Natural Frequency and Resonance

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What is a natural frequency? What is resonance?

The definition of these items are:

- Natural Frequency: All physical structures have natural frequencies.These are the frequencies at which the structure will tend to vibrate when subjected to certain external forces. These frequencies are dependent on the way mass and stiffness are distributed within the structure.
- Resonance: Resonance is a phenomenon in which a dynamic force drives a structure to vibrate at its natural frequency. When a structure is in resonance, a small force can produce a large vibration response.

What does this mean in practice? When a dynamic force is applied to a physical object, it will vibrate. When a force is applied at the object's *natural frequency*, it goes into *resonance*, and a higher amplitude vibration response is created.

An analogy with a guitar may help. Pluck a string on a guitar and it will make the same sound each time. That is the guitar string vibrating at its natural frequency! The natural frequency is a property of the object itself: it will always vibrate at the same frequency independent of how hard or where it is plucked. A force had to be applied to cause the string to resonate and be heard.

All physical objects have multiple natural frequencies and can resonate under the right conditions. Sometimes the natural frequencies are excited by external forces acting on the object, which creates vibration. These vibrations may be so small that they cannot be seen by the human eye. Sometimes, they are quite large and easily observable as seen in *Figure 1*.

Figure 1: Everyday objects, like structures holding street lights, can resonate.

Resonance can cause discomfort (vibration in steering column caused by resonance) or be catastrophic (resonance in airplane wing leads to failure).

Single Degree of Freedom Example

A mass-spring-damper system is a simplified representation that is useful for understanding natural frequencies and resonant behavior in real world objects.

This is referred to as a Single Degree of Freedom (SDOF) system, because it has only one natural frequency/mode of vibration. A real world object has many natural frequencies.

A diagram of a mass-spring-damper system is shown in *Figure 2.*

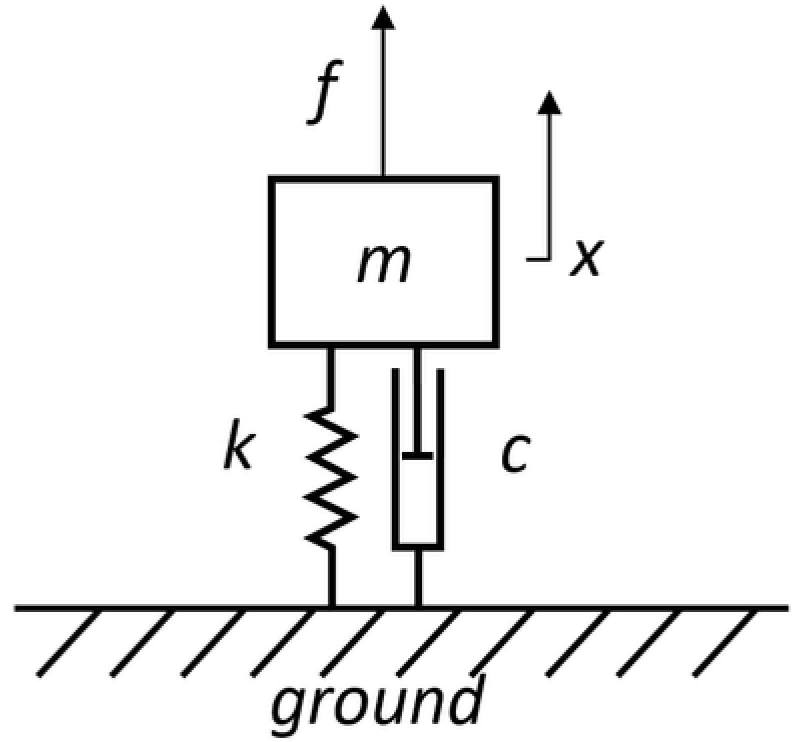


Figure 2: Mass-spring-damper system.

The system consists of:

- Mass (m)
- Stiffness (k)
- Damping (c)

The natural frequency (w_n) is defined by *Equation 1.*

$$\omega_n = \sqrt{k/m}$$

Equation 1: Natural frequency of mass-spring system

The natural frequency is an inherent property of the object. There are only two ways in which the natural frequency can be changed: either change the mass, or change the stiffness.

Amplitude Response

A force (f) can be applied to the object and the frequency response in displacement (x) or acceleration (a), can be plotted as shown in *Figure 3*.

