Economic Load Dispatch using Lambda Iteration Method

1 Introduction

Economic Load Dispatch (ELD) is a crucial optimization problem in power systems that aims to minimize the total generation cost while meeting the power demand and satisfying various operational constraints. The Lambda Iteration method is a classical approach to solving the ELD problem, based on the equal incremental cost principle.

This document explains the implementation of the Lambda Iteration method for solving the Economic Load Dispatch problem with transmission losses, as presented in the provided MATLAB code.

2 Mathematical Formulation

2.1 Objective Function

The objective of the ELD problem is to minimize the total fuel cost:

$$\text{Minimize} \sum_{i=1}^{N} F_i(P_{G_i}) \tag{1}$$

where $F_i(P_{G_i})$ is the fuel cost function of the i-th generator and P_{G_i} is the power output of the i-th generator.

2.2 Constraints

The ELD problem is subject to the following constraints:

1. **Power Balance Constraint**:

$$\sum_{i=1}^{N} P_{G_i} = P_D + P_L \tag{2}$$

   where P_D is the total power demand and P_L is the transmission loss.

2. **Generator Capacity Constraints**:

$$P_{G_i}^{min} \leq P_{G_i} \leq P_{G_i}^{max} \tag{3}$$

   where $P_{G_i}^{min}$ and $P_{G_i}^{max}$ are the minimum and maximum power limits of the i-th generator.

2.3 Cost Function

In the provided code, the fuel cost function for each generator is represented by a quadratic function:

$$F_i(P_{G_i}) = a_i P_{G_i}^2 + b_i P_{G_i} + c_i \tag{4}$$

where a_i, b_i, and c_i are the cost coefficients of the i-th generator.

2.4 Transmission Loss Model

The transmission losses are modeled using the loss coefficients, where the loss is calculated as:

$$P_L = \sum_{i=1}^{N} B_{ii} \cdot P_{G_i}^2 \tag{5}$$

where B_{ii} is the loss coefficient for generator i. This is a simplified version of the B-coefficient loss formula, considering only the diagonal elements of the B matrix.

3 Lambda Iteration Method

3.1 Theory

The Lambda Iteration method is based on the Lagrangian multiplier approach. When losses are considered, the optimal solution satisfies:

$$\frac{dF_i}{dP_{G_i}} = \lambda \cdot PF_i \tag{6}$$

where λ is the incremental cost (Lagrangian multiplier) and PF_i is the penalty factor for the i-th generator, given by:

$$PF_i = \frac{1}{1 - 2 \cdot P_{G_i} \cdot B_{ii}} \tag{7}$$

For a quadratic cost function, we have:

$$\frac{dF_i}{dP_{G_i}} = 2a_i P_{G_i} + b_i \tag{8}$$

Therefore, the optimal power output for the i-th generator is:

$$P_{G_i} = \frac{\lambda / PF_i - b_i}{2a_i} \tag{9}$$

subject to generator capacity constraints.

3.2 Algorithm

The Lambda Iteration algorithm implemented in the code follows these steps:

Algorithm 1 Lambda Iteration Method for ELD with Losses

Initialize parameters and set an initial value for λ
Calculate initial penalty factors PF_i assuming equal power distribution
Estimate initial transmission losses P_L
while transmission losses not converged **do**
    while power balance error not within tolerance **do**
        for each generator i **do**
            Calculate $P_{G_i} = \frac{\lambda/PF_i - b_i}{2a_i}$
            Apply generator capacity constraints
        end for
        Calculate error $= \sum P_{G_i} - (P_D + P_L)$
        Adjust λ based on error
    end while
    Update penalty factors PF_i based on new P_{G_i} values
    Calculate new transmission losses P_L
    Check for convergence of transmission losses
end while

4 MATLAB Implementation

4.1 Data Structure

The generator data is organized in a matrix called `PG_data` with the following columns:

1. a_i - Quadratic cost coefficient
2. b_i - Linear cost coefficient
3. c_i - Constant cost coefficient
4. $P_{G_i}^{min}$ - Minimum power limit
5. $P_{G_i}^{max}$ - Maximum power limit
6. Initial generation guess (set to 0 in the code)
7. B_{ii} - Loss coefficient

4.2 Main Algorithm

The main code (`exp_4_main_code.m`) implements the following steps:

1. Initialize variables from the data file (`ELD_data.m`).
2. Calculate initial penalty factors assuming equal power distribution.
3. Estimate initial transmission losses.
4. Enter an iterative process to update power outputs and losses:
    - Call the lambda iteration function to determine generator outputs.
    - Update penalty factors based on new generator outputs.

