Autonomous Underwater Vehicle: A Surveillance Protocol

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Preface

This project is an attempt at combining all of my major skills into creating a simulation, imaging, perception and control pipeline for Autonomous Underwater Vehicles (AUV). As such, creating this project involves creating a number of pipelines.

The first pipeline is the signal simulation pipeline. The signal simulation pipeline involves sea-floor point-cloud creation and simulating the signals received by the sensor arrays of the AUV. The signals recorded by the sensor-arrays on the AUV contains information from the surrounding environment. The imaging pipeline performs certain operations on the recorded signals to obtain acoustic images of the surrounding environment. To that end, this pipeline involves the topics of signal processing, linear algebra, signals and systems.

As such, the second pipeline is the imaging pipeline. The inputs to the imaging pipeline is the signals recorded by the different sensor-arrays of the AUV, in addition to the parameters of the AUV and its components. This pipeline involves match-filtering, focusing and beamforming operations to create acoustic images of the surrounding environment. Depending on the number of ULAs present, the imaging pipeline is responsible for creating multiple acoustic images in real-time. Thus, this pipeline involves the topics of Digital Signal Processing, Match-Filtering, Estimation and Detection Theory and so on.

The images created by the imaging pipeline are fed to the perception-to-control pipeline. This pipeline takes in the image formed created from the ULA signals, parameters of AUV and its components, and some historical data, it provides instructions regarding the movement of the AUV. The mapping from the inputs to the controls is called policy. Learning policies is a core part of reinforcement learning. Thus, this pipeline mainly involves the topics of reinforcement learning. And since we'll be using convolutional neural nets and transformers for learning the policies, this pipeline involves a significant amount of machine and deep learning.

The final result is an AUV that is primarily trained to map an area of the sea-floor in a constant surveillance mode. The RL-trained policy will also be trained to deal with different kinds of sea-floor terrains: those containing hills, valleys, and path-obstructing features. Due to the resource constrained nature of the marine vessel, we also prioritize efficient policies in the policy-training pipeline.

The project is currently written in C++. And since there is non-trivial amount of training and adaptive features in the pipelines, we'll be using LibTorch (the C++ API of PyTorch) to enable computation graphs, backpropagation and thereby, learning in our AUV pipeline. However, for the sections where a computation graph is not required we will be writing templated STL code.

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Part I AUV Components & Setup

Chapter 1

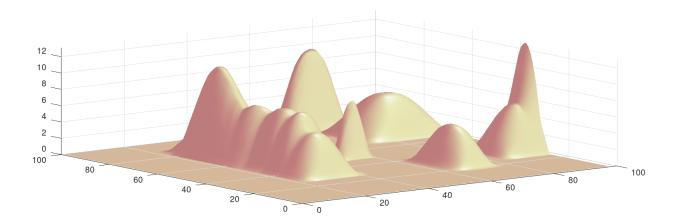
Underwater Environment

Overview

All physical matter in this framework is represented using point-clouds. Thus, the sea-floor also is represented using a number of 3D points. In addition to the coordinates, the points also have the additional property of "reflectivity". It is the impulse response of that point.

Sea-floors in real-life are rarely flat. They often contain valleys, mountains, hills and much richer geographical features. Thus, training an agent to function in such environments call for the creation of similar structures in our simulations.

To simplify things, we shall take a more constrained and structured approach. We start by creating different classes of structures and produce instantiations of those structures on the sea-floor. These structures are defined in such a way that the shape and size can be parameterized to enable creation of random sea-floors.



1.1 Underwater Hills

The most basic approach to creating this is to create a flat seafloor, where all the points have the same height. While this is a good place to start, it is good to bring in some realism to the seafloor. To that end, we shall have some rolling hills as the sea-floor. Each "hill"

is created using the method outlined in Algorithm 1. The method involves deciding the location of the hills, the dimension of the hills and then designing a hill by combining an exponential function and a cosine function. We're aiming to essentially produce gaussian-looking sea-floor hills. After the creation, this becomes the set of points representing the lowest set of points in the overall seafloor structure.

Algorithm 1 Hill Creation

```
1: Input: Mean vector m, Dimension vector d, 2D points P
  2: Output: Updated P with hill heights
  3: num\_hills \leftarrow numel(\mathbf{m}_x)
  4: H \leftarrow \text{Zeros tensor of size } (1, \text{numel}(\mathbf{P}_x))
 5: for i=1 to num_hills do
6: x_{\text{norm}} \leftarrow \frac{\frac{\pi}{2}(\mathbf{P}_x - \mathbf{m}_x[i])}{\mathbf{d}_x[i]}
            y_{\text{norm}} \leftarrow \frac{\frac{\pi}{2}(\mathbf{P}_y - \mathbf{m}_y[i])}{\mathbf{d}_y[i]}
  7:
           h_x \leftarrow \cos(x_{\text{norm}}) \cdot e^{\frac{|x_{\text{norm}}|}{10}}
           h_y \leftarrow \cos(y_{\text{norm}}) \cdot e^{\frac{|y_{\text{norm}}|}{10}}
  9:
            h \leftarrow \mathbf{d}_z[i] \cdot h_x \cdot h_y
10:
            Apply boundary conditions:
11:
            if x_{\text{norm}} > \frac{\pi}{2} or x_{\text{norm}} < -\frac{\pi}{2} or y_{\text{norm}} > \frac{\pi}{2} or y_{\text{norm}} < -\frac{\pi}{2} then
12:
13:
            end if
14:
            H \leftarrow H + h
15:
16: end for
17: \mathbf{P} \leftarrow \text{concatenate}([\mathbf{P}, H])
```

1.2 Scatterer Definition

The sea-floor is represented by a single object of the class ScattererClass.

```
Class Declaration
  template <typename T>
  class ScattererClass
  public:
      // members
      std::vector<std::vector<T>> coordinates;
      std::vector<T> reflectivity;
11
      // Constructor
      ScattererClass() {}
14
      // Constructor
      ScattererClass(std::vector<std::vector<T>> coordinates_arg,
                   std::vector<T>
                                   reflectivity_arg):
                    coordinates(std::move(coordinates_arg)),
18
                    reflectivity(std::move(reflectivity_arg)) {}
19
      // Save to CSV
```

```
void save_to_csv();

;
```

