

$$\dot{\theta} = \omega$$

$$\omega = \frac{d\theta}{dt} = \dot{\theta} \quad \text{--- (1)}$$

$$PE = mg(l - l \cos \theta)$$

$$KE = \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot m l^2 \dot{\theta}^2$$

$$L = KE - PE = \frac{1}{2} m l^2 \dot{\theta}^2 - mg(l - l \cos \theta)$$

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Euler-Lagrange eqn

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = T$$

$$m l^2 \ddot{\theta} + mg l \sin \theta = T$$

$$\ddot{\theta} = \frac{-g \sin \theta}{l} + \frac{T}{m l}$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial \theta} & \frac{\partial f_1}{\partial \dot{\theta}} \\ \frac{\partial f_2}{\partial \theta} & \frac{\partial f_2}{\partial \dot{\theta}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -g \cos \theta & 0 \end{bmatrix}$$

Eqbm. pts:  $(0, 0) \text{ and } (\pi, 0)$

$$A_{(0,0)} = \begin{bmatrix} 0 & 1 \\ g & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}$$

$$A_{(\pi,0)} = \begin{bmatrix} 0 & 1 \\ -g & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}$$

$$\frac{d}{dt} \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ g & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} T$$

$$\dot{x} = Ax + Bu$$

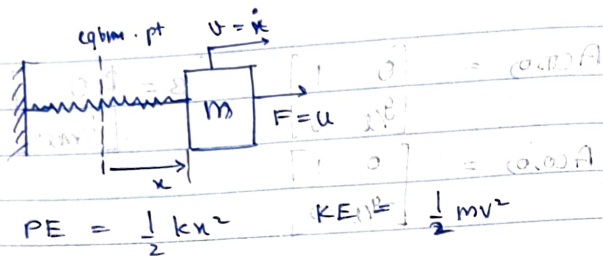
$$\dot{x} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} \quad x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \quad T = u$$

$$u = \ddot{\theta} + \ddot{\theta}$$

$$\ddot{\theta} + \ddot{\theta} = \ddot{\theta}$$

$$\begin{bmatrix} \frac{1}{m} & \frac{1}{m} \\ \frac{1}{m} & \frac{1}{m} \end{bmatrix} = J$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = B \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A$$



$$PE = \frac{1}{2} kx^2 \quad KE = \frac{1}{2} m\dot{x}^2$$

$$[7] \quad \begin{bmatrix} 0 & 1 \\ -k/m & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 \\ u \end{bmatrix}$$

$$\therefore L = KE - PE = \frac{1}{2} m\dot{x}^2 - \frac{1}{2} kx^2$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} = \frac{\partial L}{\partial x} = -kx$$

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Euler Lagrange eqn

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = u$$

$$m\ddot{x} + kx = u$$

$$\ddot{x} = \frac{-kx + u}{m} \quad (2)$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial \dot{x}} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial \dot{x}} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ -k/m & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}$$

$$\therefore \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u$$

$$\dot{x} = AX + BU \quad A = \begin{bmatrix} 0 & 1 \\ -k/m & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} \quad x = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$x(0) = \begin{bmatrix} x(0) \\ \dot{x}(0) \end{bmatrix} \quad u(0) = u(0)$$

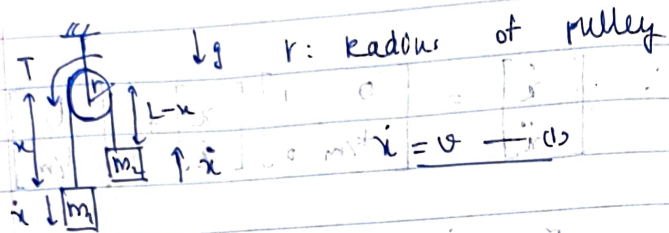
$$(T = 1.5 \text{ sec}) \quad T = \frac{1}{\omega_n} = \frac{1}{\sqrt{k/m}} = \frac{1}{\sqrt{1000/0.1}} = 0.0316 \text{ sec}$$

