# AI1103 Assignment-4

## V Rahul - AI20BTECH11030

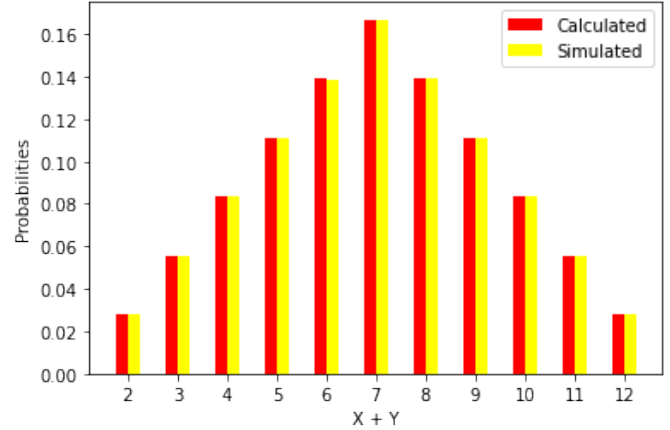Download all python codes from

and latex-tikz codes from

Fig. 0: Plot of PMF for X+Y

### PROBLEM GATE 2021 (ST), Q.15

A fair die is rolled twice independently. Let X and Y denote the outcomes of the first and second roll, respectively. Then $E(X + Y \mid (X − Y)^2 = 1)$ equals

### SOLUTION

X and Y are two independent random variables that can take the values 1, 2, 3, 4, 5, 6.

$$\Pr(X = k) = \frac{1}{6}, 1 \le k \le 6 \qquad (0.0.1)$$

$$\Pr(Y = k) = \frac{1}{6}, 1 \le k \le 6 \qquad (0.0.2)$$

On using convolution for discrete random variables, $\Pr(X + Y = n)$

$$= \sum_{k=n-6}^{n-1} \Pr(X = k, Y = n - k), 1 \le k \le 6 \qquad (0.0.3)$$

Since X and Y are independent random variables,

$$= \sum_{k=n-6}^{n-1} \Pr(X = k) \times \Pr(Y = n - k), 1 \le k \le 6 \qquad (0.0.4)$$

$$= \sum_{k=n-6}^{n-1} \frac{1}{6} \times \frac{1}{6}, 1 \le k \le 6 \qquad (0.0.5)$$

$$= \sum_{k=n-6}^{n-1} \frac{1}{36}, 1 \le k \le 6 \qquad (0.0.6)$$

$\Pr(X − Y = n)$

$$= \sum_{k=n+1}^{n+6} \Pr(X = k, Y = k - n), 1 \le k \le 6 \qquad (0.0.7)$$

Since X and Y are independent random variables,

$$= \sum_{k=n+1}^{n+6} \Pr(X = k) \times \Pr(Y = k - n), 1 \le k \le 6 \qquad (0.0.8)$$

$$= \sum_{k=n+1}^{n+6} \frac{1}{6} \times \frac{1}{6}, 1 \le k \le 6 \qquad (0.0.9)$$

$$= \sum_{k=n+1}^{n+6} \frac{1}{36}, 1 \le k \le 6 \qquad (0.0.10)$$

$$(X − Y)^2 = 1 \qquad (0.0.11)$$

$$X − Y = +1, \ X − Y = −1 \qquad (0.0.12)$$

$E\left(X + Y \mid (X − Y)^2 = 1\right)$

$$= \sum n \times \Pr\left(X + Y = n \mid (X − Y)^2 = 1\right) \qquad (0.0.13)$$

$$= \sum n \times \frac{\Pr\left(X + Y = n, (X − Y)^2 = 1\right)}{\Pr\left((X − Y)^2 = 1\right)} \qquad (0.0.14)$$
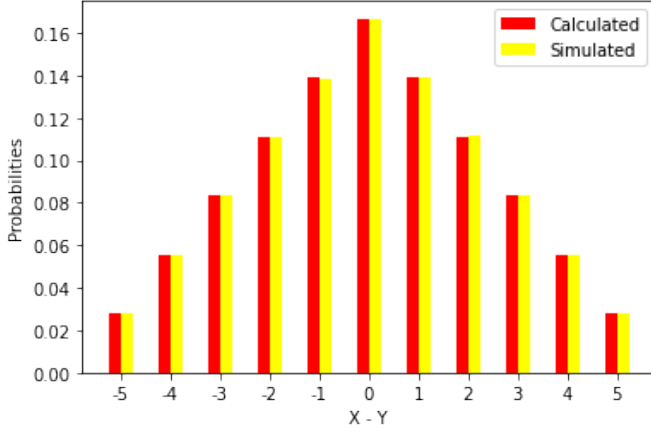
Fig. 0: Plot of PMF for X-Y

$$= \sum n \times \frac{\Pr(X + Y = n, (X - Y) = 1)}{\Pr((X - Y) = 1)}$$
$$\times \Pr\left((X - Y) = 1 | (X - Y)^2 = 1\right)$$
$$+ \sum n \times \frac{\Pr(X + Y = n, (X - Y) = -1)}{\Pr((X - Y) = -1)} \quad (0.0.15)$$
$$\times \Pr\left((X - Y) = 1 \,|\, (X - Y)^2 = 1\right)$$

$$= \frac{\Pr\left((X - Y) = 1 \,|\, (X - Y)^2 = 1\right)}{\Pr((X - Y) = 1)}$$
$$\times \sum n \times \Pr(X + Y = n, (X - Y) = 1)$$
$$+ \frac{\Pr\left((X - Y) = -1 \,|\, (X - Y)^2 = 1\right)}{\Pr((X - Y) = -1)} \quad (0.0.16)$$
$$\times \sum n \times \Pr(X + Y = n, (X - Y) = -1)$$

Using equations (0.0.6) and (0.0.10) in (0.0.16)
We get,
$E\left(X + Y \,|\, (X - Y)^2 = 1\right)$

$$= \left(\frac{\frac{1}{2}}{\frac{5}{36}}\right) \times \left(\frac{35}{36}\right) + \left(\frac{\frac{1}{2}}{\frac{5}{36}}\right) \times \left(\frac{35}{36}\right) \quad (0.0.17)$$
$$= 7 \quad (0.0.18)$$