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INF FAKULTÄT FÜR INFORMATIK

Continuous Modeling

Stray Planets, and the Swine Flu in a High School

Agenda

Illustrate what the consequences of limited ODE integration accuracy are, and how to mitigate them.

(You) develop and extend a continuous model for the progression of an epidemic.

Integration Accuracy and Planets

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Initial Conditions

State of the model at the beginning of the simulation:

$(u_1, v_1)=(0, 0)$

$(u_2, v_2)=(0, 10)$

$(x_1, y_1)=(0, 0)$

$(x_2, y_2)=(1, 0)$

$m_1=1000$

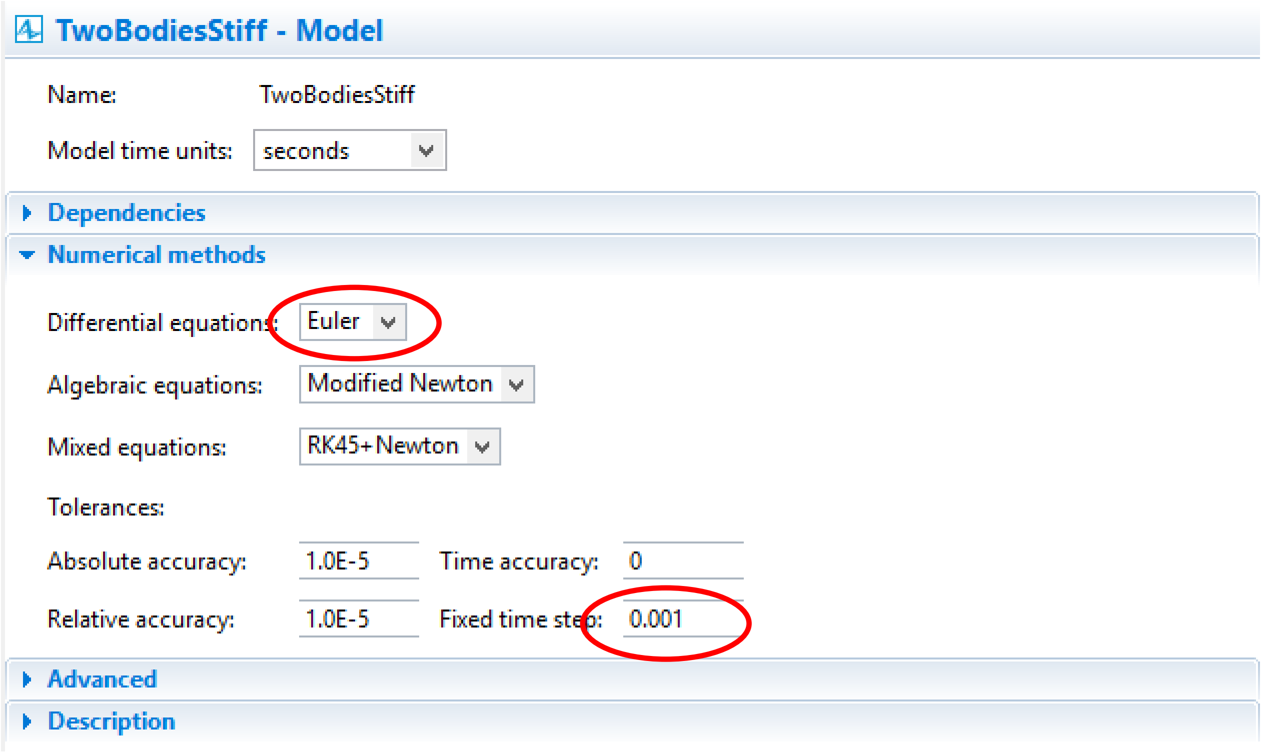
$m_2 = 1$

$\Gamma=1$

Standard Integration Parameters

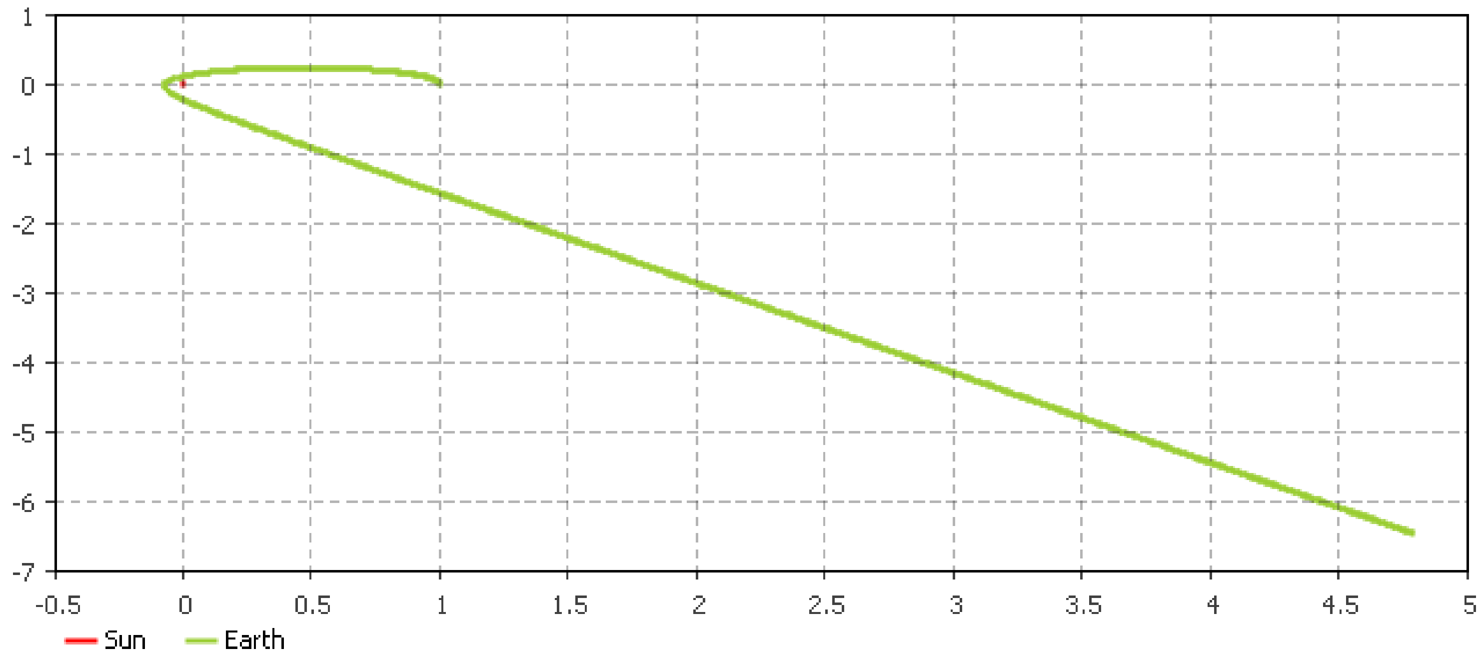
Standard parameters

- Integration method: Euler
- Integration time step: 0.001



Integration Results

Planet spins of into space

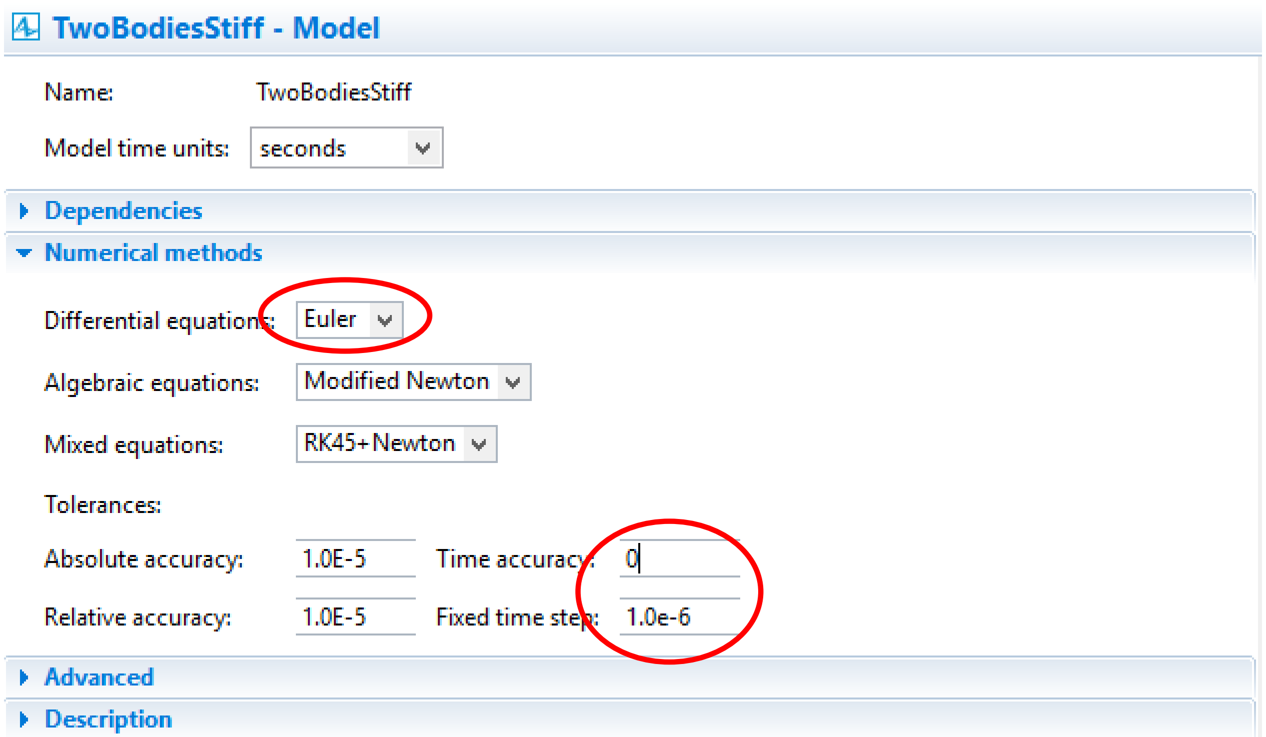


Is this physically possible?

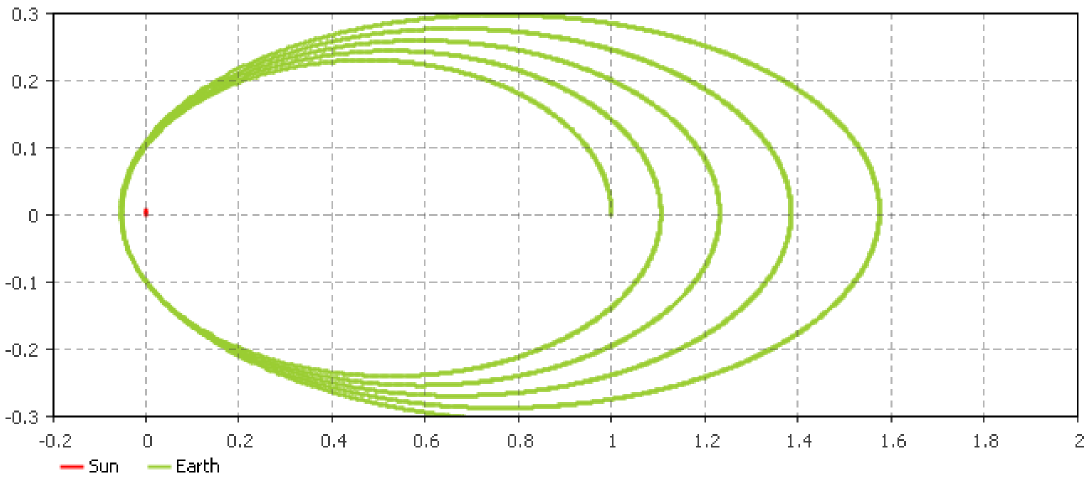
Changed Integration Parameters

New parameters

- Integration method: Euler
- Integration time step: 0.0000001 (must be greater than time accuracy)



