

Quiz 2

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Q1

Let the matrix be

$$M = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 2 & 4 \\ 4 & 3 \end{bmatrix}_{4 \times 2}$$

$$M^T M = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 2 & 1 & 4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 2 & 4 \\ 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 24 \\ 24 & 30 \end{bmatrix}_{2 \times 2}$$

Eigenvalues

$$[M^T M - \lambda I_2] = \begin{bmatrix} 25 - \lambda & 24 \\ 24 & 30 - \lambda \end{bmatrix}$$

$$|M^T M - \lambda I_2| = (25 - \lambda)(30 - \lambda) - 24^2$$

$$\begin{aligned}\therefore (\lambda - 25)(\lambda - 30) &= 24^2 \\ \Rightarrow \lambda^2 - 55\lambda + 750 - 24^2 &= 0 \\ \Rightarrow \lambda^2 - 55\lambda + 174 &= 0\end{aligned}$$

$$\begin{aligned}\therefore \lambda &= \frac{55 \pm \sqrt{55^2 - 4 \cdot 174}}{2} \\ &= \frac{55 \pm 48.26}{2} \\ &= 51.63, \underline{3.37}\end{aligned}$$

$$\therefore \text{MTM} \times \underset{\substack{\uparrow \\ \text{eigenvector}}}{q} = 51.63 q$$

Let $q = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\therefore \begin{bmatrix} 25 & 24 \\ 24 & 30 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 51.63 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$25x + 24y = 51.63x \Rightarrow 24y = 26.63x$$

$$24x + 30y = 51.63y \Rightarrow 24x = 21.63y$$

~~$x = \frac{24y}{26.63}$; Normalised x, y is~~

~~$\frac{24y}{26.63} \times 24 = 21.63y$~~

~~$\frac{24}{26.63} = 1$~~

~~$y = \frac{26.63}{24} = 1.11$~~

Normalized e_1

$$e_1 = \begin{bmatrix} 1 \\ 1.45 \\ 1.11 \\ 1.45 \end{bmatrix}$$

Similarly $e_2 = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\therefore \begin{bmatrix} 25 & 24 \\ 24 & 30 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 3.37 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore 25x + 24y = 3.37x \Rightarrow 24y = -21.63x$$

$$24x + 30y = 3.37y \Rightarrow 24x = -26.63y$$

Let $x = 1 \Rightarrow y = \frac{-21.63}{24} \approx -0.90$

Normalized e_2

$$e_2 = \begin{bmatrix} 1 \\ 1.81 \\ -0.9 \\ 1.81 \end{bmatrix}$$

\therefore Since k is low \rightarrow discarding
to reach target dimension of 1

∴ Answer :

$$M. \& [e] = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 2 & 4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1.48 \\ 1.11 \\ 1.48 \end{bmatrix}_{2 \times 2}$$

$$= \begin{bmatrix} 3.22 \\ 1.48 \\ 3.11 \\ 6.44 \\ 1.48 \\ 7.33 \\ 1.48 \end{bmatrix} \therefore \text{Ans}$$

Q2

Dimension Reduction

or
Order Reduction

Q3

There's a tradeoff between ~~loss~~ of ~~loss~~ loss of information due to dimensional reduction and the reduction of dimensions itself to same time.

Q4

~~Q4~~ First of all, it seems as though guys are more interested in sci-fi movies and not at ~~all~~ all interested in romantic movies. ~~Q4~~ Girls, on the other hand, ~~seem~~ ~~to~~ ~~be~~ while having reviewed ~~the~~ some sci-fi movies, are still more interested in romantic movies going by their interest scores.

This is validated, by the SVD decomposition obtained, the singular values indicate that that there are 2 major concepts that can be gleaned from the data while the third one is comparatively insignificant.

So, it can be intuited that the 2 major concepts are sci-fi ~~no~~ movies and romantic movies. The third category seems to have arisen from girls' ~~a~~ negligible interest in sci-fi movies.