SNF-ROM: Projection-based nonlinear reduced order modeling with smooth neural fields

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1. Motivation and Overview

Access to high-fidelity numerical PDE solutions remains prohibitively expensive for problems that require repeated model evaluations. We propose smooth neural field reduced order model (SNF-ROM) that achieves **up to a 199× speed-up** over the full-order PDE computation while maintaining high levels of accuracy.

2. Learning low-dimensional spatial representations

The **full-order model (FOM)** applies a high-dimension spatial discretization to the PDE problem. **Linear ROM** attempts to fit a subspace to the FOM trajectory, which is suboptimal for advection-dominated problems. **Nonlinear ROMs** approximate the trajectory with a nonlinear combination of the ROM DoFs.

$$\frac{\partial \boldsymbol{u}}{\partial t} = \mathcal{L}(\boldsymbol{x}, t, \boldsymbol{u}; \boldsymbol{\mu})$$

Full order model (FOM)

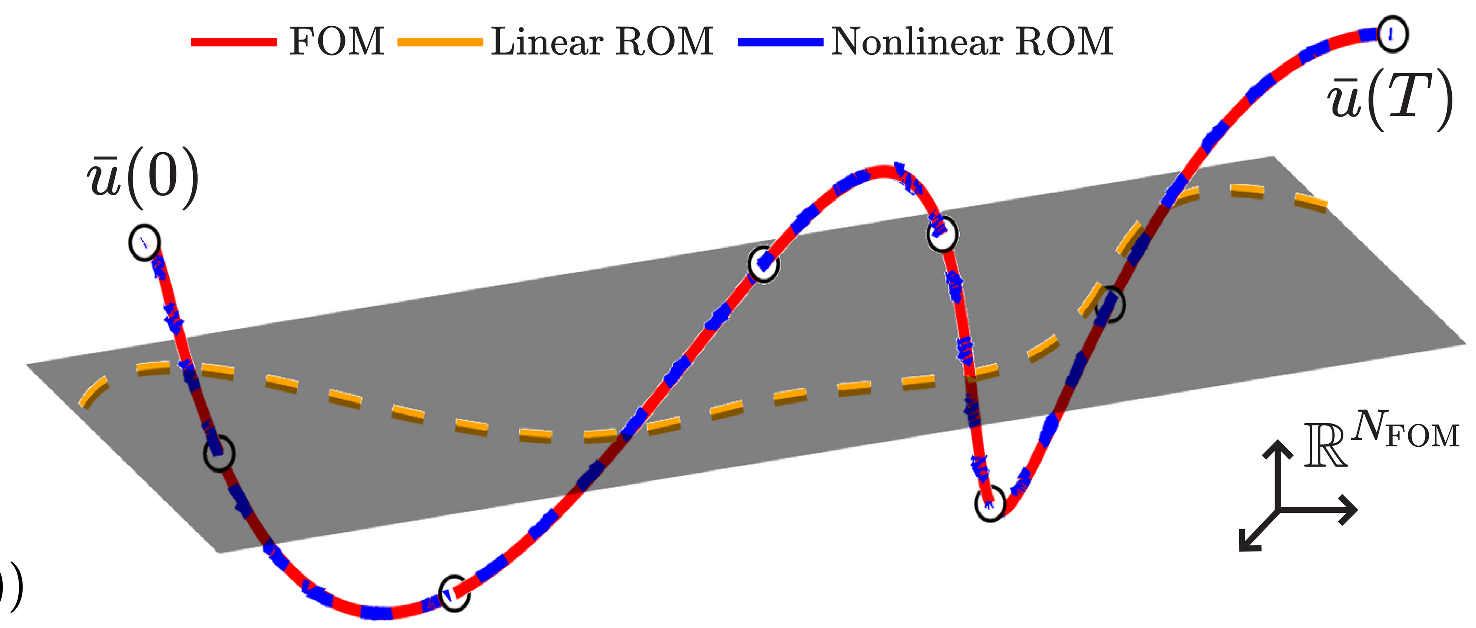
$$\boldsymbol{u}(\boldsymbol{x}, t; \boldsymbol{\mu}) \approx g_{\text{FOM}}(\boldsymbol{x}, \bar{u}(t; \boldsymbol{\mu})) = \boldsymbol{\Phi} \cdot \bar{u}(t; \boldsymbol{\mu})$$