Integration Results

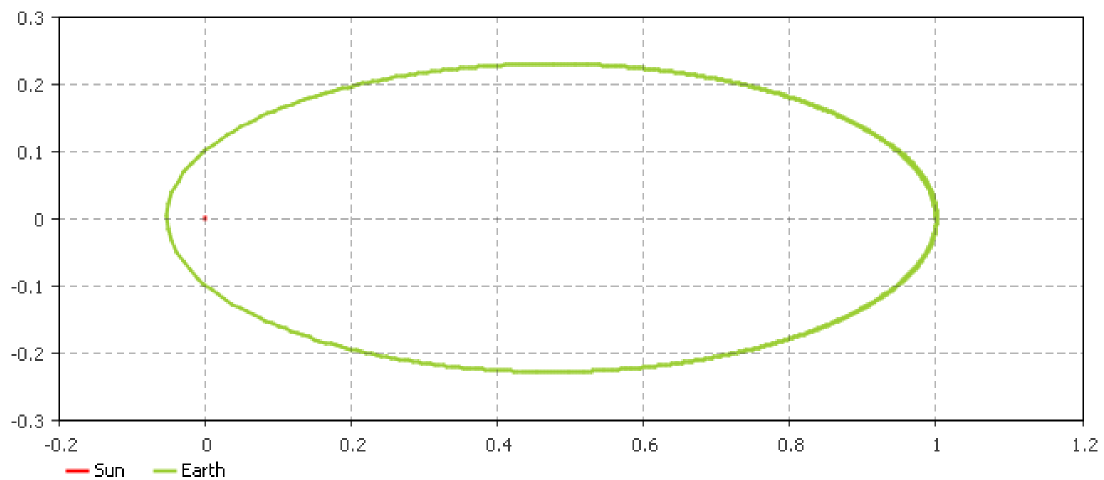


Euler

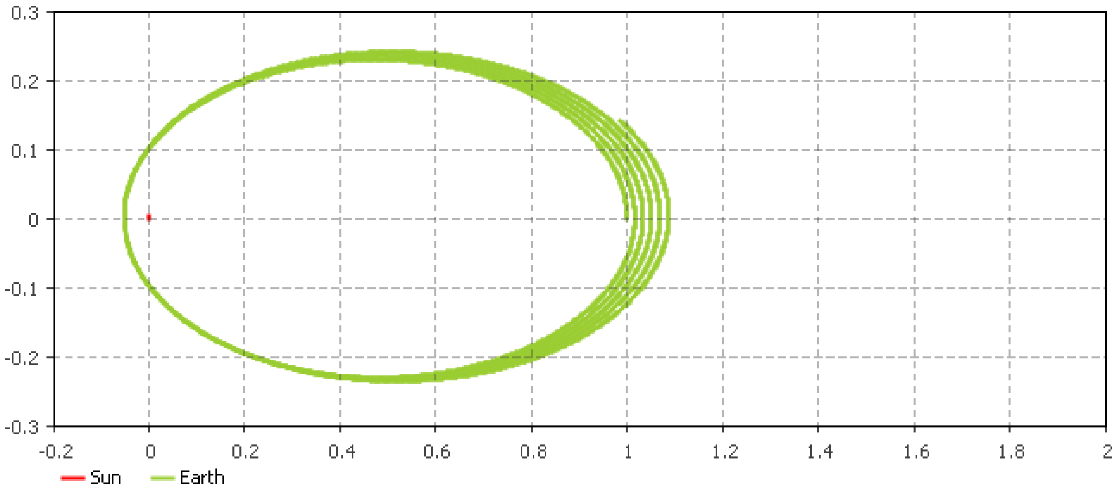
Time step = 1e−6

Euler

Time step = 1e−8



Integration Results

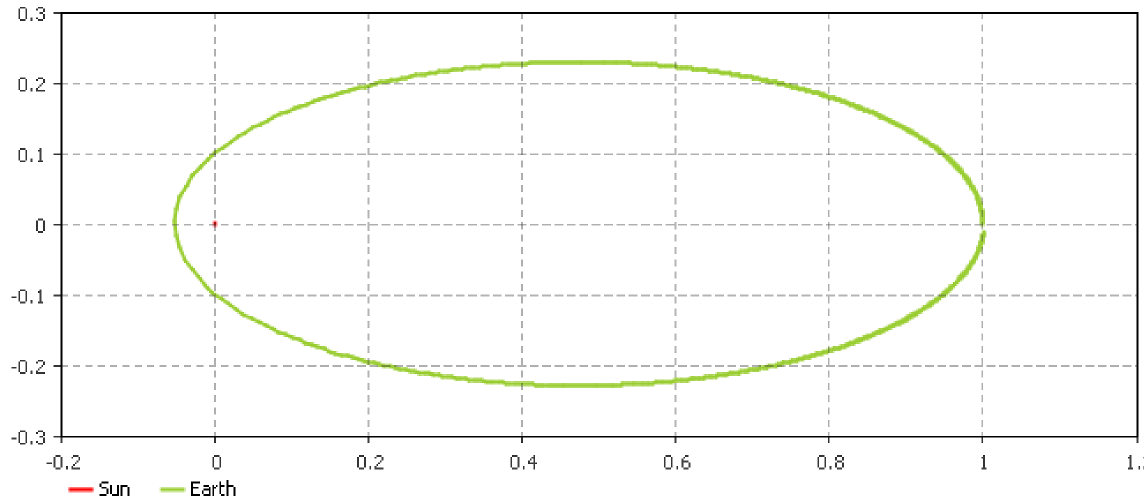


Runge–Kutta 4

Time step = 1e–6

Runge–Kutta 4

Time step = 1e–7



A New AnyLogic Element Type

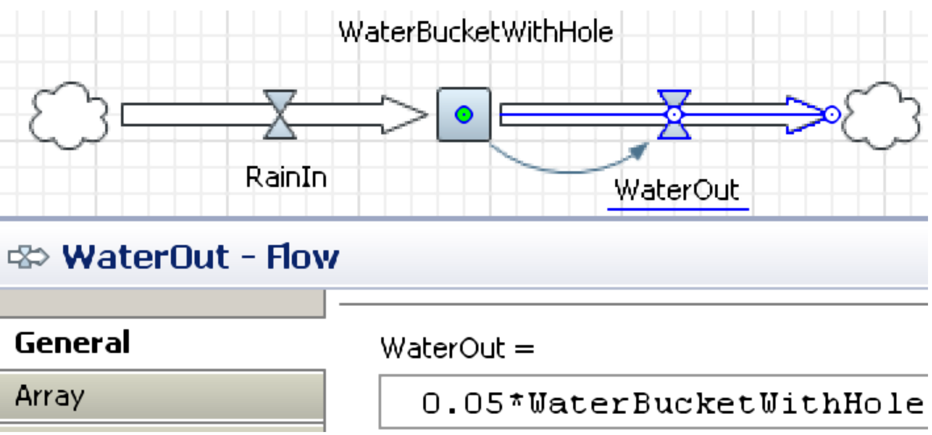
Flow Variables

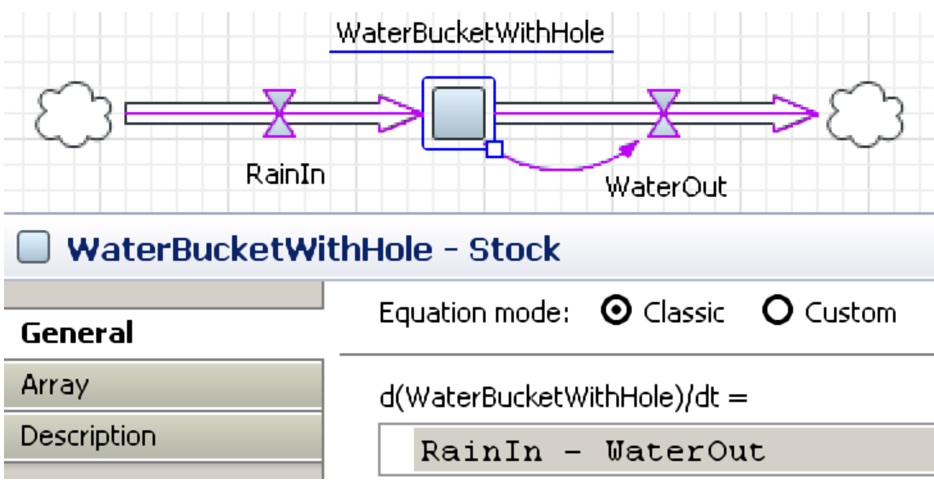
… have a value that automatically updates based on a mathematical expression (like auxiliary variables)

But Flow Variables

- Can only be accessed by stock variables
- Are used when an actual flow of a quantity (water / energy / patients / …) occurs

→ Are usually used for systems described by a balance equation





An Epidemic

The SIR Model

The SIR model is a classical model in epidemiology

- S – susceptible individuals (may get infected)
- I – infected/infectious individuals (are infected and spread the disease)
- R – recovered individuals (are healthy and cannot be infected)

It is a model of how an infectious disease spreads in a closed population, such as a High School

The model can also incorporate

- Vaccinations
- Population dynamics

