Tackling Many Objectives

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What is SUSTech?

- Southern University of Science and Technology (SUSTech)
- The youngest public university in China
- Started officially in 2012 as a research intensive international university
- Only 6 year old

Where is SUSTech?

- Shenzhen.
- But where is Shenzhen?

Shenzhen: Silicon Valley in China

"Shenzhen is the city that most resembles the Silicon Valley."

--- *Business Week (2014)*

The most competitive city economically in China

- ✓ High technology
- ✓ Finance services
- ✓ Logistics industry
- ✓ Culture industry

Department of Computer Science and Engineering (CSE)

- Only established in 2016
- Currently 20 tenure track faculty members, recruited from all over the world (many more researchers)
- Growth plan: 55 tenure track faculty members

OPAL (OPtimisation And Learning) Group

- **5 tenure track professors**
  - **Ran Cheng, Hisao Ishibuchi, Yuhui Shi, Ke Tang, Xin Yao**
- **~10 research professors and postdoc researchers.**
- **International recognition**
  - **3 IEEE fellows (H-indices: 90, 66, 47): ~120K Google Scholar citations;**
    - **Hisao Ishibuchi, Yuhui Shi, Xin Yao;**
  - **IEEE Computational Intelligence Society (CIS) *Evolutionary Computation Pioneer Award* (Xin) and *Fuzzy Systems Pioneer Award* (Hisao)**
  - **IEEE CIS *Early Career Award* (Ke Tang)**
  - **IEEE CIS *Outstanding PhD Dissertation Award* (Ran Cheng)**
  - **JSPS Award (Hisao)**
  - **Royal Society Wolfson Research Merit Award (Xin)**
  - **Royal Society Newton Advanced Fellowship (Ke)**

Some Research Topics

计算机科学与工程系
Department of Computer Science and Engineering

- **Evolutionary learning and optimisation**
  - Multi-objectivity
  - Dynamics
  - Uncertainty
- **Ensemble machine learning**
  - Online learning
  - Class imbalance learning
- **Neural network learning**
- **Real-world applications**
  - Smart logistics
  - Search-based software engineering
  - Fault diagnosis
  - Reconfigurable computing architectures

We Have Many Openings

- There is nothing more exiting than being part of a growing story.
- We recruit
  - Faculty members
  - Postdoctoral research fellows
  - PhD students
  - Visitors
- We teach in English.
- Email: xiny@sustc.edu.cn

Outline

- Introduction
- Objective Reduction
- Alternative Dominance Relationship
- Improved Two-Archive Algorithm (Two_Arch2)
- Conclusions and Future Work

What is multi-objective optimisation?

- **More than one objective to be optimised, with or without constraints.**

$$
\begin{aligned}
&\text{min/max } f_m(\boldsymbol{x}), && m = 1, 2, \cdots, M \\
&\text{subject to } g_j(\boldsymbol{x}) \geq 0, && j = 1, 2, \cdots, J \\
&\qquad h_k(\boldsymbol{x}) = 0, && k = 1, 2, \cdots, K \\
&\qquad \underbrace{x_i^{(L)}}_{\text{lower bound}} \leq x_i \leq \underbrace{x_i^{(U)}}_{\text{upper bound}}, && i = 1, 2, \cdots, n
\end{aligned}
$$

Multi-objective Evolutionary Algorithms

- **MOEAs have been widely used in the last 20 years for multi-objective optimisation.**
- **They can provide a *set* of non-dominated solutions in a *single* run without requiring the set of weights.**
- **They do not require the objective functions to be convex, smooth, or even continuous.**
- **They can handle nonlinear constraints.**
- **They can deal with uncertainty and dynamics.**

An Indicator of MOEA’s Impact

- **Deb, K., Pratap. A, Agarwal, S., and Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transaction on Evolutionary Computation*, 6(2), 181-197. (29,491 Google Scholar citations)**

Wow!

- **That’s impressive.**

Unfortunately

- **NSGA-II and other early MOEAs work well only with 2 or 3 objectives.**
- **They do not work well when the number of objectives goes beyond that.**
- **There is a scalability issue in terms of the number of objectives.**
- **In this talk, we consider Many Objective Optimisation, indicating the number of objectives is greater than three.**

Two Possible Approaches to Problem-Solving

1. Develop more sophisticated solutions to complex problems.

2. Simplify a complex problem so that an existing solution can be applied.

Can we simplify MaOPs into MOPs?

Can we reduce the number of objectives?

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Objective Reduction

- If two objectives are positively correlated, we need to optimise only one of them.
- There are many methods that could be used to reduce the number of objectives.
- We give one example here.

Nonlinear Correlation Information Entropy (NCIE)

- **NCIE is an entropy measure.**
- **NCIE firstly divides variables *X* and *Y* into *b*b* uniform rank grids. Then, the probabilities p_{ij} can be approximated by counting the samples in those grids. In other words, p_{ij} in the *ij*-th grid can be calculated by the number of solutions in *ij*-the grid (n_{ij}/N).**
- **Parameter *b* can be set as *N^0.5*.**

$$H^r(X) = -\sum_{i=1}^{b} \frac{n_i}{N} log_b(\frac{n_i}{N})$$

$$H^r(X,Y) = -\sum_{i=1}^{b}\sum_{j=1}^{b} \frac{n_{ij}}{N} log_b(\frac{n_{ij}}{N})$$

$$NCIE(X,Y) = H^r(X) + H^r(Y) - H^r(X,Y)$$

Objective Reduction Based on NCIE

- **Correlation analysis is based on the matrix of modified NCIE R^N of *the non-dominated population*.**

$$R^N = \{Sgn(cov_{ij})NCIE_{ij}\}, (1 \le i, j \le m)$$

- **Objective selection aims to choose the most conflicting objectives.**
  - **Our approach is applied in every generation of MOEAs to update the correlation information among objectives.**

H. Wang and X. Yao, "Objective Reduction Based on Nonlinear Correlation Information Entropy," *Soft Computing*, June 2016, Volume 20, Issue 6, pp 2393–2407.

