Lab -5

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In this lab, we study some realistic effects in the SIR Model and analyze their role in real-life applications.

I. INTRODUCTION

In this lab report, we investigate the effects of various factors on the spread of infectious diseases using the Susceptible-Infected-Recovered (SIR) model. The SIR model is a widely used mathematical model that helps in understanding the dynamics of infectious diseases by categorizing the population into three groups: Susceptibles (S), Infected (I), and Recovered (R). By analyzing the interactions between these groups, we can gain insights into the duration, intensity, and potential mitigating strategies for epidemics. In this study, we consider factors such as vaccination rates, immunization delays, lockdown measures, and human behavior to analyze their impact on the spread of diseases in real-life scenarios.

II. MODEL

To explain the spread of influenza/disease, we analyze the problem by the SIR Model. The population can be categorized into three parts in accordance with the disease:

- **Susceptibles (S)**, which have no immunity from the disease.
- **Infected (I)**, which are suffering the disease and may infect others too.
- **Recovered (R)**, which have recovered and therefore will not be infected further from the disease.

Their interactions can be given as:

$$\frac{dS}{dt} = -R_0 SI$$

$$\frac{dI}{dt} = R_0 SI - I$$

$$\frac{dR}{dt} = I$$

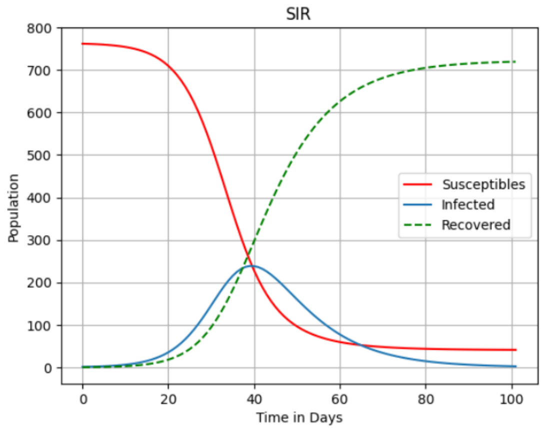

FIG. 1: SIR Model

The above-given expressions can be written as:

$$S(T + \Delta T) = S(T) - R_0 SI\Delta T$$

$$I(T + \Delta T) = I(T) + R_0 SI\Delta t - I\Delta T$$

$$R(T + \Delta T) = R(T) + I\Delta T$$

The model can be seen to be represented as a graph as follows:

III. RESULTS

A. SIR Model with 15% vaccination daily

In the analysis that follows, we expand the conventional SIR model to include a vaccination program that targets susceptible people and has a daily vaccination rate of 15%. Our aim is to find out how the duration and severity of an epidemic are impacted by this vaccination program. The graphical representation can be seen in Fig 2:

- **Susceptible (S) Population:**
  The susceptible curve (S) initially starts at its maximum value, representing the entire population being susceptible to the disease. As vaccination is introduced at a rate of 15% per day, the susceptible population gradually decreases.

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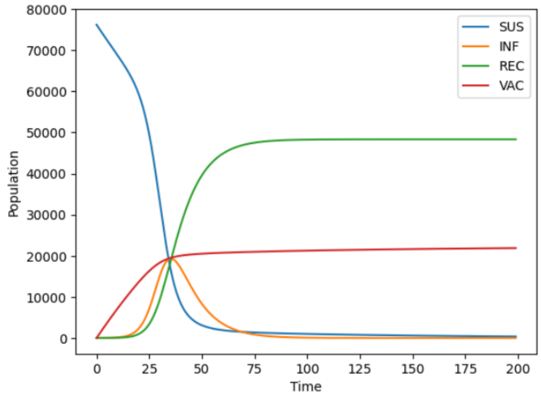

FIG. 2: SIR Model with 15% vaccination daily

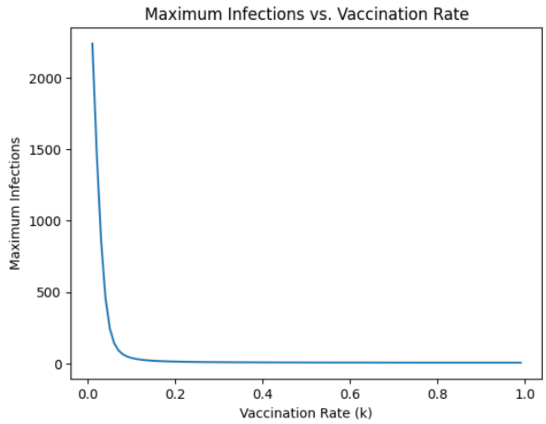

FIG. 3: Increase in Vaccination decreases Intensity of Epidemic

- **Infectious (I) Population:**
The infectious curve (I) grows quickly as the disease spreads across the vulnerable population after starting out at a low value. However, the pace of the rise in infectious people is slowed down since vaccination lowers the population that is susceptible to infection.

- **Recovered (R) Population:**
The recovered curve (R) begins at zero and steadily rises over time as individuals recover from the infection or gain immunity through vaccination.