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The SIR Model

We assume a closed population of 1000 individuals

- Initially, only one of them is infected
- No one is resistant

Everybody has 10 contacts to other people per day

For the given disease, when meeting an infected person one has an infection risk of 0.08.

An infected individual needs on average 10 days to recover

Defining the Equations

Initial Values:

$$S = 999,\ I = 1,\ R = 0$$

Differential Equations:

$$\frac{dS}{dt} = -InfectionRate$$

$$\frac{dI}{dt} = InfectionRate - RecoveryRate$$

$$\frac{dR}{dt} = RecoveryRate$$

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Defining the Equations

Infection Rate

- The given disease has an infection risk of 0.08, when meeting an infected person
- The contact rate in our population is 10 contacts per day

$$InfectionRate = InfectionRisk * EncounterRate * S * P(Person\ is\ sick)$$

$$= InfectionRisk * EncounterRate * S * \frac{I}{S+I+R}$$

Recovery Rate

- An infected individual needs on average 10 days to recover

$$RecoveryRate = I * 1/DiseaseDuration$$

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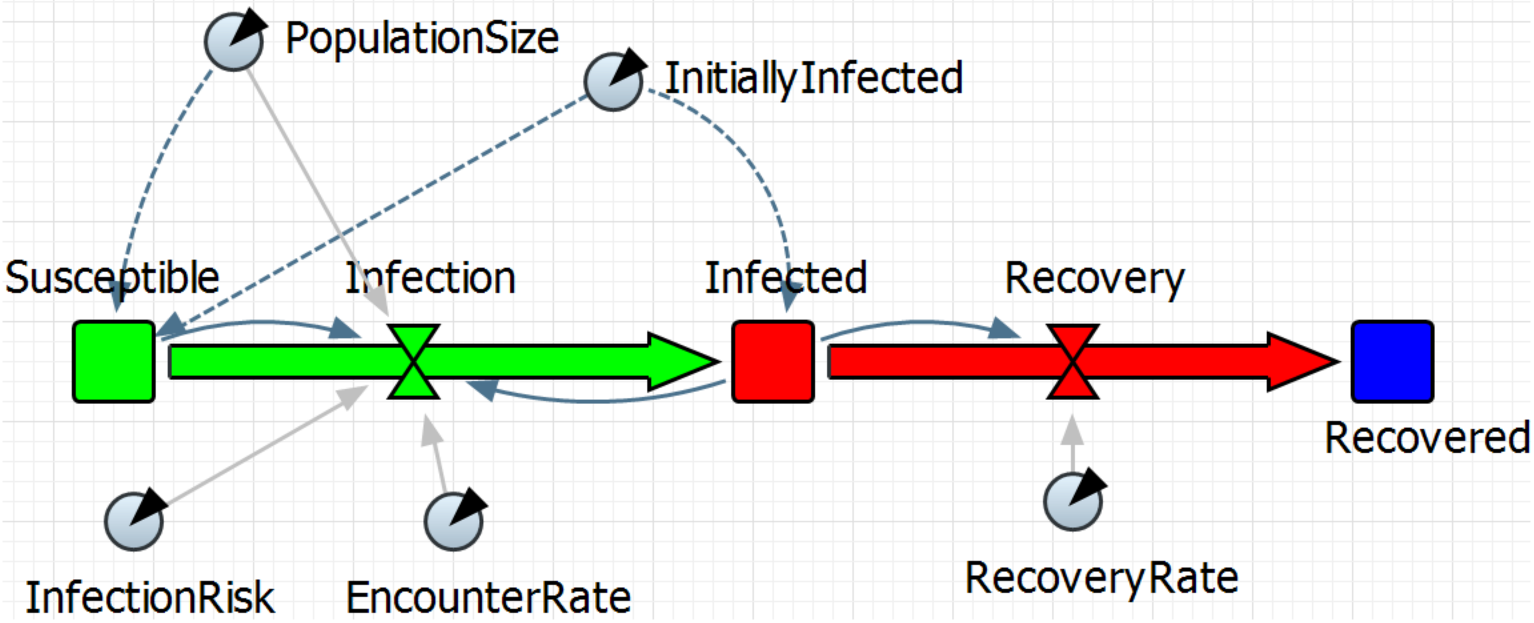
AnyLogic Model



