

EE3900 Gate Assignment-4

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Download all python codes from

<https://github.com/vrahul02/EE3900/tree/main/Gate-Assignment-4/Codes>

and latex-tikz codes from

<https://github.com/vrahul02/EE3900/tree/main/Gate-Assignment-4/Gate-Assignment-4.tex>

PROBLEM GATE EC-1998 Q.1.16

The z -transform of the time function $\sum_{k=0}^{\infty} \delta(n-k)$ is

- 1) $\frac{z-1}{z}$
- 2) $\frac{z}{(z-1)^2}$
- 3) $\frac{z-1}{z}$
- 4) $\frac{(z-1)^2}{z}$

SOLUTION

Definition 1. The z -transform of a function is defined as

$$x[n] \xrightarrow{Z} X(z) \quad (0.0.1)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \quad (0.0.2)$$

$x(n)$

$$= \sum_{k=0}^{\infty} \delta(n-k) \quad (0.0.3)$$

$$= \delta(n) + \delta(n-1) + \delta(n-2) + \dots \quad (0.0.4)$$

$$= u(n) \quad (0.0.5)$$

Thus,

$$\mathcal{Z}(x(n)) = \mathcal{Z}(u(n)) \quad (0.0.6)$$

Definition 2. The $u[n]$ function is defined as

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.7)$$

Using (1) and (2)

$$X(z) = \sum_{n=-\infty}^{\infty} u[n]z^{-n} \quad (0.0.8)$$

$$= \sum_{n=0}^{\infty} (z^{-1})^n \quad (0.0.9)$$

$$= \frac{1}{1-z^{-1}}, \text{ ROC} = |z^{-1}| < 1 \quad (0.0.10)$$

$$= \frac{z}{z-1}, \text{ ROC} = |z| > 1 \quad (0.0.11)$$

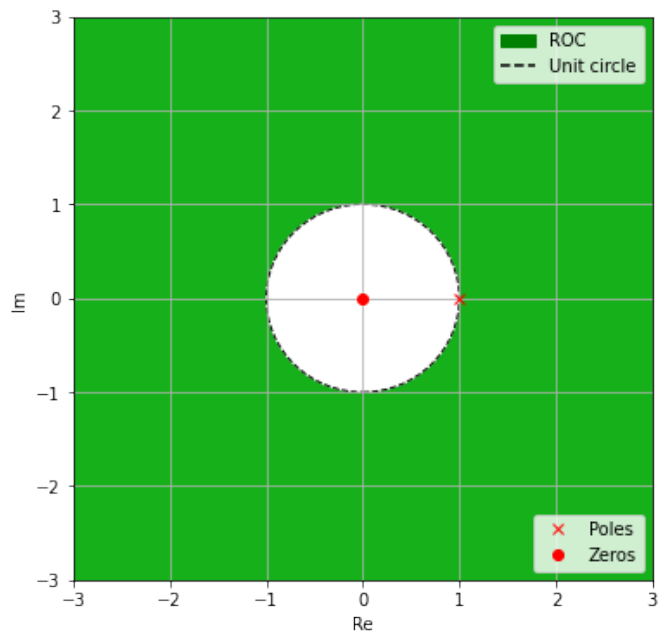


Fig. 4: Pole-zero plot of the system

$$\mathcal{Z}(x(n)) = \frac{z}{z-1} \quad (0.0.12)$$

Thus option 3) is correct