

$$\frac{\partial \mathbf{u}}{\partial t} = \mathcal{L}(\mathbf{x}, t, \mathbf{u}; \boldsymbol{\mu})$$

Full order model (FOM)

$$\mathbf{u}(\mathbf{x}, t; \boldsymbol{\mu}) \approx g_{\text{FOM}}(\mathbf{x}, \bar{\mathbf{u}}(t; \boldsymbol{\mu})) = \boldsymbol{\Phi} \cdot \bar{\mathbf{u}}(t; \boldsymbol{\mu})$$

Linear POD-ROM

$$\bar{\mathbf{u}}(t; \boldsymbol{\mu}) \approx g'_{\text{ROM}}(\bar{\mathbf{u}}(t; \boldsymbol{\mu})) = \bar{\mathbf{u}}_0 + \mathbf{P} \cdot \tilde{\mathbf{u}}(t; \boldsymbol{\mu})$$

Nonlinear ROM

$$\mathbf{u}(\mathbf{x}, t; \boldsymbol{\mu}) \approx g_{\text{ROM}}(\mathbf{x}, \tilde{\mathbf{u}}(t; \boldsymbol{\mu})) = \text{NN}_{\theta}(\mathbf{x}, \tilde{\mathbf{u}}(t; \boldsymbol{\mu}))$$



