Mathematics

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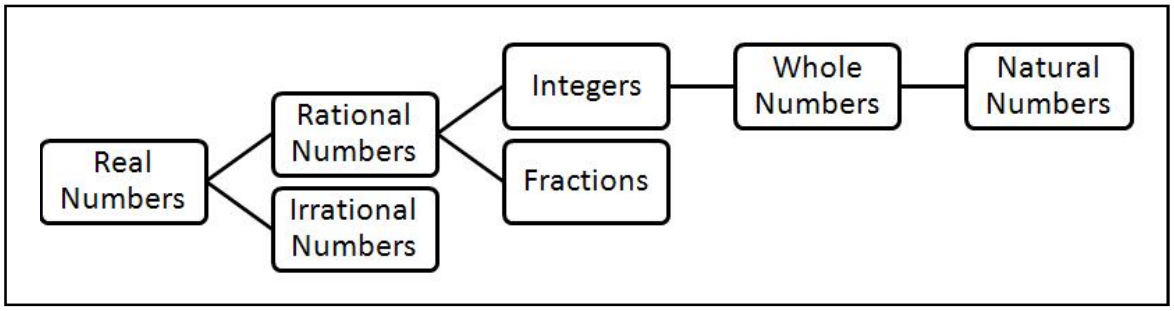
1. NUMBER SYSTEM

A numeral is a symbol that represents a particular quantity. The most commonly used numeral system is the Hindu Arabic numeral system which has numerals like 0,1,2,3,4,5, 6,7,8 and 9. If you remember from your early school years, one other numeral system is Roman numeral system which has numerals like I, V, X, C etc. A number is a combination of numerals. For eg. 34 is a number made by the numerals 3 and 4. Lets understand the different type of numbers through the Number Tree in the following figure.

Number Tree:



Natural numbers: Set of all numbers that can be counted from 1 to positive infinity. These are also called the set of positive integers.(1,2,3,....., ∞)

Whole numbers: Set of all integers from 0 to positive infinity. They are also called the set of non negative integers.(1,2,3, ..., ∞)

Integers: Set of numbers that can be represented on the number line, both positive and negative including 0, which do not have a decimal component. You can also say they are rational numbers which do not have any decimal part like -3, -4, 0, 5, 7 etc

Rational Numbers:

All numbers that can be expressed in the form $\frac{p}{q}$, where p, q are integers and $q \neq 0$.

Eg: $\frac{-5}{7}$, $1\frac{2}{3}$

If $p < q$, it is a proper fraction and
if $p > q$, it is an improper fraction.
An improper fraction can be written as an integer along with a fraction, which is nothing but the quotient followed by the remainder/divisor. This representation is called a mixed fraction.

Eg: $8/3 = 2\frac{2}{3}$

Irrational Numbers:
Those numbers that CANNOT be expressed in the form $\frac{p}{q}$.

Eg: pi, $\sqrt{2}, \sqrt{3}$ +1

Real Numbers :
All the above numbers which can be commonly seen and identified and can be represented on a number line.
Eg: - 10, 2.77, 0, 1, 7
Certain types of natural numbers have special significance.

Even numbers: Numbers which are divisible by 2.
Eg: 2, 4, 6 etc. Whenever you need to denote an even number, you could consider it to be 2n.

Odd numbers: Numbers which are not divisible by 2.
Eg: 13, 67 etc. Whenever you need to denote an odd number, you could consider it to be 2n + 1 or 2n - 1.

Factor: A number which completely divides a given number without leaving a remainder.
Eg: 3 is a factor of 6.

Prime numbers: Numbers which have exactly two distinct factors - 1 and the number itself.
Eg: 3, 5 and 7 etc.

Composite numbers: Numbers which have more than 2 factors.
Eg: 15, 66 etc

Note:
(i) 1 is neither a prime number nor a composite number.
(ii) 2 is the only even number which is prime.
(iii) There are 15 prime numbers from 1 to 50. They are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,37,41,43 and 47.
(iv) There are 10 prime numbers from 51 to 100. They are 53, 59, 61, 67, 71, 73, 79, 83, 89, 97. So there are totally 25 prime numbers from 1 to 100.

Relations betweeen odd and even numbers:
odd ± odd = even
odd ± even = odd
even ± even = even
odd * odd = odd
odd * even = even
even * even = even

Place Value:
In the Decimal System a number 34732 can be represented as
3 x 10000 + 4 x 1000 + 7 x 100 + 3 x 10 + 2 x 1= 34732
So, what is the place value of 7 in the above number?
Since, 7 is in the hundred's place, it can be represented as
7 x 100 = 700 which is the place value.

Divisibility Rules
Let us understand how to check whether a given number is divisible by any of the below factors.
by 2 : If the last digit of a number is divisible by 2 or is 0, then the number is divisible by 2.
Eg: 48, 2380, 13456 etc.

by 3: Add all the digits of the number. If the sum is divisible by 3, then the number is divisible by 3.
Eg: Take a number 2347. The sum of the digits of this number is equal to 16. Thus the number is not divisible by 3 as 16 is not divisible by3.

by 4: If the last two digits of the number are divisible by 4 or are 00, then the number is divisible by 4.
Eg: 84, 1660, 12756 etc.

by 5: If the last digit of the number is divisible by 5 i.e. if the last digit is either 0 or 5, then the number is divisible by 5.
Eg: 85, 60 etc.

by 6: If the number is divisible by 2 and 3, then the number is divisible by 6.
Eg: A number like 144 is divisible by both 2 and 3 and thus is divisible by 6 too.

by 8: If the last three digits of a number are divisible by 8 or are 000, then the number is divisible by 8.
Eg: 3064, 43128, 146512 etc.

by 9: If the sum of the digits of the number is divisible by 9, then the number is divisible by 9.
Eg: Take number 2547. The sum of the digits of this number is equal to 18. Thus the number is divisible by9 as18 is divisible by 9.

by 11: A number is divisible by 11, if the difference between the sum of the digits in the even positions and the sum of the digits in the odd positions is either 0 or a multiple of 11.
Eg: 6393189 is divisible by 11 because the difference between the sum of the digits in the even places and the odd places is equal to 25-14 = 11 which is a multiple of 11.

by 12: If the number is divisible by 3 and 4, then the number is divisible by 12. You need to note that though 2 and 6 are also factors of 12, it is not enough if the number is divisible by 2 and 6. You need to check the divisibility by mutually prime factors - numbers which do not have any factor in common.
Eg: 45636, 832272, etc

On the same lines, check divisibility by 8 and 3 to find divisibility by 24.

LCM&HCF:
HCF stands for Highest Common Factor (also called the GCD- Greatest common divisor) of given numbers. LCM stands for the Least Common Multiple of given numbers.

Let us first understand what the term multiple means before proceeding further.

Multiple:
Multiple of a number is derived by multiplying the given number by a positive integer.
For example,
Multiples of 25 = 25, 50, 75, 100, etc.
Multiples of 60 = 60, 120, 180, 240, etc.

As we know
Factors of 60 = 60, 30, 20, 15, 12, 10, 6, 5, 4, 3, 2, 1

LCM of two numbers 24 and 36:
Multiples of 24: 24, 48, **72**, 116, ...
Multiples of 36: 36, **72**, 108, 144, ...
LCM of two numbers is the least or smallest common multiple, i.e. 72 in this case.

HCF of two number 24 and 36:
Factors of 24: 1, 2, 3, 4, 6, 8, **12**, 24
Factors of 36: 1, 2, 3, 4, 6, 9, **12**, 18, 36
HCF of two numbers is their highest common factor, ie. 12 in this case.

There are two commonly used methods apart from the basic definition to find the HCF and LCM of two or more numbers.

Division Method:
This method is most often used when the given numbers are large.
LCM:
Start dividing the numbers by a common prime factor. Next divide the quotients by their common factor. Continue this process till all the numbers reduce to 1. The LCM of the numbers is then the product of all the prime numbers obtained and the common factors used to divide.
Example: LCM of 24 and 36
Divide 24, 36 by 2, You will get
12, 18. Next divide by 2. You get
6, 9 Next divide by 3. You get
2, 3 Next divide by 2. You get
1, 3 Finally dividing by 3, you get
1, 1
Note that 2 does not divide 3 completely, so let the prime number 3 carry forward as it is. This is the same with all the problems where you calculate the LCM of two or more numbers.

Thus, as per the method mentioned above, the LCM of 24 and 36 = 2x2x3x2x3 = 72.

HCF:
Divide the larger number by the smaller number. Next, the Divisor becomes the Dividend and remainder becomes the divisor. Continue this process until the remainder is zero. The last divisor is the HCF of the two numbers.

HCF of 24 and 36:

$$\begin{array}{r|l|l} 24 & 36 & 1 \\ & -24 & \\ \hline & 12 & 24 \quad 2 \\ & & -24 \\ \hline & & (0) \end{array}$$

So, the HCF of the two numbers 24 and 36 is 12.
Note: If the HCF of two numbers is 1, then the two numbers are said to be co-prime numbers or relatively prime numbers.

Prime factorisation method:
LCM :
Express the two numbers as products of prime factors.
Take the highest power of each prime number across both the numbers and find their product. That gives the LCM of the two numbers.
Example: Let us find the LCM of 24 and 36.
$24 = 3x2^3$
$36 = 3^2x2^2$

Thus, the LCM of the two numbers is $= 3^2x2^3 = 72$.

HCF

Express both the numbers in terms of the products of their prime factors. Now take the lowest power of each prime number across both the numbers and find their product.

Let us find the HCF of 24 and 36.

$24 = 3x2^3$

$36 = 3^2x2^2$

Thus the HCF of the two numbers is $2^2x3 = 12$.

Note:

1) For any two given numbers (A and B), the product of the numbers is equal to the product of their LCM and HCF.

   A x B = LCM x HCF.

   However, do note that the HCF is also a factor of the LCM since the HCF is a factor of each of the numbers whose multiple is the LCM.

2) LCM and HCF of fractions:

$$\text{LCM of fractions} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

$$\text{HCF of fractions} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$$

Note: This will work only when the fractions are in their simplest form. If not, bring the fractions to their simplest form before applying this formula.

Eg: Let us find the HCF and LCM of 2/5, 3/8 and 4/5.

Solution:

LCM is LCM(2,3,4)/HCF(5,8,5) =12/1=12

HCF is HCF(2,3,4)/LCM(5,8,5) =1/40

Important Identities & Formulae:

(1) $(a+b)^2 = a^2 + b^2 + 2ab$

(2) $(a-b)^2 = a^2 + b^2 - 2ab$

(3) $(a+b)^2 = (a-b)^2 + 4ab$

(4) $a^2 - b^2 = (a-b)(a+b)$

(5) $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

(6) $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(7) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

(8) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

(9) Sum of first n natural numbers $= \frac{n(n+1)}{2}$

(10) Sum of first n odd numbers $= n^2$

(11) Sum of first n even numbers $= n(n+1)$

(12) Sum of the squares of first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$

(13) Sum of the cubes of first n natural numbers $= \left[\frac{n(n+1)}{2}\right]^2$

(14) Following are some important formaulae for Indices

1. a^m x a^n = a^{m+n}	**2. $(a^m)^n$ = a^{mn}**
3. a^m / a^n = a^{m-n}	**4. a^{-m} = 1/a^m**
5. a^0 = 1	**6. $(ab)^m$ = $a^m b^m$**
7. $\sqrt[m]{a}$ = $a^{1/m}$	**8. $\sqrt[m]{a}^n$ = $a^{n/m}$**

SOLVED EXAMPLES

1. **Arrange the following fractions in descending order** $\frac{26}{23}, \frac{30}{27}, \frac{38}{35}, \frac{19}{16} \& \frac{22}{19}$

   Solution:

   Note that the difference between the numerator and the denominator is same for all the fractions. So, the fraction with the smallest denominator, i.e. $\frac{19}{16}$ is largest and $\frac{38}{35}$ is smallest.

   Hence, the order is: $\frac{19}{16}, \frac{22}{19}, \frac{26}{23}, \frac{30}{27}, \frac{38}{35}$

2. **What least number must be subracted from 1500 to get a number which is exactly divisible by 13?**

   Solution:

   Dividing 2000 by 13, we get 11 as remainder. If 11 is subtracted from 2000, the resulting number will be exactly divisible by 13.

3. **What least number must be added to 5000 to obtain a number exactly divisible by 23?**

   Solution:

   Dividing 5000 by 23, we get 9 as remainder. To obtain the next multiple of 23, we need to add (23 - 9) i.e. 14.

4. **The traffic lights at three different road-crossings change after every 48 sec, 72 sec and 96 sec respectively. If they all change simultaneously at 8am, then at what time will they change next simultaneously?**

   Solution:

   Interval of change = LCM of (48, 72, 96) sec = 288 sec.

   The lights will change simultaneously after every 288sec i.e. 4 min 48 sec.

   Next simultaneous change will take place at 8:04:48 am

5. **The HCF of two numbers is 15 and their LCM is 180. If one of the numbers is 60, then find the other.**

   Solution:

   We know that product of the two numbers = product of HCF and LCM.

   The other number $= \frac{15 \times 180}{60} = 45$

6. **Simplify** $\frac{57 \times 57 \times 57 + 18 \times 18 \times 18}{57 \times 57 - 57 \times 18 + 18 \times 18}$

   Solution:

   The given expression is equivalent to

   $$\frac{(57)^3 + (18)^3}{(57)^2 - 57 \times 18 + (18)^2}$$

   We know that, $\frac{a^3 + b^3}{a^2 - ab + b^2} = a + b$

   In the above example a = 57 and b = 18

   $\therefore$ The expression is equal to (57 + 18)=75.