1.3 Sea-Floor Setup Script

Following is the function that will setup the sea-floor script.

```
void fSeaFloorSetup(
      ScattererClass<double>& scatterers
   ){
3
      // auto save_files
                            {false};
                                           {false};
      const auto save_files
      const auto
                   hill_creation_flag
                                          {true};
      // sea-floor bounds
      auto
             bed_width
                            {100.00};
      auto
             bed_length
                            {100.00};
11
      // creating tensors for coordinates and reflectivity
      vector<vector<double>>
                              box_coordinates;
14
      vector<double>
                                   box_reflectivity;
16
      // scatter density
      // auto bed_width_density {static_cast<double>( 10.00)};
18
      // auto
                 bed_length_density {static_cast<double>( 10.00)};
19
      auto
             bed_width_density {static_cast<double>( 5.00)};
20
             bed_length_density {static_cast<double>( 5.00)};
      auto
22
      // setting up coordinates
23
                      {svr::linspace<double>(
             xpoints
      auto
         0.00,
          bed_width,
26
          bed_width * bed_width_density
      )};
29
      auto
             ypoints
                        {svr::linspace<double>(
          0.00,
30
          bed_length,
31
          bed_length * bed_length_density
33
      if(save_files) fWriteVector(xpoints, "../csv-files/xpoints.csv");
                                                                           // verified
34
      if(save_files) fWriteVector(ypoints, "../csv-files/ypoints.csv");
                                                                           // verified
35
      // creating mesh
37
      auto [xgrid, ygrid] = meshgrid(std::move(xpoints), std::move(ypoints));
38
      if(save_files) fWriteMatrix(xgrid, "../csv-files/xgrid.csv"); // verified
39
                                                                           // verified
      if(save_files) fWriteMatrix(ygrid, "../csv-files/ygrid.csv");
41
      // reshaping
                     {reshape(xgrid, xgrid.size()*xgrid[0].size())};
      auto X
            Y
                     {reshape(ygrid, ygrid.size()*ygrid[0].size())};
      auto
                                                                           // verified
      if(save_files) fWriteVector(X, "../csv-files/X.csv");
45
                                          "../csv-files/Y.csv");
                                                                           // verified
      if(save_files) fWriteVector(Y,
46
47
      // creating heights of scatterers
      if(hill_creation_flag){
49
```

```
50
          // setting up hill parameters
51
                  num_hills
          auto
52
          // setting up placement of hills
                                                                                  // verified
          auto
                  points2D
                                         {concatenate<0>(X, Y)};
                  min2D
                                         {min<1, double>(points2D)};
                                                                                  // verified
          auto
56
                  max2D
                                         {max<1, double>(points2D)};
                                                                                  // verified
          auto
                 hill_2D_center
                                         \{min2D + \setminus
          auto
                                         rand({2, num_hills}) * (max2D - min2D)}; // verified
60
          // setup: hill-dimensions
61
                                        {transpose(vector<double>{5, 5, 2})}; // verified
                  hill_dimensions_min
          auto
          auto
                  hill_dimensions_max
                                         {transpose(vector<double>{30, 30, 10})}; // verified
63
          auto
                 hill_dimensions
                                         {hill_dimensions_min + \
                                         rand({3, num_hills}) * (hill_dimensions_max -
                                             hill_dimensions_min)};
                                                                                   // verified
66
          // function-call: hill-creation function
67
          fCreateHills(hill_2D_center,
                       hill_dimensions,
                       points2D);
70
          // setting up floor reflectivity
                                                {std::vector<double>(Y.size(), 1.00)};
          auto floorScatter_reflectivity
74
          // populating the values of the incoming argument
75
          scatterers.coordinates = std::move(points2D);
76
          scatterers.reflectivity = std::move(floorScatter_reflectivity);
77
78
       }
79
       else{
80
81
          // assigning flat heights
82
                         {std::vector<double>(Y.size(), 0)};
83
          // setting up floor coordinates
85
          auto
                  floorScatter_coordinates
                                                {concatenate<0>(X, Y, Z)};
86
                  floorScatter_reflectivity
                                                {std::vector<double>(Y.size(), 1)};
          auto
          // populating the values of the incoming argument
89
          scatterers.coordinates = std::move(floorScatter_coordinates);
90
          scatterers.reflectivity = std::move(floorScatter_reflectivity);
91
92
       }
93
94
       // printing status
95
       std::cout << format("> Finished Sea-Floor Setup \n");
96
   }
97
```

Chapter 2

Transmitter

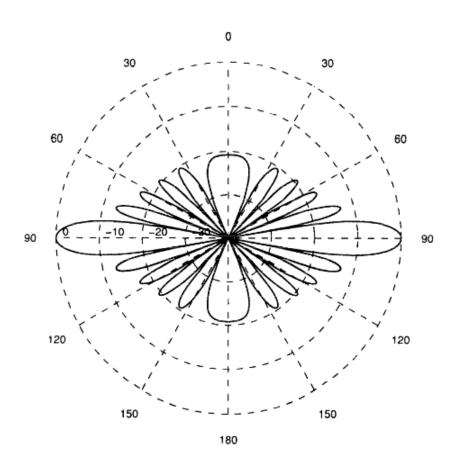


Figure 2.1: Beampattern of a Transmission Uniform Linear Array

Overview

Probing systems are those systems that send out a signal, listen to the reflection and infer qualitative and quantitative qualities of the environment, matter or object, it was trying to infer information about. The transmitter is one of the most fundamental components of probing systems. As the name suggests, the transmitter is the equipment responsible for sending out the probing signal into the medium.

A transmitter is any device or circuit that converts information into a signal and sends it out onto some media like air, cable, water or space. The components of a transmitter are usually as follows

- 1. Input: Information containing signal such as voice, data, video etc
- 2. Process: Encode/modulate the information onto a carrier signal, which can be electromagnetic wave or mechanical wave.
- 3. Transmission: The signal is then transmitted onto the media with electro-mechanical equipment.

Transmitters are of many kinds. But the ones that we will be considering will be directed transmitters, which means that these transmitters have an associated beampattern. To the uninitiated, this means that the power of the transmitted signal is not transmitted in all directions equally. A beampattern is a graphical representation of the power received by an ideal receiver when placed at different angles.

Transmitters made out of a linear-array of individual transmitters use beamforming to "direct" the major power of the transmitter. These kind of systems have well studied beampatterns which can be utilized in our simulations. These kind of studies and inculcating that in our pipelines produce accurate signal simulation pipelines. For now, we stick to a very simple model of a transmitter. We assume that the transmitter sends out the power equally into a particular cone from the AUV position.