- **Effect on Epidemic Duration:**
The duration of the pandemic is extended by the implementation of a vaccination program. This extension is visible as a slower fall in infectious cases, which is brought on by the vaccination-related decline in the vulnerable population. As a result, the epidemic lasts longer than it would have in the absence of vaccination.

- **Effect on Epidemic Intensity:**
The peak of the infectious curve (I) is lower due to the vaccination program. This reduction in the peak indicates a decrease in the epidemic's intensity. As the vaccination rate accelerates, the peak further diminishes, highlighting the effectiveness of vaccination in lightening the impact of the epidemic.

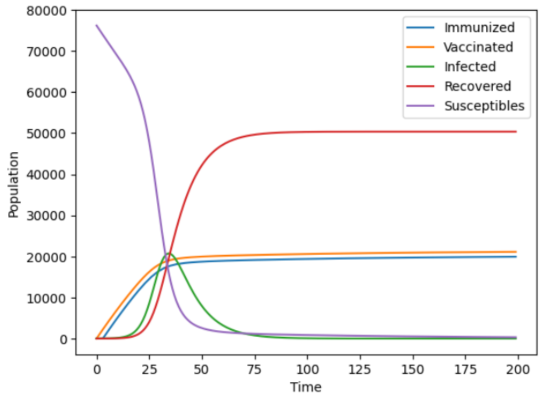

FIG. 4: Daily 15% are vaccinated and immunization begins after three days.

B. Daily 15% are vaccinated and immunization begins after three days.

Coming to the next abstract that adds one more parameter, that being that the immunization begins after three days. To solve the given problem we break the vaccinated compartment into three compartments as the problem can be understood in reference to three days. If we understand the dynamics around three days then we can extend the model further to any number of days as the behavior will be identical then. In the three compartments, we will define the change in the number of people with the following rules.

On the first day a specific number of people enter the first vaccinated compartment, while these are in the compartment and we are given the information that they are immunized only after the third day we know that these are prone to get infected until the third day, after which all the uninfected people remaining in this compartment will be transferred to the immunized compartment. But until the third day, there will be some amount of people moving from the vaccinated to the infected compartment.

Similarly, we will implement these rules in the next two compartments as well and we realize that the dynamics of the compartments are just like the one-compartment multiple dosage problem for each compartments and we obtain a curve referencing our intuition as well. For better comprehension of values, the time axis for the readings has been a little shifted in the figure.

We also infer that the number of vaccinated and immunized will be almost similar but immunized will be always less than vaccinated because of this movement of vaccinated people into the infected compartment. All of which can be seen in Fig 4

Duration and Intensity of the Epidemic
The duration of the epidemic is until the time when no infected remains in the population(infected¡1). Increasing the rate of vaccination will decrease the duration of the epidemic because vaccination of more people will lead to a decrease in the number of susceptible very quickly.

If the peak of the curve for the number of infected people is recognized as the intensity of the epidemic then we see that an increase in the vaccination rate will cause a decrease in the intensity of the epidemic. This phenomenon happens because the pool of susceptible is decreased by immunization due to vaccination. Less number of people getting infected will cause the peak of the infected to be less than the previous values.

C. All children are vaccinated two days before a boy comes down with the flu and immunization begins after four days

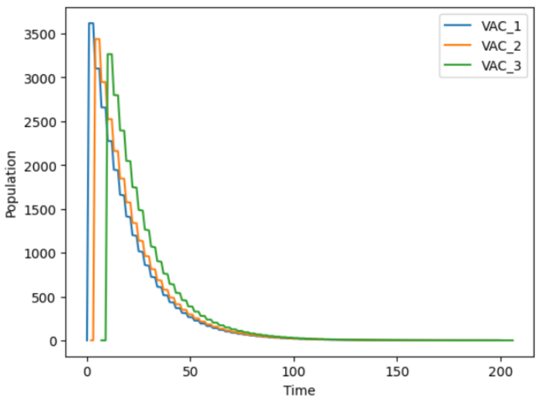

FIG. 5: All children are vaccinated two days before a boy comes down with the flu and immunization begins after four days

Finally coming to the last case (Fig 7), In this case, while the boys get immunized there are chances that a few children have already gotten infected. Now as we complete day four no more susceptible are left so no more infections will happen assuming that recovered and vaccinated can never get infected again. In this case, this will be the duration of the epidemic (2 days). Even though there are infected people who are still not recovered in

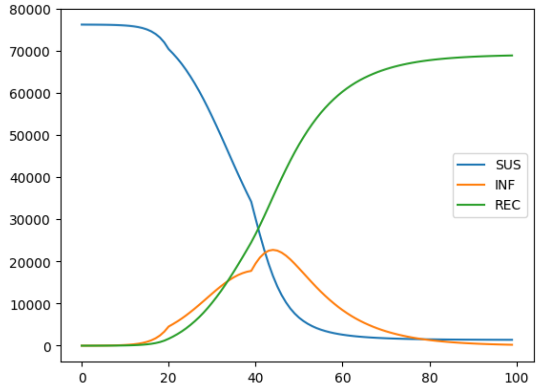

FIG. 6: We considered lockdown to be active since the start, we will analyze the plot for different parameters on shift of the graph based on the changes in the parameters i.e. t_0, t_1, A.

the population we can declare the epidemic over. The intensity of the epidemic will depend on the transmission rate and recovery rate of the disease like the SIR model.