Linear POD-ROM

$$\bar{u}(t; \boldsymbol{\mu}) \approx g'_{\text{ROM}}(\bar{u}(t; \boldsymbol{\mu})) = \bar{u}_0 + \mathbf{P} \cdot \bar{u}(t; \boldsymbol{\mu})$$

Nonlinear ROM

$$\boldsymbol{u}(\boldsymbol{x}, t; \boldsymbol{\mu}) \approx g_{\text{ROM}}(\boldsymbol{x}, \tilde{u}(t; \boldsymbol{\mu})) = \text{NN}_\theta(\boldsymbol{x}, \tilde{u}(t; \boldsymbol{\mu}))$$

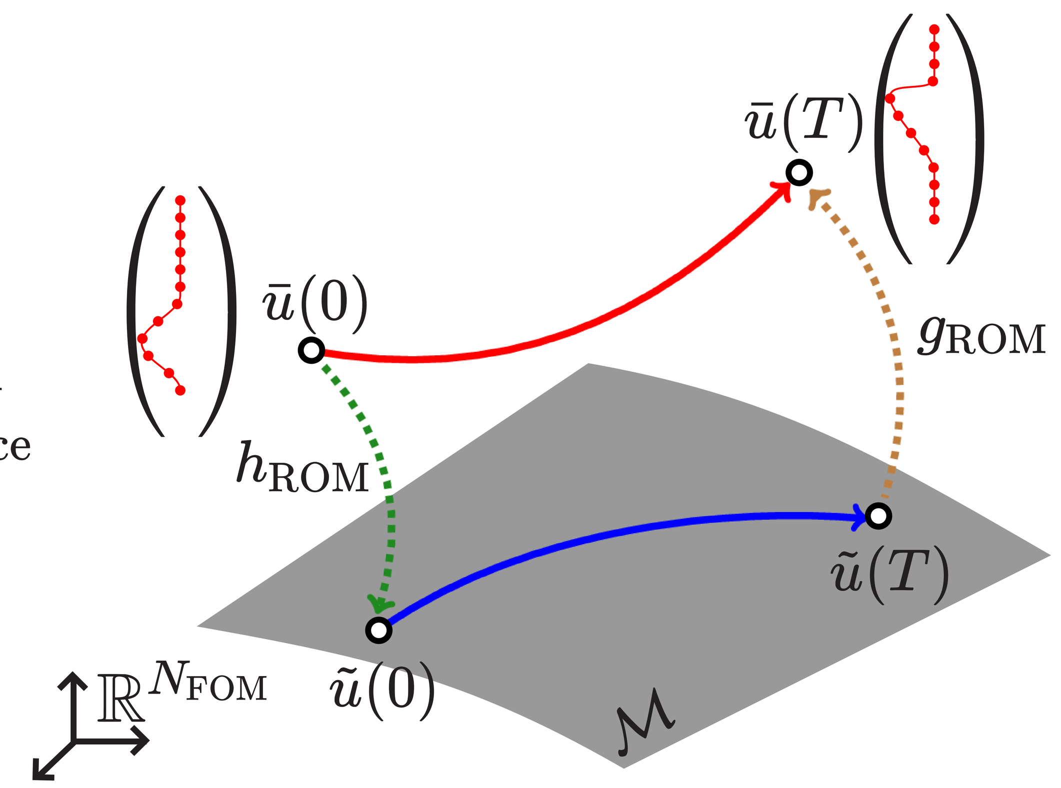


3. Equation-based dynamics-evaluation

Dynamics calculation is carried out following the governing PDE system as follows:

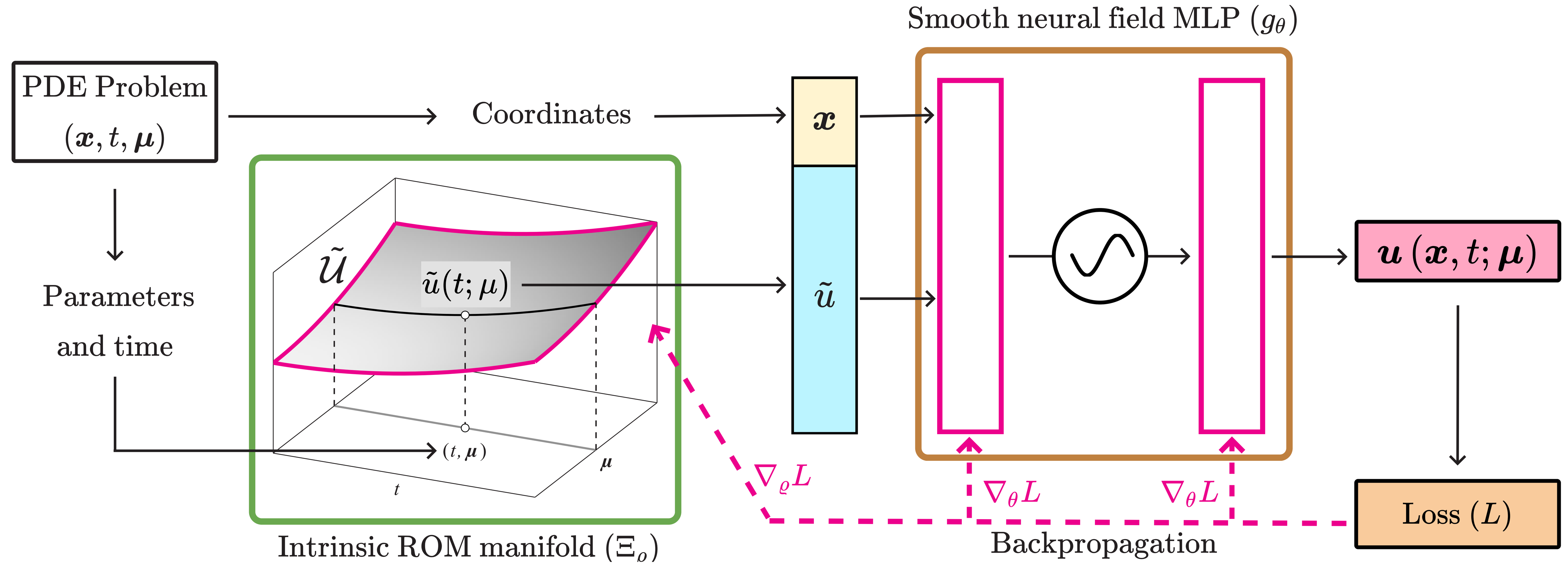
- The PDE is projected onto the reduced manifold
- The RHS is evaluated; spatial derivates are computed **non-intrusively** with automatic differentiation
- The reduced system is evolved with continuous Galerkin projection [2]

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\bar{u}(t=0) \arrow[r, "\frac{\mathrm{d}\bar{u}}{\mathrm{d}t} = \mathcal{L}(\bar{u}, t)"] \arrow[r, bend right] \arrow[d, "\text{Manifold projection}"'] & \bar{u}(t=T) \\
\tilde{u}(t=0) \arrow[r, "\mathbf{J}_g \frac{\mathrm{d}\tilde{u}}{\mathrm{d}t} = \mathcal{L}(g_{\text{ROM}}(\tilde{u}), t)"'] & \tilde{u}(t=T) \arrow[u, "\text{Model inference}"']
\end{tikzcd}$$