2.1 Transmission Signal

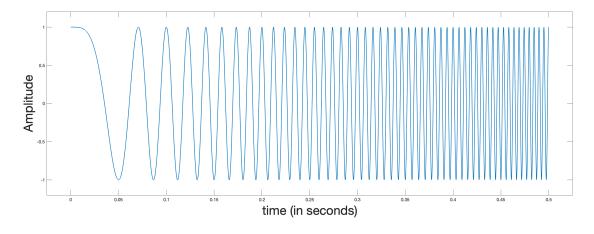


Figure 2.2: Linear Frequency Modulated Wave

The resolution of any probing system is fundamentally tied to the signal bandwidth. A higher bandwidth corresponds to finer resolution $\frac{\text{speed-of-sounds}}{2*\text{bandwidth}}$. Thus, for perfect resolution, an infinite bandwidth is in order. However, infinite bandwidth is impossible for obvious reasons: hardware limitations, spectral regulations, energy limitations and so on.

This is where Linear Frequency Modulation (LFM), also called a "chirp," becomes valuable. An LFM signal linearly sweeps a limited bandwidth over a relatively long duration. This technique spreads the signal's energy in time while retaining the resolution benefits of

the bandwidth. After matched filtering (or pulse compression), we essentially produce pulses corresponding to a base-band LFM of same bandwidth. Overall, LFM is a practical compromise between finite bandwidth and desired performance.

One of the best parts about the resolution depending only on the bandwidth is that it allows us to deploy techniques that would help us improve SNRs without virtually increasing the bandwidth at all. Much of the noise in submarine environments are in and around the baseband region (around frequency, 0). Since resolution depends purely on bandwidth, and LFM can be transmitted at a carrier-frequency, this means that processing the returns after low-pass filtering and basebanding allows us to get rid of the submarine noise, since they do not occupy the same frequency-coefficients. The end-result, thus, is improved SNR compared to use baseband LFM.

Due to all of these advantages, LFM waves are ubiquitous in probing systems, from sonar to radar. Thus, for this project too, the transmitter will be using LFM waves as probing signals, to probe the surrounding submarine environment.

2.2 Transmitter Class Definition

The transmitter is represented by a single object of the class TransmitterClass.

```
template <typename T>
   class TransmitterClass{
   public:
       // A shared pointer to the configuration object
5
       std::shared_ptr<svr::AUVParameters> config_ptr;
6
       // physical/intrinsic properties
8
       std::vector<T>
                         location;
                                               // location tensor
9
                         pointing_direction; // pointing direction
       std::vector<T>
10
11
       // basic parameters
12
       std::vector<T>
                                               // transmitted signal (LFM)
                         signal;
      Т
                         azimuthal_angle;
                                               // transmitter's azimuthal pointing direction
      Т
                         elevation_angle;
                                               // transmitter's elevation pointing direction
      Т
                         azimuthal_beamwidth; // azimuthal beamwidth of transmitter
16
      Т
                         elevation_beamwidth; // elevation beamwidth of transmitter
17
      Τ
                         range;
                                               // a parameter used for spotlight mode.
18
       // transmitted signal attributes
20
      Т
                         f_low;
                                               // lowest frequency of LFM
21
      Т
                                               // highest frequency of LFM
                         f_high;
22
      Т
                                               // center frequency of LFM
23
                                               // bandwidth of LFM
      Т
                         bandwidth;
24
                         speed_of_sound {1500}; // speed of sound
25
       // shadowing properties
27
                         azimuthQuantDensity;
                                                   // quantization of angles along the
       int
28
          azimuth
                                                   // quantization of angles along the
       int
                         elevationQuantDensity;
          elevation
      Т
                                                   // range-cell size when shadowing
                         rangeQuantSize;
30
       Т
                         azimuthShadowThreshold; // azimuth thresholding
31
       Т
                         elevationShadowThreshold; // elevation thresholding
```

```
33
      // shadowing related
                                              // box indicating whether a scatter for a
      std::vector<T>
                         checkbox;
35
          range-angle pair has been found
      std::vector<std::vector<std::vector<T>>> finalScatterBox; // a 3D tensor where the
          third dimension represnets the vector length
      std::vector<T> finalReflectivityBox; // to store the reflectivity
37
       // constructor
      TransmitterClass() = default;
40
41
       // Deleting copy constructors/assignment
42
      TransmitterClass(const TransmitterClass& other)
                                                              = delete;
43
      TransmitterClass& operator=(TransmitterClass& other) = delete;
45
       // Creating move-constructor and move-assignment
      TransmitterClass(TransmitterClass&& other)
                                                               = default;
      TransmitterClass& operator=(TransmitterClass&& other) = default;
48
49
      // member-functions
50
      auto updatePointingAngle(std::vector<T> AUV_pointing_vector);
51
       auto subset_scatterers(const ScattererClass<T>& seafloor,
```