7. **The sum of the squares of first twelve natural numbers is**

Solution:

$$1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$\therefore 1^2 + 2^2 + ... + 12^2 = \frac{12(13)(25)}{6} = 650.$$

8. **If $(6)^{15}x(10)^5x(15)^6 = 2^p x 3^q x 5^r$, then find the value of p+q+r.**

Solution:

We know that 6 = 2x3 ; 10 = 2x5 and 15 = 3x5 and we know that $(ab)^m = a^m b^m$ Thus $(6)^{15}x(10)^5x(15)^6 = (2x3)^{15}x\ (2x5)^5x\ (3x5)^6 = 2^{15} x\ 3^{15} x\ 2^5 x\ 5^5 x\ 3^6 x\ 5^6$ and we also know that $a^m x\ a^n = a^{m+n}$.

So, $(6)^{15}x(10)^5x(15)^6 = 2^{15+5} x\ 3^{15+6} x\ 5^{5+6} = 2^{20} x\ 3^{21} x\ 5^{11}$.

Thus p+q+ r = 20+21+11 = 52

9. **Find the number which is nearest to 3105 and exactly divisible by 21.**

Solution:

On dividing 3105 by 21, we get 18 as remainder.

Since 21 - 18 = 3, the nearest number to 3105 is obtained by adding 3.

So 3105 + 3 = 3108.

10. **Which of the following is a prime number?**

A. 279 B.111 C. 343 D. 797

Solution:

Clearly, 279 and 111 are divisible by 3, so they are not prime. 343 is divisible by 7. But 797 is not divisible by any number below 30 which is the nearest square root to 797. Hence, the prime number is 797.

* * *

PRACTICE EXERCISE

1. The greatest fraction among :

$\frac{2}{7}, \frac{3}{7}, \frac{1}{7}, \frac{8}{21}$ and $\frac{6}{7}$ is

A. $\frac{6}{7}$ B. $\frac{8}{21}$

C. $\frac{1}{7}$ D. $\frac{3}{7}$

2. The value of $\sqrt[3]{9} \times \sqrt[3]{192}$ is

A. 92 B. 19

C. 12 D. 30

3. If $x = 8 + \sqrt{63}$, then the reciprocal of x is

A. $\frac{1}{8-\sqrt{63}}$ B. $8 - \sqrt{63}$

C. $\sqrt{63}$ D. 16

4. The value of $\sqrt{3\sqrt{3\sqrt{3.....\infty}}}$ is

A. 0 B. 1

C. $3\sqrt{3}$ D. 3

5. Find the value of $\frac{3^{n+1} - 3^n}{3^n + 3^{n-1}}$

A. $\frac{2}{3}$ B. $\frac{3}{2}$

C. $\frac{1}{2}$ D. 2

6. If $x^2 + \frac{1}{x^2} = 34$, then $x + \frac{1}{x}$ is

A. +6 B. -6
C. 36 D. Both A & B

7. Sum of first 50 natural numbers is

A. 2550 B. 1275
C.1460 D. 2750

8. Find the LCM and HCF of the following

(i) 37,222
(ii) 64,232
(iii) 28,624

9. What is the HCF of $2\frac{2}{5}, 3\frac{3}{7}$ and $2\frac{2}{9}$?

A. $\frac{4}{315}$ B. $\frac{4}{75}$

C. $\frac{315}{4}$ D. None of these

10. HCF of two numbers is equal to 15. If the sum of the two numbers is equal to 120, how many pairs of these two numbers are possible.

A. 1 B. 2
C. 3 D. 4

11. The sum of three prime numbers is 40. If one of them exceeds another by 24, then one of the numbers is

A. 3 B. 37
C. 31 D. 29

12. What least value must be assigned to* so that the number 78236 * 2 is divisible by 8?

A. 1 B. 2
C. 3 D. 4

13. The number nearest to 98217 which is exactly divisible by 537 is

A. 99345 B. 98271
C. 98324 D. 98571

14. The value of $(256)^{0.16} \times (256)^{0.09}$ is

A. 56 B. 16
C. 4 D. 25

15. Which one of the following when divided by 17 gives the quotient 17 and the remainder 7?

A. 289 B. 296
C. 313 D. 292

16. When n is divided by 5, the remainder is 2. What is the remainder when 3n is divided by 5?

A. 2 B. 1
C. 3 D. 4

17. What is the units digit of $(16)^{250}$?

A. 0 B. 5
C. 6 D. 2

18. How many numbers upto 1000 (excluding it) are divisble by 4, 5 and 8.

A. 24 B. 25
C. 26 D. None

19. The product of 4 prime numbers is 910. The sum of these prime numbers is
    A. 27 B. 25
    C. 26 D. None

20. Find the sum of the powers of prime factors in the expression $12^{10} \times 5^{15} \times 7^{19}$?
    A. 44 B. 64
    C. 28 D. 22

21. $16^5 + 2^{15}$ is divisible by
    A. 31 B. 13
    C. 27 D. 33

22. A number when divided by 185 leaves the remainder 74. What is the remainder if it is divided by 37?
    A. 7 B. 0
    C. 2 D. 5

23. What is the remainder when 98786736 is divided by 18
    A. 3 B. 0
    C. 9 D. 13

24. Find the sum of all even numbers from 1 to 100.
    A. 1250 B. 2500
    C. 2550 D. 3250

25. Find the sum of all odd numbers upto 50.
    A. 875 B. 625
    C. 490 D. 525

* * *

2. AVERAGES

Average is a statistical measure. It is a value that represents a group of observations. An average is the sum of all the observations divided by the total number of observations.

$$\text{Average} = \frac{\text{Sum of all Observations}}{\text{Total Number of Observations}}$$

For example, if the runs scored by Ajinkya Rahane in the recent series are 23, 34, 45, 0 and 68, his average score in the series is = 170/5 = 34. The average is also called the Mean.

Weighted Average: Weighted average is used when two or more groups whose individual averages are known are combined together. In this case, each group may have different number of members, so their weights are different!!

Suppose, in a class of NALSAR students, 20 people score 25 marks, 15 people score 35 marks and another 15 score 40 marks in the subject Basics of Patent Laws, The average score of the class is given by

$$\text{Average} = \frac{20\times 25+15\times 35+15\times 40}{20+15+15}$$

$$= \frac{500+525+600}{50} = 32.5$$

In this particular case, the numbers 20, 15 and 15 are called the weights of the averages 25, 35 and 40 respectively. The average thus obtained is called the weighted average.

SOLVED EXAMPLES

1. **The average age of the boys at Abhyaas Law Prep is 20 years and the average age of the girls of the same class is 18 years. What is the average age of the entire class, if the ratio of boys and girls is 2 : 3?**

Solution:

$$\text{Average age} = \frac{\text{Total Age of all students}}{\text{Total number of students}}$$

$$= \frac{20\times 2+18\times 3}{2+3} = \frac{94}{5} = 18.8 \text{ years}$$

2. **In Colony A, if there are 3 dogs on an average in each family and Colony B has 4 dogs per family. what is the average number of dogs per family across both the colonies if there are 40 families in Colony A and 60 families in Colony B?**

Solution:

The ratio of the families in Colony A : Colony B is 2 : 3.

Average no. of dogs per family

$$= \frac{\text{Total no. of dogs}}{\text{Total no. of families}}$$

$$= \frac{3\times 2+4\times 3}{2+3} = \frac{18}{5} = 3.6$$

3. **If the monthly salaries of five friends in a company are 40, 45, 55, 60 and 40 thousands respectively, then what is their average salary?**
Solution:

$$\text{Average} = \frac{40+45+55+60+40}{5}$$

$$= \frac{240}{5} = 44$$

The average salary is 44 thousands.

4. **The average of 12 positive integers is X. If each integer is increased by 70%, the average is increased by:**
Solution.

Let 12 positive integers be $X_1, X_2, X_3, ... X_{12}$.
According to the given condition:

New integers are $1.7X_1, 1.7X_2, 1.7X_3, ... 1.7X_{12}$

New average

$$= \frac{1.7X_1 + 1.7X_2 + 1.7X_3 + ... + 1.7X_{16}}{12} = 1.7X$$

Increase in average = 1.7X - X = 0.7X i.e. 70%

Note: When each member of the group is increased or decreased by a certain value, then the average also increases or decreased by the same value.

5. **Krishnam goes to Hyderabad from Indore at a speed of 50km/hr and returns with a speed of 60 km/hr. What is her average speed during the whole journey?**
Solution:
Let the distance between Indore and Hyderabad be 'd' kms.

Time taken from Indore to Hyderabad

$$= \left(\frac{d}{50}\right) \text{ hours}$$

Time taken from Hyderabad to Indore

$$n\left(\frac{n}{nn}\right) \text{ hours}$$

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time consumed}}$$

$$= \frac{d+d}{\frac{d}{50}+\frac{d}{60}}. = \frac{2\times 50\times 60}{50+60} = 54.5 \text{ km / hr}$$

6. **Rahul Dravid makes a score of 87 runs in his 14th innings and thus increases his average by 3. Find his average after 14th innings.**
Solution:
Let the average after 13 innings be 'A'. Then, average after 14 innings = A + 3

$$\left(\frac{13A+87}{14}\right) = (A+3) \Rightarrow A = 45$$

Average after 14 innings = A+3 = 45 + 3 = 48

7. **The average weight of a group of 16 students is 48 kgs. When a student is replaced with a teacher, the average weight is increased by 5 kgs. What is the difference between the weight of the student and the teacher?**
Solution:
Let the weight of student and the teacher be S kgs and T kgs respectively.
According to the given condition:
Total weight of 16 students = $16 \times 48 = 768$ kgs
Total weight of 15 students and a teacher

= 768 - S + T = 768 +(T - S) kgs

$\therefore 48+5=\frac{\{768+(T-S)\}}{16}$

$\Rightarrow 16\times 53=768+(T-S)$

$\Rightarrow T-S=848-768=80$ kgs.

The same problem can be solved in a simpler manner using a single step. The difference in weight between the student and teacher has contributed to a difference of 5 in the average weight. So the difference is 5 * 16 = 80 Kgs

8. **The captain of a hockey team of 11 members is 26 years old and the goal keeper is 3 years older than him. If the ages of these two are excluded, the average age of the remaining players is one year less than the average age of the whole team. What is the average age of the team?**

Solution:

Let the average age of the whole team be x years.

$\therefore$ 11x - (26 + 29) = 9(x -1)

$\Rightarrow$ 11x - 9x = 46

$\Rightarrow$ 2x = 46

$\Rightarrow$ x = 23.

So, average age of the team is 23 years.

9. **The average monthly income of Pallavi and Kia is Rs 5050. The average monthly income of Kia and Reshma is Rs. 6250 and the average monthly income of Pallavi and Reshma is Rs. 5200. The monthly income of Pallavi is:**

Solution:

We have,

Pallavi + Kia = (5050 x 2) = 10100 .... (i)

Kia + Reshma = (6250 x 2) = 12500 .... (ii)

Pallavi + Reshma = (5200 x 2) = 10400 .... (iii)

Adding (i), (ii) and (iii), we get

2(Pallavi + Kia + Reshma) = 33000 or Pallavi + Kia + Reshma = 16500 .... (iv)

Subtracting (ii) from (iv), we get

Pallavi = 4000.

$\therefore$ Pallavi's monthly income = Rs. 4000.

10. **The average height of 8 persons increases by 1.5 inches when a new person comes in place of one of them of height 65 inches. What might be the height of the new person?**

Solution:

Total increase in height = 8 x 1.5 inches = 12 inches.

Height of new person = 65 + 12 inches = 77 inches.

* * *

PRACTICE EXERCISE

1. What is the average of all even numbers between 3 and 25.
   A. 12 B. 13
   C. 15 D. 14

2. What is the arithmetic mean of the given set of data ? {12, 14, 13, 8, 7, 4, 6, 2 and 15}
   A. 8 B. 9
   C. 7 D. 11

3. What is the average of first 27 natural numbers?
   A. 15 B. 14
   C. 14.5 D. 15.5

4. If the average of 5,20,14,16,9, x ,13 and 8 is 12, what is the value of ' x '?
   A. 12 B. 11
   C. 13 D. 15

5. The average of first 13 multiples of 13 is
   A. 117 B. 65
   C. 78 D. 91

6. The average of seven observations was found to be 35. Later on, it was detected that among the given observations 50 was misread as 15. What is the correct average?
   A. 45 B. 38
   C. 40 D. 42

7. The sum of certain group of numbers is 4,680. If the average of the group is 39, find how many numbers are there in the group?
   A. 110 B. 115
   C. 120 D. 125

8. The average of seven numbers is 26. If four numbers are 22, 30, 32, and 20 and the remaining three numbers are consecutive, which is the lowest of the consecutive numbers?
   A. 25 B. 26
   C. 24 D. 27

9. Eight friends go for a dinner buffet at the newly launched Chutneys restaurant in Madhapur. Six of them bring Rs. 300 each. The seventh person brings in Rs.100 more than the average of eight persons and the eighth person brings Rs. 900. What is the total sum brought in for the dinner buffet?
   A. Rs. 3200 B. Rs. 2400
   C. Rs. 3000 D. Rs. 2800

10. If the average of a, a^{-1} is 'N', then the average of a^2, a^{-2} is :
    A. N^2 B. $2N^2 - 1$
    C. $2N^2$ D. $2N$

11. The average of 'A' numbers is equal to the average of 'A+1' numbers. Hence, the A+1th number is:
    A. 0
    B. A
    C. the average of A numbers
    D. twice the average of A numbers.