D. Short lockdown with immediate effect (ideal situation)

At first, let's keep A to be a constant $(=\frac{1}{2})$. At first, let's analyze behaviour of t_0 and t_1.

Case 1: When $t_0 = 10$,$t_1 = 20$
The curve flattens out i.e. the number of infected people when lockdown in introduced will decrease which aligns with out intuition. As the number of people from the pool of susceptibles decrease there will be less people to get infected. Also this will cause a delay in the time at which peak occurs as for some time the rate at which infected people are increasing is lowered.

Case 2: When $t_0 = 20$,$t_1 = 40$
As the peak initially lied in the time interval t_0 to t_1, a lockdown at this time will cause the peak to shift to a lower value and the curve to flatten out. This means that the number of infected which was increasing have less number of susceptibles to infect when lockdown is introduced so the peak is not as high as before and as the susceptibles are supplied steadily due to lockdown not being hundred percent efficient we get a flattened curve.

Case 3: Case 3: When $t_0 = 40$,$t_1 = 80$
There is not much effect in this case on the peak of the infected or the time at which the peak is achieved hence this will not be a very effective way of utilizing the lockdown.

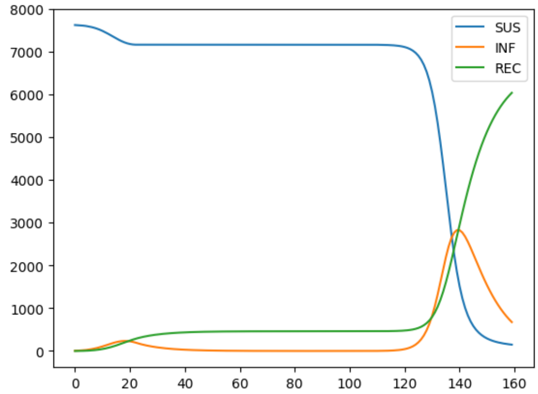

FIG. 7: When human behaviour is considered, peaks are shifted, which shows a more realistic model of spread of disease under lockdown.

Now, when we have analyzed the effect of t_1 and t_2, let's study the effect of the parameter A.

As of A, we would expect a sudden decrease in the number of susceptibles available with increase in A (effectivity of lockdown). If increased effective lockdown is implemented in the begining, then the peak is delayed more. However, we might expect that the number of infected at peak will be higher because we restricted the pool of susceptibles before the peak more than the previous case so after the lockdown the pool will be larger than the previous case and will cause a larger peak. If a lockdown with increased restrictions is implemented during the time when peak is going to occur then the peak would be shifted to the right, later, as well as the peak will be less in terms of the number of people infected.

E. Short lockdown taking the human behaviour into account

Here, we take human behaviour towards the rules of lockdown and carelessness in following them is also taken into account. This model will give us a better understanding and will map more accurately with the actual data.

Initially, θ_t is at a low value (people do not take it seriously) but slowly increases. In the initial stages, people are cautious, so θ_t is at its higher value. In time, other important factors take over, and they venture out and people become more relaxed.

We need to figure out the revised model by incorporating the factors discussed above to the ideal model (Lockdown with immediate effect).

In this case, we observe that there will be two peaks, one when the the lockdown in running which will be relatively quiet small and will occur soon before the lockdown is hundred percent efficient because after that point there will be no susceptibles after that point and the number of infected will decrease to a constant value however after the lockdown is relased we will observe a new peak after some days because the large pool of susceptibles is only now released for the infecteds to infect so it will be like the normal SIR model. By this effect we have just delayed the peak so that meanwhile we can prepare or work on creating medicines and vaccines to fight the epidemic.

IV. CONCLUSIONS

Through our analysis of the SIR model under various conditions, we have observed some of these key findings:

- Increasing the vaccination rate leads to a decrease in both the duration and intensity of the epidemic, highlighting the importance of vaccination programs in controlling the spread of infectious diseases.
- Introducing a short lockdown with immediate effect can help in delaying the peak of the epidemic and reducing the number of infected individuals. However, the effectiveness of the lockdown depends on the timing and the level of restrictions imposed.
- Taking human behavior into account, such as adherence to lockdown rules and carelessness in following preventive measures, can provide a more realistic representation of the disease spread under lockdown conditions.
- The implementation of lockdown measures and vaccination programs can help buy time for the development of medicines and vaccines, which are crucial in combating epidemics.

Further research can be conducted to refine the model and incorporate additional factors, such as varying transmission rates and population demographics, to enhance its applicability in real-world scenarios.

[1] A. Shiflet and G. Shiflet, *Introduction to Computational Science: Modeling an Simulation for the Sciences*, Princeton University Press.3, 276 (2006).

[2] A. Einstein and N. Rosen, Phys. Rev.**48**, 73 (1935).