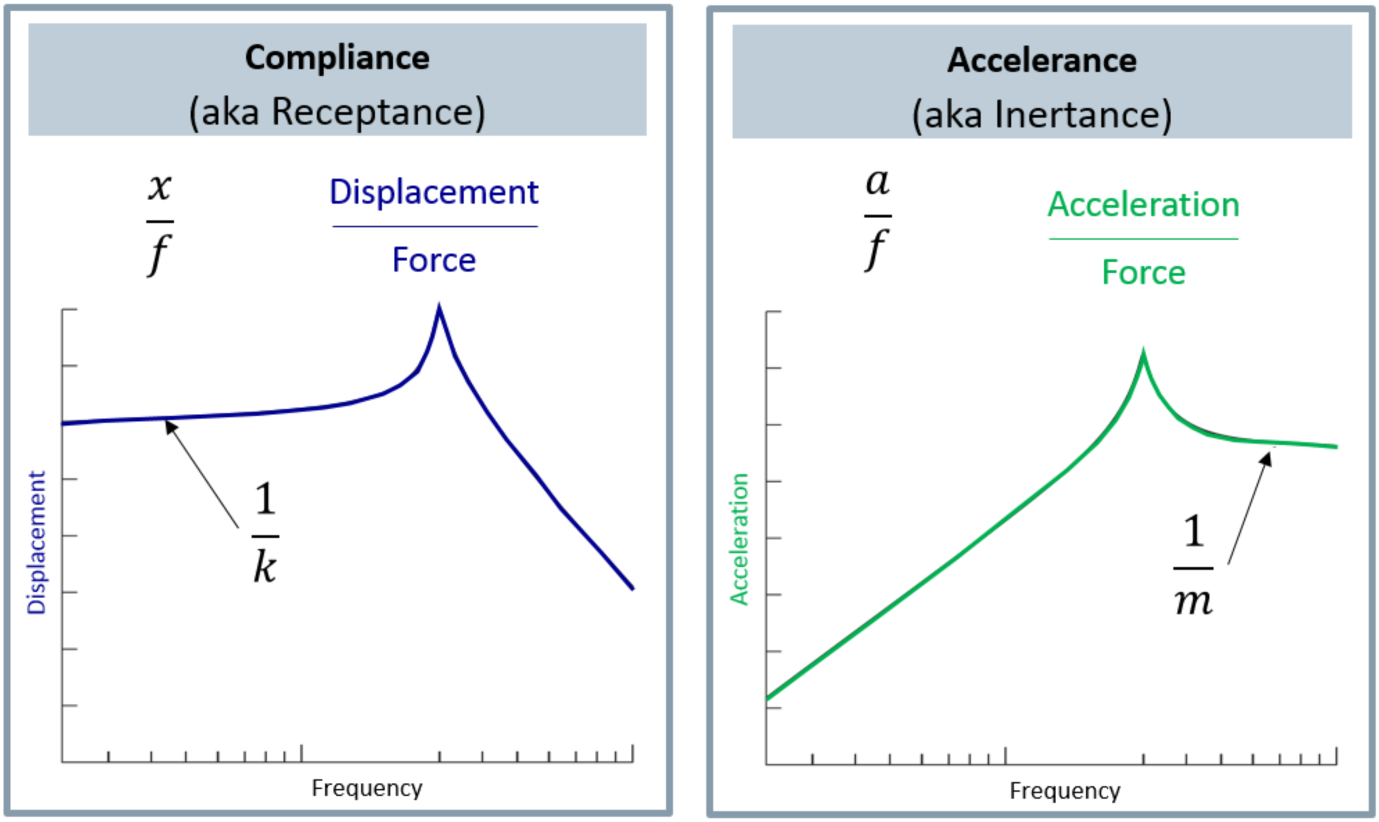


Figure 3: Left - Displacement response (compliance) graph of mass-spring-damper system due to force as function of frequency. Right – Acceleration response (accelerance) graph of same.

The largest displacement/acceleration of the mass occurs at the system's natural frequency. Other amplitude response observations include:

- Compliance plot - Below the resonant peak, the amplitude of the response is nearly constant, approximately 1/k. This comes from Hooke's law where force equals the product of stiffness and displacement (f=kx). Below the resonant frequency, the response of the system can be said to be stiffness dominated.
- Accelerance plot - Above the resonant peak, the amplitude is nearly a constant value of 1/m (really -1/m if phase is accounted for) as shown in *Figure 3*. This behavior is due to Newton's second law where force is the product of mass and acceleration (f=ma). Above the resonant frequency, the response of the system can be said to be dominated by the mass.

Knowing about these stiffness or mass regions can be useful in reducing vibration levels away from the resonance.

Phase Response

Applying the force through a moving base, and observing the mass response, yields some interesting phase relationships as shown in *Figure 4*.

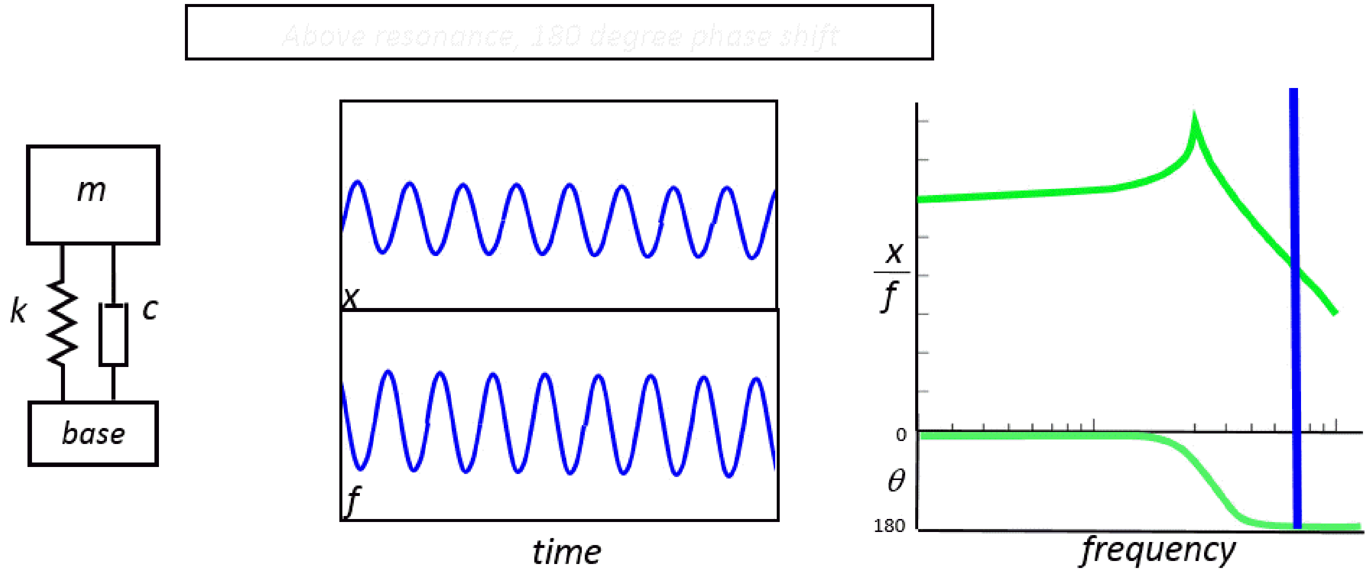


Figure 4: SDOF system response below, at, and above natural frequency of system.

The following can be observed:

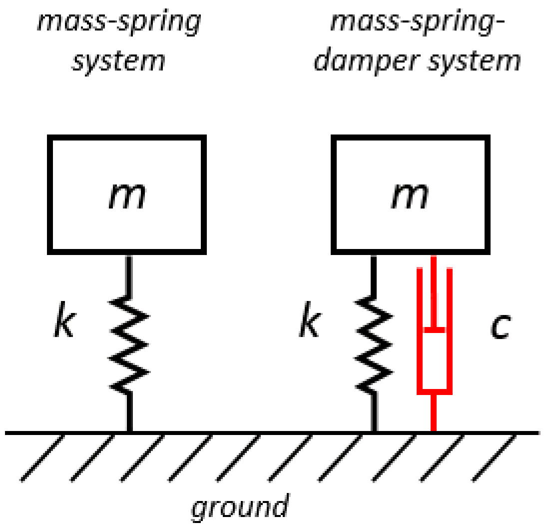
- Below the natural frequency, the base and mass move together in phase.
- At the natural frequency, the base and mass move 90 degrees apart, which creates a kind of "bucking" motion causing the high levels of vibration.
- Above the resonant frequency, the base and mass move out of phase.

Real world objects, from cars to airplanes to washing machines, can be thought of a collection of mass, stiffness, and damping elements. They have many natural frequencies. Finite element models, used in calculating natural frequencies virtually, use this approach. The models consist of a collection of elements composed of mass (mass density) and stiffness (Young's modulus).

Damping

Damping is the way a system naturally dissipates energy. Think back to the guitar example: does the guitar string oscillate forever after it is plucked? No! Energy is dissipated in the form of friction and sound which causes the string to return to rest after it has been plucked.

In the single degree of freedom example covered in the previous section, the mass-spring system (m and k) would stay in motion forever if there was no damper (c) present as shown in *Figure 5*.



The higher the damping, c, the sooner the response of the system decays to zero. The system response amplitude at the resonant frequency is reduced by increased damping. At the resonant frequency, the response of the system can be said to be damping dominated.

More information about damping, and how to calculate it, can be found in the Knowledge base article: **How to determine damping from a FRF**.