12. The average of 5 quantities is 10 and the average of 3 of them is 9. What is the average of the remaining 2?
    A. 11 B. 12
    C. 11.5 D. 12.5

13. The average weight of a class of 40 students is 45 kgs, Average weight of 25 boys in that class is 48kgs. What is the average weight of the 15 girls in the class?
A. 44 kgs B. 42 kgs
C. 39 kgs D. 40 kgs

14. The average height of 50 girls was 54 inches. Two new girls with heights 65 inches and 69 inches joined the group. What is the average height of 52 girls?
A. 54.5 inches B. 55 inches
C. 54 inches D. 53.5 inches

15. The average of seven numbers is 40. The total of four of them is 178. The remaining three numbers are in the ratio of 1 : 2 : 3. These three numbers are:
A. 15,30,45 B. 16,32,48
C. 17,34,51 D. 18,36,54

16. In a class there are X boys with an average age of 18 years. One boy of 17 years leaves the class and another boy of 11 years joins. If the average age now is 16 years, then what is the value of X?
A. 3 B. 5
C. 6 D. 8

17. The average marks obtained by 21 students in a patents law examination are 45. The average marks of first 10 students are 50 and that of the last 10 are 40. How many marks does the 11th student scored?
A. 50 B. 45
C. 0 D. 40

18. The average age of 8 students is 14 years. If the age of the teacher is included, the average age is increased by 5 years. What is the age of the teacher?
A. 59 years B. 67 years
C. 56 years D. 68 years

19. Two years ago, the average age of Tina and Rudrama was 22 years. If Nikhil joins them, their average age at present becomes 26 years. What is the present age of Nikhil?
A. 30 years B. 25 years
C. 20 years D. 24 years

20. In the month of August, the average daily temperature of Warangal is $29.3^{o}C$. What is the total temperature recorded in the town that month?
A. 879^{0}C B. 908.3^{0}C
C. 903.8^{0}C D. 897^{0}C

21. The average marks in CLAT 2010 across India is 93.25. If the average marks of boys are 95 and that of the girls are 90. What is the percentage of boys in all?
A. 25% B. 50%
C. 60% D. 65%

22. The average of p, q, r and s is 24. Three fourths of the sum of p, q and r is 60. Find the value of s?
A. 18 B. 14
C. 20 D. 16

23. The average runs scored by Indian and Australian players is 60 and 54 respectively. If the number of players in each team is 11. What is the average score of all the players?
    A. 57.5 B. 58
    C. 59 D. 57

24. A retailer mixes two varieties of dal. One variety costs Rs. 105 per kilogram and another Rs. 90 per kilogram. What is the average cost of the resulting mixture?
    A. Rs. 92.5
    B. Rs. 90.5
    C. Rs. 97.5
    D. Cannot be determined

25. A formula1 race driver covered a distance of 1280 km in first 8 hours and 520 km in the next 4 hours at the recently launched Buddh circuit in India. What is the average speed of the driver?
    A. 123.5 km/hr B. 150 km/hr
    C. 162.5 km/hr D. 180 km/hr

* * *

6. RATIO, PROPORTION&MIXTURES

Ratio is the relationship that one quantity bears with the other. In simple words, ratio is nothing but a fraction. The ratio of a and b is denoted as **a:b** and is read as **"a is to b"**. A ratio compares two numbers.

Consider the two quantities A and B such that A : B = 3 : 4. The same thing can be expressed in multiple ways.

i. The ratio B : A is 4 : 3.
ii. A is $\frac{3}{4}$th of B.
iii. B is 1.33 times A.
iv. B is 33.33% more than A.
v. A is 25% less than B.

Since a ratio is a comparison of two quantities that have the same units, a ratio is only a number or a fraction without any units. If both the terms of a ratio are multiplied or divided by the same (non-zero) quantity, then the value of the ratio remains the same. In terms of notations.

$\frac{a}{b} = \frac{ka}{kb}$ (Here, k is any non-zero real number.)

Propotion:

The equality of two ratios is called proportion.

a, b, c, d are said to be in proportion if **a:b::c:d.**

In a proportion, the first and fourth terms are known as **extremes,** while the second and third terms are known as **means.**

So in the proportion a:b::c:d, a and d are extremes and b and c are means of the proportion

Note that in any proportion,

Product of extremes =Product of means

ie a* x *d* = *b* x *c

Direct Propotion:

Two variables are said to be *directly proportional* if one is a constant multiple of the other:

$y = kx$

where, *k* is a constant. In other words, when *x* increases, *y* increases and when *x* decreases, *y* also decreases.

When speed is constant, Distance is directly proportion to the time in the relation D = S x T.

Inverse Propotion:

Two variables are said to be *inversely proportional* if the product of both the variables is always a constant

$xy = k$ *(or)* $x = k/y$

In this case, as *x* increases *y* decreases and vice versa.

When Distance is constant, Speed and Time are inversely proportional in the relation S = D/T.

Continued Proportion:

Four quantities a, b, c, d are said to be in a continued proportion, if a: b = b: c = c: d or

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

Three quantities are said to be in continued proportion, if $a:b = b:c$ or $ac = b^2$ In this relationship, b is said to be the mean proportional between a and c and c is said to be a third proportional to a and b.

Mixtures:

Mixtures and Alligations are simple extension of the concept of ratio and proportion and averages.

When a shopkeeper mixes different quantities of two different qualities and prices, the price of the resultant mixture is nothing but the weighted average of the two quantities. The weights here are the quantities.

Suppose W_1 kgs of cheaper quality rice of price R_1 rupees/kg is mixed with W_2 kgs of dearer quality rice of R_2 rupees/kg, then the resultant mean or average price/kg of the rice is given by

$$\text{Mean price} = \frac{W1 \times R1 + W2 \times R2}{W1 + W2}$$

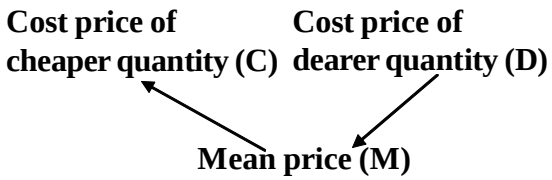
Let us now look at a simpler way to find the ratio in which the cheaper and dearer quantities should be mixed so that the resultant price is equal to a given mean price.

Assume that

Cost price of cheaper quantity = C

Cost price of dearer quantity = D

Mean price = M



The ratio at which the cheaper quantity and the dearer quantity should be mixed to result in a mean price (M) is given by

$W_1 : W_2 = (D-M):(M-C)$

The arrows in the figure suggest an easy way to calculate the ratio.

SOLVED EXAMPLES

1. **Ajay and Bhama got 175 and 225 marks respectively in in their term test. What is the ratio of their marks?**

   Solution: $A : B = \frac{175}{225} = \frac{7}{9} = 7 : 9$

2. **Chintan scored 140 marks out of 200 and Yadav scored 105 marks out of 120. What is the ratio of the percentage marks scored by each?**

   Solution: $C : Y = \frac{140}{200} \div \frac{105}{120} = 4 : 5$

3. **The incomes of Nikhil and Yana are in the ratio 3 : 2 and their expenditures in the ratio of 5 : 3. If each saves Rs. 2,000, then find their incomes?**
   Solution:
   Let 3a, 2a be the incomes, and 5b and 3b be the expenditures of Nikhil and Yana respectively.
   We have 3a - 5b = 2a- 3b = 2000
   Solving, a = 4000, b = 2000
   Nikhil's income = Rs. 12,000
   Yana's income = Rs. 8,000.

4. **5 kg of gram flour is mixed with 500 gm of sugar extract. What is the ratio of sugar extract to the rest of the mixture after adding 1.5 kg of proteinex?**
   Solution :
   We first need to express all quantities in single unit.
   Gram flour = 5 kg
   Proteinex = 1.5 kg
   Sugar extract = 500 gm = 0.5 kg
   Total weight of the mixture = 7 kg
   Ratio of sugar extract to the rest of mixture
   $= \frac{0.5}{6.5} = 1 : 13$

5. **At Davis Polk & Wardwell law firm,the number of employees recently got reduced in the ratio of 15:8 and the wages are increased in the ratio of 14: 25. What is the ratio between the old wage bill and new wage bill?**
   Solution:
   Let the number of employees initially be 15 and the wage per head be Rs. 14
   The old wage bill is $15 \times 14 = 210$.
   Then the number of employees becomes 8 and the wage per head becomes Rs. 25.
   $\Rightarrow$ The new wage bill = Rs. 25×8 = Rs. 200.
   $\Rightarrow$ The ratio of the wage bills = 210 : 200 = 21 : 20
   $\Rightarrow$ Wage bill ratio = 21 : 20.

6. **On a controversial world map, 1.5 inch represents 225 miles. What is the actual distance between two cities if they are 3.5 inches apart on the map?**
   Solution:
   The relationship between the distances on the map is directly proportional to the actual distances on the earth. So,
   1.5 inch = 225 miles
   3.5 inches = 3.5 x 225/1.5 = 525 miles.

7. **From a total of Rs. 159000, Rs. 5000 is to be divided between Rani and Yana in the ratio of 2 : 3. The rest of the money is to be divided among Rani, Yana and Rajitha in the ratio of 5 : 3 : 3. How much money did Rani and Yana get respectively?**

Solution :

Rs. 5,000 is to be divided between Rani and Yana in the ratio of 2 : 3.
Rani's share = Rs. 2,000;
Yana's share = Rs. 3,000

Amount left is 1,54,000 and it is to be divided in the ratio of 5 : 3 : 3.
Rani's share = Rs. 70,000
Yana's share = Rs. 42,000
Rajitha's share = Rs. 42,000
Rani's total share = Rs. 72,000
Yana's total share = Rs. 45,000

8. **A's income is $\frac{2}{3}$rd of B's income. B's income is 75% of C's income. What is the ratio of C's income to A's income?**

Solution:

B's income = $\frac{3}{2}$ of A's income.

C's income = $\frac{4}{3}$ of B's income = $\frac{4}{3}\times\frac{3}{2}$ of A's income

Required ratio = 2 : 1.

9. **The ratio of the number of boys and girls in a college is 7 : 8. If the percentage increase in the number of boys and girls be 20% and 10% respectively, what will be the new ratio of boys and girls?**

Solution:

Originally, let the number of boys and girls in the college be 7x and 8x respectively.
Their increased number is (120% of 7x) and (110% of 8x).

$\Rightarrow \left(\frac{120}{100}\times 7x\right)$ and $\left(\frac{110}{100}\times 8x\right)$

$\Rightarrow \frac{42x}{5}$ and $\frac{44x}{5}$

$\therefore$ The required ratio = $\left(\frac{42x}{5} : \frac{44x}{5}\right)$ =21:22

10. **A class of students attended a charity show with entrance ticket of Rs 3.70 per girl and Rs 4.50 per boy. If the strength of the class and the amount collected from the class are 160 and Rs 640 respectively, find the number of girls and boys from the class (respectively) who attended the show.**

Solution:

Alligation is here applicable to the ticket price of a boy and that of a girl
The Mean(Average) ticket price = 640/ 160=Rs 4 per student

Let the number of boys be B and girls be G, then

B/G = (4 - 3.7)/(4.5-4) =3/5
Now let B= 3k and G=5k
B+G = 3k+5k = 8k =160
Therefore K=20 and
Number of Boys= B= 60
Number of Girls= G= 100

* * *

PRACTICE EXERCISE

1. The diference between two positive numbers is 44 and the ratio between them is 7:3. What is the product of the two numbers
   A. 2121 B. 2561
   C. 2541 D. None

2. Three friends are walking from their school to a swimming pool. Their speeds are in the ratio of 3:6:4. The ratio of time taken to reach the pool by these friends respctively will be
   A. 3:5:4 B. 4:2:3
   C. 20:12:15 D. 20:5:12

3. If a:b=6:7 and b:c=3:5 find a:b:c
   A. 6:21:4 B. 18:21:35
   C. 18:21:20 D. 30:21:35

4. If x:y=11:6 and y:z=8:3 find x:z
   A. 11:3 B. 88:21
   C. 44:9 D.77:24

5. If the ratio of men to women in a tour party of 54 members is 5:1, how many more women need to join the tour party to make this ratio 3:2?
   A. 25 B. 22
   C. 21 D. 26

6. Rs.1800 is distributed among A, B&C in the ratio of 3:5:4. The difference in shares of B&C is
   A. 400 B. 600
   C. 300 D. 150

7. A bag contains 50paise,25paise,10paise coins in the ratio 3:6:5, amounting to Rs.210. Find the number of 10paise coins.
   A. 200 B. 360
   C. 300 D. None

8. What is the third propotional to 13 and 52?
   A. 184 B. 126
   C. 208 D. 206

9. The ratio $11\frac{9}{10}$: 6.8 is equivalent to
   A. 7 : 4 B. 8 : 5
   C. 11 : 5 D. 17 : 1

10. Sandesh can earn Rs.580 in 2 days. How long will it take for him to earn Rs. 2,030?
    A. 9 days B. 8 days
    C. 7 days D. 6 days

11. If x : y = 3 : 2, what is the value of (6x + 8y) : (5x - 7y)
    A. 13 : 4 B. 14 : 1
    C. 15 : 2 D. 34 : 1

12. In traingle ABC
    $\angle B = 90^0, \angle A = 60^0$ and AC = 8 cm, then what is the value of AB : BC?
    A. $\sqrt{3}:2$ B. $2:\sqrt{3}$
    C. $1:\sqrt{3}$ D. $\sqrt{3}:\sqrt{5}$

13. If the ratio of a : $\frac{1}{b} = 7:5$, then what is the ratio of b : $\frac{1}{a} = ?$

A. 5 : 7 B. 7 : 5

C. $7:\frac{1}{5}$ D. $5:\frac{1}{7}$

14. The ratio of $9^{2.5}:3^3$ is same as

A. 2:1 B. 9:1

C. 4:1 D. 9:6

15. If $0.8:5::x:15$, then x is equal to

A. 18 B. 10.8

C. 2.4 D. 1.8

16. The ratio of three numbers is $3:4:5$ and the sum of their squares is 800. The sum of the numbers is:

A. 72 B. 48

C. 224 D. 108

17. If $\frac{a}{3} = \frac{b}{7} = \frac{c}{4}$, then $\frac{a+b+c}{b}$ is equal to:

A. 5 B. $\frac{1}{2}$

C. 2 D. $\frac{1}{5}$

18. The ratio of third propotional to 8 and 12 and the mean propotional between 4 and 36 is

A. 2:3 B. 1:2

C. 3:2 D. 9:1

19. If 35% of a number is equal to one-fifth of another number, what is the ratio of first number to the second number?

A. $2:5$ B. $5:3$

C. $4:7$ D. $3:7$

20. If half the original distance is covered in triple the original time then the ratio of the original and the final speed is:

A. 3 : 1 B. 1 : 6

C. 6 : 1 D. 1 : 3

21. In a solution of 42 L, the ratio of milk and water is 5 : 2. If 8 L of water is added to the solution, then the ratio of milk and water in the resulting solution will be:

A. 2 : 1 B. 3 : 1

C. 3 : 2 D. None of these

22. In a company, the ratio between the number of male employees to the female employees was 4 : 1. If the difference between the numbers of male employees to the female employees was 60, what was the total number of employees in the company?

