AI1103 Assignment-5

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Let X be a binomial random variable with parameters $\left(11, \dfrac{1}{3}\right)$. At which value(s) of k is $\Pr(X = k)$ maximized?

1) k=2
2) k=3
3) k=4
4) k=5

SOLUTION

X has a binomial distribution :

$$\Pr(X = k) = {}^{n}C_k(q)^{n-k}(p)^{k} \quad (0.0.1)$$

Where,

- n=11
- $p = \dfrac{1}{3}$
- $q = 1 - p = 1 - \dfrac{1}{3} = \dfrac{2}{3}$

$$\Pr(X = k) = {}^{11}C_k \left(\frac{2}{3}\right)^{11-k} \left(\frac{1}{3}\right)^{k} \quad (0.0.2)$$

For Pr(X = k) to be maximized

$$\Pr(X = k) \geq \Pr(X = k + 1) \quad (0.0.3)$$

$$\frac{\Pr(X = k)}{\Pr(X = k + 1)} = \frac{{}^{11}C_k \left(\frac{2}{3}\right)^{11-k} \left(\frac{1}{3}\right)^{k}}{{}^{11}C_{k+1} \left(\frac{2}{3}\right)^{10-k} \left(\frac{1}{3}\right)^{k+1}} \geq 1 \quad (0.0.4)$$

$$\frac{2(k + 1)}{11 - k} \geq 1 \quad (0.0.5)$$

$$\implies k \geq 3 \quad (0.0.6)$$

$$\Pr(X = k) \geq \Pr(X = k - 1) \quad (0.0.7)$$

$$\frac{\Pr(X = k)}{\Pr(X = k - 1)} = \frac{{}^{11}C_k \left(\frac{2}{3}\right)^{11-k} \left(\frac{1}{3}\right)^{k}}{{}^{11}C_{k-1} \left(\frac{2}{3}\right)^{12-k} \left(\frac{1}{3}\right)^{k-1}} \geq 1 \quad (0.0.8)$$

$$\frac{12 - k}{2k} \geq 1 \quad (0.0.9)$$

$$\implies k \leq 4 \quad (0.0.10)$$

From (0.0.6) , (0.0.10) and since k is an integer
$\Pr(X = k)$ is maximized for k=3, k=4
Thus options 2) and 3) are correct