EE3900 Gate Assignment-3

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Download all python codes from

```
https://github.com/vrahul02/EE3900/tree/main/
    Gate−Assignment−3/Codes
```

and latex-tikz codes from

```
https://github.com/vrahul02/EE3900/tree/main/
    Gate−Assignment−3/Gate−Assignment−3.
    tex
```

Problem Gate EC-2002 Q.2.4

If the impulse response of a discrete-time system is $h[n] = -5^n u[-n-1]$, then the system function H(z) is equal to

1) $\frac{-z}{z-5}$ and the system is stable
2) $\frac{z}{z-5}$ and the system is stable
3) $\frac{-z}{z-5}$ and the system is unstable
4) $\frac{z}{z-5}$ and the system is unstable

Solution

Definition 1. *The z-transform of a function is defined as*

$$h[n] \overset{\mathcal{Z}}{\rightleftharpoons} H(z) \tag{0.0.1}$$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n} \tag{0.0.2}$$

Definition 2. *The $u[n]$ function is defined as*

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & otherwise \end{cases} \tag{0.0.3}$$

Lemma 0.1. *If $h[n] = a^n u[n]$, then $h[n] \overset{\mathcal{Z}}{\rightleftharpoons} H[z] = \frac{1}{1-az^{-1}}$ with $ROC = |z| > a$*

Proof. Using the formula for the sum of an infinite GP, we get:

$$h[n] = \begin{cases} a^n & n \geq 0 \\ 0 & otherwise \end{cases} \tag{0.0.4}$$

$$\mathcal{Z}\{h[n]\} = H[z] = \sum_{n=-\infty}^{\infty} h[n] z^{-n} \tag{0.0.5}$$

$$= \sum_{n=-\infty}^{0} 0 \times z^{-n} + \sum_{n=0}^{\infty} (az^{-1})^n \tag{0.0.6}$$

$$= \frac{1}{1-az^{-1}}, ROC = |az^{-1}| < 1 \tag{0.0.7}$$

$$= \frac{1}{1-az^{-1}}, ROC = |z| > a \tag{0.0.8}$$

□

Lemma 0.2. *If $h[n] = -a^n u[-n-1]$, then $h[n] \overset{\mathcal{Z}}{\rightleftharpoons} H[z] = \frac{1}{1-az^{-1}}$ with $ROC = |z| < a$*

Proof. Using the formula for the sum of an infinite GP, we get:

$$h[n] = \begin{cases} -a^n & n \leq -1 \\ 0 & otherwise \end{cases} \tag{0.0.9}$$

$$\mathcal{Z}\{h[n]\} = H[z] = \sum_{n=-\infty}^{\infty} h[n] z^{-n} \tag{0.0.10}$$

$$= \sum_{n=-\infty}^{\infty} \left\{-a^n u[-n-1]\right\} z^{-n} \tag{0.0.11}$$

$$= \sum_{n=-\infty}^{-1} -a^n z^{-n} + \sum_{n=-1}^{\infty} 0 \times z^{-1} \tag{0.0.12}$$

$$= -\sum_{n=1}^{\infty} (a^{-1}z)^n = 1 - \sum_{n=0}^{\infty} (a^{-1}z)^n \tag{0.0.13}$$

$$= 1 - \frac{1}{1-a^{-1}z}, ROC = |a^{-1}z| < 1 \tag{0.0.14}$$

$$H[z] = \frac{1}{1-az^{-1}}, ROC = |z| < a \tag{0.0.15}$$

□

We are given h[n] as,

$$h[n] = -5^n u[-n-1] \tag{0.0.16}$$

Then the z-transform of h[n] using (0.1) and (0.2) is,

$$H(z) = \frac{1}{1 - 5z^{-1}}, |z| < 5 \tag{0.0.17}$$

$$H(z) = \frac{z}{z - 5}, |z| < 5 \tag{0.0.18}$$

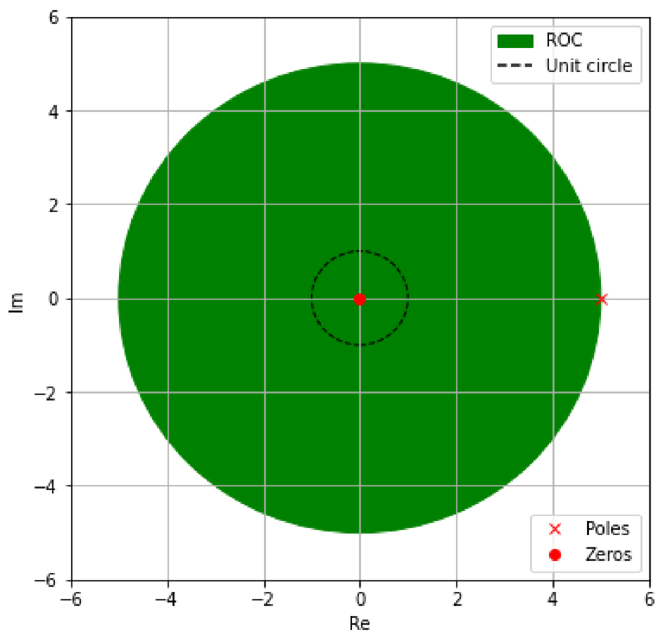

Fig. 4: Pole-zero plot of the system

Definition 3. *An discrete-time system is BIBO stable if and only if its impulse response sequence $h[n]$ is absolutely summable, i.e,*

$$S = \sum_{n=-\infty}^{\infty} |h[n]| < \infty \tag{0.0.19}$$

Using (3) and (2)

$$S = \sum_{n=-\infty}^{\infty} |-5^n u[-n-1]| \tag{0.0.20}$$

$$= \sum_{n=-\infty}^{-1} 5^n + \sum_{n=-1}^{\infty} 0 \times 5^n \tag{0.0.21}$$

$$= -\sum_{n=1}^{\infty} 5^{-n} \tag{0.0.22}$$

$$= 1 - \sum_{n=0}^{\infty} (5^{-1})^n \tag{0.0.23}$$

Using the formula for the sum of an infinite GP, we get:

$$S = 1 - \frac{1}{1 - 5^{-1}} \tag{0.0.24}$$

$$= \frac{-1}{4} < \infty \tag{0.0.25}$$

So for a bounded input we get bounded output. Thus it is a stable system.
Thus option 2) is correct