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INF

FAKULTÄT FÜR
INFORMATIK

Comparing Systems

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How to Compare Systems

Perform the same number of replications for both systems and record the desired result sequences Y_1 and Y_2, then compute confidence intervals.

Y_1	Y_2
1	3
4	5
2	1
…	…

Approach 1:

- Compute C.I. for each sequence separately
- If the two C.I. do not overlap, evaluate positions

Problem: difficult to interpret combined level of confidence

Approach 2:

- Compute single C.I. for the difference sequence
- If C.I. does not include the origin, evaluate position

Y_1-Y_2
-2
-1
1
…

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Correlated Sampling – Why does it help?

Approach 2 is easier to compute and potentially more efficient

- C.I. width depends on data variance
- So, for A2 it depends on the variance of the *differences* :

$$\operatorname{var}(Y_1 - Y_2) = \frac{\sigma_1^2}{R} + \frac{\sigma_2^2}{R} - \frac{2\rho_{12}\sigma_1\sigma_2}{R}$$

Variance of Y_1 *Variance of Y_2*

Correlation coefficient between Y_1 and Y_2

→ If results between both models correlate, variance and width of difference C.I. get smaller

Consequence: Fewer replications for desired level of accuracy

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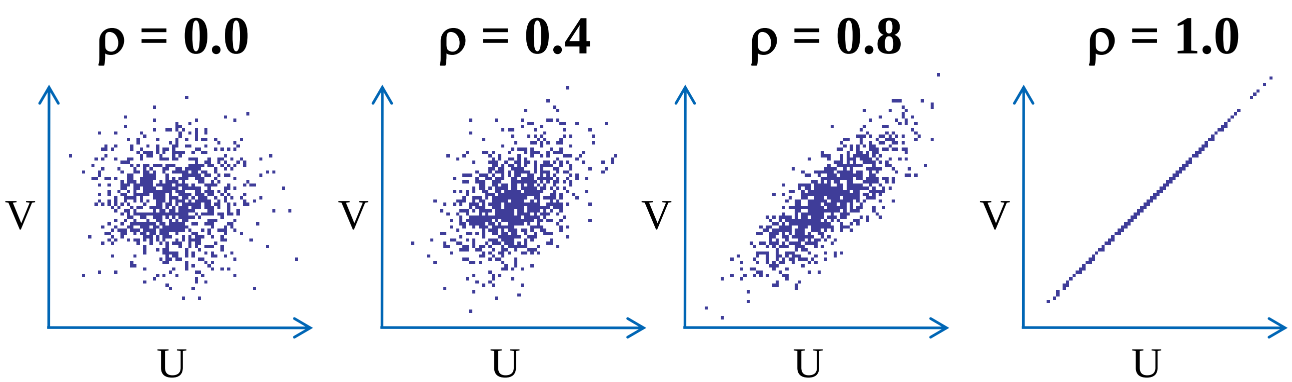
Review: Correlation

The correlation coefficient ρ describes the statistical relationship between two data sequences U and V

- $\rho > 0$ for a positive linear relationship between U and V



How do we create a positive correlation?

- Ensure that the models to be compared behave similarly in the same replication
- Do **NOT** manipulate the models or re–order the results!

Y_1	Y_2
100	105
345	314
57	64
…	…

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Correlated Sampling –Achieving a Positive Correlation

Idea:

- Use the same random numbers in both models

More precisely:

- For each model, each replication uses different random numbers
- In each replication, both models use the same RNs

Then,

- Different replications will be independent (Required for C.I.)
- Every pair of results Y_{r1}, Y_{r2} will be positively correlated

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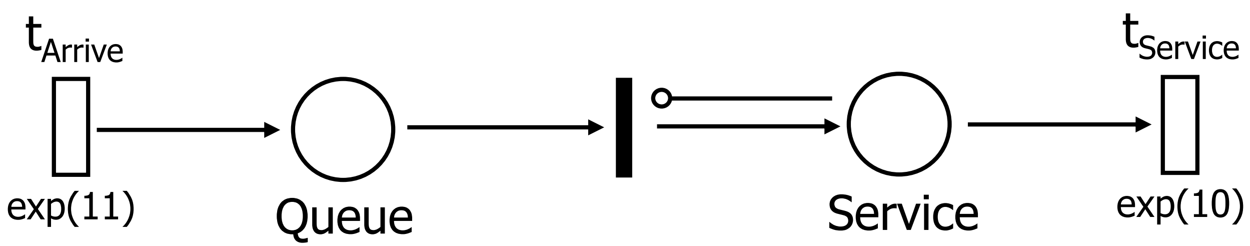
Correlated Sampling

