INF

FAKULTÄT FÜR
INFORMATIK

Confidence Intervals and Replications in AnyLogic

The Medical Practice – A Business Perspective

Computing Confidence Intervals

The method of *independent* replications:

- Run the simulation n times
- Use independent sets of random numbers
- Make the observations Y_r, $r = 1, ..., n$
- Compute an estimate $\hat{\theta}$ for the measure of interest
- Compute an estimate S^2 for the variance of this estimator

Confidence Interval Computation

Estimator for mean value

$$\hat{\theta} = \frac{1}{n}\sum_{r=1}^{n} Y_r$$

Estimator for standard deviation

$$\hat{\sigma}(\hat{\theta}) \approx \sqrt{\frac{S^2}{n}} = \sqrt{\frac{\sum_{r=1}^{n}(Y_r - \hat{\theta})^2}{n(n-1)}}$$

Symbols:

- n … number of replications
- Y_r … value of the measured variable at the r*th* replication

Confidence Interval Computation

Confidence Interval

$$\hat{\theta} - \hat{\sigma}(\hat{\theta}) \cdot t_{\alpha/2,f} \leq \theta \leq \hat{\theta} + \hat{\sigma}(\hat{\theta}) \cdot t_{\alpha/2,f}$$

Symbols:

- $t_{\alpha/2,f}$ … value of the inverse Student's t–CDF ("*TINV*")
  - $f = n - 1$ …degrees of freedom
  - α … level of significance

Confidence Interval Interpretation

A useful result could look like this:

$$0.1 \leq \theta \leq 0.6 \quad with\ \alpha = 0.01$$

And means: "The true value of θ lies inside the interval *[0.1, 0.6]* with probability ***0.99*** $(= 1 - \alpha)$"

Pitfall:

- To obtain the necessary value of the t–distribution, use $\alpha/2$, not α!

Experiments in AnyLogic

Experiment Types in AnyLogic PLE

Simulation

- Simulates the model exactly once (So far, we only used this)

Parameter Variation

- Multiple replications for multiple versions of a model

Optimization

- Automatically optimize model parameters with respect to a user–defined objective function

Experiments in AnyLogic

To compute confidence intervals, we need to:

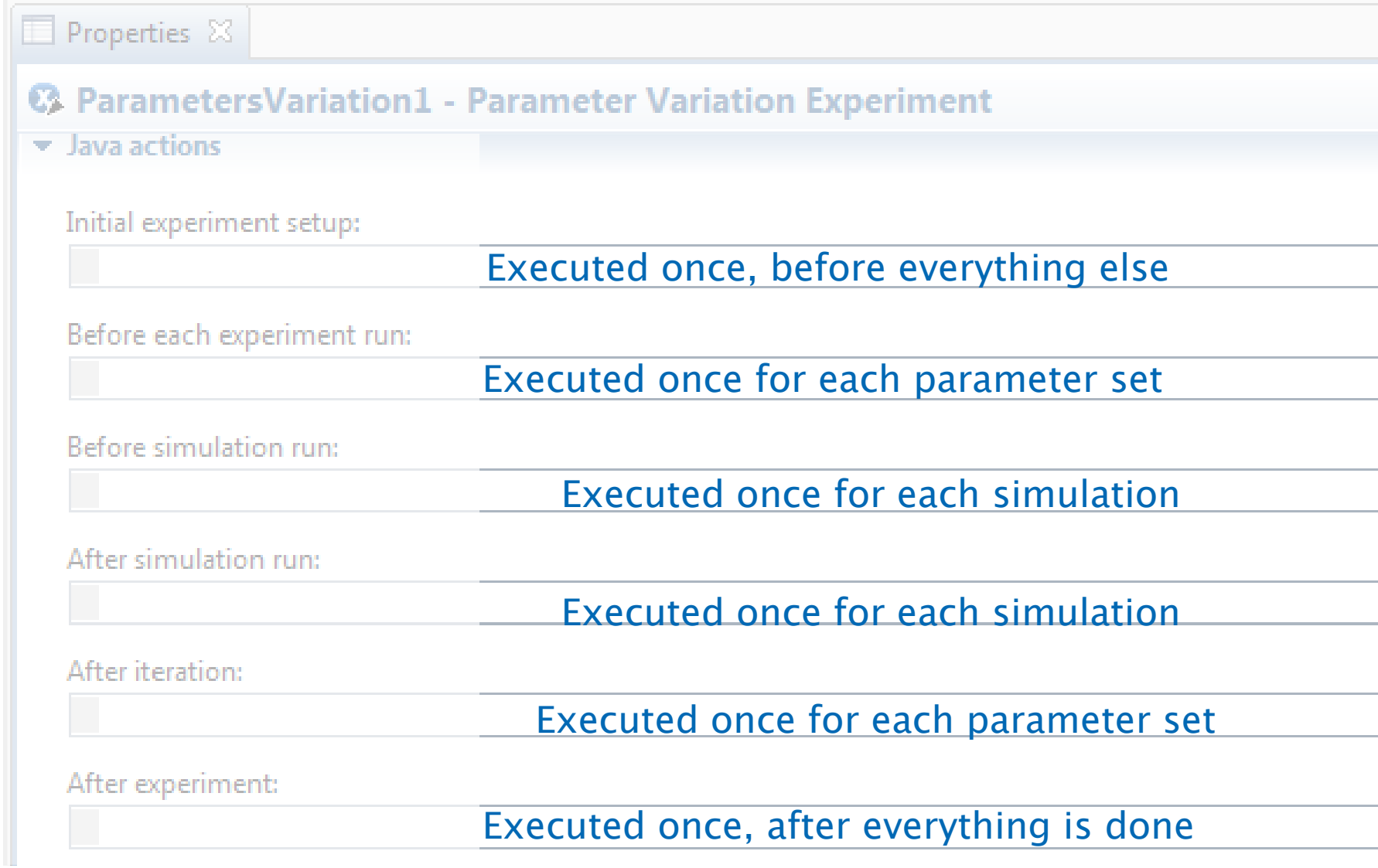
- Run the model multiple times
- Analyse the results statistically

