaptive grid simulations. Furthermore, the learned **manifold is smooth**, enabling time-evolution with physics-based dynamics.



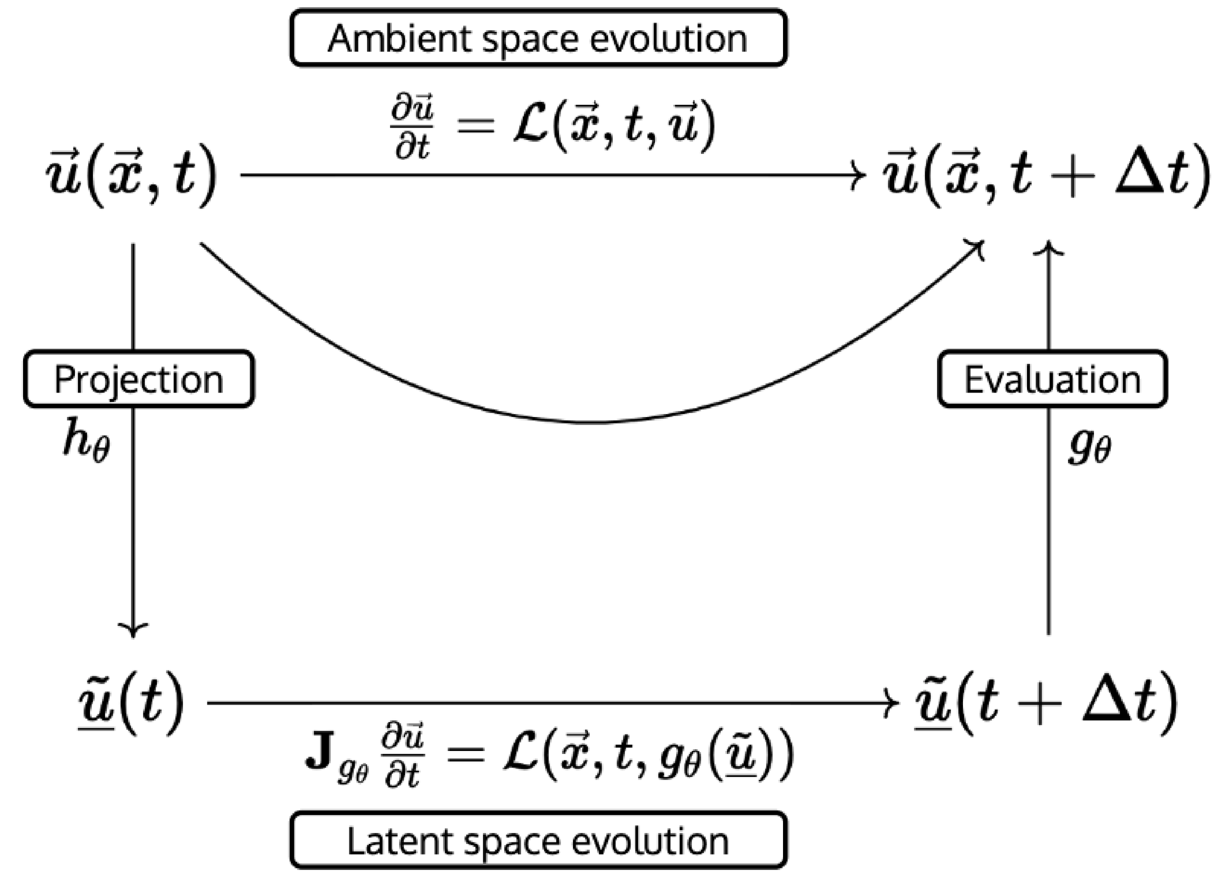
Method: physics-based dynamics

Dynamics calculation is carried out following the governing PDE system:

$$\frac{\partial}{\partial t}\vec{u}(\vec{x},t) = \mathcal{L}(\vec{x},t,\vec{u},\vec{\nabla}\vec{u},\dots)$$