A. 100 B. 160

C. 180 D. None of these

23. If $5a^2b$, $6a^2b^2$, x and $24ab^3$ are in proportion then express x in terms of a and b?

A. $32ab^3$ B. $24a^2b^2$

C. $32a^2b^2$ D. $20ab^2$

24. The number 175 is divided into 4 parts p, q, r and s such that p = 2q = 3r = 4s. What is the value of p + s?
    A. 105 B. 84
    C. 115 D. 120

25. If the area of a big circle 'A' is sixteen times the area of small circle 'B', what is the ratio between the circumference of circle 'A' to that of circle 'B'?
    A. 2 : 1 B. 4 : 1
    C. 8 : 1 D. 16 : 1

* * *

2. PERCENTAGE

Percentage or percent in the simplest sense means per hundred (100).

Why do we need percentages? Can we not compare two quantities directly without using percentages? Lets look at a scenario. Ram obtains 75 marks in a class test and his brother Shyam obtains 80 marks in his test. Who has fared better?

At first glance, one would say Shyam has fared better. However, we cannot compare the two unless we know the total marks of each test. In fact, Ram has obtained 75 marks out of 100 marks paper, whereas Shyam has obtained 80 marks in a paper that carries 150 marks. Immediately you can make out that Ram has performed better. Ram has obtained 75% marks whereas Shyam has obtained only 53.33%.

Directly comparing the marks of Ram and Shyam is like comparing oranges and apples. We need a common ground for comparison and that common ground is per 100 or per cent

Something like 32 % is a fraction with numerator as 32 and denominator as 100.
32% = 32/100 = 0.32

Every fraction can be represented as percentage and also as a decimal value. Similarly a decimal can be represented as a percentage and also as a fraction. This conversion is a very useful tool and simplifies calculations as we shall see later.

Conversion of a fraction or a decimal into a percent

A fraction or a decimal can be converted into a percentage by multiplying it by 100.

So, the fraction $\frac{1}{5}$ expressed as percentage is $\frac{1}{5}\times 100 = 20\%$

Converting a percentage into a fraction:

A percentage when divided by 100 is converted into a fraction. So, 40% converted into a fraction is $\frac{40}{100} = \frac{2}{5}$ or 0.4.

Successive Percentage changes:

If a number is first changed (increased/ decreased) by a% and subsequently, this is changed (increased/decreased) by b%, then

$$\text{Net percent change} = a \pm b \pm \frac{a \times b}{100}\%$$

If a or b show decrease, then put (-) ve sign before it, otherwise (+) ve sign.

Concept of Multiplying Factor:

Another important concept to simplify calculations related to percentages is the multiplying factor.

Consider the following calculation that is done when 50 has to be increased by 10%.

Increased value = 50 + 10% of 50

$$= 50 + \frac{10}{100}\times 50 = 50 + 5 = 55$$

Another way of looking at it -
Increased value = 50 + 10% of 50
= 50 (1+0.1). In this case, 1.1 is called the Multiplying Factor (MF) corresponding to 10% and can simplify a lot of calculations.
The Multiplying Factor (MF) corresponding to x % increase is nothing but $\left(1+\frac{x}{100}\right)$ and is $\left(1-\frac{x}{100}\right)$ for an x% decrease.

SOLVED EXAMPLES

1. **Bhima's income is 50% of Anand's income and Anand's income is 40% of Chitan's income. If Chitan's income is Rs 10,000 per month, then what is the sum of Bhima's income and Anand's income per month?**
Solution:
Bhima's income = 0.5 x A
Anand's income = 0.4 x C = 0.4 x 10000 = 4000 per month
Bhima's income = 0.5 x 4000 = 2000 per month
Thus the sum of the monthly income of Bhima and Anand is equal to Rs. 6000

2. **Stalin's salary is 20% more than Kanimozhi's salary. By what percentage is Kanimozhi's salary less than Stalin's salary?**
Solution:
Let Kanimozhi's salary be 100. Then, Stalin's salary is 100 x 1.2 = 120.
Kanimozhi's salary is $\frac{(120-100)}{120}\times 100$
$= \frac{20\times 100}{120}$% less than Stalin's salary.
Hence, Kanimozhi's salary is 16.67% less than Stalin's salary.

3. **If the price of the sugar increased by 10% in the first year and decreased by 5% in the next year. Find the net percentage increase/decrease of the sugar at the end of the second year.**
Solution:
The percentage increase/decrease of the sugar at the end of the second year
= 10% - 5% - (10x5)/100 %
= 4.5% increase in total.

4. **P is 15% more than Q, Q is 20% more than R, R is 10% less than S. By what percentage P is more than R?**
Solution:
Let us assume that S = 100
R = $0.9 \times 100 = 90$,
Q = 90×1.20 and
P = $90 \times 1.20 \times 1.15$
R = 90, Q = 108 and P = 124.2
$\Rightarrow$ P is more than R in percentage by
$= \frac{124.2 - 90}{90} \times 100 = 38\%$

5. **If the cost of a Radio increases from Rs. 600 in 2000 to Rs. 840 in 2001, then calculate the absolute value change and the percentage change of the Radio between the two years.**
Solution:
The absolute value change
= [Final value - orginal Value]
= 840 - 600 = Rs. 240
The percentage change
$= \frac{\text{Final value-original value}}{\text{original value}} \times$
$100 = \frac{240}{600} \times 100 = 40\%$

6. **The price of diesel increases by 20%. By what percentage should Venkat reduce his consumption of diesel to keep fuel expenditure constant?**
Solution:
$\text{Reduction in consumption} = \frac{20}{100 + 20} \times 100$
$= 16.67\%$

7. **The price of mangoes decreased by 20% on a given day. By what percentage should the sold quantity increase so as to keep the revenue constant for the mango seller?**
Solution:
$\text{Increase in quantity} = \frac{20}{100 - 20} \times 100 = 25\%$

8. **Due to 30% increase in the selling price of the apples, a man can buy 9 apples less for Rs. 390. What is the original price of the apples?**
Solution:
Let the original price of the apples be Rs. x.
According to the given condition:
$\Rightarrow \frac{390}{x} - \frac{390}{1.3x} = 9$
$\Rightarrow \frac{1.3 - 1}{1.3x} = \frac{9}{390} \Rightarrow \frac{3}{13x} = \frac{3}{130} \quad x = 10$
Hence, the original price of the apples is Rs. 10.

9. **If the population of India has increased by 10% in 2000-2010 and is expected to increase by another 8% in 2011-2020, find the overall percentage change in India's population between 2000-2020**

Solution:

The percentage increase in the population between 2000-2020 is

= 10%+ 8% + (10x8)/100 %

= 18.8 % increase in total.

10. **Three candidates contested an the recently held Hissar election and received 1100, 7600 and 11700 votes respectively. What percentage of the total votes did the winning candidate get if there were no invalid votes?**

Solution:

Total number of votes polled = (1100 + 7600 + 11700) = 20400.

$$\therefore \text{Required percentage} = \left(\frac{11700}{20400} \times 100\right)\%$$

=57.3 %.

PRACTICE EXERCISE

1. Bingo offers 25% extra on a chips packet of 18 grams. What will be the total weight of the new pack with this offer?

   A. 20.4 grams B. 19.9 grams
   C. 22.4 grams D. 22.5 grams

2. Swapan brought certain number of chocolates from Switzerland, he gave 30% of them to his neighbours and 40% of the remaining to his relatives. He is left with 84 chocolates. How many chocolates did he bring in all?

   A. 190 B. 300
   C. 200 D. 250

3. Ravan's salary is increased by 35% and he now gets Rs. 54,000pm. What was his salary before the increase?

   A. Rs. 18,000
   B. Rs. 40,000
   C. Rs. 45,000
   D. Cannot be determined

4. Rajitha's height is 25% more than that of Harika's. By what percentage Harika's height is less than that of Rajitha?

   A. 18% B. 20%
   C. 25% D. 20.6%

5. In a colony there are 60 Ganesh pandals, out of them 24 have ecofriendly Ganeshas. What is the percentage of ecoriendly Ganeshas in that colony?

   A. 40% B. 45%
   C. 35% D. 30%

6. Priya lost 35% of her total salary in a train robbery. She spends 40% of the remaining and is left with Rs. 3,705. What is her total salary?
   A. Rs. 9,000 B. Rs. 9,500
   C. Rs. 10,500 D. Rs.10,000

7. Sudheer types 22% faster than Ramesh. How much slower does Ramesh types than Sudheer?
   A. 17.09% B. 18.20%
   C. 19.06% D. 18.03%

8. The price of wheat in the market has risen by 15%. By what percentage a family must reduce its consumption of wheat such as not to increase their expenditure on wheat?
   A. 13.04% B. 12.28%
   C. 13.5% D. 12.5%

9. What is 33% of 33.33?
   A. 13 B. 11
   C. 12 D. 17

10. In an examination, 85% marks are required to get scholarship.Radhika got 864 marks and failed to get the scholarship by 13% of the required marks. What were the minimum marks required to get scholarship?
   A. 1300 B. 1,000
   C. 1,020 D. 1,350

11. Krishnam's salary is 75% of her sister's salary. She spends 40% of her salary on rent and 20% of the remaining on food. If she is left with Rs. 3,600. What is her sister salary?
   A. Rs. 9,600 B. Rs. 10,000
   C. Rs. 7,500 D. Rs. 8,800

12. Chitt's salary is first decreased by 40% then increased by 20% and finally increased by 20%. His present salary is what percent of his starting salary?
   A. Same B. 10.5% more
   C. 13.6% lesser D. Data Insufficient

13. In Abhyaasa school, 75% of the students pass and 30 fail. The number of students from Lavasa school is 20 more and 15 more students passed than Abhyaasa school. The pass percentage of Lavasa school is?
   A. 75 B. 70
   C. 85 D. 80

14. By how much $\frac{7}{5}$ is more than 0.2% of 600?
   A. 0.54 B. 0
   C. 0.8 D. 0.32

15. If 75% of a certain number is added to 100, the result is the same number itself. Which of the following is the number?
   A. 300 B. 400
   C. 250 D. 350

16. Sirisha bought a designer saree for Rs. 10,200 after 15% discount. If so, what would be the actual marked price of the saree?
   A. Rs. 11,800 B. Rs. 12,400
   C. Rs. 10,500 D. Rs. 12,000

17. A reduction of 12.5% in the price of a table brouht down the price to Rs. 4,375. The original price of table was
   A. Rs. 6000 B. 5400
   C. Rs. 5000 D. 5200

18. A mango tree was planted 2 years ago. It increases at the rate of 20% per annum. If at present, the height of the tree is 540cm, what was its height when the tree was planted?
A. 225cm B. 375cm
C. 295cm D. 395cm

19. Ajay's salary is first increased by 20% and then decreased by 25%. The result is the same as Srinivas's salary increased by 25% and then decreased by 20%. Find the ratio of their original salries.
A. 10:9 B. 1:2
C. 2:1 D. 9:10

20. A student has to secure 35 % to pass. He gets 158 marks and fails by 17 marks. The maximum marks are
A. 200 B. 800
C. 500 D. 1000

21. The length and breadth of a square are increased by 40% and 25% respectively. The area of the resulting rectangle exceeds the area of the square by
A. 65% B. 57.5%
C. 72.5% D. 75%

22. If x is 80% of y, then what percent of $2x$ is y?
A. 40% B. $62\frac{1}{2}\%$
C. 80% D. 160%

23. 4000 candidates appeared for the CLAT 2011. Out of these, one-fourth were girls. If 80% of the girls and 95% of the boys failed to get through. What is the number of successful candidates?
A. 250 B. 310
C. 350 D. 270

24. If 41% of a number is less than 76% of the same number by 105, what is the number?
A. 300 B. 400
C. 500 D. None of these

25. The diameter of a circle is increased by 24%. What is the percentage increase in its area?
A. 25.55% B. 24%
C. 53.76% D. 42.25%

* * *

PROFIT, LOSS & DISCOUNT

Profit, Loss and Discount is an important and simple concept of arithmetic that is very useful in our day-to-day life. Let us first look at a few terms that are commonly used.