Objective Selection: An Example

- **Select the most conflicting objective**
- **Remove the objectives that are positively correlated to the selected objective**

	f_1	f_2	f_3	f_4	f_5
f_1	1.0000	0.4959	0.4244	0.5348	-0.3552
f_2	0.4959	1.0000	0.3972	0.4686	-0.3381
f_3	0.4244	0.3972	1.0000	0.4765	-0.4352
f_4	0.5348	0.4686	0.4765	1.0000	-0.4488
f_5	-0.3552	-0.3381	-0.4352	-0.4488	1.0000
$\sum NCIE < 0$	-0.3552	-0.3381	-0.4352	-0.4488	-1.5773

- ✓ f_5 is selected, because it has the most conflicting degree with other objectives.
- ✓ There is no objective positively correlated to f_5, thus, there is not a redundant objective with f_5 in the remaining objectives.
- ✓ f_4 is selected, because it has the largest absolute sum of NCIEs to other objectives. f_1 , f_2, and f_3 are omitted, they are all positively correlated to f_4 .
- ✓ Output $\{f_5, f_4\}$

Objective reduction can remove redundant objectives, but what if there is no redundancy?

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Why Are Many Objectives Hard?

- The number of non-dominated solutions increases exponentially as the number of objectives grows.
- As a result, there is no selection pressure in MaOEAs to drive the evolutionary search.
- Can we use alternative dominance relationship other than Pareto dominance?

Θ-dominance --- Intuition

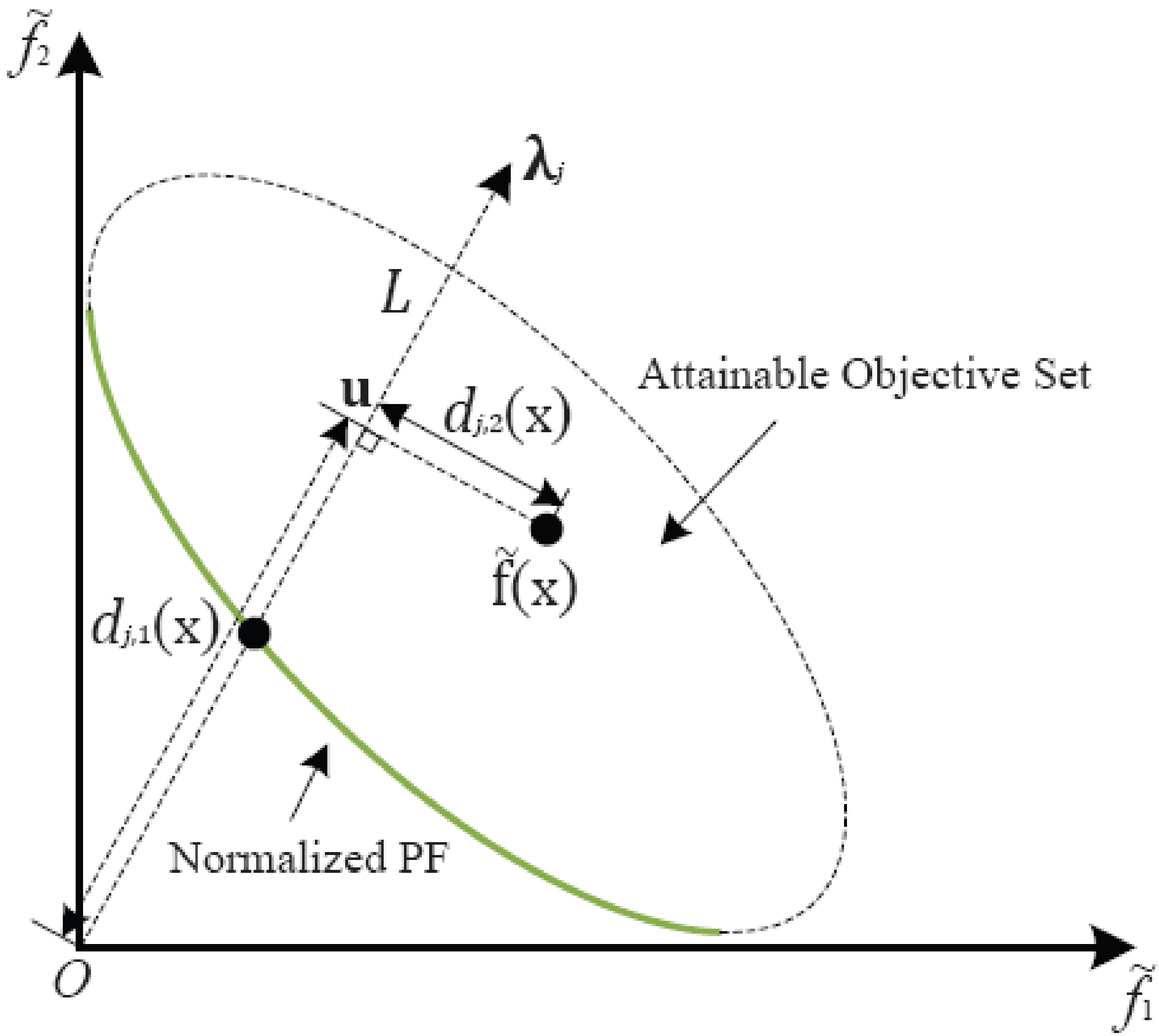

Fig. 3. Illustration of distances $d_{j,1}(\mathbf{x})$ and $d_{j,2}(\mathbf{x})$.

- ***f*'s are normalised objective functions.**
- **λ is the reference direction (point).**

•Y. Yuan, H. Xu, B. Wang and X. Yao, "A New Dominance Relation Based Evolutionary Algorithm for Many-Objective Optimization," *IEEE Transactions on Evolutionary Computation*, 20(1):16-37, February 2016.

