Light propagation through cosmic structures and new numerical approaches in cosmology

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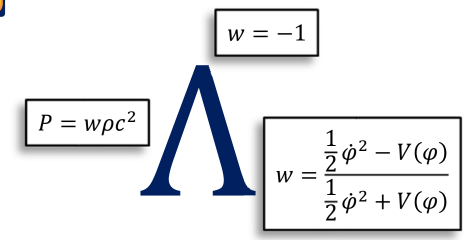
under the supervision of Jean-Michel Alimi and Yann Rasera

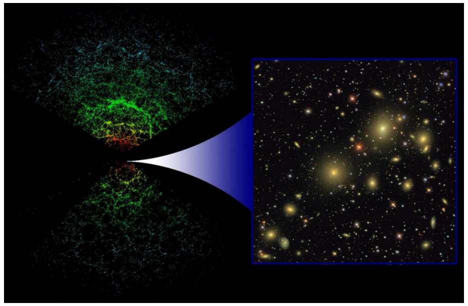
Ph.D. Defense

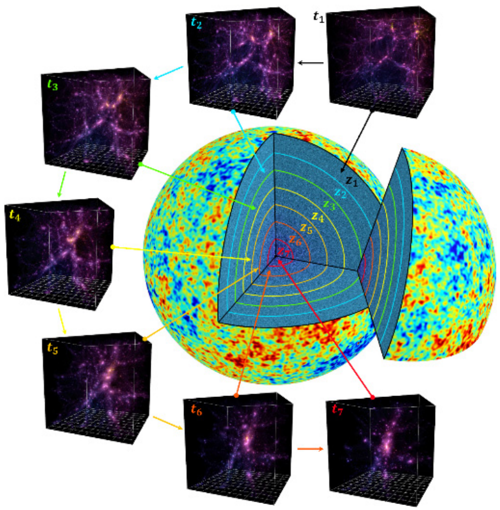
November 12th, 2014

In memory of Franck and Arnaud Reverdy

Part 1: Introduction and outline



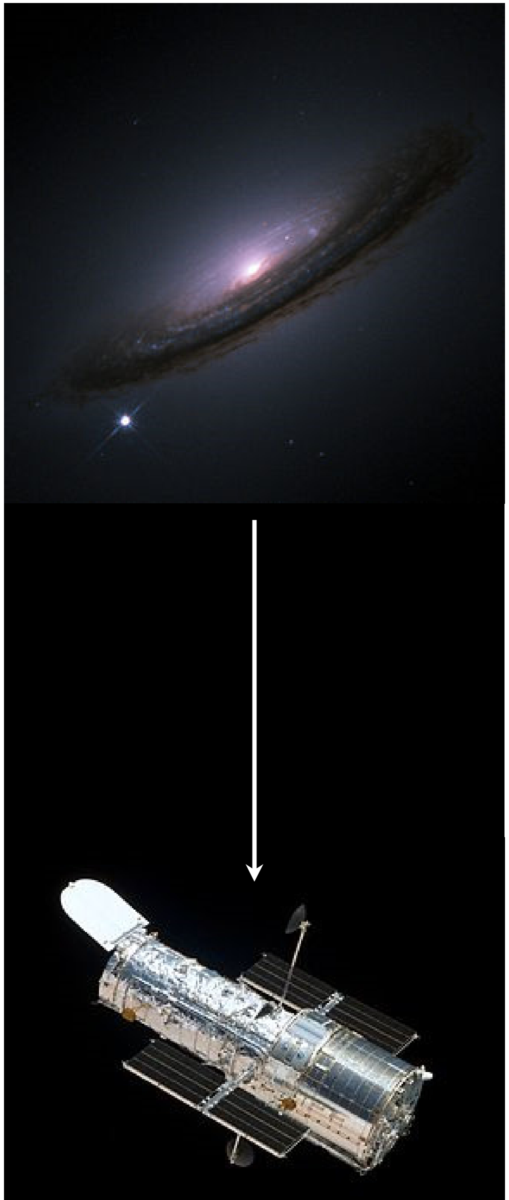




Part 2

The theoretical context of homogeneous cosmology

Interpretation framework



GR equation

$$G_{\mu\nu} = \frac{8\pi G}{c^4} \mathrm{T}_{\mu\nu}$$

- $G_{\mu\nu}$: Einstein tensor
- $\mathrm{T}_{\mu\nu}$: Stress-energy tensor

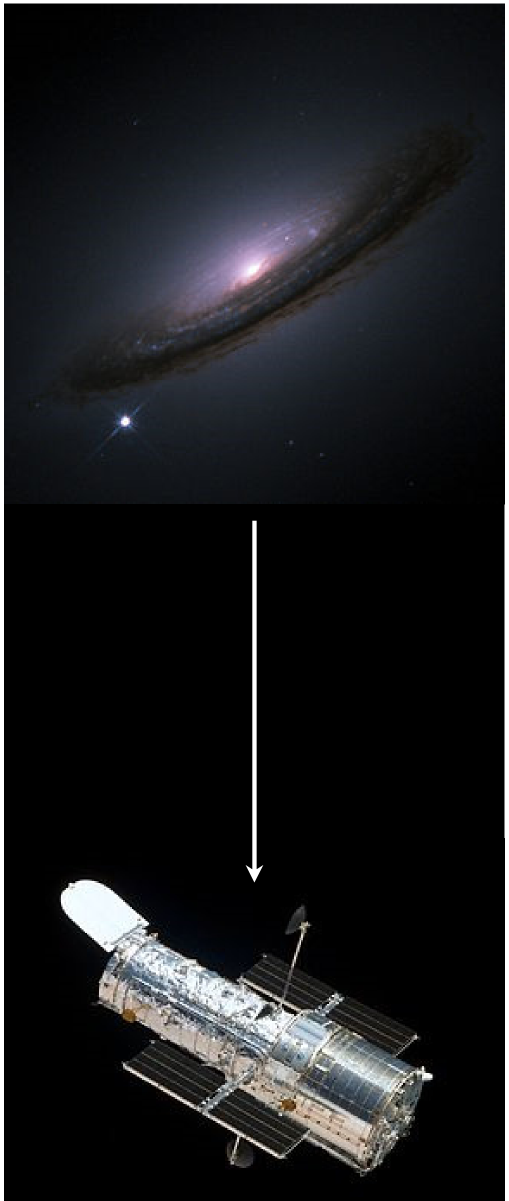
No symmetry = no simplifications

$$G_{\mu\nu} = \begin{pmatrix} G_{tt} & G_{tx} & G_{ty} & G_{tz} \\ G_{tx} & G_{xx} & G_{xy} & G_{xz} \\ G_{ty} & G_{xy} & G_{yy} & G_{yz} \\ G_{tz} & G_{xz} & G_{yz} & G_{zz} \end{pmatrix} \quad T_{\mu\nu} = \begin{pmatrix} T_{tt} & T_{tx} & T_{ty} & T_{tz} \\ T_{tx} & T_{xx} & T_{xy} & T_{xz} \\ T_{ty} & T_{xy} & T_{yy} & T_{yz} \\ T_{tz} & T_{xz} & T_{yz} & T_{zz} \end{pmatrix}$$

- 10 scalar equations
- 6 independent scalar equations (Bianchi identities)
- At each position in space-time

▸ **Interpretation of observational data is difficult**

Interpretation framework



Cosmological principle

At very large scales:

- the Universe is homogeneous
- the Universe is isotropic

Fluid description

The contents can be modeled as a mix of perfect fluids:

- of density ρ_X
- of pressure P_X
- of state equation $w_X = P_X / \rho_X c^2$

GR equations for a FLRW metric

$$G_{\mu\nu} = \begin{pmatrix} G_{tt} & G_{tx} & G_{ty} & G_{tz} \\ G_{tx} & G_{xx} & G_{xy} & G_{xz} \\ G_{ty} & G_{xy} & G_{yy} & G_{yz} \\ G_{tz} & G_{xz} & G_{yz} & G_{zz} \end{pmatrix} \quad T_{\mu\nu} = \begin{pmatrix} \rho c^4 & T_{tx} & T_{ty} & T_{tz} \\ T_{tx} & P & T_{xy} & T_{xz} \\ T_{ty} & T_{xy} & P & T_{yz} \\ T_{tz} & T_{xz} & T_{yz} & P \end{pmatrix}$$

▸ **Standard interpretation framework**

Interpretation framework

Evolution equation

$$\frac{H}{H_0} = \Omega_R a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda$$

- a: the scale factor
- $H = \dot{a}/a$: the Hubble parameter ($h = H/(100km/s/Mpc)$)
- $\Omega_X = \rho_c/\rho$: density parameter
- ρ_c: critical density (flat Universe)
- R: radiation
- M: matter
- k: curvature
- Λ: cosmological constant

Redshift z (general)

$$1 + z = \frac{\nu_S}{\nu_O} = \frac{\left(g_{\mu\nu} k^\mu k^\nu\right)_S}{\left(g_{\mu\nu} k^\mu k^\nu\right)_O}$$

Redshift z (in FLRW)

$$1 + z = \frac{a_0}{a}$$

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Cosmological distances

Comoving distance

$$\chi = \int_{\eta_S}^{\eta_O} c\mathrm{d}\eta = \int_{t_S}^{t_O} c\,\frac{\mathrm{d}t}{a(t)}$$

- t: cosmic time
- η: conformal time $\mathrm{d}\eta = \frac{\mathrm{d}t}{a}$

Angular diameter distance

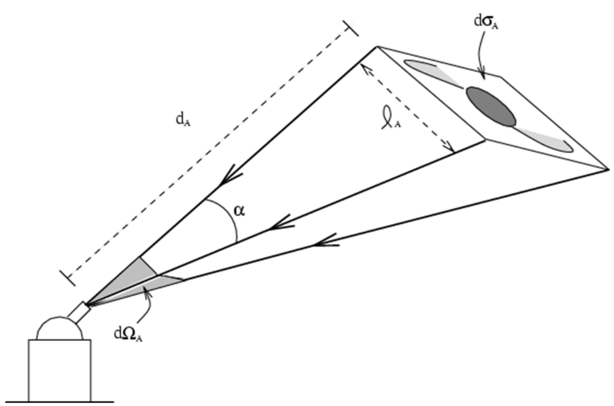
$$d_A = \frac{x}{\theta} \Rightarrow d_A = \frac{\chi}{1+z}$$

- x: object size
- θ: observed angle

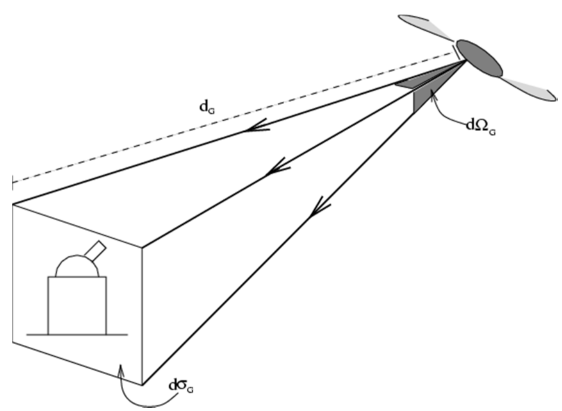
Luminosity distance

$$d_L = \sqrt{\frac{L}{4\pi F}}$$

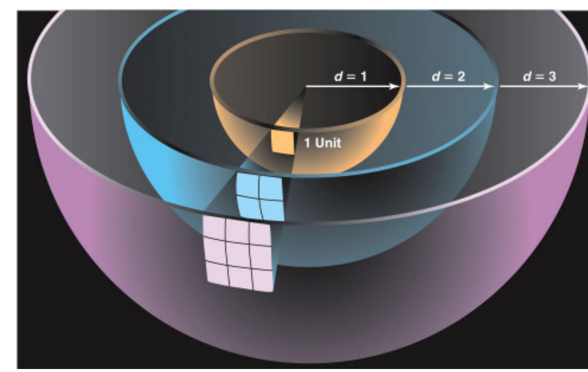
- L: intrinsic luminosity
- F: flux



Angular diameter distance



Luminosity distance

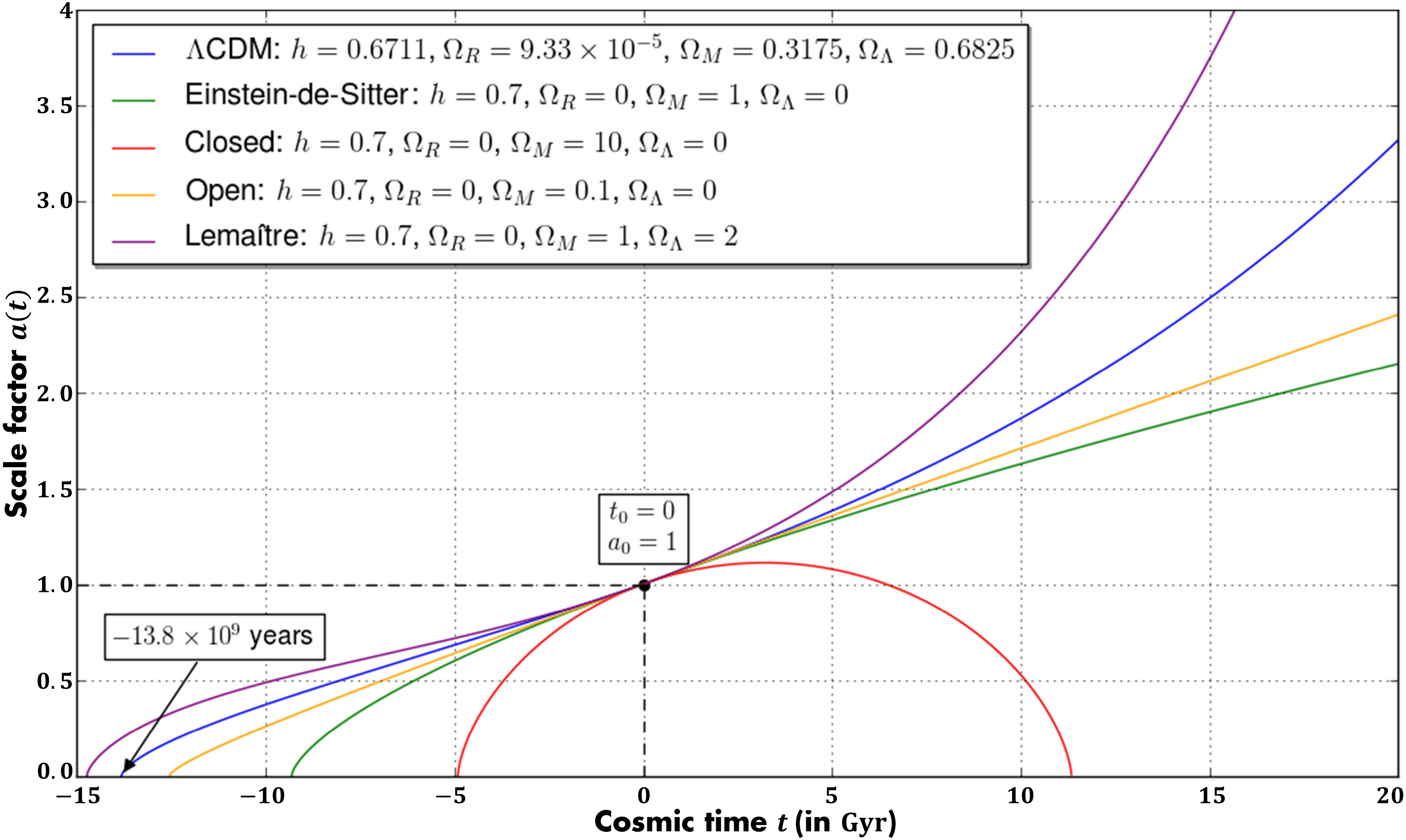


Flux dilution

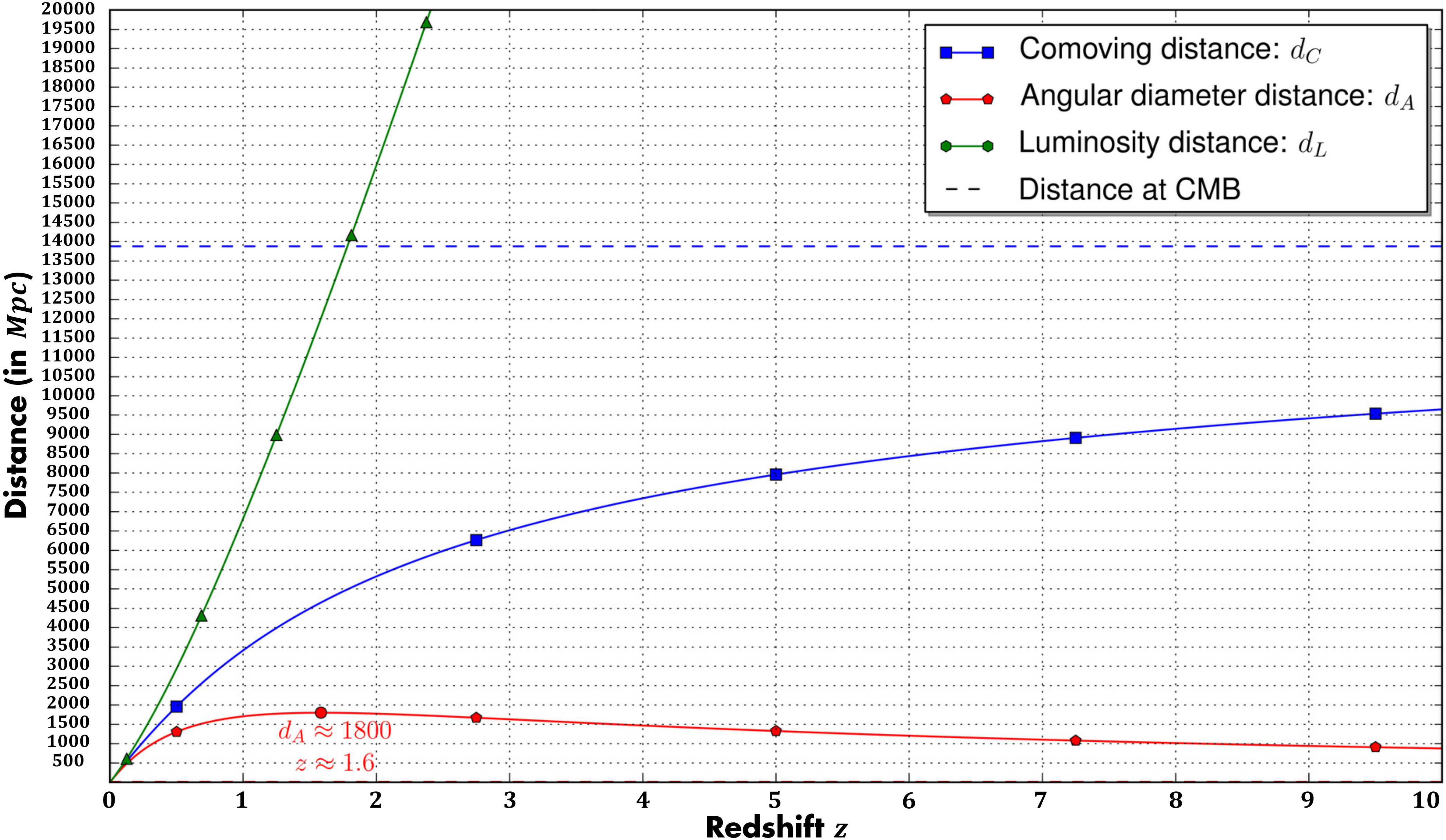
Etherington relation

$$d_L = (1+\mathrm{z})^2 \mathrm{d_A}$$

Examples of FLRW universes



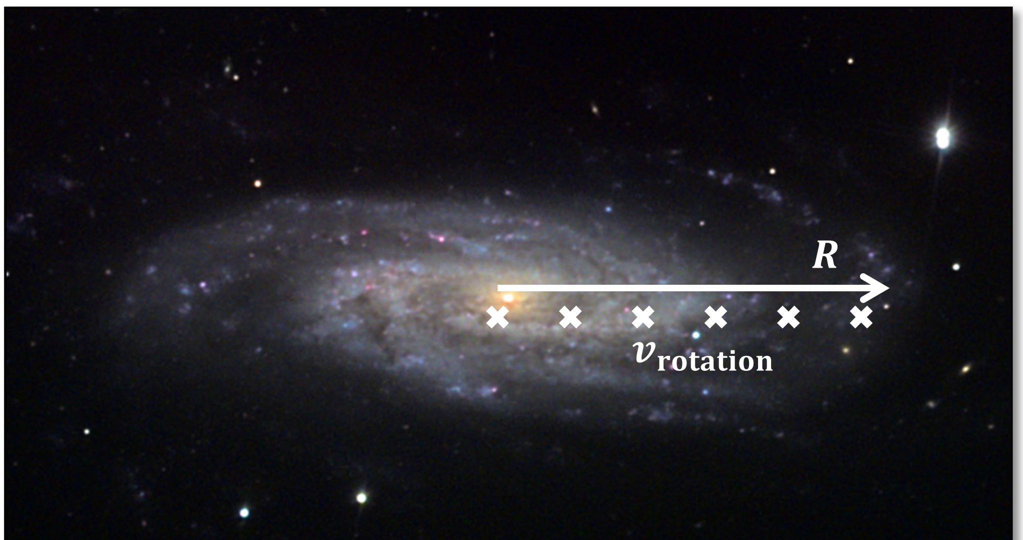
Cosmological distances in ΛCDM



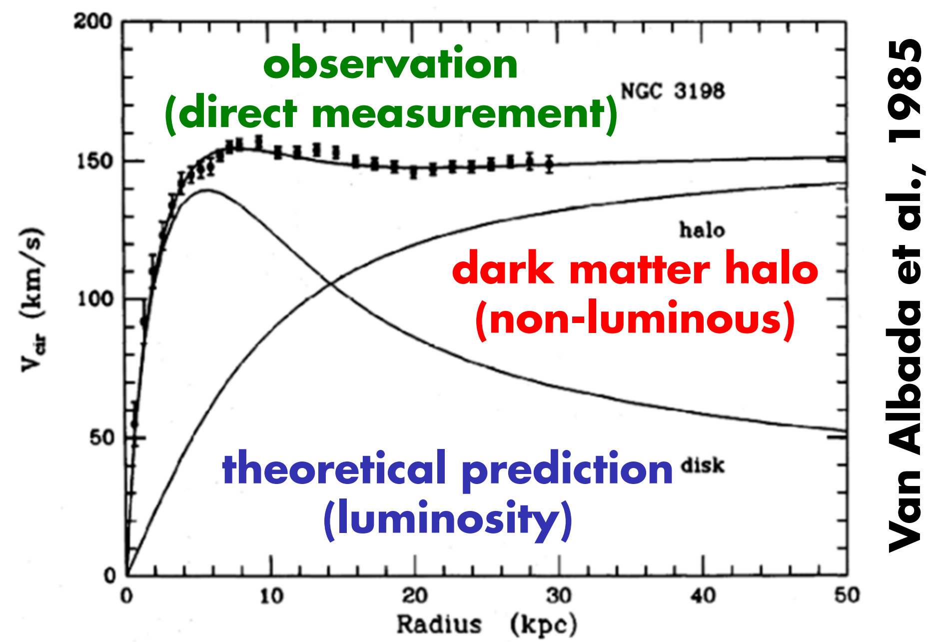
The invisible Universe: dark matter

Dark matter

- Historically introduced for clusters dynamics (Zwicky ~1930) and galactic dynamics (Rubin ~1970)



