Equations

2024-05-15

This Markdown is used exclusively for any equations required for the dissertation writing

## 1D

Moffat Model:

Where A = amplitude, = position of the maximum, = core width, = power index.

Moffat Distribution:

To obtain the Negative Log Likelihood. First I take the Likelihood of the parameters of f(x).

Next I take the log of the likelihood and apply the log law .

Taking the negative of this yields the Negative Log Likelihood. Let NLL = Negative Log Likelihood

By applying more log laws, this can be expanded. Let b = Let c = Let d =

. . .

c can be split into two parts Let k = Let l =

Substituting the parameters back in.

In this form it is easier to find the partial derivatives.

In order to take the partial derivative with respect to I utilised the chain rule.

Let u =

Let v =

u =

NLL =

Resulting in

To take the partial derivative with respect to it is slightly more complex, as exists twice within the Negative Log Likelihood. However, we can find the derivative to the two parts seperately.

Let u =

Taking out The partial derivative with respect to here is .

For u =

Combined this becomes

To obtain the partial derivative with respect to beta, make use of the digamma function in R. = digamma()

## 2D

The 2D Moffat Model has this form:

Where A = amplitude, = x-position of the maximum, = y-position of the maximum, = core width, = power index.

This is a symmetric uncorrelated function.

Similarly to the 1D Moffat, the amplitude must be greater than 0. Alpha can’t be equal to zero and Beta must be greater than 0.

This two must be converted to the moffat distribution:

Moffat Distribution:

Gradient of the negative log likelihood with respect to each parameter

## Spatial Mixture Models

Creating a mixture model that uses two 2D moffat functions.

Where is the mixing proportion parameter and are the component densities. Which in the case of a Moffat density looks like this.

The mixture model in full:

Let = Let =

The Negative Log Likelihood:

To find the partial derivatives of the Negative Log Likelihood Throughout the derivations the chain rule, quotient rule and product rule are utilised Chain Rule: Quotient Rule: Product Rule:

Let:

For :

Simplifying to:

A similar method is used to find the partial derivative with respect to , and .

For the partial derivative with respect to .

Similarly for

For the partial derivative with respect to Let Let So Apply the product rule.

To find d’ the quotient and chain rules are applied

This can be simplified:

Leading to:

This can be simplified:

Resulting in:

Similarly for

For the partial derivative with respect to . Rearrange the initial equation and Let

The Product rule is employed to find the derivative of the first term.

Similarly for