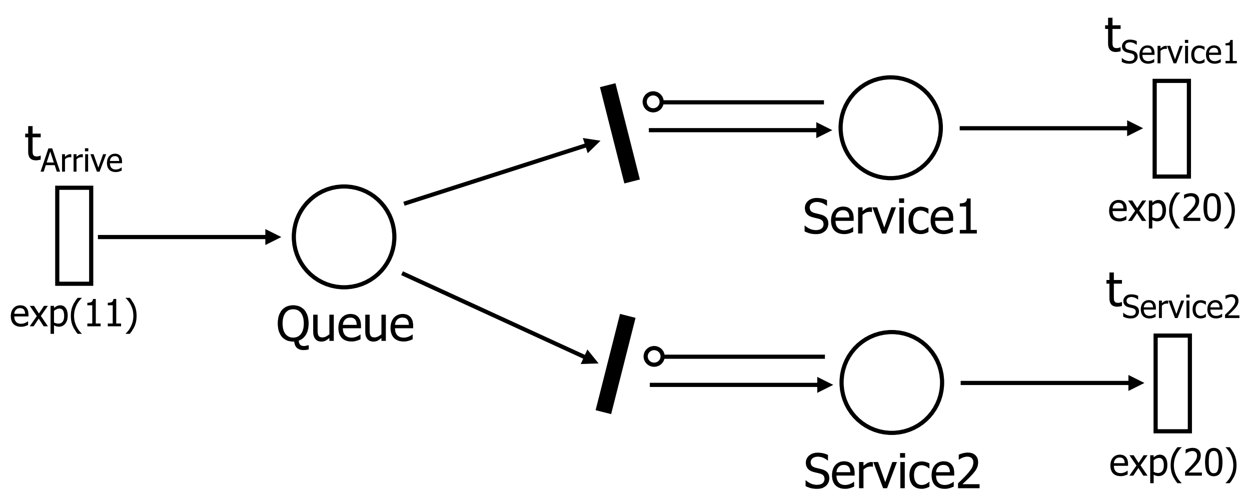
Algorithm for comparing two systems:

- Compute $D_r = Y_{r1} - Y_{r2}$
- Compute $D = \frac{1}{R} \sum D_r$
- Compute $S^2 = \frac{1}{R-1} \sum (D_r - D)^2$
- Compute $\sigma = \frac{S}{\sqrt{R}}$
- Choose α and compute the confidence interval

Example

Example: Compare one– and two–server models





Example

Do a simulation study to make the comparison

We will do the following experiment:

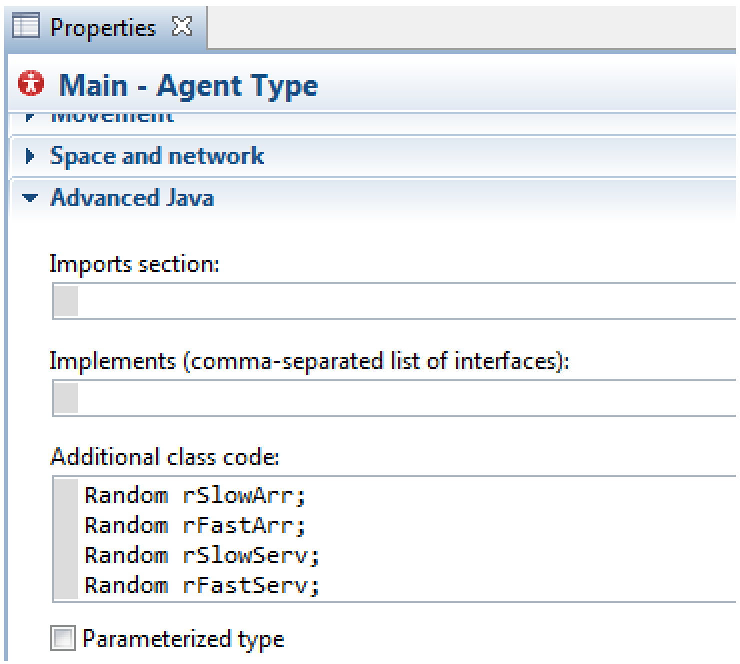
- Compute average queue length at $T = 10{,}000$
- Use $R = 15$ replications