Θ-dominance --- Definition

Definition 7: Given two solutions $\mathbf{x}, \mathbf{y} \in S_t$, $\mathbf{x}$ is said to θ-dominate $\mathbf{y}$, denoted by $\mathbf{x} \prec_\theta \mathbf{y}$, iff $\mathbf{x} \in C_j$, $\mathbf{y} \in C_j$, and $\mathcal{F}_j(\mathbf{x}) < \mathcal{F}_j(\mathbf{y})$, where $j \in \{1, 2, \ldots, N\}$.

$$\mathcal{F}_j(\mathbf{x}) = \bar{d}_{j,1}(\mathbf{x}) + \theta d_{j,2}(\mathbf{x})$$

Y. Yuan, H. Xu, B. Wang and X. Yao, "A New Dominance Relation Based Evolutionary Algorithm for Many-Objective Optimization," *IEEE Transactions on Evolutionary Computation*, 20(1):16-37, February 2016.

Balancing Convergence and Diversity

- The form of $F_j(x)$ indicates that the balance between convergence and diversity is very important in MaOEAs.
- **Why not manipulating the balance explicitly?**
  - **Y. Yuan, H. Xu, B. Wang, B. Zhang and X. Yao, "Balancing Convergence and Diversity in Decomposition-Based Many-Objective Optimizers," *IEEE Transactions on Evolutionary Computation*, 20(2):180-198, April 2016.**
- **Don't know how to strike the balance?**
  - **B. Li, K. Tang, J. Li and X. Yao, ``Stochastic Ranking Algorithm for Many-Objective Optimization Based on Multiple Indicators,'' *IEEE Transactions on Evolutionary Computation*, 20(6):924-938, December 2016.**

What if alternative dominance relationships still do not provide a satisfactory solution to a MaOP?

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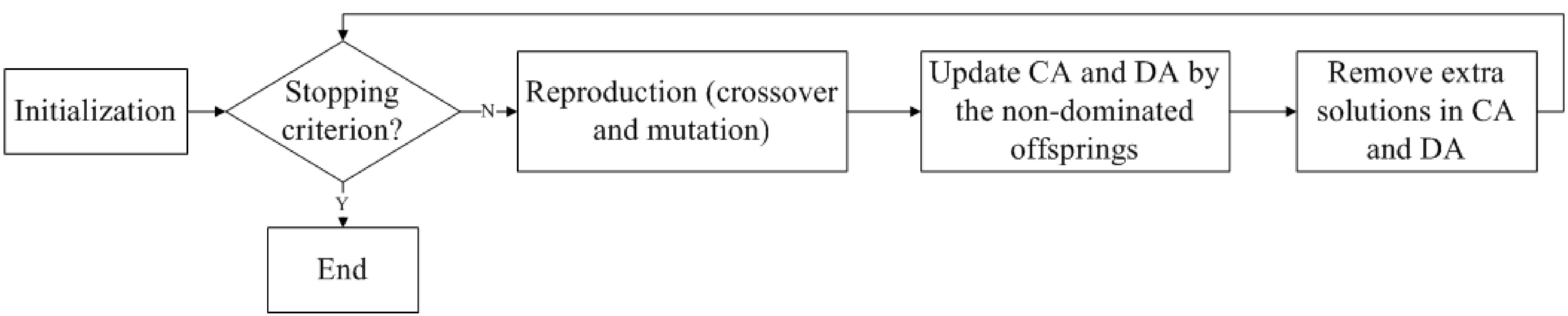
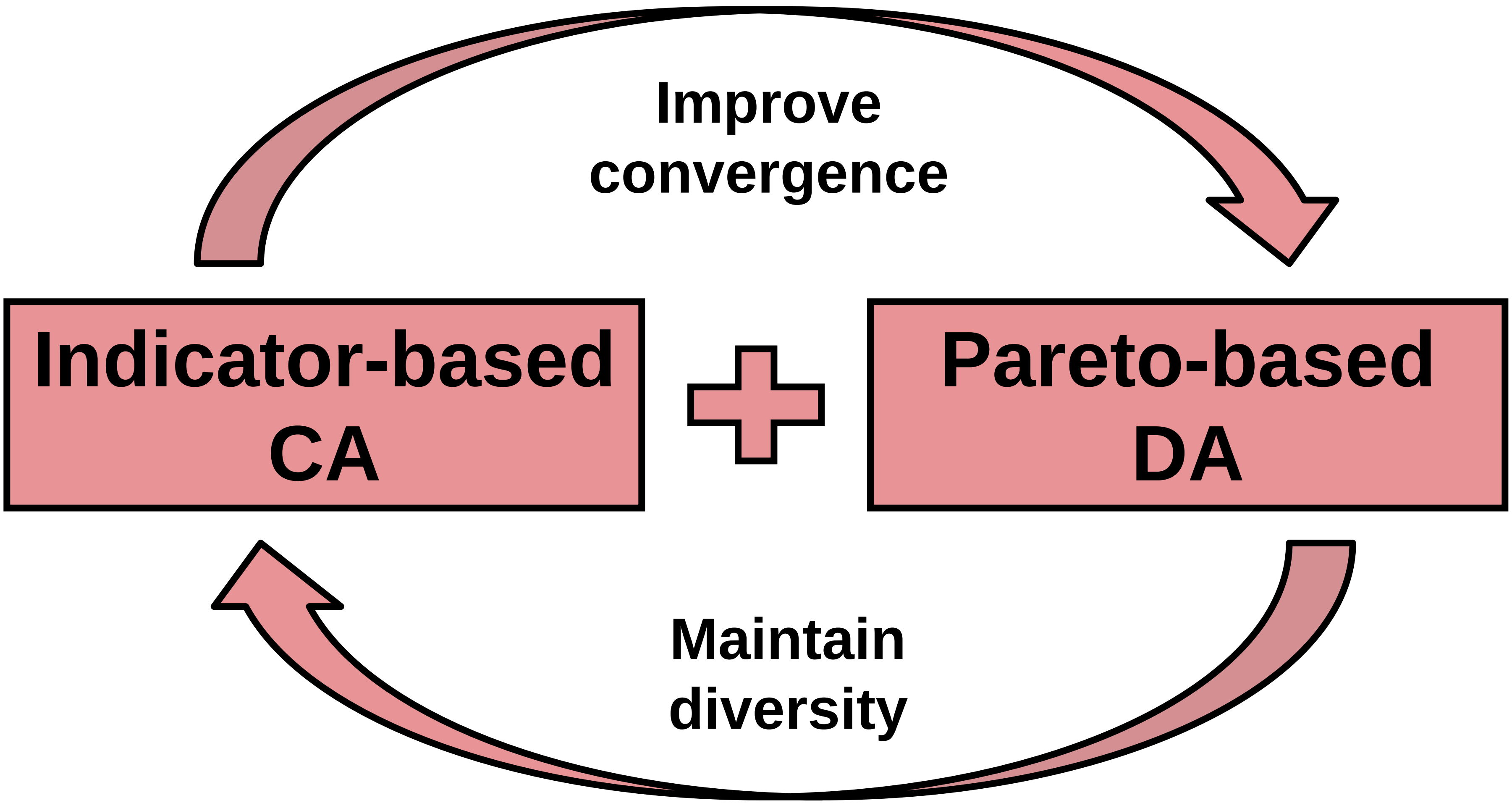
Two-Archive Algorithm