2.3 Transmitter Setup Scripts

The following script shows the setup-script

```
template
       typename
                  Τ,
                     std::enable_if_t<
       typename
                  =
          std::is_same_v<T, double> ||
          std::is_same_v<T, float>
6
   >
   void fTransmitterSetup(
       TransmitterClass<T>& transmitter_fls,
       TransmitterClass<T>& transmitter_portside,
10
       TransmitterClass<T>& transmitter_starboard
11
   ){
12
       // Setting up transmitter
                                     {160e3};
                                                           // sampling frequency
       Т
              sampling_frequency
14
       Т
                                     {50e3};
                                                           // first frequency of LFM
              f1
       Т
              f2
                                     {70e3};
                                                           // second frequency of LFM
       Τ
                                     \{(f1 + f2)/2.00\};
                                                           // finding center-frequency
17
                                     {std::abs(f2 - f1)}; // bandwidth
       Τ
              bandwidth
18
                                                           // time of recording
19
              pulselength
                                     {5e-2};
       // building LFM
                             {svr::linspace<T>(
       auto
              timearray
          0.00,
          pulselength,
          std::floor(pulselength * sampling_frequency)
       )};
26
                             {f2 - f1/pulselength}; // calculating frequency-slope
       auto
              K
       auto
              Signal
                             {cos(2 * std::numbers::pi * \
                              (f1 + K*timearray) * \
29
```

```
// frequency at each time-step, with f1
                              timearray)};
30
                                  = 0
31
       // Setting up transmitter
32
                                                {std::vector<T>(3, 0)};
       auto
              location
                                                                              // location of
33
          transmitter
              azimuthal_angle_fls
                                                {0};
                                                                              // initial
          pointing direction
              azimuthal_angle_port
                                                {90};
                                                                              // initial
          pointing direction
              azimuthal_angle_starboard
                                                {-90};
                                                                              // initial
36
          pointing direction
37
              elevation_angle
                                                {-60};
                                                                              // initial
38
          pointing direction
30
              azimuthal_beamwidth_fls
                                                {20};
                                                                              // azimuthal
           beamwidth of the signal cone
                                                                              // azimuthal
              azimuthal_beamwidth_port
                                                {20};
41
          beamwidth of the signal cone
       Т
              azimuthal_beamwidth_starboard
                                                {20};
                                                                              // azimuthal
42
          beamwidth of the signal cone
43
                                                                              // elevation
      Т
              elevation_beamwidth_fls
                                                {20};
          beamwidth of the signal cone
              elevation_beamwidth_port
                                                {20};
                                                                              // elevation
45
          beamwidth of the signal cone
              elevation_beamwidth_starboard
                                                                              // elevation
       Τ
                                                {20};
46
          beamwidth of the signal cone
47
              azimuthQuantDensity
                                                {10}; // number of points, a degree is split
       int
48
           into quantization density along azimuth (used for shadowing)
              elevationQuantDensity
                                                {10}; // number of points, a degree is split
          into quantization density along elevation (used for shadowing)
              rangeQuantSize
                                                {10}; // the length of a cell (used for
50
           shadowing)
51
              azimuthShadowThreshold
      Т
                                                {1};
                                                       // azimuth threshold
                                                                                (in degrees)
52
              {\tt elevationShadowThreshold}
                                                {1};
       Т
                                                       // elevation threshold (in degrees)
53
55
       // transmitter-fls
56
       transmitter_fls.location
                                            = location;
                                                                              // Assigning
57
          location
       transmitter_fls.signal
                                            = Signal;
                                                                              // Assigning
58
           signal
       transmitter_fls.azimuthal_angle
                                           = azimuthal_angle_fls;
                                                                             // assigning
           azimuth angle
       transmitter_fls.elevation_angle
                                            = elevation_angle;
                                                                              // assigning
60
           elevation angle
       transmitter_fls.azimuthal_beamwidth = azimuthal_beamwidth_fls;
                                                                              // assigning
61
           azimuth-beamwidth
       transmitter_fls.elevation_beamwidth = elevation_beamwidth_fls;
                                                                              // assigning
62
           elevation-beamwidth
       // updating quantization densities
       transmitter_fls.azimuthQuantDensity
                                               = azimuthQuantDensity;
                                                                           // assigning
64
           azimuth quant density
       transmitter_fls.elevationQuantDensity = elevationQuantDensity;
                                                                           // assigning
65
           elevation quant density
```

```
transmitter_fls.rangeQuantSize
                                              = rangeQuantSize;
                                                                          // assigning
           range-quantization
       transmitter_fls.azimuthShadowThreshold = azimuthShadowThreshold; //
67
           azimuth-threshold in shadowing
       transmitter_fls.elevationShadowThreshold = elevationShadowThreshold; //
           elevation-threshold in shadowing
       // signal related
69
       transmitter_fls.f_low
                                                             // assigning lower frequency
                                              = f1;
70
       transmitter_fls.f_high
                                              = f2;
                                                             // assigning higher frequency
71
72
       transmitter_fls.fc
                                              = fc;
                                                             // assigning center frequency
       transmitter_fls.bandwidth
                                              = bandwidth; // assigning bandwidth
73
74
75
       // transmitter-portside
76
       transmitter_portside.location
                                                 = location;
                                                                               // Assigning
77
           location
                                                  = Signal;
                                                                               // Assigning
       transmitter_portside.signal
           signal
       transmitter_portside.azimuthal_angle
                                                 = azimuthal_angle_port;
                                                                              // assigning
79
           azimuth angle
       transmitter_portside.elevation_angle
                                                 = elevation_angle;
                                                                              // assigning
           elevation angle
       transmitter_portside.azimuthal_beamwidth = azimuthal_beamwidth_port; // assigning
81
           azimuth-beamwidth
       transmitter_portside.elevation_beamwidth = elevation_beamwidth_port; // assigning
           elevation-beamwidth
       // updating quantization densities
83
       transmitter_portside.azimuthQuantDensity = azimuthQuantDensity;
                                                                               // assigning
84
           azimuth quant density
       transmitter_portside.elevationQuantDensity = elevationQuantDensity;
85
                                                                               // assigning
           elevation quant density
       transmitter_portside.rangeQuantSize
                                                  = rangeQuantSize;
                                                                               // assigning
           range-quantization
       transmitter_portside.azimuthShadowThreshold = azimuthShadowThreshold; //
87
           azimuth-threshold in shadowing
       transmitter_portside.elevationShadowThreshold = elevationShadowThreshold; //
           elevation-threshold in shadowing
       // signal related
20
       transmitter_portside.f_low
                                                                                // assigning
                                                  = f1;
           lower frequency
       transmitter_portside.f_high
                                                  = f2;
                                                                               // assigning
91
           higher frequency
       transmitter_portside.fc
                                                  = fc;
                                                                               // assigning
92
           center frequency
       transmitter_portside.bandwidth
                                                  = bandwidth;
                                                                               // assigning
93
           bandwidth
04
95
       // transmitter-starboard
96
       transmitter_starboard.location
                                                                                   //
                                                     = location;
97
           assigning location
       transmitter_starboard.signal
                                                     = Signal;
                                                                                   //
           assigning signal
       transmitter_starboard.azimuthal_angle
                                                     = azimuthal_angle_starboard; //
99
           assigning azimuthal signal
       transmitter_starboard.elevation_angle
                                                      = elevation_angle;
100
       transmitter_starboard.azimuthal_beamwidth
                                                      = azimuthal_beamwidth_starboard;
101
       transmitter_starboard.elevation_beamwidth
                                                      = elevation_beamwidth_starboard;
       // updating quantization densities
```

```
transmitter_starboard.azimuthQuantDensity
                                                       = azimuthQuantDensity;
                                                                                       //
104
            assigning azimuth-quant-density
                                                       = elevationQuantDensity;
       transmitter_starboard.elevationQuantDensity
105
       transmitter_starboard.rangeQuantSize
                                                       = rangeQuantSize;
106
       transmitter_starboard.azimuthShadowThreshold = azimuthShadowThreshold;
107
       transmitter_starboard.elevationShadowThreshold = elevationShadowThreshold;
108
       // signal related
109
       transmitter_starboard.f_low
                                                       = f1;
                                                                                       //
110
           assigning lower frequency
111
       transmitter_starboard.f_high
                                                       = f2;
           assigning higher frequency
       transmitter_starboard.fc
                                                       = fc;
112
           assigning center frequency
       transmitter_starboard.bandwidth
                                                       = bandwidth;
113
            assigning bandwidth
114
    }
```