Both tasks are tedious to do manually

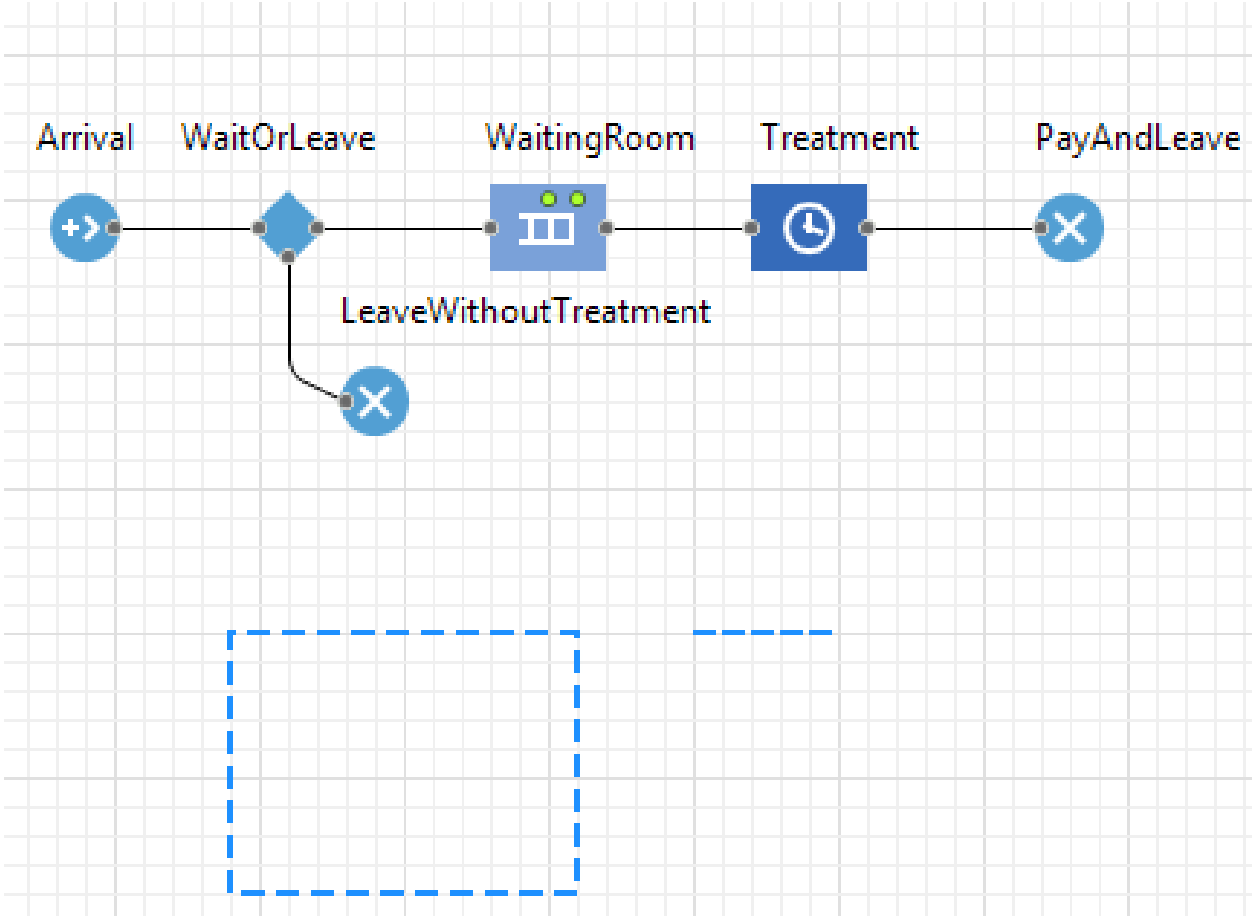
AnyLogic experiments enable us to write custom Java code to be executed:

- After each replication has finished (e.g. to store the results)
- After all replications of a parameter set (iteration) have finished (e.g. to automatically compute confidence intervals)

Parameter Variation Experiment



Example: The Waiting Room Model



Example: The Waiting Room Model

We again use the waiting room model with

- Inter–Arrival times: exponential(1/5.0) minutes
- Treatment duration: normal(2, 4.5) minutes

