OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG

INF FAKULTÄT FÜR INFORMATIK

Boundary Conditions in Continuous Models

Coffee and a Bungee Jump

Agenda

Implement a continuous model from the lecture in AnyLogic

Develop and animate a simulation model for a bungee jump

The Coffee Problem

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The Coffee Problem

An early morning situation:

- You want to drink a cup of (white) coffee
- The (hot) coffee is already in the cup
- You still need to add (cold) milk
- You need to go to the bathroom first
- You would like your coffee to be as hot as possible

Question:

- Is it better to pour in the milk before going to the bathroom, or after you return?

Governing Equations

Cooling depends on the heat transport constant c

$$\frac{dT_{coffee}}{dt} = -c \cdot (T_{coffee} - T_{room})$$

Pouring in the milk changes the temperature immediately

$$T^*_{coffee} = \frac{(T_{coffee} \cdot V_{coffee} + T_{milk} \cdot V_{milk})}{(V_{coffee} + V_{milk})}$$

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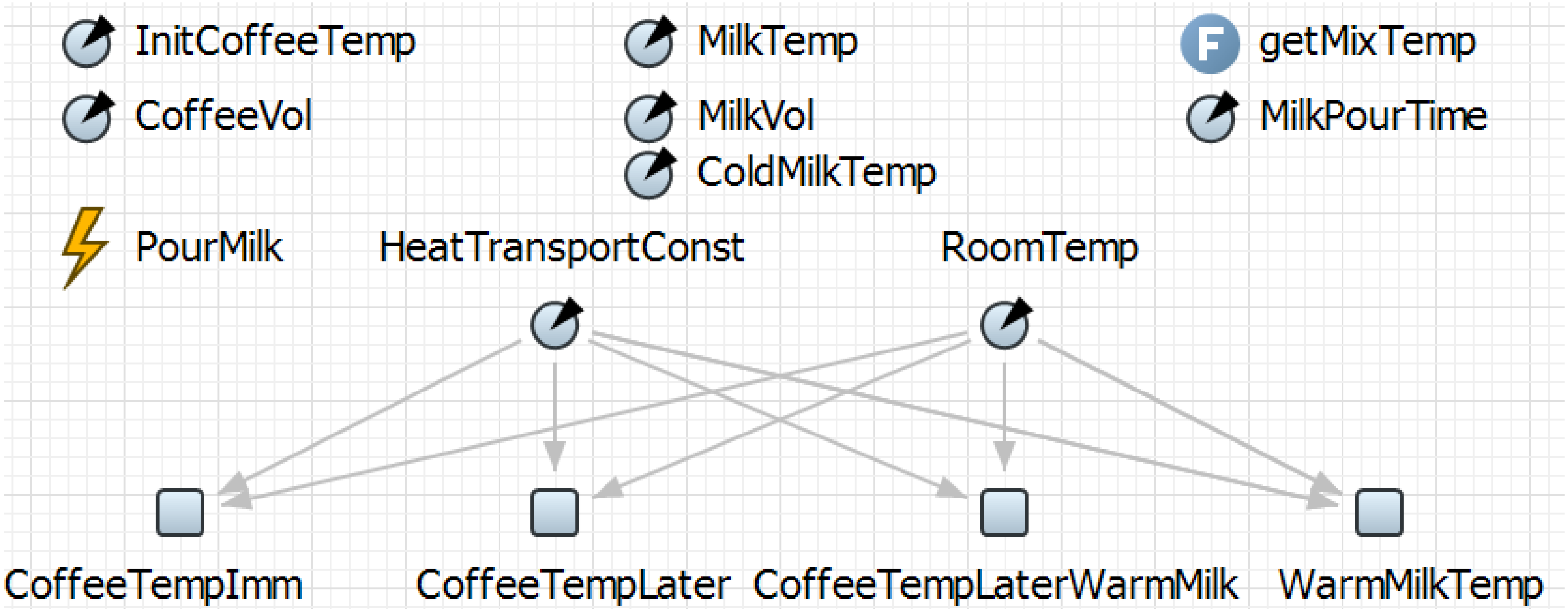
Problem Parameters