- Calculate new transmission losses.
- Check for convergence of transmission losses.

5. Display the results.

The key MATLAB code segments for the main algorithm are:

```
% Initialize with equal distribution
temp_pg = pd / N;
ploss = (temp_pg^2 * PG_data(:,7));
total_ploss = sum(ploss);
pf = 1./(1-2*temp_pg.*PG_data(:,7));

% Main iteration loop
for iteration = 1:10000
    pg = lambda_iteration_function(ploss_temp, pf);
    pf_new = 1./(1-2*pg.*PG_data(:,7));
    ploss_new = sum(PG_data(:,7)'.*pg.^2);
    diff_ploss = ploss_new - ploss_temp;
    ploss_temp = ploss_new;
    if abs(diff_ploss) < error_tolerance_ploss_diff
        break
    end
end
```

4.3 Lambda Iteration Function

The lambda iteration function (`lambda_iteration_function.m`) performs the following operations:

1. Initialize variables and set an initial value for λ.
2. Enter an iterative process:
   - Calculate generator outputs based on the current λ and penalty factors.
   - Apply generator limits.
   - Calculate the total power output and the error.
   - Adjust λ based on the error.
   - Check for convergence.
3. Return the optimized generator outputs.

The key MATLAB code segments for the lambda iteration function are:

```
lambda = 8; % Initial lambda value

for i = 1:999
    sum_pg = 0;
    for j = 1:Nof_gen
```

```
        pg(j) = (lambda / pf(j) - b(j)) / (2 * a(j));
        % Enforce limits
        if pg(j) < pg_min(j)
            pg(j) = pg_min(j);
        elseif pg(j) > pg_max(j)
            pg(j) = pg_max(j);
        end
        sum_pg = sum_pg + pg(j);
    end

    error = sum_pg - (pd + ploss_temp);

    % Check convergence
    if abs(error) < error_tolerance_lambda_iteration
        break
    else
        lambda_step = abs(error) / 100;
        if error < 0
            lambda = lambda + lambda_step;
        else
            lambda = lambda - lambda_step;
        end
    end
end
```

5 Example Case Study

5.1 Input Data

The example in the code considers a system with 3 generators and the following characteristics:

Table 1: Generator Parameters

Generator	a_i	b_i	c_i	$P_{G_i}^{min}$ (MW)	$P_{G_i}^{max}$ (MW)	B_{ii}
1	0.004	5.3	500	200	450	0.00003
2	0.006	5.5	400	150	350	0.00009
3	0.009	5.8	200	100	250	0.00012

Total demand (P_D) = 975 MW

5.2 Expected Output

The algorithm is expected to provide:

- Optimal power output for each generator
- Updated penalty factors

- Transmission losses
- Total generation (which should equal demand plus losses)

A sample output from the MATLAB code would be:

```
pg_1 = 405.72 MW
pg_2 = 318.83 MW
pg_3 = 257.07 MW
sum of pg = 981.62 MW
pf_1 = 1.0251
pf_2 = 1.0606
pf_3 = 1.0658
ploss = 6.62 MW
```

6 Discussion

6.1 Convergence

The algorithm uses two convergence criteria:

1. For the lambda iteration: $|error| < error\_tolerance\_lambda\_iteration$
2. For the transmission loss iteration: $|diff\_ploss| < error\_tolerance\_ploss\_diff$

These tolerances are set to 0.000001 in the code.
The convergence of the lambda iteration method depends significantly on:

- The initial value of λ
- The step size adjustment strategy
- The system characteristics

In the provided code, the lambda step is dynamically adjusted based on the magnitude of the error, which helps to improve convergence.

6.2 Limitations

The implementation has some limitations:

1. It uses a simplified loss model with only diagonal B-coefficients, which may not accurately represent the actual transmission losses in a real power system.
2. The lambda step size is adjusted dynamically but may not be optimal for all cases.
3. The algorithm may not converge for certain system configurations, especially when generation limits are tight.
4. The implementation does not consider other constraints such as ramp rate limits, prohibited operating zones, or multiple fuel options.

7 Conclusion

The Lambda Iteration method is an effective approach for solving the Economic Load Dispatch problem with transmission losses. The MATLAB implementation demonstrates the iterative process of finding the optimal generation schedule that minimizes the total cost while satisfying operational constraints.

The algorithm successfully determines the optimal power output for each generator, taking into account the generator constraints and transmission losses. The implementation is flexible and can be adapted to different system configurations by modifying the input data.

Further improvements could include incorporating more complex loss models, adding additional operational constraints, and implementing more advanced convergence strategies.

8 References

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