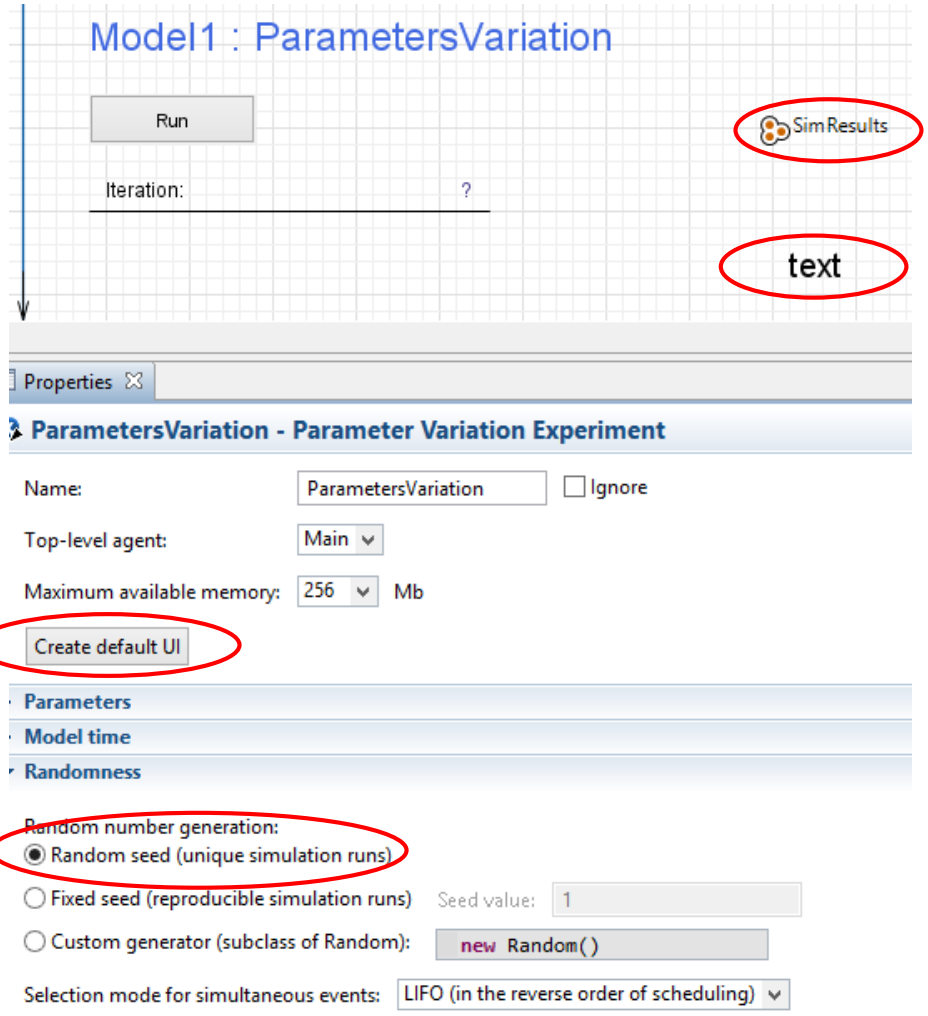
We want to know the waiting queue length at T=10,000

This time, we will compute a confidence interval

We will use experiments and custom code to automate the computation

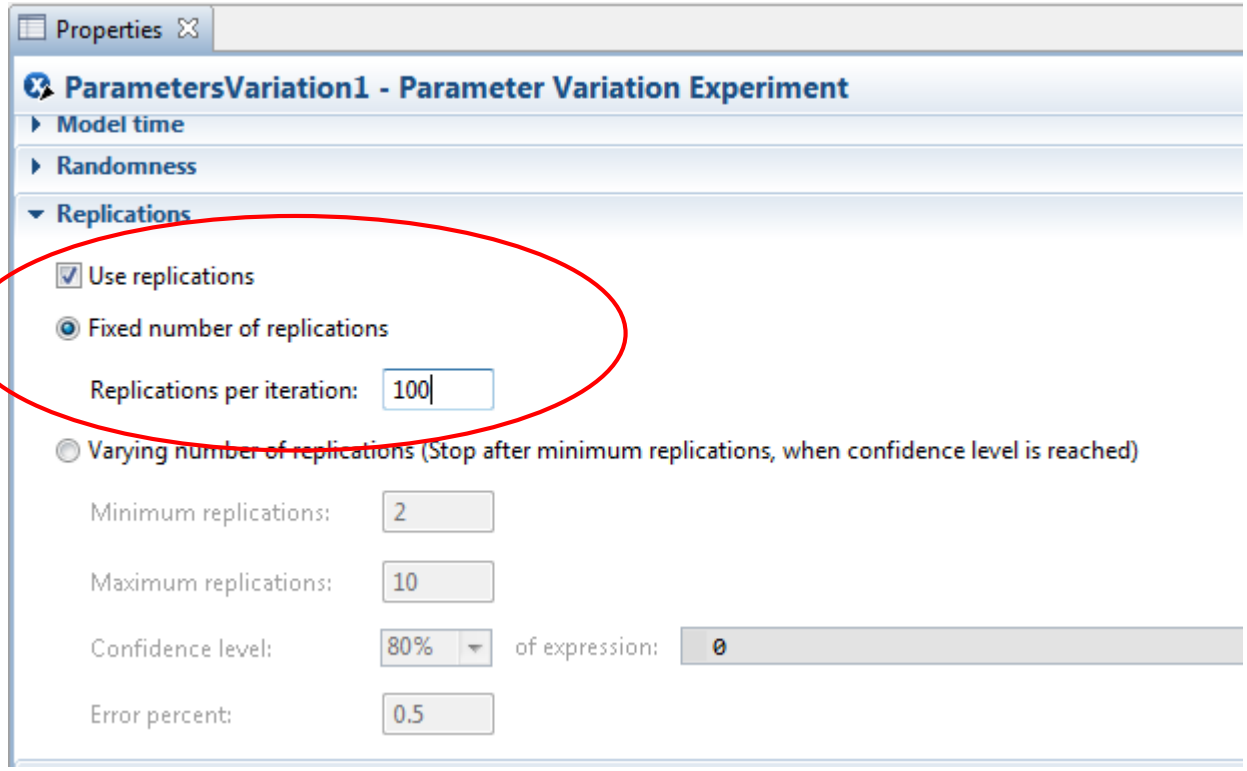
Experiment Setup

- Create a “Parameter Variation Experiment”
- Let AnyLogic “Create default UI”
- Add an (*int*) collection variable to store the replications’ results
- Add a *text* for displaying the results
- Select “unique simulation runs”, otherwise all replications will have the same result!