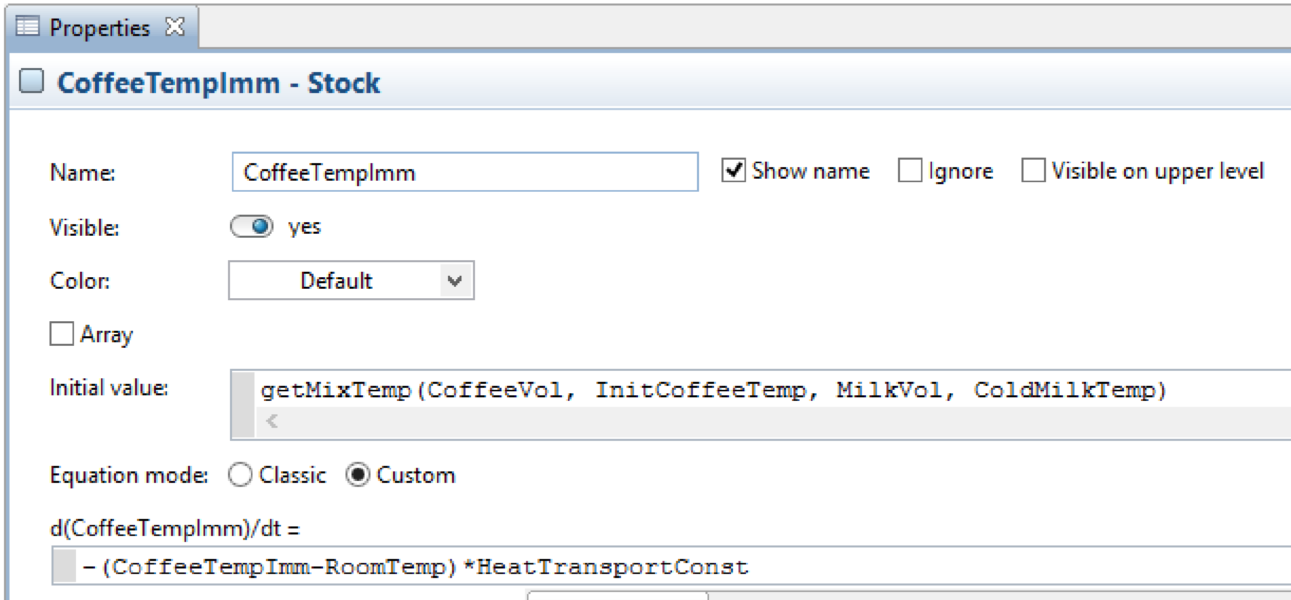
Simulate using the following parameters:

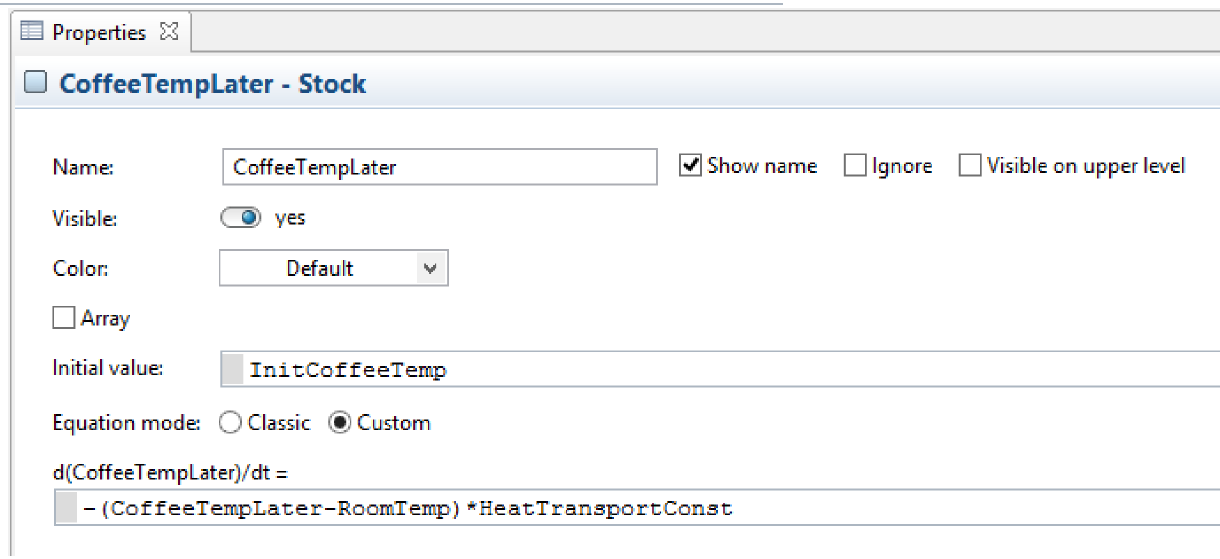
- dt = 0.3 Minutes
- Initial temperature of coffee = 90 degrees Celsius
- Temperature of surroundings = 22 degrees Celsius
- Temperature of milk = 8 degrees Celsius
- Volume of coffee = 0.2 liters
- Volume of milk = 0.05 liters
- Heat transport constant c = 0.5 per minute

