AI1103 Assignment-4

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Download all python codes from

https://github.com/vrahul02/AI1103−Probability−and−Random−Variables/tree/main/Assignment−4/Codes

and latex-tikz codes from

https://github.com/vrahul02/AI1103−Probability−and−Random−Variables/tree/main/Assignment−4/Assignment−4.tex

Problem GATE 2021 (ST), Q.15

A fair die is rolled twice independently. Let X and Y denote the outcomes of the first and second roll, respectively. Then $E(X+Y \mid (X-Y)^2 = 1)$ equals

Solution

X and Y are two independent random variables that can take the values 1, 2, 3, 4, 5, 6.

$$(X-Y)^2 = 1 \tag{0.0.1}$$

$$X - Y = +1,\ X - Y = -1 \tag{0.0.2}$$

X+Y=n	3	5	7	9	11
Outcomes	(1,2), (2,1)	(2,3), (3,2)	(3,4), (4,3)	(4,5), (5,4)	(5,6), (6,5)

$$\mathrm{E}\left(X+Y \mid (X-Y)^2 = 1\right)$$

$$= \Sigma\, n \times \Pr\left(X+Y=n \mid (X-Y)^2 = 1\right) \tag{0.0.3}$$

$$= \Sigma\, n \times \frac{\Pr\left(X+Y=n, (X-Y)^2 = 1\right)}{\Pr\left((X-Y)^2 = 1\right)} \tag{0.0.4}$$

$$= \Sigma\, n \times \frac{\Pr\left(X+Y=n, (X-Y=1) \cup (X-Y=-1)\right)}{\Pr\left((X-Y=1) \cup (X-Y=-1)\right)} \tag{0.0.5}$$

$$= 3 \times \frac{2}{10} + 5 \times \frac{2}{10} + 7 \times \frac{2}{10} + 9 \times \frac{2}{10} + 11 \times \frac{2}{10} \tag{0.0.6}$$

$$= 7 \tag{0.0.7}$$