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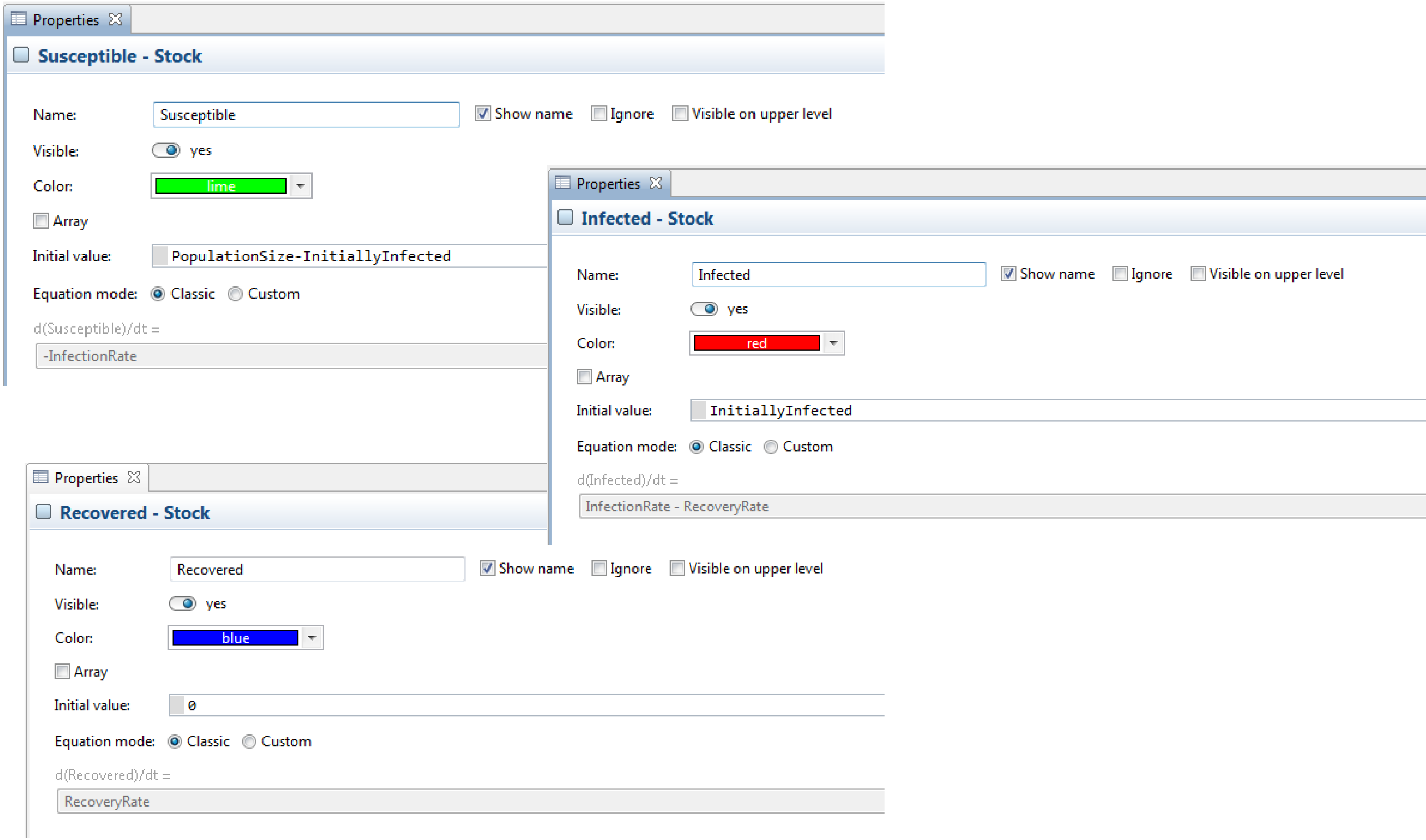
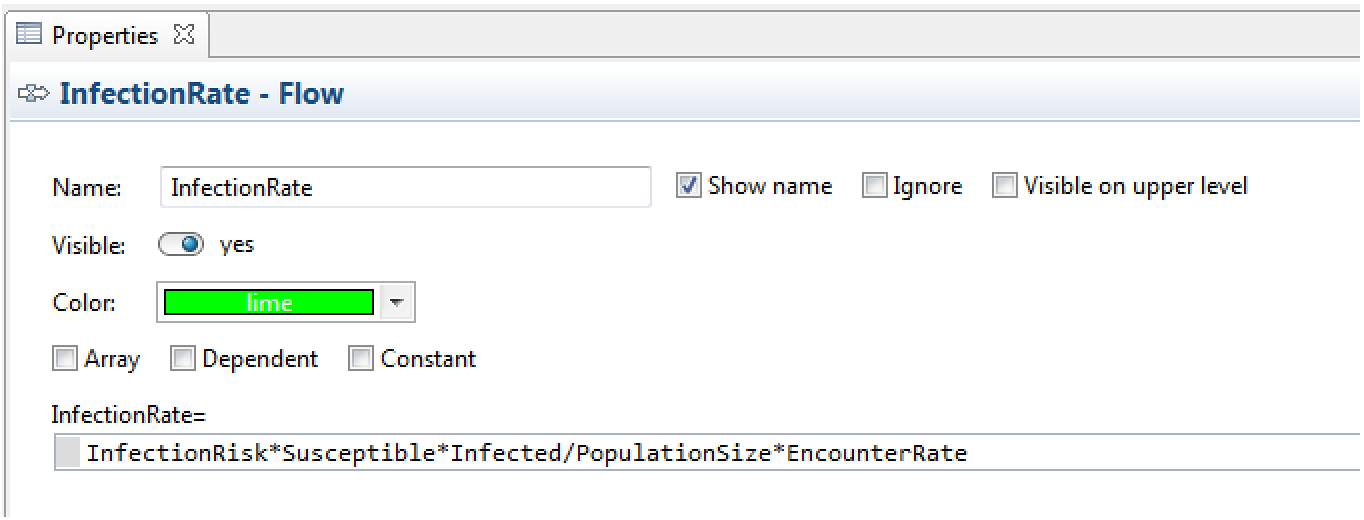
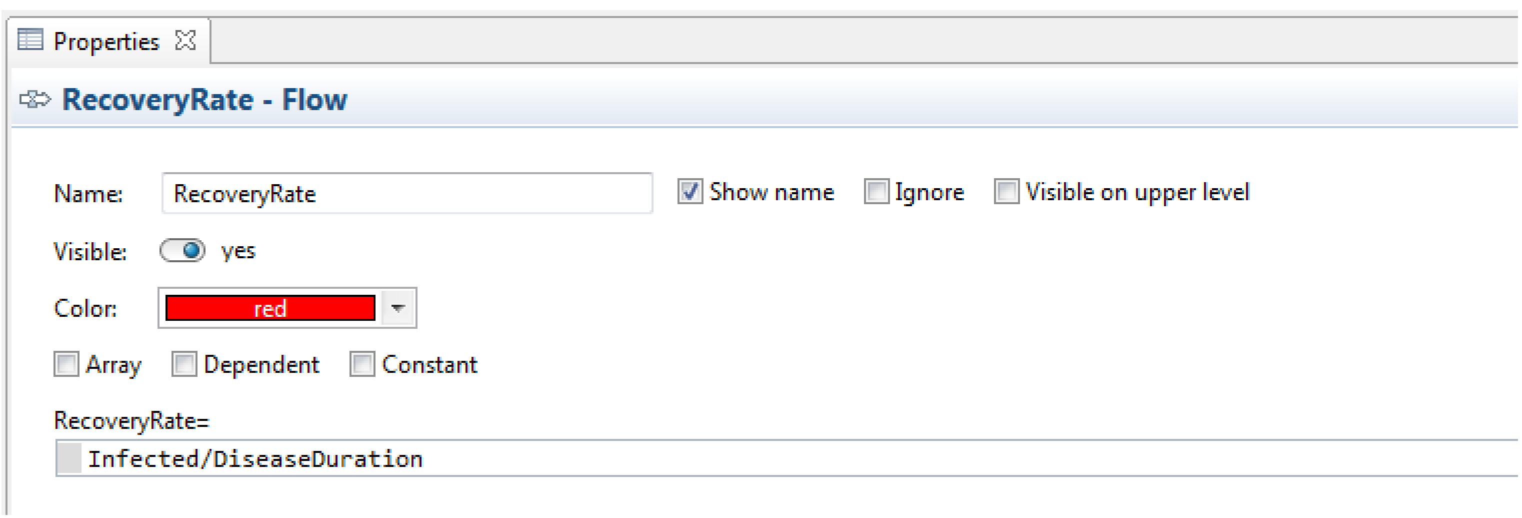
Stock Variables