AnyLogic Model



Stock Variables





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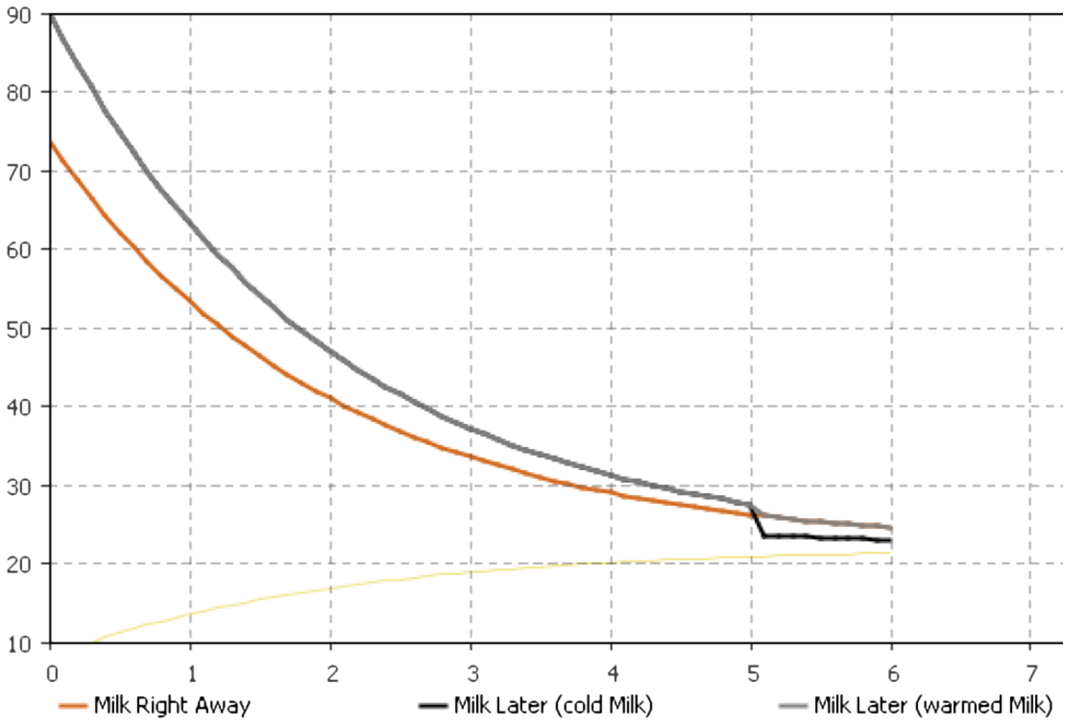
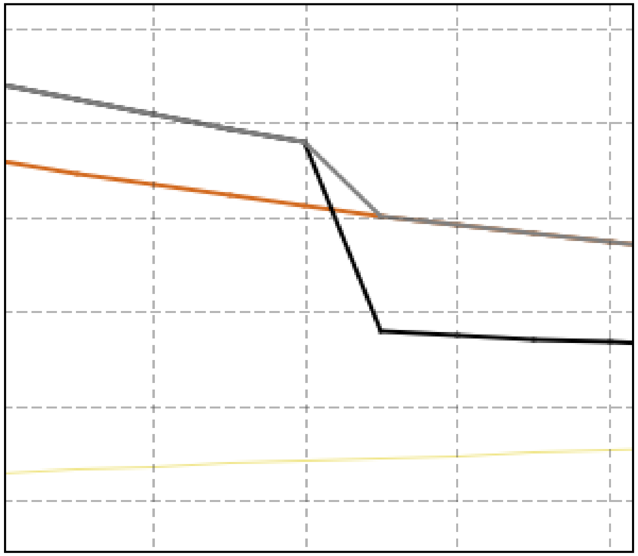
Simulation Result

