Modeling and Simulation, MC312
Lab-7 & 8
Due Date: October 31, 2023

In this lab, we will look at the one-dimensional and two dimensional random walk problems. These constitute one of the most basic and important models of stochastic processes with widespread applicability in physical and natural processes. An important objective of this lab is to review your understanding of probability, random processes and the limit theorems.

1. Imagine a one-dimensional discrete lattice with unit spacing. At each time instant, the walker moves to the right with a probability p and to the left with a probability $q = 1 - p$. We first consider an **unbiased random walk** $p = q = 1/2$ problem.
   (a) Simulate this process for n steps and look at the trajectory of the random walker. Trajectory here implies the position of the random walker at steps $1,\ 2, \cdots,\ n$. Plot a few trajectories. Are they the same? If not, why?
   (b) Numerically obtain the expected position of the random walker at each time step. Comment based on your observation.
   (c) Numerically obtain the variance of the random walker at each time step. Comment based on your observation.
   (d) Plot the probability distribution $P_n(m)$, which is the probability that the random walker is at location m after n steps.
2. **Biased random walk** Assume that the random walker has a preferred direction ($p \neq q$). Repeat all the steps of the previous problem and record how the observations change.
3. **(2D random walk on a lattice)** For this part, you should refer to module 9.5 of the book. I also suggest following the book's documentation for proper code development.
   (a) Using the pseudo-code randomWalkPoints, develop the 2D random walk code with parameter n for the number of steps.
   (b) By referring to animateWalk animate your random walk. This kind of visualization technique is essential while conducting simulations of the kind we are doing here.
   (c) Plot the average distance traveled versus the number of steps. Plot it to clearly show the dependence of the average distance traveled on the number of steps.

(d) Modify your code for problem 8 in the Projects section. Based on the understanding developed, note the key observations and provide an appropriate explanation for the observations you make.

Your report should discuss the simulation details also from a Monte Carlo Simulation perspective.