Cost Price(CP):
CP of an item is the amount of money spent to produce or purchase an item. In all, CP is the amount spent to make an item ready for sale.

Selling Price(SP):
SP of an item is the amount obtained when an item is sold.

Profit/Loss:
This is the difference between the selling price and the cost price.
If the SP is greater than CP, it is called profit, if SP is lesser than CP, then it is loss.

Profit/Loss %:
Since absolute Profit/Loss does not give a complete indication for comparison, we need to calculate Profit/Loss percentage. It is nothing but the profit/loss as a percentage of the CP. The Profit/Loss is always computed on the CP because CP in the investment over which one realises a profit or loss.

Profit Margin:
Profit Margin is used in percentage terms only and is the profit as a percentage of SP.

Marked Price:
This is the price of the product as displayed on the label.Typically this is higher than the CP since every sale is intended for profit.

Discount:
Most often there is a reduction given on the marked price before selling it to a customer. Actual discount is a fraction of the marked price.

To summarise or put in formulae -

1. $\text{Profit \%} = \frac{\text{Profit}}{\text{CP}} \times 100 = \left(\frac{\text{SP}}{\text{CP}} - 1\right) \times 100$

2. $\text{Loss \%} = \frac{\text{Loss}}{\text{CP}} \times 100 = \left(1 - \frac{\text{SP}}{\text{CP}}\right) \times 100$

3. $\text{SP} = \frac{(100 + \text{Profit\%}) \times \text{CP.}}{100}$ or $\frac{(100 - \text{Loss\%}) \times \text{CP.}}{100}$

4. $\text{CP} = \frac{(100 \times \text{S.P})}{100 + \text{Profit\%}}$ or $\frac{100 \times \text{S.P.}}{(100 - \text{Loss\%})}$

5. If marked price is MP and discount percent is d %, then

   $\text{SP} = \frac{\text{M.P.}(100 - \text{d})}{100}$; $\text{M.P.} = \frac{100 \times \text{S.P.}}{(100 - \text{d})}$

6. If two items are sold, each at Rs. X, one at a gain of P% and the other at a loss of P%, then the overall loss percentage = $\frac{P^2}{100}\%$

7. $CP \xrightarrow{m\%} MP \xrightarrow{d\%} SP$

8. $p = m - d - \frac{m \times d}{100}$ where m, d and P are the mark- up, discount and profit (all in percents)

Don't panic if you have difficulty remembering all the above formulae. As long as you understand the concepts of Profit/ Loss and Multiplying factor, you can solve all problems easily.

SOLVED EXAMPLES

1. **Sai buys an ipod at a certain price. Later he sells it to Kishore for Rs. 4,200 and earns a profit of 20%. What was the cost price of the ipod to Sai?**

Solution:

Let the CP of the ipod be Rs. x.

$\Rightarrow$ Profit = SP - CP = 4200 - x.

Profit percentage = $\frac{(4200-x)}{x} \times 100$

$\Rightarrow 20 = \frac{(4200-x)}{x} \times 100 \Rightarrow$ x= Rs. 3,500

Alternative method:

Using the method of MF (multiplying factor)

$CP = \frac{SP}{MF} = \frac{4200}{1.2} = 3{,}500; CP = Rs.3{,}500$

2. **Prasad sold his bike at a gain of 15%. If he had sold it at Rs. 4000 more, then he would have made a gain of 35%. What is the cost price of the bike?**

Solution:

Let the cost price of the bike be Rs. X

SP = 1.15X

According to the given condition:

New, SP + 4000 = 1.35X

Putting the value of SP, we get

$\Rightarrow$ 1.35X = 1. 15X + 4000

$\Rightarrow$ 0.2X = 4000

X = 10,000.

Hence, the cost price of the bike is Rs. 10,000.

3. **Kumari bought a computer for Rs. 25,000 and sold it to Nitil at a loss of 20%. Nitil then sold it to Chakradhar at a loss of 15%. Find the amount paid by Chakradhar to purchase the computer.**

Solution:

Since Kumari incurred a loss of 20%, she sold it at 80% of the CP.

SP of Kumari = 25000 x 0.8 = Rs. 20,000

$\Rightarrow$ CP of Nitil = Rs. 20,000

Similarly, Nitil sold it at a loss of 15%. Hence, he sold it at 85% of the C.P.

$\Rightarrow$ SP of Nitil = 20000 x 0.85 = 17,000

$\Rightarrow$ CP of Chakradhar = Rs. 17,000.

4. **The cost price of 20 crackers is equal to the selling price of 18 crackers. What is the profit percentage?**

Solution:

Let the CP of each cracker be Re. 1.

CP of 20 crackers = Rs. 20

SP of 18 crackers = Rs. 20

SP of each cracker = Rs. $\frac{20}{18}$

$\Rightarrow$ SP > CP

Profit = $\frac{20}{18}-1=\frac{1}{9}$;

Profit percentage = $\frac{1}{9}\times 1\times 100 = 11.11\%$

5. **Bimlesh, a Mobile dealer, gives a discount of 25% on his Mobiles and yet makes a profit of 25%. How much was his marked price above the cost price?**

Solution:

Let the MP of Mobile be Rs. 100.

SP of the Mobile = $100 \times (1\text{-}0.25)$ = Rs. 75

Using the concept of MF:

CP = $\frac{SP}{MF} = \frac{75}{1.25}$ = Rs. 60

CP of the Mobile = Rs. 60 Percentage value of the Markup

= $\frac{100-60}{60}\times 100 = 66.67\%$

6. **Sandeep bought 8 bats for a rupee, but sells only 6 bats for a rupee. Find his profit percentage.**

Solution:

Cost price of 1 bat = Rs. $\frac{1}{8}$.

Selling price of 1 bat = Rs. $\frac{1}{6}$.

MF = $\frac{SP}{CP} = \frac{8}{6} = 1.3333...$

Hence, profit percentage $33\frac{1}{3}\%$

7. **Krishnan purchases apples at Rs. 20 per dozen and sells them at Rs. 25 for every 10 apples. Find the gain or loss percentage.**

Solution:

Cost price = Rs. $\frac{20}{12}$.

Selling price = Rs. $\frac{25}{10}$

MF = $\frac{SP}{CP} = \frac{300}{200} = 1.5$

So, profit percentage = 50%.

8. **In a book shop, the marked price of a book is worked out in such a way that it generates a profit of 25%. What should be the discount allowed on the marked price such that the profit made on the sale of a book is 20%?**
Solution :
Let the CP of the book be Rs. 100.
$\Rightarrow$ MP = 125 and SP = 120
Now, Discount = 125 - 120 = 5
$\Rightarrow$ Discount Percentage = 5 x 100/125 = 4%

9. **A generous businessman allows a discount of 20% to his customers and still makes a profit of 5%. Find the marked price of a product which costs him Rs. 10,000.**
Solution:
CP = Rs. 10,000
$\Rightarrow$ SP = 1.05 x 10,000 = Rs. 10,500
Since the shopkeeper gives 20% discount on marked price, hence MP x 0.8 = SP
$\Rightarrow$ MP = 10,500/0.8 = 13,125
Hence, the marked Price = Rs. 13,125

10. **On wednesday in 'Big Bazaar', there was a huge rush. They had an offer where in for every purchase of Rs. 6,000, customers got gifts worth Rs. 2,000 absolutely free. What was the discount % offered?**
Solution:
Discount Percentage

$$= \frac{2000}{6000+2000} \times 100 = 25\ \%$$

PRACTICE EXERCISE

1. A sweet vendor sells his sweets at 85% of his cost price. What percent does he gain or lose?
A. 15% gain B. 15% loss
C. 12.5% loss D. 25% loss

2. Anand sells an article at 25% gain. Had he sold it for Rs.260 more, he would have gained 38%. What is the cost price of the article?
A. Rs. 2000 B. Rs. 1580
C. Rs. 1720 D. Rs. 1000

3. A Tote handbag is marked at Rs. 2,250. During off-season, a discount is given and it is sold for Rs. 2,000. What is the discount percentage?
A. 11% B. $16\frac{2}{3}\%$
C. 11.11% D. 9%

4. Navya sells two gift hampers for Rs. 204 each, gaining 30% on the one and losing 30% on the other. What is his profit/loss percentage on the transaction?
A. 4% loss B. 4%gain
C. No proit/No loss D. 9% loss

5. The cost price of 12 cars is equal to the selling price of 9 cars. What is the profit percentage?
A. 25% B. 30%
C. 22% D. 19%

6. A watch dealer allows his customer a discount of 20% and still gains 10%. What is the marked price of a watch which costs the dealer Rs.850?
   A. Rs.1,300 B. Rs. 1020
   C. Rs. 1,275 D. Rs. 950

7. Vilas gives a discount of 20% on a refrigerator and yet makes a profit of 4%. How much much was his marked price above his cost price?
   A. 20% B. 30%
   C. 40% D. 25%

8. Haripriya solds a mobile at a loss of 10%. If she had bought it for 10% less and sold it for Rs. 3600 more, she would have gained 20%. What is the cost price of the mobile?
   A. Rs. 20,000 B. Rs. 15,000
   C. Rs. 36,000 D. Rs. 25,000

9. Hrithik buys a laptop and sells it for Rs.21,000 at a loss of 30%. For how much did he buy the laptop?
   A. Rs. 23,000 B. Rs. 25,750
   C. Rs. 30,000 D. Rs. 27,000

10. A vendor sells 15 chocosticks for 1 rupee and gains 20%. How many chocosticks did he buy for a rupee?
   A. 12 B. 18
   C. 15 D. 16

11. Sandeep buys a piece of land and 2 years later, sells it for Rs. 75,000. If he makes a profit of Rs. 9,500 on the transaction, what is profit percentage?
   A. 14.50% B. 12.65%
   C. 16.25% D. 18.75%

12. Kiren bought 12 pens for Rs. 60 and sold them at the rate of Rs. 6 per pen. What was her gain percentage?
   A. 22% B. 25%
   C. 20% D. 16.6%

13. Find the profit or loss percentage when a shopkeeper marks his goods 20% above the cost price and then allows a discount of 20% on the marked price?
   A. 4% loss B. 9% profit
   C. 9% loss D. 4% profit

14. Laxmi sold her scooty to Manasa at 20% profit. Manasa spent Rs.1000 to get it painted and sold it to Neethika at 10% profit. If Neethika paid Rs.27,500 to Manasa for the scooty, how much did Gouthami pay for the scooty?
   A. Rs.24,500 B. Rs.20,000
   C. Rs.25,000 D. Rs.24,000

15. Nagdeve bought 11 chocolates for one rupee. How many should she sell for Rs.3 to earn 10% profit?
   A. 33 B. 36
   C. 30 D. 35

16. Sushanth, a trader sold goods at a gain of 20%. If he had sold it for Rs. 50 more he would have made a gain of Rs. 25%. What is the cost price of the goods?
   A. Rs. 1300 B. Rs. 1000
   C. Rs. 1500 D. Rs. 1200

17. A vendor marks-up the price at 30% above the cost price and then allows customers a discount of 20%. What is his profit percentage?
   A. 6% B. 4%
   C. 7% D. 5%

18. A Planner is marked at Rs. 1,250. During off-season, a discount is given and it is sold for Rs. 1,100. What is the discount percentage?
A. 13.67% B. 12%
C. 14.5% D. 15%

19. Mr. Rahane gains 15% by selling his bike for Rs. 27,025. If he had sold it for Rs. 21,150, what would be his profit/loss percentage?
A. 10%, Profit B. 10%, Loss
C. 9% Profit D. 9% Loss

20. A bottle vendor sells his bottles at 20% discount. If the selling price is Rs. 1240, what is the marked price of bottles?
A. Rs. 1550 B. Rs. 1540
C. Rs. 1480 D. Rs. 1496

21. Tyson sold a guitar for Rs. 2250 and gained $\frac{1}{8}$ th of its cost price. What is the cost price?
A. Rs. 2000 B. Rs. 2200
C. Rs. 2500 D. Rs. 2220

22. A shopkeeper marks his goods 35% above the cost price and offers a discount of 15%. What is his gain percentage?
A. 25% B. 20%
C. 23.5% D. 21.5%

23. A bookseller buys 9 books for Rs. 350, but sells 8 books for Rs.350. What is his profit percentage?
A. 15% B. 12.5%
C. 17.5% D. 18%

24. Asmita sold her bracelet at a profit of 10% to Nakshatra who sold it to Nikhila by gaining 15% on it. If Nikhila had paid Rs. 2277 for it. At what price Asmita must have bought it?
A. Rs. 2170 B. Rs. 1800
C. Rs. 1920 D. Rs. 2200

25. A single discount equivalent to a series discount of 20%, 10% and 5% is:
A. 35% B. 33.5%
C. 34.7% D. 31.6%

* * *

SIMPLE INTEREST&COMPOUND INTEREST

When someone lends money he/she loses the chance to utilize this money by investing it and getting a return on it. Consequently, the lender expects a return from the money lent to the borrower. Thus, money borrowed today is repaid with a higher amount tomorrow. This gives rise to the concept of interest.