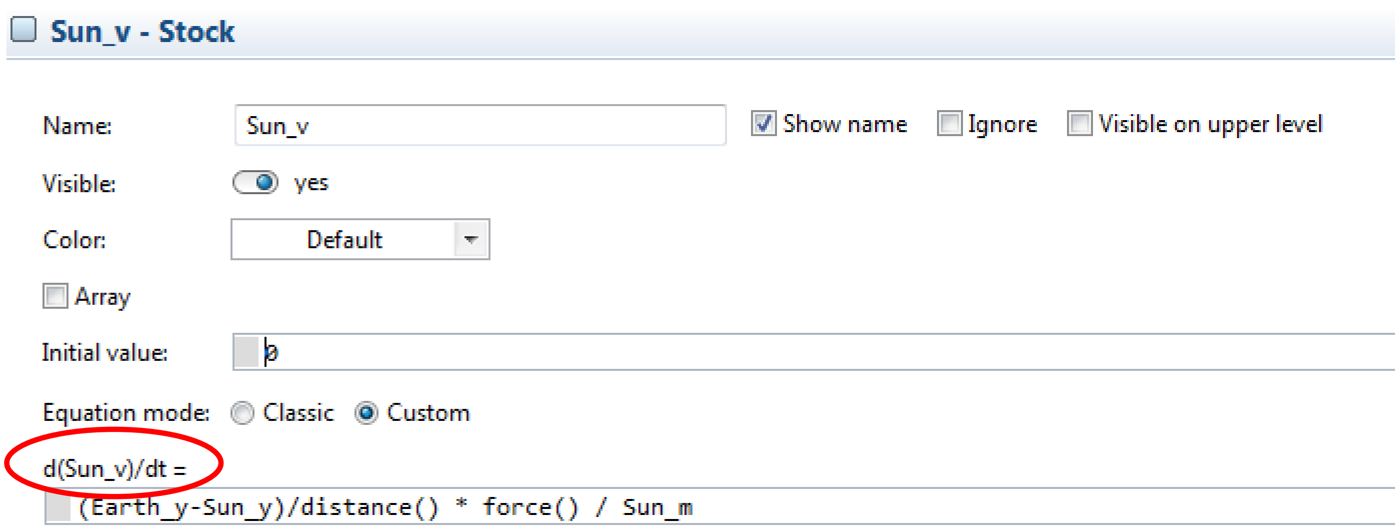




A Modeling Pitfall – Conditions in Differential Equations

Conditions in Differential Equations

Stock variables in AnyLogic are defined by mathematical expressions (the equation's left-hand side is fixed).



Consequence: Something like this is not possible:

```
if (c > 0) dx/dt = c; else dx/dt = 0;
```

So how incorporate conditional expressions?

Conditions in Differential Equations

Workaround for some cases: the min() and max() functions

```
if (c > 0)
  dx/dt = c;
else
  dx/dt = 0;
```

Is semantically identical to

```
dx/dt = max(c, 0);
```

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Conditions in Differential Equations

General Solution: the *ternary operator "?"* (C/C++/Java/…):

```
if (condition)
  x = a;
else
  x = b;
```

can be written as

```
x = (condition) ? a : b;
```

e.g.

```
dx/dt = (c > 0) ? c : 0;
```

… or, if you don't like that approach, write a separate function.