Chapter 3

Uniform Linear Array

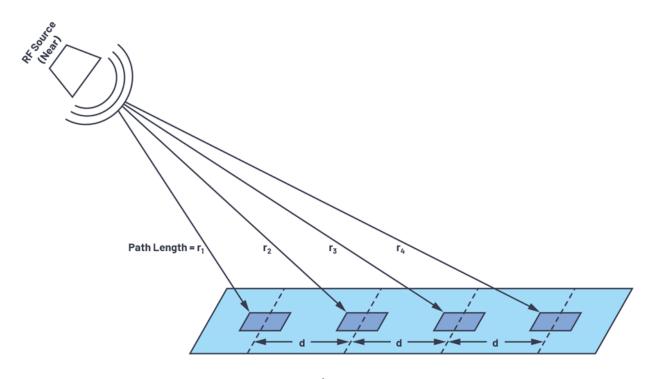


Figure 3.1: Uniform Linear Array

Overview

A Uniform Linear Array (ULA) is a common antenna or sensor configuration in which multiple elements are arranged in a straight line with equal spacing between adjacent elements. This geometry simplifies both the analysis and implementation of array signal processing techniques. In a ULA, each element receives a version of the incoming signal that differs only in phase, depending on the angle of arrival. This phase difference can be exploited to steer the array's beam in a desired direction (beamforming) or to estimate the direction of arrival (DOA) of multiple sources. The equal spacing also leads to a regular phase progression across the elements, which makes the array's response mathematically tractable and allows the use of tools like the discrete Fourier transform (DFT) to analyze spatial frequency content.

The performance of a ULA depends on the number of elements and their spacing. The spacing is typically chosen to be half the wavelength of the signal to avoid spatial aliasing, also called grating lobes, which can introduce ambiguities in DOA estimation. Increasing the number of elements improves the array's angular resolution and directivity, meaning it can better distinguish closely spaced sources and focus energy more narrowly. ULAs are widely used in radar, sonar, wireless communications, and microphone arrays due to their simplicity, predictable behavior, and compatibility with well-established signal processing algorithms. Their linear structure also makes them easier to implement in hardware compared to more complex array geometries like circular or planar arrays.

3.1 ULA Class Definition

The following is the class used to represent the uniform linear array

```
template <typename T>
   class ULAClass
   public:
      // intrinsic parameters
      std::size_t
                                                                            // number of
                                   num_sensors;
          sensors
                                   inter_element_spacing;
                                                                            // space between
          sensors
      std::vector<std::vector<T>> coordinates;
                                                                            // coordinates
          of each sensor
                                    sampling_frequency;
                                                                            // sampling
9
          frequency of the sensors
                                   recording_period;
                                                                            // recording
10
          period of the ULA
      std::vector<T>
                                                                            // location of
                                    location;
11
          first coordinate
      // derived
      std::vector<T>
                                    sensor_direction;
14
      std::vector<std::vector<T>> signal_matrix;
      std::size_t
                                   num_samples;
17
      // decimation related
18
                                                                               // the new
      int
                                    decimation_factor;
19
          decimation factor
                                   post_decimation_sampling_frequency;
                                                                               // the new
20
          sampling frequency
      std::vector<T>
                                   lowpass_filter_coefficients_for_decimation; //
21
          filter-coefficients for filtering
      // imaging related
23
      T range_resolution;
                                       // theoretical range-resolution = $\frac{c}{2B}$
24
      T azimuthal_resolution;
                                       // theoretical azimuth-resolution =
          $\frac{\lambda}{(N-1)*inter-element-distance}$
      T range_cell_size;
                                     // the range-cell quanta we're choosing for
26
          efficiency trade-off
          azimuth_cell_size;
                                      // the azimuth quanta we're choosing
      std::vector<T> azimuth_centers; // tensor containing the azimuth centeres
28
      std::vector<T> range_centers; // tensor containing the range-centers
29
                                       // the frame-size corresponding to a range cell in a
      int frame_size;
          decimated signal matrix
```

```
31
       std::vector<std::vector<complex<T>>> mulfFTMatrix; // the matrix containing the
32
          delays for each-element as a slot
       std::vector<complex<T>>
                                            matchFilter; // torch tensor containing the
33
          match-filter
            num_buffer_zeros_per_frame;
                                                           // number of zeros we're adding
          per frame to ensure no-rotation
       std::vector<std::vector<T>> beamformedImage;
                                                           // the beamformed image
35
       std::vector<std::vector<T>> cartesianImage;
                                                            // the cartesian version of
          beamformed image
37
       // Decimating Related
38
       std::vector<std::complex<T>> basebanding_signal;
39
40
       // Artificial acoustic-image related
41
       std::vector<std::vector<T>> currentArtificialAcousticImage; // acoustic image
           directly produced
43
44
       // Basic Constructor
45
       ULAClass() = default;
46
47
       // constructor
48
       ULAClass(const int
                             num_sensors_arg,
49
               const auto
                             inter_element_spacing_arg,
               const auto& coordinates_arg,
51
               const auto& sampling_frequency_arg,
52
               const auto& recording_period_arg,
               const auto& location_arg,
               const auto& signalMatrix_arg,
55
               const auto& lowpass_filter_coefficients_for_decimation_arg):
56
                  num_sensors(num_sensors_arg),
                  inter_element_spacing(inter_element_spacing_arg),
58
                  coordinates(std::move(coordinates_arg)),
59
                  sampling_frequency(sampling_frequency_arg),
60
                  recording_period(recording_period_arg),
                  location(std::move(location_arg)),
62
                  signal_matrix(std::move(signalMatrix_arg)),
63
                  lowpass_filter_coefficients_for_decimation(std::move(lowpass_filter_coefficients_for_decimation)
66
          // calculating ULA direction
67
          sensor_direction = std::vector<T>{coordinates[1][0] - coordinates[0][0],
68
                                            coordinates[1][1] - coordinates[0][1],
                                            coordinates[1][2] - coordinates[0][2];
70
71
          // normalizing
                  norm_value_temp
                                    {std::norm(std::inner_product(sensor_direction.begin(),
                                                                sensor_direction.end(),
74
                                                                sensor_direction.begin(),
75
76
                                                                0.00));
          // dividing
78
          if (norm_value_temp != 0) {sensor_direction = sensor_direction /
79
              norm_value_temp;}
80
       }
81
82
       // // deleting copy constructor/assignment
```

```
// ULAClass<T>(const ULAClass<T>& other)
                                                                  = delete;
84
       // ULAClass<T>& operator=(const ULAClass<T>& other)
                                                                  = delete;
85
       ULAClass<T>(ULAClass<T>&& other)
                                                               = delete;
86
       ULAClass<T>& operator=(const ULAClass<T>& other)
                                                               = default;
87
88
       // member-functions
              buildCoordinatesBasedOnLocation();
       void
90
              buildCoordinatesBasedOnLocation(const std::vector<T>& new_location);
       void
91
```