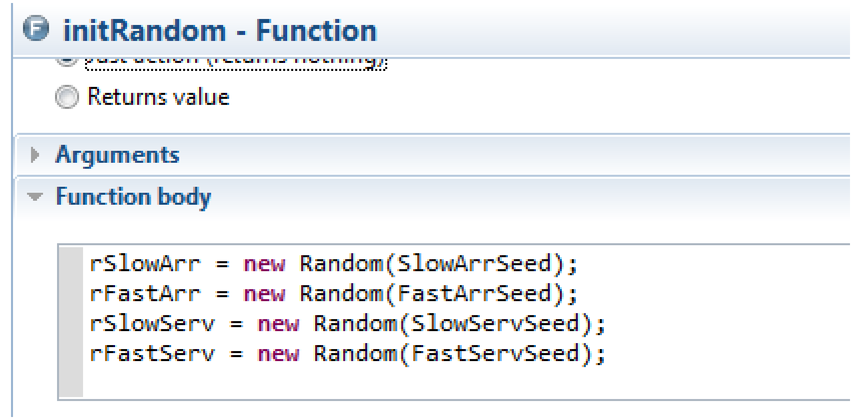
AnyLogic Model Using Correlated Sampling

Controlling Random Numbers in AnyLogic

Declare four separate random number streams

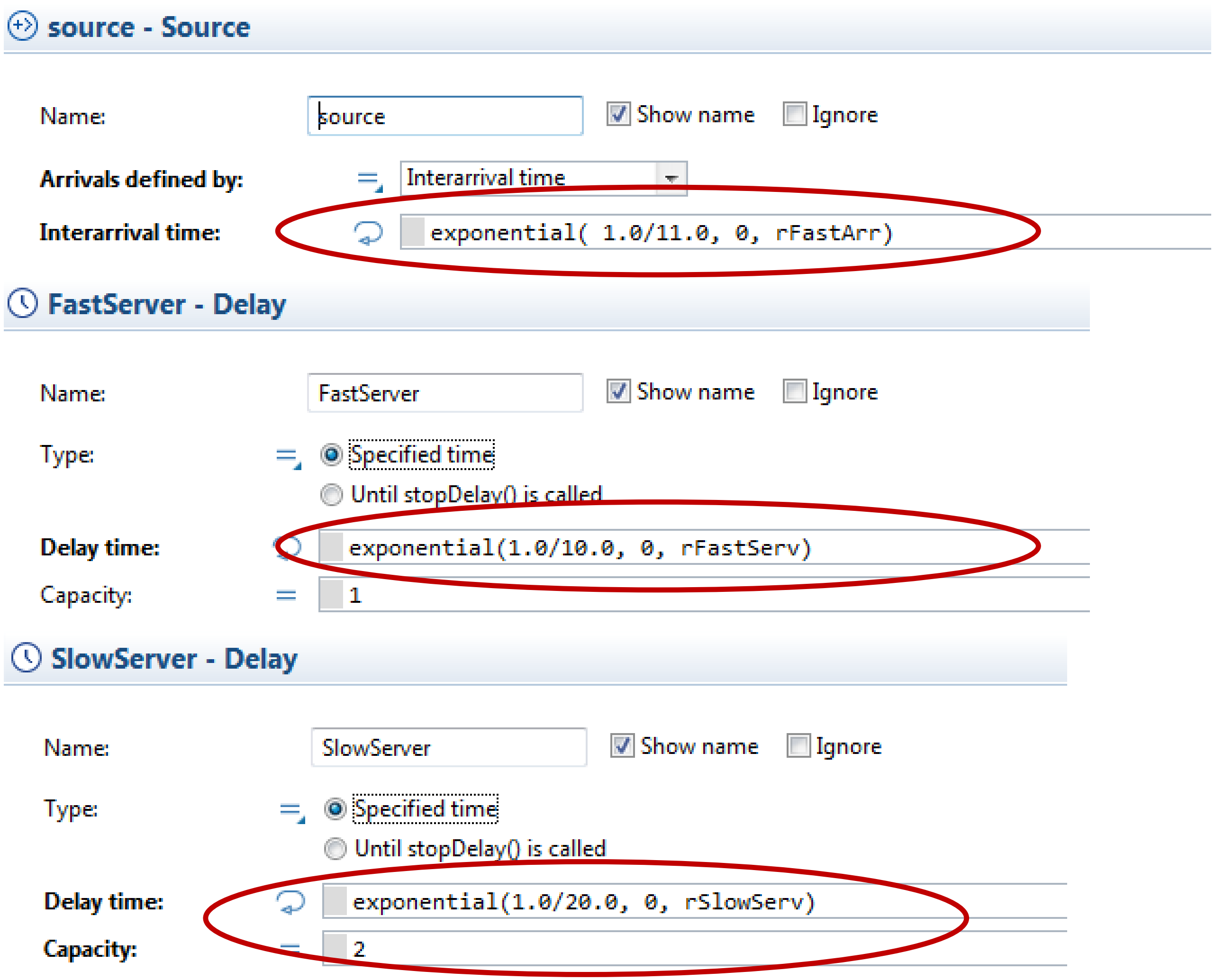
Initialize the random number streams in a function