- Two-Archive algorithm (Two_Arch) maintains two archives (CA and DA) to promote convergence and diversity separately.
  - K. Praditwong and X. Yao, "A New Multi-objective Evolutionary Optimisation Algorithm: The Two-Archive Algorithm," *Proc. of the 2006 International Conference on Computational Intelligence and Security (CIS'2006)*, 3-6/11/2006, Ramada Pearl Hotel, Guangzhou, China. IEEE Press, Volume 1, pp.286-291.



<u>Improved</u> Two-Archive Algorithm:

Main Idea



Two_Arch2: Main Steps

Step 1: Initialization.
Step 2: Output <u>DA</u> if the stopping criterion is met, otherwise continue.
Step 3: Generate new solutions from CA and DA by crossover and mutation.
Step 4: Update CA and DA separately, go Step 2.

- **H. Wang, L. Jiao and X. Yao, "Two_Arch2: An Improved Two-Archive Algorithm for Many-Objective Optimization," *IEEE Transactions on Evolutionary Computation*, 19(4):524-541, August 2015.**

Convergence Archive (CA)

- **The quality indicator $I_{\varepsilon+}$ in IBEA is used in selection of CA. $I_{\varepsilon+}$ is an indicator that describes the minimum distance that one solution needs to dominate another solution in the objective space.**

$$I_{\varepsilon+}(x_1, x_2) = min_{\varepsilon}(f_i(x_1) - \varepsilon \leq f_i(x_2), 1 \leq i \leq m)$$

- **The fitness is assigned as below, the solution with the smallest fitness is removed from CA first.**

$$F(x_1) = \sum_{x_2 \in P \setminus \{x_1\}} -e^{-I_{\varepsilon+}(x_2, x_1)/0.05}$$

Diversity Archive (DA)

- **Update DA**
  - **When DA overflows, boundary solutions (solutions with maximal or minimal objective values) are firstly selected.**
  - **In the iterative process, the most different solution from the current DA is added until reaching the size.**
- **L_p-norm distance is adopted as the similarity measure in DA.**
- **DA is used as the final output of Two_Arch2.**

Degraded Euclidean Distance (Distance Concentration) in High-Dimensional Space

- **The Euclidean distance (L_2-norm) degrades its similarity indexing performance in a high-dimensional space.**
- **Most of existing diversity maintenance methods use the Euclidean distance to measure similarity among solutions for MaOPs.**

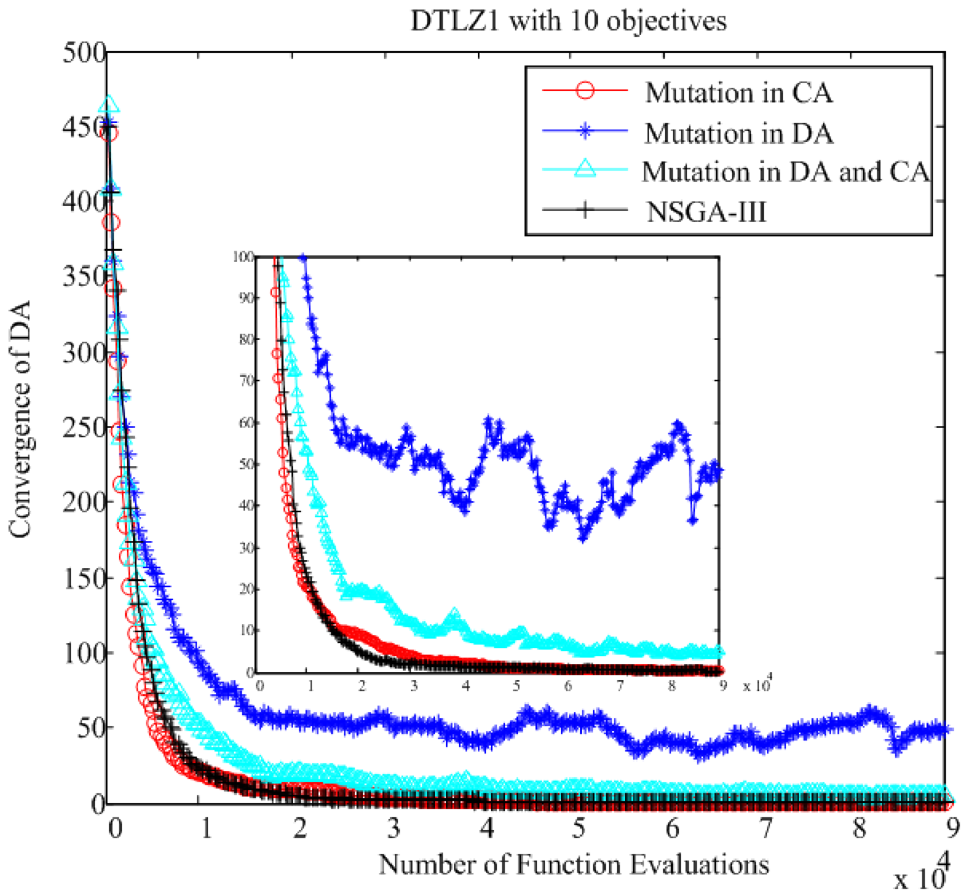
C. C. Aggarwal, A. Hinneburg and D. A. Keim, "On the surprising behavior of distance metrics in high dimensional space." Springer, 2001.