NGC 3198 (Credit : HST)



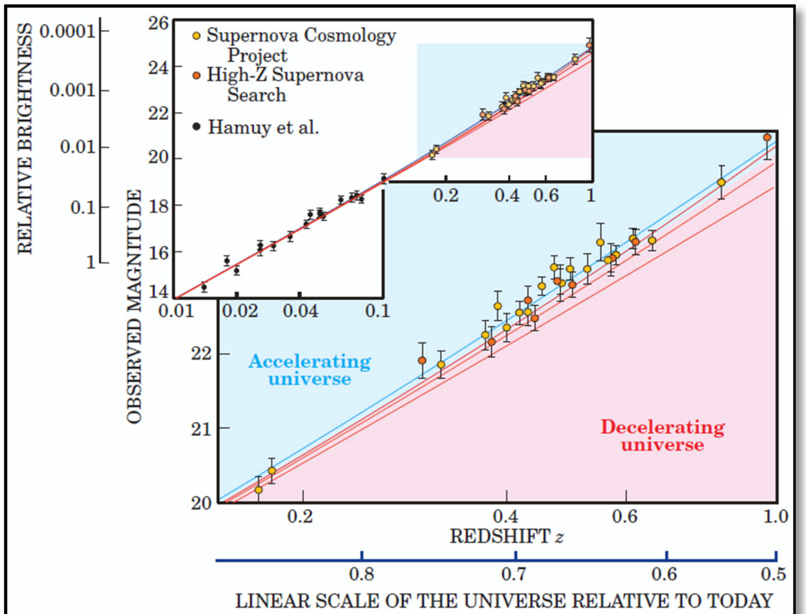
Approaches

- Ontological: dark matter…
- Legislative: MOND theory…

The invisible Universe: dark energy

Dark energy

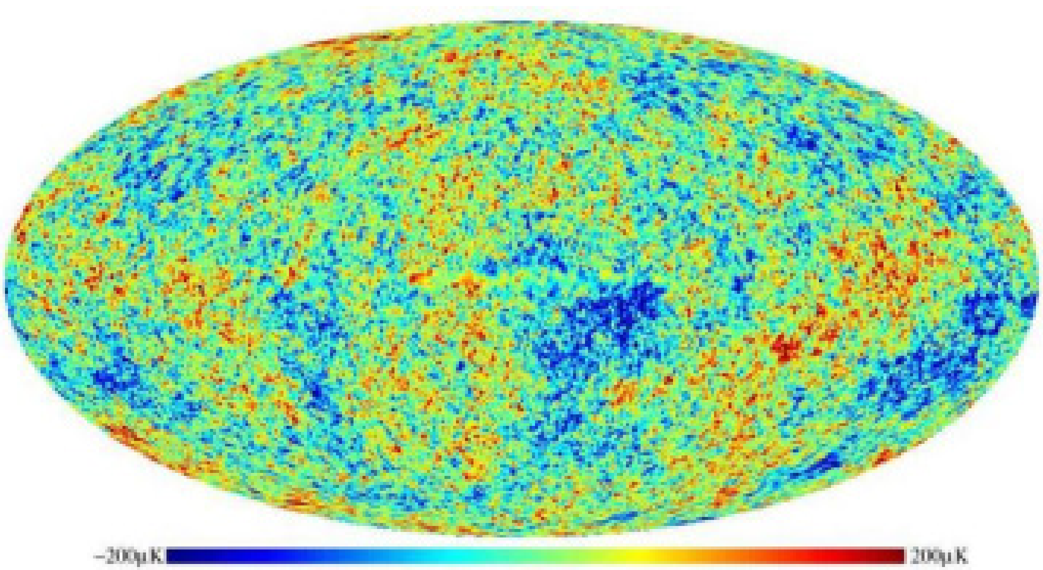
- Model the accelerated expansion (1998)

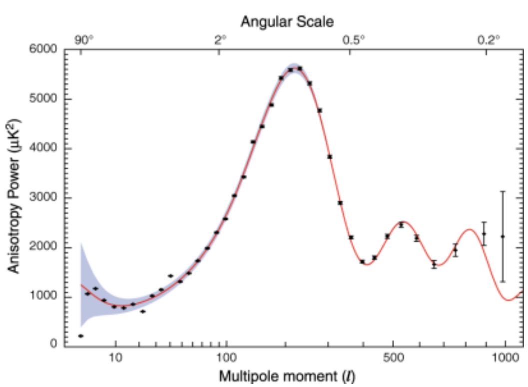


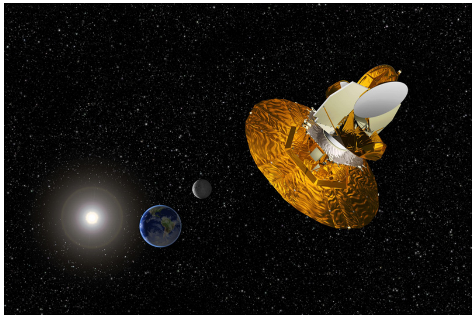
Supernovae measurements

Solutions

- Ontological: dark energy...
- Legislative: extension of GR...
- Paradigmatic: backreaction...







Cosmic Microwave Background

ΛCDM, RPCDM, WCDM: definition

Exploring the nature of dark energy

▸ **3 realistic models of dark energy to explore its imprints**

Concordance model: ΛCDM

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} \mathrm{T}_{\mu\nu}$$

- Λ: dark energy ⇒ cosmological constant
- $\Omega_M = 0.2573,\ \Omega_{DE} = 0.7426,\ \sigma_8 = 0.8$

Ratra-Peebles quintessence: RPCDM

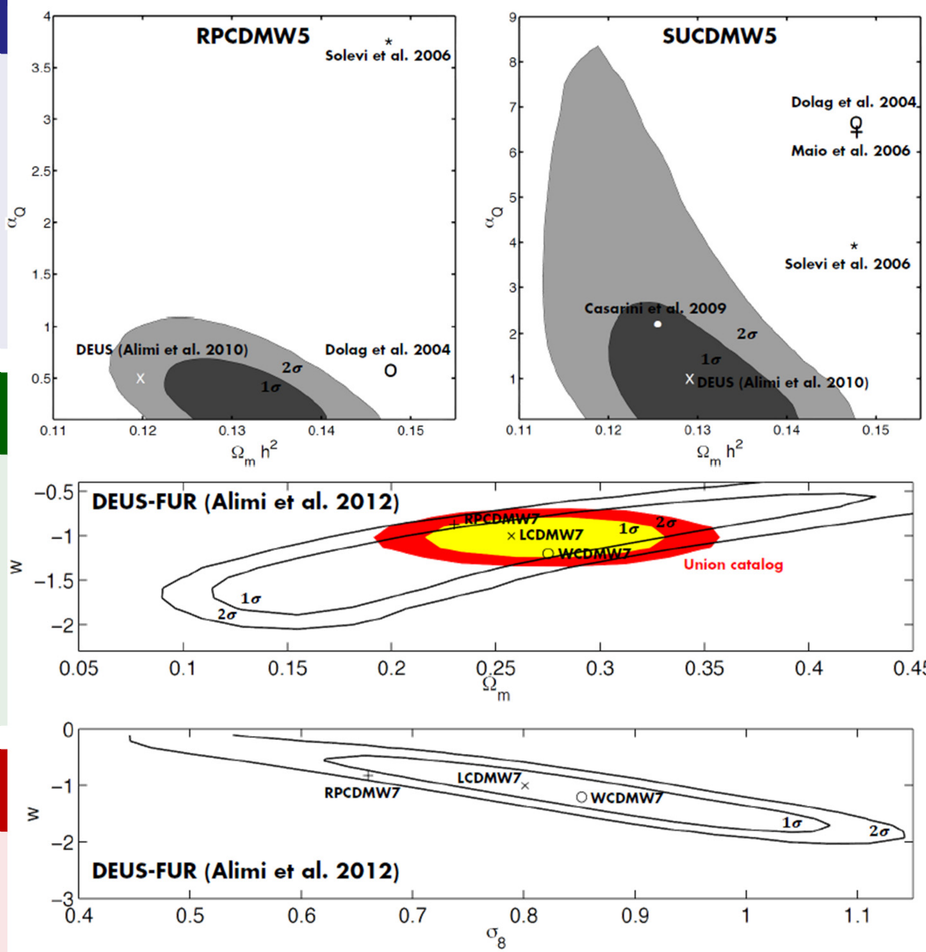
$$w_Q = \frac{P_Q}{\rho_Q} = \frac{\frac{\dot{Q}^2}{2} - V(Q)}{\frac{\dot{Q}^2}{2} + V(Q)} \text{ avec } V_{RP}(Q) = \frac{\lambda^{\alpha+4}}{Q^\alpha}$$

- $\Omega_M = 0.23,\ \Omega_{DE} = 0.77,\ \sigma_8 = 0.66$

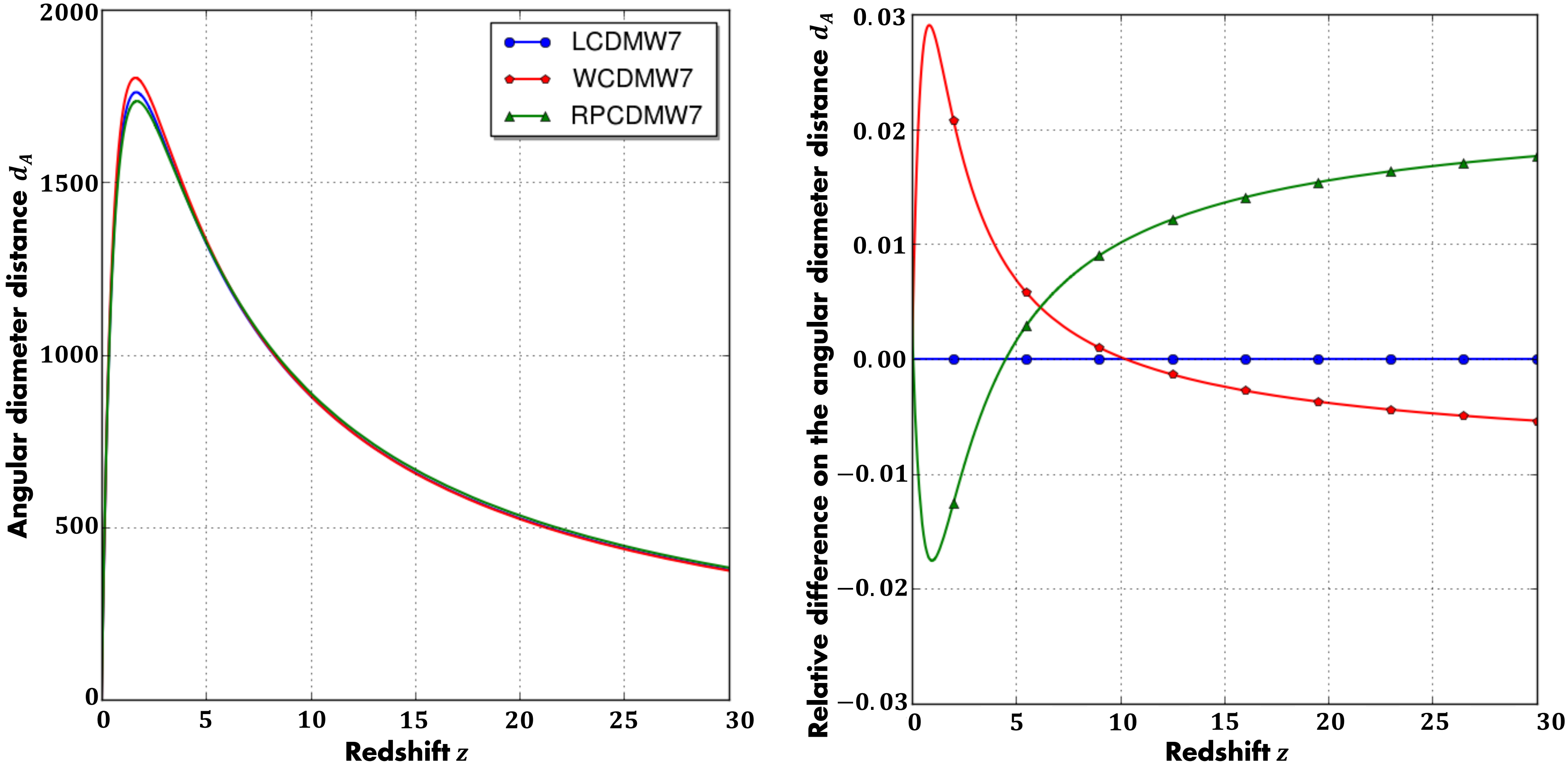
Phantom dark energy: WCDM

$$w_{DE} = -1.2$$

- $\Omega_M = 0.275,\ \Omega_{DE} = 0.725,\ \sigma_8 = 0.852$



ΛCDM, RPCDM, WCDM: d_A



Angular diameter distances

- 3 natures of dark energy ⇒ 3 different $a(t)$ ⇒ 3 different homogeneous $d_A(z)$

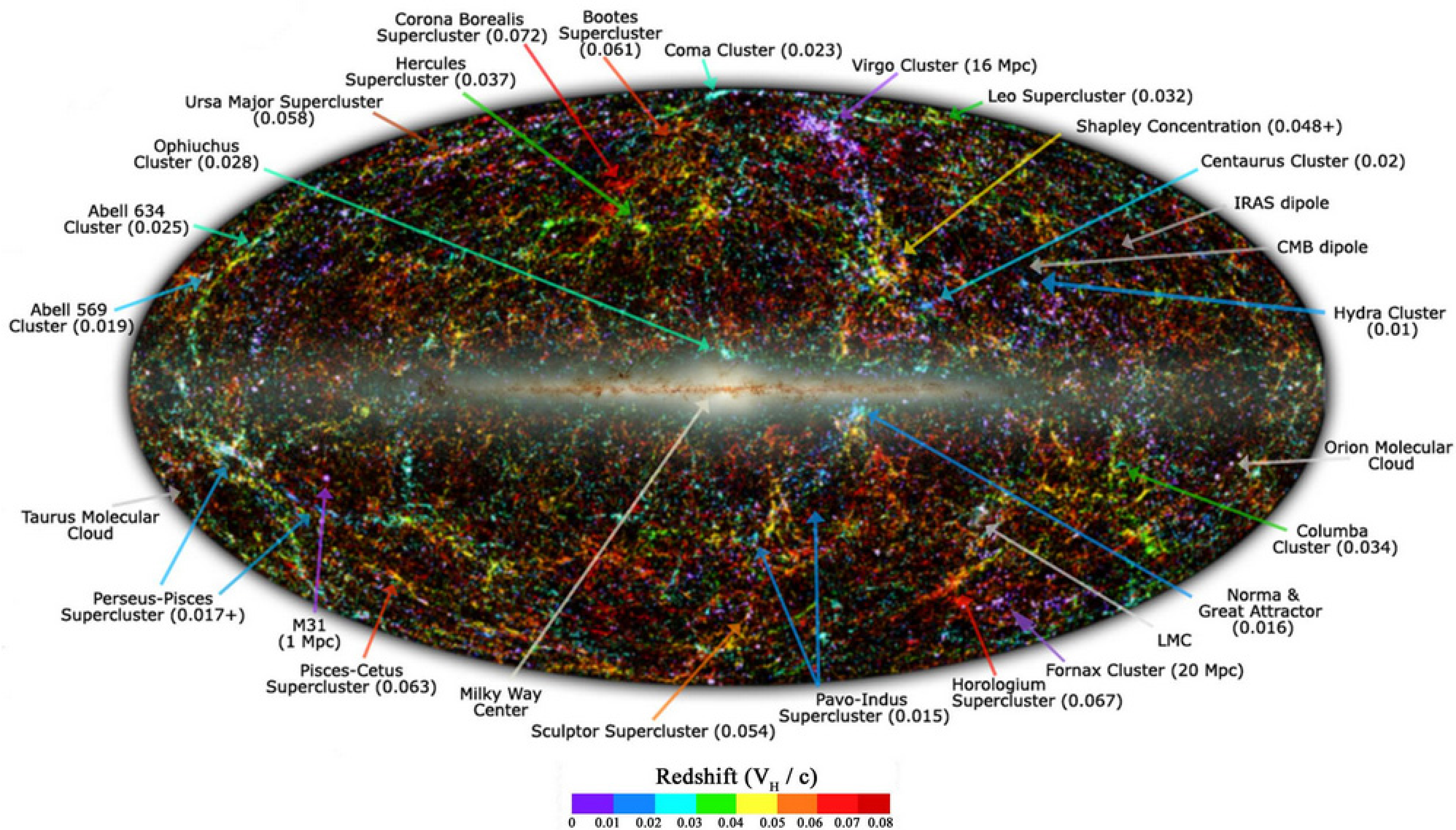
Homogeneity to inhomogeneity

Take into account matter field inhomogeneities...