Following are few terms we need to know with regards to the concept of interest.

Principal - The amount of money invested **(P)**

Time or Period - The duration for which the money is invested **(T or N)**

Rate of Interest - The rate at which interest is calculated. This is expressed as a percentage **(R)**

Amount - The money repaid at the end of the time period.

Interest - The difference between the amount of money deposited and the amount repaid at the end of the time period **(I)**

Amount (A) = Principal (P) + Interest (I)

Interest is calculated in two different ways. Simple Interest (SI) and Compound Interest (CI). Let us understand them in detail.

SIMPLE INTEREST

The interest calculated for the given time duration only on the original principal, is called Simple Interest.

$$\text{Simple Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$

$$= \frac{P \times R \times T}{100}$$

Amount = Principal + Interest = P + I

Compound Interest:

In case of simple interest, the lender receives a fixed amount of money for every defined period of time. In the case of compound interest, the interest amount received after every given period of time is reinvested at the same rate of interest as agreed upon initially, In other words, the amount received at the end of time period 1 becomes principal for period 2, and so on.

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^{T} \text{ Where}$$

P = Principal or sum being borrowed
R = Rate of interest per time period
T = Number of time-periods

Note:

I. If only the term interest is used, then consider it to be SI.
II. If the interest is given by bank, then consider it to be CI.
III. Population growth can be calculated similarly with growth rate as Rate of Interest on compounding basis.
IV. For a single year, CI is the same as SI.
V. The difference between CI and SI for the second year is the interest on the SI of the first year.

SOLVED EXAMPLES

1. **Calculate the interest that will be paid on a principal of Rs. 10,000 borrowed at a rate of 12% for a period of 4 years and 6 months?**

   Solution:

   $SI = \frac{P \times R \times T}{100}$;

   P = 10,000, R = 12 and T = 4.5

   So, $SI = \frac{(10000 \times 12 \times 4.5)}{100}$

   = Rs. 5400

2. **If a certain sum amounts to Rs. 1296 in 2 years, and Rs. 1344 in 3 years, find the principal and rate of interest under SI?**

   Solution:

   Amount after 2 years = Rs. 1296.

   Amount after 3 years = Rs. 1344.

   In SI, interest amount remains the same for every year.

   Therefore, interest for 1 year = 1344-1296 = Rs. 48.

   Hence, the principal = 1296-2(48)

   = Rs. 1200.

   Rate of interest = $\frac{48 \times 100}{1200 \times 1} = 4\%$

3. **Find the simple interest on a principal of Rs. 8,500 at the rate of 6% per annum for a period of 5 years.**

   Solution :

   Simple Interest = $\frac{P \times R \times T}{100}$ = 8500 x 0.06x5

   = Rs. 2,550

4. **Find the CI on Rs.10,000 at 8% p.a. for 2 years, compounding being done annually.**

   Solution:

   P = Rs.10,000, R = 8% and T = 2 years

   $10000\left(1+\frac{8}{100}\right)^2$ = Amount = Rs. 11664.

   Interest = Amount - Principal

   = Rs. (11664 - 10000) = Rs. 1664.

5. **Find amount for a principal of Rs. 50,000 at 8% per annum, compounded semi-annually for 2 years.**

   Solution:

   Here T = 2 years x 2 = 4 time-periods

   Similarly, R= $\frac{8}{2}$ =4% (for half year)

   P = 40,000,

   A = $50000\left(1+\frac{4}{100}\right)^4$ =Rs. 58492.9

6. **Find CI on Rs. 15,000 at 12% for 9 months compounded quarterly.**

   Solution:

   T = 3 time-periods,

   R = 3% per period and

   P = Rs. 15,000

   Amount = $15000\left(1+\frac{3}{100}\right)^3$ = Rs. 16,391

   (approximately)

   CI = Amount - Principal = 16391 - 15000

   = Rs. 1391

7. **Find the amount after 3 years if the principal is Rs. 10,000 and rate are 10%, 8%, 12% for each year respectively. Consider it as a case of compound interest**

Solution:

Amount

$$=10{,}000\left(1+\frac{10}{100}\right)\left(1+\frac{8}{100}\right)\left(1+\frac{12}{100}\right)$$

$$= 10{,}000\times 1.1\times 1.08\times 1.12 = \text{Rs. } 13{,}305.6$$

8. **Samrat took a certain amount of loan from a bank at the rate of 8% p.a. S.I. and gave the same amount to Sathish as a loan at the rate of 10% p.a. If at the end of 12 years, he made a profit of Rs. 3600 in the deal, then what was the original amount?**

Solution:

Let the original amount be Rs. x. Then

$$\frac{x\times 12\times 10}{100} - \frac{x\times 12\times 8}{100} = 3600$$

$$\Rightarrow x = 15000$$

= Rs. 15,000.

9. **Praveen Kumar invested Rs. 14,000 for 3 years and Rs. 12,000 for 4 years at the same rate of simple interest. If the total interest from these investements is Rs. 6000, then what was the rate of interest?**

Solution:

Let the rate of interest be R%.

$$\frac{14000\times 3\times R}{100}+\frac{12000\times 4\times R}{100} = 6000 \text{ or}$$

1000R = 6000 or R = 6%

10. **The difference between the CI and SI on a certain amount at 10% per annum for 2 years, compounded annually, is Rs. 468. Find the principal.**

Solution:

Let the principal be Rs. P

$$\text{SI} = P\times 10\times\frac{2}{100} = \frac{20}{100}\text{P}$$

and CI = Amount - P

$$= P\left(1+\frac{10}{100}\right)^2 - P = \frac{21}{100}\times P$$

$\because$ CI - SI = Rs. 468

$$\Rightarrow \frac{21}{100}P - \frac{20}{100}P = \text{Rs. } 468$$

$$\Rightarrow \text{P} = 46{,}800$$

PRACTICE EXERCISE

1. Rahul lent Rs. 72,000 to Digvijay to help him open a book store. Bharav plans to repay Rahul at the end of 3 years with 5% interest compounded annually. How much will Rahul receive at the end of two years?
   A. Rs. 83,349 B. Rs. 82,000
   C. Rs. 82,000 D. Rs. 85,200

2. On what sum money will simple interest be Rs. 700 in 4 years at $3\frac{1}{2}\%$ rate annually?
   A. Rs. 4,000 B. Rs. 4,540
   C. Rs.5,500 D. Rs. 5,000

3. Saroja lends Rani a sum of Rs. 14,000. After three years Rani pays back Saroja an amount of Rs. 17,500. How much was the rate of simple interest?
   A. $5\frac{2}{3}\%$ B. $5\frac{2}{3}\%$
   C. $8\frac{1}{3}\%$ D. $12\frac{1}{5}\%$

4. If a certain principal amount increase to Rs. 6,000 in 2 years at a certain fixed rate of simple interest and Rs.8,000 in 6 years at the same rate of interest, then what is the rate of simple interest?
   A. 5 B. 10
   C.15 D. 20

5. The difference between the compound interest and the simple interest for 2 years on a certain sum at 20% rate of interest is Rs. 800. Find the principal.
   A. Rs. 2,00,000 B. Rs. 2,000
   C. Rs. 20,000 D. Rs. 10,500

6. After how many years approximately would the amount payable on a loan be twice the principal, if the principal is lent at the rate of 20% per annum, compounded quarterly?
   A. 5 B. 6.5
   C. 8 D. 3.5

7. It takes n years, for Rs. 62,500 to amount to Rs.1,08,000, at 12% per annum compounded annually. Find the value of n.
   A. 7 B. 5
   C. 4 D. 3

8. At what rate of compound interest, would Rs.60,000 amount to Rs. 66,150 in 2 years, interest being compounded annually?
   A. 4% B. 5%
   C. 6% D.8%

9. The difference between simple and compound interest on Rs. 5,000 for 3 years at the annual interest rate of 10% is
   A. Rs. 300 B. Rs. 250
   C. Rs. 155 D. Rs. 150

10. On what sum of money will simple interest be Rs.444 at 6% for two and a half year?
    A. Rs. 2,500 B. Rs.2,960
    C. Rs. 3,240 D. Rs. 4,200

11. Find the difference between the simple interest and compound interest on Rs. 4,500 for 2 years at 5% per annum, if compound interest to be calculated based on semi annual compounding.
    A. Rs. 95 B. Rs.74
    C. Rs. 86 D. Rs. 91

12. If a sum of money at a certain rate of simple interest doubles in 5 years and at a different rate of interest becomes three times in 8 years, find out the lesser rate of interest.
A. 20% B. 19.23%
C. 16. 67% D. 25.%

13. The population of a town increases each year by 4% of its total at the beginning of the year. If the population on 1st January 1985 was 5,00,000, what was it on 1st January 1988?
A. 5,05,000 B. 5,62,432
C. 5,60,000 D. 5,26,342

14. In how many years will Rs. 3,000 amount to Rs.3,993 at the rate of 10% per annum compound interest?
A. 5 B. 4
C. 3 D. 2

15. A particular sum of money amounted to Rs. 27,280 and Rs. 29,920 at the end of 2nd year & 3rd year, respectively. Find the rate, if interest is simple interest.
A. 15% B. 16%
C. 18% D. 12%

16. If the difference of CI and SI on a certain sum of money for 3 years at 5% per annum is Rs. 122. Find the sum.
A. Rs. 16,000 B. Rs. 16,500
C. Rs. 16,383 D. Rs. 16,400

17. A sum of money becomes twice in 5 years at CI. In how many years will it become eight times?
A. 25 years B. 75 years
C. 40 years D. 15 years

18. Raja lent a sum of money to Shekhar at simple interest. The sum tripled itself in 35 years. What was the rate of interest?
A. 8.13% B. 5.71%
C. 6.27% D. 6.67%

19. The simple interest on a certain sum for 2 years is Rs. 150 and the compound interest for the same sum for the same period is Rs. 153. Find the rate of interest.
A. 4% B. 5%
C. 6% D. 8%

20. A sum of money amounts to Rs. 4,840 in 2 years and Rs. 5,324 in 3 years. Find rate of interest and the principal under CI.
A. 8%, Rs. 4,500 B. 12%, Rs. 3,800
C. 10%, Rs. 4,000 D. 8%, Rs. 4,000

21. Find the compound interest on Rs. 25,000 for 3 years if the rates of interest for 3 years are 4%, 5% and 10%, respectively.
A. Rs. 5,080 B. Rs. 6,040
C. Rs. 4,050 D. Rs.5,030

22. A sum of Rs. 7,200 was invested for two years at Compound interest and is amounted to Rs.7,776 in first year. What was the interest for the second year?
A. Rs. 622 B. Rs.576
C. Rs. 550 D. Rs. 650

23. In how many years will a certain amount of money triple itself at 18% simple interest?
A. $9\frac{1}{9}$ years B. $11\frac{1}{11}$ years
C. $10\frac{1}{8}$ years D. $11\frac{1}{9}$ years

24. A person, who had borrowed some money from his friend, had to pay an interest of Rs. 50 at the end of first year, failing which the total interest payable at the end of second year was Rs. 105. Find the rate of interest and charged and the sum he borrowed.
    A. 12%, Rs. 600 B. 10%, Rs.500
    C. 15%, Rs.750 D. 16%, Rs. 800

25. From a sum, one-third is invested at 3%, one-sixth at 6% and the rest at 8%. If the simple interest for 2 years from all these investments is Rs. 900, find the sum.
    A. Rs. 4,000 B. Rs. 5,000
    C. Rs. 7,500 D. Rs. 8,400

* * *

TIME, SPEED & DISTANCE

Distance covered in unit time is called Speed. In other words, Speed is the rate at which distance is covered.

$$\text{Speed} = \frac{\text{Distance covered}}{\text{Time taken}}$$

Speed is measured in meters/second or kilometres/hour.

From the chapter of Ratio and Proportion, we can say

1. Speed is directly proportional to distance. If the speed is doubled, then distance travelled in the same time will also be doubled.

2. Distance and time are directly proportional. If distance to be travelled is doubled, then the time taken would also be doubled at the same speed.

3. Time is inversely proportional to speed. If the distance remains the same and speed is doubled, then the time taken to travel the same distance becomes half of the original time taken at the original speed.

Conversion of m/s to km/h and vice-versa:

$$1 \text{ km/hr} = \frac{1000m}{3600s} = \frac{5}{18} \text{ m/s}$$

If the speed is given in km/hr and needs to be converted to m/s, then you need to multiply by

Similarly, if the speed is given in m/s and it needs to be converted into km/hr, then multiply it by $\frac{18}{5}$

Average Speed:

Average speed is defined as the ratio of the total distance covered to the total time travelled.

Average speed=Total distance/ Total time

Suppose a body is travelling at a speed of *u* m/s for **half the time** and *v* m/s for the other half, then the average speed of the body is given by

Average speed = $(u + v)/2$ *m/s.*

Similarly, if the body is travelling at a speed of *u m/s* for the first **half of the distance** and *v* m/s for the second half of the distance, then the average speed is given by

Average speed = $2uv/(u+v)$ *m/s*

Relative Speed:

Relative speed is the speed of a moving object in relation to another moving or stationary object.

When you are sitting in a moving train, you observe that the speed of your train increases when another train moving in the opposite direction crosses it. The speed appears decreasing when another train moving in the parallel track in the same direction passes it. Hence, this apparent change in speed of the train is nothing but its speed in relation to the other train or relative speed.

