

AI1103 Assignment-5

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Download all python codes from

<https://github.com/vrahul02/AI1103-Probability-and-Random-Variables/tree/main/Assignment-5/Codes>

and latex-tikz codes from

<https://github.com/vrahul02/AI1103-Probability-and-Random-Variables/tree/main/Assignment-5/Assignment-5.tex>

$$\frac{2(k+1)}{11-k} \geq 1 \quad (0.0.5)$$

$$\implies k \geq 3 \quad (0.0.6)$$

$$\Pr(X = k) \geq \Pr(X = k-1) \quad (0.0.7)$$

$$\frac{\Pr(X = k)}{\Pr(X = k-1)} = \frac{{}^{11}C_k \left(\frac{2}{3}\right)^{11-k} \left(\frac{1}{3}\right)^k}{{}^{11}C_{k-1} \left(\frac{2}{3}\right)^{12-k} \left(\frac{1}{3}\right)^{k-1}} \geq 1 \quad (0.0.8)$$

$$\frac{12-k}{2k} \geq 1 \quad (0.0.9)$$

$$\implies k \leq 4 \quad (0.0.10)$$

PROBLEM CSIR UGC NET EXAM DEC 2012, Q.104

Let X be a binomial random variable with parameters $\left(11, \frac{1}{3}\right)$. At which value(s) of k is $\Pr(X = k)$ maximized?

- 1) $k=2$
- 2) $k=3$
- 3) $k=4$
- 4) $k=5$

SOLUTION

X has a binomial distribution :

$$\Pr(X = k) = {}^nC_k (q)^{n-k} (p)^k \quad (0.0.1)$$

Where,

- $n=11$
- $p = \frac{1}{3}$
- $q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

$$\Pr(X = k) = {}^{11}C_k \left(\frac{2}{3}\right)^{11-k} \left(\frac{1}{3}\right)^k \quad (0.0.2)$$

For $\Pr(X = k)$ to be maximized

$$\Pr(X = k) \geq \Pr(X = k+1) \quad (0.0.3)$$

$$\frac{\Pr(X = k)}{\Pr(X = k+1)} = \frac{{}^{11}C_k \left(\frac{2}{3}\right)^{11-k} \left(\frac{1}{3}\right)^k}{{}^{11}C_{k+1} \left(\frac{2}{3}\right)^{10-k} \left(\frac{1}{3}\right)^{k+1}} \geq 1 \quad (0.0.4)$$

From (0.0.6) , (0.0.10) and since k is an integer $\Pr(X = k)$ is maximized for $k=3, k=4$
Thus options 2) and 3) are correct