Experiment Setup

Set the number of replications to be executed



The Evaluation Code

„After simulation run“Code:

```
SimResults.add(root.WaitingRoom.size());
```

„After iteration“Code:

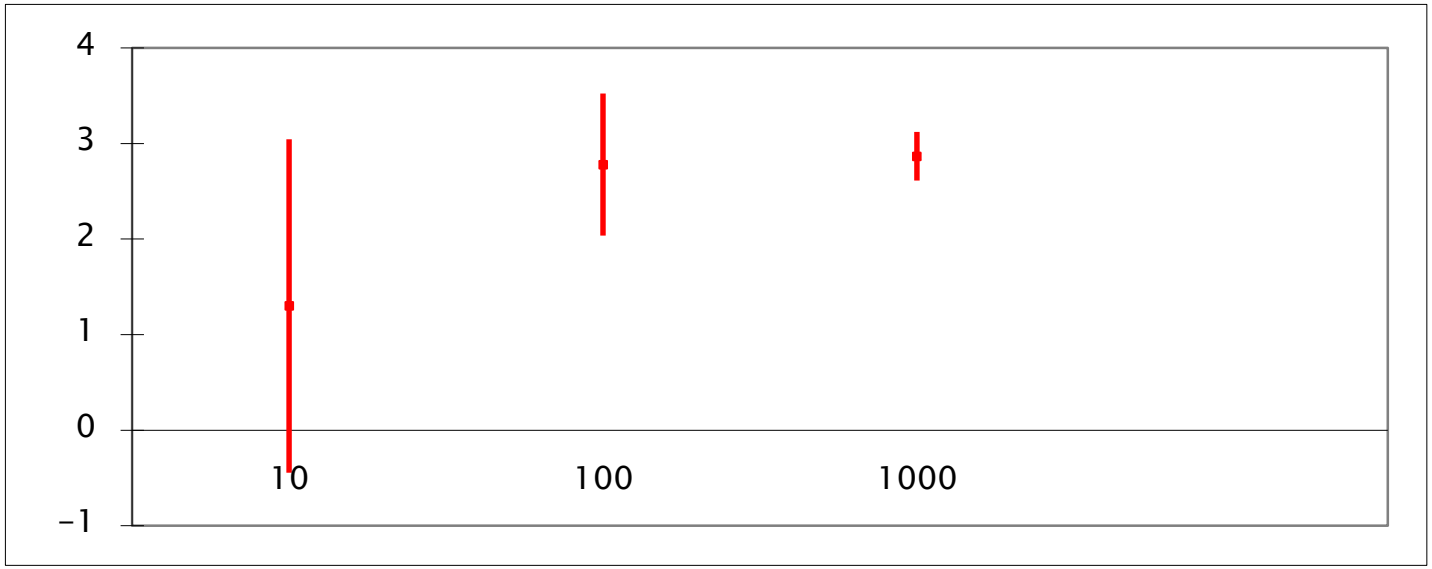
```
double sum = 0;
int n = SimResults.size();          //container holding all results
for (int idx = 0; idx < n; idx++) sum += SimResults.get(idx);
    double mean = sum / n;

double sumdiffsq = 0;      //sum of squared differences
for (int idx = 0; idx < n; idx++)
        sumdiffsq += (SimResults.get(idx)- mean)*(SimResults.get(idx) - mean);

double S_squared = sumdiffsq / (n-1);


double tVal = 2.87130765 ; //t-value for alpha = 0.005; 99 D.o.F.
double ci_min = mean - tVal * sqrt( S_squared / n);
double ci_max = mean + tVal * sqrt( S_squared / n);

text3.setText( format(ci_min) +" <= theta <= " + format(ci_max) );
```

Results

Queue lengths at time T=10000:

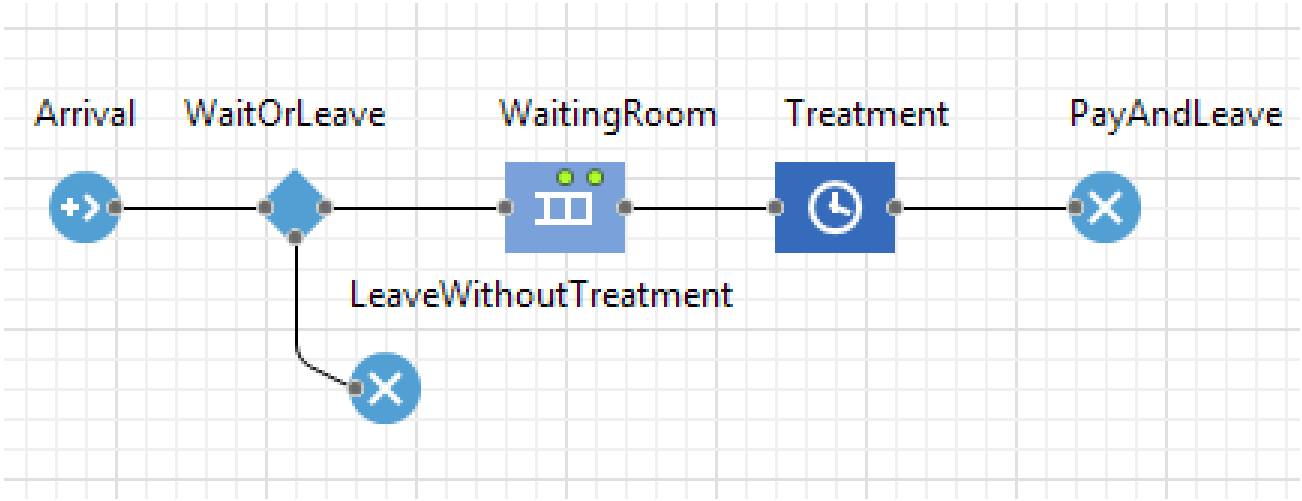
Seed	1	2	3	4	5	6	7	8	9	10
Length	3	0	1	1	0	3	1	0	4	0

Confidence intervals for n=10, 100, 1000:



And now from a Business Perspective…

An Actual Parameter Variation Experiment



Imagine the following situation:

- A waiting room costs 2€ per hour and seat in maintenance
- Each successfully treated patient pays 20€
- Patients without a seat will leave the practice at once

The physician of a new medical practice wants to determine an optimal waiting room size (i.e. the one with maximum expected profit).