Relative speed of a body moving at speed X km/h. in relation to another body moving at speed Y km/h is given by:
(X - Y) km/hr if the bodies are moving in same direction; or
(X + Y) km/hr if the bodies are moving in the opposite direction.

Train-related Problems:
You will encounter various problems related to trains. In all the cases, you need to remember the following Basic concept
Speed = Distance/Time or
Time = Distance/Speed

In case both the objects are moving, then the relative speed should be considered Distance (when two trains need to cross each other) = Sum of the lengths of the trains In this case,
Time = Time taken by the two trains to cross each other.
This will become clearer when we solve a few problems.

Boats and Streams:
The problems related to Boats and Streams are based on the concept of Relative Speed. The direction along the stream (water) is called downstream and, the direction against the stream is called upstream.

If the speed of a boat in still water is *p* km/h and the speed of the stream is *q* km/h, then
Downstream speed = (*p*+*q*) km/h
Do remember that speed increases if the boat is travelling in the direction of the stream.
Upstream speed = (*p*-*q*) km/h

The speed of the boat decreases if it has to travel against the direction of the stream.

If the downstream speed is u km/h and upstream speed is v km/h, then Speed of

$$\text{boat in still water} = \left[\frac{u+v}{2}\right] \text{km/h}$$

$$\text{Speed of stream} = \left[\frac{u-v}{2}\right] \text{km/h}$$

SOLVED EXAMPLES

1. **If a bird is moving at a speed of 15 km/hr, how much distance will it cover in 36 seconds?**

   Solution:

   Given, that the ball is moving at a speed of 15 km/hr. This is equal to 15 x 5/18 m/s =75/18 m/s.

   We know that the distance covered

   = Speed x Time of travel Distance covered

   = 75/18 x 36 = 150 m.

2. **Raju and Ramya cover the same distance with the speeds 45 km/hr and 54 km/hr respectively. Find the distance travelled by each, if one takes 20 min longer than the other.**

   Solution:

   Let the required distance be x km.

   $$\frac{x}{45}-\frac{x}{54}=\frac{20}{60}$$

   or $\frac{6x-5x}{270}=\frac{1}{3}\Rightarrow x=90$

   Hence, the required distance = 90 km.

3. **Shreyas travelled from Bangalore to Hyderabad by his car. For the first half of the distance he travelled with 40 miles/ hr and for the next half of the distance he travelled at 60 miles/hr. What is the average speed of Shreyas in the journey?**

   Solution:

   Let the speeds s_1 =40miles/hr and s_2 =60miles/hr.

   Average speed when half the distance with speed s_1 and half the distance with speed

   $$s_2=\frac{2\,s_1s_2}{s_1+s_2}=\frac{2\times40\times60}{40+60}=48\text{miles/hr.}$$

4. **What is the average speed if a person travels half the time at speed of 50 kmph, 1/4th of the time at 40 kmph and rest of the time at 60kmph?**

   Solution:

   In this case the average speed will be the weighted average

   $$\frac{1}{2}\times50+\frac{1}{4}\times40+\frac{1}{4}\times60=50\text{ kmph.}$$

5. **Rajan covers the distance between his house and office on his old scooter. At an average speed of 50 km/hr, he is late by 10min. However, with a speed of 60 km/hr, he reaches his office 5 min earlier. Find the distance between his house and office.**

   Solution:

   Let the distance be x km.

   Time taken to cover x km at 50 km/hr = $\frac{x}{50}$ hrs.

   Time taken to cover x km at 60 km/hr = $\frac{x}{60}$ hrs.

   Difference between the times taken = 25 min

   $$\therefore\frac{x}{50}-\frac{x}{60}=\frac{15}{60}\text{ or }6x-5x=75$$

   $$\Rightarrow x=125km$$

   Hence, the required distance is 75km.

6. **A hare makes 9 leaps in the same time as a dog makes 4. But the dog's leap is $2\frac{1}{3}$ m while hare's is only 1 m. How many leaps will the dog have to make before catching up with the hare if the hare has a head start of 16 m ?**

Solution:

Distance covered by dog in 4 leaps

$= 4 \times \frac{7}{3} = \frac{28}{3}$ m

Distance covered by hare in 9 leaps = 9 X 1 = 9m

Distance gained by the dog in 4 leaps = $\frac{1}{3}$ m.

Hence, for 1 m gain he has to make 12 leaps. Number of leaps required by the dog to gain 16m = 12 x 16=192 leaps.

7. **A boat took a total of 4 hours to row 6 kms downstream and to return to the starting point. If speed of the boat is 4 kmph, for how much time was the boat moving downstream and for how much time upstream?**

Solution:

If s is the speed of the stream, we have

$$\frac{6}{4+s} + \frac{6}{4-s} = 4 \Rightarrow \frac{24-6s+24+6s}{16-s^2} = 4$$

$\Rightarrow 48 = 64 - 4s^2 \Rightarrow s^2 = 4 \Rightarrow s = 2$ kmph

Thus time taken downstream = 6/6 = 1 hour and time taken upstream = 6/2 = 3 hours.

8. **The speed of a boat in still water is 10 m/s and the speed of the stream 6 m/s. If the boat is moving upstream and again downstream, what is the ratio of time taken to cover upstream and downstream distances?**

Solution:

As the distance travelled in both the directions will be the same, the effective speeds are in the ratio,

10-6 : 10+6 = 4:16 = 1:5

Hence the times taken are in the reverse ratio i.e. 5:1

9. **If the length of a train and a platform are 400 m and 650 m respectively, find the time taken by the train moving at 63 km/hr to cross the platform.**

Solution:

The total distance that the train needs to travel = Length of the train + Length of the platform. Hence, the time taken by the train

$$= \left\{\frac{(400+650)}{17.5}\right\} s = 60s.$$

[Since 63 km/hr = 17.5 m/s]

10. **A train running at 72 km/hr takes 15s to cross a platform and 8s to pass a man running in the same direction at a speed of 9km/hr. Find the length of the train and the platform.**

Solution:

Let the length of the train = X m. Let the length of the platform = Y m.

Speed of the train relative to the man = 63 km/hr –17.5 m/s

In passing the man, the train covers its own length with relative speed.
Length of the train = 17.5 x 8 = 140m.
Since speed of train = 72 km/hr = 20 m/s

$\frac{X+Y}{20}$ =15 or X +Y = 300 or Y=160.

Length of the platform = 160 m.

PRACTICE EXERCISE

1. A train 140m long passes another stationary goods train in 15 secs. If the train was traveling at a speed of 20 m/s, what is the length of the stationary train?
   A. 120m B. 160m
   C. 155m D. 200m

2. Harish jogs up-hill and down-hill on the road at a speed of 4km/hr and 5km/hr respectively. If he takes 17hours to jog a distance of 80 km on a road, no part of which is leveled, How much of the distance is up-hill?
   A. 24km B. 20km
   C. 30km D. 28km

3. If the speed of Navtarang Express is increased by 2 km/hr, a railway journey of 240 km would take half an hour less of its usual time. What is the speed of the train?
   A. 20 km/hr B. 15 km/hr
   C. 30 km/hr D. 25 km/hr

4. A passenger train takes 4 hours more to cover a distance of 450 km than an express train. If the speed of passenger train is 50 km/hr, what is the speed of the express train?
   A. 100 km/hr B. 90 km/hr
   C. 150 km/hr D. 120 km/hr

5. A Yatri boat takes twice as much time to sail upstream as it would have taken to sail downstream of a river. If the speed of the water strea, is 4 km/hr, what is the speed of the boat in still water?
   A. 15 km/hr B. 16 km/hr
   C. 12 km/hr D. 20 km/hr

6. Sanchayan covers a distance of 15 km in 12 minutes. If his speed decreases by 15 km/hr, what is the time taken by him to cover the same distance?
   A. 13min B. 15min
   C. 11min D. 14min

7. Surya can row 45km upstream in 2hours. If the stream flows at a rate of 2 km/hr, what is the speed of the boat in still water?
   A. 40km/hr B. 25km/hr
   C. 30km/hr D. 24.5km/hr

8. A Reva Car travelled the first half of its route at an average speed of 35km/hr. If its average speed for the entire trip was 42km/hr, what was the average speed of the Reva Car for the second half of its route?
   A. 45km/hr B. 38km/hr
   C. 38.5km/hr D. 40km/hr

9. A car covers 100 km at a speed of 50 km/

hr, another 90 km at a speed of 60 km/hr and remaining 60 km to reach another city at a speed of 40 km/hr. What is the average speed of the car?

A. 50.0 km/hr B. 55.3 km/hr
C. 45.2 km/hr D. 53.5 km/hr

10. Anamika can row 20 km upstream in 5 hours. If the stream flows at 2 km/hr, what is Anamika's speed in still water?

A. 6 km/hr B. 2 km/hr
C. 5 km/hr D. 4 km/hr

11. Soha covers a distance of 35 km partly by walking and partly by cycling. She walks at a rate of 4 km/hr and cycles at a rate of 5 km/hr. If she had walked at 5 km/hr and cycles at 4 km/hr, she would have covered 2 km more in the same time. What is the time taken to cover the distance?

A. 6 hours B. 8 hours
C. 4 hours D. 7 hours

12. Ramu walks at 6 km/hr to reach the bus stand but he misses the bus by 2 min. If he had walked at 7 km/hr, he would have reached the bus stand 1 min before the arrival of the bus. How far is the bus stand?

A. 7 km B. 5 km
C. 2.1 km D. 1.4 km

13. Banta is travelling by car at the rate of 40 km/hr. After every 80 km, he rests for 20 min. How long will he take to cover a distance of 240 km?

A. 6 hr 40 mins B. 6 hrs
C. 6 hrs 20 mins D. 7 hrs

14. Salman starts from a place at 7 a.m. with a speed of 10 km/hr. Malaika starts from the same place at 9 a.m. with a speed of 15 km/hr. If both Salman and Malaika move along the same path in the same direction, when will Malaika catch up with Salman?

A. 1 P.M. B. 2 P.M.
C. 3 P.M. D. None of these

15. Nirupam can run 660 m in 81 sec and Jain in 88 sec. If Jain has a 60 m head start, by how much time will Jain win?

A. 1 sec B. 2 sec
C. 7 sec D. 11 sec

16. Vadlamani completes a journey of 112 km and at the end of it he realises that if he had increased his speed by 3.2 km/hr, he would have completed the journey 4 hours earlier. At what speed he travelled initially?

A. 8 km/hr. B. 6 km/hr.
C. 10 km/hr. D. 5 km/hr.

17. A railway train, 180 m long crosses a 120 m long tunnel in 12 seconds. In how much time this train, moving with the same speed, cross another train 195 m long train, moving in the opposite direction with a speed of 60km/hr?

A. 6 secs B. 15 secs
C. 9 secs D. 20 secs

18. A bus can finish a journey in 10 hr at a speed of 48 km/hr. To cover the same distance in 8 hr, the speed must be increased by:

A. 10 km/hr B. 12 km/hr
C. 16 km/hr D. 18 km/hr

19. Richa can cover a certain distance in 1 hr 21 min by covering half the distance at 4 km/hr and the rest at 5 km/hr. The total distance is:
A. 9 km B. 6 km
C. 5 km D. 4.5 km

20. Walking at three-fourths of his normal speed, a person is late by 3 hr. The usual time is:
A. 4.5 hr B. 9 hr
C. 7.5 hr D. 8 hr

21. The speeds of Nirmal and Bimal are in the ratio 3 :4. Nirmal takes 30 min more than Bimal to reach the destination. How much time does Nirmal take to reach the destination?
A. 1 hr B. 2 hr
C. 1 hr 30 mins. D. 2 hr 30 mins.

22. A man sees a train passing over a bridge2 km long. The length of the train is half that of the bridge. If the train clears the bridge in 2 min, the speed of the train is.
A. 45 km/hr B. 60 km/hr
C. 30 km/hr D. 90 km/hr

23. If a man walks at a speed of 5 km/hr, he reaches the station 5 minutes early. If he walks at 4 km/hr,he misses the train by 10 minutes. Find the distance between station and his house?
A. 3 km B. 5 km
C. 6 km D. 8 km

24. A thief escaped from a cop's hand and runs at a speed of 8km/hr. If the cop chases him at 15 km/hr and takes12 minutes to catch him. Find the distance covered by the thief?
A. 1 km B. 2 km
C. 3 km D. 6 km

25. A farmer traveled a distance if 56 km in 9 hrs. He traveled partly on foot at the rate of 4 km/hr and partly on a bicycle at the rate of 9km/hr. what is the total distance traveled on foot?
A. 12 Km B. 15 Km
C. 16 Km D. 20 Km

* * *

Doing work is synonymous to performing any activity. It could be constructing a road, packing certain items, eating food or filling/emptying a tank.

In simple terms,

Work Done = Number of people x Rate of doing work x Amount of time
i.e. Work Done = number of men x Hours /Day x number of days.
Work is measured in man-hours or man-days.

Concept of Unit Work

Work is generally considered as 1 unit.
While solving problems related to Time & Work, it is generally assumed that if a person takes X days to complete a certain work, then he does equal amount of work on each day which is equal to 1/X.

On the same note, if a person completes 1/X of the work in 1 day, then he will take X days to finish the entire work.

In other words, the number of days required to complete the work is a reciprocal of the amount of work done in one day.

$\therefore$ The number of days to complete the work

$$= \frac{1}{\text{work done in one day}}$$

Work done can also be considered as the product of the rate at which work is done and the time taken to finish it.

i.e. Total Work Done = Work Rate x Time

CONCEPT OF EFFICIENCY

If A is twice as efficient as B, it implies that A takes half the time as B to do the same work or in other words,
A can do twice the amount of work that B can do in a certain time.

