

# EE3900 Assignment-3

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Download all python codes from

<https://github.com/vrahul02/EE3900/tree/main/Assignment-3/Codes>

and latex-tikz codes from

<https://github.com/vrahul02/EE3900/tree/main/Assignment-3/Assignment-3.tex>

## PROBLEM RAMSEY TANGENT AND NORMAL Q.20

Find the condition that the line  
 $(l \ m)\mathbf{x} + n = 0$  should touch the circle  
 $\left\|x - \begin{pmatrix} a \\ b \end{pmatrix}\right\| = r.$

### SOLUTION

The vector equation of a line can be expressed as

$$\mathbf{x} = \mathbf{q} + \mu \mathbf{m} \quad (0.0.1)$$

If  $\mathbf{n}$  is the normal vector of a line, equation of that line can be written as

$$\mathbf{n}^T \mathbf{x} = c \quad (0.0.2)$$

Here

$$\mathbf{n} = \begin{pmatrix} l \\ m \end{pmatrix} \quad (0.0.3)$$

$$c = -n \quad (0.0.4)$$

The general equation of a second degree can be expressed as :

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (0.0.5)$$

Here

$$\mathbf{u} = \begin{pmatrix} -a \\ -b \end{pmatrix} \quad (0.0.6)$$

$$f = a^2 + b^2 - r^2 \quad (0.0.7)$$

The point of contact  $\mathbf{q}$  of a line with a normal vector  $\mathbf{n}$  to the conic in (0.0.5) is given by:

$$\mathbf{q} = \mathbf{V}^{-1} (\kappa \mathbf{n} - \mathbf{u}) \quad (0.0.8)$$

$$\kappa = \pm \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\mathbf{n}^T \mathbf{V}^{-1} \mathbf{n}}} \quad (0.0.9)$$

We know that, for a circle,

$$\mathbf{V} = \mathbf{I} \quad (0.0.10)$$

and from the properties of an Identity matrix,

$$\mathbf{I}^{-1} = \mathbf{I} \quad (0.0.11)$$

$$\mathbf{I} \mathbf{x} = \mathbf{x} \quad (0.0.12)$$

Solving for the point of contact using the above equations we get,

$$\kappa = \pm \sqrt{\frac{(-a \ -b) \begin{pmatrix} -a \\ -b \end{pmatrix} - a^2 - b^2 + r^2}{(l \ m) \begin{pmatrix} l \\ m \end{pmatrix}}} \quad (0.0.13)$$

$$= \pm \frac{r}{\sqrt{l^2 + m^2}} \quad (0.0.14)$$

$$\mathbf{q} = \pm \frac{r}{\sqrt{l^2 + m^2}} \begin{pmatrix} l \\ m \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} \quad (0.0.15)$$

$$(0.0.16)$$

Since point of contact  $\mathbf{q}$  lies on tangent it satisfies the line equation of tangent

$$\mathbf{n}^T \mathbf{q} = c \quad (0.0.17)$$

$$(l \ m) \times \left( \pm \frac{r}{\sqrt{l^2 + m^2}} \begin{pmatrix} l \\ m \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} \right) = -n \quad (0.0.18)$$

$$\pm r \sqrt{l^2 + m^2} + al + bm = -n \quad (0.0.19)$$

$$\pm r \sqrt{l^2 + m^2} = -n - al - bm \quad (0.0.20)$$

On squaring both sides

$$r^2(l^2 + m^2) = (n + al + bm)^2 \quad (0.0.21)$$

This is the condition of tangency