3.2 ULA Setup Scripts

The following script shows the setup-script for Uniform Linear Arrays

```
template
       typename
                 Τ,
       typename
                  = std::enable_if_t<
          std::is_same_v<T, double> ||
          std::is_same_v<T, float>
6
   >
   void fULASetup(
8
       ULAClass<T>&
9
                      ula_fls,
       ULAClass<T>&
                      ula_portside,
10
       ULAClass<T>&
                      ula_starboard)
       // setting up ula
              num_sensors
                                        {static_cast<int>(32)};
                                                                          // number of sensors
       auto
14
       Т
              sampling_frequency
                                        {static_cast<T>(160e3)};
                                                                          // sampling frequency
              inter_element_spacing
                                        {1500/(2*sampling_frequency)};
                                                                          // space between
16
           samples
                                                                          // sampling-period
17
              recording_period
                                        {10e-2};
       auto num_samples
                                        {static_cast<std::size_t>(
          std::ceil(
19
              sampling_frequency * recording_period
2.0
          )
22
       )};
23
       // building the direction for the sensors
24
              ULA_direction
                                      {std::vector<T>({-1, 0, 0})};
       auto
       auto
              ULA_direction_norm
                                        {norm(ULA_direction)};
26
                                        {ULA_direction = ULA_direction/ULA_direction_norm;}
       if (ULA_direction_norm != 0)
2.7
                                        ULA_direction * inter_element_spacing;
      ULA_direction
28
       // building coordinates for sensors
30
              ULA_coordinates
31
          transpose(ULA_direction) * \
          svr::linspace<double>(
              0.00,
              num_sensors -1,
35
              num_sensors)
36
       };
38
       // coefficients of decimation filter
39
              lowpassfiltercoefficients {std::vector<T>{0.0000, 0.0000, 0.0000, 0.0000,
40
           0.0000, 0.0000, 0.0001, 0.0003, 0.0006, 0.0015, 0.0030, 0.0057, 0.0100, 0.0163,
           0.0251, 0.0364, 0.0501, 0.0654, 0.0814, 0.0966, 0.1093, 0.1180, 0.1212, 0.1179,
```

```
0.1078, 0.0914, 0.0699, 0.0451, 0.0192, -0.0053, -0.0262, -0.0416, -0.0504,
           -0.0522, -0.0475, -0.0375, -0.0239, -0.0088, 0.0057, 0.0179, 0.0263, 0.0303,
           0.0298, 0.0253, 0.0177, 0.0086, -0.0008, -0.0091, -0.0153, -0.0187, -0.0191,
           -0.0168, -0.0123, -0.0065, -0.0004, 0.0052, 0.0095, 0.0119, 0.0125, 0.0112,
           0.0084, 0.0046, 0.0006, -0.0031, -0.0060, -0.0078, -0.0082, -0.0075, -0.0057,
           -0.0033, -0.0006, 0.0019, 0.0039, 0.0051, 0.0055, 0.0050, 0.0039, 0.0023, 0.0005,
           -0.0012, -0.0025, -0.0034, -0.0036, -0.0034, -0.0026, -0.0016, -0.0004, 0.0007,
           0.0016, 0.0022, 0.0024, 0.0023, 0.0018, 0.0011, 0.0003, -0.0004, -0.0011,
           -0.0015, -0.0016, -0.0015}};
41
      // assigning values
42
      ula_fls.num_sensors
                                                       = num_sensors;
43
      ula_fls.inter_element_spacing
                                                       = inter_element_spacing;
44
      ula_fls.coordinates
                                                       = ULA_coordinates;
45
      ula_fls.sampling_frequency
                                                       = sampling_frequency;
46
      ula_fls.recording_period
                                                       = recording_period;
47
      ula_fls.sensor_direction
                                                       = ULA_direction;
      ula_fls.lowpass_filter_coefficients_for_decimation = lowpassfiltercoefficients;
49
      ula_fls.num_samples
                                                       = num_samples;
50
51
52
      // assigning values
53
      ula_portside.num_sensors
                                                              = num_sensors;
54
      ula_portside.inter_element_spacing
                                                              = inter_element_spacing;
55
      ula_portside.coordinates
                                                              = ULA_coordinates;
      ula_portside.sampling_frequency
                                                              = sampling_frequency;
57
      ula_portside.recording_period
                                                              = recording_period;
58
      ula_portside.sensor_direction
                                                              = ULA_direction;
59
      ula_portside.lowpass_filter_coefficients_for_decimation = lowpassfiltercoefficients;
60
61
      ula_portside.num_samples
                                                               = num_samples;
62
      // assigning values
      ula_starboard.num_sensors
                                                              = num_sensors;
65
      ula_starboard.inter_element_spacing
                                                              = inter_element_spacing;
66
      ula_starboard.coordinates
                                                              = ULA_coordinates;
67
      ula_starboard.sampling_frequency
                                                              = sampling_frequency;
68
      ula_starboard.recording_period
                                                              = recording_period;
      ula_starboard.sensor_direction
                                                              = ULA_direction;
70
      ula_starboard.lowpass_filter_coefficients_for_decimation = lowpassfiltercoefficients;
71
72
      ula_starboard.num_samples
                                                               = num_samples;
73
   }
```