Similarity in High-Dimensional Space

- **The fractional distances (L_p-norm, p<1) perform better in a high-dimensional space.**
- **$L_{1/m}$-norm is employed in Two_Arch2, where *m* is the number of objectives.**

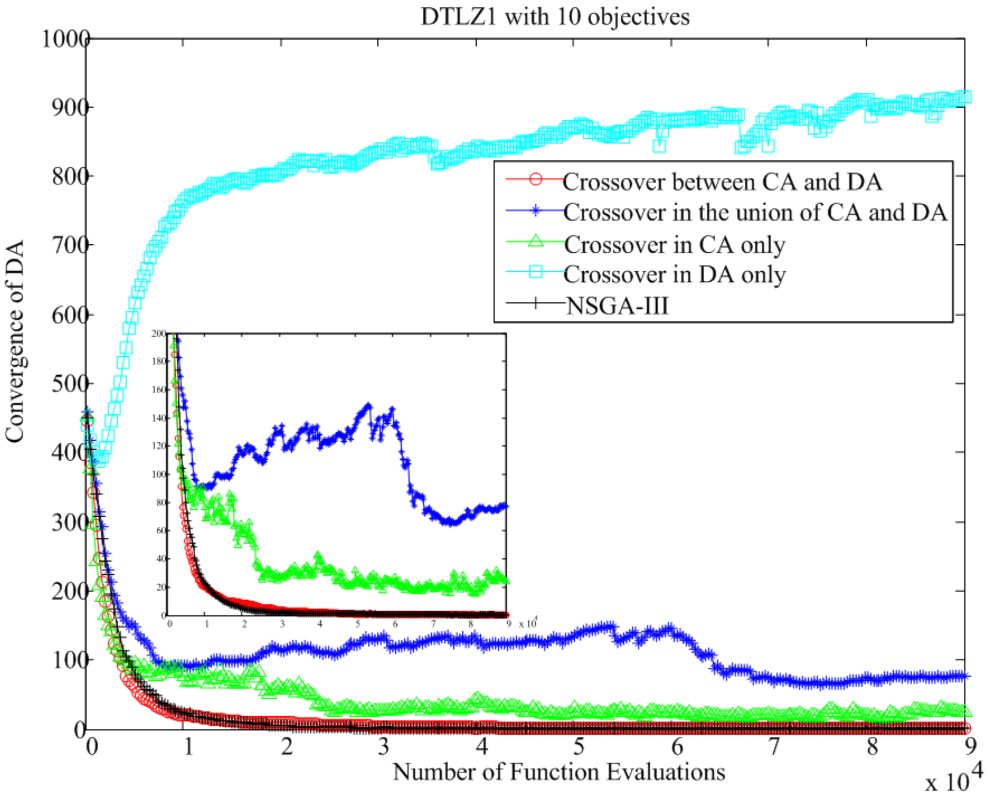
Interaction between CA and DA: Mutation

- Mutation to DA does not speed up convergence, and disturbs the guidance of CA to DA.
- Mutation is applied to CA only in Two_Arch2.