Flow Variables





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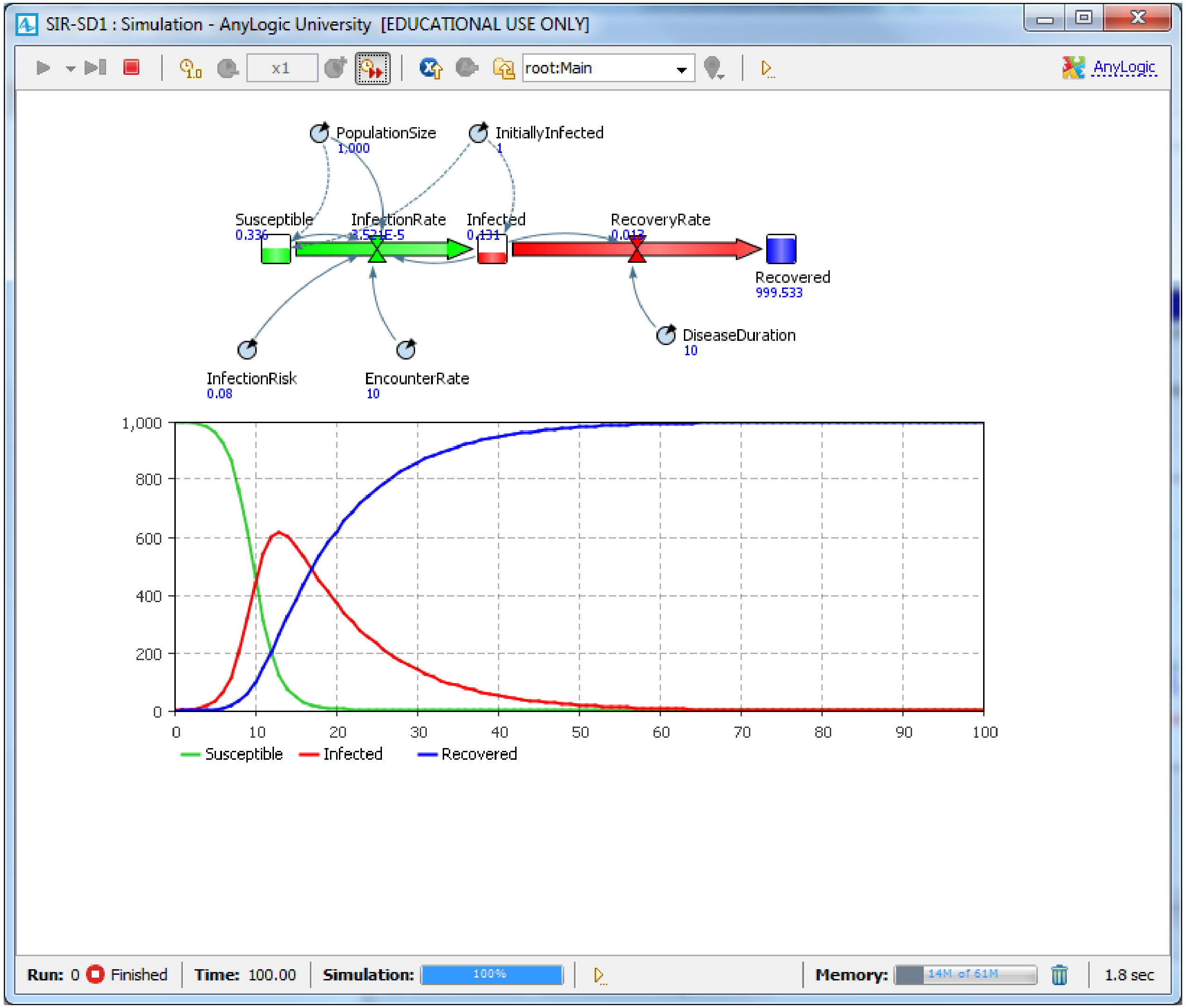
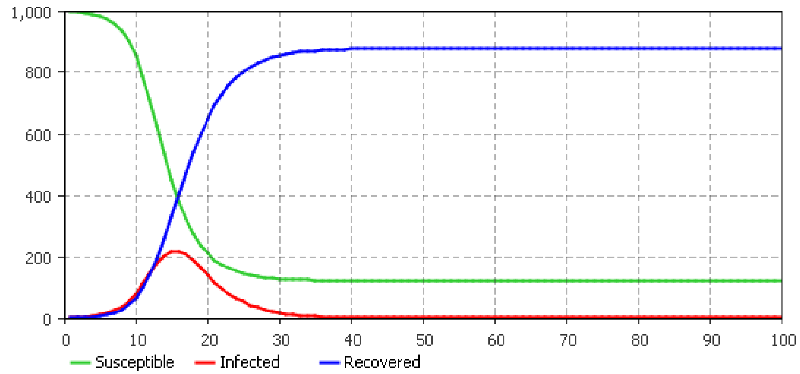
AnyLogic Simulation Result

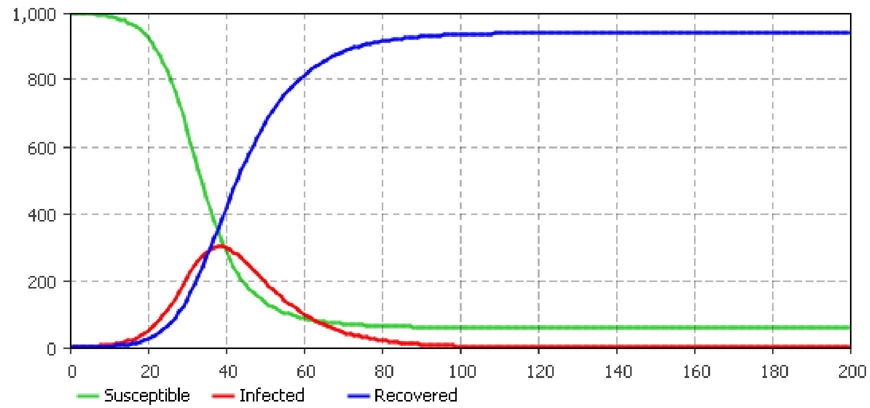


Scenarios

Patients recover in 3, not in 10 days → not everyone is infected



Infection risk is decreased to 0.03, instead of 0.08 → disease is slowed down and controlled



