EE3900 Assignment-5

V Rahul - AI20BTECH11030

Download all python codes from

```
https://github.com/vrahul02/EE3900/tree/main/Assignment−5/Codes
```

and latex-tikz codes from

```
https://github.com/vrahul02/EE3900/tree/main/Assignment−5/Assignment−5.tex
```

Problem Quadratic forms Q.2.24

Find the coordinates of the focus, axis, equation of the directrix and latus rectum of the parabola $y^2 = 8x$.

Solution

The parabola $y^2 = 8x$ can be written as

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -8 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{0.0.1}$$

$$\therefore \mathbf{u} = \begin{pmatrix} -8 \\ 0 \end{pmatrix} \tag{0.0.2}$$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{0.0.3}$$

Since the parabola is symmetric about x-axis, it is the axis of the parabola.
Vertex of the parabola is v=$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Let the focus be f=$\begin{pmatrix} a \\ 0 \end{pmatrix}$.
Then the point of intersection of directrix and x-axis will be $\begin{pmatrix} -a \\ 0 \end{pmatrix}$.
Since directrix of the parabola is perpendicular to the axis, the equation of the directrix will be

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = -a \tag{0.0.4}$$

The point of intersection of latus rectum and x-axis will be $\begin{pmatrix} a \\ 0 \end{pmatrix}$.
Since latus rectum of the parabola is perpendicular to the axis, the equation of the latus rectum will be

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = a \tag{0.0.5}$$

Let $\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $c = -a$.
Let $\mathbf{x}$ be any general point on the parabola.
By the definition of a parabola, the distance between $\mathbf{x}$ and the focus is equal to the perpendicular distance between $\mathbf{x}$ and the directrix.
So we can write

$$\|\mathbf{x} - \mathbf{f}\| = \frac{\left|\mathbf{n}^\top \mathbf{x} - c\right|}{\|\mathbf{n}\|} \tag{0.0.6}$$

$$\implies \|\mathbf{x} - \mathbf{f}\|^2 \|\mathbf{n}\|^2 = \left|\mathbf{n}^\top \mathbf{x} - c\right|^2 \tag{0.0.7}$$

$$\implies (\mathbf{x} - \mathbf{f})^\top (\mathbf{x} - \mathbf{f}) \|\mathbf{n}\|^2 = (\mathbf{n}^\top \mathbf{x})^2 - 2c\mathbf{n}^\top \mathbf{x} + c^2 \tag{0.0.8}$$

$$\|\mathbf{n}\|^2 \mathbf{x}^\top \mathbf{x} - 2\|\mathbf{n}\|^2 \mathbf{f}^\top \mathbf{x} + \|\mathbf{n}\|^2 \|\mathbf{f}\|^2 = \mathbf{x}^\top \mathbf{n}\mathbf{n}^\top \mathbf{x} - 2c\mathbf{n}^\top \mathbf{x} + c^2 \tag{0.0.9}$$

$$\implies \mathbf{x}^\top (\|\mathbf{n}\|^2 \mathbf{I} - \mathbf{n}\mathbf{n}^\top) \mathbf{x} + 2(c\mathbf{n} - \|\mathbf{n}\|^2 \mathbf{f})^\top \mathbf{x} + \|\mathbf{n}\|^2 \|\mathbf{f}\|^2 - c^2 = 0 \tag{0.0.10}$$

Putting values of $\mathbf{n}$, $\mathbf{f}$ and c, we get

$$a = 2 \tag{0.0.11}$$

Thus focus $\mathbf{f} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$
Equation of directrix is given by

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = -2 \tag{0.0.12}$$

Equation of latus rectum is given by

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 2 \tag{0.0.13}$$

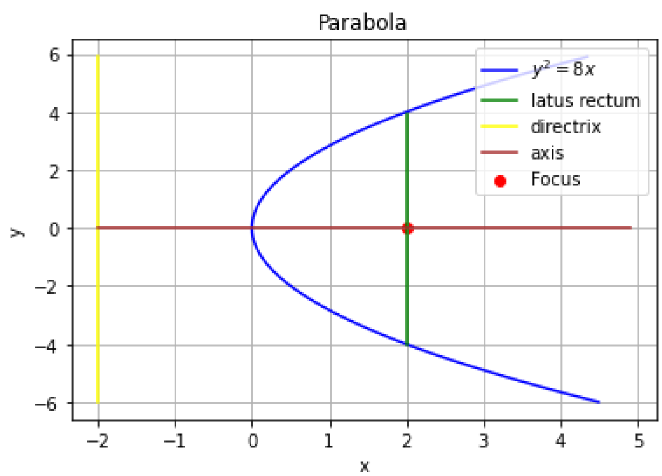

Fig. 0: Plot of the parabola