Mode Shapes

The SDOF example system had one natural frequency. Structures in the real world are more complex, and have multiple degrees of freedom (MDOF). As a result, real world structures have many natural frequencies. The structure vibrates differently at each of these natural frequencies. How it moves at a particular frequency is called a mode shape.

Two modes of an aircraft (selected from many modes) are shown in *Figure 6*. Each mode shape is unique, with different parts of the aircraft participating in the mode.

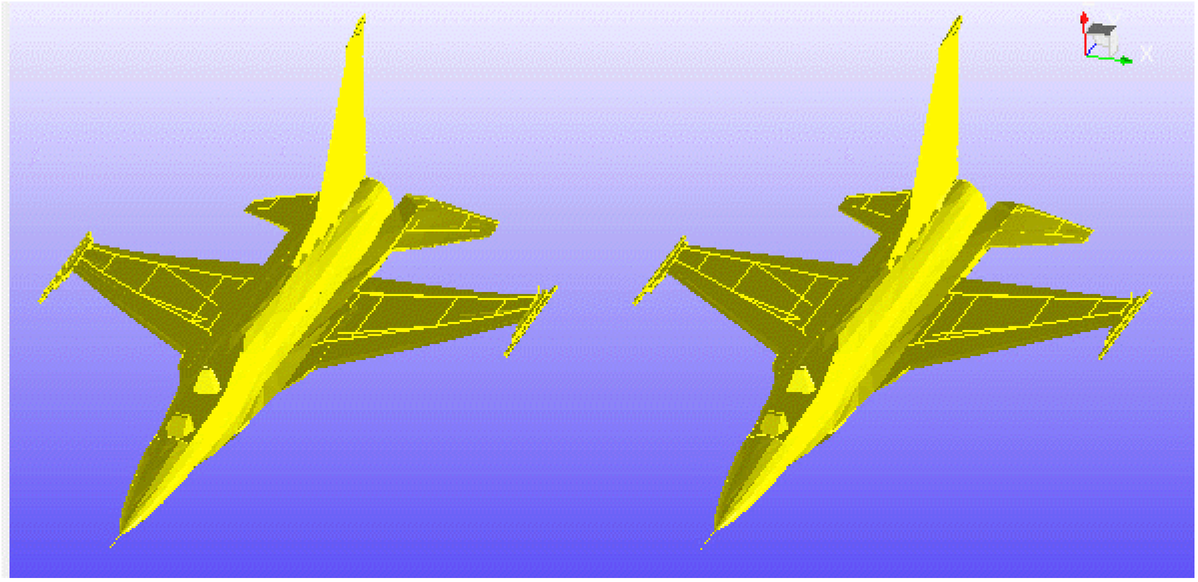


Figure 6: Left – Lower frequency global mode of aircraft, Right – Higher frequency local mode of aircraft tail.

Why is it important to understand modes and mode shapes? Mode shapes give valuable insight into how a structure behaves when operating at its natural frequencies. The shape can show the engineer where to constrain/modify a structure to reduce the vibration response, or how to shift the natural frequency so it does not coincide with the frequency of an excitation.

In *Figure 6*, for example, the tail wing mode (*right side*) would need to be modified to change the natural frequency. Changing the nose of the aircraft would have no effect on the natural frequency. The shape gives insight into how to tackle a dynamic issue.

At higher frequencies, generally speaking, modes become local in natural, rather than global. In a global mode, the entire structure participates (*mode shape on left in Figure 6*), while in a local mode, only part of the structure participates (*mode shape on right in Figure 6*).

It is also typical that mode shapes become more complex at higher natural frequencies as seen in *Figure 7*.

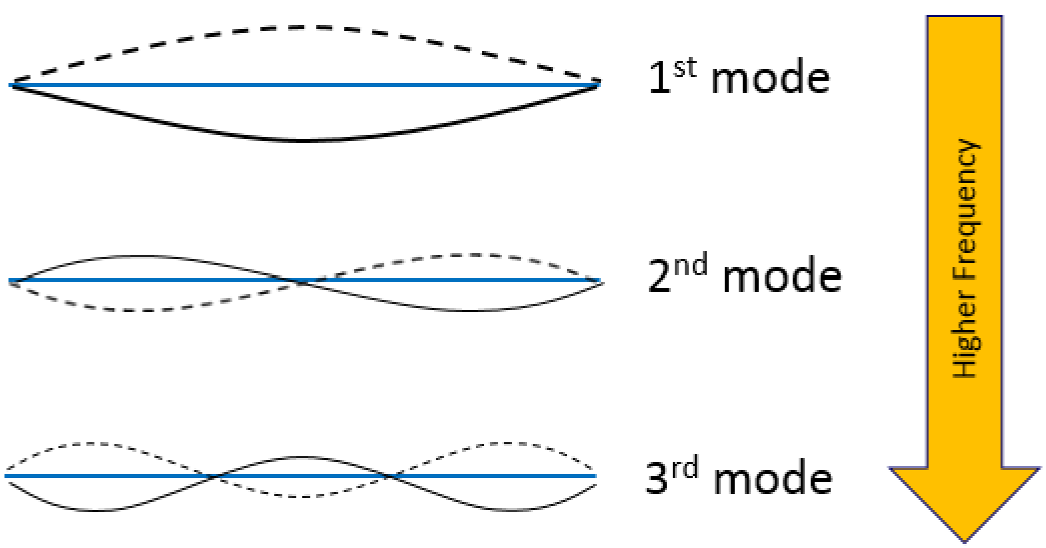


Figure 7: Mode shape of a simply-supported beam becomes more complex at higher frequencies.

The fact that modes become more complex and localized at higher frequencies has implications for structural dynamic simulations and tests. Simulations require a finer mesh and more elements, increasing solution times. Tests will require more locations to be measured on the structure.

Mitigating the Effects of Resonance

Knowing how destructive mechanical resonance can be, what can be done to avoid it? Options include:

- mass/stiffness modifications
- damping changes
- tuned absorbers

Mass and Stiffness Fixes

To avoid resonance, the forcing frequency applied to the structure should not be at or near a natural frequency. If the forcing frequency cannot be changed, then the natural frequency of the structure needs to be modified. This can only be done by altering the mass or stiffness (see *Equation 1*).

The guitar is a good example of how changing the mass or stiffness of a system effects the natural frequency. The strings on a guitar have different thicknesses. The strings which produce lower notes are thicker (more mass) than those which produce higher notes (less mass). As the mass of a guitar string increases, the natural frequency decreases (*Figure 8*).