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Introducing Vaccinations

Assume that the government has started a large scale vaccination program to prevent the spread of the disease

Five people not having had the disease yet are vaccinated per day

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Introducing Vaccinations

Assume that the government has started a large scale vaccination program to prevent the spread of the disease

Five people not having had the disease yet are vaccinated per day

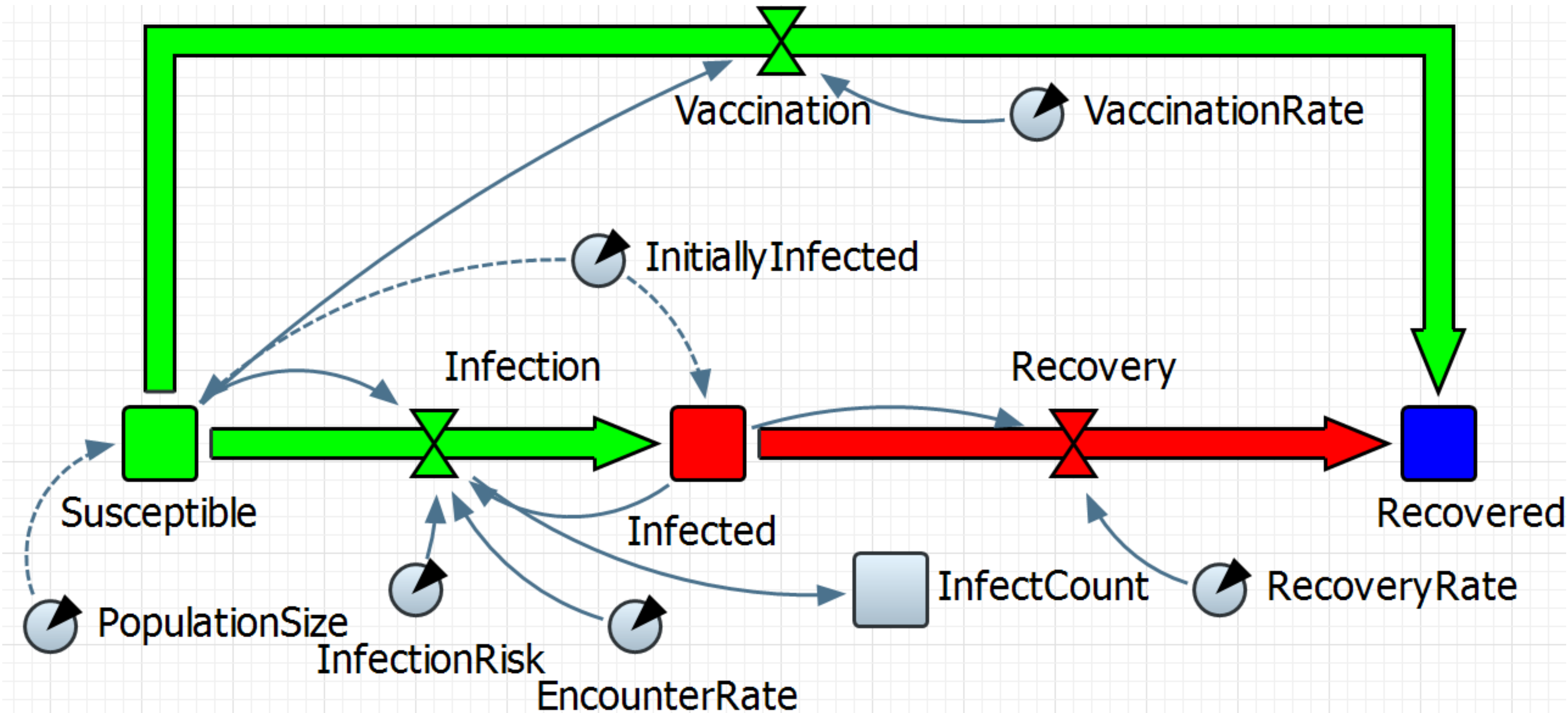
$$\frac{dS}{dt} = -InfectionRate - VaccinationRate$$

$$\frac{dR}{dt} = RecoveryRate + VaccinationRate$$

Vaccination Rate

$$VaccinationRate = S > 0 \; ? \; 5 : 0$$

AnyLogic Model Including Vaccinations



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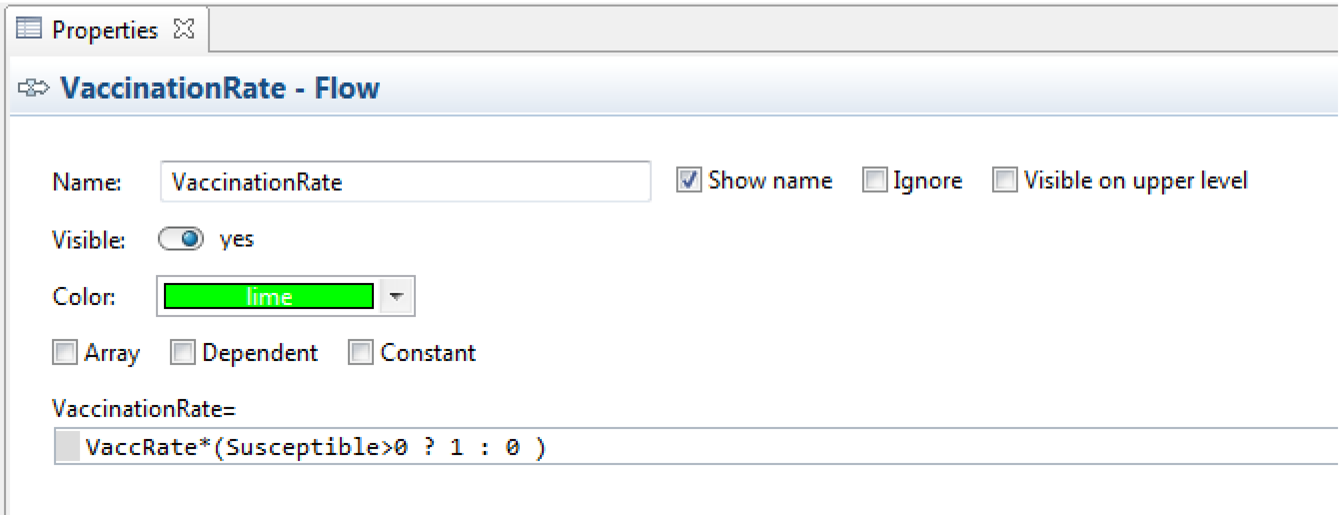
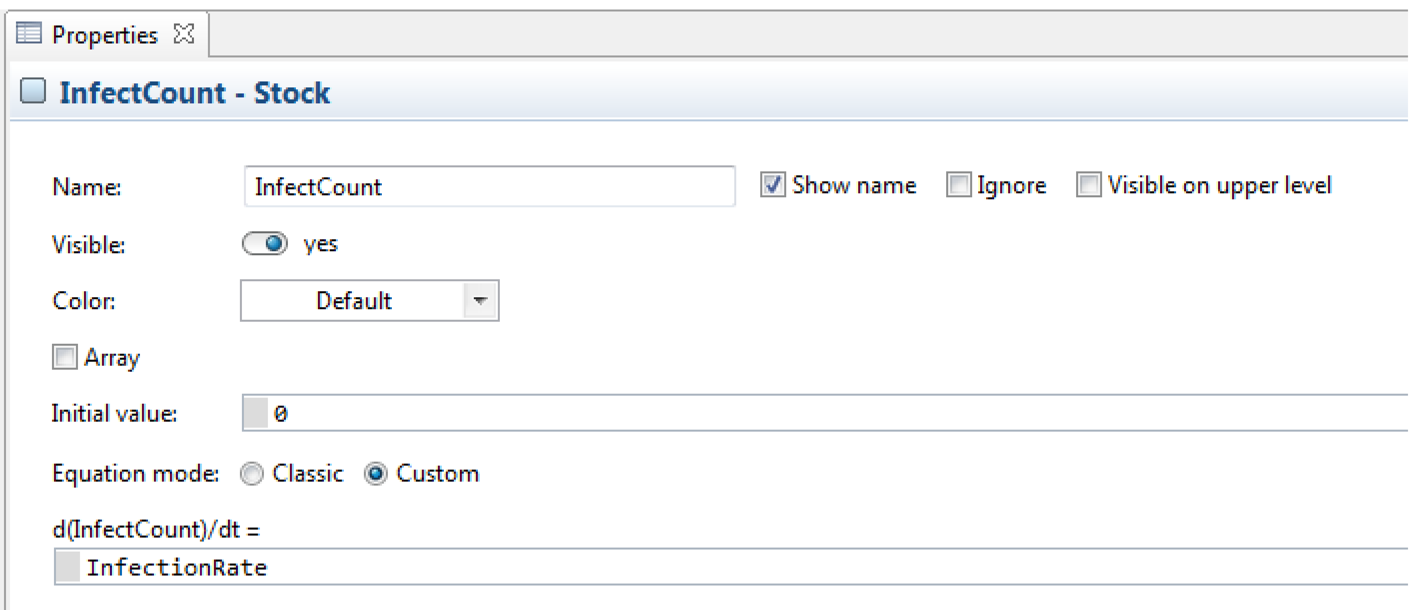
New Stock and Flow Variable