Part 3

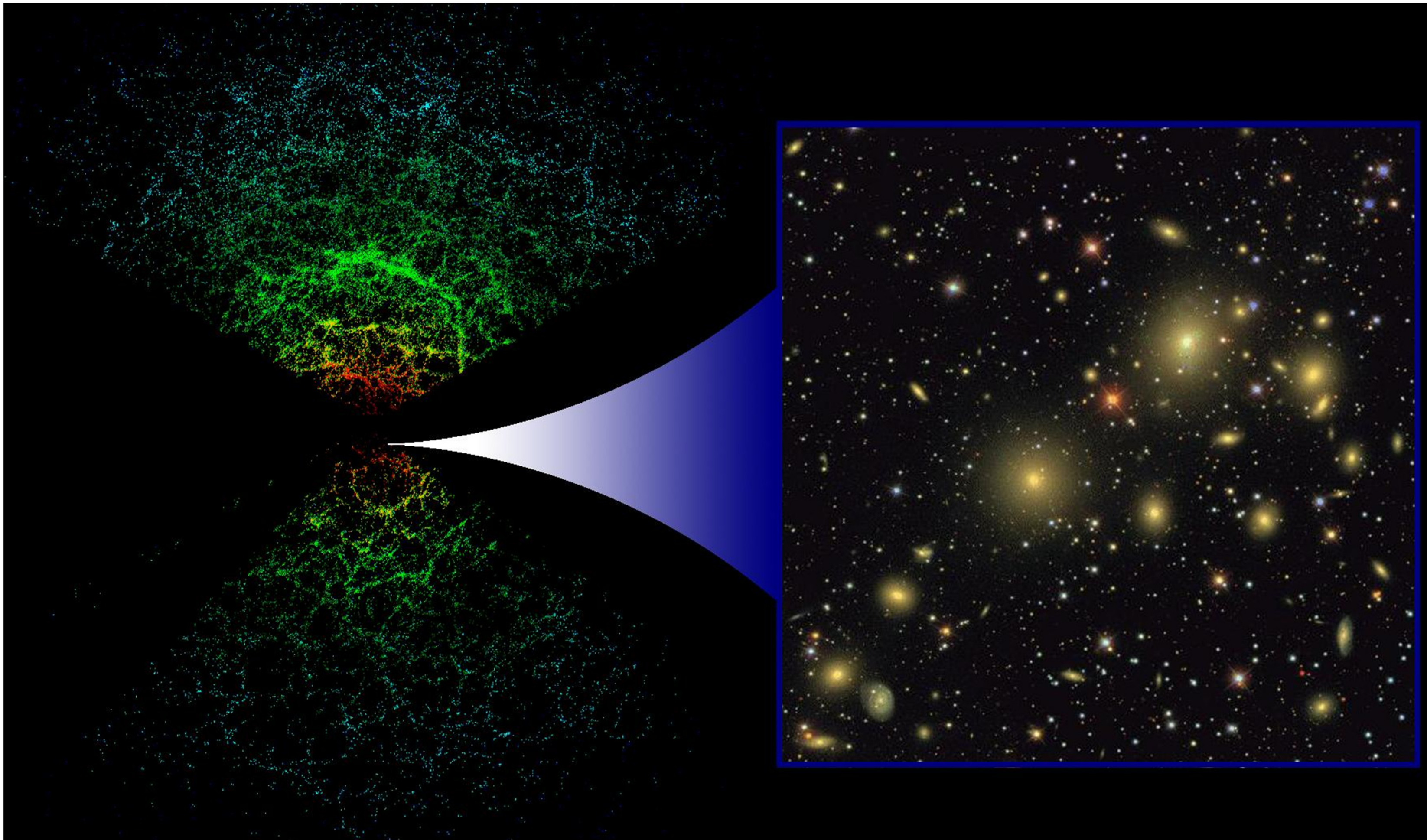
A structured Universe

The local Universe



Sources: 2dF Galaxy Redshift Survey, 2MASS Redshift Survey

Cosmic structures



Large scale structures (LSS)

- **Imprints of dark energy on LSS?**
- **Nature of dark energy from LSS?**
- **Effects of inhomogeneities on light propagation/observations?**

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Gravitational collapse

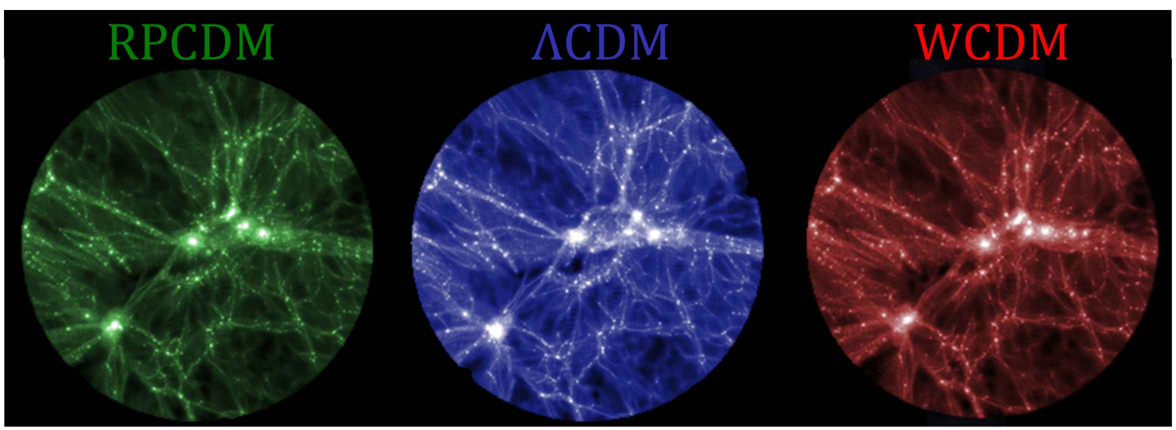
Vlasov-Poisson equations

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{\vec{p}}{a^2}\cdot\frac{\partial f}{\partial \vec{x}} - \vec{\nabla}\Phi\cdot\frac{\partial f}{\partial \vec{p}} = 0$$

$$\nabla^2\Phi = 4\pi G a^2 \bar{\rho}\delta \qquad \delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

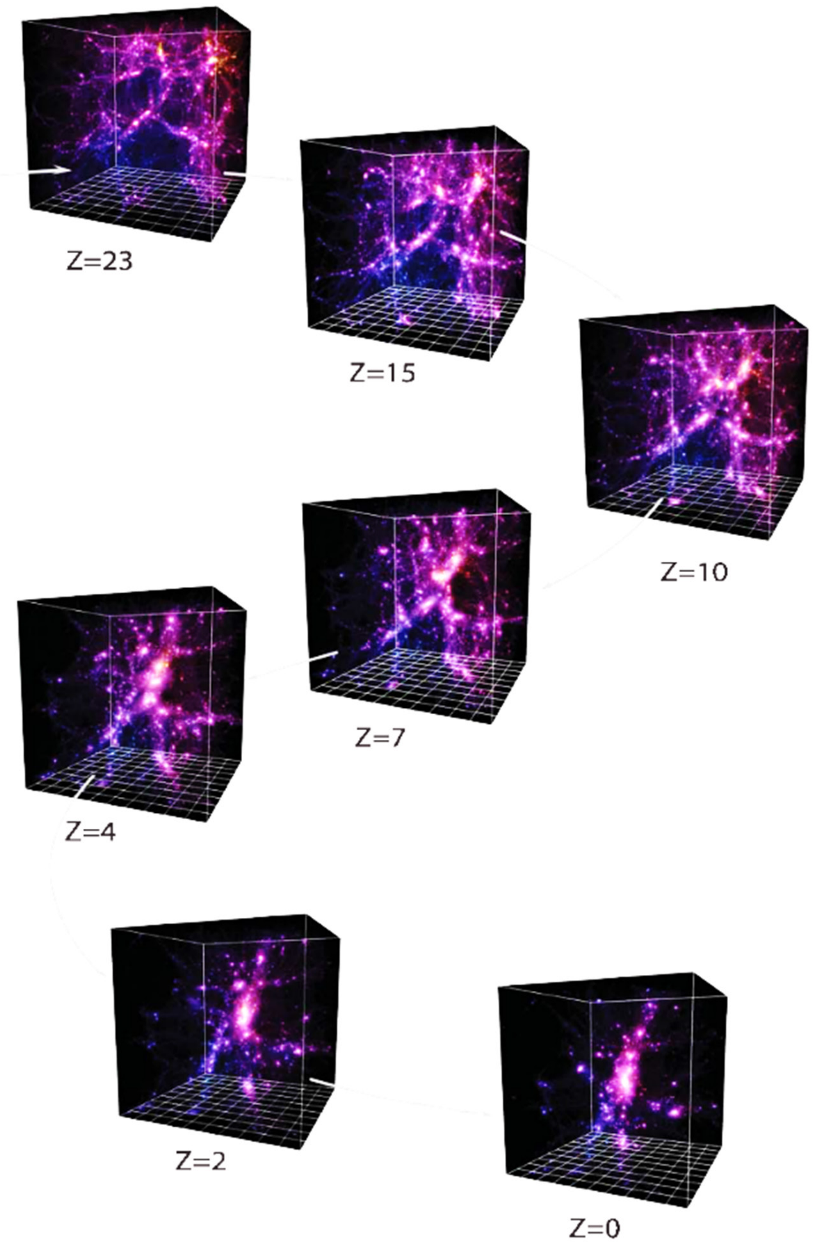
2 regimes

- linear regime $\delta \ll 1$
- non-linear regime $\delta > 1$



Numerical simulation

- **Need of HPC to explore non-linearities**



Simulation of LSS formation

N-body simulation

- N-body simulations of interacting (dark matter) particles in an FLRW metric

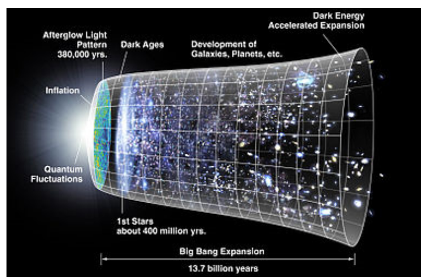
$$\frac{\mathrm{d}\vec{x}}{\mathrm{d}t} = \vec{v} \qquad \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -\vec{\nabla}\Phi$$

$w = -1$

$P = w\rho c^2$ Λ $w = \dfrac{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)}{\frac{1}{2}\dot{\varphi}^2 + V(\varphi)}$

Cosmological model

\+



Homogenous metric of an expanding Universe

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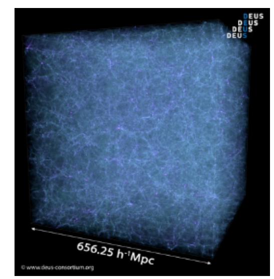
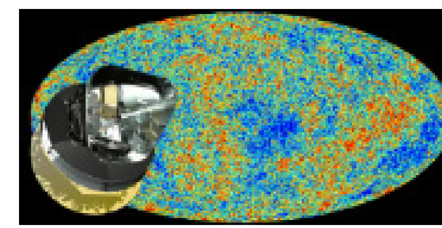


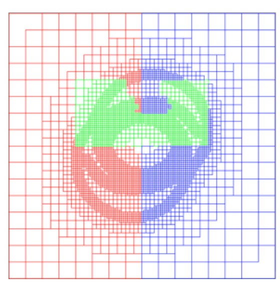
Matter particles in a simulation box

\+



Initial conditions compatible with observations

\+ **Poisson equation solver (quasi-newtonian gravity) (RAMSES [Teyssier, 2002])**

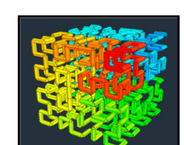


\+

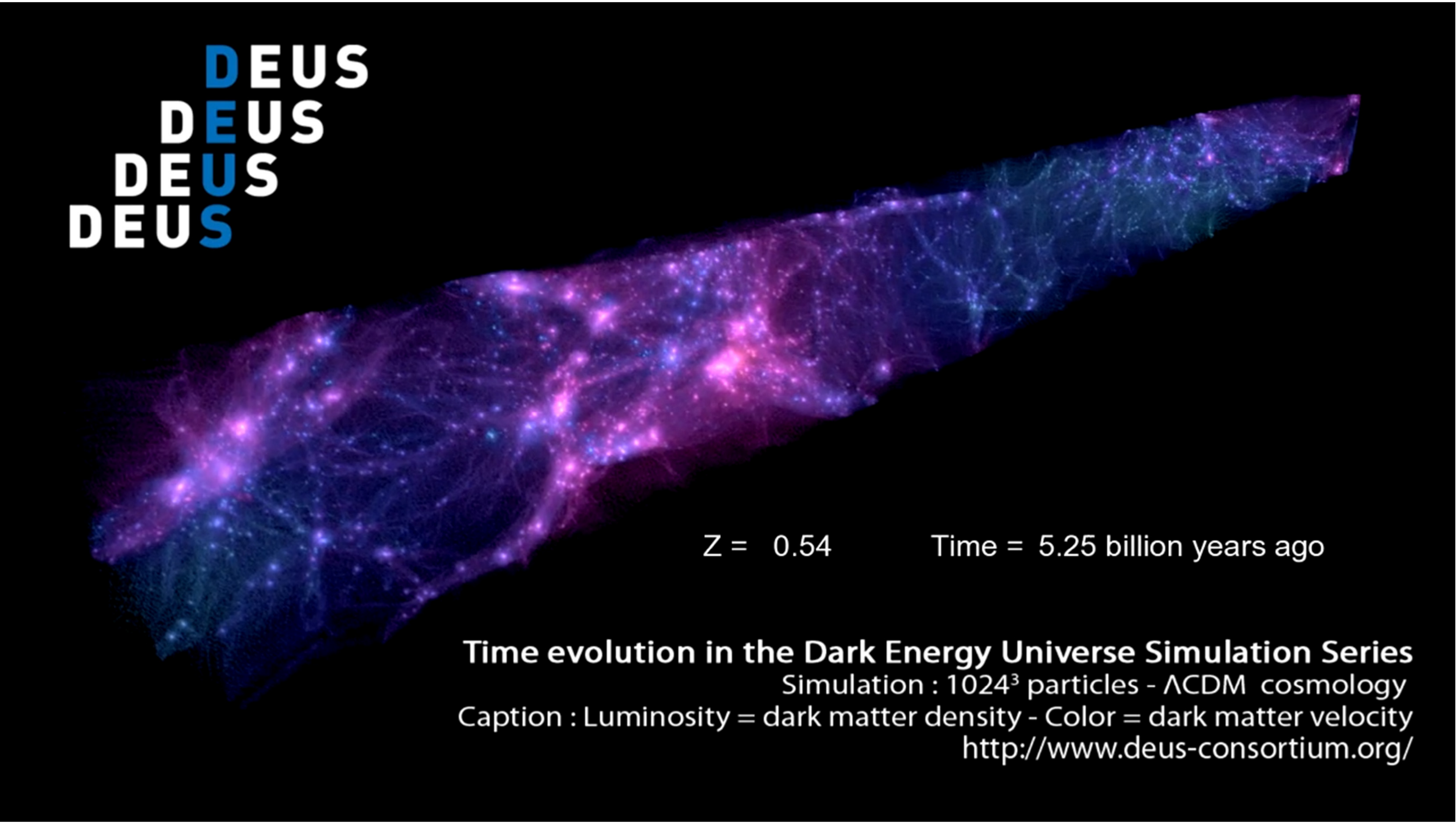
Supercomputer

⬇

Adaptive Mesh Refinement (AMR)
Parallelization based on Space Filling Curve (SFC)



Simulation example



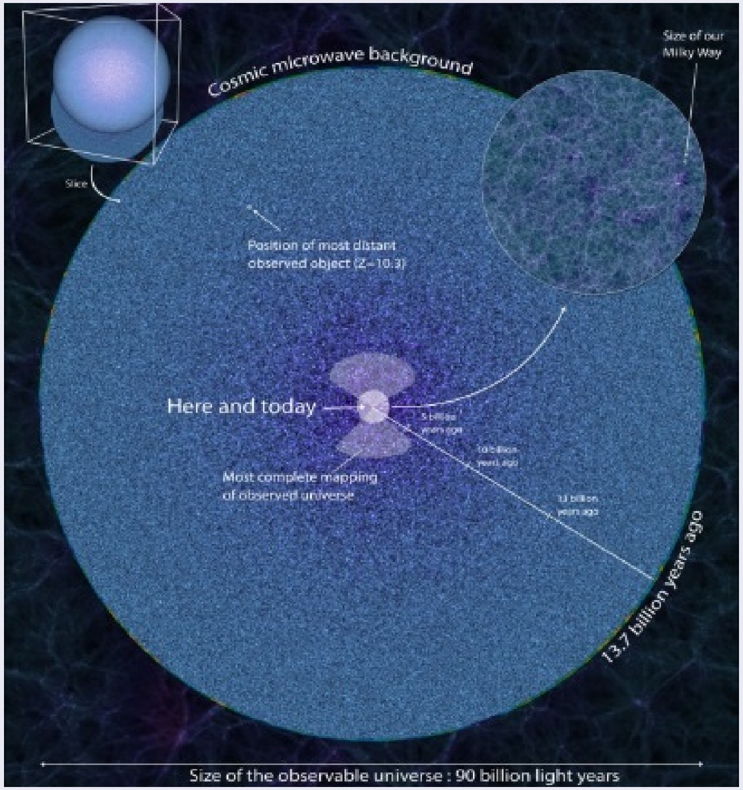
The Full Universe Run (DEUS-FUR)

Problematic

How to investigate LSS formation using cosmological simulations?

DEUS-FUR (Alimi *et al.*, 2012)

- Full Universe Run
- Observable Universe



DEUS-FUR: motivations

- Largest physical box: maximal volume
- Large number of halos (statistics)
- Extreme events (most massive halos)
- Redshift space (light propagation)

DEUS-FUR: some numbers

- 3 models: $\Lambda CDM, RPCDM, WCDM$
- $L_{box} = 21\ Gpc/h$
- $N_{part} = 8192^3 \approx 550 \times 10^9$
- $M_{part} \approx 10^{12} M_{\odot}/h$
- $z_{ini} = 100$
- 5 virtual observers per model
- 31 redshifts of output

DEUS-FUR: a numerical experiment

Run characteristics

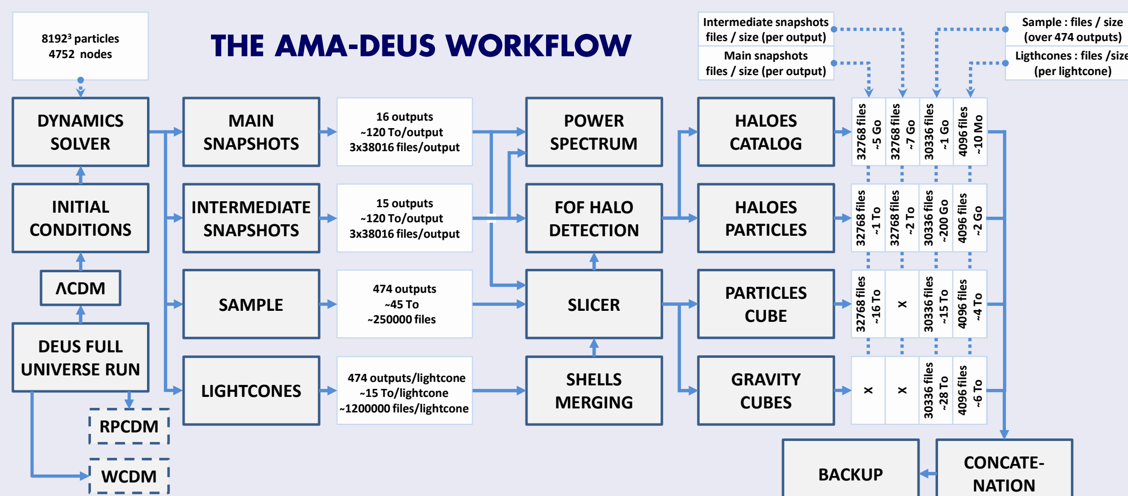
- CURIE supercomputer
- 50 million computing hours
- 4752 nodes/38016 tasks/300 TB of memory
- >1.6 PB of archived data

Pushing current codes to their limit

▸ **Very important lessons learnt regarding the future of simulations**

Lesson: need of workflows

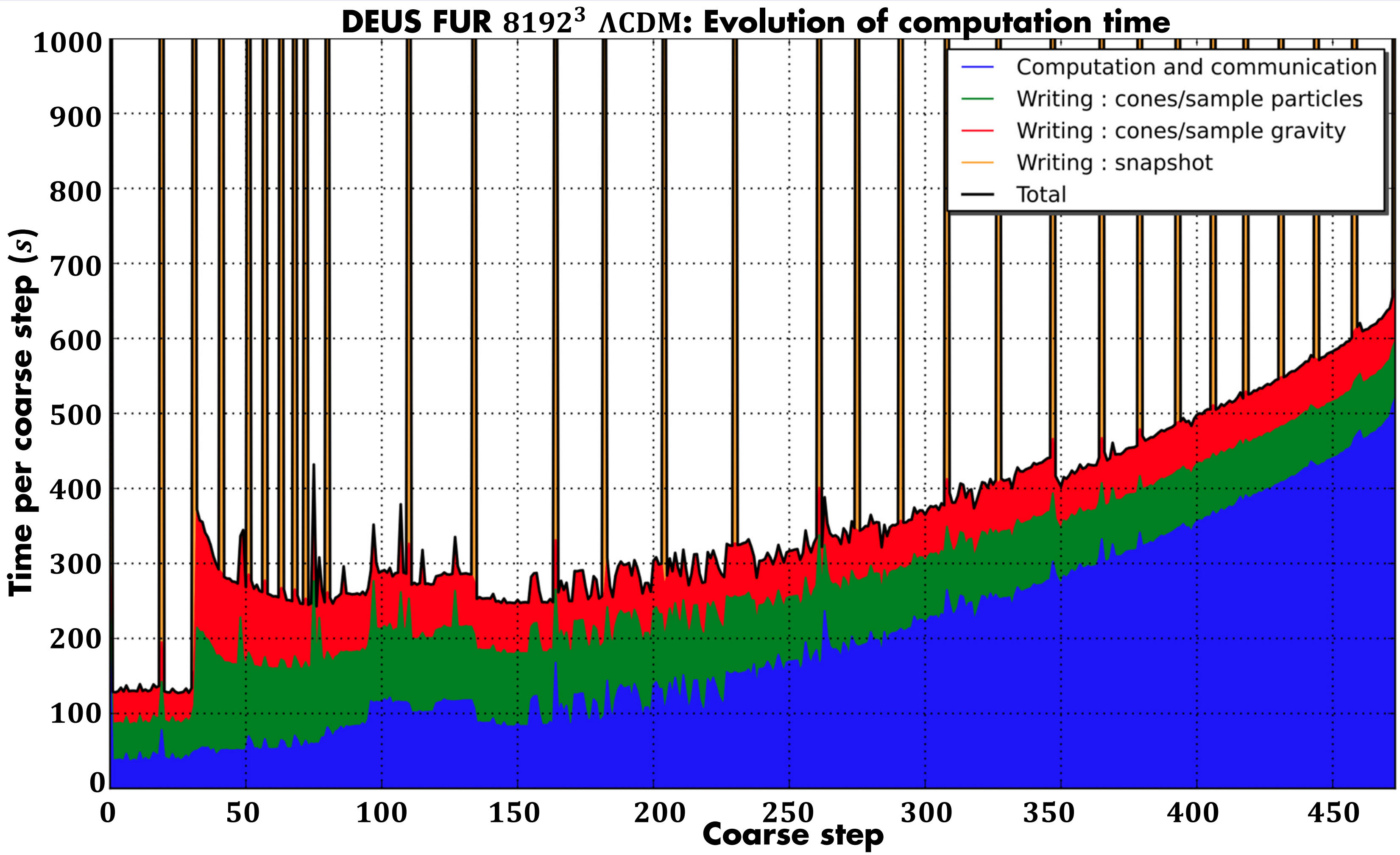
- Multiplicity of codes: pre/post processing phase are demanding
- Post-processing: on-the-fly/quasi-on-the fly/after
- Management of data



