EE3900 Gate Assignment-4

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Download all python codes from

```
https://github.com/vrahul02/EE3900/tree/main/
    Gate−Assignment−4/Codes
```

and latex-tikz codes from

```
https://github.com/vrahul02/EE3900/tree/main/
    Gate−Assignment−4/Gate−Assignment−4.
    tex
```

Problem Gate EC-1998 Q.1.16

The z-transform of the time function $\sum_{k=0}^{\infty} \delta(n-k)$ is

1) $\frac{z-1}{z}$
2) $\frac{z}{(z-1)^2}$
3) $\frac{z}{z-1}$
4) $\frac{(z-1)^2}{z}$

Solution

Definition 1. *The z-transform of a function is defined as*

$$x[n] \overset{\mathcal{Z}}{\rightleftharpoons} X(z) \tag{0.0.1}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \tag{0.0.2}$$

x(n)

$$= \sum_{k=0}^{\infty} \delta(n-k) \tag{0.0.3}$$

$$= \delta(n) + \delta(n-1) + \delta(n-2) + ... \tag{0.0.4}$$

$$= u(n) \tag{0.0.5}$$

where $u(n)$ denotes the unit-step function

Definition 2. *The $u[n]$ function is defined as*

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & otherwise \end{cases} \tag{0.0.6}$$

Thus,

$$\mathcal{Z}(x(n)) = \mathcal{Z}(u(n)) \tag{0.0.7}$$

Using (1) and (2)

$$X(z) = \sum_{n=-\infty}^{\infty} u[n]z^{-n} \tag{0.0.8}$$

$$= \sum_{n=0}^{\infty} (z^{-1})^n \tag{0.0.9}$$

Using the formula for the sum of an infinite GP, we get:

$$X(z) = \frac{1}{1-z^{-1}}, ROC = |z^{-}1| < 1 \tag{0.0.10}$$

$$= \frac{z}{z-1}, ROC = |z| > 1 \tag{0.0.11}$$

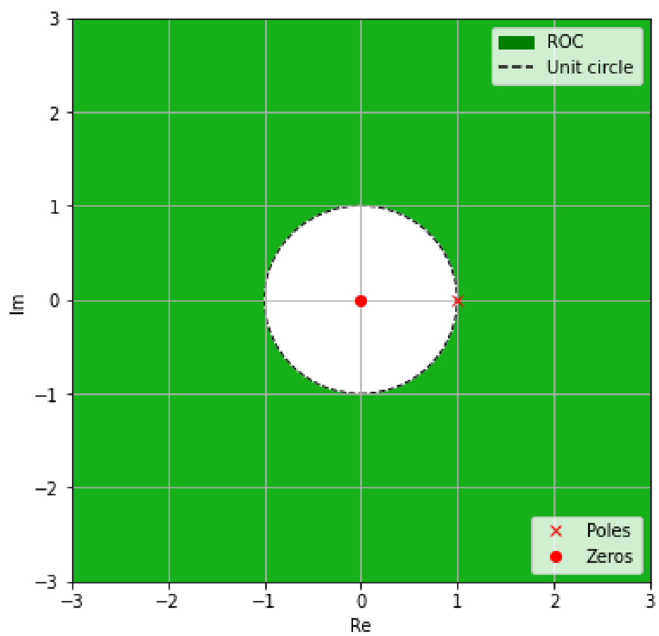

Fig. 4: Pole-zero plot of the system

$$\mathcal{Z}(x(n)) = \frac{z}{z-1} \tag{0.0.12}$$

Thus option 3) is correct

Definition 3. *The fourier-transform of a function is defined as*

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] \exp^{-j\omega n} \tag{0.0.13}$$

Using (3) and (2)

$$X(\omega) = \sum_{n=-\infty}^{\infty} u[n] \exp^{-j\omega n} \tag{0.0.14}$$

$$= \sum_{n=0}^{\infty} (\exp^{-j\omega})^{n} \tag{0.0.15}$$

Using the formula for the sum of an infinite GP, we get:

$$X(\omega) = \frac{1}{1 - \exp^{-j\omega}}, |\exp^{-j\omega}| < 1 \tag{0.0.16}$$

$$= \frac{\exp^{j\omega}}{\exp^{j\omega} - 1}, |\exp^{j\omega}| > 1 \tag{0.0.17}$$