1. **a**. Let $a, b, c, d \in \mathbb{Z}$. If g is injective, $g(a, b) = g(c, d)$ should only be true when $a = c$ and $b = d$. Let us check for this property.

$$g(a, b) = g(c, d)$$
$$(2a, a - b) = (2c, c - d)$$

$$2a = 2c \qquad\qquad a - b = c - d$$
$$a = c \qquad\qquad a - c = b - d$$

Let us substitute a for c in the second equation.

$$c - c = b - d$$
$$0 = b - d$$
$$b - d$$

Thus, we have proven that g is injective.

However, g is not surjective, as the first element of any of its outputs can never be odd. Since it is not surjective, it is also not bijective.

b. Let $a, b, c, d \in \mathbb{Z}$. If h is injective, $h(a, b) = h(c, d)$ should only be true when $a = c$ and $b = d$. Let us check for this property.

$$h(a, b) = h(c, d)$$
$$\frac{a + b}{b} = \frac{c + d}{d}$$
$$\frac{a}{b} + \frac{b}{b} = \frac{c}{d} + \frac{d}{d}$$
$$\frac{a}{b} + 1 = \frac{c}{d} + 1$$
$$\frac{a}{b} = \frac{c}{d}$$

One set of possible values is $a = 1, b = 2, c = 2$ and $d = 4$. These values make the statement true, but $a \neq c$ and $b \neq d$, meaning that h is not injective.

Now, let us check for surjectivity. Let j and $k \in \mathbb{Z}$. Following this, $j - k \in \mathbb{Z}$. Now, plug in:

$$h(j - k, k)$$
$$\frac{(j - k) + k}{k}$$
$$\frac{j}{k}$$

$\frac{j}{k} \in \mathbb{Q}$, so h is surjective. Since h is not injective, it is also not bijective.

2. **a**. $f(A) \Rightarrow [f(2), f(5)] \Rightarrow [-2 \cdot 2 + 1, -2 \cdot 5 + 1] \Rightarrow [-3, -9] \Rightarrow$ **[-9,-3]**

b. $x = -2f^{-1}(x) + 1 \Rightarrow x - 1 = -2f^{-1}(x) \Rightarrow \frac{1-x}{2} = f^{-1}(x) \Rightarrow f^{-1}(x) = \frac{1-x}{2}$ $f^{-1}(A) \Rightarrow [f^{-1}(2), f^{-1}(5)] \Rightarrow [\frac{1-2}{2}, \frac{1-5}{2}] \Rightarrow [-\frac{1}{2}, -2] \Rightarrow$ **[-2, $\mathbf{\frac{1}{2}}$]**

3. **a**. To prove $f^{-1}(f(A)) = A$, we must prove that $f^{-1}(f(A)) \subseteq A$, as we can assume $A \subseteq f^{-1}(f(A))$. Let $x \in f^{-1}(f(A))$, then we can say $f(x) \in f(A)$ and that there exists some $a \in A$, such that $f(x) = f(a)$. Since f is an interjection, $x = a$ and then $x \in A$. This proves that $f^{-1}(f(A)) \subseteq A$, so, combining both, $f^{-1}(f(A)) = A$ if f is an interjection.

b. To prove $f(f^{-1}(C)) = C$, we must prove that $C \subseteq f(f^{-1}(C))$, as we can assume $f(f^{-1}(C)) \subseteq C$. (continued on page 2)

4. **a**. We will first prove that $f(S \cup T) \subseteq f(S) \cup f(T)$. Let $y \in f(S \cup T)$ and $x \in S \cup T$. Since $x \in S \cup T$, $x \in S \vee x \in T$, then $y \in f(S) \vee y \in f(T)$, so $y \in f(S) \cup y \in f(T)$. This proves that $f(S \cup T) \subseteq f(S) \cup f(T)$. Next, we will prove that $f(S) \cup f(T) \subseteq f(S \cup T)$. Let $a \in f(S) \cup f(T)$, then $a \in f(S) \vee f(T)$, then let $b \in S \vee T$, then $b \in S \cup T$, so $a \in f(S \cup T)$. This proves that $f(S) \cup f(T) \subseteq f(S \cup T)$. Combining $f(S \cup T) \subseteq f(S) \cup f(T)$ and $f(S) \cup f(T) \subseteq f(S \cup T)$, we can conclude that $f(S \cup T) = f(S) \cup f(T)$.

b. Let $x \in S \cap T$ and $y \in f(S \cap T)$. Thus, $x \in S \wedge x \in T$ and $y \in f(x)$. Then, $y \in f(S) \wedge y \in f(T)$, so, $y \in f(S) \cap f(T)$. This proves that $f(S \cap T) \subseteq f(S) \cap f(T)$.

c. We have already proved that $f(S \cap T) \subseteq f(S) \cap f(T)$, in order to prove equality, we have to prove $f(S) \cap f(T) \subseteq f(S \cap T)$. Let $y \in f(S) \cap f(T)$, then $y \in f(S) \wedge y \in f(T)$. We can then let $x_s \in S$ and $x_t \in T$, such that $x_s \neq x_t$ and $f(x_s) = f(x_t) = y$. If we do so, our left side remains valid, but $x_s, x_t \notin S \cap U$. This means that we can not prove that $f(S) \cap f(T) \subseteq f(S \cap T)$, so, generally, $f(S \cap T) \neq f(S) \cap f(T)$.

d. Let us continue off the previous part. To prove equality, we have to prove $f(S) \cap f(T) \subseteq f(S \cap T)$. Let $y \in f(S) \cap f(T)$, then $y \in f(S) \wedge y \in f(T)$. We can then let $x_s \in S$ and $x_t \in T$, such that $f(x_s) = f(x_t) = y$. Since f is injective and $f(x_s) = f(x_t) = y$, we can say that $x_s = x_t$, so $x_s, x_t \in S \cap U$. As a result, $y \in f(S \cap U)$, so $f(S) \cap f(T) \subseteq f(S \cap T)$. Since we know this along with our proof from 4b, we can say that when f is injective, $f(S \cap T) = f(S) \cap f(T)$.