Experiment Plan

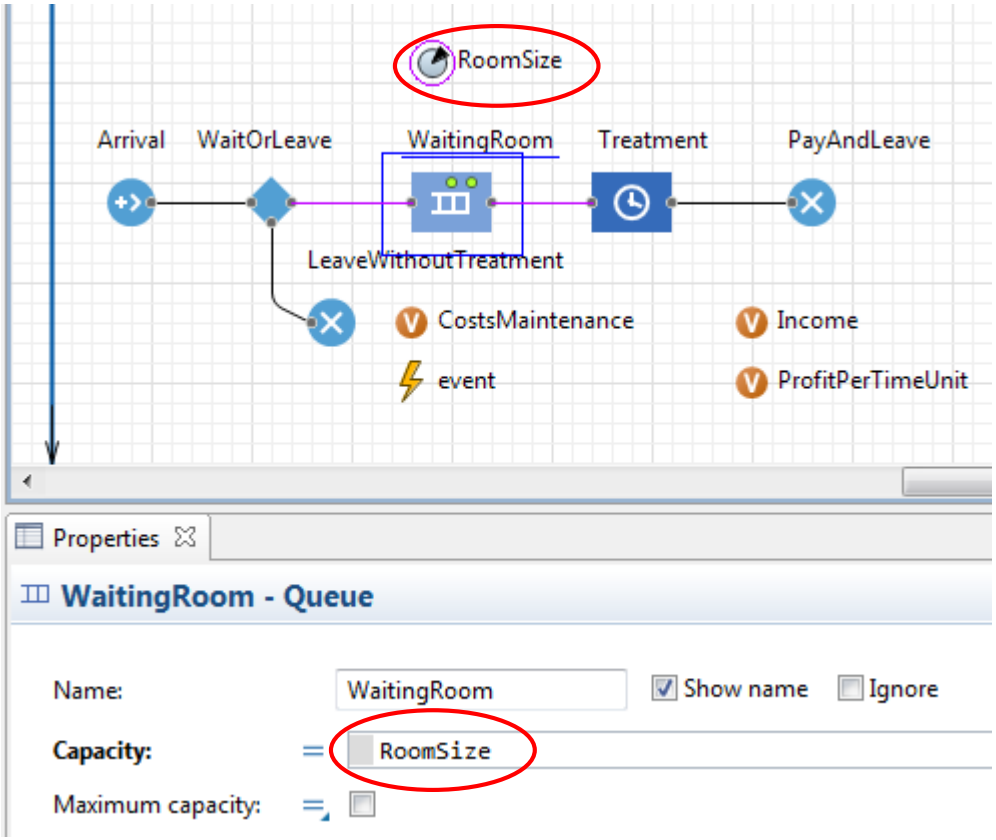
We will

- Vary the parameter *RoomSize* in 20 steps from 1 to 20
  → 20 iterations
- For each iteration, conduct 100 replications and compute a confidence interval for the *ProfitPerTimeUnit*
- Plot each confidence interval and mean against the *RoomSize*

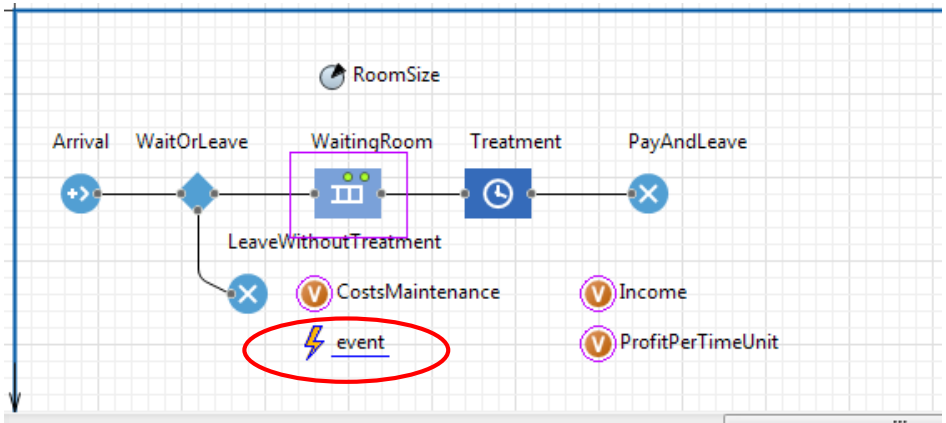
→ We will already have 20*100 = 2000 simulation runs!

Changes to the Model – Parameters

Only parameters can be manipulated (i.e. varied) by the experiment. Inside a model, a parameter can be used the same way as a plain variable.



