EE3900 Assignment-3

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PROBLEM RAMSEY TANGENT AND NORMAL Q.20

Find the condition that the line $\begin{pmatrix} l & m \end{pmatrix}\mathbf{x} + n = 0$ should touch the circle $\left\Vert x - \begin{pmatrix} a \\ b \end{pmatrix} \right\Vert = r$.

SOLUTION

If $\mathbf{n}$ is the normal vector of a line, equation of that line can be written as

$$\mathbf{n}^T\mathbf{x} = c \tag{0.0.1}$$

Here

$$\mathbf{n} = \begin{pmatrix} l \\ m \end{pmatrix} \tag{0.0.2}$$

$$c = -n \tag{0.0.3}$$

Center of the given circle is $\mathbf{o} = \begin{pmatrix} a \\ b \end{pmatrix}$ and its radius is r.

The condition for a given line to touch a circle is: Distance of the line from the center of the circle, must be equal to its radius.

Formula for Distance of a line $\mathbf{n}^T\mathbf{x} = c$ from a point $\mathbf{x}$ is given as

$$d = \frac{\left| \mathbf{n}^T\mathbf{x} - c \right|}{\Vert n \Vert} \tag{0.0.4}$$

$$r = \frac{\left| \mathbf{n}^T\mathbf{o} - c \right|}{\Vert n \Vert} \tag{0.0.5}$$

$$= \frac{\left| \begin{pmatrix} l & m \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} + n \right|}{\sqrt{l^2 + m^2}} \tag{0.0.6}$$

$$= \frac{|al + bm + n|}{\sqrt{l^2 + m^2}} \tag{0.0.7}$$

On squaring both sides,

$$r^2(l^2 + m^2) = (n + al + bm)^2 \tag{0.0.8}$$

This is the condition of tangency