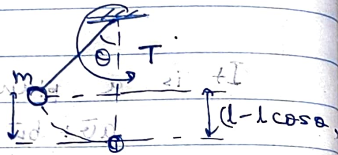
$\cancel{\dot{\theta} = d\theta}$

$$\omega = \frac{d\theta}{dt} = \dot{\theta} \quad \text{— (1)}$$



$$PE = mg(l - l\cos\theta)$$

$$KE = \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot ml^2 \cdot \omega^2$$

$$= \frac{ml^2 \cdot \dot{\theta}^2}{2}$$

$$L = KE - PE = \frac{ml^2 \dot{\theta}^2}{2} - mg(l - l\cos\theta)$$

Euler – Lagrange eqn

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = T$$

$$ml^2 \cdot \ddot{\theta} + mgl\sin\theta = T$$

$$\ddot{\theta} = -\frac{g}{l}\sin\theta + \frac{T}{ml^2} \quad \text{— (2)}$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial \theta} & \frac{\partial f_1}{\partial \dot{\theta}} \\ \frac{\partial f_2}{\partial \theta} & \frac{\partial f_2}{\partial \dot{\theta}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l}\cos\theta & 0 \end{bmatrix}$$

Eqbm. pts : $\overset{\theta\ \dot{\theta}}{(0, 0)}$ & $\overset{\theta\ \dot{\theta}}{(\pi, 0)}$

$$A(\pi, 0) = \begin{bmatrix} 0 & 1 \\ g/l & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1/ml^2 \end{bmatrix}$$

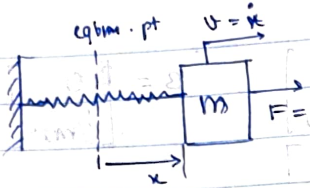
$$A(0, 0) = \begin{bmatrix} 0 & 1 \\ -g/l & 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ g/l & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/ml^2 \end{bmatrix} [T]$$

$$\dot{x} = Ax + Bu$$

$$\dot{x} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} \qquad x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \qquad T = u$$

(inputs)



$PE = \frac{1}{2} k x^2 \qquad KE = \frac{1}{2} m v^2$

$\partial KE = \partial \frac{1}{2} m . \dot{x}^2$

$\therefore \; L = KE - PE = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} k x^2$

$\cancel{\frac{\partial L}{\partial x}} \qquad \dot{x} = \frac{dx}{dt} = v \quad \text{—(1)}$

<u>Euler Lagrange eqn</u>

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = u$$

$$m\ddot{x} + kx = u$$

$$\ddot{x} = \frac{-kx}{m} + \frac{u}{m} \quad \text{—(2)}$$

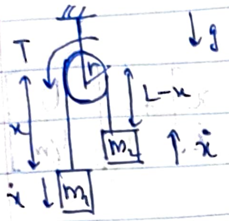
$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial \dot{x}} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial \dot{x}} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ -k/m & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}$$

$$\therefore \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u$$

$\cancel{A} \qquad \dot{X} = AX + BU$

$$\dot{X} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} \qquad X = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$



r: Radius of pulley

$$\dot{x} = v \quad \text{— (1)}$$

$$PE = -m_1 g x - m_2 g (L-x)$$

$$KE = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \dot{x}^2$$

$$L = KE - PE = \frac{1}{2}(m_1+m_2)\dot{x}^2 + m_1 g x + m_2 g (L-x)$$

Euler - Lagrange eqn

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = \frac{T}{r} \quad (\because F \cdot r = T)$$

$$(m_1+m_2)\ddot{x} - m_1 g + m_2 g = \frac{T}{r}$$

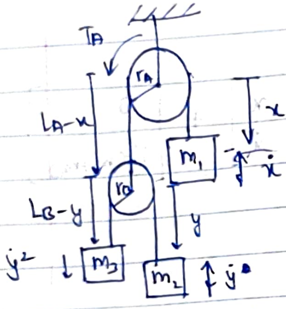
$$\ddot{x} = \frac{(m_1-m_2)g}{(m_1+m_2)} + \frac{T}{r(m_1+m_2)} \quad \text{— (2)}$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_2}{\partial \dot{x}} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial \dot{x}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = A$$

$$B = \begin{bmatrix} 0 \\ \frac{1}{r(m_1+m_2)} \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/r(m_1+m_2) \end{bmatrix} T$$

$$\dot{X} = Ax + Bu$$



$m_1 < m_2 + m_3$ &

$m_3 > m_2$

$$PE = -m_2 g(y + L_A - x) - m_3 g(L_B + L_A - y - x) - m_1 g x$$

$$KE = \frac{1}{2}(m_2 + m_3)\dot{y}^2 + \frac{1}{2} m_1 \dot{x}^2$$

$$L = KE - PE = \frac{1}{2}(m_2 + m_3)\dot{y}^2 + \frac{1}{2} m_1 \dot{x}^2 + m_2 g(L_A + y - x) + m_1 g x + m_3 g(L_B + L_A - y - x)$$

$$\dot{x} = v_x \quad -(1)$$

$$\dot{y} = v_y \quad -(2)$$

Euler- Lagrange eqn

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = \frac{T_A}{r_A}$$

$$m_1 \ddot{x} - (-m_2 g + m_1 g - m_3 g) = \frac{T_A}{r_A}$$

$$\ddot{x} = \frac{(m_1 - m_2 - m_3)g}{m_1} + \frac{T_A}{m_1 r_A} \quad -(3)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = \frac{T_A}{r_A} + \frac{T_B}{r_B}$$

$$(m_2 + m_3)\ddot{y} - (m_2 g - m_3 g) = \frac{T_A}{r_A} + \frac{T_B}{r_B}$$

$$\ddot{y} = \left(\frac{m_2 - m_3}{m_2 + m_3}\right) g + \frac{T_A}{r_A(m_2 + m_3)} + \frac{T_B}{r_B(m_2 + m_3)} \quad -(4)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(As there are no x, $\dot{x}$, y & $\dot{y}$ terms in eqns (3) & (4))

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1 r_A} & 0 \\ \frac{1}{(m_2 + m_3) r_A} & \frac{1}{(m_2 + m_3) r_B} \end{bmatrix}$$

$$\cancel{\begin{bmatrix}\dot{x}\\ \dot{y}\\ \ddot{x}\\ \ddot{y}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ & & & \\ & & & \\ & & & \end{bmatrix}}$$

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{y}\\ \ddot{y}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ y\\ \dot{y}\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ \frac{1}{m_1 r_A} & 0\\ \frac{1}{(m_2+m_2)r_A} & \frac{1}{(m_2+m_2)r_B}\end{bmatrix}\begin{bmatrix}T_A\\ T_B\end{bmatrix}$$

$$\underline{\dot{X} = AX + BU}$$