

EE3900 Gate Assignment-3

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Download all python codes from

<https://github.com/vrahul02/EE3900/tree/main/Gate-Assignment-3/Codes>

and latex-tikz codes from

<https://github.com/vrahul02/EE3900/tree/main/Gate-Assignment-3/Gate-Assignment-3.tex>

PROBLEM GATE EC-2002 Q.2.4

If the impulse response of a discrete-time system is $h[n] = -5^n u[-n - 1]$, then the system function $H(z)$ is equal to

- 1) $\frac{-z}{z-5}$ and the system is stable
- 2) $\frac{z}{z-5}$ and the system is stable
- 3) $\frac{-z}{z-5}$ and the system is unstable
- 4) $\frac{z}{z-5}$ and the system is unstable

SOLUTION

Definition 1. The z -transform of a function is defined as

$$h[n] \stackrel{Z}{\rightleftharpoons} H(z) \quad (0.0.1)$$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} \quad (0.0.2)$$

Definition 2. The $u[n]$ function is defined as

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.3)$$

Lemma 0.1. If $h[n] = a^n u[n]$, then $h[n] \stackrel{Z}{\rightleftharpoons} H[z] = \frac{1}{1-az^{-1}}$ with $ROC = |z| > a$

Proof. Using the formula for the sum of an infinite GP, we get:

$$h[n] = \begin{cases} a^n & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.4)$$

$$\mathcal{Z}\{h[n]\} = H[z] = \sum_{n=-\infty}^{\infty} h[n]z^{-n} \quad (0.0.5)$$

$$= \sum_{n=-\infty}^0 0 \times z^{-n} + \sum_{n=0}^{\infty} (az^{-1})^n \quad (0.0.6)$$

$$= \frac{1}{1-az^{-1}}, ROC = |az^{-1}| < 1 \quad (0.0.7)$$

$$= \frac{1}{1-az^{-1}}, ROC = |z| > a \quad (0.0.8)$$

□

Lemma 0.2. If $h[n] = -a^n u[-n - 1]$, then $h[n] \stackrel{Z}{\rightleftharpoons} H[z] = \frac{1}{1-az^{-1}}$ with $ROC = |z| < a$

Proof. Using the formula for the sum of an infinite GP, we get:

$$h[n] = \begin{cases} -a^n & n \leq -1 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.9)$$

$$\mathcal{Z}\{h[n]\} = H[z] = \sum_{n=-\infty}^{\infty} h[n]z^{-n} \quad (0.0.10)$$

$$= \sum_{n=-\infty}^{\infty} \{-a^n u[-n - 1]\} z^{-n} \quad (0.0.11)$$

$$= \sum_{n=-\infty}^{-1} -a^n z^{-n} + \sum_{n=-1}^{\infty} 0 \times z^{-n} \quad (0.0.12)$$

$$= - \sum_{n=1}^{\infty} (a^{-1}z)^n = 1 - \sum_{n=0}^{\infty} (a^{-1}z)^n \quad (0.0.13)$$

$$= 1 - \frac{1}{1-a^{-1}z}, ROC = |a^{-1}z| < 1 \quad (0.0.14)$$

$$H[z] = \frac{1}{1-az^{-1}}, ROC = |z| < a \quad (0.0.15)$$

□

We are given $h[n]$ as,

$$h[n] = -5^n u[-n - 1] \quad (0.0.16)$$

Then the z -transform of $h[n]$ using (0.1) and (0.2) is,

$$H(z) = \frac{1}{1 - 5z^{-1}}, |z| < 5 \quad (0.0.17)$$

$$H(z) = \frac{z}{z - 5}, |z| < 5 \quad (0.0.18)$$

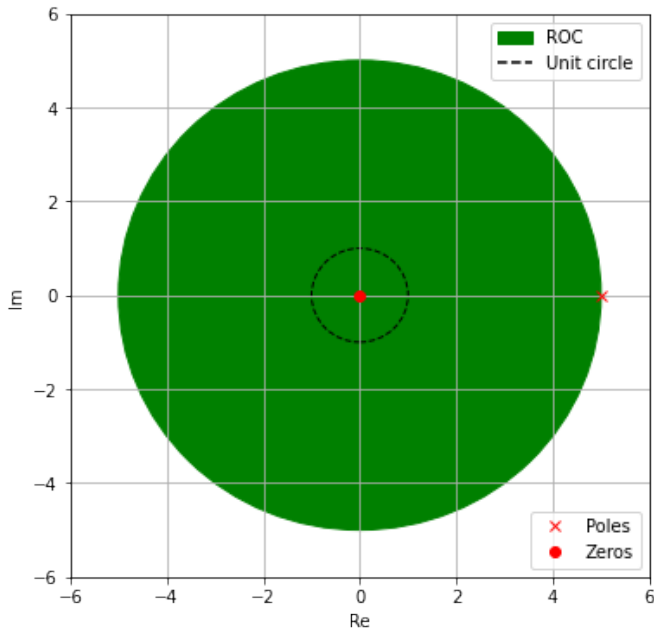


Fig. 4: Pole-zero plot of the system

The system is stable because ROC includes unit circle

Thus option 2) is correct