AI1103 Assignment-2

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Download all python codes from

https://github.com/vrahul02/AI1103−Probability−and−Random−Variables/tree/main/Assignment−2/Codes

and latex-tikz codes from

https://github.com/vrahul02/AI1103−Probability−and−Random−Variables/tree/main/Assignment−2/Assignment−2.tex

Problem Gate-28

Consider two independent random variables X and Y with identical distributions. The variables X and Y take value 0,1 and 2 with probabilities $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$ respectively. What is the conditional probability $\Pr\left(\frac{X+Y=2}{X-Y=0}\right)$?

a) 0
b) $\frac{1}{16}$
c) $\frac{1}{6}$
d) 1

Solution

The values that the random variable X can take along with its probabilities are given by

X	0	1	2
$\Pr(X)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

The values that the random variable Y can take along with its probabilities are given by

Y	0	1	2
$\Pr(Y)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

X and Y are independent random variables
Possible outcomes of the event are

$$S = \{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,1),(2,2),(2,3)\} \tag{0.0.1}$$

The outcomes where X+Y=2 is given by set

$$A = \{(0,2),(1,1),(2,0)\} \tag{0.0.2}$$

The outcomes where X-Y=0 is given by set

$$B = \{(0,0),(1,1),(2,2)\} \tag{0.0.3}$$

$$A + B = \{(1,1)\} \tag{0.0.4}$$

$$\Pr(A+B) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16} \tag{0.0.5}$$

$$\Pr(B) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} = \frac{1}{4} + \frac{1}{16} + \frac{1}{16} = \frac{6}{16} \tag{0.0.6}$$

$$\Pr(A/B) = \frac{\Pr(A+B)}{\Pr(B)} = \frac{\frac{1}{16}}{\frac{6}{16}} = \frac{1}{6} \tag{0.0.7}$$

(0.0.8)

Thus option c) is correct