A new AnyLogic Element

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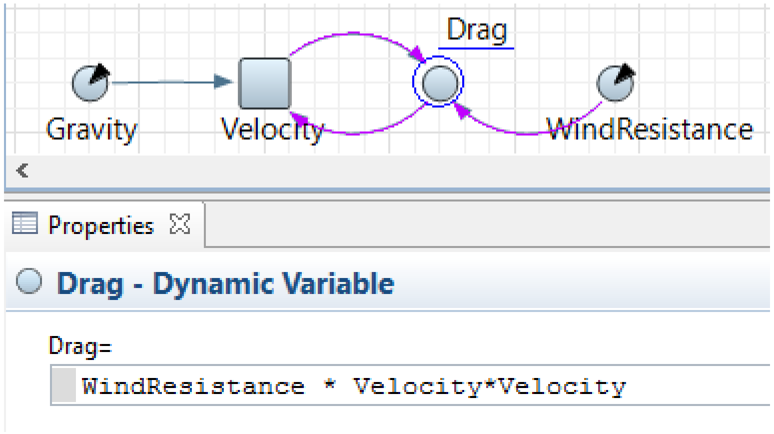
Dynamic Variables

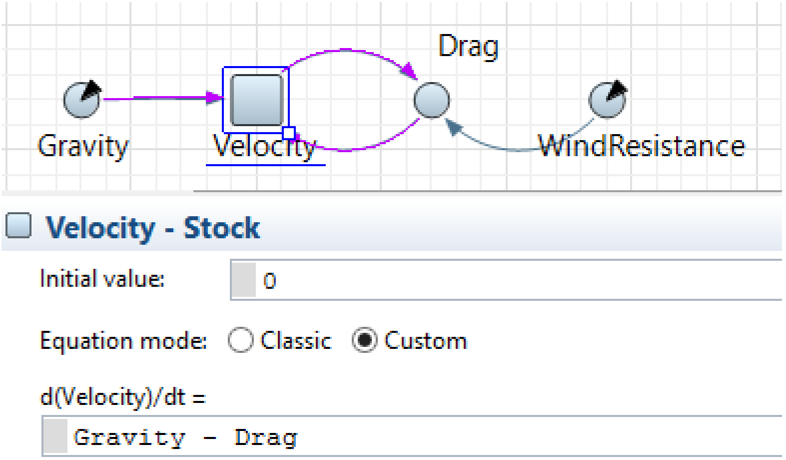
… have a value that depends on other variables (like functions)

But Dynamic Variables

- Are accessed like a variable (`myvar` instead of `myvar(...)`)
- Are recomputed whenever their “source” variables change