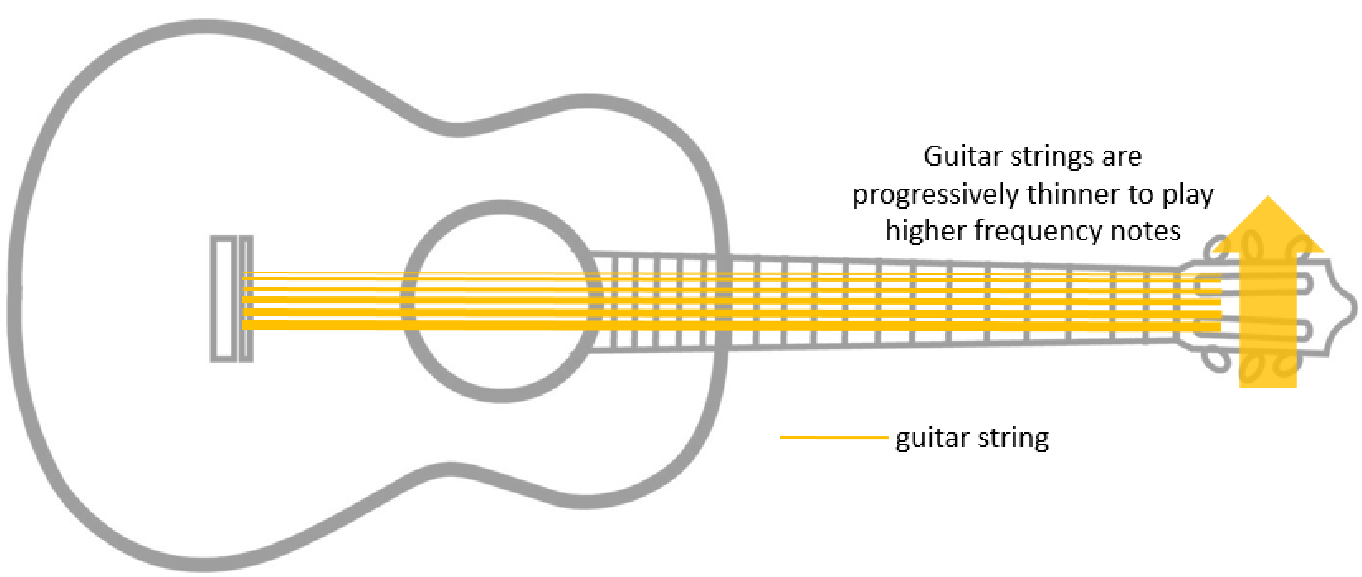


Figure 8: Progressively thinner strings create higher frequency notes on a guitar.

When tuning a guitar, pegs/knobs on the guitar are turned to tighten or loosen the strings. Tightening a string increases the stiffness, raising the natural frequency.

In a structure, increasing the stiffness to place the natural frequency above the forcing frequency helps reduce vibration.

Damping fixes

Damping can be added to reduce the severity of vibration when operating at or near a natural frequency. The plot below (*Figure 9*) shows the reduction in amplitude of the system response as damping increases.

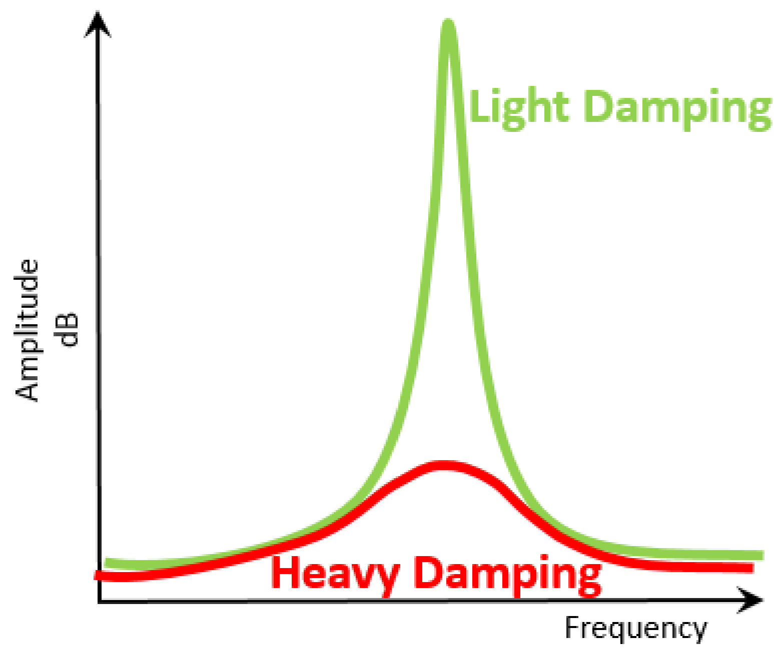


Figure 9: When damping is low (green), system amplitude response is high. When damping is high (red), system amplitude response is low.

Damping treatments are often used to reduce vibration. For example, many large bridges, such as London's Millennium Bridge, feature fluid viscous dampers to control vibration (*Figure 10*).

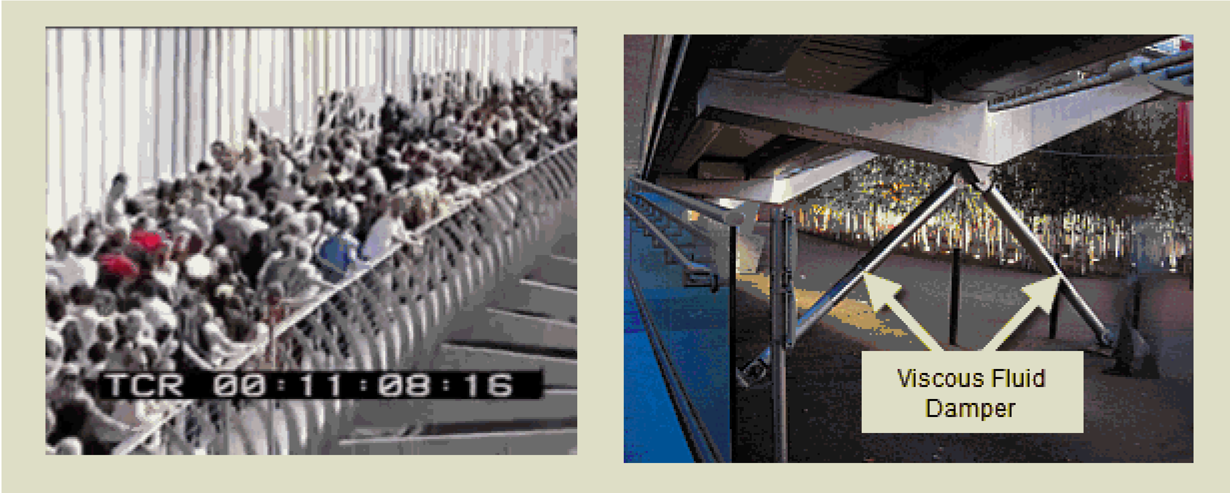


Figure 10: Fluid viscous dampers (right) are used to reduce unwanted vibration (left) on the Millennium Bridge.

The Millennium Bridge was opened to the public on June 10, 2000. Due to excessive vibration caused by pedestrian traffic, it was shutdown after two days, retrofit with 37 fluid viscous dampers, and re-opened on February 22, 2002.

Tuned Absorber

A tuned mass-spring-damper system can be used to reduce the amplitude of vibration in a dynamic system. A tuned mass damper modification is created by adding an additional mass-spring system "tuned" to the natural frequency of an existing system (*Figure 11*).

