EE3900 Gate Assignment-1

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Download all python codes from

```
https://github.com/vrahul02/EE3900/tree/main/
    Gate−Assignment−1/Codes
```

and latex-tikz codes from

```
https://github.com/vrahul02/EE3900/tree/main/
    Gate−Assignment−1/Gate−Assignment−1.
    tex
```

Problem Gate EC-2015 Q.23

Consider the sequence $x[n] = a^n u[n] + b^n u[n]$ where $u[n]$ denotes the unit-step sequence and $0 < |a| < |b| < 1$. The region of convergence (ROC) of the z-transform of $x[n]$ is

1) $|z| > |a|$
2) $|z| > |b|$
3) $|z| < |a|$
4) $|a| < |z| < |b|$

Solution

Definition 1. *The z-transform of a function is defined as*

$$x[n] \overset{\mathcal{Z}}{\rightleftharpoons} X(z) \tag{0.0.1}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \tag{0.0.2}$$

Definition 2. *The $u[n]$ function is defined as*

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & otherwise \end{cases} \tag{0.0.3}$$

Lemma 0.1. *If $x[n] = a^n u[n]$, then $x[n] \overset{\mathcal{Z}}{\rightleftharpoons} X[z] = \frac{1}{1-az^{-1}}$ with $ROC = |z| > a$*

Proof. Using the formula for the sum of an infinite GP, we get:

$$x[n] = \begin{cases} a^n & n \geq 0 \\ 0 & otherwise \end{cases} \tag{0.0.4}$$

$$\mathcal{Z}\{x[n]\} = X[z] = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \tag{0.0.5}$$

$$= \sum_{n=-\infty}^{0} 0 \times z^{-n} + \sum_{n=0}^{\infty} (az^{-1})^n \tag{0.0.6}$$

$$= \frac{1}{1-az^{-1}}, ROC = |az^{-1}| < 1 \tag{0.0.7}$$

$$= \frac{1}{1-az^{-1}}, ROC = |z| > a \tag{0.0.8}$$

□

We are given x[n] as

$$x[n] = a^n u[n] + b^n u[n] \tag{0.0.9}$$

Let

$$x_1[n] = a^n u[n] \tag{0.0.10}$$

$$x_2[n] = b^n u[n] \tag{0.0.11}$$

Then the z-transform of $x_1[n]$ and $x_2[n]$ using (0.1) is,

$$X_1(z) = \frac{1}{1-az^{-1}},\ ROC : |z| > |a| = R_1 \tag{0.0.12}$$

$$X_2(z) = \frac{1}{1-bz^{-1}},\ ROC : |z| > |b| = R_2 \tag{0.0.13}$$

Thus

$$X(z) = X_1(z) + X_2(z),\ ROC : R_1 \cap R_2 \tag{0.0.14}$$

$$X(z) = \frac{1}{1-az^{-1}} + \frac{1}{1-bz^{-1}},\ ROC : (|z| > |a|) \cap (|z| > |b|) \tag{0.0.15}$$

Since $0 < |a| < |b| < 1$

$$X(z) = \frac{1}{1-az^{-1}} + \frac{1}{1-bz^{-1}},\ ROC : |z| > |b| \tag{0.0.16}$$

Thus option 2) is correct

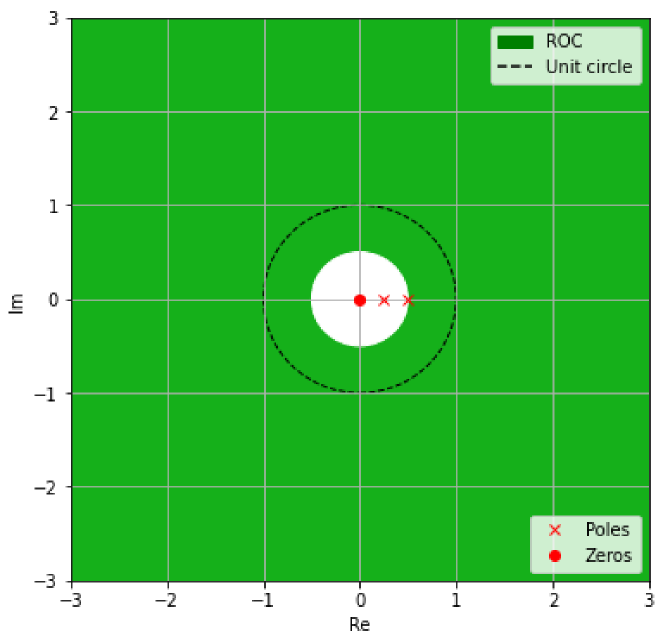

Fig. 4: Pole-zero plot of the system for a=0.25 and b=0.5