EE3900 Assignment-3

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Download all python codes from

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https://github.com/vrahul02/EE3900/tree/main/Assignment-3/Codes
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and latex-tikz codes from

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https://github.com/vrahul02/EE3900/tree/main/Assignment-3/Assignment-3.tex
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Problem Ramsey Tangent and Normal Q.20

Find the condition that the line $\begin{pmatrix} l & m \end{pmatrix} \mathbf{x} + n = 0$ should touch the circle $\left\| x - \begin{pmatrix} a \\ b \end{pmatrix} \right\| = r$

Solution

The vector equation of a line can be expressed as

$$\mathbf{x} = \mathbf{q} + \mu \mathbf{m} \tag{0.0.1}$$

If $\mathbf{n}$ is the normal vector of a line, equation of that line can be written as

$$\mathbf{n}^T \mathbf{x} = c \tag{0.0.2}$$

Where

$$\mathbf{n} = \begin{pmatrix} l \\ m \end{pmatrix} \tag{0.0.3}$$

$$c = -n \tag{0.0.4}$$

The general equation of a second degree can be expressed as :

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \tag{0.0.5}$$

Where

$$\mathbf{u} = \begin{pmatrix} -a \\ -b \end{pmatrix} \tag{0.0.6}$$

$$f = a^2 + b^2 - r^2 \tag{0.0.7}$$

The point of contact $\mathbf{q}$, of a line with a normal vector $\mathbf{n}$ to the conic in (0.0.5) is given by:

$$\mathbf{q} = \mathbf{V}^{-1} (\kappa \mathbf{n} - \mathbf{u}) \tag{0.0.8}$$

$$\kappa = \pm \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\mathbf{n}^T \mathbf{V}^{-1} \mathbf{n}}} \tag{0.0.9}$$

We know that, for a circle,

$$\mathbf{V} = \mathbf{I} \tag{0.0.10}$$

and from the properties of an Identity matrix,

$$\mathbf{I}^{-1} = \mathbf{I} \tag{0.0.11}$$

$$\mathbf{I}\mathbf{X} = \mathbf{X} \tag{0.0.12}$$

Solving for the point of contact using the above equations we get,

$$\kappa = \pm \sqrt{\frac{\begin{pmatrix} -a & -b \end{pmatrix} \begin{pmatrix} -a \\ -b \end{pmatrix} - a^2 - b^2 + r^2}{\begin{pmatrix} l & m \end{pmatrix} \begin{pmatrix} l \\ m \end{pmatrix}}} \tag{0.0.13}$$

$$= \pm \frac{r}{\sqrt{l^2 + m^2}} \tag{0.0.14}$$

$$\mathbf{q} = \pm \frac{r}{\sqrt{l^2 + m^2}} \begin{pmatrix} l \\ m \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} \tag{0.0.15}$$

$$\tag{0.0.16}$$

Since point of contact $\mathbf{q}$ lies on tangent it satisfies the line equation of tangents

$$\mathbf{n}^T \mathbf{q} = c \tag{0.0.17}$$

$$\begin{pmatrix} l & m \end{pmatrix} \times \left(\pm \frac{r}{\sqrt{l^2 + m^2}} \begin{pmatrix} l \\ m \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} \right) = -n \tag{0.0.18}$$

$$\pm r \sqrt{l^2 + m^2} + al + bm = -n \tag{0.0.19}$$

$$\pm r \sqrt{l^2 + m^2} = -n - al - bm \tag{0.0.20}$$

On squaring both sides

$$r^2 (l^2 + m^2) = (n + al + bm)^2 \tag{0.0.21}$$

This is the condition of tangency