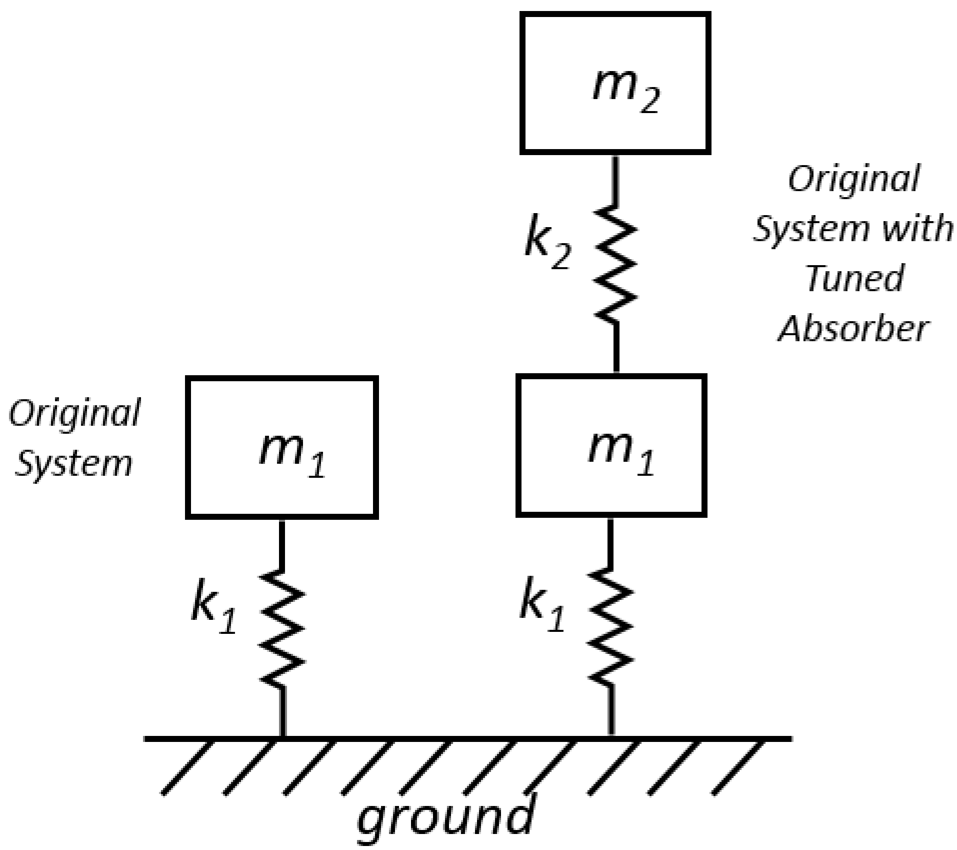


Figure 11: Left – Original system consisting of m1 and k1. Right – Original system with tuned absorber (m2, k2) applied.

Applying a tuned absorber (m2, k2) to an existing system (m1, k1) has two effects as shown in *Figure 12*:

- At the original natural frequency of the original system, the additional tuned mass-spring-damper will vibrate, but the original system will not move.
- The original natural frequency is split into two. One mode of vibration where the original and tuned system are in-phase, and one mode where the original and tuned systems are out of phase. The in-phase mode is at a lower frequency than the original system natural frequency, while the out-of-phase mode is a higher frequency that the original system natural frequency.

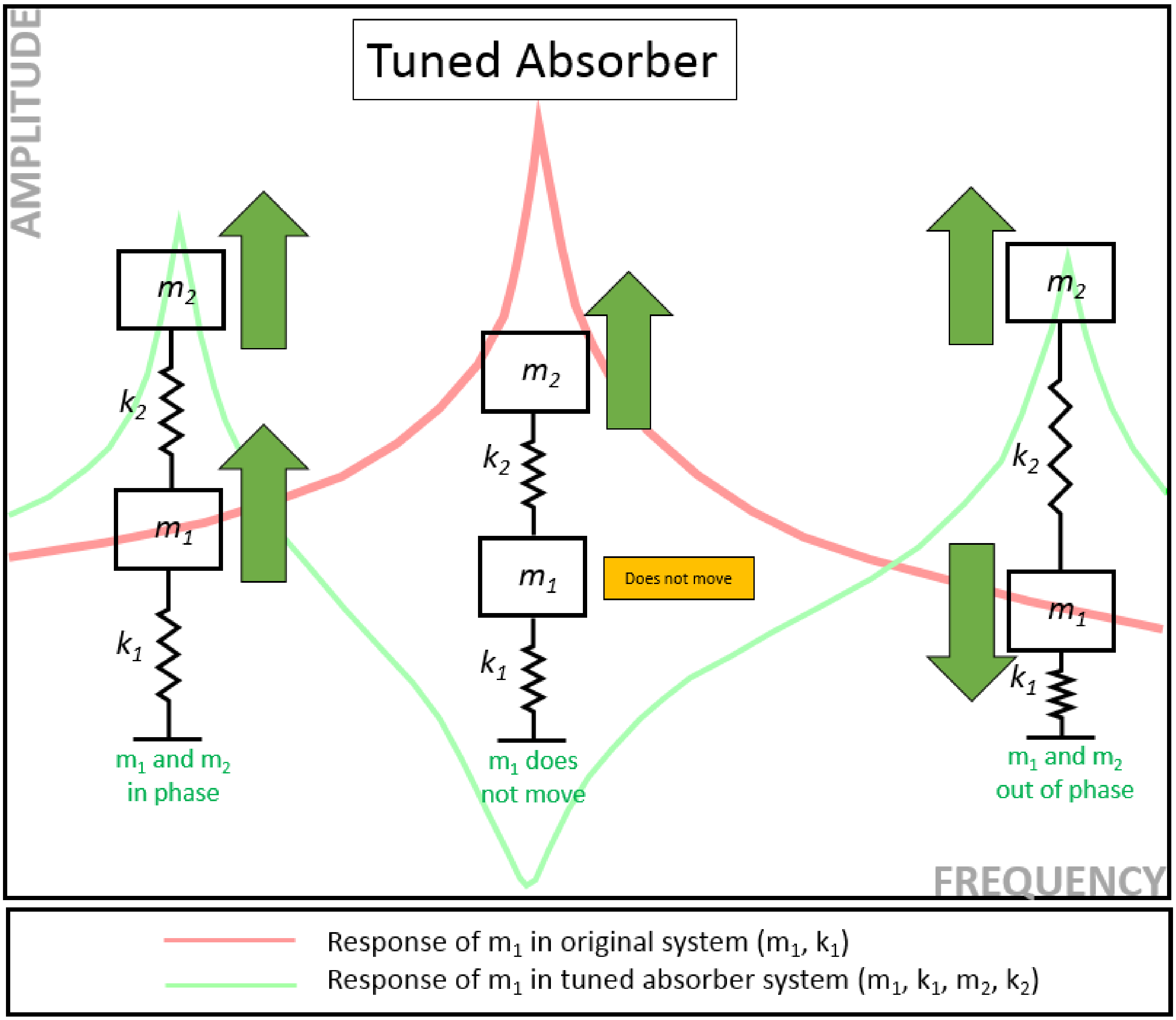


Figure 12: Original system (m1, k1) response (red) is reduced to zero (green) by introducing tuned absorber (m2, k2). New system has two modes: one above original system natural frequency, and one below.