DEUS-FUR: a numerical experiment

All numerical aspects were critical

- Computation (CPU time)
- Communications
- Memory
- Inputs and outputs



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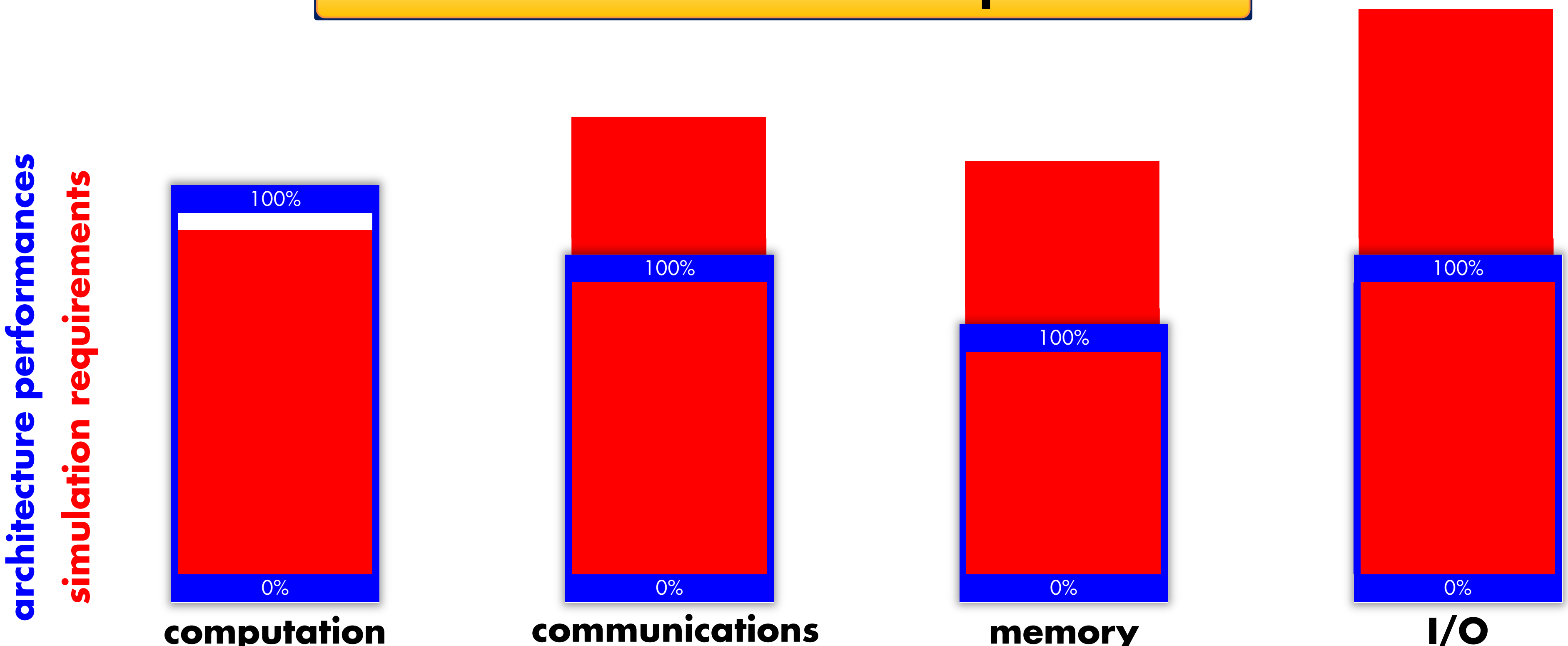
DEUS-FUR: a numerical experiment



Lesson: optimization

- Trading-based optimization when some aspects are not critical
- Architecture-centered optimization when all aspects are critical

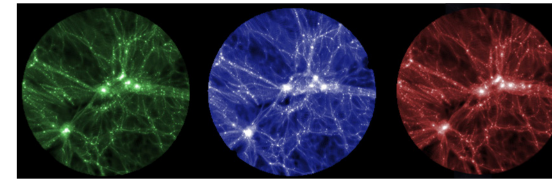
Results

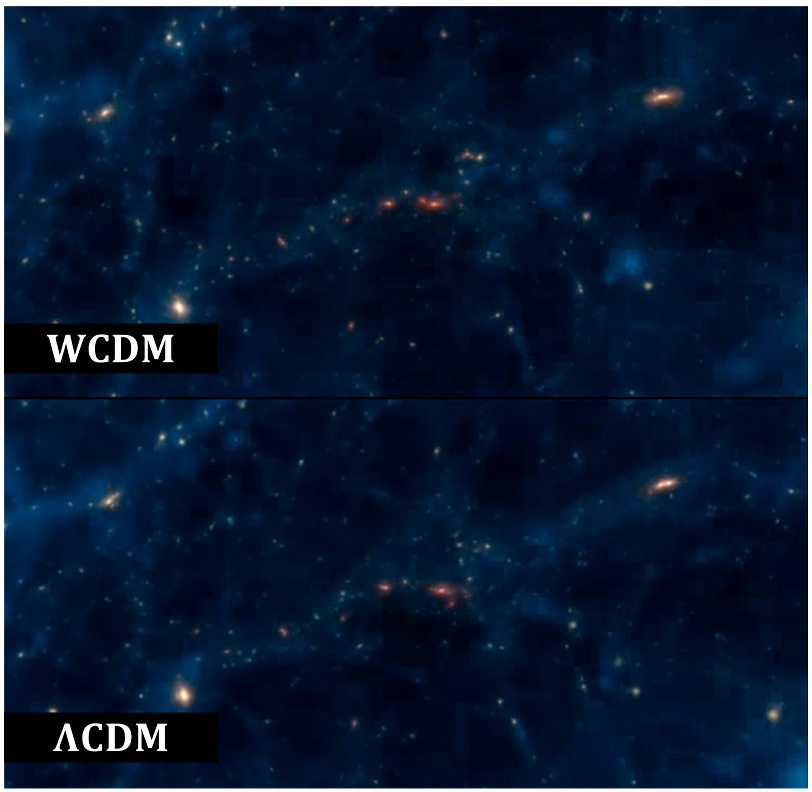
- **Physical results on the scale of the Observable Universe**
- **Pushing codes to their limits raises new questions for exascale**

Standard probes from FUR

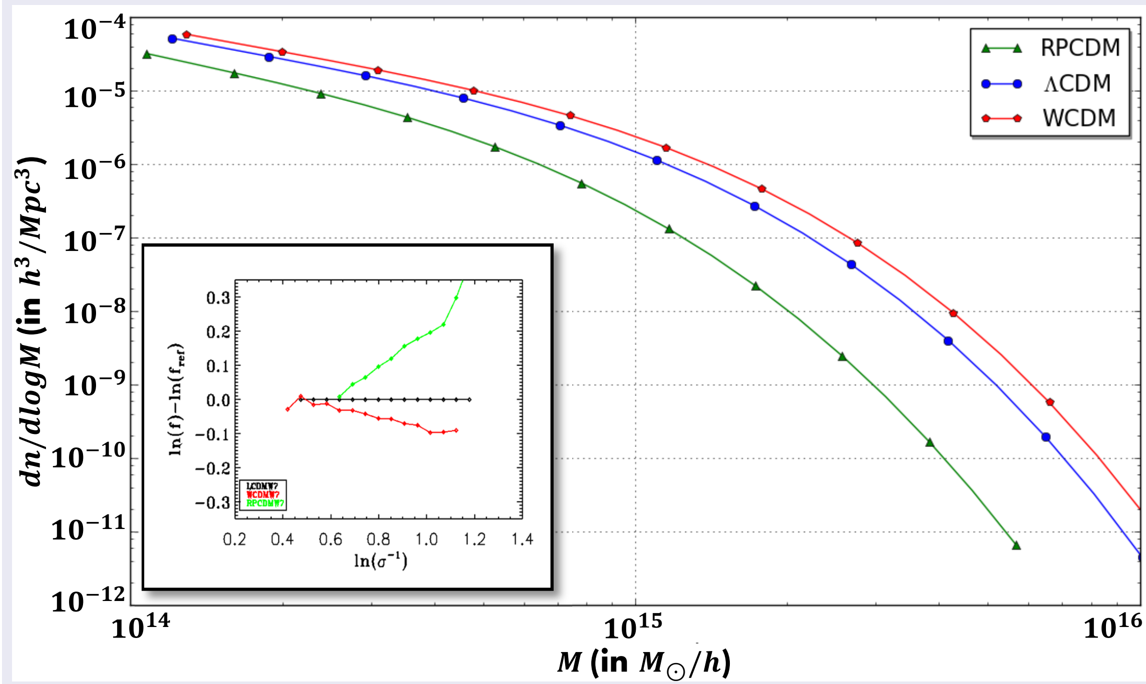
Approach

- **Simulations ⇒ non-linear regime exploration**





Mass function



From the structured Universe to the observed Universe

- **Imprints of DE on standard probes:** $MF_{\mathrm{RPCDM}} < MF_{\Lambda\mathrm{CDM}} < MF_{\mathrm{WCDM}}$
- **Beyond that: effects on light propagation and observations?**

Part 4

The effects of inhomogeneities on light propagation

Large scale structures as perturbations

Homogeneous FLRW metric

$$ds^2 = -c^2 dt^2 + a(t)^2 (dx^2 + dy^2 + dz^2)$$

Perturbed FLRW metric

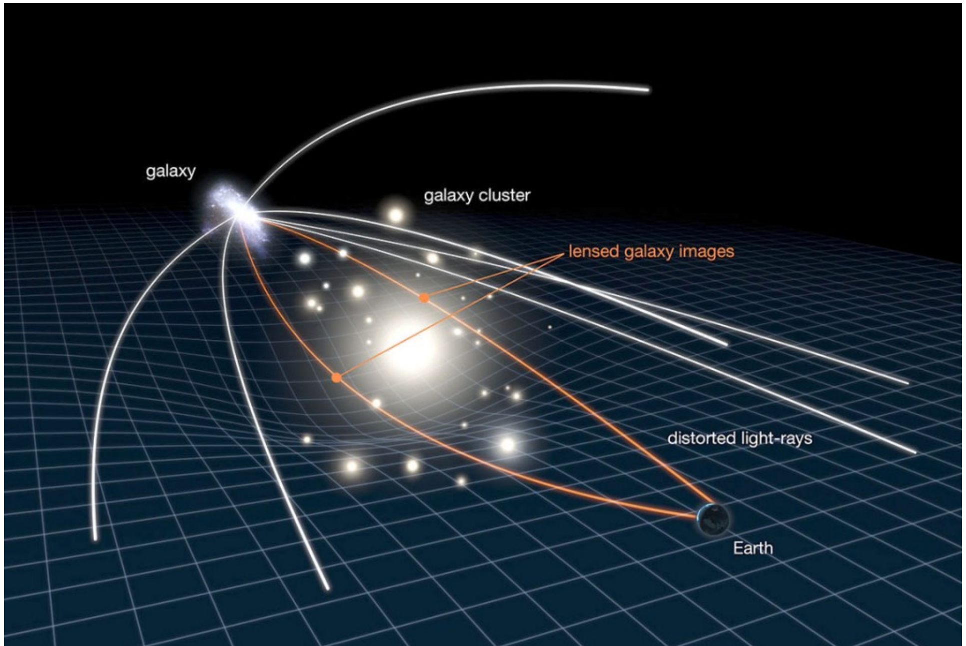
$$ds^2 = -c^2 \left(1 + 2\frac{\Phi}{c^2}\right) dt^2 + a(t)^2 \left(1 - 2\frac{\Phi}{c^2}\right) (dx^2 + dy^2 + dz^2)$$

Affine connections

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\delta} \left(\frac{\partial g_{\delta\beta}}{\partial x^{\alpha}} + \frac{\partial g_{\delta\gamma}}{\partial x^{\beta}} - \frac{\partial g_{\beta\gamma}}{\partial x^{\delta}} \right)$$

Geodesics equation

$$\frac{d^2 x^{\alpha}}{d\lambda^2} = -\Gamma^{\alpha}_{\beta\gamma} \frac{dx^{\beta}}{d\lambda} \frac{dx^{\gamma}}{d\lambda}$$



LSS as perturbations

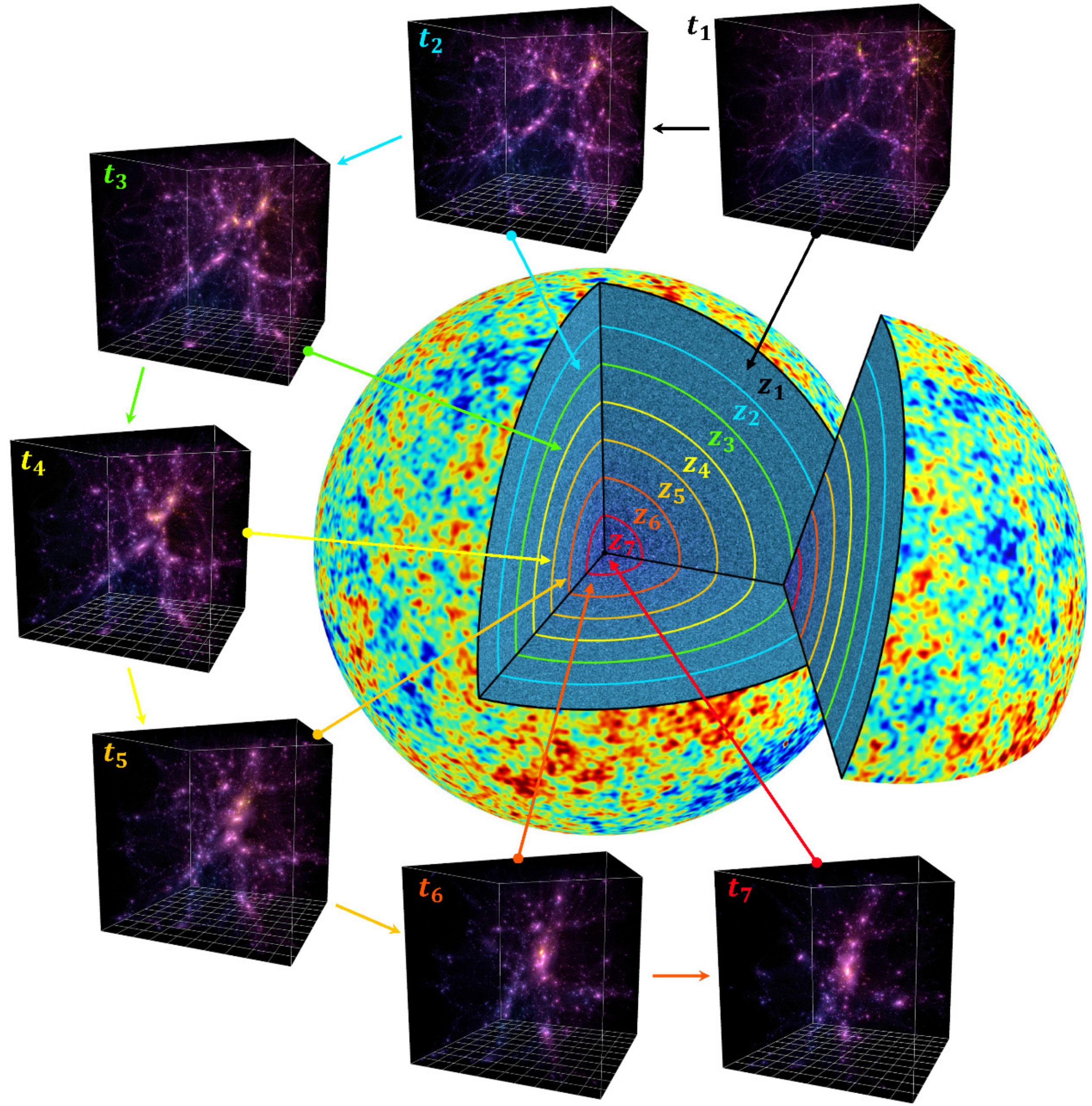
Quantifying effects using simulations

Problematic

- **How to quantify the effects of large scale structures on light propagation and on distance measurements?**

Approach using DEUS-FUR

- 1) Construct light-cones
- 2) Integrate geodesics
- 3) Compute cosmological distances



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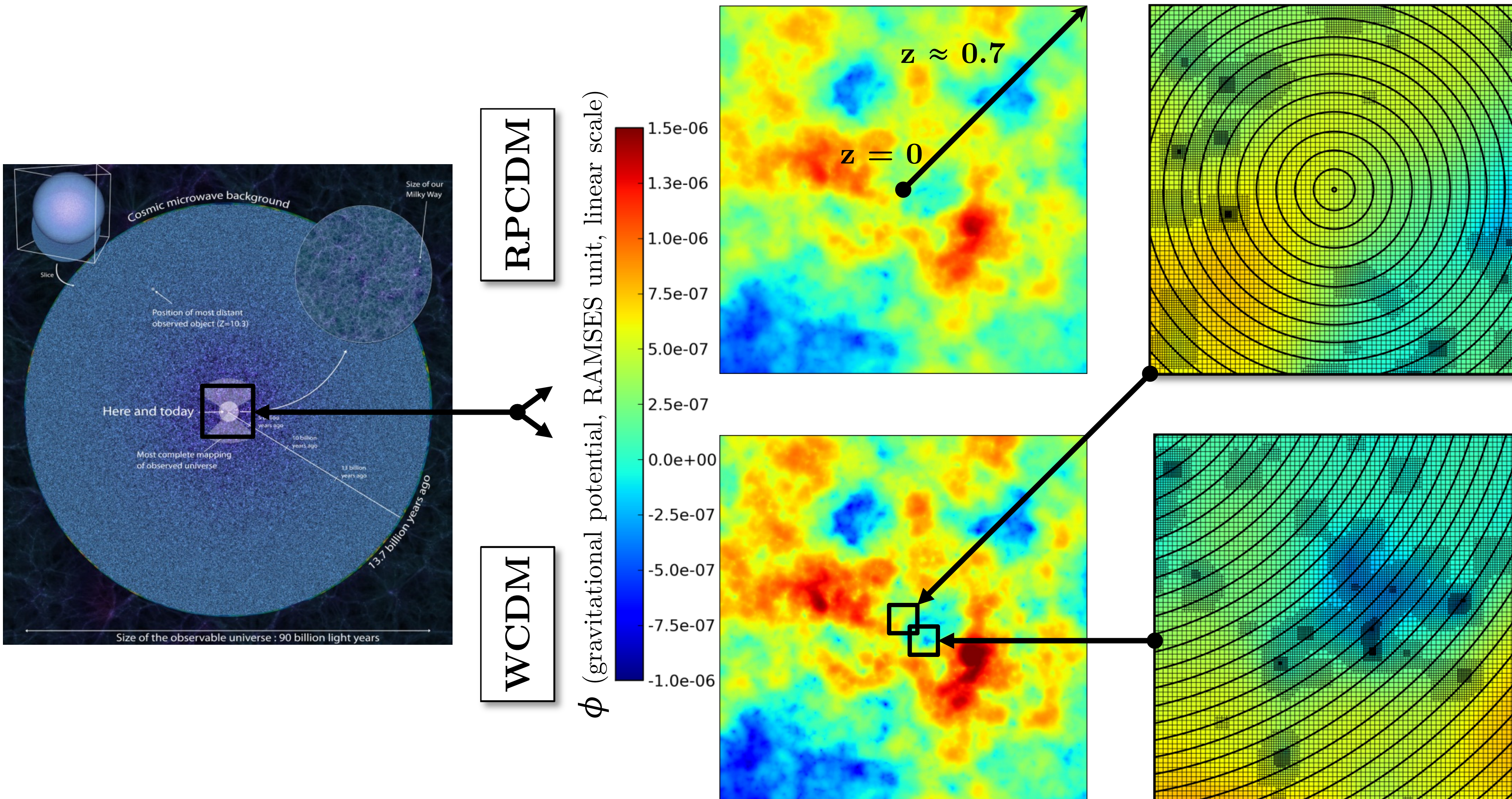
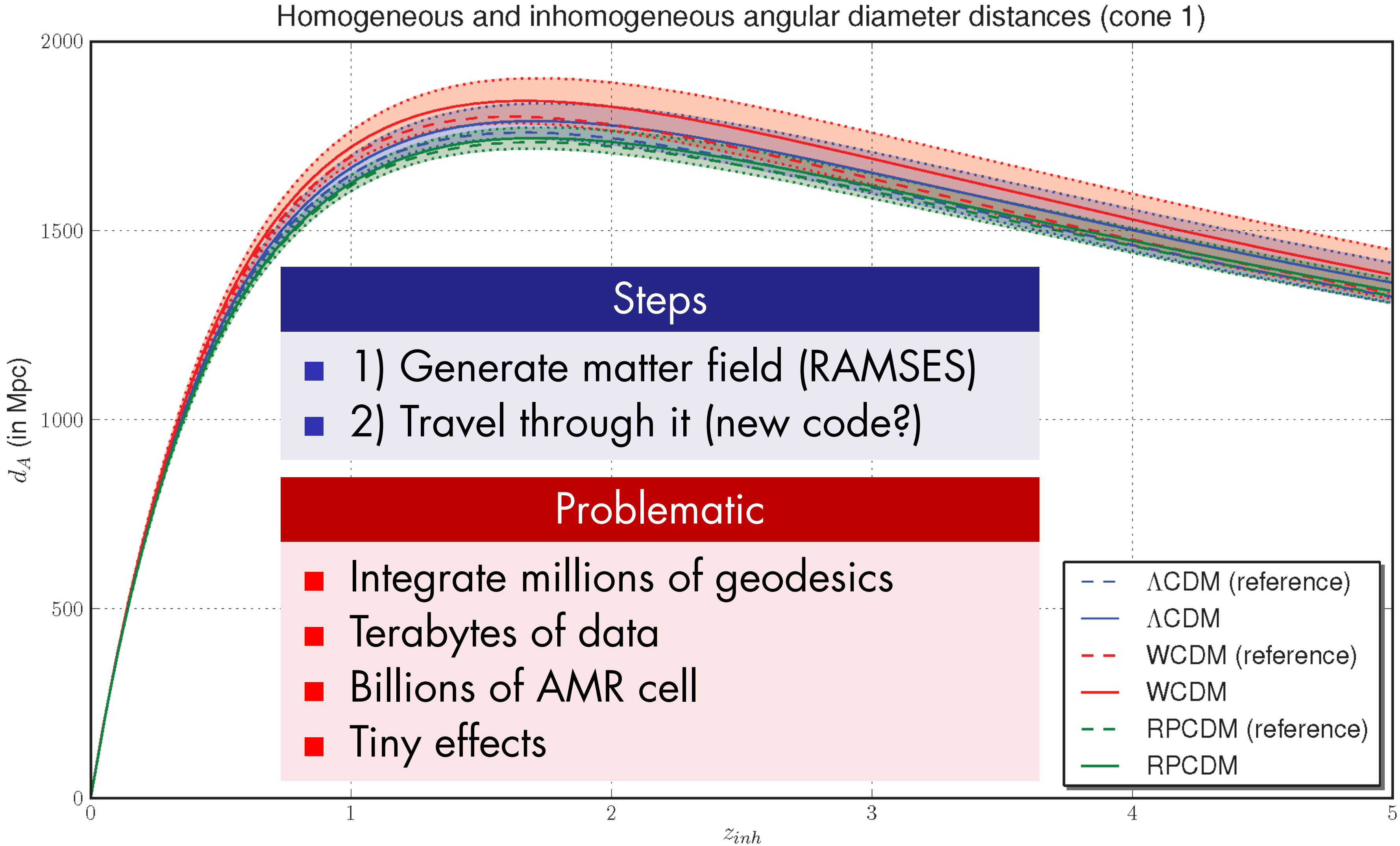
Extracted data



$$\mathbf{ds^2 = -c^2\left(1 + 2\frac{\Phi}{c^2}\right)dt^2 + a(t)^2\left(1 - 2\frac{\Phi}{c^2}\right)\left(dx^2 + dy^2 + dz^2\right)}$$

Strategy



Steps

- 1) Generate matter field (RAMSES)
- 2) Travel through it (new code?)

