Engineering Mathematics II (ED 121)

Homework 4

January 31, 2024

1. The percentage of impurities per batch in a certain chemical product is a random variable X that follows the beta distribution given by

$$f(x) = \begin{cases} 60x^3(1-x)^2 & \text{for } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

   What is the probability that a randomly selected batch will have more than 25% impurities?

2. If $X \sim \bigwedge(0, 4)$, then what is the probability that X is between 1 and 12.1825?

3. Consider two random variables X and Y with joint PMF given in Table below.
   a. Find $P(X \leq 2, Y \leq 4)$.
   b. Find the marginal PMFs of X and Y.

	$Y = 2$	$Y = 4$	$Y = 5$
$X = 1$	$\frac{1}{12}$	$\frac{1}{24}$	$\frac{1}{24}$
$X = 2$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{8}$
$X = 3$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{12}$

4. A group of 9 executives of a certain firm include 4 who are married, 3 who never married, and 2 who are divorced. Three of the executives are to be selected for promotion. Let X denote the number of married executives and Y the number of never married executives among the 3 selected for promotion. Assuming that the three are randomly selected from the nine available, what is the joint probability density function of the random variables X and Y ?

5. If Z is a standard normal random variable, what is Cov(Z, Z^2)?

6. Consider two six-sided fair dice, and let X and Y be the outcomes of the first and second dice rolls, respectively.
   1. Define the joint probability mass function $P(X = x, Y = y)$ for all possible pairs (x, y).
   2. Calculate the marginal probability mass functions $P(X = x)$ and $P(Y = y)$.
   3. Determine the probability that the sum of the two dice rolls is even, $P(X + Y \text{ is even})$.

7. Consider two random variables, X and Y, with joint probability mass function given by:

$$P(X = i, Y = j) = \frac{1}{2^{i+j}},$$

   for $i, j = 0, 1, 2, \ldots$.

1. Determine the marginal probability mass functions $P(X = i)$ and $P(Y = j)$ for all possible values of i and j.
2. Find $E(X)$, the expected value of X.
3. Calculate $Cov(X, Y)$, the covariance between X and Y.