Equations

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This Markdown is used exclusively for any equations required for the dissertation writing

## 1D

Moffat Model:

Where A = amplitude, = position of the maximum, = core width, = power index.

Moffat Distribution:

To obtain the Negative Log Likelihood. First I take the Likelihood of the parameters of f(x).

Next I take the log of the likelihood and apply the log law .

Taking the negative of this yields the Negative Log Likelihood. Let NLL = Negative Log Likelihood

By applying more log laws, this can be expanded

In order to take the partial derivative with respect to I utilised the chain rule. Let $ u = $. Let $ v = x - $ $ = -(-) $

Resulting in

A similar approach was used to take the partial derivative with respect to and . Resulting in

To obtain the partial derivative with respect to beta I will be using the Rcode of digamma

## 2D

The 2D Moffat Model has this form:

Where A = amplitude, = x-position of the maximum, = y-position of the maximum, = core width, = power index.

This is a symmetric uncorrelated function.

Similarly to the 1D Moffat, the amplitude must be greater than 0. Alpha can’t be equal to zero and Beta must be greater than 0.

Moffat Distribution:

Gradient of the negative log likelihood with respect to each parameter

## Spatial Mixture Models

Creating a mixture model that uses two 2D moffat functions.

Where is the mixing proportion parameter and are the component densities. Which in the case of a Moffat density looks like this.

The mixture model in full:

Let $ v\_i = 1 + $ Let $ c\_i = $

The Negative Log Likelihood:

To find the partial derivatives of the Negative Log Likelihood Throughout the derivations the chain rule, quotient rule and product rule are utilised Chain Rule: $ = $ Quotient Rule: $ = $ Product Rule:

Let:

For :

Simplifying to:

A similar method is used to find the partial derivative with respect to , and .

For the partial derivative with respect to .

Similarly for

For the partial derivative with respect to Let Let So Apply the product rule.

To find d’ the quotient and chain rules are applied

This can be simplified:

Leading to:

This can be simplified:

Resulting in:

Similarly for

For the partial derivative with respect to . Rearrange the initial equation and Let

The Product rule is employed to find the derivative of the first term.

Similarly for