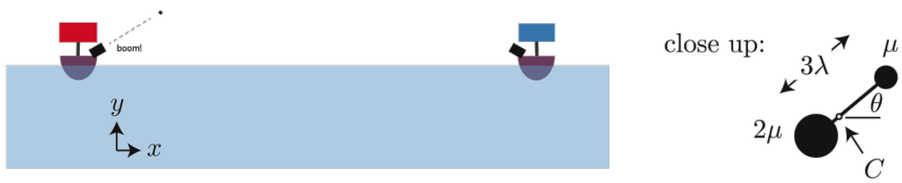
MEGR 3121 HW #6

Due at class time on Thursday 21 March 2024 via Dropbox
`https://www.dropbox.com/request/XQf4rBcpcMk3ycPxv8FU`

1. In the days of wooden ships and iron men, a variety of objects were fired from cannons during sea battles, including metal barbells. Let's analyze the linear and angular momentum of an asymmetric barbell comprising a ball with mass μ and a ball with mass 2μ separated by a bar with length 3λ, hurtling through the air and tumbling along the way.



We'll assume that the bar between the balls is effectively massless, so that the barbell's center of mass C is a distance λ from the center of the larger ball and a distance 2λ from the center of the smaller ball, and we'll assume that each ball can be treated as a point mass.

Let O denote the origin of the inertial coordinate system shown and let $x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}$ be the position of C relative to O. Let θ represent the angle shown, and assume that x, y, and θ are all changing with time as the barbell flies through the air. Let gravitational potential energy equal zero at O.

Noting that the total linear momentum, angular momentum, kinetic energy, or potential energy in a system can be obtained by summing the contributions of the system's individual components, compute

(a) the barbell's total linear momentum;

(b) the barbell's total angular momentum with respect to O;

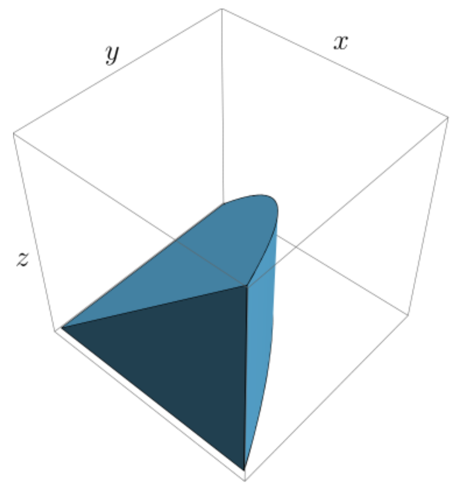
(c) the barbell's total angular momentum with respect to C;

(d) the barbell's total kinetic energy;

(e) the barbell's total potential energy.

Suppose that the barbell were replaced by a single ball with mass 3μ at the point C. Which of the quantities computed above would remain the same? Which would remain the same only in the special case in which $\dot{\theta} = 0$?

2. You've just acquired this object...



... and you're eager to begin swinging it around various axes, but you're not sure how big a motor you'll need to get the swinging performance you desire. The object has uniform density ρ, and if you define coordinate directions as shown with the point $(x, y, z) = (0, 0, 0)$ at the object's bottom-left corner, then the object occupies the region in which

$$0 \leq x \leq 1, \qquad 0 \leq y \leq 1 - x^2, \qquad 0 \leq z \leq x,$$

where x, y, and z are measured in meters.

In terms of ρ, compute the object's mass and the components of its inertia matrix.

(You're welcome to use a computer to complete the tedious integrals involved in this problem, but if you do so then you should submit a copy of your computer code with your responses.)