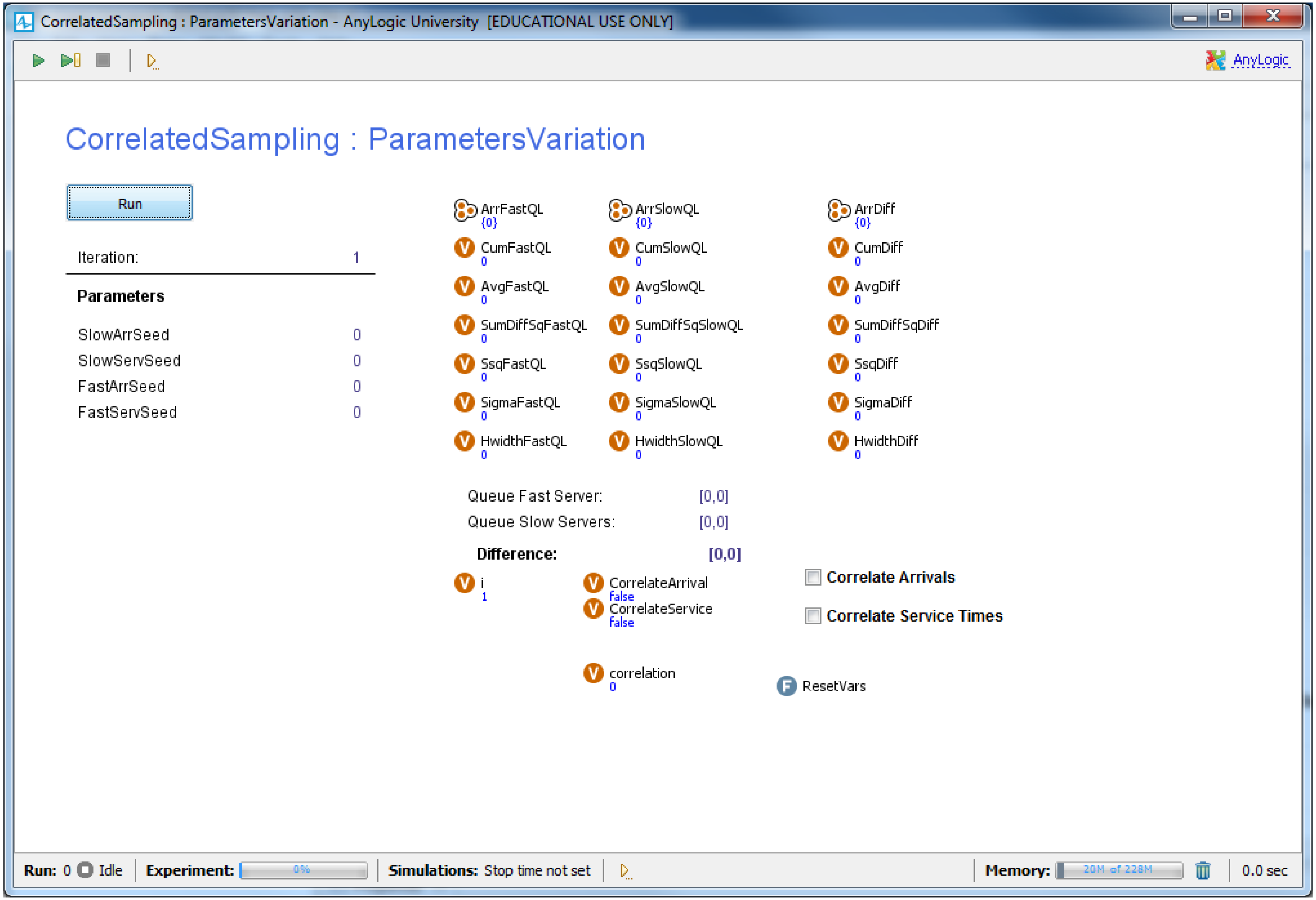


Controlling Random Numbers in AnyLogic

Assign specific random number stream to each random variable



Experiment Setup



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Controlling Random Numbers in AnyLogic

Set seeds and initialize the random number streams from the experiment

- Use the same seeds for correlated runs
- Use different seeds for independent runs

