

Highlights from NeurIPS 2019

Vancouver, Canada

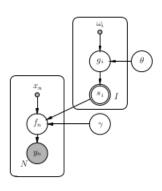
Function-Space Distributions over Kernels

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¹Gregory Benton et al. "Function-Space Distributions over Kernels". In: Advances in Neural Information Processing Systems. 2019, pp. 14939–14950.

Spectral Representation



Bochner's Theorem: Can represent a (positive definite) kernel as

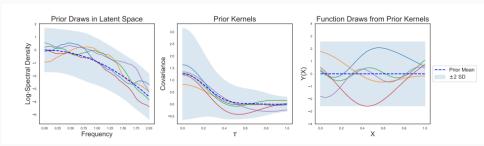
$$k(\tau) = \int_{\mathbb{R}} e^{2\pi i \omega \tau} S(\omega) d\omega,$$

with $\tau = |x - x'|$, the distance between any two inputs, and some positive finite function $S(\omega)$. Only need to learn the (unnormalized) spectral density $S(\omega)$ to learn $k(\tau)$.

Model Specification

{Hyperprior}
$$p(\phi) = p(\theta, \gamma)$$
 {Latent GP}
$$g(\omega)|\theta \sim \mathcal{GP}(\mu(\omega; \theta), k_g(\omega, \omega'; \theta))$$
 {Spectral Density}
$$S(\omega) = \exp\{g(\omega)\}$$
 {Data GP}
$$f(x_n)|S(\omega), \gamma \sim \mathcal{GP}(\gamma_0, k(\tau; S(\omega))).$$

Forward sampling



²Gregory Benton et al. "Function-Space Distributions over Kernels". In: *Advances in Neural Information Processing Systems*. 2019, pp. 14939–14950.

Inference & Prediction

Updating Hyper-Parameters: Considering the model specification in Eq. 4, we can define a loss as a function of $\phi = \{\theta, \gamma\}$ for an observation of the density, $\tilde{g}(\omega)$, and data observations y(x). This loss corresponds to the entropy, marginal log-likelihood of the latent GP with fixed data GP, and the marginal log-likelihood of the data GP.

$$\mathcal{L}(\phi) = -\left(\log p(\phi) + \log p(\tilde{g}(\omega)|\theta,\omega) + \log p(y(x)|\tilde{g}(\omega),\gamma,x)\right). \tag{7}$$

Updating Latent Gaussian Process: With fixed hyper-parameters ϕ , the posterior of the latent GP is

$$p(g(\omega)|\phi, x, y(x), f(x)) \propto \mathcal{N}(\mu(\omega; \theta), k_g(\omega; \theta))p(f(x)|g(\omega), \gamma).$$
 (9)

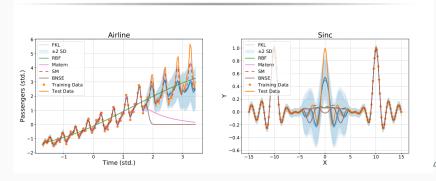
Prediction: The predictive distribution for any test input x^* is given by

$$p(f^*|x^*,x,y,\phi) = \int p(f^*|x^*,x,y,\phi,k) p(k|x^*,x,y,\phi) dk$$

$$p(f^*|x^*, x, y, \phi) \approx \frac{1}{J} \sum_{j=1}^{J} p(f^*|x^*, x, y, \phi, k_j), \quad k_j \sim p(k|x^*, x, y, \phi).$$

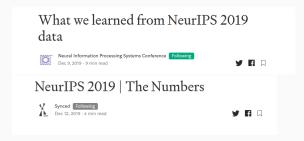
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Extrapolation and Interpolation



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- 1. Matthew D Hoffman and Yian Ma. "Langevin Dynamics as Nonparametric Variational Inference". In: 2nd Symposium on Advances in Approximate Bayesian Inference (2019) "Best Industry paper"
- Yoshua Bengio Keynote on "Going from System 1 to System 2 Deep learning"
- 3. NeurIPS Summaries (on Medium):



Trivia