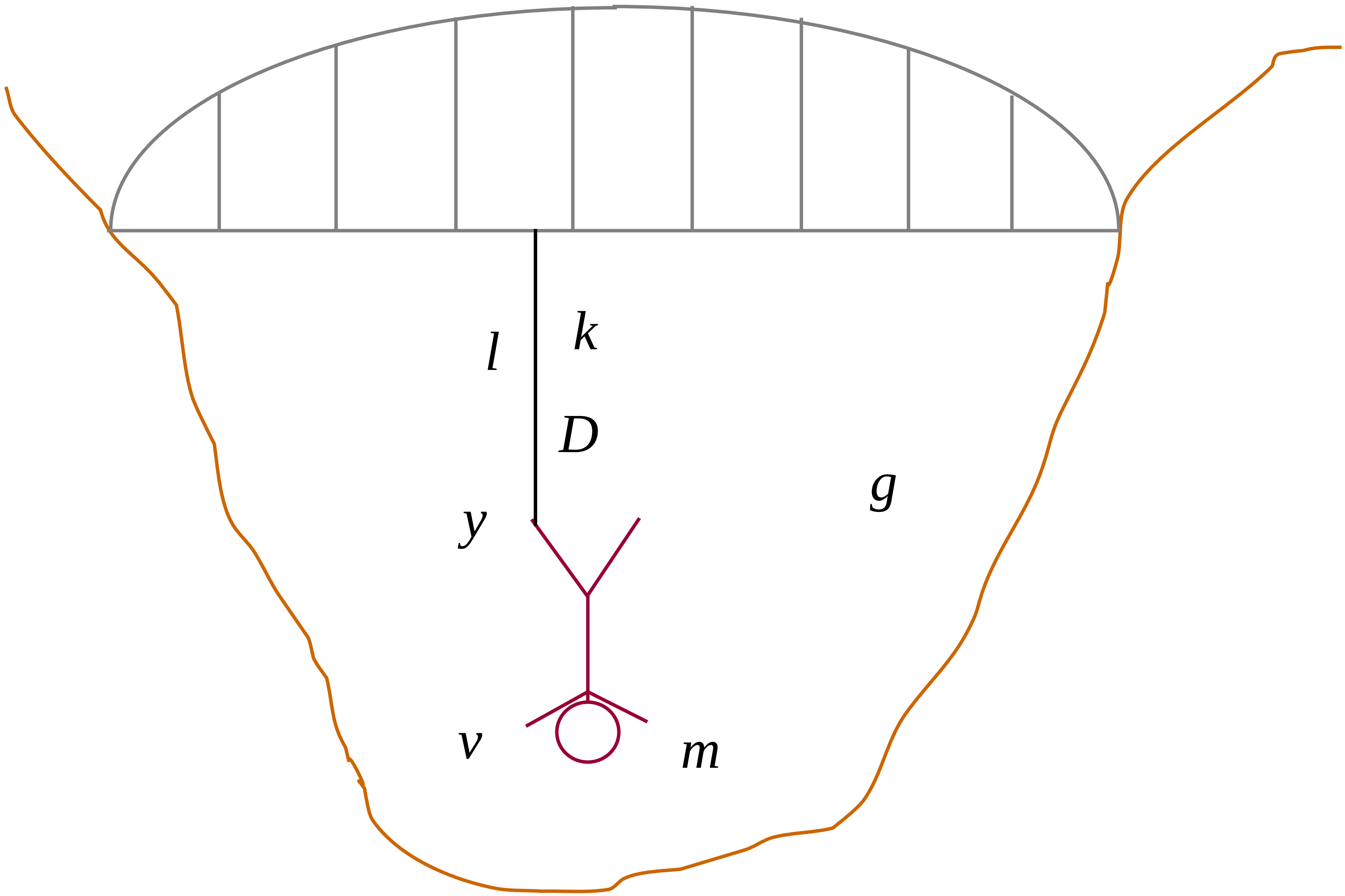
→ Are usually used to store intermediate results





The Bungee Jumper

The Bungee Jumper



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The Bungee Jumper

Definition of relevant quantities:

Rope

- Spring constant: k [N/m] (=50.0 N/m)
- Damping constant: D [N·s/m] (=10.0 N·s/m)
- Length (relaxed): l [m] (=20 m)
- Length (momentary): y [m]

Jumper

- Downward velocity: v [m/s]
- Mass: m [kg] (=60.0 kg)

System

- Acceleration (gravity): g [m/s²] (=9.81 m/s²)

Model

We need equations for position y and velocity v

Position:

- Definition of speed: $v = dy/dt$

Speed:

- Definition of acceleration: $a = dv/dt$
- Newton's Law: $acceleration = force\,/\,mass$

Result:

$$\frac{dy}{dt} = v \qquad \frac{dv}{dt} = g + F/m$$

?

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Springs and Dampers

When taut, the rope exerts two *downward* (!) forces:

1) proportional to its length of extension:

$$F_{Spring} = -k \cdot extension$$

$$\rightarrow \quad F_{Spring} = -k \cdot (y - l)$$

2) proportional to its speed of extension:

$$F_{Damping} = -D \cdot rate\ of\ extension$$

… *iff the rope is extending (rate of extension > 0) !*

$$\rightarrow \quad F_{Damping} = -max(D \cdot v, 0)$$

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The Downward Forces

Let F be the rope's *downward* force on the jumper

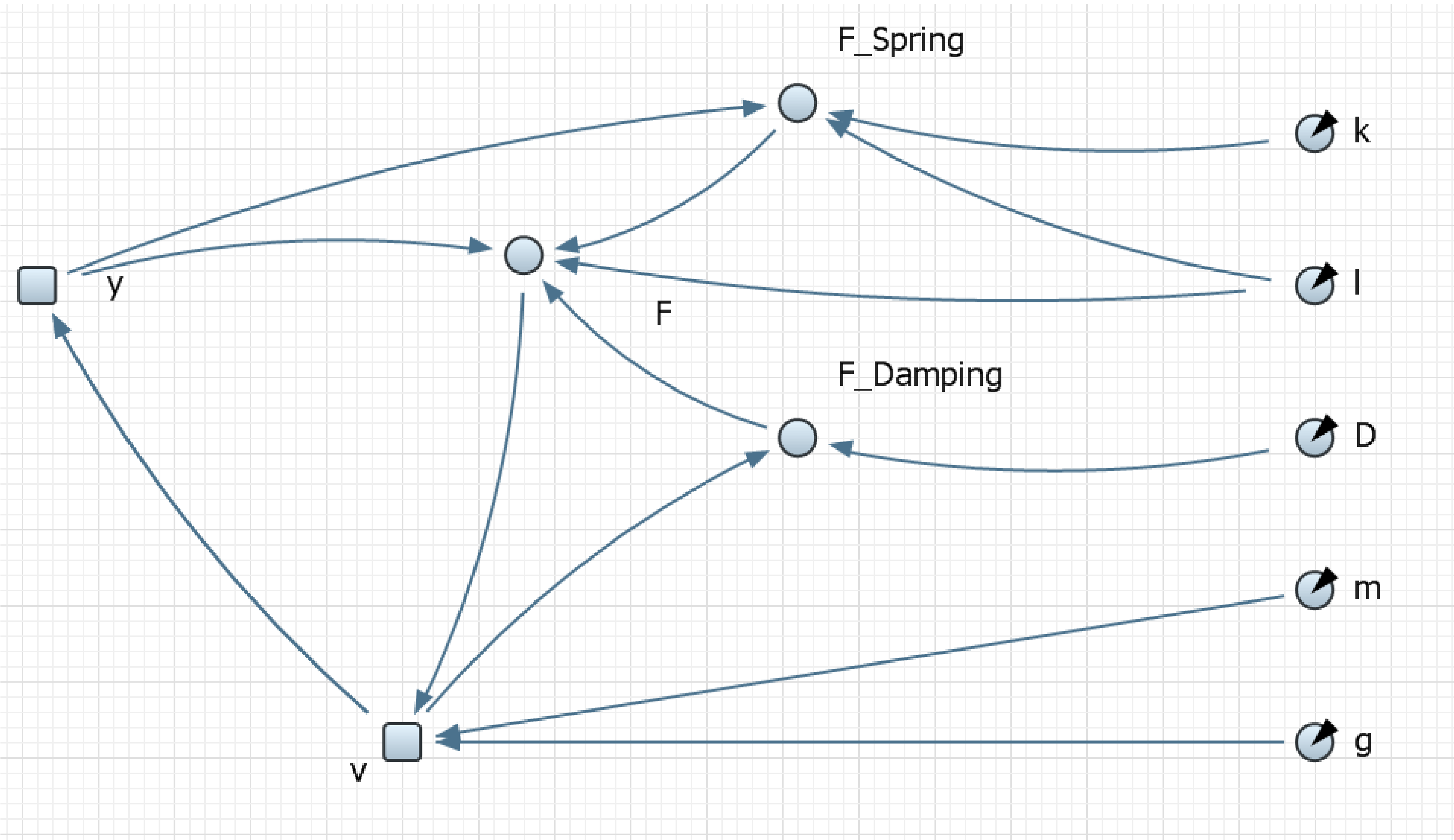
When $y < l$, the rope is slack:

$$F = 0$$

When $y > l$, the rope is taut and pulls up:

$$F = F_{Damping} + F_{Spring}$$

AnyLogic Model



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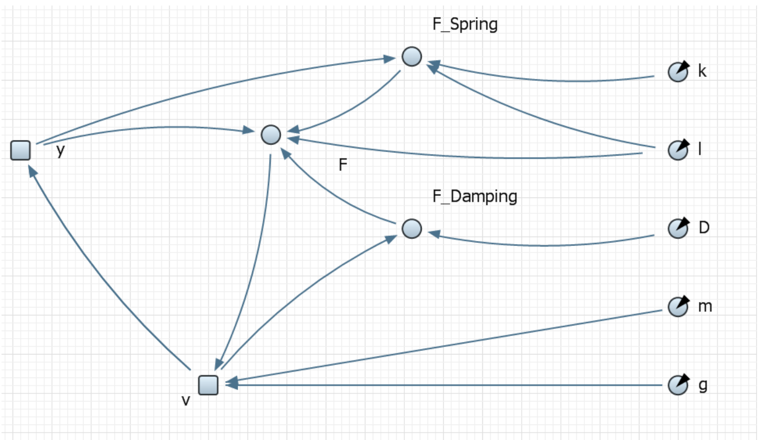
Dynamic Behavior




```
dv/dt       = g + F / m
dy/dt       = v

F_Spring    = - k * ( y - l)
F_Damping   = - max ( D * v, 0)
F           = (y < l) ? 0 : F_Spring + F_Damping
```

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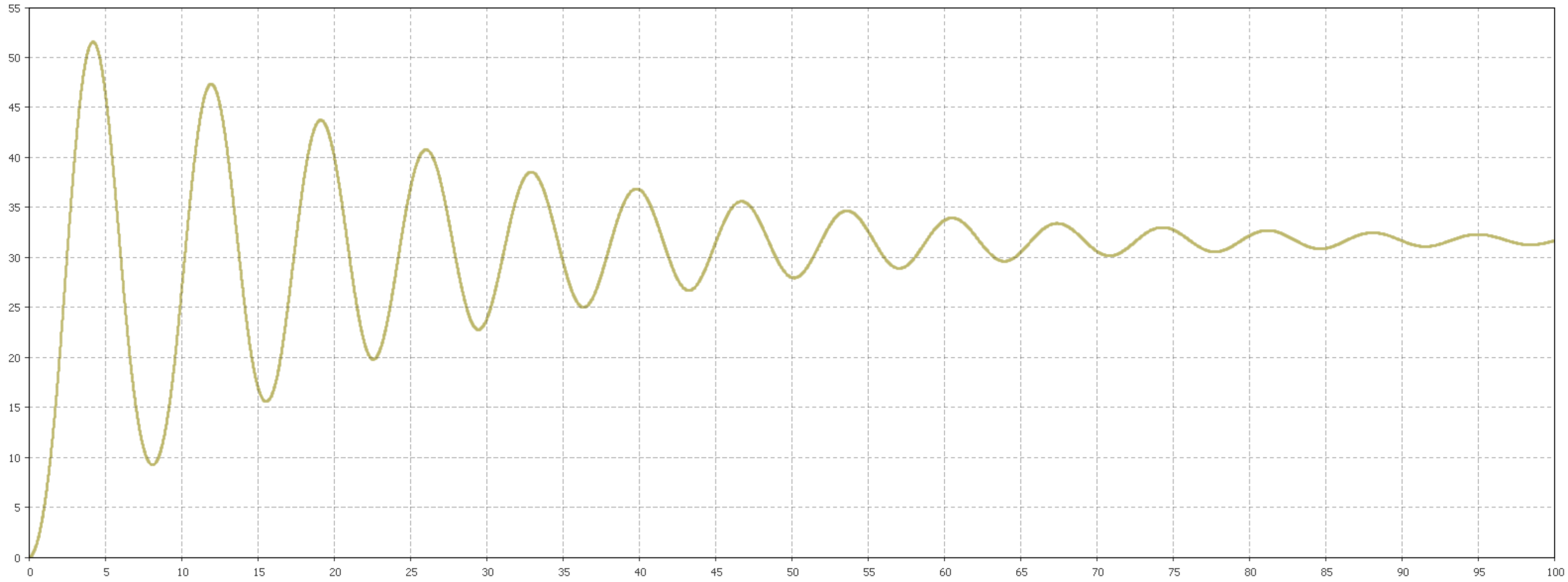
Simulation Results

Simulation result for y:

y in [m]



t in [s]

Animating the Jumper

An animation of the jumper makes the speed and distance of the jumper more graspable

Elements of the animation:

- A line from the presentation palette as “rope”
- A person from the pictures palette as “jumper”

Dynamics of the animation

- The y–position of the person is dependent on the model stock variable y
- The line offset of the second point of the line is dependent on the model stock variable y
- Color of the “rope” depends on l and y