AI1103 Assignment-2

V Rahul - AI20BTECH11030

Download all python codes from

https://github.com/vrahul02/AI1103-Probabilityand-Random-Variables/tree/main/Assignment -2/Codes

and latex-tikz codes from

https://github.com/vrahul02/AI1103-Probabilityand-Random-Variables/tree/main/Assignment -2/Assignment-2.tex

Problem Gate-28

Consider two independent random variables X and Y with identical distributions. The variables X and Y take value 0,1 and 2 with probabilities $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$ respectively. What is the conditional probability $\Pr\left(\frac{X+Y=2}{X-Y=0}\right)$?

- a) 0
- b) $\frac{1}{16}$ c) $\frac{1}{6}$ d) 1

Solution

The values that the random variable X can take along with its probabilities are given by

X	0	1	2
Pr(X)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

The values that the random variable Y can take along with its probabilities are given by

Y	0	1	2
Pr(Y)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

X and Y are independent random variables

Possible outcomes of the event are

$$S = \{(0,0), (0,1), (0,2),$$

$$(1,0), (1,1), (1,2),$$

$$(2,1), (2,2), (2,3)\}$$

$$(0.0.1)$$

The outcomes where X+Y=2 is given by set

$$A = \{(0, 2), (1, 1), (2, 0)\}$$
 (0.0.2)

The outcomes where X-Y=0 is given by set

$$B = \{(0,0), (1,1), (2,2)\}\$$
 (0.0.3)

$$A + B = \{(1, 1)\} \tag{0.0.4}$$

$$\Pr(A+B) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16} \tag{0.0.5}$$

$$Pr(B) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4}$$
$$= \frac{1}{4} + \frac{1}{16} + \frac{1}{16} = \frac{6}{16}$$
(0.0.6)

$$\Pr(A/B) = \frac{\Pr(A+B)}{\Pr(B)} = \frac{\frac{1}{16}}{\frac{6}{16}} = \frac{1}{6}$$
 (0.0.7)

Thus option c) is correct