Portfolio Optimization: A Monte Carlo Study

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1. Introduction

This project applies Monte Carlo simulation to study how portfolio risk and return behave under two investing regimes:

1. Long-only portfolios, where all asset weights are positive (no short selling).
2. Short-allowed portfolios, where some assets can have negative weights, representing short positions.

By simulating thousands of random portfolios using the expected returns, volatility, and correlations of four assets, we can visualize the opportunity set, the range of achievable combinations of risk and return.

This approach demonstrates how diversification, shorting, and random weighting influence the efficient frontier, the curve representing optimal portfolios for different risk levels.

2. Methodology

The simulation assumes asset returns follow a multivariate normal distribution:

$R \sim N(\mu, \Sigma)$
where

- μ: vector of expected returns,
- Σ: covariance matrix (computed from asset standard deviations and correlations).

Each Monte Carlo trial generates a random weight vector across the four assets, summing to one.
Two cases were analyzed:

- Case 1: No shorts allowed (weights ≥ 0)
- Case 2: Shorts allowed (weights $\in [-1, 1]$)

For each portfolio, the expected return and standard deviation (risk) are computed as:

$E[R_p] = w^T\mu, \sigma_p = w^T\Sigma w$

3. Asset Configuration

For the custom scenario (outputs_alt), four assets were selected to mimic a realistic, modern investment portfolio combining technology and fixed-income exposure:

Asset	Expected Return	Std. Dev.	Description
AAPL	0.11	0.20	Apple Inc. (large-cap tech)
NVDA	0.18	0.30	NVIDIA Corp. (semiconductors)
TSLA	0.22	0.35	Tesla Inc. (high-volatility growth)
TLT	0.05	0.10	U.S. Treasury Bond ETF (low-risk hedge)

The correlation matrix was configured as:

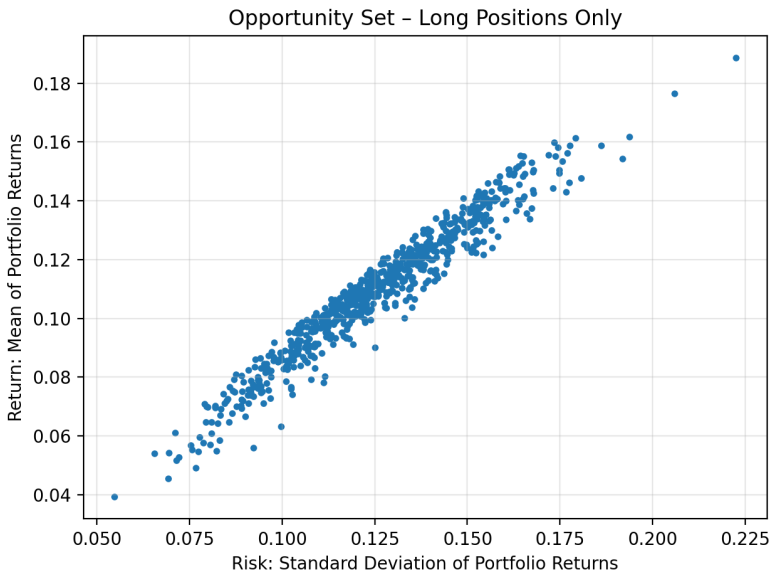
Asset Pair	Correlation
Equity–Equity (AAPL, NVDA, TSLA)	0.6–0.8
Equity–Bond (vs. TLT)	–0.2 to –0.3

This structure reflects typical real-world market behavior — equities move together, while bonds provide diversification.

4. Results and Analysis

Each figure below represents 700 randomly generated portfolios simulated using this configuration.