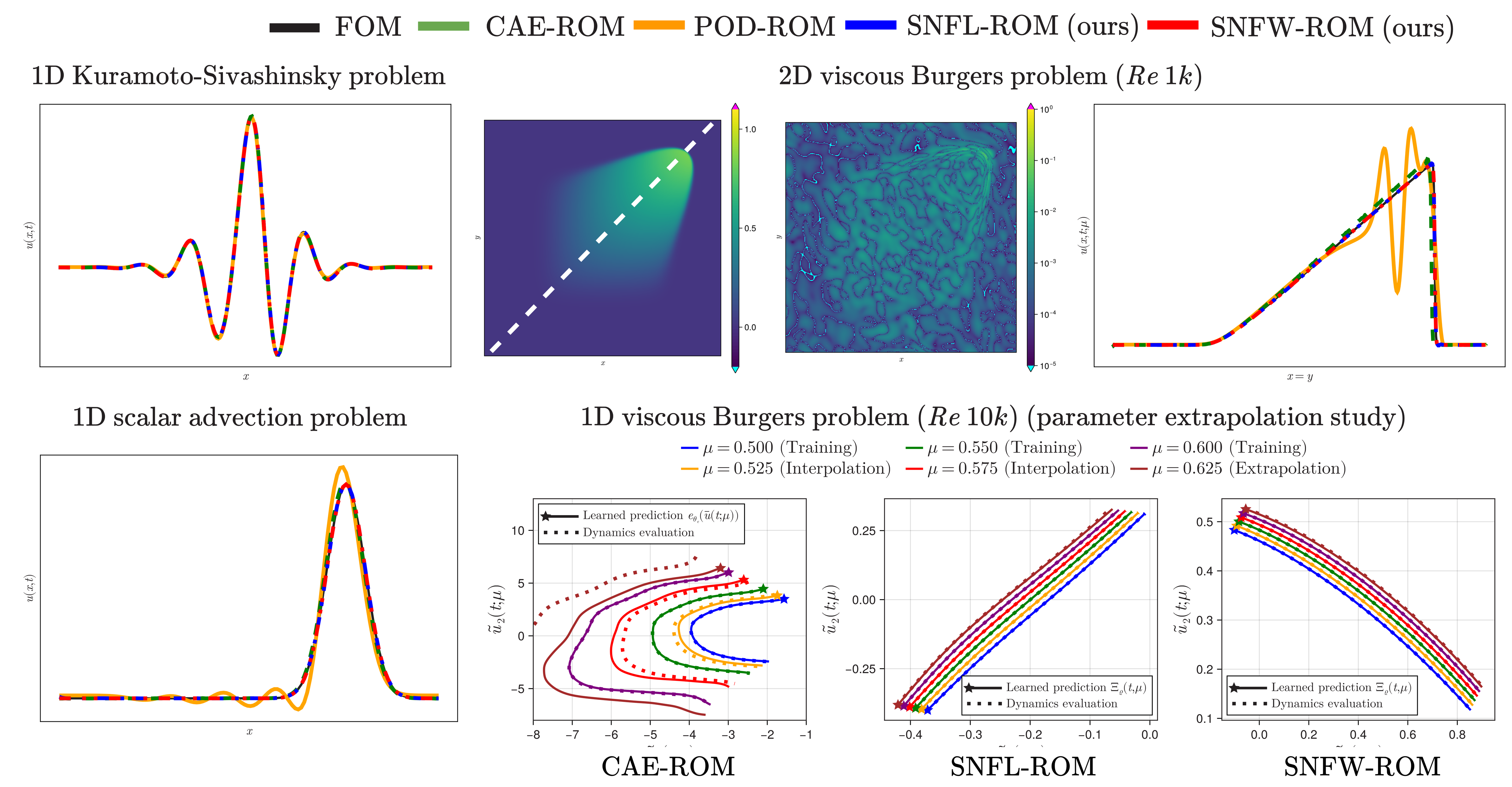


4. Model architecture

SNF-ROM directly models the reduced states as a learnable function of time. This restricts the reduced trajectories to be smooth and regular, thus stabalizing the downstream dynamics evaluation.



4. Numerical experiments



6. Conclusions

SNF-ROM is an ML reduced modeling framework that enables fast and accurate non-intrusive reduced order modeling. This is achieved by ensuring an accurate capture of reduced state dynamics, and by learning smooth reduced state trajectories that can be accurately evolved while taking large time-steps. In future, we plan to attack larger problems with multi-resolution neural field architectures that promise high accuracy and faster training.

7. References

[1] Puri V et. al. arXiv:2405.14890 [physics] (2024).
[2] Lee K et. al. J. Comp. Phys (2020).
[3] Chen P et. al. arXiv:2206.02607 [physics] (2023).