Changes to the Model – Maintenance Cost Computation



Properties

event - Event

Name: event ☑ Show name ☐ Ignore

Visible: yes

Trigger type: Timeout

Mode: Cyclic

◉ Use model time ○ Use calendar dates

First occurrence time (absolute): 0

Occurrence date: 28.08.2009 12:26:28

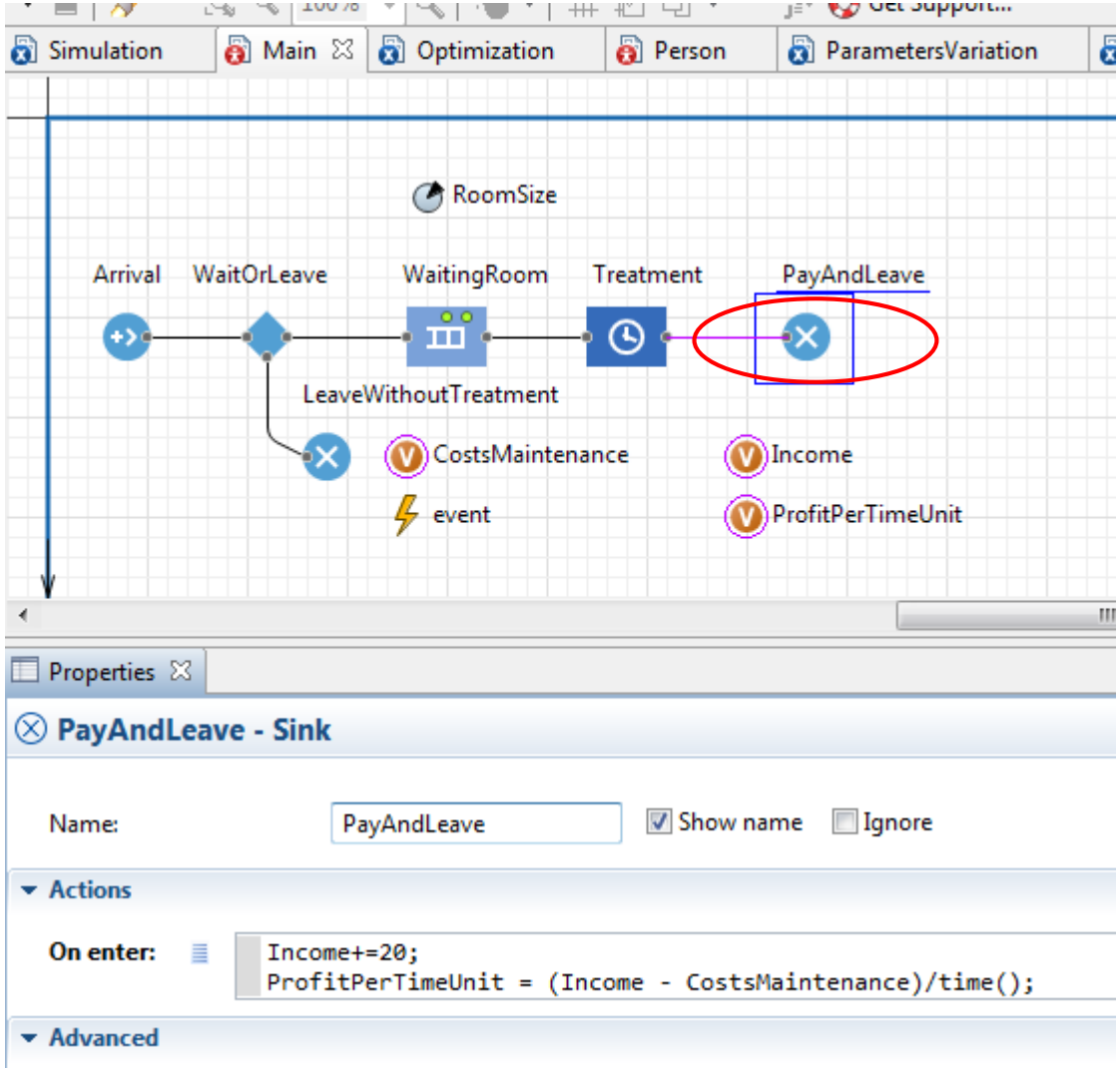
Recurrence time: 60

Action

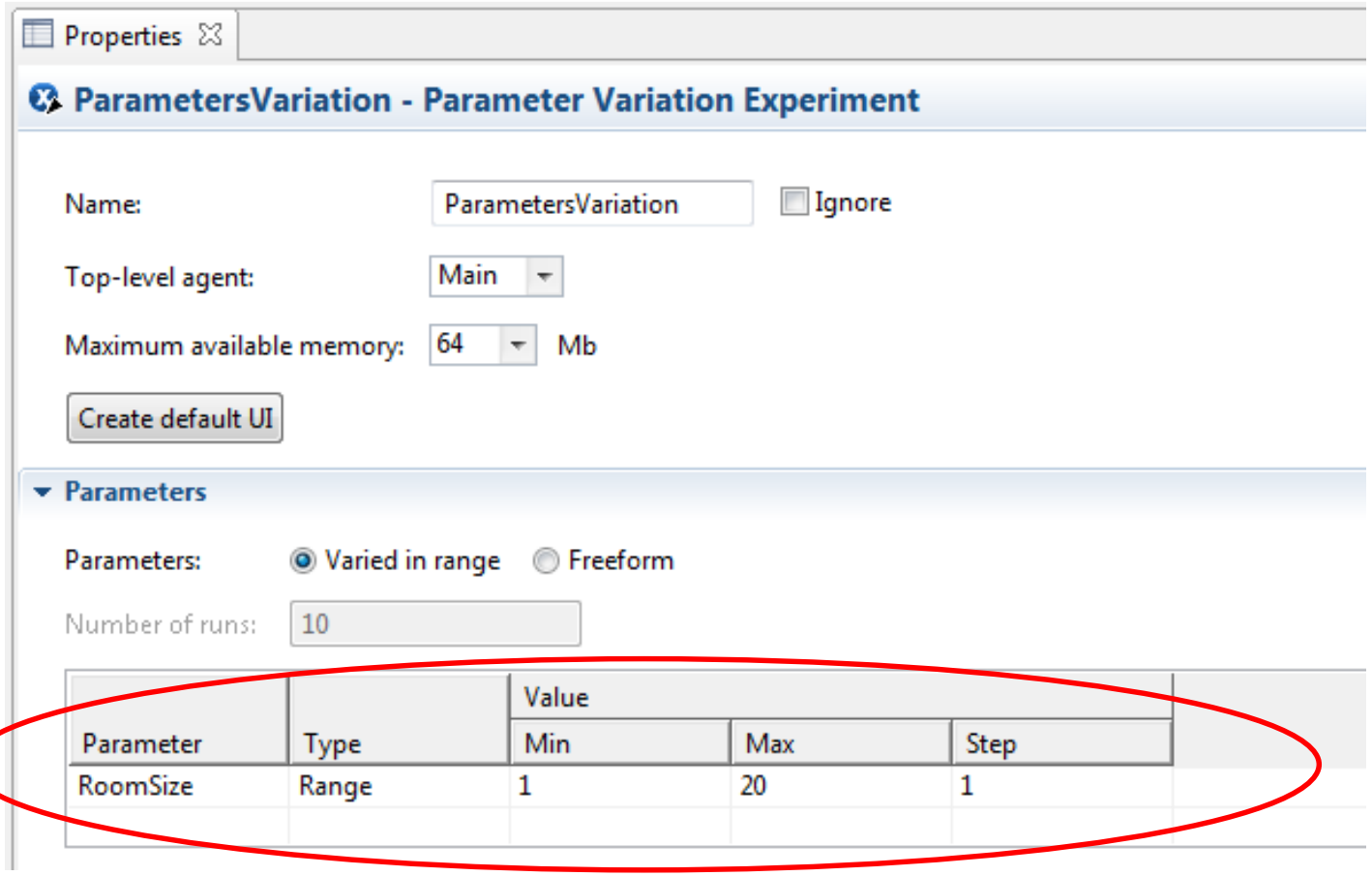
```
CostsMaintenance += 2 * WaitingRoom.capacity;

ProfitPerTimeUnit = (Income - CostsMaintenance)/time();
```