Problematic

- Integrate millions of geodesics
- Terabytes of data
- Billions of AMR cell
- Tiny effects

Need to rethink numerical cosmology

- **Numerical interlude: code architecture and optimized AMR?**

Software development for science

Simulation codes in the classical approach

- Solve a given physical problem
- Difficult to reuse them to solve a different problem

Libraries in the classical approach

- Passives libraries
- Basic functions call to do specific task
- No backreaction on the code

4 main problems

- Intertwining
- Collaborations
- Expertise
- Combinatorial explosion

Problem: intertwining

Parallelization

Numerical methods

Physics

Parallelization/numerical methods/physics are intertwined

- Impossible to just write physics
- Need knowledge of all aspects at the same time
- 10% of physics and 90% of parallelization/numerical methods
- Extensive use of copy/paste

Problem: combinatorial explosion

How does a code (itself) scale?

Source Lines Of Code (SLOC) $\propto$ **Data types** $\times$ **Algorithms** $\times$ **Architectures**

scalars (ρ)
vectors ($\vec{v}$)

mean
sum
norm

CPU (MPI)
CPU (MPI+OMP)
CPU (MPI) + GPU

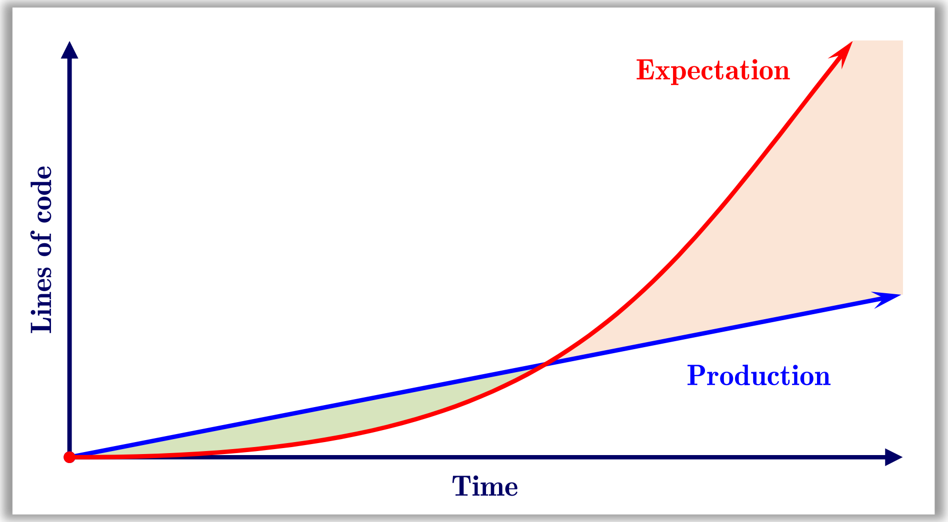
$$\underbrace{\qquad\qquad}_{\boxed{2 \times 3 \times 3 = 18}}$$

Two solutions

- 1) Hire an army of developers to code all possibilities
- 2) Each research group only writes the combinations it needs

Loosing efficiency

- Expectation: combinatorial explosion
- Production: scales linearly



The million dollar question

How to transform × in +?

Source Lines Of Code (SLOC) ∝ **Data types** × **Algorithms** × **Architectures**

Source Lines Of Code (SLOC) ∝ **Data types** + **Algorithms** + **Architectures**

- Like optimizing the scaling of an algorithm: $\mathcal{O}(P \times Q \times R) \implies \mathcal{O}(P + Q + R)$

Solution

- **Let's code a compiler for numerical cosmology!**

Typical job of a compiler

- Assembling the "code puzzle"
- Choosing the best solution
- Optimizing it
- Generating an executable

3 solutions available

- Domain specific language (DSL)
- Precompiler/translator
- Embedded domain specific language (EDSL)

EDSL using template metaprogramming

Principle

- Using C++ template system
- Take action in the instanciation process

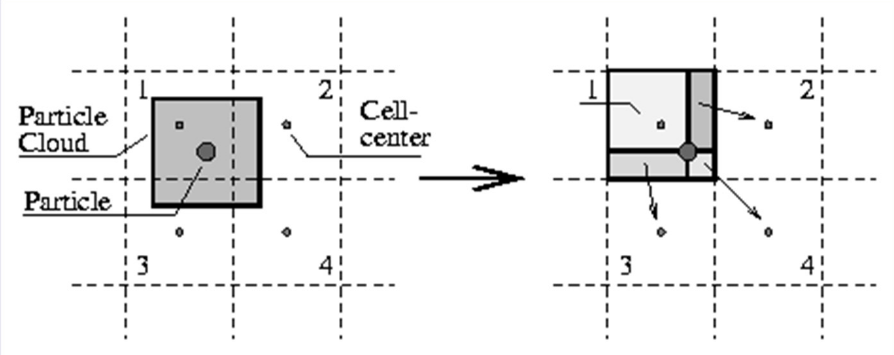
Metaprogramming on values: computation at compile-time

Ackermann function computation time		
	Compilation time	Execution time
`run_ackermann(4, 1)`	0.34 s	2.99 s
`struct_ackermann<4, 1>::value`	37.85 s	$<$ 0.01 s
`meta_ackermann<4, 1>()`	0.58 s	$<$ 0.01 s

Metaprogramming on types: example with CIC (3D interpolation)

N-dimensional linear interpolation regardless:

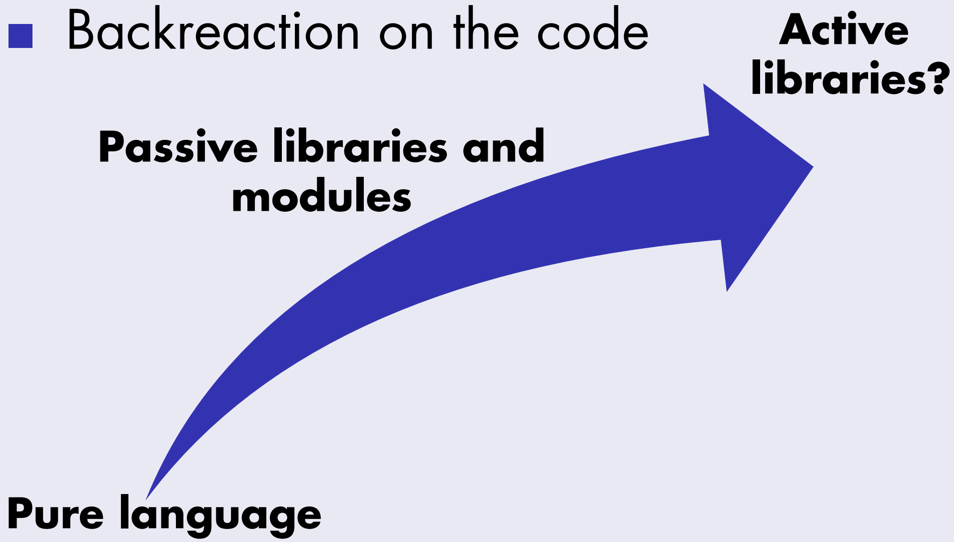
- The number of dimension N
- The type of cell
- The type of particle



EDSL & active libraries

Active libraries

- Backreaction on the code



Solves the 4 problems

- Dissociation of each aspect
- Commutativity of library modifications
- Best expertise on each aspect
- SLOC scales in $\mathcal{O}(P + Q + R)$

Parallelization | **Numerical methods** | **Physics**

Result

▸ **Performance + genericity**

A software architecture

▸ **How to create a code for cosmology using these techniques?**

Numerical cosmology revisited

Common denominators of all (most) cosmological codes

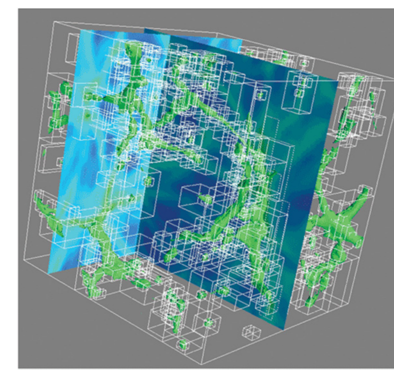
2 types of data:
- Eulerian: fields at a given point
- Lagrangian: particles moving around

3 levels of complexity (as in BLAS):
- Static: grid, interpolation, search…
- Cinematic: integration in a fixed grid
- Dynamic: complete dynamic solver

2 types of quantities

Eulerian (cell)

Lagrangian (particle)



Level 0: Utilities
Management, parallelization, files, log, time...

Level 1: Static → **Static Eulerian/Static Lagrangian**
Trees, interpolators...

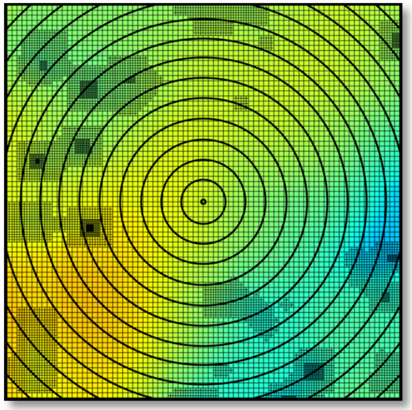
Level 2: Cinematic → **Static Eulerian/Dynamic Lagrangian**
Integrators...

Level 3: Dynamic → Dynamic Eulerian/Dynamic Lagrangian
Dynamic solvers…

Focusing on the static part

Analyzing the Full Universe Run

- For each cone, billions of AMR cells

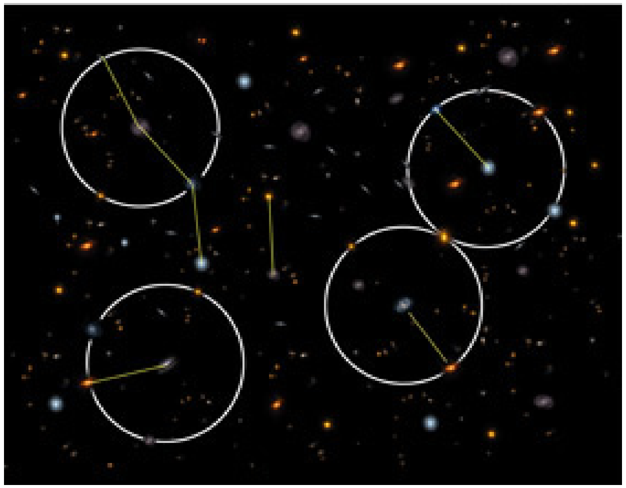


Adaptive mesh refinement & spatial trees

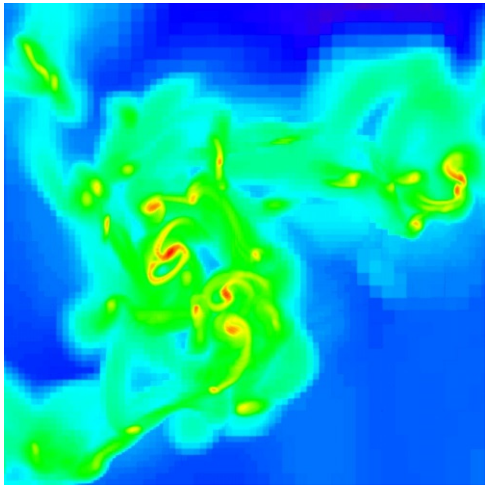
THE key data structure in numerical cosmology

Static level: goals and applications

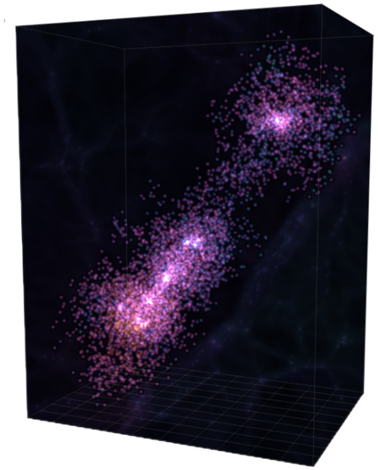
- Processing physical data distributed in space: navigation, search, detection...
- Consequently, optimizing it can have a huge impact on all sort of analysis codes



correlation calculation



interpolation of a field



structure detection

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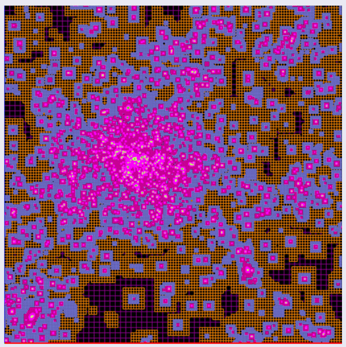
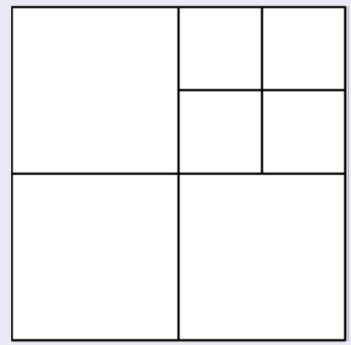
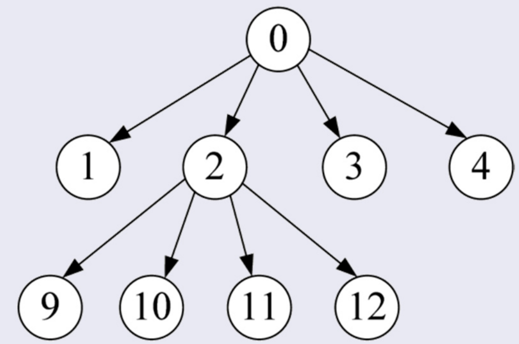
Standard AMR tree: RAMSES

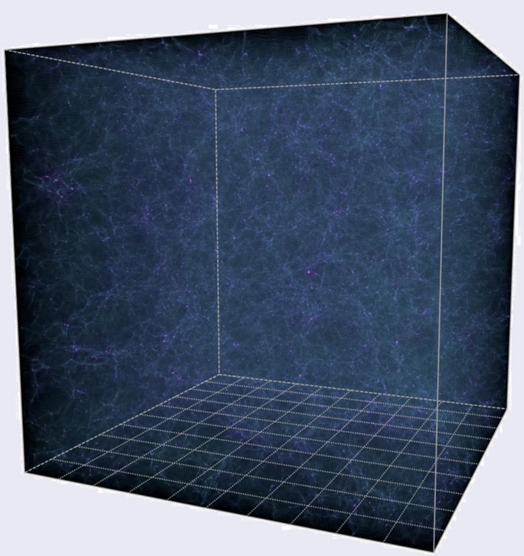
In 2 dimensions: quadtrees

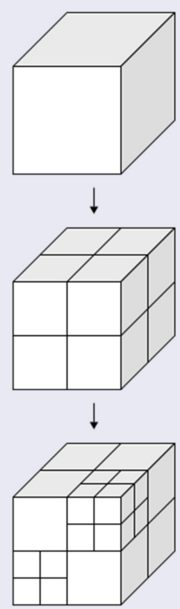


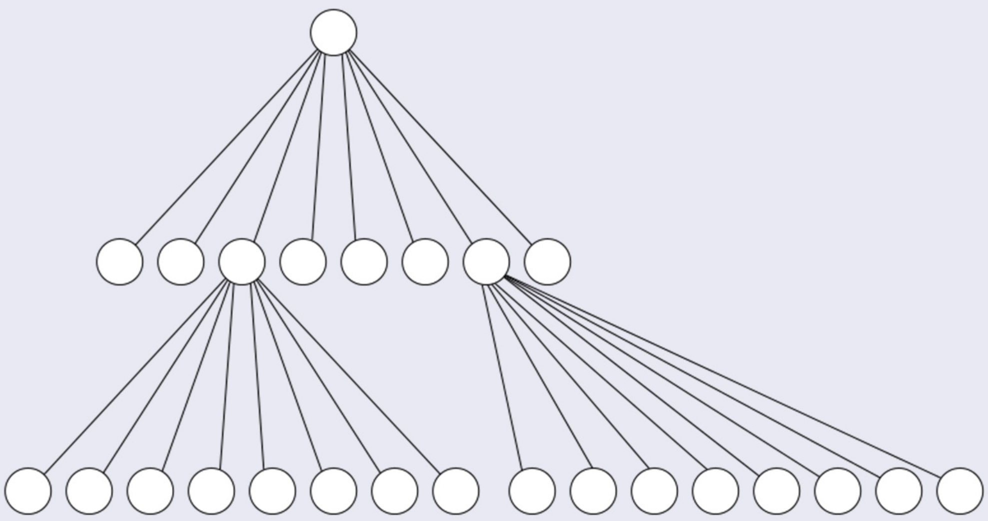




In 3 dimensions: octrees







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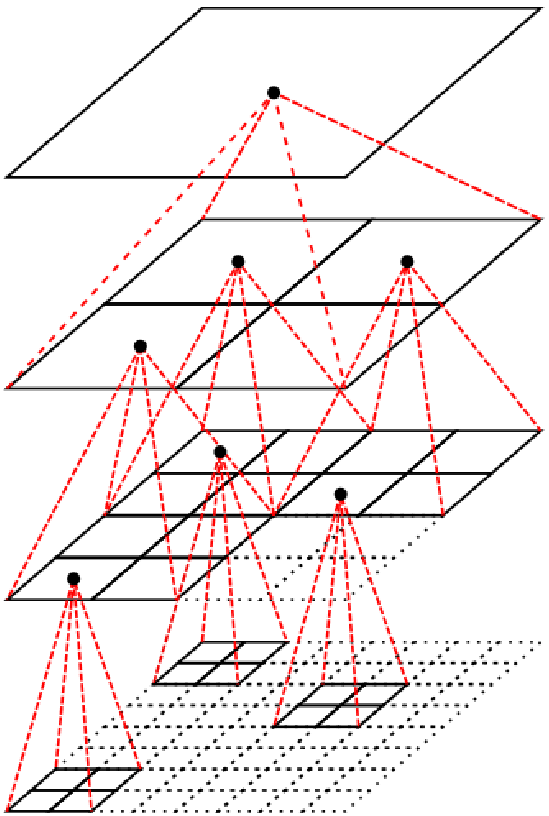
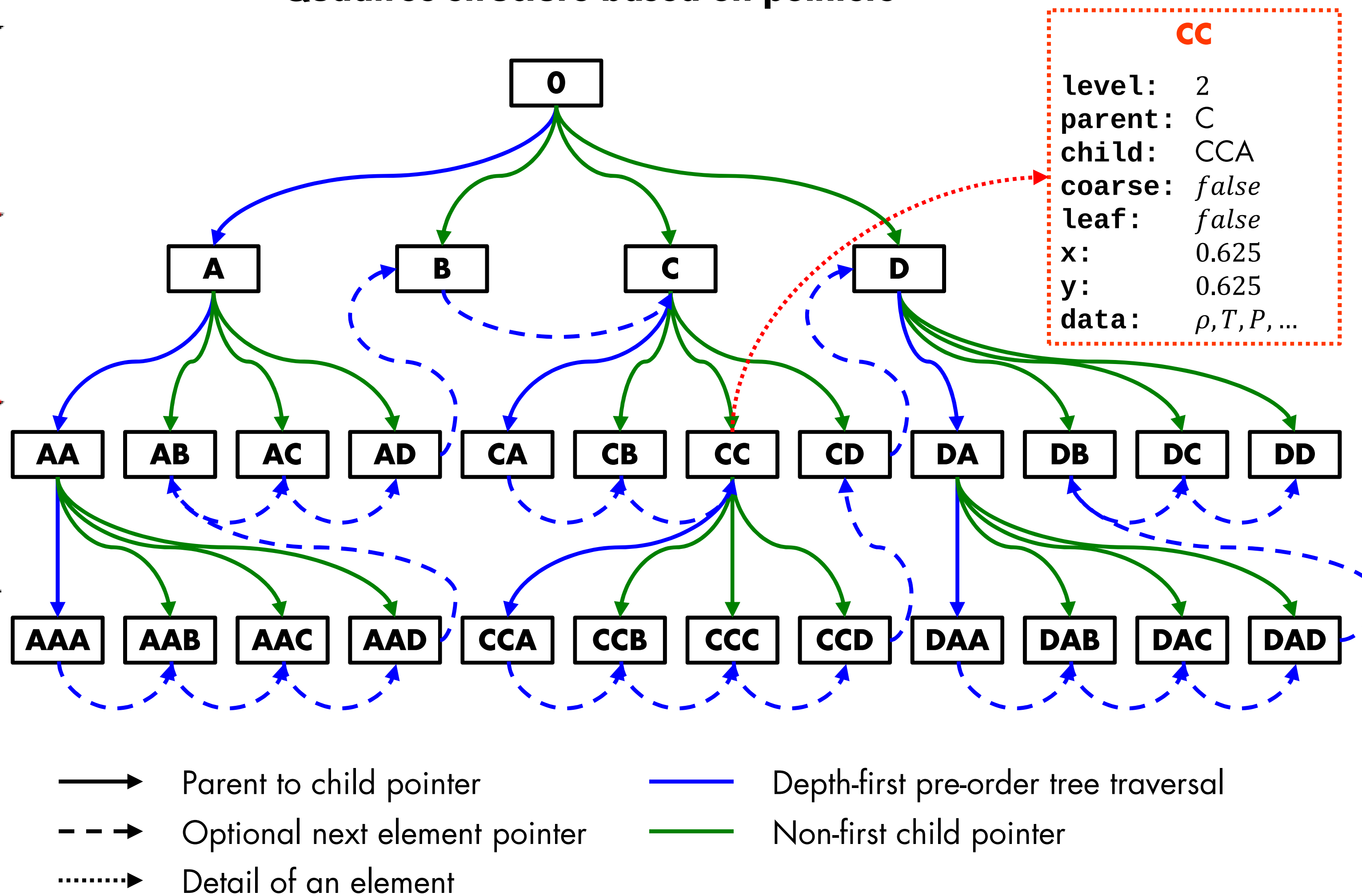
Standard tree implementation

