Direct Numerical Simulations of Flow Over Smooth and Rough Wavy Walls at $Re_\lambda = 4,780$

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Introuction

Problem:

- To model the dynamics of the atmospheric boundary layer over complex terrains with application to windfarm simulations
- LES calculations often do not resolve near-surface regions, thereby requiring additional subgrid modelling

Objectives:

- Understand physics of turbulent flows over periodic, wavy boundaries
- Study the effects of small boundary irregularities on on LES calculation

Method:

- Perform DNS of flow over a wavy wall (Smooth Wavy Wall, SWW), and a wavy wall with added roughness (Rough Wavy Wall, RWW) that an LES would not capture
- Compare Reynolds stress budgets statistics of SWW, RWW

NEK5000 Spectral Element Code

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} = -\boldsymbol{\nabla} p + \frac{1}{Re}\nabla^2 \boldsymbol{u} + \rho \boldsymbol{f}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0, \quad Re = \frac{\rho U L}{\mu}$$

NEK5000

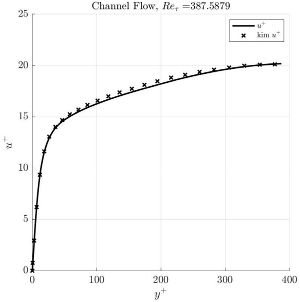
- Flexible like finite elements, accurate like spectral methods
- 7^{th} order GLL polynomial within each element
- 3^{rd} order BDF/Ext formula for time-integration (explicit handling of nonlinear term)
- Excellent scalability (32768 cores on Argonne resources)
- Linear Stokes problem (pressure-viscous decoupling) for solving Navier-Stokes

Validation Turbulence Budgets Subroutines in NEK5000

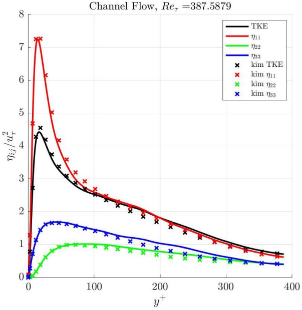
Validation on Reynolds Stress budgets subroutines in NEK5000 against channel flow dataset of Kim et al. 1998 at $Re_\tau = 394$

$$\partial_t k + \underbrace{\langle u_i \rangle k_{,i}}_{\text{convection}} = -\underbrace{\langle u'_j u'_i \rangle \langle u_{j,i} \rangle}_{\text{production}} - \underbrace{\langle u'_i k \rangle_{,i}}_{\text{turbdiff}} - \underbrace{\langle p'_{,j} u'_j \rangle}_{\text{presdiff}} + \underbrace{\frac{k_{,ii}}{Re}}_{\text{visc.diff}} - \underbrace{\frac{\langle u'_{j,i} u'_{j,i} \rangle}{Re}}_{\text{dissipation}}$$

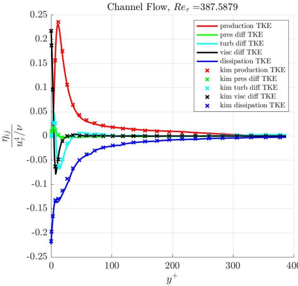
$\langle u \rangle$



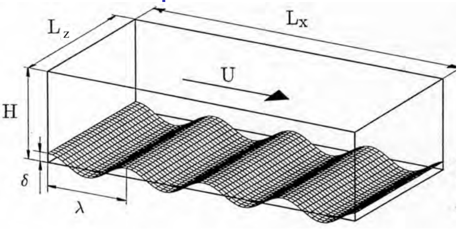
$\eta_{ij} = \langle u'_i u'_j \rangle$



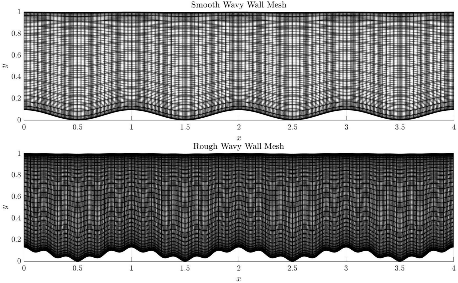
$\dot{\eta}_{ij}$



Case Setup



(SWW schematic, Maass and Schumann, 1994)

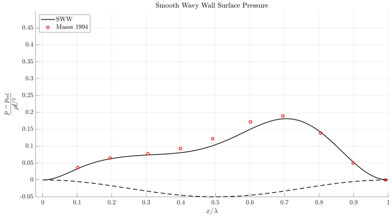


- $Re_\lambda = 4780$; wave amplitude $\delta/\lambda = 0.05$
- Periodic BC in streamwise and spanwise directions
- Flow averaged over 60 convective time units $(\overline{U}/4\lambda)$

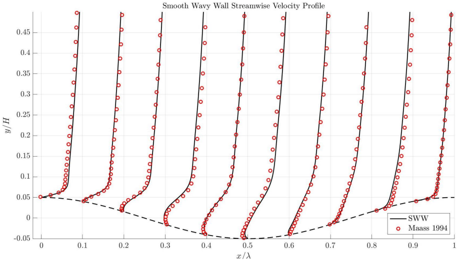
Case	Element Mesh	Order	$L_x \times H \times L_z$	λ	δ	λ'	δ'
SWW	$64 \times 16 \times 32$	7	$4 \times 1 \times 2$	1	0.05		
RWW	$128 \times 24 \times 32$	7	$4 \times 1 \times 2$	1	0.05	0.2	0.02