The effects are only possible when the tuned absorber frequency is equal to the original system frequency.

How does this reduce vibration? Consider a vehicle body mode where the engine idle forces excite the bending resonance. The bumper can be turned into tuned mass-spring system so it can vibrate and cancel the bending mode. The bumper will vibrate at idle, but the body will not, reducing the vibration felt by the driver. The newly created in-phase mode will not be excited, since it is below the idle vibration. The out-of-phase mode, which is higher than the original frequency excited at idle, can be tuned to an engine speed that is not often used in operation.

One of the benefits of a tuned absorber approach is the additional mass and stiffness changes to the structure can be minimal. In the vehicle bumper example, the mass of the bumper was already part of the structure. Adding spring/stiffness elements to the bumper was a minimal change in the overall weight of the vehicle, which helps with fuel efficiency, etc.

Tuned dynamic absorbers also are used to help reduce the swaying vibration in buildings. The Taipei 101 skyscraper contains the world's largest and heaviest tuned mass dampers, at 660 metric tons (730 short tons). The damper is tuned to the swaying mode of the building (*Figure 13*).

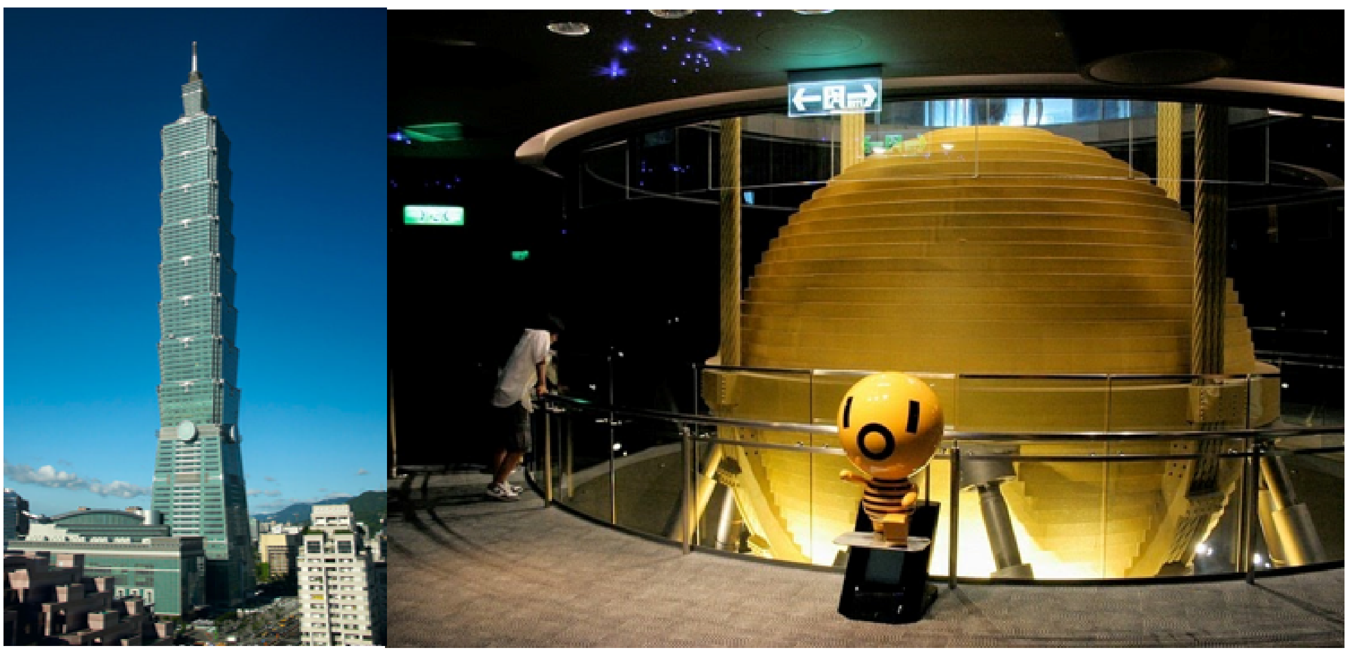


Figure 13: Tapei 101 skyscraper (left) and tuned absorber (right).

The tuned absorber is viewable by the public on an indoor observation deck at the top of the skyscraper. It cost an estimated $4 million to build.

More about modifying stiffness/mass, adding damping, applying tuned absorbers to modal models in the **Modification Prediction knowledge base article.**

Examples of Resonance

Broughton Bridge

In 1831, the Broughton suspension bridge in England (*Figure 14*) collapsed when a column of soldiers crossed the bridge marching in step.

Figure 14: Broughton bridge in 1883.

The bridge had a natural frequency near the frequency of the marching of the soldiers and began to vibrate violently. This caused a bolt in one of the supporting chains to fail which led to the collapse of the bridge.

Flutter in Aircraft

The flight envelope (Mach speed and altitude) of any aircraft is determined by the resonant frequencies of the wings and airframe structure. If the aircraft goes too fast, the aerodynamic forces can excite the natural frequencies causing a resonance, which in an aircraft is called flutter (*Figure 15*).

Figure 15: In aircraft, flutter occurs when aerodynamic forces excite modes of the airframe.

For more information read the **Ground Vibration Testing and Flutter Analysis** knowledge base article.

Beating Sound

Rolled the rear windows down while driving and heard a beating sound? That beating sound (sometimes also called buffeting) is amplified by a resonance of the vehicle air cavity (*Figure 16*). Even air can have a resonance!