CA leads convergence

Interaction between CA and DA: Crossover

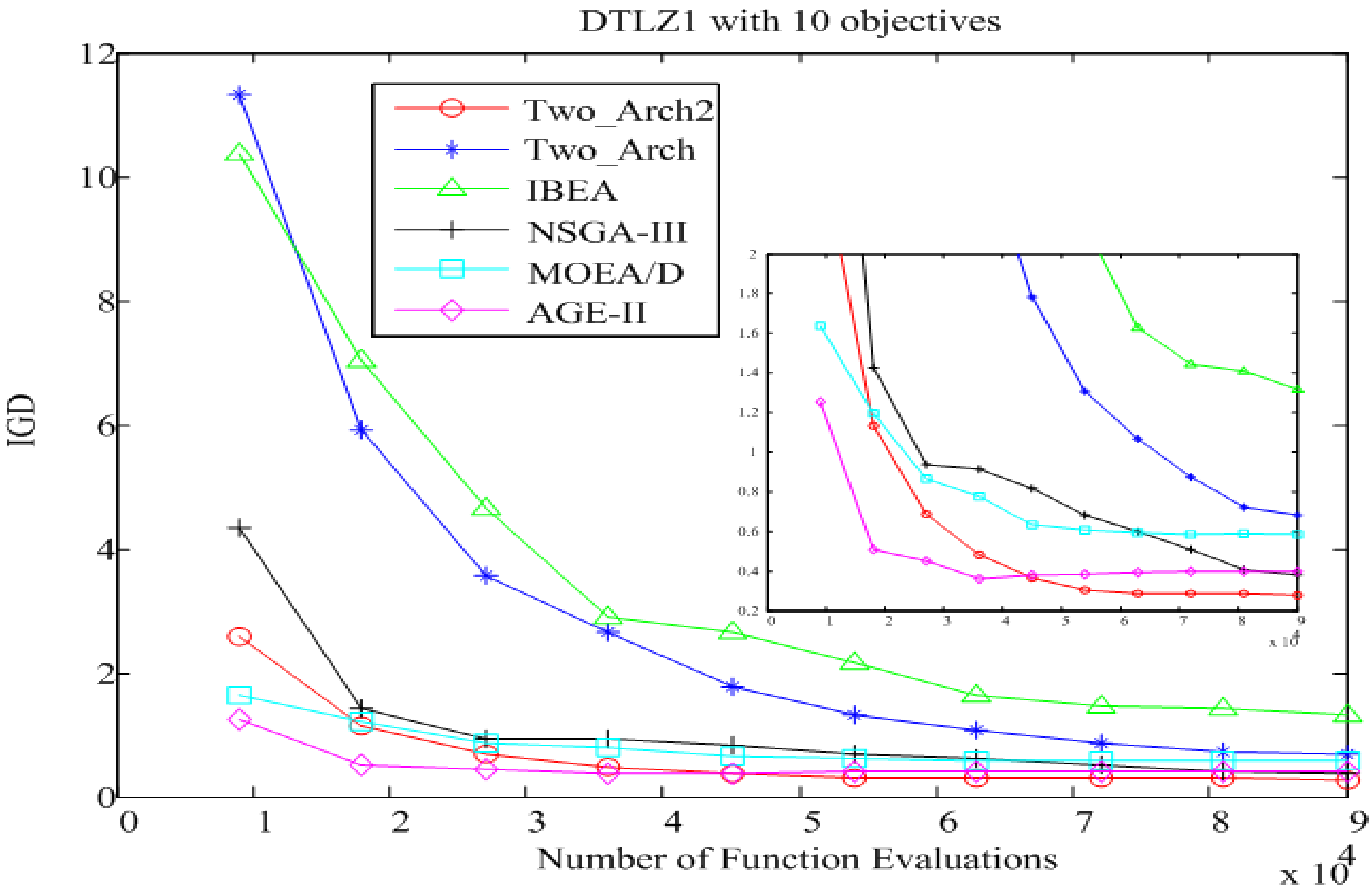


- The crossover between CA and DA has the fastest convergence speed.
- The crossover between CA and DA is employed in Two_Arch2.

Experimental Comparisons

- **Two_Arch2: Developed here**
- **Two_Arch: a reference to show the improvement of Two_Arch2 on MaOPs**
- **IBEA: indicator-based ($I_{\varepsilon+}$) MOEA with good convergence but poor diversity**
- **NSGA-III: newly-proposed MOEA with reference points for MaOPs**
- **MOEA/D: aggregation function-based MOEA**
- **AEG-II: Pareto-based MOEA with the ε-grid approximation in the objective space**

DTLZ1 with 10 Objectives



More Problems, More Objectives

- **More experimental results are in**
  - **H. Wang, L. Jiao and X. Yao, "Two_Arch2: An Improved Two-Archive Algorithm for Many-Objective Optimization,"** ***IEEE Transactions on Evolutionary Computation*****, 19(4):524-541, August 2015.**

Including Matlab code.

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Conclusions

- **There are three major approaches to dealing with a large number of objectives:**
  1. **Objective reduction**
  2. **Alternative dominance relationship**
  3. **New algorithms**

- **This talk touches on only a tiny proportion of all the work. For more comprehensive review:**
  - **B. Li, J. Li, K. Tang and X. Yao, "Many-Objective Evolutionary Algorithms: A Survey,"** ***ACM Computing Surveys*****, 48(1), Article 13, 35 pages, September 2015.**