4.1 Long Positions Only

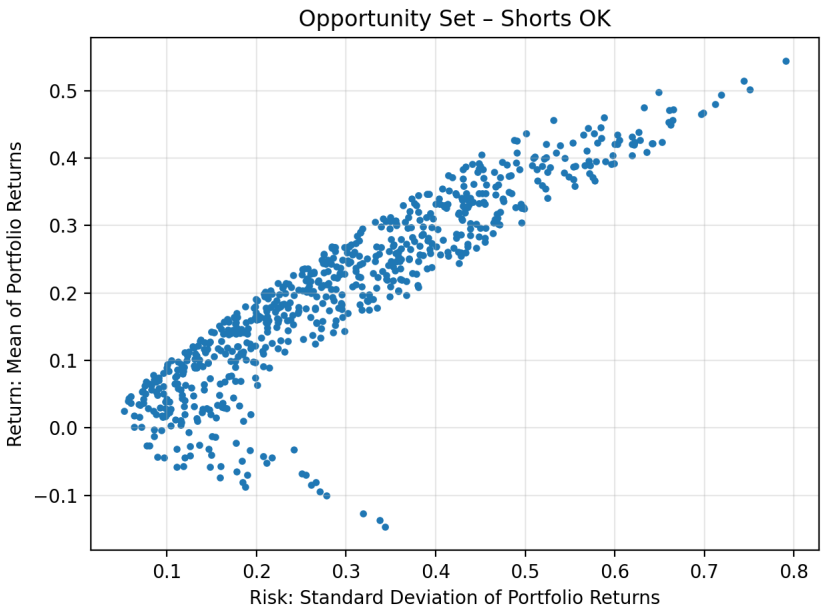


The long-only portfolios form a tight, upward-sloping curve, showing a clear risk–return tradeoff.
As standard deviation increases, expected return also rises.
This represents the efficient frontier , the best attainable portfolios under long-only constraints.

Key observations:

- The shape is relatively narrow and smooth.
- No negative returns appear since all weights are positive.
- Higher risk portfolios correspond to heavier allocations toward volatile assets (e.g., NVDA and TSLA).

4.2 Shorts Allowed



When shorting is permitted, the feasible set expands dramatically.
Portfolio returns now range from negative to very high positive values, forming a butterfly-like pattern.

Key insights:

- Shorting increases both potential gains and potential losses.

- Portfolios in the outer edges represent extreme leveraged or shorted positions.
- While some points lie far above the long-only frontier, others fall below zero — a sign of high variance and negative expected return outcomes.

Overall, shorting widens the opportunity set but introduces significant instability.

5. Discussion

The comparison between the two cases demonstrates the impact of shorting on portfolio design:

Aspect	Long-Only	Shorts Allowed
Risk Range	Moderate	Much wider
Return Range	0.04–0.18	–0.2–0.4
Shape	Narrow upward curve	Broad cone-like region
Diversification Effect	Limited	Enhanced (can hedge and leverage)

Key takeaways:

- The long-only frontier is stable but constrained — realistic for most investors.
- The shorts-allowed frontier is volatile but efficient — feasible for hedge funds or advanced traders.
- The simulation confirms Markowitz's portfolio theory — diversification reduces risk, while shorting introduces nonlinearity and higher variance.

6. Conclusion

Through Monte Carlo simulation, we observe how allowing short positions expands the feasible risk-return space of a portfolio.

- Without shorts: portfolios exhibit a steady, monotonic increase in returns with risk, ideal for conservative investors.
- With shorts: risk and return increase exponentially, demonstrating both potential and danger.

This experiment provides valuable intuition for investors about how risk tolerance and leverage affect portfolio efficiency.

It also validates stochastic programming's usefulness for exploring uncertain investment outcomes.

7. Appendix

A. Files Included

- monte_carlo_portfolio.py - main Python simulation file
- assets_alt.json - custom asset configuration
- outputs_alt/ - generated CSVs and plots (shown above)
- (Placeholder for outputs/ - baseline simulation with default assets)
- portfolio_optimization_report.pdf - this report
- README.md - documentation and usage instructions

B. AI Assistance

Parts of this project (code translation from R, structure, and figure formatting) were assisted by ChatGPT.
All code and data validation steps were independent by the author.

8. References

- Markowitz, H. (1952). Portfolio Selection. Journal of Finance.
- Venables, W. & Ripley, B. (2002). Modern Applied Statistics with S. Springer.
- Tom Miller (2025). Monte Carlo Portfolio Optimization Jump-Starter Code.