Similarly, if A is n times as efficient as B, then Number of days taken by A

$$= \frac{Number\ of\ days\ taken\ by\ B}{n}$$

However, if A is two times more efficient than B, it implies that A takes 1/3rd the time that B takes. In other words, it is equivalent to saying that A is 3 times as efficient as B.

PIPES AND CISTERNS

The concept of pipes and cisterns is an extension of the time and work. Pipes are of two types-inlet and outlet. Inlet pipes fill the tank, while outlet pipes empty the tank. Work done by an inlet pipe is treated as positive work and that done by an outlet pipe is treated as negative work.

Total tank filled = Portion of Tank filled by inlet pipes - Portion of Tank emptied by outlet pipes

The above concepts will be clearly understood if you go through the following examples.

Solved Examples:

1. **If Balajee completes 4/15th of some work in one day, in how many days can he finish it?**

   Solution:

   Work done in 1 day = 4/15

   Total days needed to complete the work

   $$= \frac{1}{\text{work done in 1day}} = \frac{15}{4} = 3.75 \text{ days.}$$

   Hence, Balajee will complete the work in 3.75days.

2. **If Venkat alone completes a project in 8 days and Anand alone completes the same project in 4 days, then in how many days will they complete the project if they both work together?**

   Solution:

   Venkat alone completes the project in 8 days.

   Hence, work completed by Venkat in 1 day = 1/8

   Anand alone completes the project in 4 days.

   Hence, work completed by Anand in 1 day = 1/4

   Hence, work completed by Venkat and Anand together in 1 day

   $$= \frac{1}{8} + \frac{1}{4} = \frac{6}{16}$$

   Number of days to complete the work

   $$= \frac{1}{\text{work done in one day}} = \frac{16}{6} = 2.67 \text{days.}$$

   Hence, Venkat and Anand working together can finish the work in 2.67 days.

3. **Sandesh and Hari together complete some work in 5 days, while Sandesh alone takes 15 days to complete it. How much time would Hari take to complete the work if he is working all alone?**

   Solution:

   Let Hari working alone take b days to complete the work.

   Hence, work completed by Hari in one day = 1/b

   Sandesh takes 15 days to complete the work alone.

   Hence, work completed by Sandesh in one day = 1/15

   Work done by Sandesh and Hari together in 1 day = Work done by Sandesh in one day + Work done by Hari in one day

   $$= \frac{1}{15} + \frac{1}{b} \quad \text{.... (i)}$$

   Also, Sandesh and Hari can together complete the work in 5 days.

   $\therefore$ Work done by Sandesh and Hari together in 1 day

   = 1/5.... (ii)

   Equating (i) and (ii),

   $$\frac{1}{15} + \frac{1}{b} = \frac{1}{5}$$

   $$\therefore \frac{1}{b} = \frac{1}{5} - \frac{1}{15}$$

   $$\therefore \frac{1}{b} = \frac{2}{15}$$

   $$\therefore b = \frac{15}{2} = 7.5 \text{ days}$$

   Hence, Hari alone will take 7.5 days to finish the work.

4. **If Ram takes 13 days to complete a job and Laxman is thrice as efficient as Ram,**

then in how many days can they finish the job together?

Solution:

Number of days taken by Ram to finish the job = 13

Laxman is thrice as efficient as Ram.

So, Laxman will take one third of time taken by Ram.

$\therefore$ Number of days taken by Laxman to finish the job = 13/3

Job completed together in 1 day

$$=\frac{1}{13}+\frac{3}{13}=\frac{4}{13}$$

Number of days to complete the job = 13/4 = 3.25days.

Hence, Ram and Laxman working together can finish the job in 3.25days.

5. **Prashanth and Srihari together can complete a piece of work in 6 days and Prashanth alone can complete the same in 14 days. In how many days can Srihari complete it alone?**

Solution:

Work done by both Prashanth and Srihari in

$$1\text{day}=\frac{1}{6}$$

Work done by Prashanth alone in $1\text{day}=\frac{1}{14}$

Work done by Srihari alone in 1day

$$=\frac{1}{6}-\frac{1}{14}=\frac{4}{42}$$

Thus, Srihari can complete the work alone in $\frac{42}{4}=10.5$ days.

6. **Ranga and Ramya can complete a piece of work in 12 days and 15 days respectively. Ranga and Ramya work together for 4 days and then Ranga leaves. How much time will Ramya take to complete the remaining work?**

Solution:

Work done by Ranga and Ramya in 1 day

$$=\frac{1}{12}+\frac{1}{15}=\frac{9}{60}=\frac{3}{20}$$

In 4 days, they will complete

$$=\frac{3}{20}\times 4=\frac{3}{5}\text{th part.}$$

Remaining part to be completed

$$=1-\frac{3}{5}=\frac{2}{5}$$

Ramya can complete a work in 15 days.

$\Rightarrow$ Time taken to complete the remaining work

$$=15\times\frac{2}{5}=6\text{ days}$$

Hence, Ramya can complete the remaining work in 6 days.

7. **An empty tank is connected to pipes A, B and C. Pipes A and B are inlet pipes and they fill the tank in 4 hours and 3 hours respectively while pipe C is an outlet pipe and it empties the tank in 2 hours. Find the time in which the tank will fill up if all the pipes are opened simultaneously.**

Solution:

Pipe A alone fills the tank in 4 hours, So, tank filled by A in 1 hour = 1/4

Pipe B alone fills the tank in 3 hours. So, tank filled by B in 1 hour = 1/3
Pipe C alone empties the tank in 2 hours. So, tank emptied by C in 1 hour = 1/2
Tank filled in 1 hour = Tank filled by A+Tank filled by B-Tank emptied by C

$$= \frac{1}{4} + \frac{1}{3} - \frac{1}{2} = \frac{1}{12}$$

$\therefore$ Number of hours to fill the tank = 12 hours

8. **Two inlet pipes can completely fill up a Petrol tank in 20 hours and 30 hours, respectively. There is a leak at the bottom that can empty the completely filled tank in 25 hours. How much time will it take to fill up the tank if the both the pipes are opened?**

Solution:
In 1 hour, the part of tank filled completely by the pipes inspite of leak is

$$= \frac{1}{20} + \frac{1}{30} - \frac{1}{25} = \frac{15+10-12}{300} = \frac{13}{300} \cong \frac{1}{23}$$

Total time taken to completely fill the tank = 23 hours.(approximately)

9. **A Monkey jumps up 4m in the first minute and jumps down 1m in the next minute. It continues the same pattern till it reaches the top of the 100m pole. Find the time taken by the monkey to reach the top of the pole.**

Solution:
In a cycle of every 2 minutes, Monkey goes up by 4m and come down by 1m i.e. it jumps up by 3m in all.

It would reach the top when it covers the last 4m in an upward jump.

The remaining 96m have to be covered in cycles of 2 min. The total cycles required is = Height to be jumped /net jump in a cycle = 96/3 =32 cycles = 64 min.

Now in 65th min, the monkey covers 4m more and reaches the top. So time taken is 65 min.

10. **A large water tank has to be filled with pipes A and B. Pipe A can fill the tank in 6 hours and pipe B can empty it in 10 hours. In how much time will these pipes fill up the tank if pipes A and B are working alternately. i.e Pipe A fills for an hour and then pipe B empties in the seconf hour and so on?**

Solution:
Pipe A can fill the tank in 6 hours. So, portion of tank filled by A in 1 hour = 1/6
Pipe B alone empties the tank in 10 hours. So, portion of tank emptied by B in 2nd hour = 1/10

$\therefore$ Portion of Tank filled by both A and B in 2 hours

$$= \frac{1}{6} - \frac{1}{10} = \frac{2}{30} = \frac{1}{15}$$

1/15th tank is filled in 2 hours. So total tank is filled in 30 hours.

1. A 120m height wall has to be built in 10 days, 20 men are employed for this purpose. It is found that after 6 days of work, only 30m of the wall has been built. How many more men must be employed to complete the work on

time?

A. 60 B. 40
C. 50 D. 70

2. Roshan can complete a piece of work in 15 hours. Roshan and Soha working together can complete the same work in 10 hours. How much time will Soha take to complete the work on her own?

A. 30 hours B. 40 hours
C. 15 hours D. 25 hours

3. Hari and Harsha both alone can complete a piece of work in 80days. They start working together but Harsha left away after 25 days. In how many days Hari will complete the remaining work?

A. 50 days B. 30 days
C. 40 days D. 55 days

4. B and C working together can complete a piece of work in 8 days. A, B and C working together can complete the same piece of work in 4 days. How much time does A alone take to complete the work?

A. 12 days B. 6 days
C. 10 days D. 8 days

5. Eighteen men can build a wall in 60 days. How many more men are required to build a similar wall in 36 days?

A. 6 men B. 12 men
C. 4 men D. 10 men

6. Seven men can clear a forest in 16 days working 6 hours a day. In how many days can Six men do the same working 8 hours a day?

A. 10 days B. 12 days
C. 13 days D. 14 days

7. A tap can fill a cistem in 4 hours and a waste pipe can empty it in 7 hours. If both the taps are opened simultaneously, in what time will the empty cistern be filled?

A. 7.67 hrs B. 8.33 hrs
C. 9.33 hrs D. 8.67 hrs

8. As part of MGNREGA scheme, a work is completed by 40 men working for 4 hours a day in 12 days. How much time will be required to complete the same work by 20 men working for 6 hours a day?

A. 16 days B. 20 days
C. 18 days D. 22 days

9. A tap can fill an empty tank in 6 hrs while another tap can empty the full tank in 12 hrs. If both the taps are opened simultaneously, in how much time would an empty tank gets filled?

A. 15 hrs B. 12 hrs
C. 10 hrs D. 14 hrs

10. Rakesh and Ramya together can repair and service a car in 4 days. If Rakesh alone can complete the work in 6 days, in how many days Ramya alone will complete the same work?

A. 24 days B. 12 days
C. 6 days D. 8 days

11. A and B can complete a certain work in 15 days and 20 days respectively. But with the help of C they can complete the same in 6 days. How long C alone will take to complete the work?

A. 16 days B. 10 days
C. 22 days D. 20 days

12. If 12 men can harvest 64 hectares of land in 16 days, how many hectares of land can 18 men harvest in 15 days?
A. 90 hectares B. 50 hectares
C. 80 hectares D. 75 hectares

13. Pipe 1 can fill a cistern in 10 minutes and Pipe 2 can empty it in 14 minutes. If both the pipes are opened simultaneously, in what time cistern would be completely full?
A. 60 min B. 40 min
C. 48 min D. 35 min

14. Amul can complete one-fourth of a piece of work in 3 days and Bhanu can complete one-third of the same work in 2 days. If they start working together, in how many days the work would be completed?
A. 7 days B. 4 days
C. 6 days D. 5 days

15. 4 men or 6 boys can complete a piece of work in 4 days. In how many days can 2 men and 3 boys complete the work?
A. 5 days B. 8 days
C. 3 days D. 4 days

16. Ashok is twice more efficient than Bhama. If Ashok takes 16 days less than Bhama to complete the same work, then in how many days working together they can complete the work?
A. 12 days B. 10 days
C. 8 days D. 6 days

17. Two pipes opened simultaneously, can fill an empty reservoir in 4 hrs. If one of the pipe can fill the reservoir in 6 hr faster than the other pipe then how much time does the slower pipe take to fill the reservoir?
A. 10 hrs B. 16 hrs
C. 14 hrs D. 12 hrs

18. Three men and Six boys can complete a piece of work in 9 days, Six men and three boys can complete the same amount of work in 6 days. How much time does a team of 2 men and 2 boys working together require to complete the work?
A. $16\frac{2}{5}$ days B. days
C. $18\frac{5}{6}$ days D. $18\frac{4}{6}$ days

19. Pipes A and B can fill a tank in 6 hours and 8 hours respectively. If they are opened at alternate hours with Pipe B opened first, what will be the time taken to completely fill the tank?
A. 7 hours B. 7 hrs 10 mins
C. 7 hrs 40 mins D. 8 hours

20. If Asmita and Babita can complete a job in 4 and 5 days respectively when working alone, then how many days will they take to complete the job together?
A. 50/9 B. 20/9
C. 1/20 D. 9/20

21. If A and B together complete a job in 10 hours and A takes 15 hours to do the job alone, in how many hours can B alone complete the job?
A. 30 B. 30
C. 16.66 D. 6.16

22. Two leaks together empty a 1250 litre tank in 6.25 minutes. What is the rate at which the tank gets empty?
A. 150 litres/min
B. 194 litres/min
C. 180 litres/min
D. 200 litres/min

23. Rakesh and Harish together have to complete a project. Rakesh alone can complete it in 8 days while Harish alone requires 16 days. If both of them decide to work on alternate days, starting with Harish, then in how many days will they be able to finish the project?
A. 10.25 days B. 11 days
C. 10 days D. 10.5 days

24. A tank is initially full. Pipe A can empty it in 4 hours while pipes B and C fill it in 8 and 12 hours respectively. If all the pipes are opened simultaneously, then after how many hours will the tank be empty?
A. 22 B. 24
C. 27 D. 29

25. A and B together complete a task in 4 days. B and C together complete the same task in 6 days while A and C together complete it in 7 days. Find the number of days taken by the least efficient person to complete the task.
A. 168 B. 168/3
C. 168/5 D. None

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