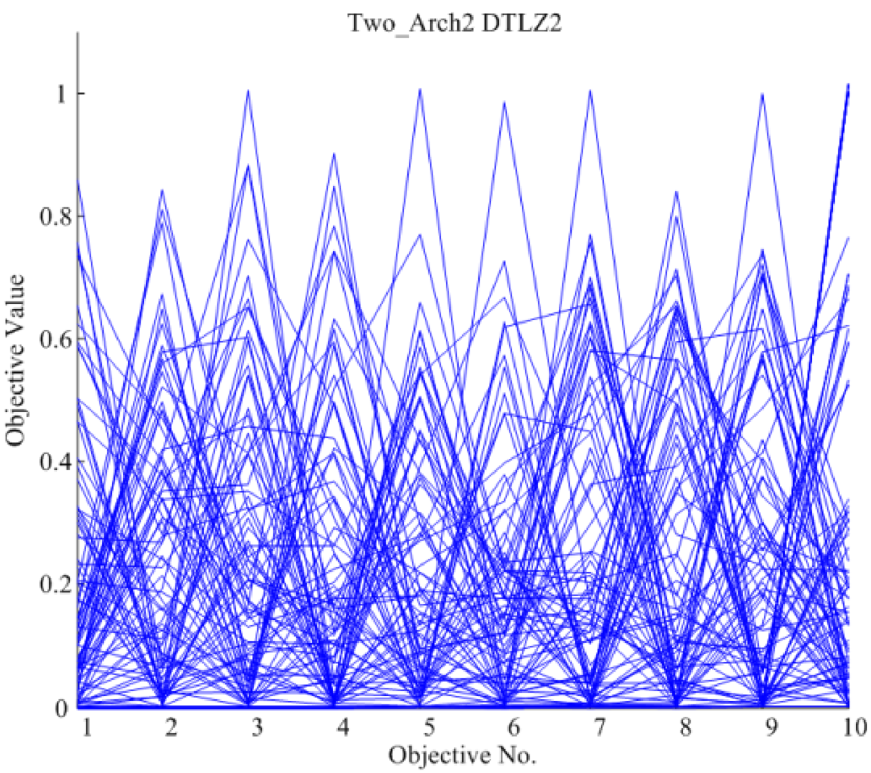
Future Work

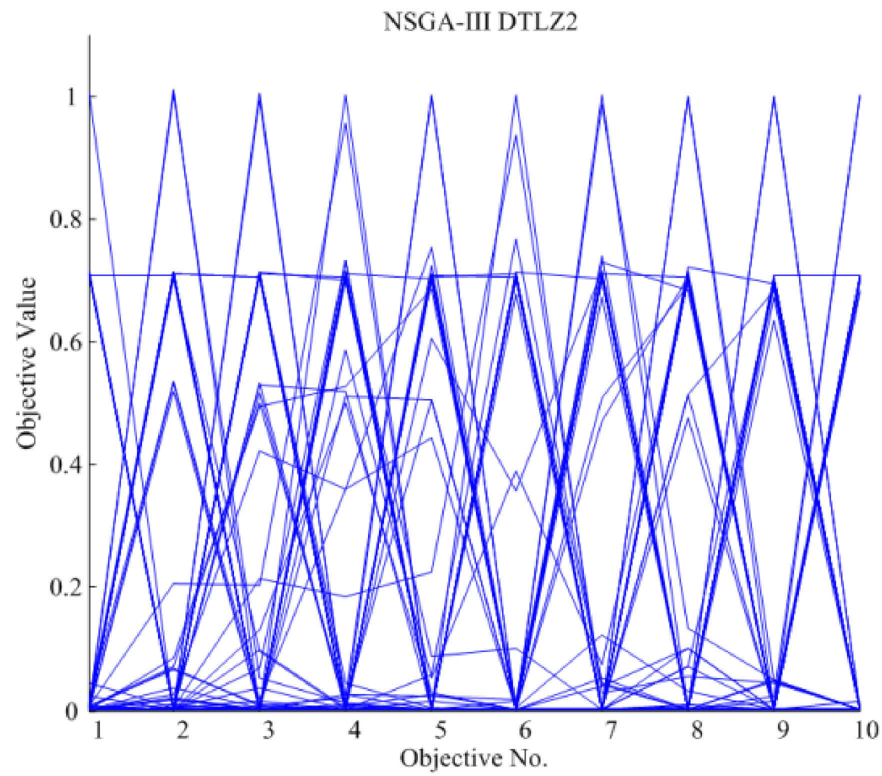
1. **Dynamic number of objectives, e.g.,**
   - R. Chen, K. Li and X. Yao, "Dynamic Multiobjectives Optimization With a Changing Number of Objectives," *IEEE Transactions on Evolutionary Computation*, vol. 22, no. 1, pp. 157-171, Feb. 2018.

2. **Constraint handling, e.g.,**
   - K. Li, R. Chen, G. Fu and X. Yao, "Two-Archive Evolutionary Algorithm for Constrained Multi-Objective Optimization," *IEEE Transactions on Evolutionary Computation*, online on 19/7/2018.
     DOI: 10.1109/TEVC.2018.2855411

Two_Arch2 vs. NSGA-III on DTLZ2 with 10 Objectives

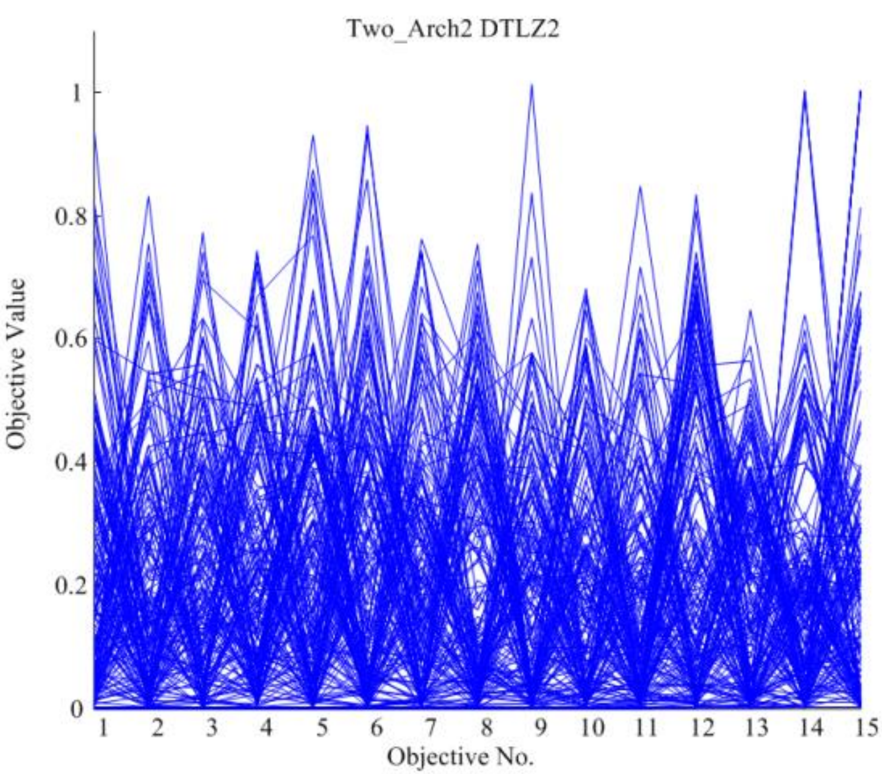
	Convergence	Diversity	Extreme point
Two_Arch2	Good	Good	Fair
NSGA-III	Good	Fair	Good

