AI1103 Assignment-1

V Rahul - AI20BTECH11030

Download all python codes from

https://github.com/vrahul02/AI1103−Probability−and−Random−Variables/tree/main/Assignment−1/Codes

and latex-tikz codes from

https://github.com/vrahul02/AI1103−Probability−and−Random−Variables/tree/main/Assignment−1/Assignment−1.tex

Problem 1.5

The probability that a student is not a swimmer is $\frac{1}{5}$. Then the probability that out of five students, four are swimmers is

1) $\binom{5}{4}\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$
2) $\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$
3) $\binom{5}{1}\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$
4) None of these

Solution

Being 'swimmer' and being 'non swimmer' are mutually exclusive,

- Probability of 'non swimmer' = q = $\frac{1}{5}$
- Probability of 'swimmer' = p = 1- q = $\frac{4}{5}$
- Total number of students = n = 5
- Total number of swimmers = l = 4

Let K be a random variable representing number of students who are 'swimmer' in a given sample.
So K has a binomial distribution :

$$\Pr(K = l) = \binom{n}{l}(q)^{n-l}(p)^l \tag{0.0.1}$$

$$\Pr(K = 4) = \binom{5}{4}\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^4 \tag{0.0.2}$$

As per the property of permutation and combination,

$$\binom{n}{l} = \binom{n}{n-l} \tag{0.0.3}$$

$$\implies \Pr(K = 4) = \binom{5}{1}\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^4 \tag{0.0.4}$$

$$\implies \Pr(K = 4) = 0.4096 \tag{0.0.5}$$

The probability that 4 students are 'swimmer' out of a random sample of 5 students is 0.4096.
Thus options 1) and 3) are correct