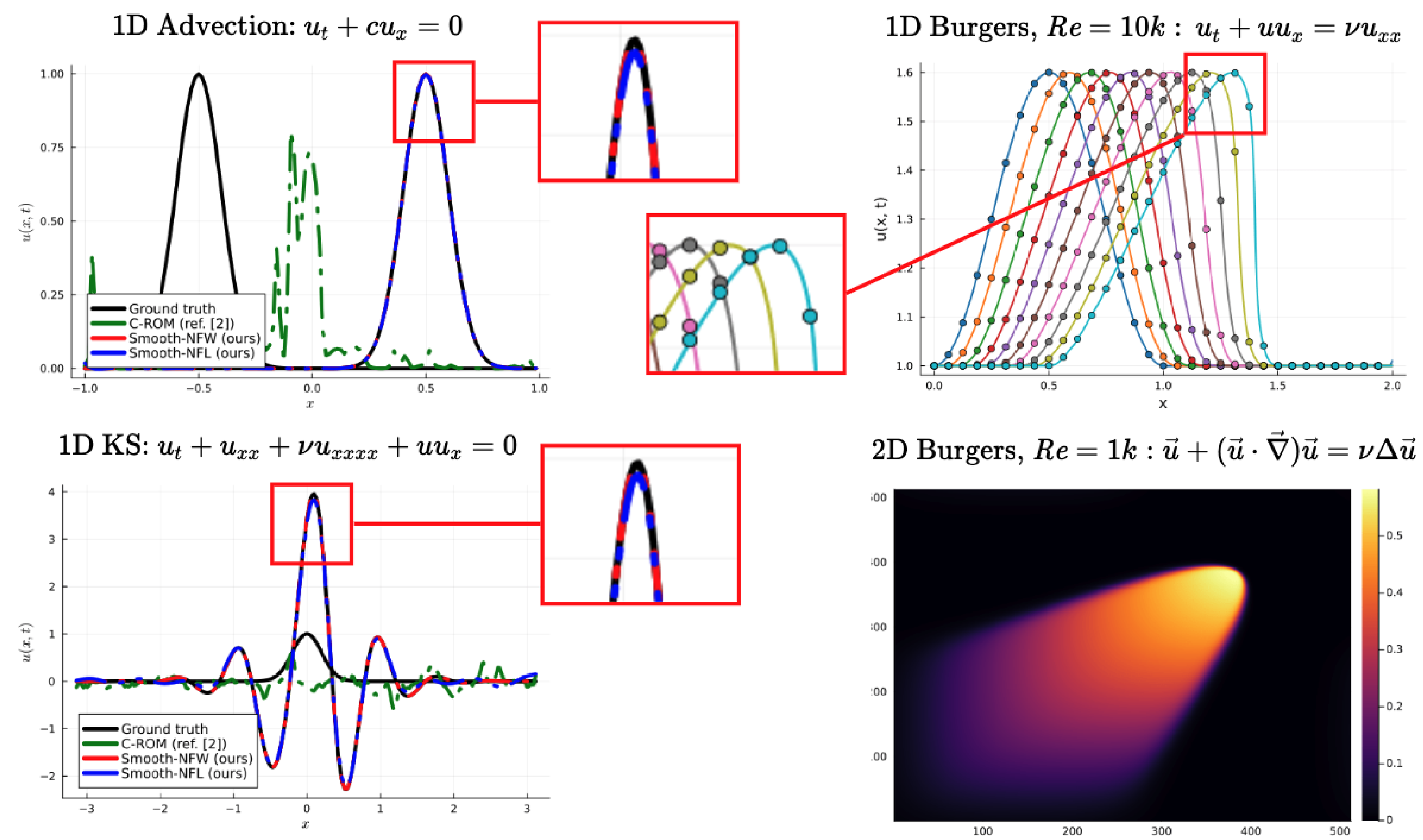
The reduced system is evolved in time as follows:

- The PDE system is projected onto the reduced manifold with Gauss-Newton iteration
- The right-hand-side term $\mathcal{L}$ is evaluated; automatic-differentiation is used for exactly computing spatial derivatives
- The reduced system is evolved with Galerkin projection [1], and the resulting reduced vector is evaluated in ambient space



Results

We demonstrate our method on a slew of advection dominated problems where we consistently achieve sub 1% accuracy.



References

[1] Puri V et. al. Manuscript in preparation
[2] Chen P et. al. arXiv:2206.02607 [physics] (2023).

Conclusion and future work

We have presented an ML reduced modeling framework that learns from point cloud simulation data, and supports computation of dynamics with automatic differentiation. Future work will consider including uncertainty metrics in reduced modeling frameworks.

Contact Information

Web `https://vpuri3.github.io/`
Email `vedantpuri@cmu.edu`
Phone `+1-347-330-1343`