Before simulation run:

```
i+=1;
root.SlowArrSeed = i;
if(CorrelateArrival)    root.FastArrSeed = i;
else                    root.FastArrSeed = i+10;
root.SlowServSeed = i+20;
if(CorrelateService) root.FastServSeed = i+20;
else                 root.FastServSeed = i+30;
root.initRandom();
```

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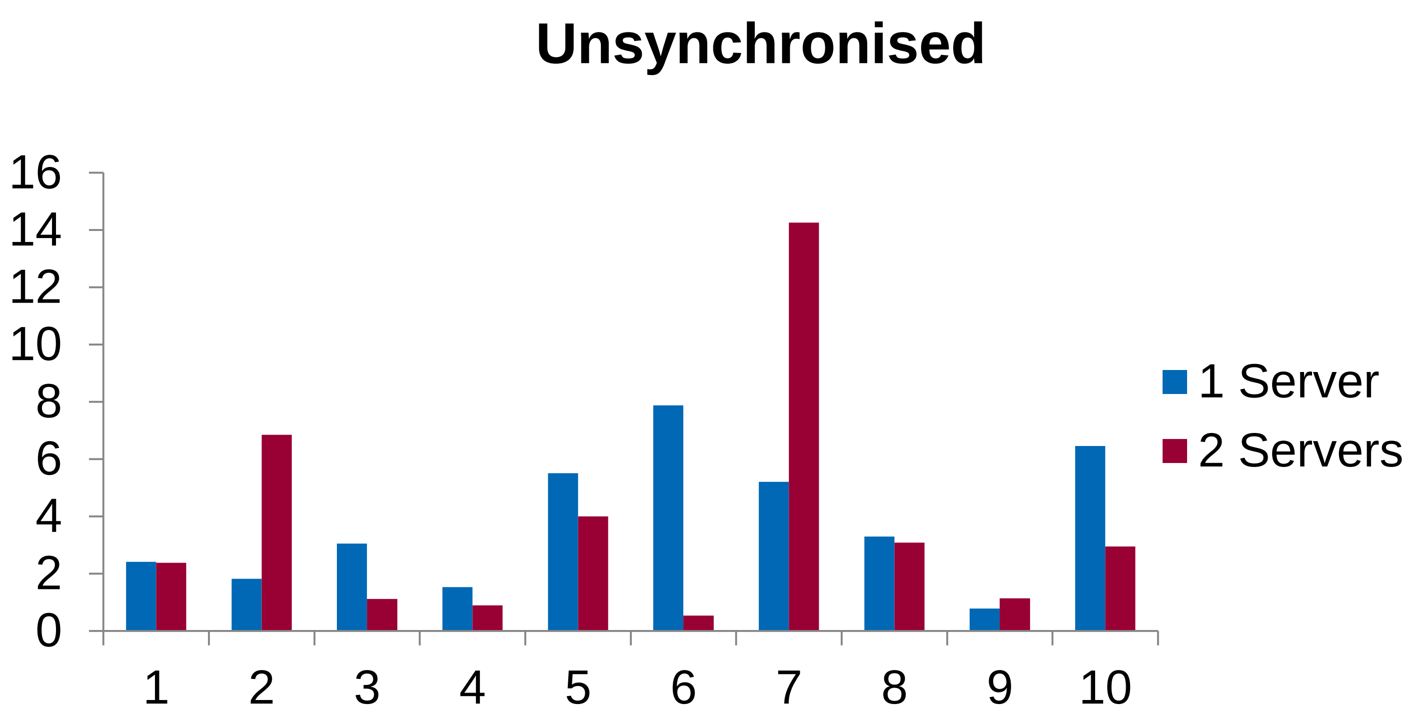
Experimental Results