Description

Changes to the Model – Income Computation



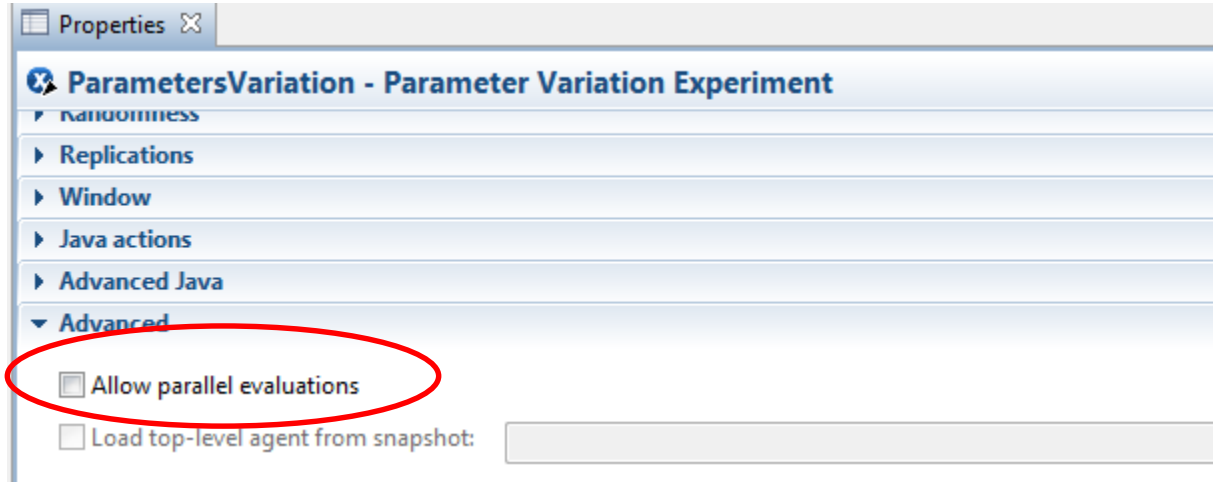
Experiment Setup



Experiment Setup – Pitfall

Using our experiment description we assume that all iterations are executed sequentially !!!

- ***Uncheck the "Allow parallel evaluations" checkbox !!***
- Otherwise you will get very strange results ☺

