

# AI1103 Assignment-4

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Download all python codes from

<https://github.com/vrahul02/AI1103-Probability-and-Random-Variables/tree/main/Assignment-4/Codes>

and latex-tikz codes from

<https://github.com/vrahul02/AI1103-Probability-and-Random-Variables/tree/main/Assignment-4/Assignment-4.tex>

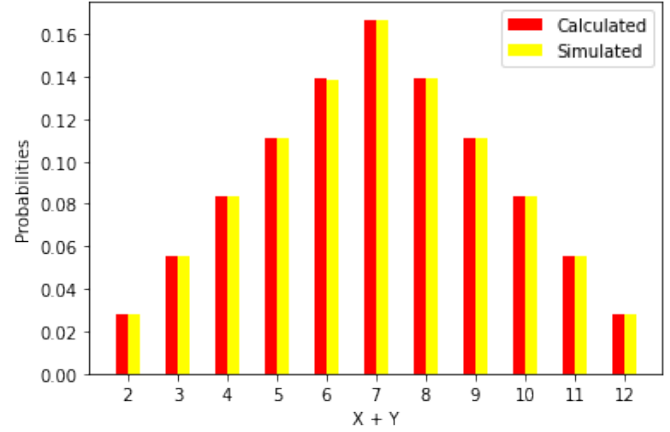


Fig. 0: Plot of PMF for X+Y

## PROBLEM GATE 2021 (ST), Q.15

A fair die is rolled twice independently. Let  $X$  and  $Y$  denote the outcomes of the first and second roll, respectively. Then  $E(X + Y | (X - Y)^2 = 1)$  equals

### SOLUTION

$X$  and  $Y$  are two independent random variables that can take the values 1, 2, 3, 4, 5, 6.

$$\Pr(X = k) = \frac{1}{6}, 1 \leq k \leq 6 \quad (0.0.1)$$

$$\Pr(Y = k) = \frac{1}{6}, 1 \leq k \leq 6 \quad (0.0.2)$$

On using convolution for discrete random variables,  
 $\Pr(X + Y = n)$

$$= \sum_{k=n-6}^{n-1} \Pr(X = k, Y = n - k), 1 \leq k \leq 6 \quad (0.0.3)$$

Since  $X$  and  $Y$  are independent random variables,

$$= \sum_{k=n-6}^{n-1} \Pr(X = k) \times \Pr(Y = n - k), 1 \leq k \leq 6 \quad (0.0.4)$$

$$= \sum_{k=n-6}^{n-1} \frac{1}{36}, 1 \leq k \leq 6 \quad (0.0.5)$$

$$= \begin{cases} n - 1 & \text{if } 2 \leq n \leq 7 \\ 13 - n & \text{if } 8 \leq n \leq 12 \end{cases} \quad (0.0.6)$$

$$\Pr(X - Y = n)$$

$$= \sum_{k=n+1}^{n+6} \Pr(X = k, Y = k - n), 1 \leq k \leq 6 \quad (0.0.7)$$

Since  $X$  and  $Y$  are independent random variables,

$$= \sum_{k=n+1}^{n+6} \Pr(X = k) \times \Pr(Y = k - n), 1 \leq k \leq 6 \quad (0.0.8)$$

$$= \sum_{k=n+1}^{n+6} \frac{1}{36}, 1 \leq k \leq 6 \quad (0.0.9)$$

$$= \begin{cases} n + 6 & \text{if } -5 \leq n \leq 0 \\ 6 - n & \text{if } 1 \leq n \leq 5 \end{cases} \quad (0.0.10)$$

$$(X - Y)^2 = 1 \quad (0.0.11)$$

$$X - Y = +1, X - Y = -1 \quad (0.0.12)$$

$$E(X + Y | (X - Y)^2 = 1) = \sum n \times \Pr(X + Y = n | (X - Y)^2 = 1) \quad (0.0.13)$$

$$= \sum n \times \frac{\Pr(X + Y = n, (X - Y)^2 = 1)}{\Pr((X - Y)^2 = 1)} \quad (0.0.14)$$

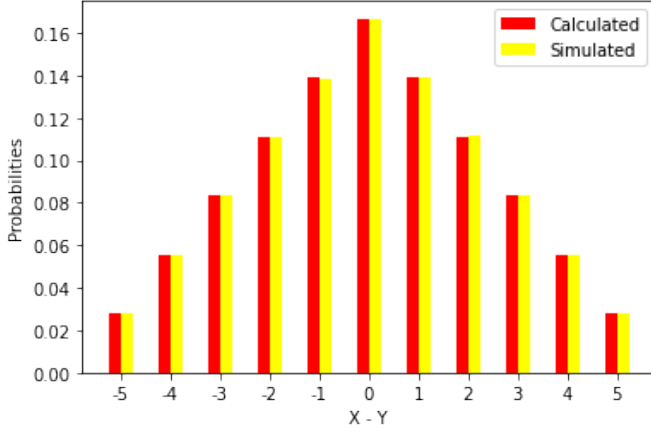


Fig. 0: Plot of PMF for X-Y

$$\begin{aligned}
 &= \sum n \times \frac{\Pr(X + Y = n, (X - Y) = 1)}{\Pr((X - Y) = 1)} \\
 &\quad \times \Pr((X - Y) = 1 | (X - Y)^2 = 1) \\
 &+ \sum n \times \frac{\Pr(X + Y = n, (X - Y) = -1)}{\Pr((X - Y) = -1)} \\
 &\quad \times \Pr((X - Y) = -1 | (X - Y)^2 = 1) \quad (0.0.15)
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\Pr((X - Y) = 1 | (X - Y)^2 = 1)}{\Pr((X - Y) = 1)} \\
 &\times \sum n \times \frac{\Pr(X + Y = n, (X - Y) = 1)}{\Pr((X - Y) = 1 | (X - Y)^2 = 1)} \\
 &+ \frac{\Pr((X - Y) = -1 | (X - Y)^2 = 1)}{\Pr((X - Y) = -1)} \\
 &\times \sum n \times \Pr(X + Y = n, (X - Y) = -1) \quad (0.0.16)
 \end{aligned}$$

Using equations (0.0.5) and (0.0.9) in (0.0.16)

We get,

$$E(X + Y | (X - Y)^2 = 1)$$

$$= \left(\frac{\frac{1}{2}}{\frac{5}{36}}\right) \times \left(\frac{35}{36}\right) + \left(\frac{\frac{1}{2}}{\frac{5}{36}}\right) \times \left(\frac{35}{36}\right) \quad (0.0.17)$$

$$= 7 \quad (0.0.18)$$