EE3900 Assignment-1

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PROBLEM VECTORS Q.2.28

Verify if $A = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, B = \begin{pmatrix} 6 \\ 4 \end{pmatrix}, C = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$ are points on a line.

SOLUTION

The direction vectors of AB and BC are

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \tag{0.0.1}$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \tag{0.0.2}$$

If $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ form a line, then, AB and BC should have the same direction vector. Hence, there exists a k such that

$$\mathbf{B} - \mathbf{A} = k \times (\mathbf{C} - \mathbf{B}) \tag{0.0.3}$$

$$\begin{pmatrix} 3 \\ 3 \end{pmatrix} = k \times \begin{pmatrix} 2 \\ 2 \end{pmatrix} \tag{0.0.4}$$

$$\implies k = \frac{2}{3} \tag{0.0.5}$$

Thus the points A, B, C are collinear and forms a line.
An alternative method is to create the matrix

$$\mathbf{M} = \begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix}^T \tag{0.0.6}$$

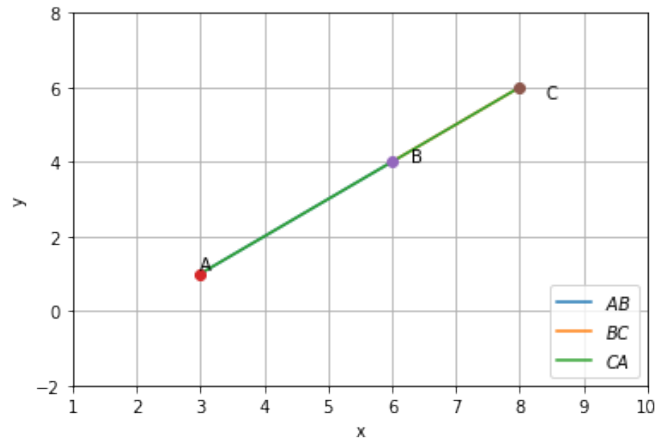If rank(M)=1, the points are collinear. The rank of a matrix is the number of nonzero rows left after doing row operations. In this problem,

$$\mathbf{M} = \begin{pmatrix} 3 & 3 \\ 5 & 5 \end{pmatrix} \xleftarrow{R_1 \to R_1 - \frac{3R_2}{5}} \begin{pmatrix} 0 & 0 \\ 5 & 5 \end{pmatrix} \tag{0.0.7}$$

$$\implies rank(\mathbf{M}) = 1 \tag{0.0.8}$$

as the number of nonzero rows is 1.
Thus the points A, B, C are collinear and forms a line.

Fig. 0: Plot of vectors