Individual replication results, unsynchronised



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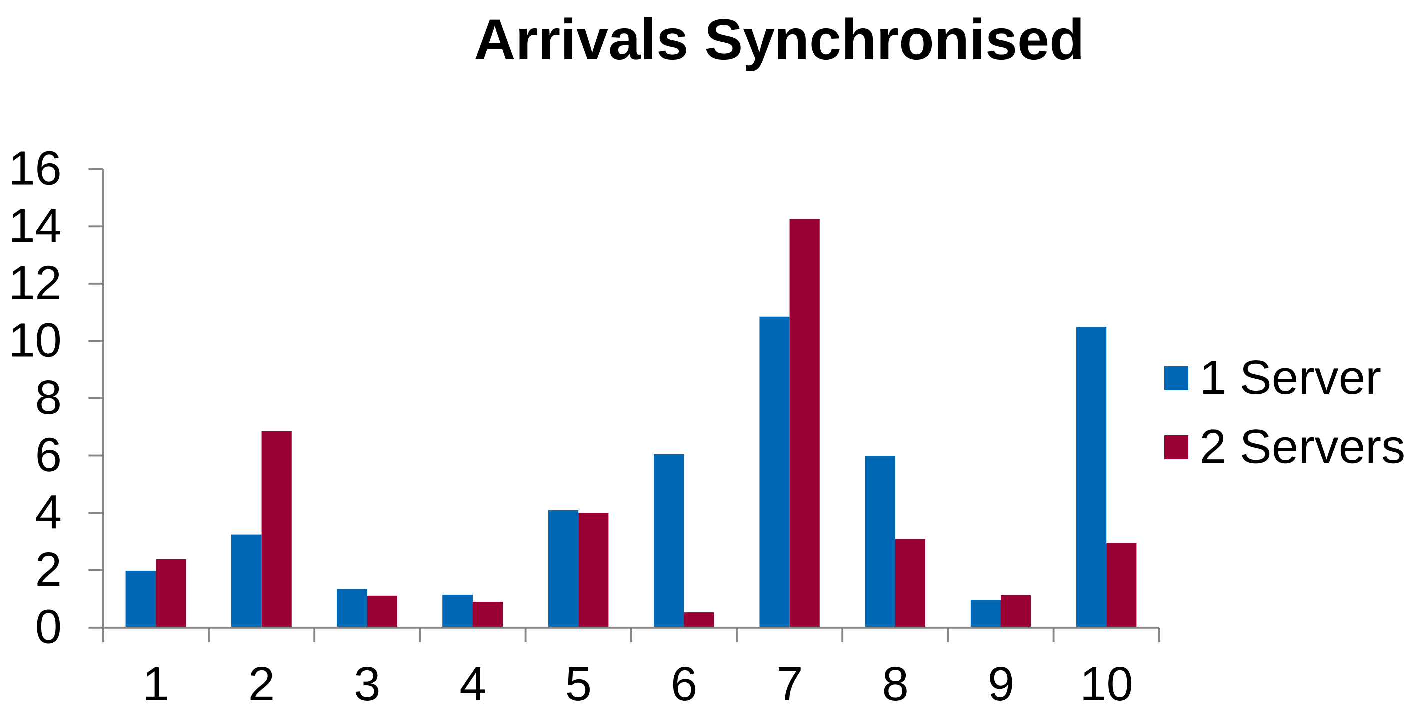
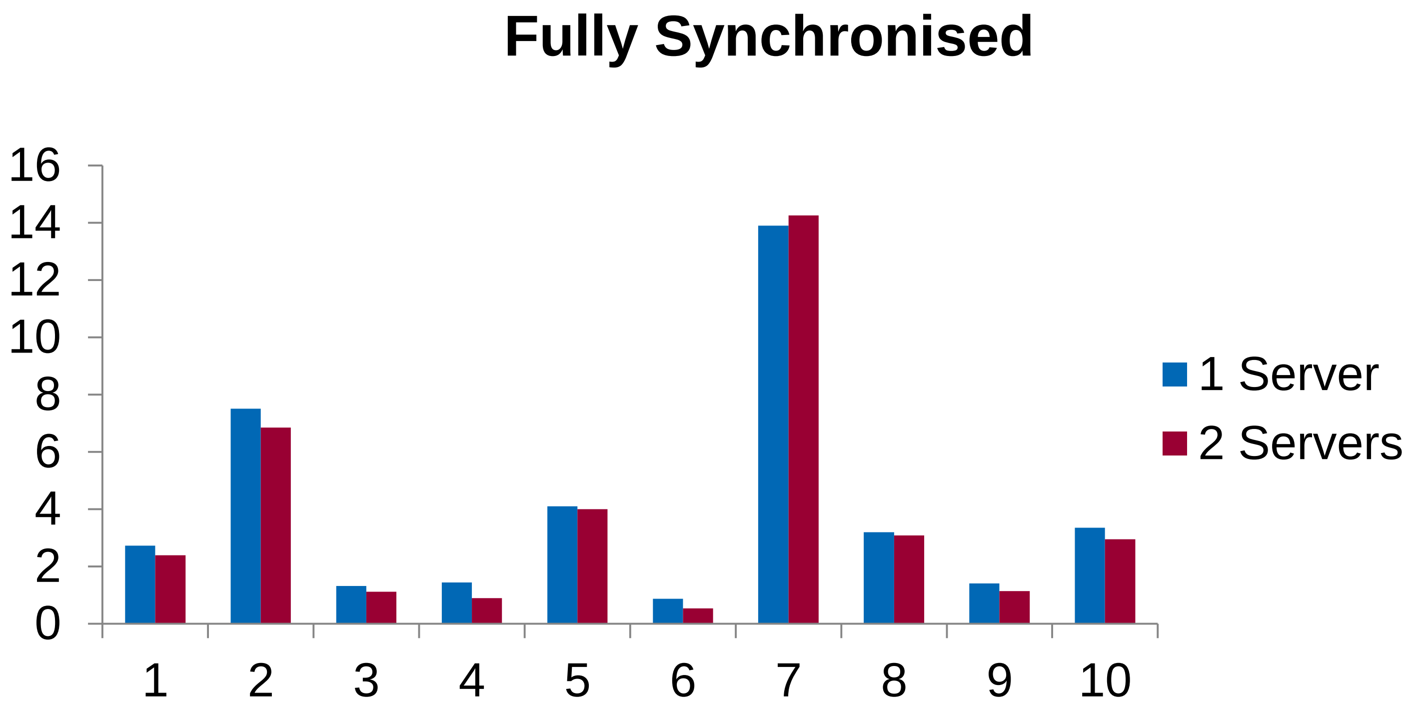
Experimental Results

Individual replication results, only arrivals synchronised



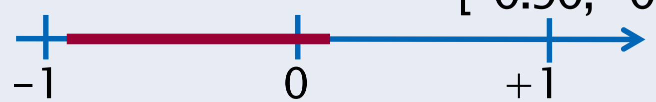
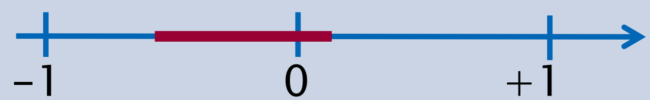
Experimental Results

Individual replication results, fully synchronised



Results

For 100 Replications:

	Sample		C.I. ($\alpha = 0.05$)	
RN	Corr.	Variance	Width	Interval
Independent	0.02	6.46	1.01	[−0.90, 0.11] 
Arrivals Synchronized	0.54	2.75	0.66	[−0.54, 0.12] 
Fully Synchronized	0.99	0.05	0.09	[−0.35, −0.26]