Chapter 4

Autonomous Underwater Vehicle

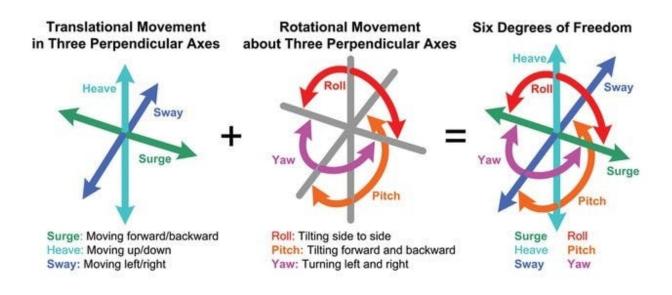


Figure 4.1: AUV degrees of freedom

Overview

Autonomous Underwater Vehicles (AUVs) are robotic systems designed to operate underwater without direct human control. They navigate and perform missions independently using onboard sensors, processors, and preprogrammed instructions. They are widely used in oceanographic research, environmental monitoring, offshore engineering, and military applications. AUVs can vary in size from small, portable vehicles for shallow water surveys to large, torpedo-shaped platforms capable of deep-sea exploration. Their autonomy allows them to access environments that are too dangerous, remote, or impractical for human divers or tethered vehicles.

The navigation and sensing systems of AUVs are critical to their performance. They typically use a combination of inertial measurement units (IMUs), Doppler velocity logs

(DVLs), pressure sensors, magnetometers, and sometimes acoustic positioning systems to estimate their position and orientation underwater. Since GPS signals do not penetrate water, AUVs must rely on these onboard sensors and occasional surfacing for GPS fixes. They are often equipped with sonar systems, cameras, or other scientific instruments to collect data about the seafloor, water column, or underwater structures. Advanced AUVs can also implement adaptive mission planning and obstacle avoidance, enabling them to respond to changes in the environment in real time.

The applications of AUVs are diverse and expanding rapidly. In scientific research, they are used for mapping the seafloor, studying marine life, and monitoring oceanographic parameters such as temperature, salinity, and currents. In the commercial sector, AUVs inspect pipelines, subsea infrastructure, and offshore oil platforms. Military and defense applications include mine countermeasure operations and underwater surveillance. The development of AUVs continues to focus on increasing endurance, improving autonomy, enhancing sensor payloads, and reducing costs, making them a key technology for exploring and understanding the underwater environment efficiently and safely.

4.1 AUV Class Definition

The following is the class used to represent the uniform linear array

```
template <</pre>
      typename T,
      typename = std::enable_if_t<std::is_floating_point_v<T>>
3
4
  class AUVClass{
   public:
      // Intrinsic attributes
8
      std::vector<T> location;
                                             // location of vessel
Q
                                             // velocity of the vessel
      std::vector<T> velocity;
10
      std::vector<T> acceleration;
                                             // acceleration of vessel
11
      std::vector<T> pointing_direction; // AUV's pointing direction
      // uniform linear-arrays
      ULAClass<T> ULA_fls;
                                              // front-looking SONAR ULA
      ULAClass<T>
                        ULA_portside;
                                              // mounted ULA [object of class, ULAClass]
16
      ULAClass<T>
                        ULA_starboard;
                                              // mounted ULA [object of class, ULAClass]
17
      // transmitters
19
      TransmitterClass<T> transmitter_fls;
                                                    // transmitter for front-looking SONAR
2.0
      TransmitterClass<T> transmitter_portside;
                                                    // portside transmitter
21
      TransmitterClass<T> transmitter_starboard; // starboard transmitter
23
      // derived or dependent attributes
24
25
      std::vector<std::vector<T>> signalMatrix_1;
                                                             // matrix containing the
          signals obtained from ULA_1
      std::vector<std::vector<T>> largeSignalMatrix_1;
                                                             // matrix holding signal of
26
          synthetic aperture
      std::vector<std::vector<T>> beamformedLargeSignalMatrix; // each column is the
          beamformed signal at each stop-hop
28
      // plotting mode
29
      bool plottingmode; // to suppress plotting associated with classes
```

```
31
       // spotlight mode related
       std::vector<std::vector<T>> absolute_coords_patch_cart; // cartesian coordinates of
33
          patch
34
       // Synthetic Aperture Related
       std::vector<std::vector<T>> ApertureSensorLocations; // sensor locations of
36
          aperture
       // functions
38
       void syncComponentAttributes();
39
       void init(
40
          svr::ThreadPool&
                                                                          thread_pool,
41
          svr::FFTPlanUniformPoolHandle<T,</pre>
                                                       std::complex<T>>& fph_match_filter,
42
          svr::IFFTPlanUniformPoolHandle<std::complex<T>, T>&
                                                                          ifph_match_filter);
43
       void simulate_signal(
                    ScattererClass<T>&
          const
                                                               seafloor,
          svr::ThreadPool&
                                                               thread_pool,
46
          svr::FFTPlanUniformPoolHandle<T, std::complex<T>>& fft_pool_handle,
47
          svr::IFFTPlanUniformPoolHandle<std::complex<T>, T>& ifft_pool_handle);
48
       void subset_scatterers(
          const ScattererClass<T>&
                                        seafloor,
50
          svr::ThreadPool&
                                        thread_pool,
51
                                        fls_scatterer_indices,
          std::vector<std::size_t>&
```

4.2 AUV Setup Scripts

The following script shows the setup-script for Uniform Linear Arrays

```
template <</pre>
      typename
                 Τ,
      typename
                 =
                     std::enable_if_t<
          std::is_same_v<T, double> ||
          std::is_same_v<T, float>
   >
   void fAUVSetup(AUVClass<T>& auv) {
8
      // building properties for the auv
             location
                                {std::vector<T>{0, 50, 30}}; // starting location
11
      auto
      auto
              velocity
                                {std::vector<T>{5, 0, 0}}; // starting velocity
              pointing_direction {std::vector<T>{1, 0, 0}}; // pointing direction
      auto
      // assigning
      auv.location
                            = std::move(location);
                                                             // assigning location
16
                            = std::move(velocity);
17
      auv.velocity
                                                             // assigning velocity
      auv.pointing_direction = std::move(pointing_direction); // assigning pointing
          direction
19
      // signaling end
20
      std::cout << format("> Completed AUV-setup\n");
21
22
   }
23
```

Part II Signal Simulation Pipeline

Chapter 5

Signal Simulation

Overview

The signal simulation pipeline is the pipeline responsible for simulating/modeling the signals sampled by the ULA-sensors under a real sub-marine environment. This chapter, and the subsequent ones, deal with the assumptions, mathematics, physics and code that goes into the design and creation of the pipeline.

