AI1103 Assignment-1

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PROBLEM 1.5

The probability that a student is not a swimmer is $\frac{1}{5}$. Then the probability that out of five students, four are swimmers is

a) $\binom{5}{4} \times \left(\frac{4}{5}\right)^4 \times \left(\frac{1}{5}\right)$
b) $\left(\frac{4}{5}\right)^4 \times \left(\frac{1}{5}\right)$
c) $\binom{5}{1} \times \left(\frac{4}{5}\right)^4 \times \left(\frac{1}{5}\right)$
d) None of these

SOLUTION

A student can either be 'swimmer' or 'non swimmer'. Given that the probability that a student is 'non swimmer' is $\frac{1}{5}$ and since being 'swimmer' and being 'non swimmer' are mutually exclusive,

- Probability of 'swimmer' = $\Pr(S) = \frac{1}{5}$
- Probability of 'non swimmer' = $\Pr(N)$ = 1- $\Pr(S) = \frac{4}{5}$

We can use binomial distribution to find out the probability that out of five students, four are 'swimmer'.
Let X be a random variable representing number of students who are 'swimmer' in a given sample. Picking different number of students is an example of a Bernoulli trial.
So X has a binomial distribution :

$$\Pr(X = y) = \binom{n}{x} \times (n)^{n-y} \times (s)^y \qquad (0.0.1)$$

Where

- n = Total number of students = 5
- l = Probability that a student is 'swimmer' = $\frac{4}{5}$
- r = Probability that a student is 'non swimmer' = $\frac{1}{5}$

So,

$$\Pr(X = y) = \binom{n}{y} \times \left(\frac{1}{5}\right)^{n-y} \times \left(\frac{4}{5}\right)^y \qquad (0.0.2)$$

$$Pr(4\ are\ swimmer) = \Pr(X = 4) \qquad (0.0.3)$$

$$\Pr(X = 4) = \binom{5}{4} \times \left(\frac{1}{5}\right)^1 \times \left(\frac{4}{5}\right)^4 \qquad (0.0.4)$$

As per the property of binomial theorem,

$$\binom{n}{y} = \binom{n}{n-y} \qquad (0.0.5)$$

$$\implies \Pr(X = 4) = \binom{5}{1} \times \left(\frac{1}{5}\right)^1 \times \left(\frac{4}{5}\right)^4 \qquad (0.0.6)$$

$$\implies \Pr(X = 4) = 0.4096$$

Thus options a) and c) are correct
The probability that 4 students are 'swimmer' out of a random sample of 5 students is 0.4096.