AI1103 Assignment-1

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PROBLEM 1.5

The probability that a student is not a swimmer is $\frac{1}{5}$. Then the probability that out of five students, four are swimmers is

a) $\binom{5}{4} \times \left(\frac{4}{5}\right)^4 \times \left(\frac{1}{5}\right)$

b) $\left(\frac{4}{5}\right)^4 \times \left(\frac{1}{5}\right)$

c) $\binom{5}{1} \times \left(\frac{4}{5}\right)^4 \times \left(\frac{1}{5}\right)$

d) None of these

SOLUTION

Probability of 'non swimmer' = $\Pr(T) = \frac{1}{5}$
Since being 'swimmer' and being 'non swimmer' are mutually exclusive,
Probability of 'swimmer' = $\Pr(S) = 1\text{-}\Pr(T) = \frac{4}{5}$

Let K be a random variable representing number of students who are 'swimmer' in a given sample. So K has a binomial distribution :

$$\Pr(K = l) = \binom{n}{l} \times (n)^{n-l} \times (s)^l \qquad (0.0.1)$$

Where

- n = Total number of students = 5
- s = Probability that a student is 'swimmer' = $\frac{4}{5}$
- t = Probability that a student is 'non swimmer' = $\frac{1}{5}$
- l= Total number of swimmers = 4

$$\Pr(K = 4) = \binom{5}{4} \times \left(\frac{1}{5}\right)^1 \times \left(\frac{4}{5}\right)^4 \qquad (0.0.2)$$

As per the property of permutation and combination,

$$\binom{n}{l} = \binom{n}{n-l} \qquad (0.0.3)$$

$$\implies \Pr(K = 4) = \binom{5}{1} \times \left(\frac{1}{5}\right)^1 \times \left(\frac{4}{5}\right)^4 \qquad (0.0.4)$$

$$\implies \Pr(K = 4) = 0.4096$$

Thus options a) and c) are correct
The probability that 4 students are 'swimmer' out of a random sample of 5 students is 0.4096.