A rotated measurement plane for the ORT Cosmology Experiment

V R Marthi & J N Chengalur

National Centre for Radio Astrophysics, Tata Institute of Fundamental Research

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The Ooty Radio Telescope(ORT) lies along the 11^o north-south slope of a hill near Ooty in the Nilgiris. The slope of the hill is the same as its latitude - 11^o, making the north-south telescope equatorial mounted. The telescope itself consists of 1056 dipoles, each of approximately 0.5 m length. The telescope is 530 m long and 30 m wide.

The central radio frequency(RF) of the ORT is 326.5 MHz. The 1056 dipoles elements are grouped into 22 modules - each module combines signal from 48 dipoles in a Christmas tree network. In Phase-I of the modernisation of the ORT, 40 half-modules are digitised in the field and the signals are transported over optical fibers to the central building. The correlations of these digitised signals give us the visibilities. The visibilities and the intensity distribution in the sky are related through a two-dimensional Fourier transform pair. The fundamental requirement in this 2D FT is that the plane on which the measurements are made and the plane which is measured(the sky plane) must be parallel to each other. Therefore, any coordinate system in which the pair of planes are parallel is valid. This short note describes a rotated coordinate system in which all measurements are made in a plane parallel to the axis of the telescope and, additionally, the effects of sky curvature can be completely ignored in the FT relation. This latter advantage issues from the fact that the coordinate system is chosen in such a manner as to avoid any baseline component normal(the w-component) to the sky-plane.

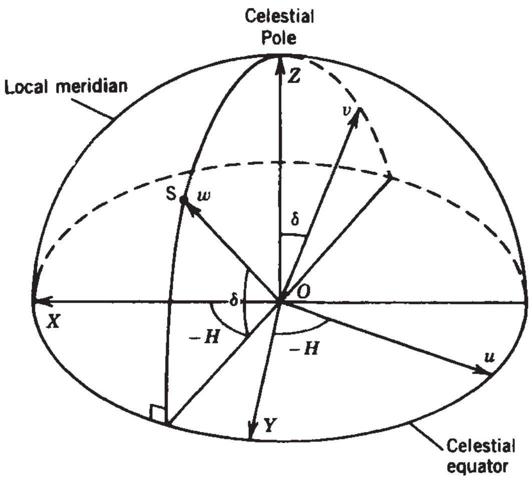

Figure 1: The projection in which the uvw axes are defined with respect to the source. This projection is followed at the GMRT as it is a non-planar array. The projection for the ORT can be obtained by (i) rotating the uvw-axes to align the v-axis with the celestial north and the u-axis with east, and (ii) projecting the source from the celestial sphere to a tangent plane parallel to the uv-plane. This figure is taken from [1].

Since the ORT is a planar array, we can tie our measurement coordinate system(or baseline coordinate system) to the array. As a brief but useful digression in this context, this is not possible with the GMRT - a non-planar array - for which the measurement coordinate system is generally referred to the source. Consequently, the w-component cannot be avoided. However, to appreciate the simplicity in the case of ORT, we will start with the right-handed Cartesian measurement coordinate system referred to the source. The array is placed on the XZ plane. X is along east and Z is along celestial north. Y, therefore, is normal to the tangent plane at the equator. Looking at the array from the source, u and v are the orthogonal components of the baselines in the plane parallel to the tangent-plane at the source(with the source itself at the origin) and the component towards the source is w. The w-term can be avoided by absorbing the uniform phase gradient along the array into a uniform inter-element path-length gradient. However this applies only to the phase-centre. For a large field of view, points in the celestial sphere sufficiently away from the phase-centre(but still within the large primary beam) will have significant w-components. Flux from these sources leads to errors when referring the visibility to the phase-centre, as the additional phase contribution from the w-component will not have been taken into account.

We redefine the measurement-sky plane systems, taking particular advantage of the equatorial mount and planar configuration of the ORT. A simple projection would hence be to have the sky projected onto a plane parallel to the tangent-plane at the earth's equator (i.e. parallel to the rotation axis) and with origin at $\delta = 0$, $\alpha = \alpha_0]$. The array itself is placed on the equatorial tangent-plane on the surface of the earth. The w-axis of the measurement-plane is therefore normal to the sky-plane as well.

We can derive the projection rigorously, from the following considerations: Fig. 1 shows the uvw coordinates tied to the source. The coordinate transformation from the $[X, Y, Z]$ to $[u, v, w]$ is

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} sin(HA) & cos(HA) & 0 \\ -sin(\delta)cos(HA) & sin(\delta)sin(HA) & cos(\delta) \\ cos(\delta)cos(HA) & -cos(\delta)sin(HA) & sin(\delta) \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{1}$$

For the ORT, the axis of the array points to the celestial north(Z) and the array lies on the XZ plane. However, when the array is tracking a source from rise to set, there is no relative motion between the array and the source. Therefore the hour-angle of the source is irrelevant, and can be set to zero($HA = 0.0$). Referring to Fig. 2, we have $X = Y = 0$. So eqn. 1 reduces to

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 & 1.0 & 0 \\ -sin(\delta) & 0 & cos(\delta) \\ cos(\delta) & 0 & sin(\delta) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ Z \end{bmatrix} \tag{2}$$

By rotating the tangential sky-plane to align it parallel to the measurement plane, the radio source will have $m = cos(\delta)$. Eqn. 2 reduces to

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ Z \\ 0 \end{bmatrix} \tag{3}$$

The w-term has been obliterated because of the rotation, and the v-term has been simplified to triviality. The m term picks up offsets due to the fixed rotation of δ radians of the plane, which is precisely $m = cos(\delta)$. To summarise, we have