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$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1000}{0.1}} = 100 \text{ rad/sec}$$

$$A = \begin{bmatrix} 0 & 1 \\ -1000 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = 0$$



$$PE = -m_1 g x - m_2 g (L-x)$$

$$KE = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \dot{x}^2$$

$$L = KE - PE = \frac{1}{2} (m_1 + m_2) \dot{x}^2 + m_1 g x + m_2 g (L-x)$$

Euler-Lagrange eqn

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = \frac{T}{r} \quad (\because F \cdot r = T)$$

$$(m_1 + m_2) \ddot{x} - m_1 g + m_2 g = \frac{T}{r}$$

$$\ddot{x} = \frac{(m_1 - m_2)g}{(m_1 + m_2)} + \frac{T}{r(m_1 + m_2)} \quad (3)$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial \dot{x}} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial \dot{x}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = A$$

$$B = \begin{bmatrix} 0 \\ \frac{r}{r(m_1 + m_2)} \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{r(m_1 + m_2)} \end{bmatrix} T$$

$$\dot{X} = Ax + Bu$$

$$(A - B^{-1}A + B^{-1}A) p_{cm} - (A - A + B^{-1}A) p_{cm} = 0$$

$$-x_{cm} L + \frac{1}{2} (m_1 + m_2) L = 0 \Rightarrow x_{cm} = \frac{1}{2} L$$

$$-x_{cm} L + \frac{1}{2} (m_1 + m_2) L = 0 \Rightarrow x_{cm} = \frac{1}{2} L$$

$$m_1 g + (A - B^{-1}A) p_{cm} = 0 \Rightarrow p_{cm} = -m_1 g$$

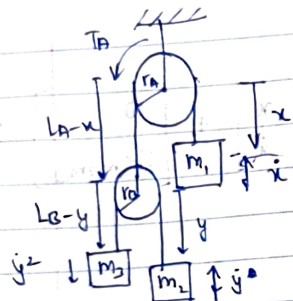
$$(A - B^{-1}A) p_{cm} = 0 \Rightarrow p_{cm} = 0$$

$$m_1 g + (A - B^{-1}A) p_{cm} = 0 \Rightarrow p_{cm} = -m_1 g$$

$$\frac{\partial L}{\partial x} = \frac{1}{2} (m_1 + m_2) \dot{x}^2 = 0$$

$$\frac{\partial L}{\partial \dot{x}} = (m_1 + m_2) \dot{x} = 0$$

$$\frac{\partial L}{\partial x} + \frac{\partial L}{\partial \dot{x}} = 0$$



$$m < m_1 + m_3 \text{ if}$$

$$m_3 > m_1 \text{ if}$$

$$L_A + L_B = \text{const}$$

$$PE = -m_2 g (y + L_A - u) - m_3 g (L_B + L_A - y - u) - m_1 g x$$

$$KE = \frac{1}{2} (m_2 + m_3) \dot{y}^2 + \frac{1}{2} m_1 \dot{x}^2$$

$$L = KE - PE = \frac{1}{2} (m_2 + m_3) \dot{y}^2 + \frac{1}{2} m_1 \dot{x}^2$$

$$\begin{aligned} \ddot{x} &= \frac{v_x}{r_A} - (c) \\ \ddot{y} &= \frac{v_y}{r_B} - (c) \end{aligned} \quad \begin{aligned} &+ m_2 g (L_A + y - u) + m_1 g u \\ &+ m_3 g (L_B + L_A - y - u) \end{aligned}$$

Euler-Lagrange eqn

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = \frac{T_A}{r_A}$$

$$m_1 \ddot{x} - (-m_2 g + m_1 g - m_3 g) = \frac{T_A}{r_A}$$

$$\ddot{x} = \frac{(m_1 - m_2 - m_3) g}{m_1} + \frac{T_A}{m_1 r_A} \quad \text{--- (3)}$$

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{y}} \right) - \frac{\partial L}{\partial y} = \frac{T_A}{r_A} + \frac{T_B}{r_B}$$

$$(m_2 + m_3) \ddot{y} - (m_2 g - m_3 g) = \frac{T_A}{r_A} + \frac{T_B}{r_B}$$

$$\ddot{y} = \frac{(m_2 - m_3) g}{(m_2 + m_3)} + \frac{T_A}{r_A (m_2 + m_3)} + \frac{T_B}{r_B (m_2 + m_3)} \quad \text{--- (4)}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

CA there are no

$x$  or  $\dot{x}$ ,  $y$  or  $\dot{y}$

terms in eqns

(3) & (4)

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1 r_A} & 0 \\ 0 & \frac{1}{(m_2 + m_3) r_B} \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1 r_A} & 0 \\ \frac{1}{(m_1+m_2)r_A} & \frac{1}{(m_1+m_2)r_D} \end{bmatrix} \begin{bmatrix} T_A \\ T_D \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1 r_A} & 0 \\ \frac{1}{(m_1+m_2)r_A} & \frac{1}{(m_1+m_2)r_D} \end{bmatrix} \begin{bmatrix} T_A \\ T_D \end{bmatrix}$$

$$\underline{\dot{x} = Ax + Bu}$$