Engineering Mathematics II (ED 121)

Homework 5

Release Date: 13.02.2024
Due Date: 18.02.2024

1. The probability density function of X and Y is given by $f(x, y) = c(y^2 - x^2)e^{-y}, \quad -y \leq x \leq y,\ 0 \leq y \leq \infty$
(a) Find c.
(b) Find the marginal densities of X and Y.
(c) Find $E[X]$.

2. An ambulance travels back and forth, at a constant speed, along a road of length L. At a certain moment of time an accident occurs at a point uniformly distributed on the road. [That is, its distance from one of the fixed ends of the road is uniformly distributed over $(0, L)$.] Assuming that the ambulance's location at the moment of the accident is also uniformly distributed, compute, assuming independence, the distribution of its distance from the accident.

3. The random variables X and Y have joint density function. $f(x, y) = 12xy(1 - x), \quad 0 < x < 1, 0 < y < 1$ and equal to 0 otherwise. (a) Are X and Y independent? (b) Find $E[X]$.
(c) Find $E[Y]$. (d) Find $Var(X)$. (e) Find $Var(Y)$.

4. The gross weekly sales at a certain restaurant is a normal random variable with mean \$2200 and standard deviation \$230. What is the probability that
(a) the total gross sales over the next 2 weeks exceeds \$5000;
(b) weekly sales exceed \$2000 in at least 2 of the next 3 weeks? What independence assumptions have you made?

5. The joint density function of X and Y is given by $f(x, y) = xe^{-x(y+1)}, \ \ x > 0, y > 0$ (a) Find the conditional density of X, given $Y = y$, and that of Y, given $X = x$.

6. Let X and Y have the joint density function

$$f(x, y) = \begin{cases} x + y & \text{if } 0 < x, y < 1 \\ 0 & \text{elsewhere .} \end{cases}$$

What is the covariance between X and Y ?

7. Let the probability density function of a random variable X be

$$f(x) = \begin{cases} 630x^4(1 - x)^4 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

What is the exact value of $P(|X - \mu| \leq 2\sigma)$? What is the approximate value of $P(|X - \mu| \leq 2\sigma)$ when one uses the Chebychev inequality?

8. Let X and Y be discrete random variables with joint probability function

$$f(x,y) = \begin{cases} \frac{1}{21}(x+y) & \text{for } x = 1,2,3; y = 1,2. \\ 0 & \text{elsewhere.} \end{cases}$$

What are the marginal probability density functions of X and Y ?

9. The score distribution of an exam is modelled by a random variable X with range $\Omega_X = [0, 110]$ (with 10 points for extra credit). Give an upper bound on the proportion of students who score at least 100 when the average is 50 ? When the average is 25?

10. Let $Y = X_1 + X_2 + \cdots + X_{15}$ be the sum of a random sample of size 15 from the distribution whose density function is

$$f(x) = \begin{cases} \frac{3}{2}x^2 & \text{if } -1 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

What is the approximate value of $P(-0.3 \le Y \le 1.5)$ when one uses the central limit theorem?