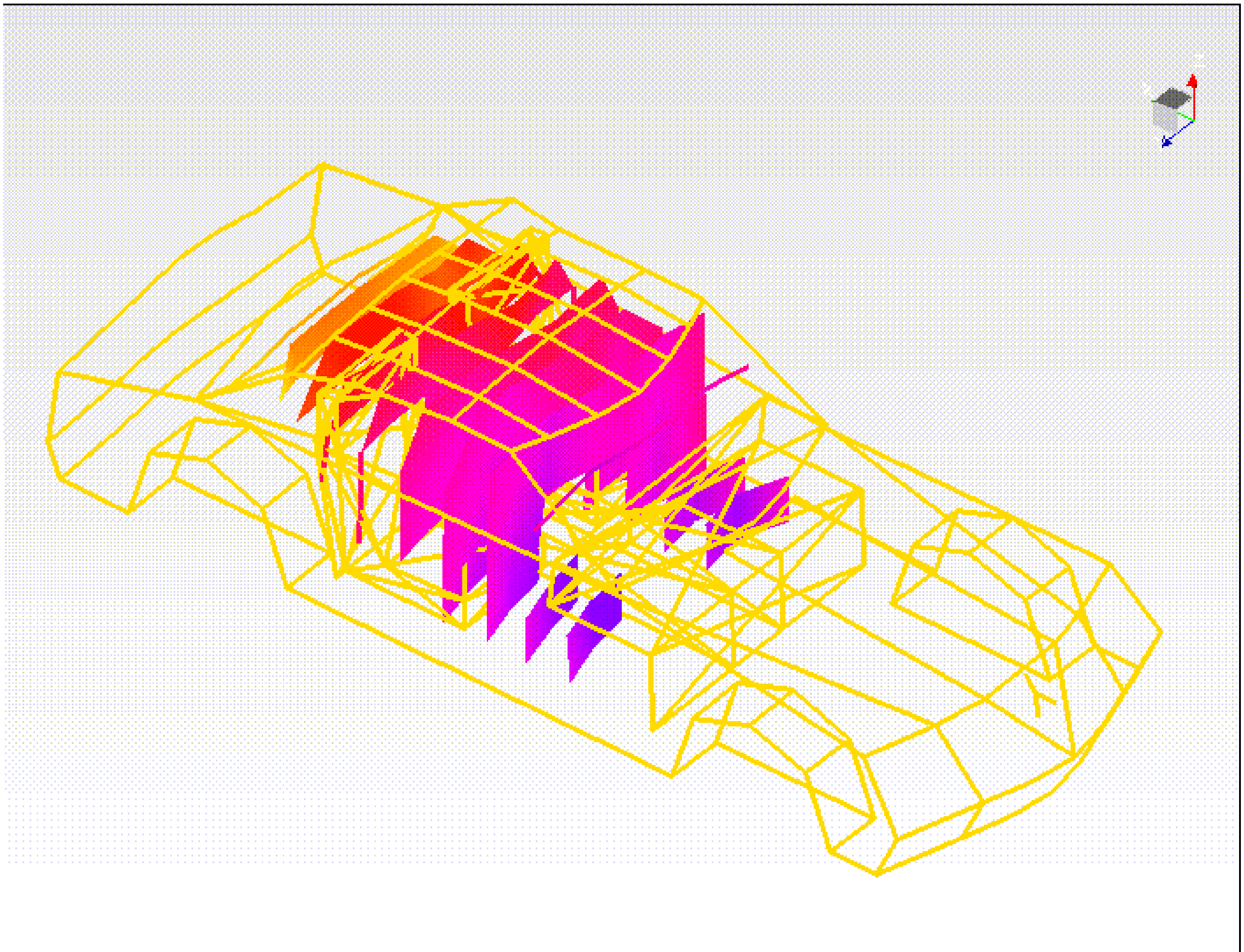


Figure 16: Experimentally measured acoustic mode of vehicle air cavity.

The air cavity inside the vehicle is trapped in a specific volume/geometry. As a result, the air cavity has natural frequencies which can be excited and cause increases in the sound level in the car.

Opera Singer and Glass

A popular example of resonance from movies has an opera singer hitting a specific note, causing glasses in the audience to shatter (*Figure 17*), due to resonance.

Figure 17: Glass shattering due to sound waves exciting resonance.

The TV show “Mythbusters” confirmed that this is possible on the episode “Breaking Glass” which was originally broadcast on May 18, 2005.

Brake Squeal

Ever pressed on the brake in a car and heard a high frequency squeal? This is resonance at work. A brake rotor mode, probably around 3000 Hz or so, was excited by braking (*Figure 18*).

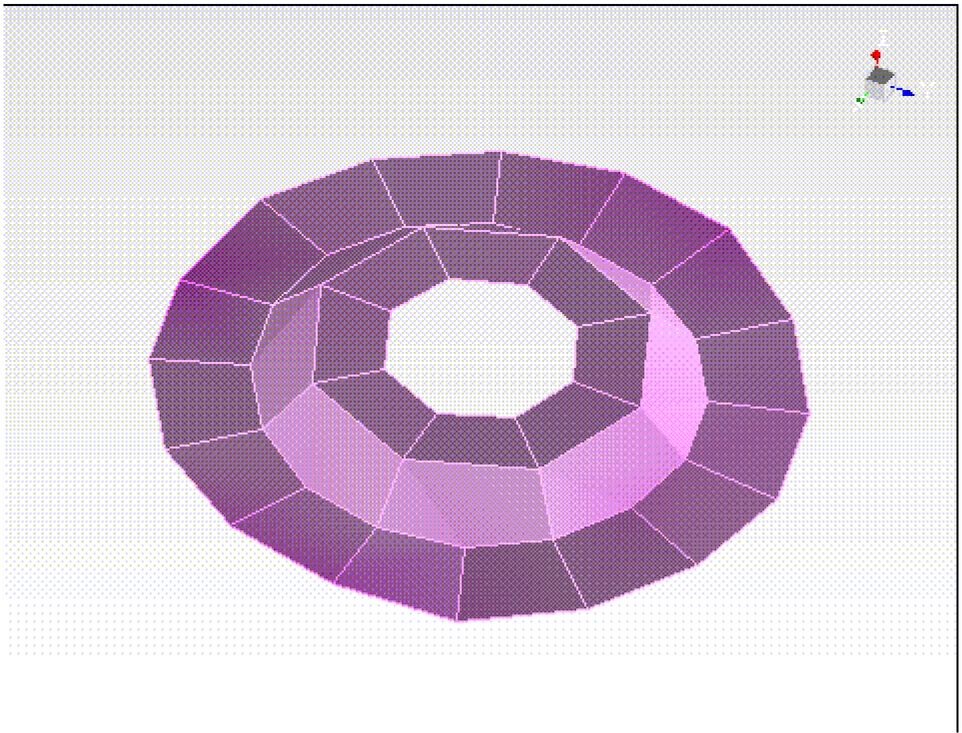


Figure 18: Brake rotor has a resonance which can create audible squeal during braking.

Brake rotors are symmetric structures, which can create multiple modes at the same frequency. To reduce squeal, the rotor design should be modified to keep the modes well separated so they cannot cross excite each other.

Helmholtz Resonator Air Induction

The German scientist, Hermann von Helmholtz (a friend of Ernest Werner Siemens), created the **Helmholtz resonator in the 1850s.** A Helmholtz resonator is a air-based tuned absorber that is designed for a specific frequency. When blowing over the lip of an empty bottle, the resonant frequency of the trapped volume of air can be heard.

The larger trapped volume of air acts like the spring (it gets compressed into the volume). The smaller volume at the mouth, which is open to the air, acts like a mass. See the diagram in *Figure 19*.