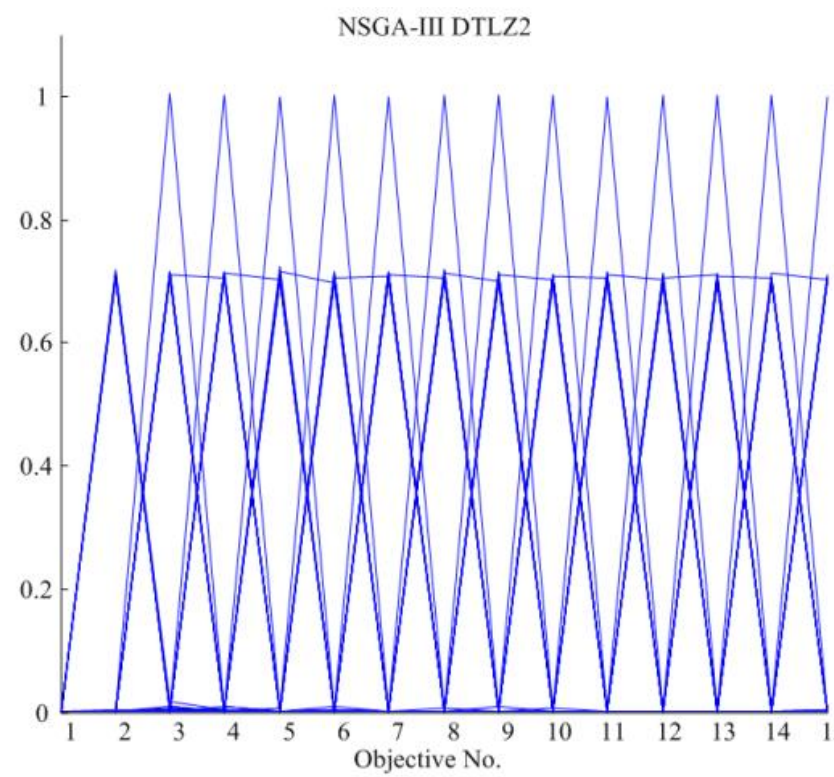




Two_Arch2 vs. NSGA-III on DTLZ2 with 15 Objectives

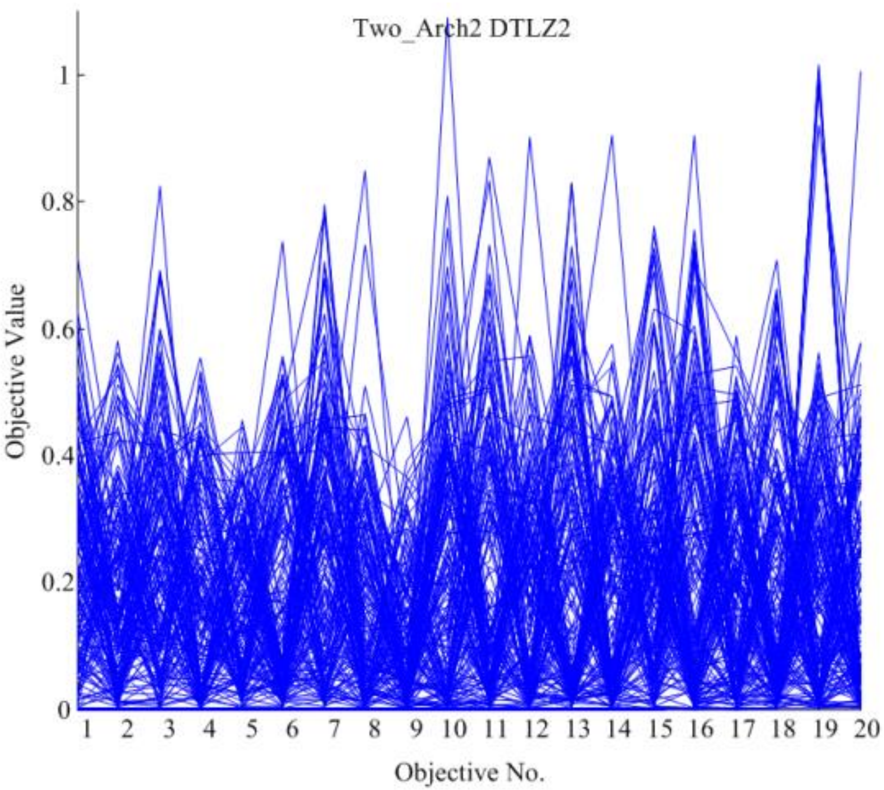
	Convergence	Diversity	Extreme point
Two_Arch2	Good	Good	Poor
NSGA-III	Good	Fair	Good

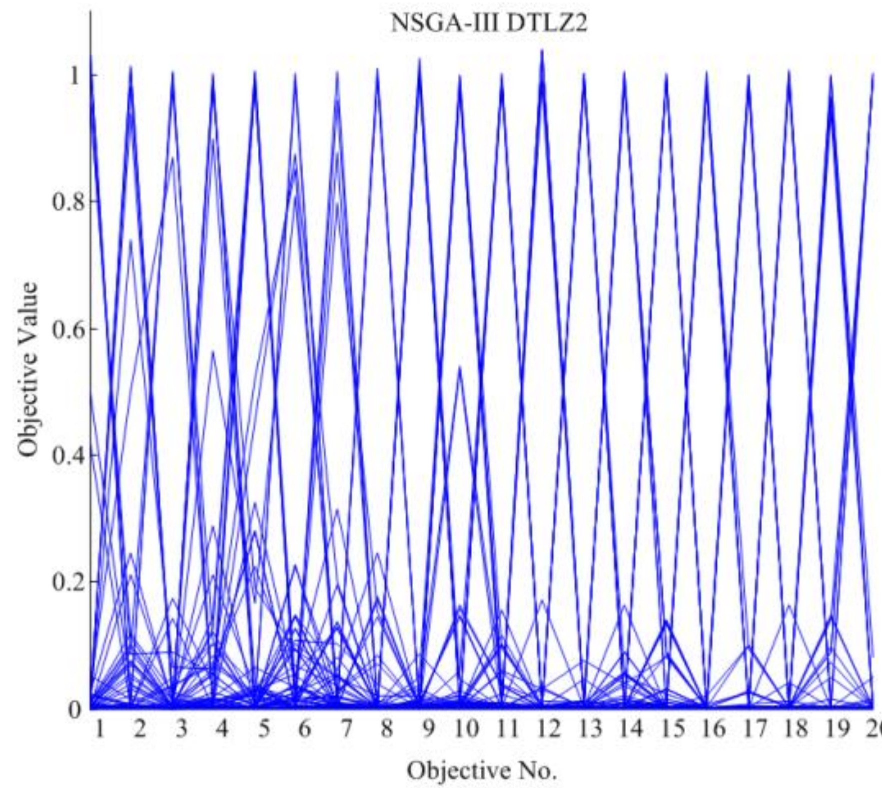




Two_Arch2 vs. NSGA-III on DTLZ2 with 20 Objectives

	Convergence	Diversity	Extreme point
Two_Arch2	Good	Good	Poor
NSGA-III	Good	Fair	Good





Two_Arch2 vs. NSGA-III

	Two_Arch2	NSGA-III
Convergence methodology	$I_{\varepsilon+}$	Pareto dominance
Convergence degeneration	No	No
Diversity maintenance	$L_{1/m}$-norm-based distance	Minimal perpendicular distances to reference points
Diversity degeneration	No	Increase with the dimension of objective space
Manual Settings	None	Reference points