AMR tree



Quadtree structure based on pointers



Properties

- Data structure implemented as a tree: indirection + large memory footprint

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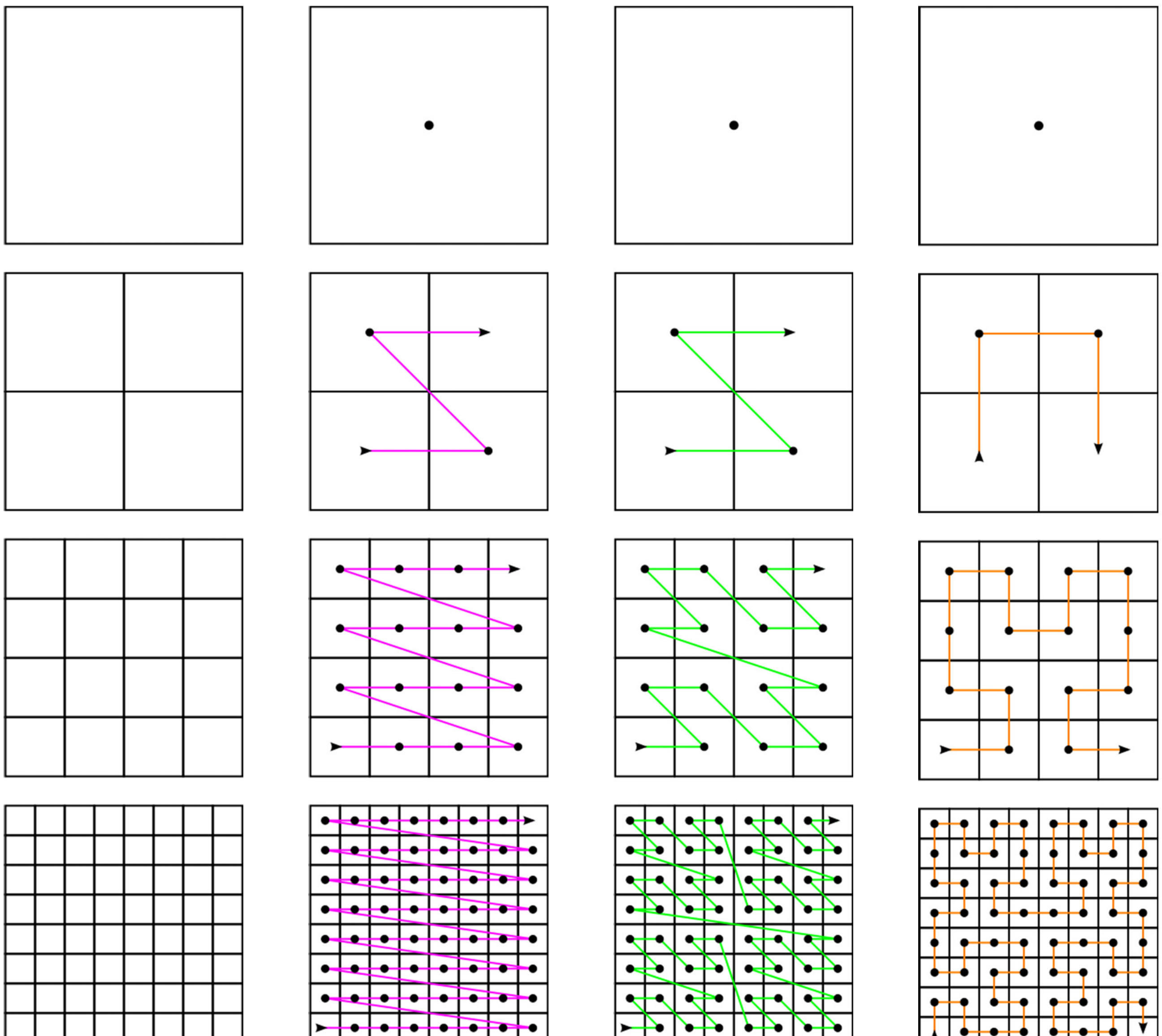
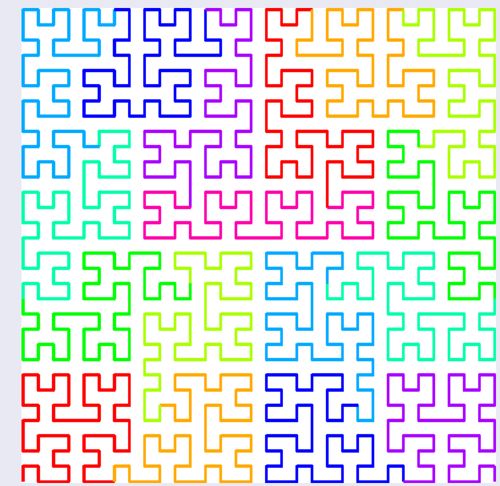
Space filling curves

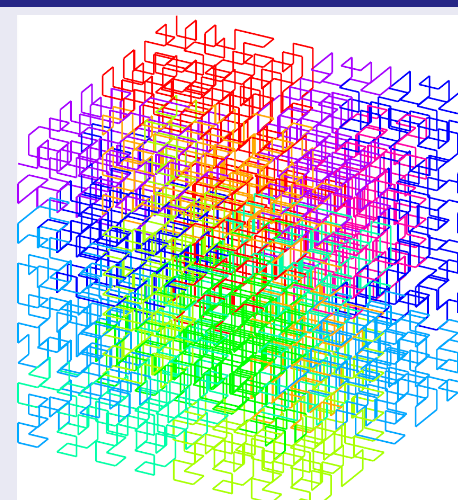


Refinement **Linear** **Z-curve** **Hilbert curve**

2D parallelization



3D parallelization



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Toward a new algorithm

Standard tree implementation

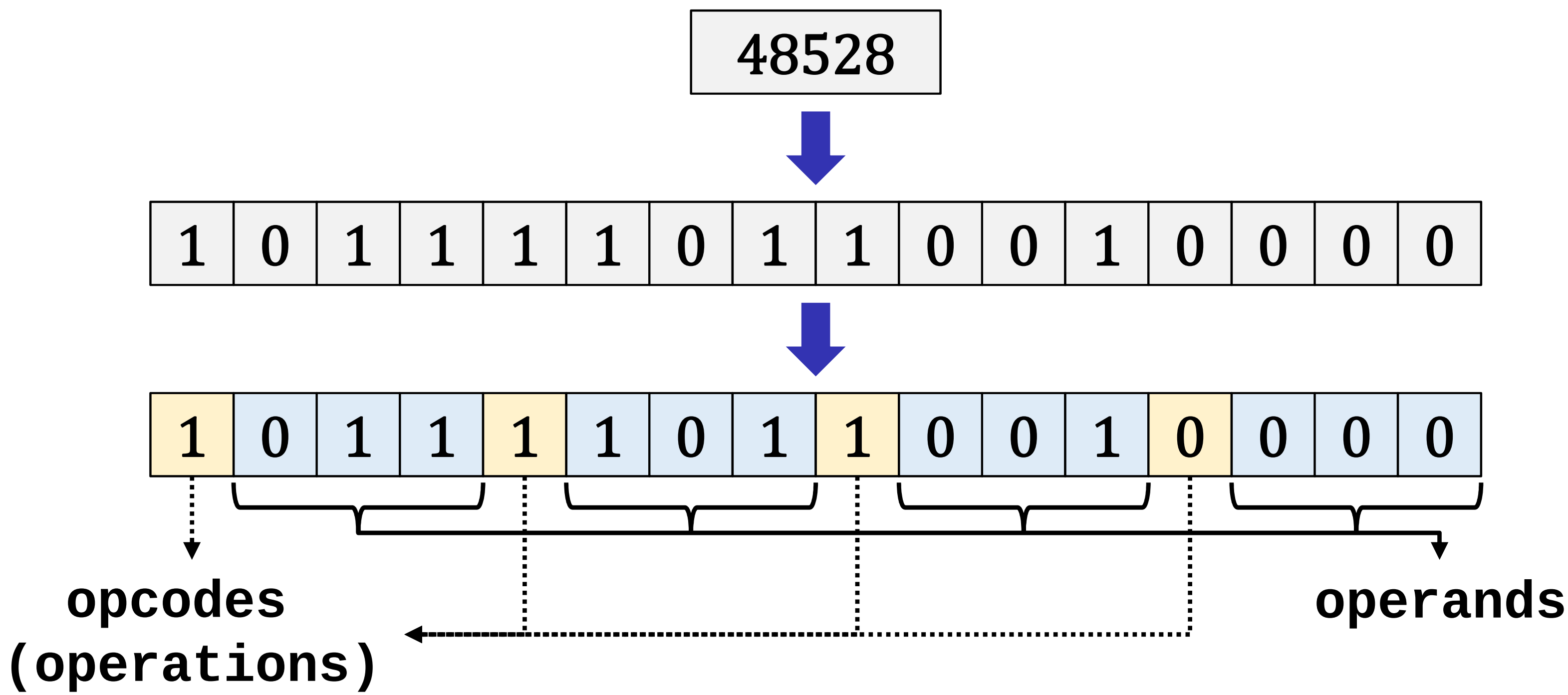
- CPU: indirections and cache problem
- Memory: large memory footprint

Space filling curves

- Necessity for parallelization
- Hilbert curve is complex to handle

Here it comes...

▸ **Key idea: duality programming language/integer representation**



Mini-language definition

An instruction is defined exactly by

- 1 opcode
- 1 operand

2 opcodes available

- ▸ **`goto`** encoded as **`1`**
- ■ **`stop`** encoded as **`0`**

$2^{N_{dim}}$ operands available as directions: $N_{dim} = 2$ example

- ↙ **`lower left`** encoded as **`00`**
- ↘ **`lower right`** encoded as **`01`**
- ↖ **`upper left`** encoded as **`10`**
- ↙ **`upper right`** encoded as **`11`**

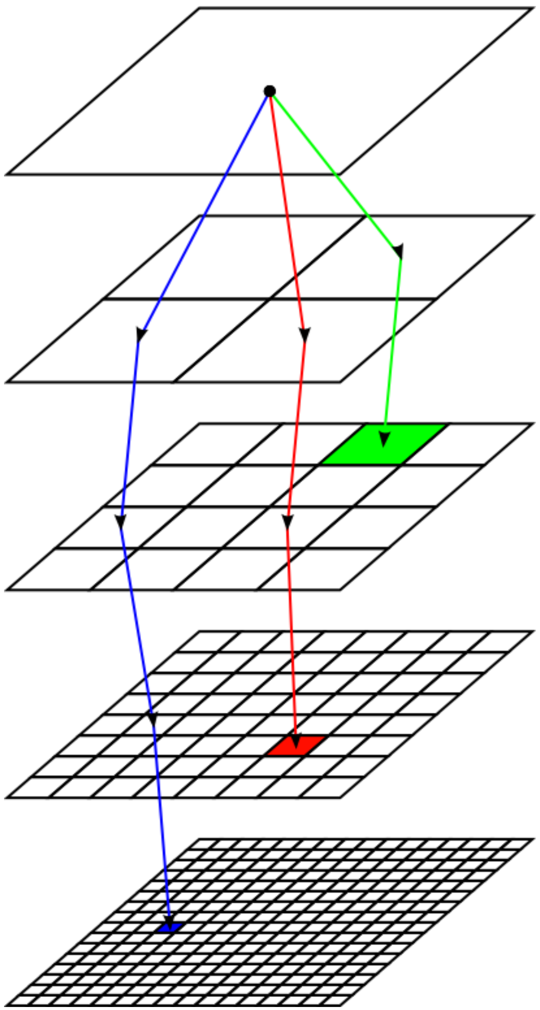
Additional rules

- The first **`stop`** terminates the program ⇒ fill with **`0`** after it
- Start from the most significant bit

Indice encoding

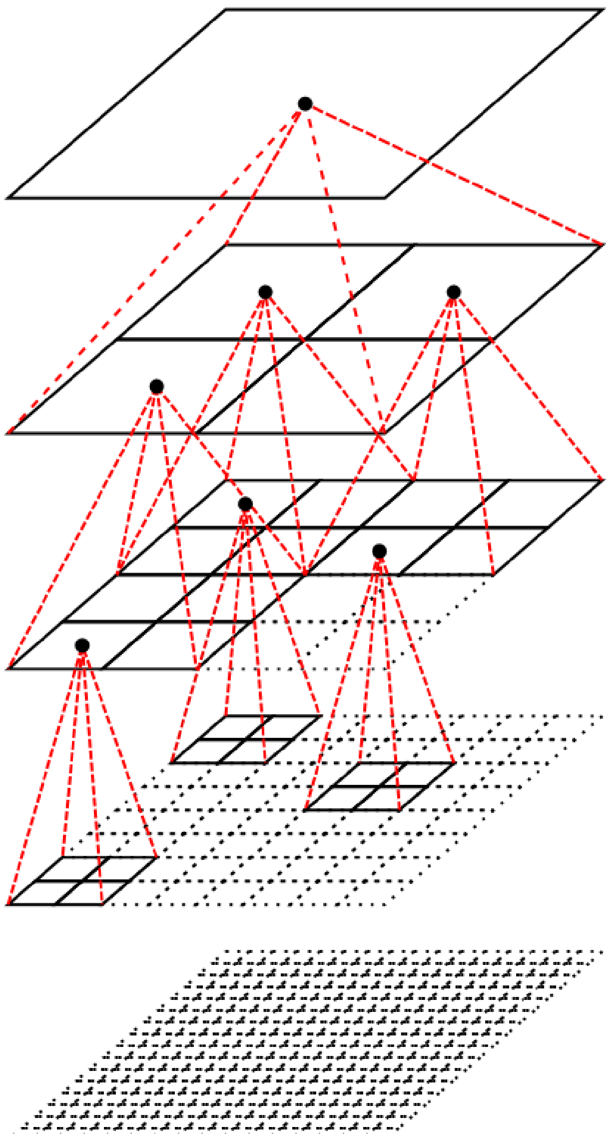
From a program to an integer

For each cell: program ⇒ associated binary code ⇒ binary representation ⇒ integer

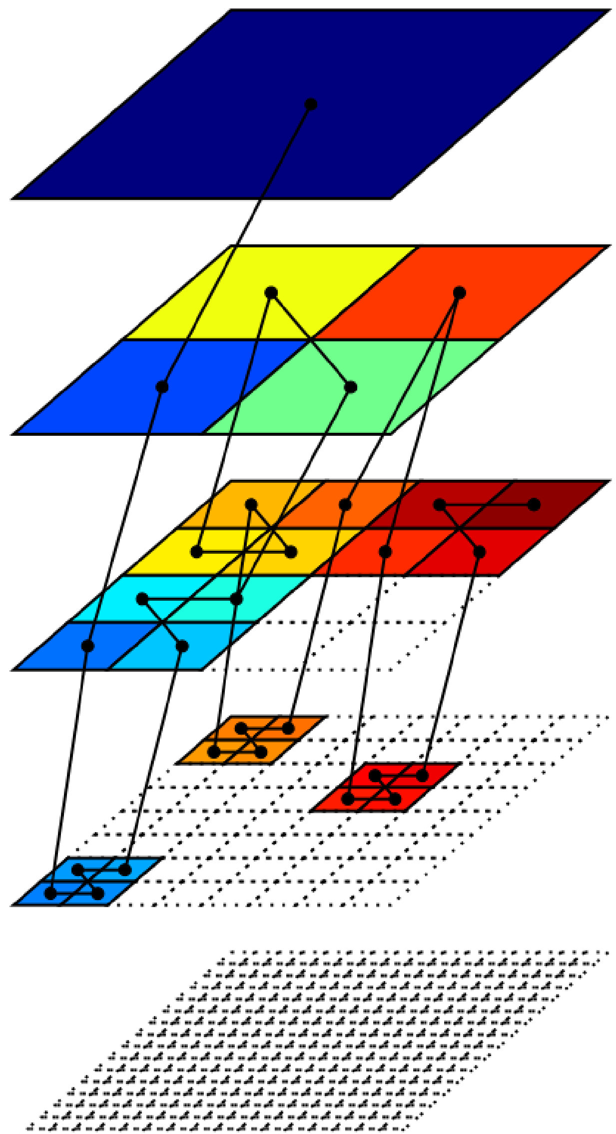


PROGRAM	CODE	BINARY	INT
GOTO UPPER RIGHT GOTO UPPER LEFT STOP	1 11 1 10 0	1111100000000000	63488
GOTO LOWER RIGHT GOTO UPPER LEFT GOTO LOWER RIGHT STOP	1 01 1 10 1 01 0	1011101010000000	47744
GOTO LOWER LEFT GOTO UPPER LEFT GOTO UPPER RIGHT GOTO UPPER RIGHT STOP	1 00 1 10 1 11 1 11 0	1001101111110000	39920

Global information: emerging tree/SFC



AMR tree



indexing representation

16-bit binary code	key	data
0000000000000000	00000	...
1000000000000000	32768	...
1001000000000000	36864	...
1001001000000000	37376	...
1001001010000000	37504	...
1001001100000000	37632	...
1001001110000000	37760	...
1001010000000000	37888	...
1001100000000000	38912	...
1001110000000000	39936	...
1010000000000000	40960	...
1100000000000000	49152	...
1101000000000000	53248	...
1101010000000000	54272	...
1101100000000000	55296	...
1101101000000000	55808	...
1101101010000000	55936	...
1101101100000000	56064	...
1101101110000000	56192	...
1101110000000000	56320	...
1110000000000000	57344	...
1111000000000000	61440	...
1111001000000000	61952	...
1111001010000000	62080	...
1111001100000000	62208	...
1111001110000000	62336	...
1111010000000000	62464	...
1111100000000000	63488	...
1111110000000000	64512	...

internal representation

EDSL & AMR: MAGRATHEA

Emerging properties

- Implicit tree structure
- Space-filling curve (multilevel Z-curve)
- Depth-first pre-order tree traversal
- Navigation using binary search

Some advantages

- Memory efficiency with1 integer/cell
- Data locality & cache efficiency
- Usage of cryptographic functions
- Parallelization using array distribution
- Genericity of a N-dimensional AMR
- Multilevel curve