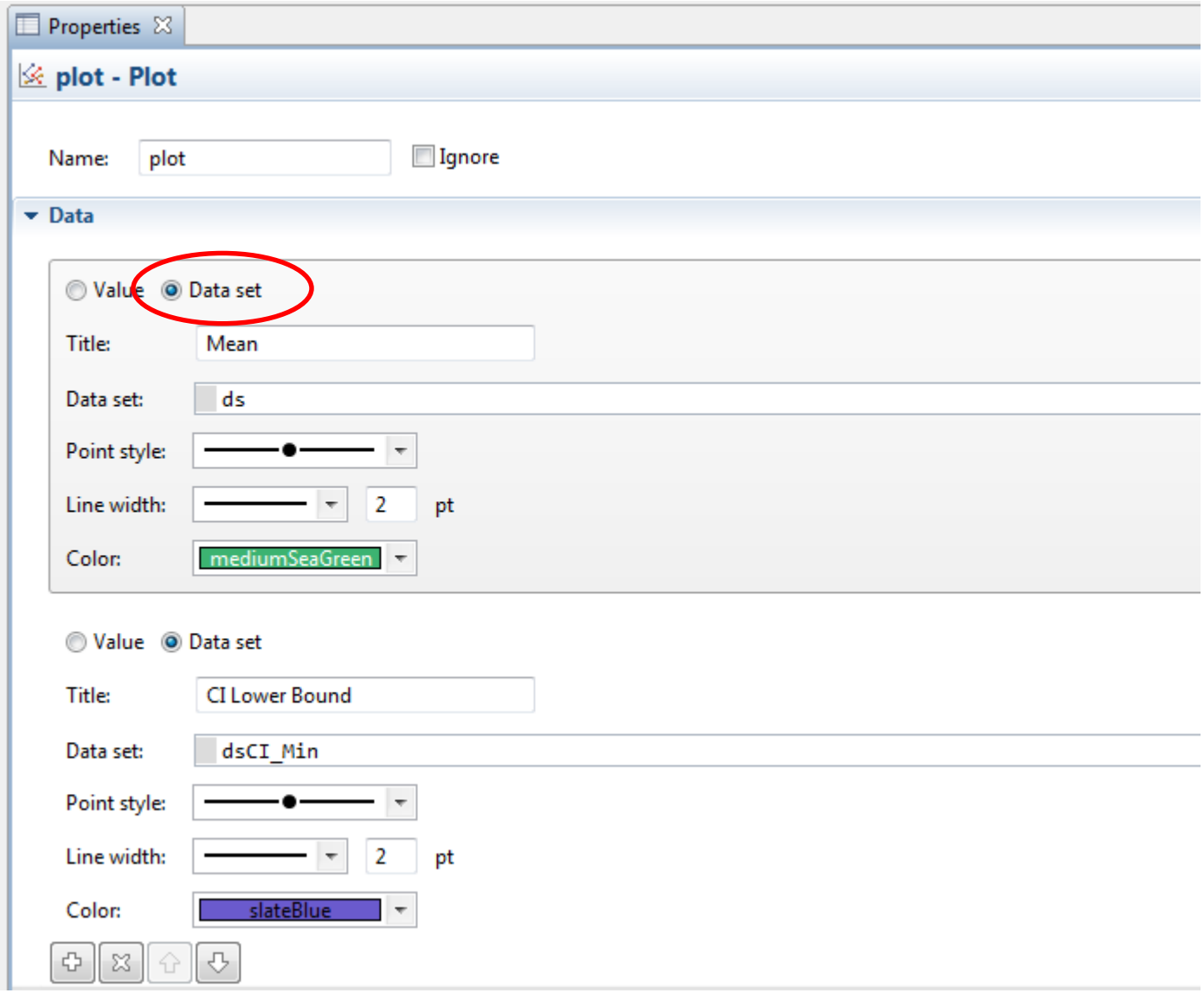


If you want to make use of AnyLogic's multi–thread ability, be sure to understand the implications of multi–thread programming (race conditions, synchronization, …)

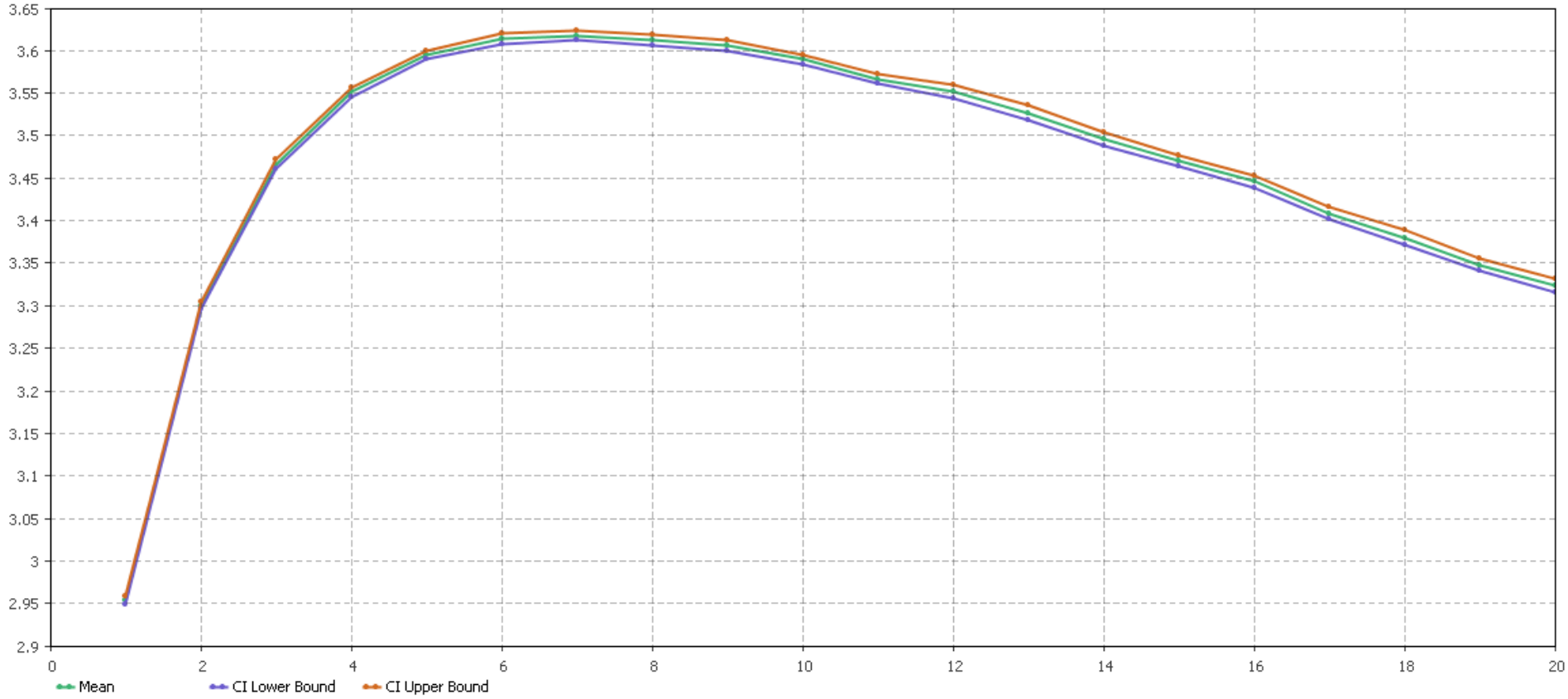
C.I. Computation – "After Iteration" Code

```
/**
   ...
   The usual confidence interval computation code

*/
double ci_min = mean - tVal * sqrt( S_squared / n);
double ci_max = mean + tVal * sqrt( S_squared / n);

dsCI_Min.add(RoomSize, ci_min);
dsCI_Max.add(RoomSize, ci_max);
ds.add( RoomSize, mean);
simResults.clear();//for the next iteration
```

Diagram Setup



Results



So, what is the optimal waiting room size?

Learning Goals

You are now able to create an experiment to answer the following questions:

What is the expected value of the following measures:

- Shield energy level after two hours
- Number of antimatter particles to hit the shield
- Total time of unemployment of the father
- Amount of money spent on damaged school property

Approach:

- Do at least 100 replications
- Compute the confidence intervals