Validation of Smooth Wavy Wall DNS

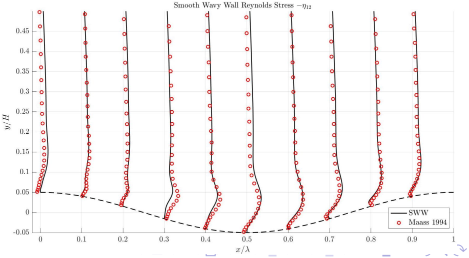
- Mesh refinement near wall
  - ≈ 2 points within $y^+ \leq 1$, 2 elements within $y^+ \leq 10$
  - $\Delta x^+ = \Delta z^+ \approx 5$
- Flow separation at crest of wave, reattachment at $x/\lambda = 0.69$
- Residual of TKE budgets is 2 orders of magnitude than TKE



$\langle u_1 \rangle$



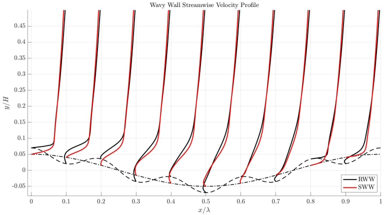
$\langle -u_1' u_2' \rangle$



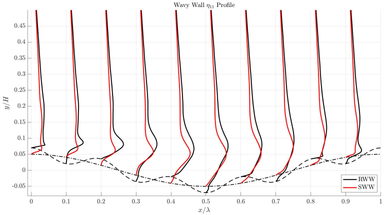
Mean Flow

- Flow separates at crest of wave ($x/\lambda = 0$) and reattaches at $x/\lambda = 0.69$
- Outer region ($y/H > 0.3$) where flow is not effected by wavy wall; inner region is $y/H < 0.1$; and buffer region in between
- Flow is largely undisturbed in the outer region - Townsend's hypothesis holds!

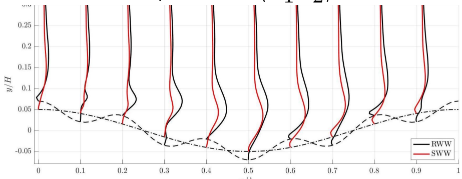
$$\langle u_1 \rangle$$



$$\eta_{11} = \langle u_1' u_1' \rangle$$

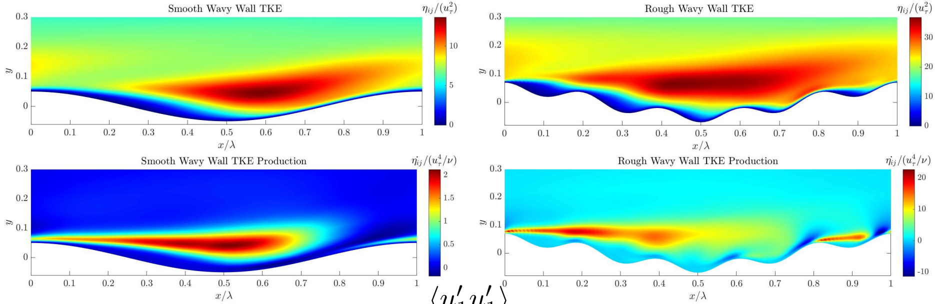


$$-\eta_{12} = -\langle u_1' u_2' \rangle$$

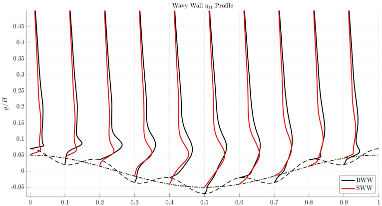


Turbulent Kinetic Energy Budgets

- Roughness disrupts inner flow and causes larger TKE and budgets
- Correlation between production term and $\langle u_1' u_1' \rangle$

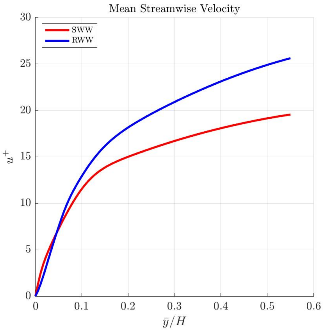


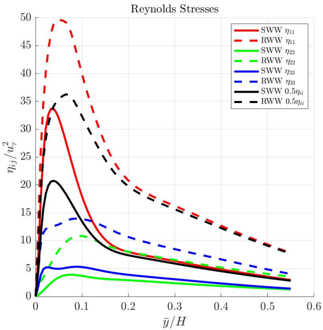
$\langle u_1' u_1' \rangle$



Homogenized Statistics

- Spatially average statistics over lateral directions (including sinosoudal waves) to obtain a single profile
- $Re_\tau = 142, 104$ for SWW, RWW respectively





Conclusion

- Extensive computations on flow over smooth and rough wavy walls have been done
- Results of Maass and Schumann have been reproduced with a high-order code
- Reynolds Stress budgets are computed for both Smooth Wavy Wall, and Rough Wavy Wall cases
- The effects of roughness have been observed to remain in the inner flow
- Work is underway to model the effects of roughness in subgrid models for Large Eddy Simulations of high Reynolds number wall-bounded flows

Acknowledgements

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