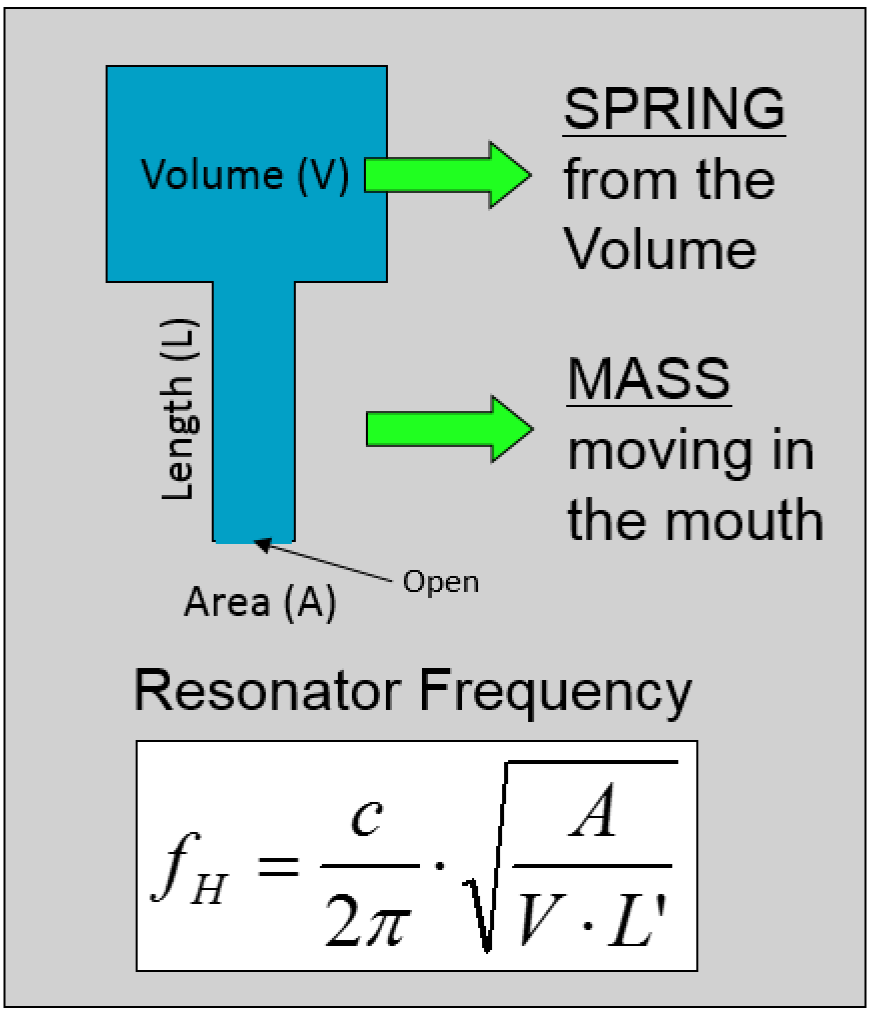


Figure 19: A Helmholtz resonator creates a tuned absorber (frequency fh) with a trapped volume of air (V) which acts like a spring, and an open volume of air (defined by A and L).

In ducted systems, the Helmholtz resonator can be used as tuned absorber to reduce sound levels at a specific frequency. In *Figure 20*, the air in the exhaust system has a 330 Hz resonant frequency. The sound pressure level at the outlet is greatly reduced by introducing a Helmholtz resonator attached to the duct. The resonator "absorbs" energy from the exhaust system and reduces sound at the outlet.

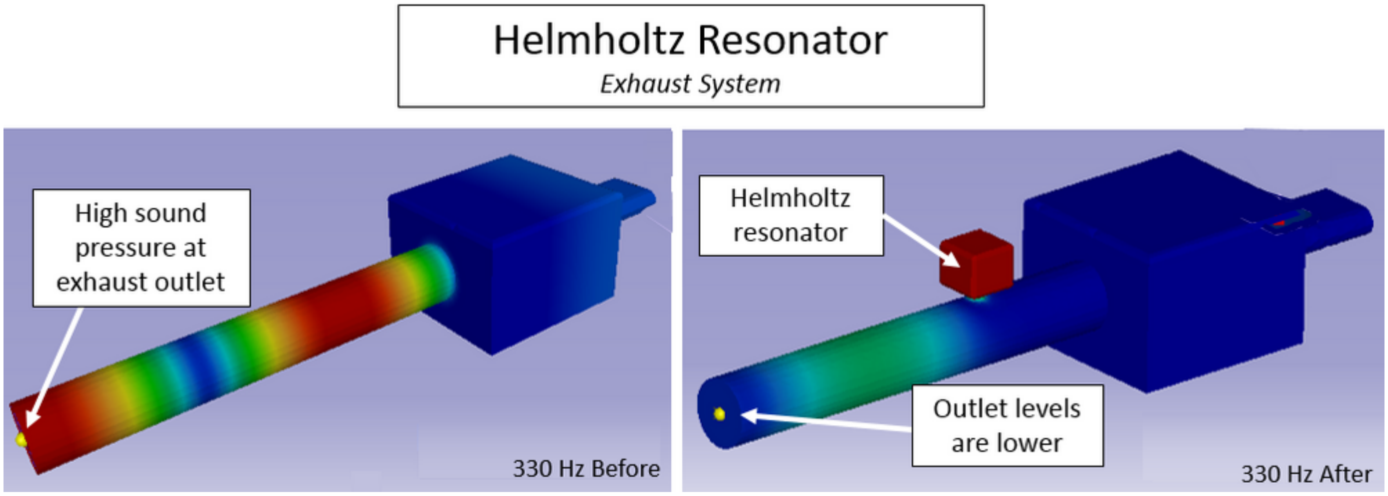


Figure 20: Left - A 330 Hz air cavity mode creates high pressure at outlet. Right - With a Helmholtz resonator installed, the outlet pressure is greatly reduced.

These resonators can be commonly found in air induction systems, HVAC ducts, and exhaust systems.

Final Thoughts: Resonance - Not always a bad thing!

Resonance is not always a bad thing! Sometimes amplifying the response of a system can be useful to increase the its performance. Some examples include:

- The human ear can hear sound better between **1000 and 4000 Hertz due to an acoustic resonance** of the air in the ear canal.
- Tuning into a radio station (at least in the analog days) is done through resonance. The electrical circuit is tuned to the radio frequency by adjusting the frequency of a capacitor.
- Vibrating bowls used in manufacturing applications to sort parts rely on the resonance of the bowl to create large vibration with minimal energy input.
- Microwave ovens use the natural frequency of water molecules to heat items quicker than a traditional oven.
- In the medical field, magnetic resonance imaging (MRI) is used to view inside the human body without requiring invasive procedures. A MRI is tuned to resonate with certain tissues.

Hope this article was helpful for understanding natural frequencies and resonance.

Questions? Email **nicholas.divincenzo@siemens.com**

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- **Simcenter Testlab Impact Testing**
- **What is Frequency Response Function (FRF)?**
- **How to calculate damping from a FRF?**

- **What modal impact hammer tip should I use?**
- **Modal Tips: Roving hammer versus roving accelerometer**
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