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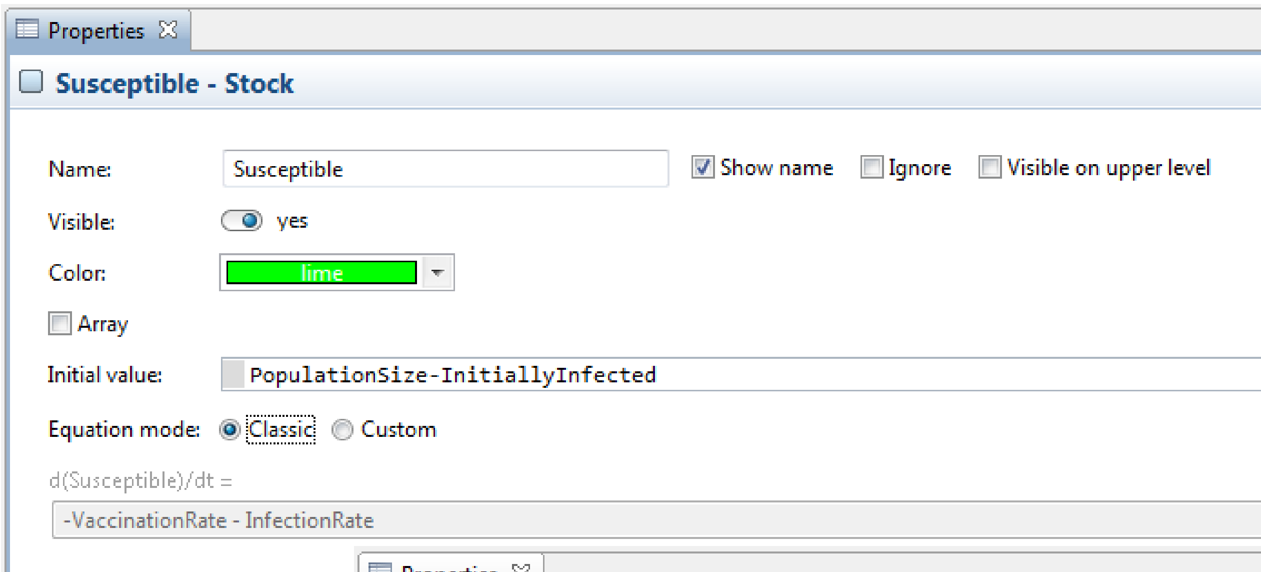
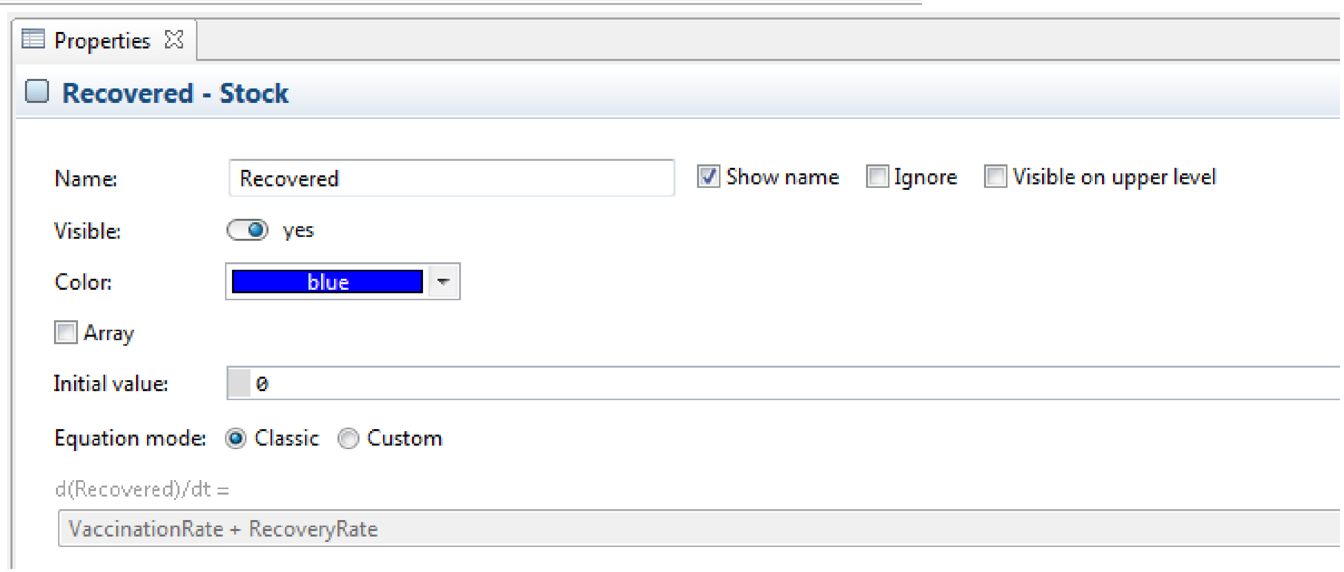
Modified Stock Variables





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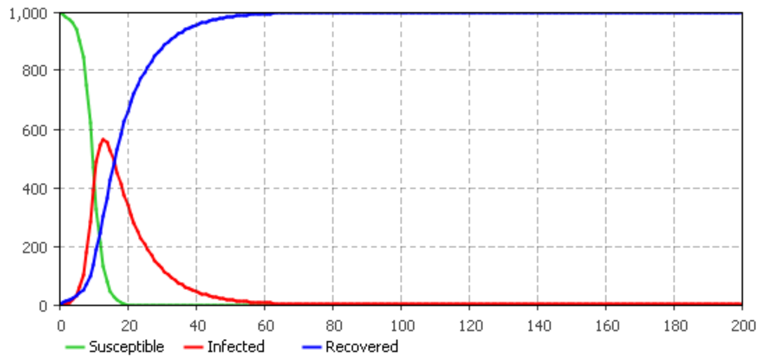
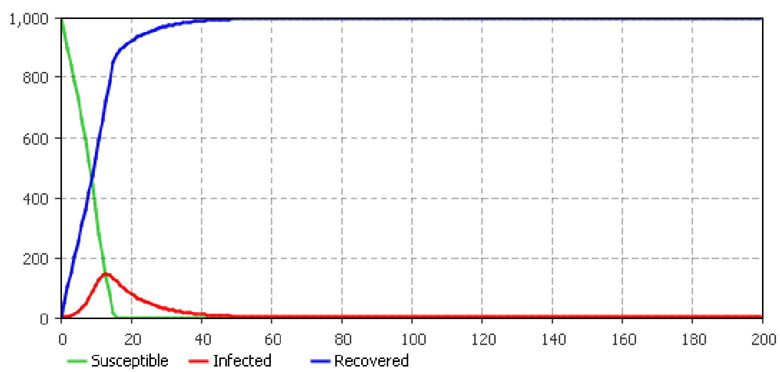
Scenarios