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ Z \\ 0 \end{bmatrix}, \begin{bmatrix} l \\ m \\ n \end{bmatrix} = \begin{bmatrix} cos(\alpha - \alpha_0) \\ cos(\delta) \\ sin(\delta) \end{bmatrix} \tag{4}$$

and

$$ul + vm + wn = vm = Z.cos(\delta) \tag{5}$$

which makes the brightness distribution-visibility Fourier transform pair a one-dimensional transform. Appropriate inter-element distances of the baselines (Z) can be directly used in the FT relation

$$\mathbf{V_{ij} = S(m).e^{i\ 2\pi z_{ij}.cos(\delta)}} \tag{6}$$

References

1. *Interferometry and Synthesis in Radio Astronomy*, eds. Thompson, A. R., Moran, J. M., Swenson Jr., G. W., 2^{nd}.ed, Wiley-VCH.

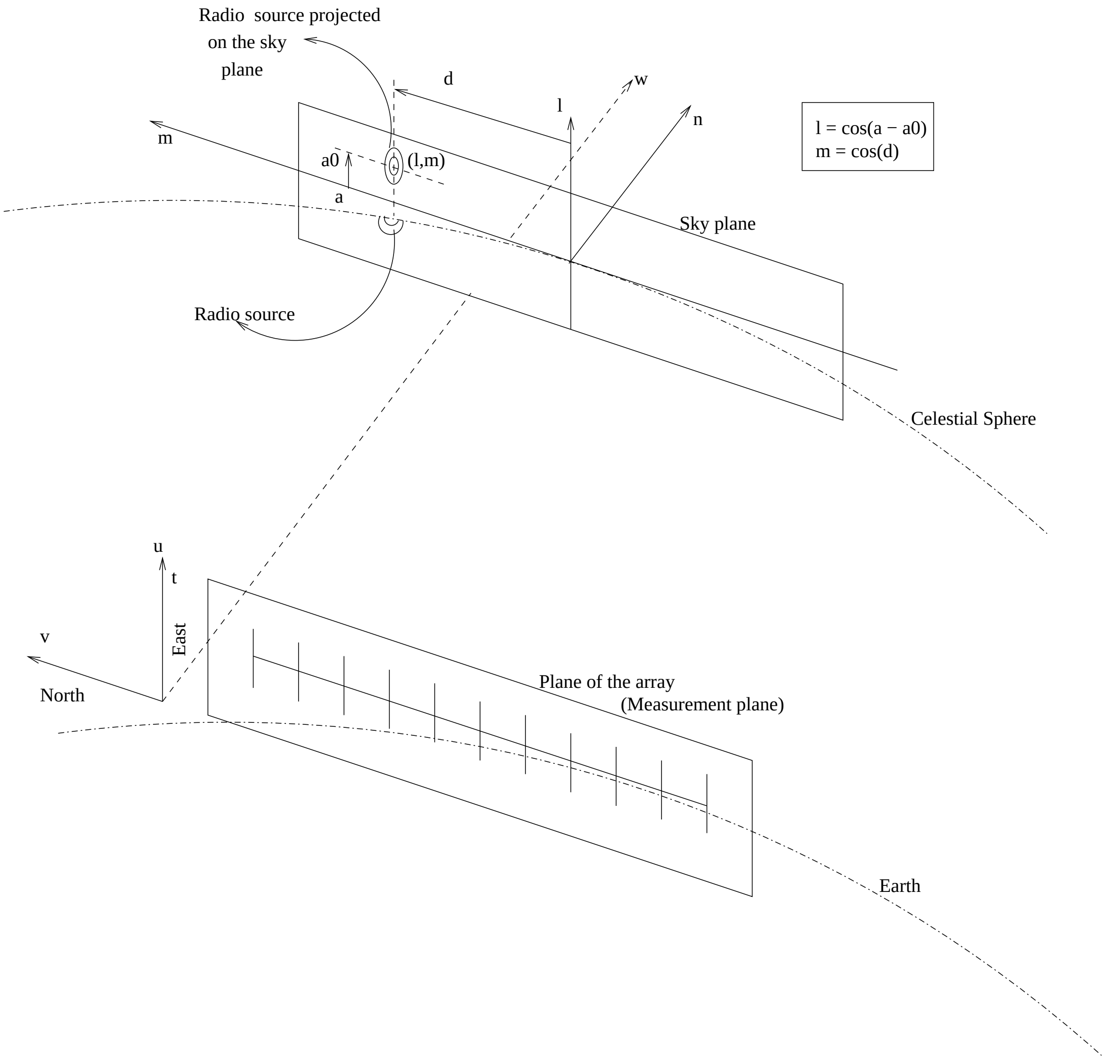

Figure 2: The projection to be used for the ORT Cosmology Experiment. The radio source, imagined to be on the celestial sphere, is projected on a tangent plane which is parallel to the measurement plane. The w-axis is normal to these planes, which relieves the visibility measurements of any w-component. When the array is phased to observe the radio source shown here, the centre of the primary beam should more appropriately called the measurement centre instead of the phase centre. The measurement centre is therefore $l = cos(\alpha - \alpha_0), m = cos(\delta)$, where α is the right ascension(or hour angle) and δ is the declination of the pointing centre. α_0 is the right ascension(or hour angle) of the radio source. However, for the ORT array configuration, l is of no importance as the array cannot resolve in the east-west direction. The visibility for the radio source shown above is therefore $\mathbf{V_{ij}} = \mathbf{S}.\mathbf{e}^{\mathbf{i}\ \mathbf{2\pi v_{ij}.cos}(\delta)}$, a simple one-dimensional Fourier transform relation.