Some limitations

- Fixed maximum refinement level
- Time for insertion/deletion

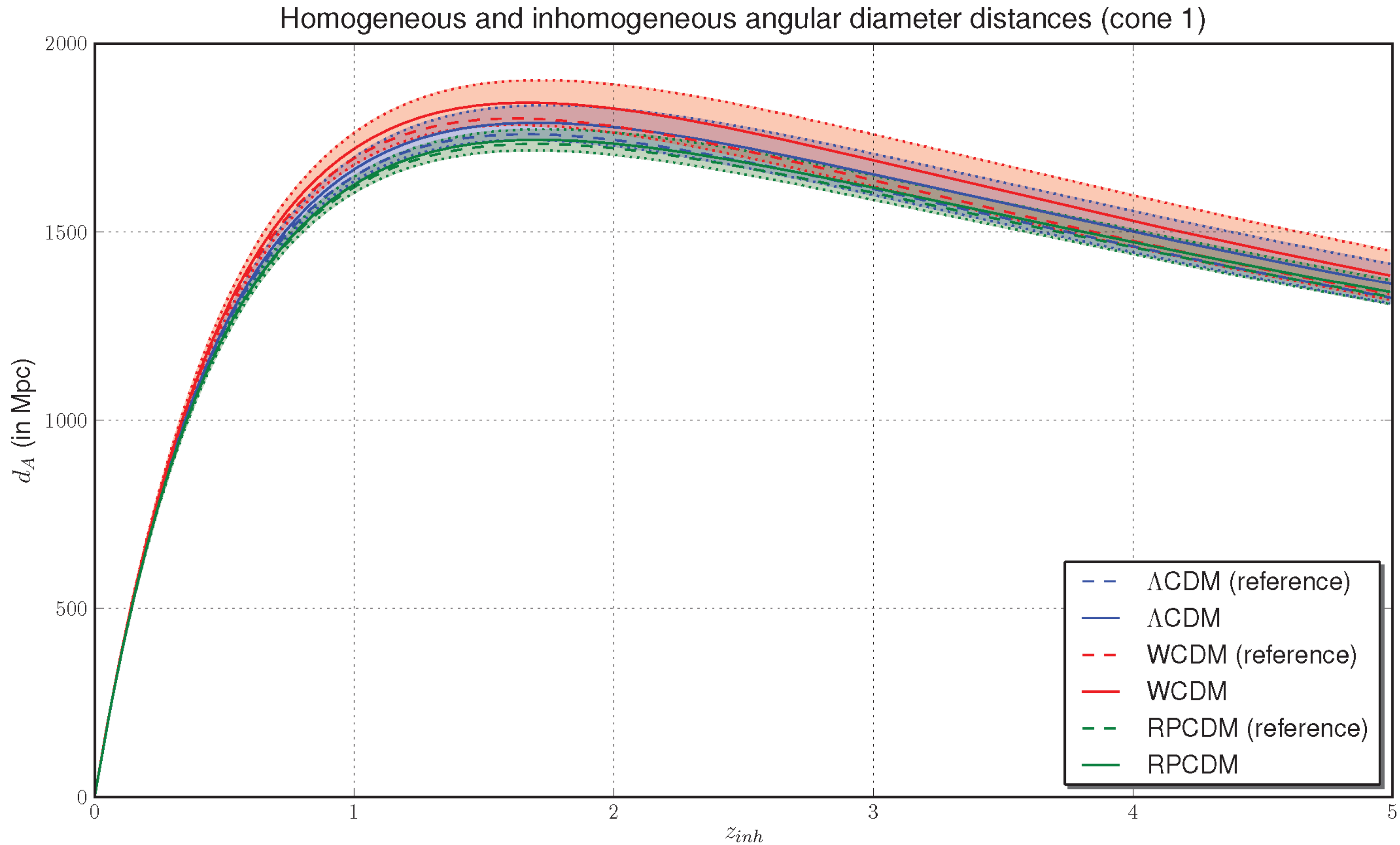
MAGRATHEA

Multi-cpu Adaptive Grid Refinement for THEoretical astrophysics

Summary

- Active library based on C++11
- Demonstrator of EDSL for cosmology
- Management of files, I/O, parallelization, integration, cells…
- Implementation of AMR

Coming back to physics



Back to physics

- **Using these approaches we can measures tiny effects on d_A**

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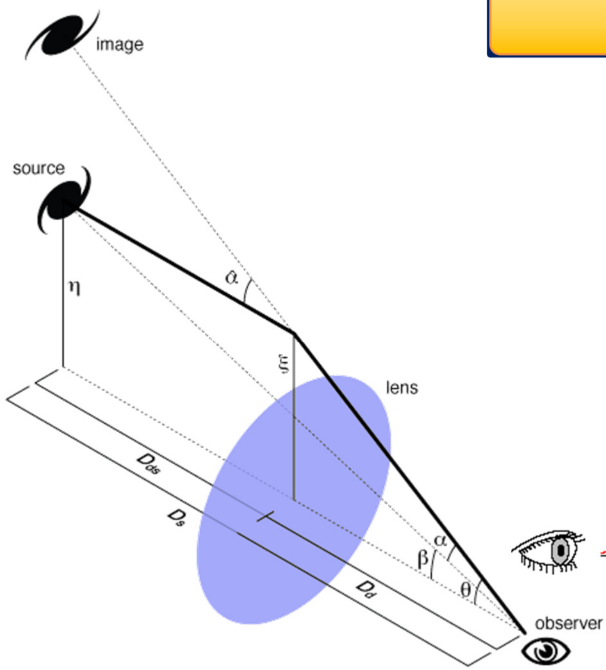
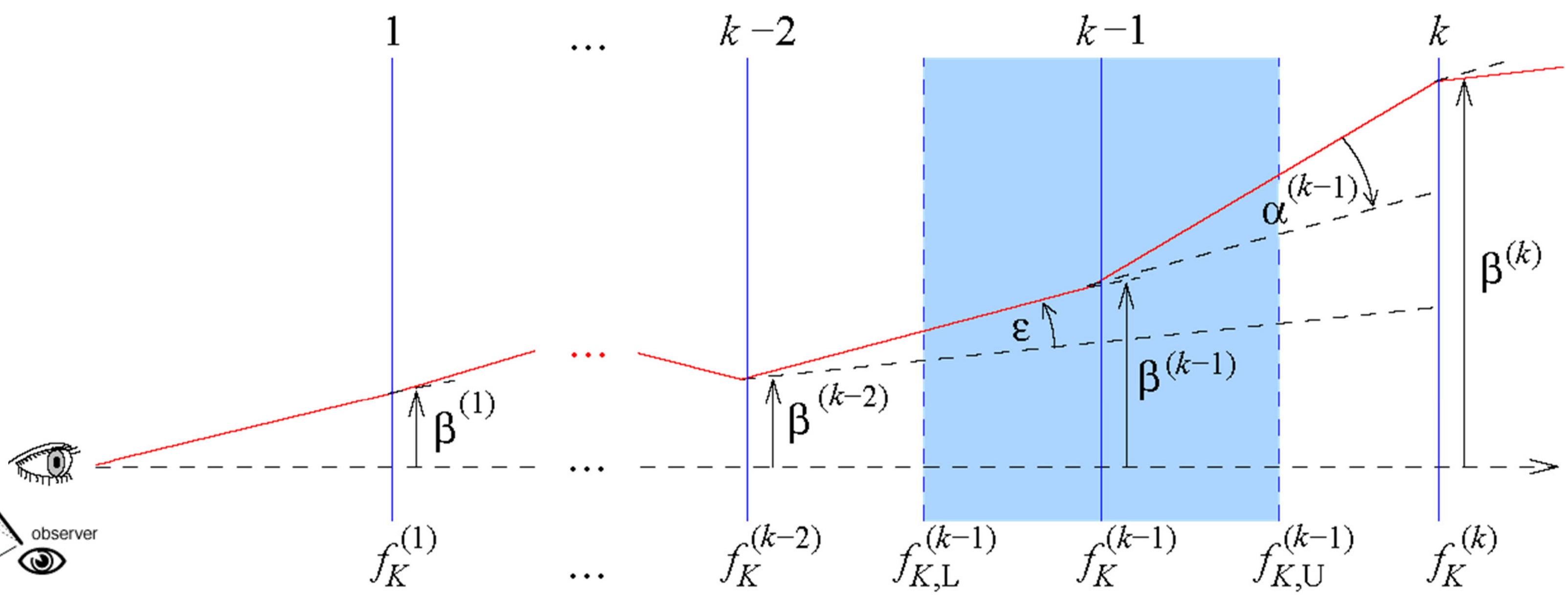
Standard approaches



Deviation from thin lens



Multiple lens plane approximation (from Hilbert *et al.* 2009)

Some limitations of standard approaches

- Born approximation
- Multiple-lens plane approximation
- Geometric approach
- Cone constructed using replica
- Focused on one type of analysis

Approach used here

- Direct integration of geodesics
- Potential directly from simulations
- No replica using FUR

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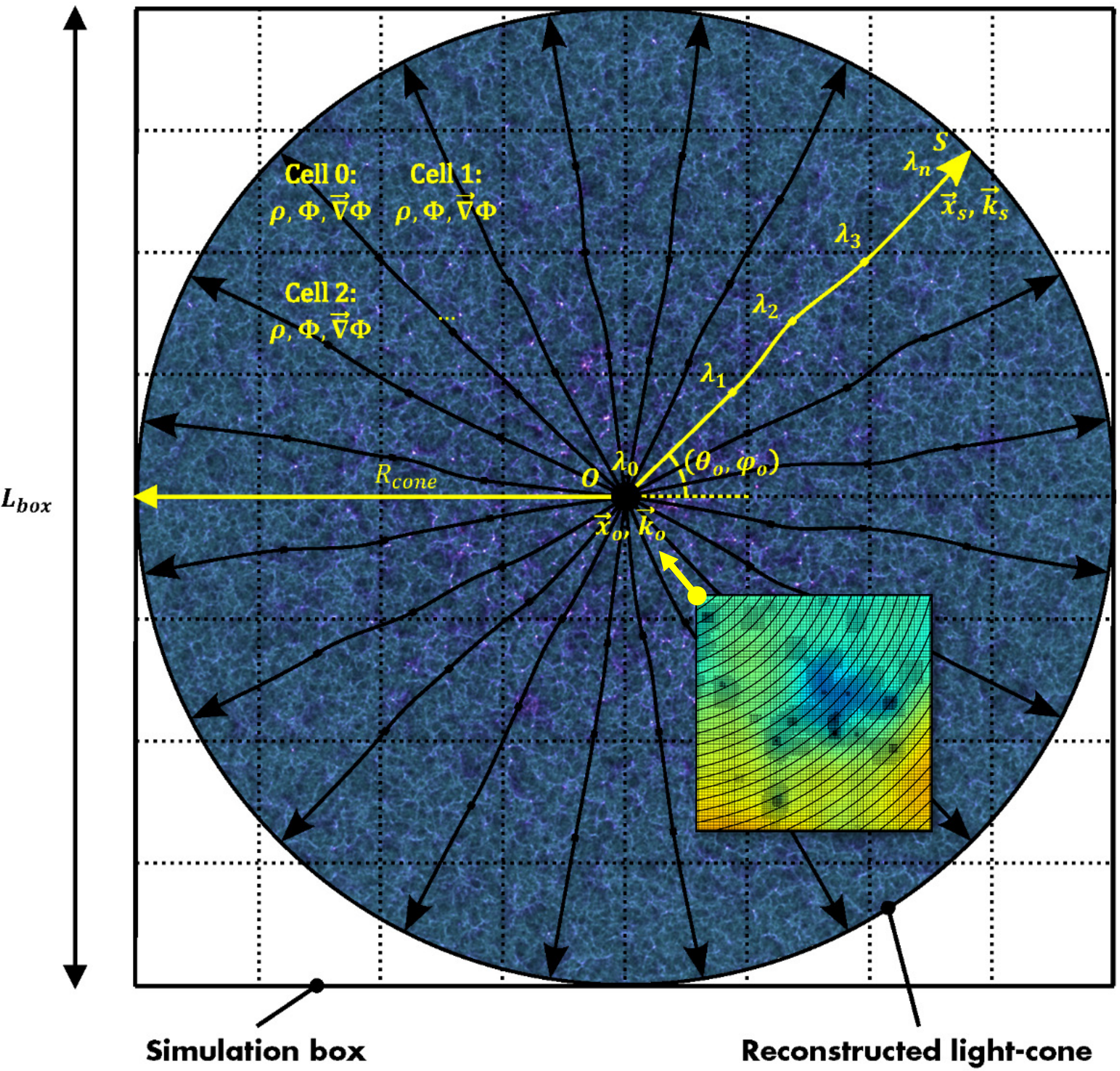
Integration of geodesics



Generic approach

- **Direct integration of geodesics**

Integration of geodesics

$$\frac{\mathrm{d}^2\eta}{\mathrm{d}\lambda^2} \approx -\frac{2a'}{a}\frac{\mathrm{d}\eta}{\mathrm{d}\lambda}\frac{\mathrm{d}\eta}{\mathrm{d}\lambda} - \frac{2}{c^2}\frac{\mathrm{d}\Phi}{\mathrm{d}\lambda}\frac{\mathrm{d}\eta}{\mathrm{d}\lambda} + 2\frac{\partial\Phi}{\partial\eta}\left(\frac{\mathrm{d}\eta}{\mathrm{d}\lambda}\right)^2$$

$$\frac{\mathrm{d}^2x}{\mathrm{d}\lambda^2} \approx -\frac{2a'}{a}\frac{\mathrm{d}\eta}{\mathrm{d}\lambda}\frac{\mathrm{d}x}{\mathrm{d}\lambda} + \frac{2}{c^2}\frac{\mathrm{d}\Phi}{\mathrm{d}\lambda}\frac{\mathrm{d}x}{\mathrm{d}\lambda} - 2\frac{\partial\Phi}{\partial x}\left(\frac{\mathrm{d}\eta}{\mathrm{d}\lambda}\right)^2$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}\lambda^2} \approx -\frac{2a'}{a}\frac{\mathrm{d}\eta}{\mathrm{d}\lambda}\frac{\mathrm{d}y}{\mathrm{d}\lambda} + \frac{2}{c^2}\frac{\mathrm{d}\Phi}{\mathrm{d}\lambda}\frac{\mathrm{d}y}{\mathrm{d}\lambda} - 2\frac{\partial\Phi}{\partial y}\left(\frac{\mathrm{d}\eta}{\mathrm{d}\lambda}\right)^2$$

$$\frac{\mathrm{d}^2z}{\mathrm{d}\lambda^2} \approx -\frac{2a'}{a}\frac{\mathrm{d}\eta}{\mathrm{d}\lambda}\frac{\mathrm{d}z}{\mathrm{d}\lambda} + \frac{2}{c^2}\frac{\mathrm{d}\Phi}{\mathrm{d}\lambda}\frac{\mathrm{d}z}{\mathrm{d}\lambda} - 2\frac{\partial\Phi}{\partial z}\left(\frac{\mathrm{d}\eta}{\mathrm{d}\lambda}\right)^2$$

DEUS-FUR + MAGRATHEA

- Full-sky light cones from FUR
- Billions of AMR cells/observer
- 1 million photon/observer
- Minimum number of approx

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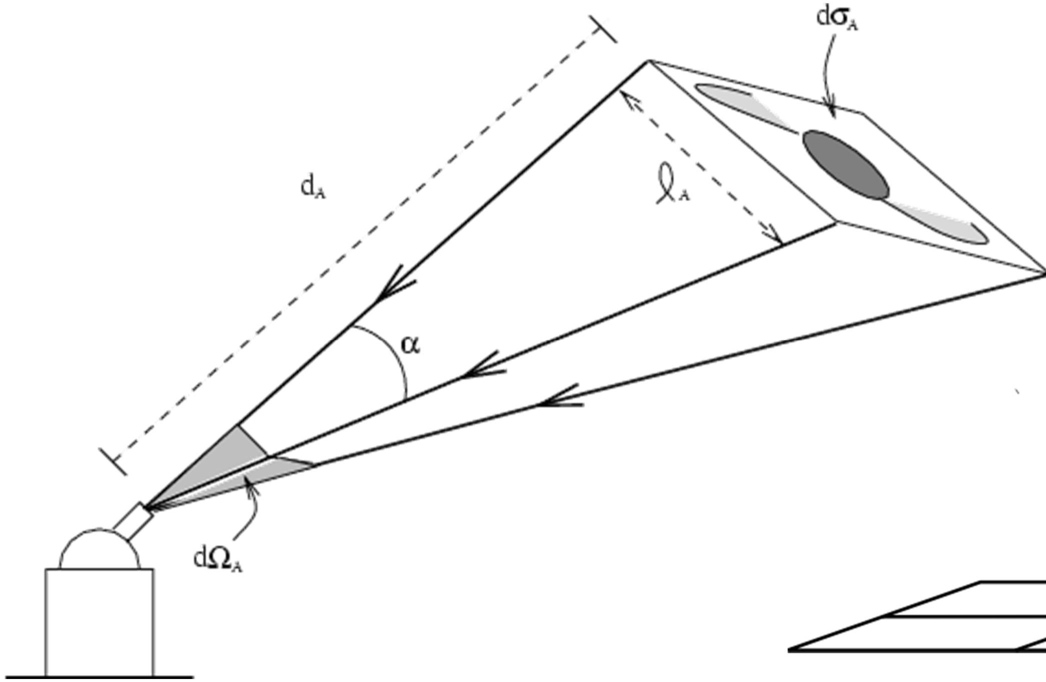
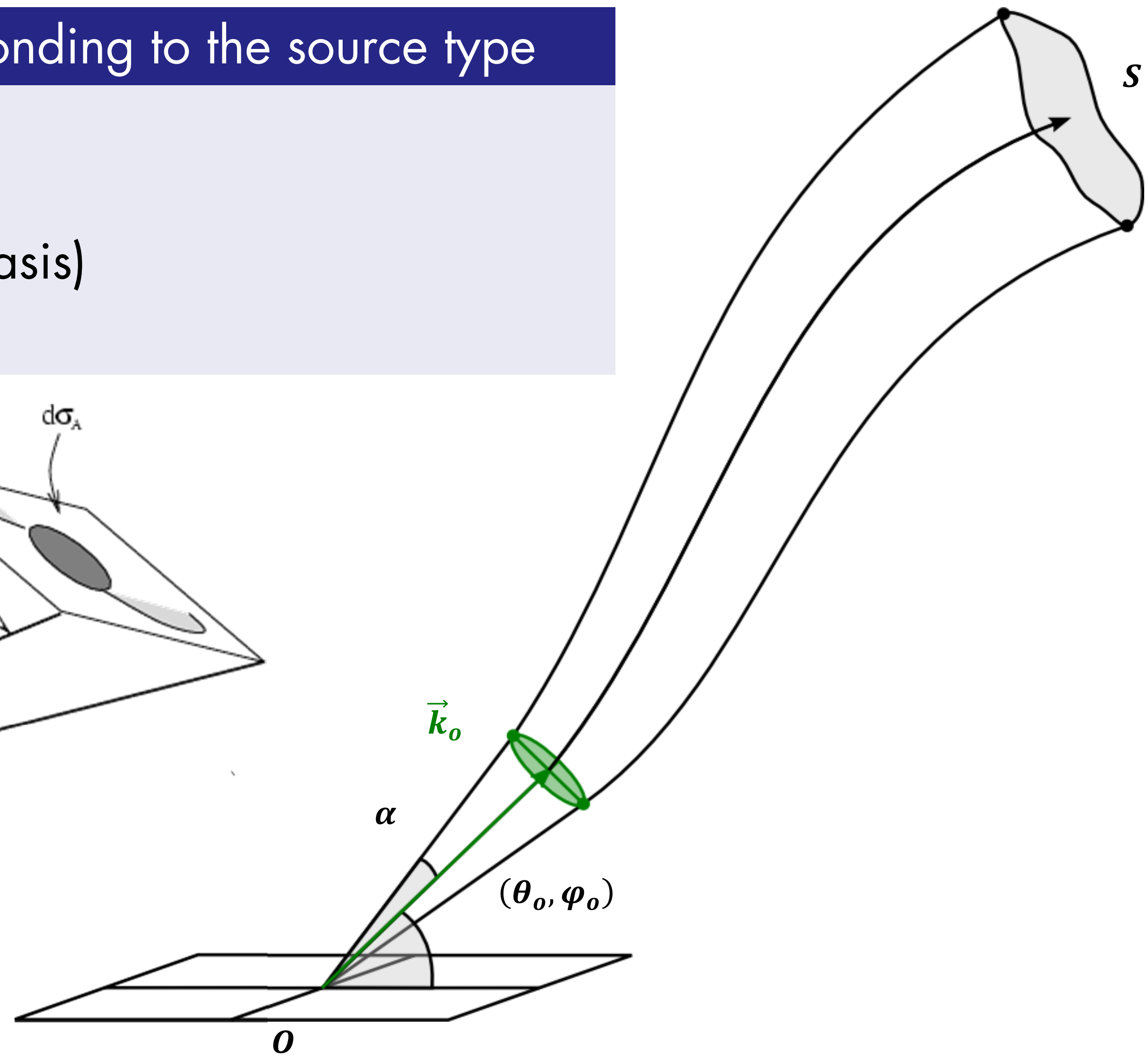
Stop conditions