Vaccination rate of 5 per day → reduces total number of infections to 904



Vaccination rate increased to 50 per day → reduces total number of infections to 230



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Introducing Population Dynamics

Population is now also affected by birth and death dynamics

- The birth and death rates are proportional to the population, with a proportionality factor of 0.01 for both rates, maintaining the overall population size
- All newborn are initially susceptible, and the deaths are equally likely in all three groups of people

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$$BirthRate = (S + I + R) * BirthLikelihood$$

$$\frac{dS}{dt} = -InfectionRate - VaccinationRate + BirthRate - S * DeathRisk$$

$$\frac{dI}{dt} = InfectionRate - RecoveryRate - I * DeathRisk$$

$$\frac{dR}{dt} = RecoveryRate + VaccinationRate - R * DeathRisk$$

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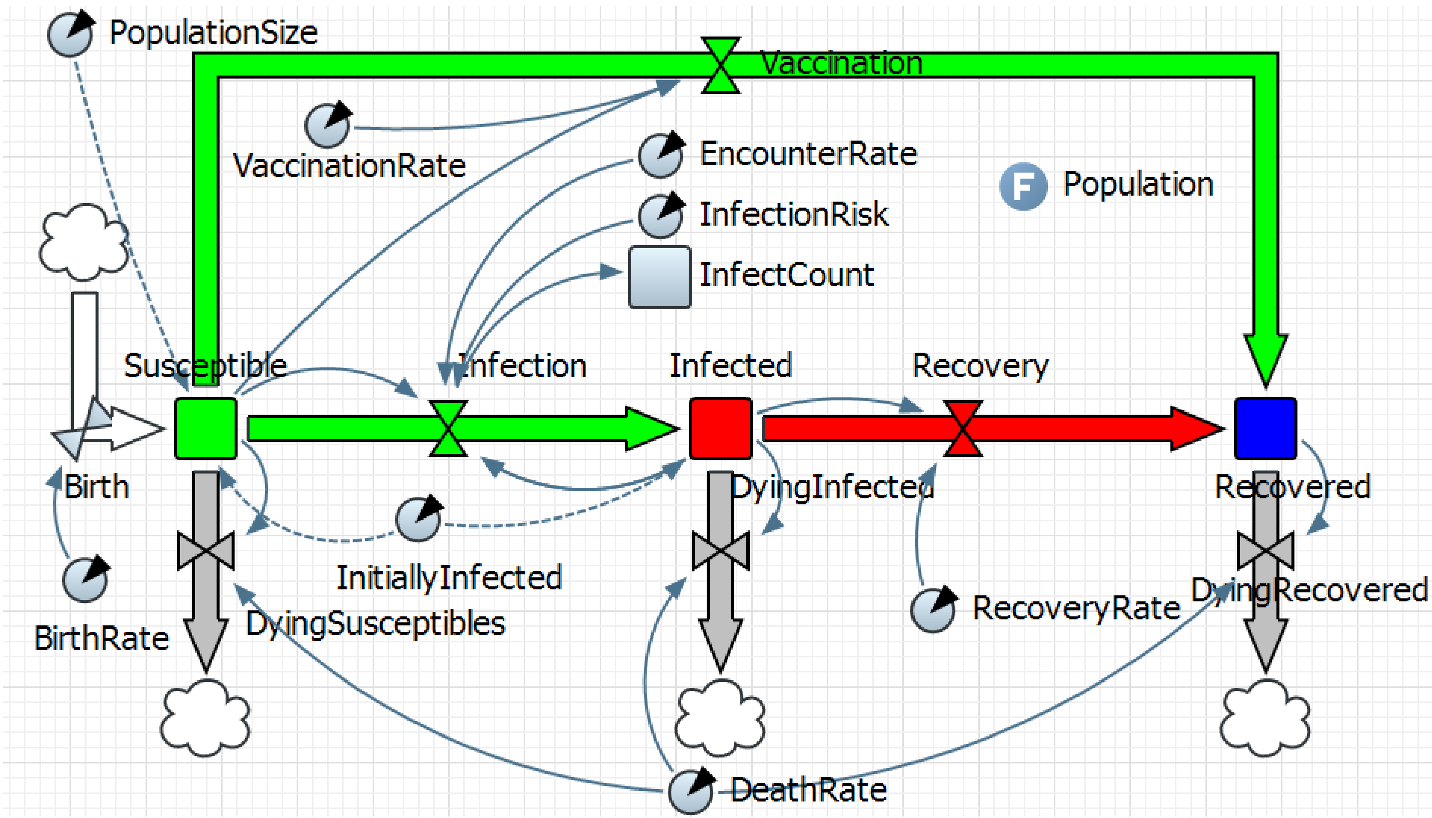
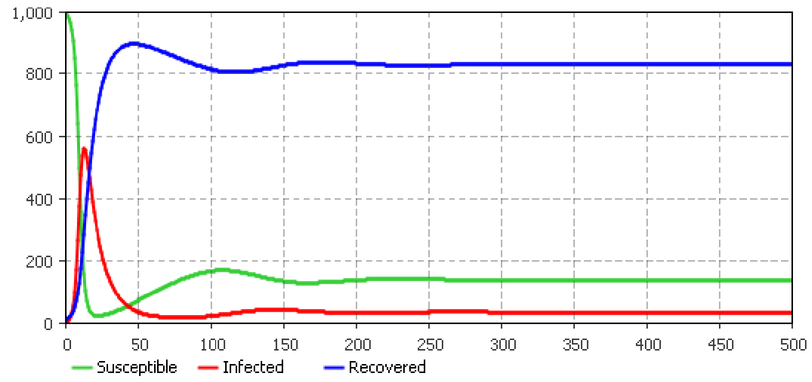
AnyLogic Model Including Population Dynamics

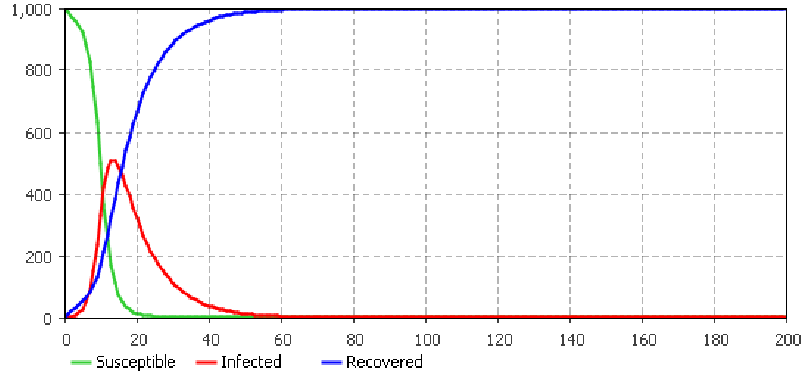


Scenarios

Constant birth and death rate of 0.01 → the disease will become a permanent threat



A campaign increasing the vaccination rate to 10 per day counters population dynamics → the disease is exterminated



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Learning Goals

Practical experience with converting a textual description into a set of differential equations

- … just like question one in the exam!

Assignment 2 gives you further opportunity for practice, and to extend the SIR model