Different possibillities corresponding to the source type

- Affine parameter
- Time
- Perpendicular plane (Sachs basis)
- **Redshift**

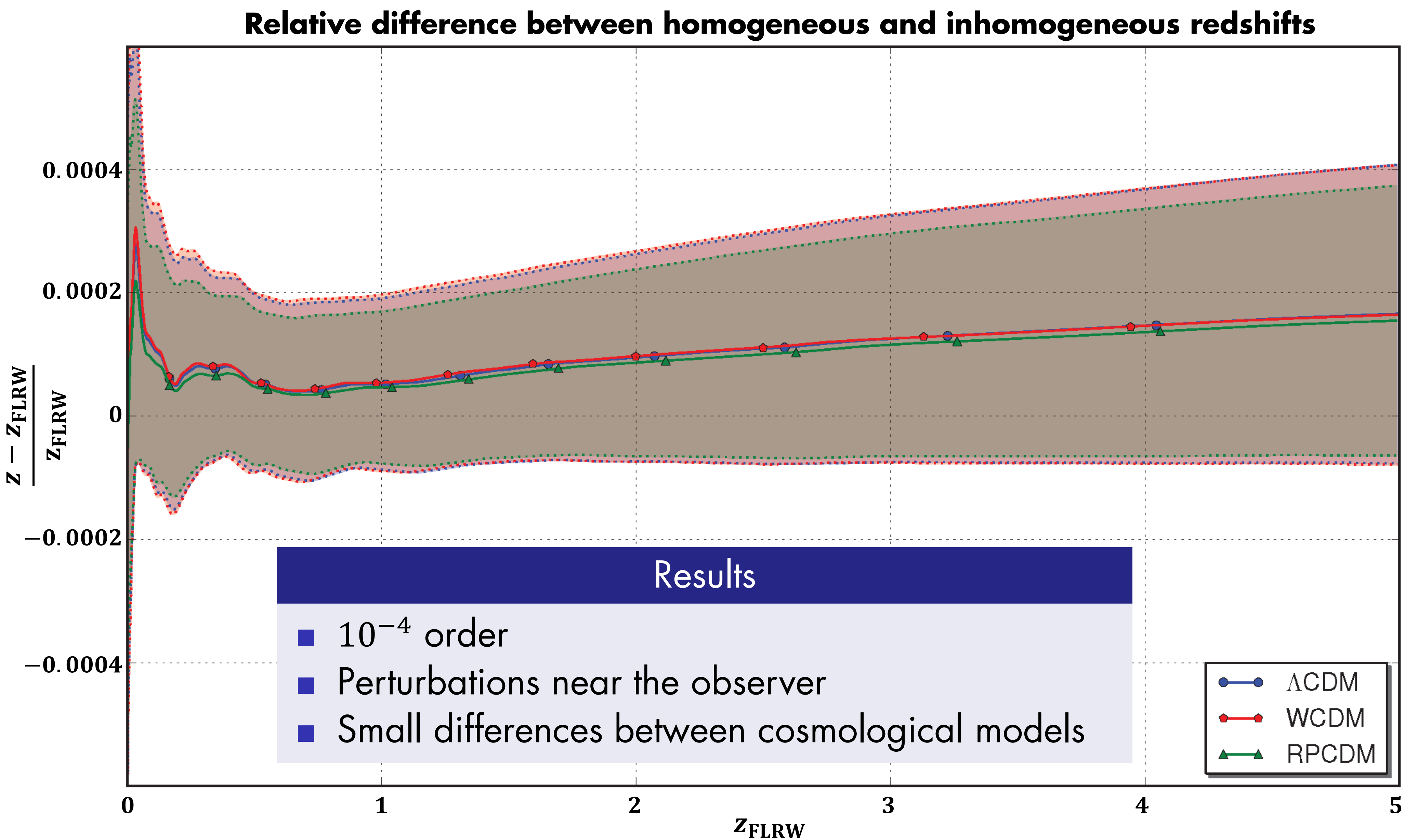


Unperturbed angular diameter distance



Perturbed angular diameter distance

Effects on redshift

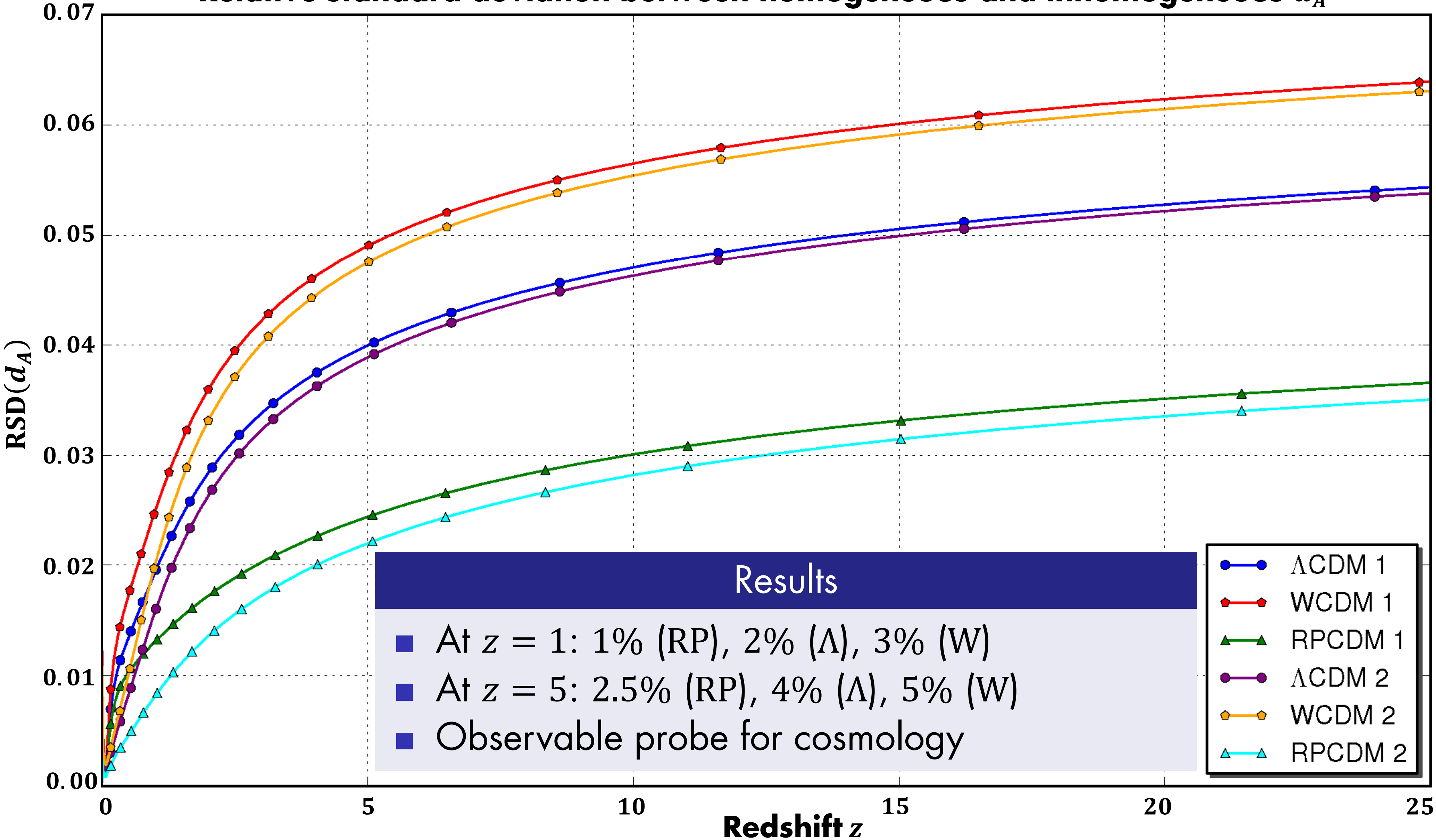


Results

- 10^{-4} order
- Perturbations near the observer
- Small differences between cosmological models

Effects on angular diameter distance

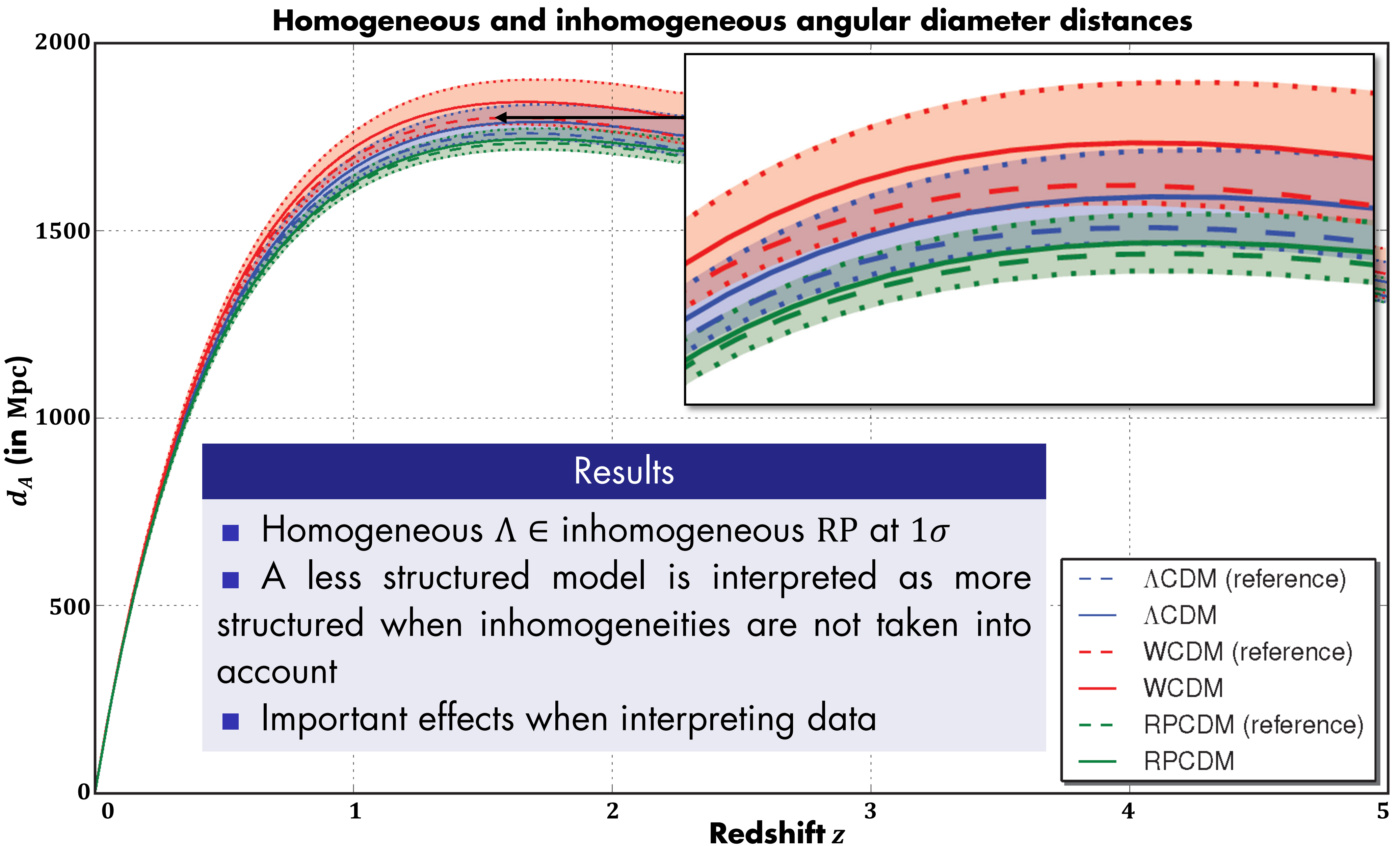
Relative standard deviation between homogeneous and inhomogeneous d_A



Results

- At $z = 1$: 1% (RP), 2% (Λ), 3% (W)
- At $z = 5$: 2.5% (RP), 4% (Λ), 5% (W)
- Observable probe for cosmology

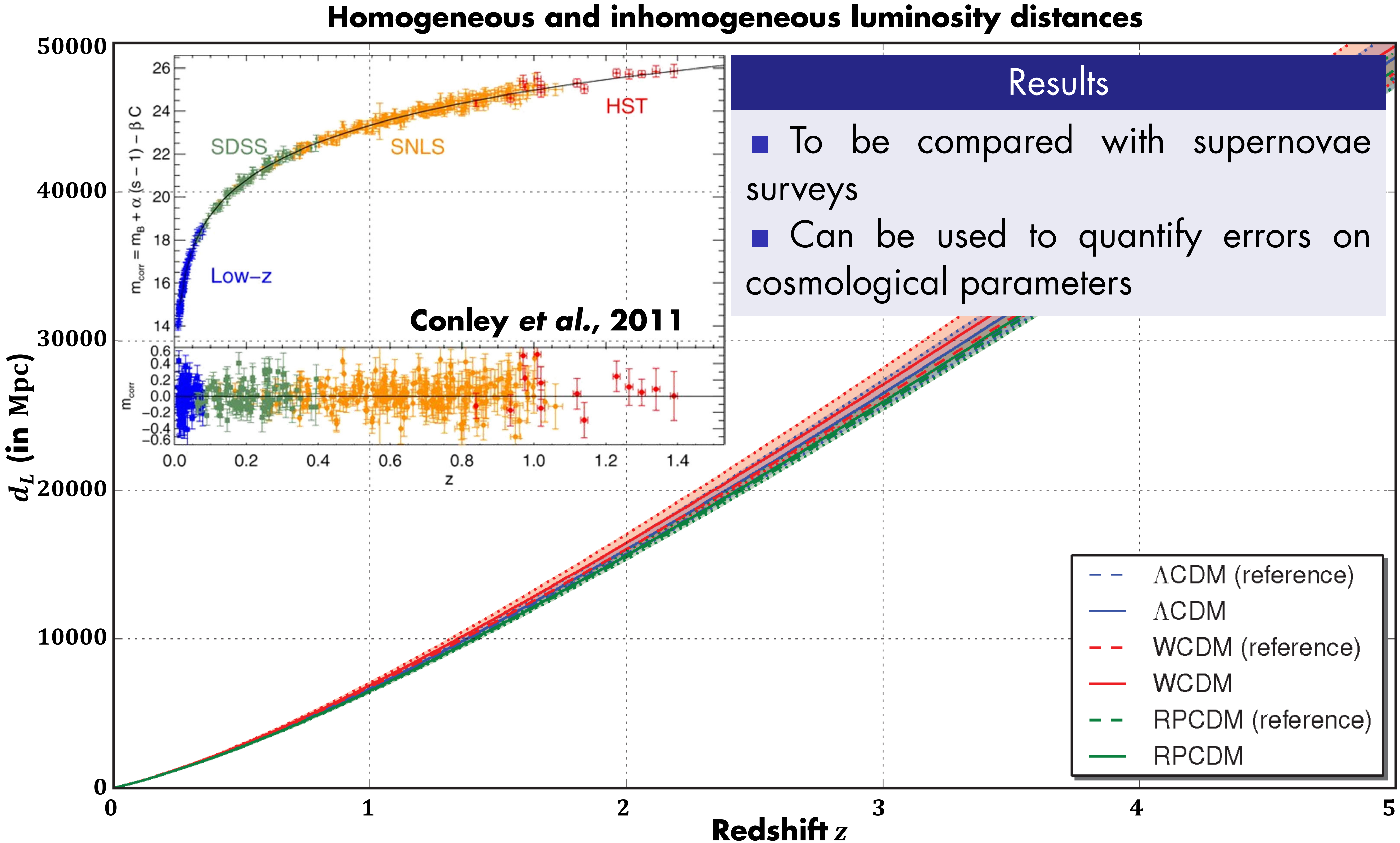
Effects on angular diameter distances



Results

- Homogeneous $\Lambda \in$ inhomogeneous RP at 1σ
- A less structured model is interpreted as more structured when inhomogeneities are not taken into account
- Important effects when interpreting data

Effects on luminosity distance



Results

- To be compared with supernovae surveys
- Can be used to quantify errors on cosmological parameters

Part 5

Conclusions and perspectives

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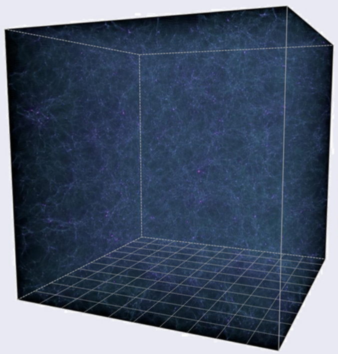
Numerics: summary & conclusions

Full Universe Run

- Observable Universe
- 80 000 cores on Curie



Optimizations & lessons

- Intertwining
- Complexity

Parallelization

Numerical methods

Physics

EDSL & metaprogramming

- Genericity
- Performance
- MAGRATHEA library

Eulerian (cell)

Lagrangian (particle)

Level 0: Utilities
Management, parallelization, files, log, time...

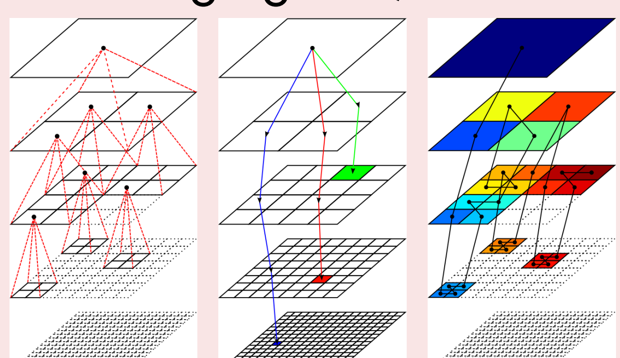
Level 1: Static

Level 2: Cinematic

Level 3: Dynamic

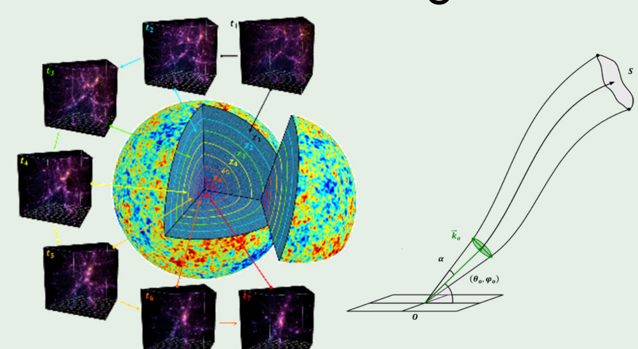
AMR (static)

- Program-integer duality
- Emerging tree/SFC



3D raytracing (cinematic)

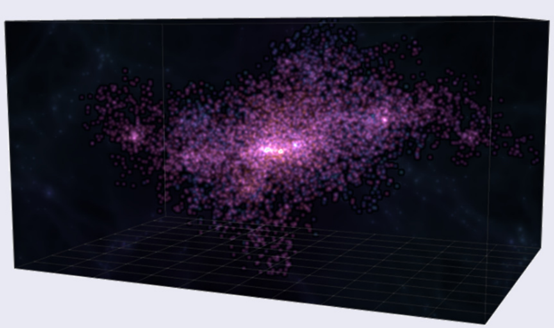
- Cone construction
- Geodesics integration



Physics: summary & conclusions

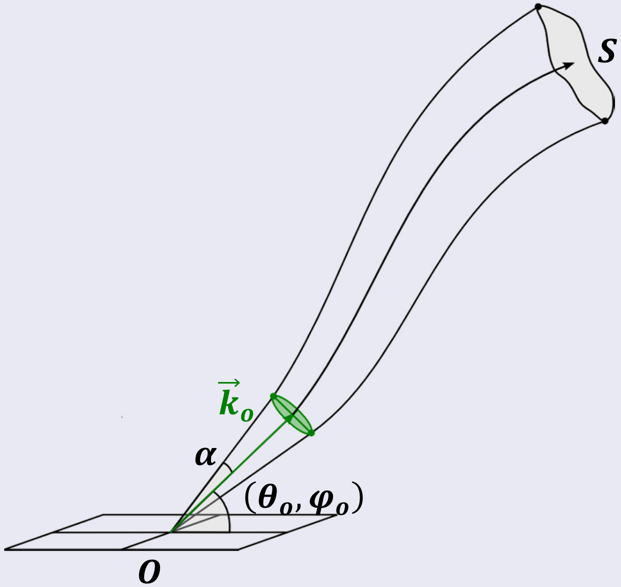
Statistics from FUR

- Characteristics of most massive halos over the volume of the observable Universe
- Mass function: $MF_{RP} < MF_{\Lambda} < MF_{W}$
- Most massive halo in $\Lambda\mathrm{CDM}: 1.07 \times 10^{16} M_{\odot}/h$



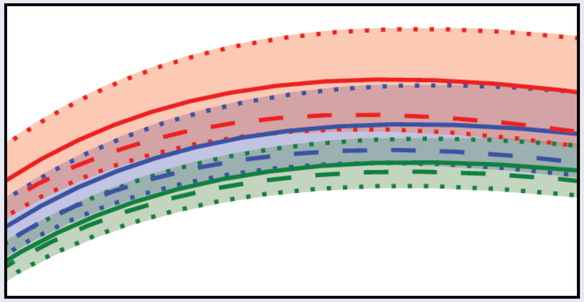
Geodesics integration

- Generic method using direct integration + light-cones constructed from FUR
- Raising the question of stop condition in an inhomogeneous matter field



Effects on d_A

- Effects of inhomogeneous matter field at $z = 1: 1\%$ (RP), 2% (Λ), 3% (W)
- Need to take inhomogeneities into account to interpret observations
- Distribution of d_A is a probe for cosmology



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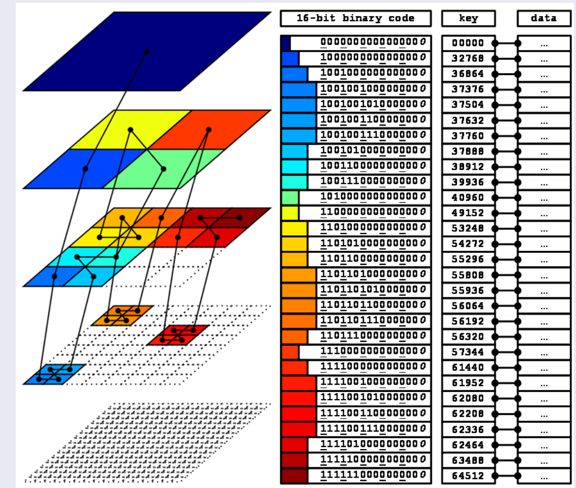
Perspectives

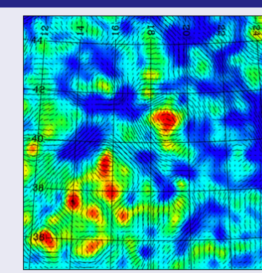
Numerics

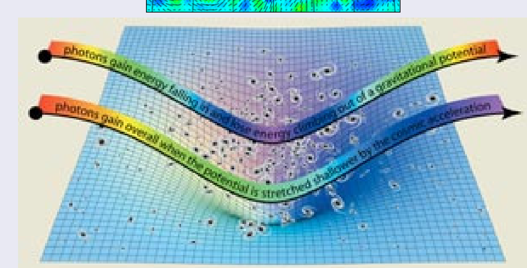
- Integration of geodesics on more resolved simulations
- Detailed study of the tree algorithm performances
- Generalization of the tree algorithm
- Extension of the MAGRATHEA library to the dynamic part



Physics

- Detailed Monte-Carlo study on cosmological parameters
- Exploration and comparison of several stop conditions
- Use of the integrator to study weak-lensing and ISW…





…and good luck to PHILAE