A disclaimer that goes without saying is that signal-simulation is a world of its own. There's a reason that comsol, flexcompute and other numerical-simulation based companies exist. To write a signal simulation, from scratch, while these entities exist, and to make any case that this competes with those, would be flirting with delusion.

To that end, we don't write general-purpose signal simulation pipeline. However, the effort in the signal-simulator direction is purely for application-specific reasons. This is something I can talk about. One of the major in-house signal simulation pipelines yours truly developed at Naval Physical and Oceanographic Laboratory did just that. The aim of that pipeline was not to re-invent the wheel. But to create one that existed at the right speed-fidelity trade-off that the institute operated in. The pipeline created during my time there had several toggles corresponding to the different information to consider during simulation. The more information pertaining to the environment, is involved, the higher the compute and time required. Thus, mid-to-high fidelity pipelines often involve writing well-tuned GPU-supported C++ (think, CUDA). And this is important when you have pipelines downstream whose outputs depend on the signal accuracy, and by association, signal simulator fidelity.

To that end, understanding what this pipeline is not, is perhaps just as important as what it is. The core priority of this signal simulator pipeline is to produce signals for navigation. Navigation does not require high-accuracy signals owing to the very simple fact that decisions made from high-accuracy signals and low-accuracy signals tend to be the same as long as environment-topology information is not lost. To grossly oversimplify what I mean by that, the outcome of your driving does't change whether you have high-definition LI-DAR mapping the surrounding environment to the millimeter level or if you're just driving with your eyes. Thus fidelity of simulator is not a priority and I will not be putting in the kind of effort I put in at NPOL, for this reason (also because I don't want OPSEC to be

mad).

To put it simply, the signal simulation pipeline is quite trivial as far as signal simulators are concerned. But it'll work perfectly for our purposes. And thus, we'll be choosing the simplest of systems and one I like to call, "the EE engineer's best friend": the infamous Linear Time Invariant systems.

Part III Imaging Pipeline

Part IV Perception & Control Pipeline

Appendix A

Application Specific Tools

A.1 CSV File-Writes

```
#pragma once
  /*-----
  writing the contents of a vector a csv-file
  template <typename T>
                                        inputvector,
  void fWriteVector(const vector<T>&
                  const string&
                                            filename){
      // opening a file
      std::ofstream fileobj(filename);
      if (!fileobj) {return;}
11
      // writing the real parts in the first column and the imaginary parts int he second
      if constexpr(std::is_same_v<T, std::complex<double>> ||
14
                 std::is_same_v<T, std::complex<float>> ||
                 std::is_same_v<T, std::complex<long double>>){
          for(int i = 0; i<inputvector.size(); ++i){</pre>
17
             // adding entry
             fileobj << inputvector[i].real() << "+" << inputvector[i].imag() << "i";</pre>
             // adding delimiter
             if(i!=inputvector.size()-1) {fileobj << ",";}</pre>
                                      {fileobj << "\n";}
             else
      }
      else{
          for(int i = 0; i<inputvector.size(); ++i){</pre>
             fileobj << inputvector[i];</pre>
             if(i!=inputvector.size()-1) {fileobj << ",";}</pre>
29
                                      {fileobj << "\n";}
30
          }
      // return
34
35
      return;
```

```
writing the contents of a matrix to a csv-file
   template <typename T>
40
   auto fWriteMatrix(const std::vector<std::vector<T>> inputMatrix,
41
                  const string
                                                filename){
42
      // opening a file
44
      std::ofstream fileobj(filename);
      // writing
      if (fileobj){
48
          for(int i = 0; i<inputMatrix.size(); ++i){</pre>
49
             for(int j = 0; j<inputMatrix[0].size(); ++j){</pre>
                 fileobj << inputMatrix[i][j];</pre>
51
                 if (j!=inputMatrix[0].size()-1) {fileobj << ",";}</pre>
                                                {fileobj << "\n";}
                 else
             }
          }
55
      }
56
      else{
          cout << format("File-write to {} failed\n", filename);</pre>
59
60
  }
61
   /*-----
   writing complex-matrix to a csv-file
63
64
  template <>
   auto fWriteMatrix(const std::vector<std::vector<std::complex<double>>> inputMatrix,
67
                  const string
68
      // opening a file
69
      std::ofstream fileobj(filename);
70
71
      // writing
72
      if (fileobj){
          for(int i = 0; i<inputMatrix.size(); ++i){</pre>
             for(int j = 0; j<inputMatrix[0].size(); ++j){</pre>
                 if (j!=inputMatrix[0].size()-1) {fileobj << ",";}</pre>
                                                {fileobj << "\n";}
                 else
78
             }
79
          }
      }
81
      else{
82
          cout << format("File-write to {} failed\n", filename);</pre>
83
84
   }
85
```

A.2 Thread-Pool

```
#pragma once
namespace svr {
class ThreadPool {
public:
```

```
// Members
        boost::asio::thread_pool
                                thread_pool;
                                            // the pool
        std::vector<std::future<void>> future_vector; // futures to wait on
        // Special-Members
        ThreadPool(std::size_t num_threads) : thread_pool(num_threads) {}
        ThreadPool(const ThreadPool& other)
                                        = delete;
11
        ThreadPool& operator=(ThreadPool& other) = delete;
        // Member-functions
        void
                             converge();
        template <typename F> void push_back(F&& func);
                             shutdown();
18
     private:
19
        template<typename F>
20
        std::future<void> _wrap_task(F&& func) {
21
           std::promise<void> p;
22
23
           auto f = p.get_future();
           boost::asio::post(thread_pool,
              [func = std::forward<F>(func), p = std::move(p)]() mutable {
                 func();
                 p.set_value();
              });
30
           return f;
31
        }
     };
33
34
     /*-----
35
     Member-Function: Add new task to the pool
36
     -----*/
37
     template <typename F>
38
     void ThreadPool::push_back(F&& func)
30
     {
        future_vector.push_back(_wrap_task(std::forward<F>(func)));
41
42
     /*-----
     Member-Function: waiting until all the assigned work is done
45
     void ThreadPool::converge()
46
47
        for (auto &fut : future_vector) fut.get();
        future_vector.clear();
49
50
     /*----
     Member-Function: Shutting things down
     void ThreadPool::shutdown()
54
55
        thread_pool.join();
57
58
  }
```

A.3 FFTPlanClass

```
#pragma once
  namespace svr
     template <typename T>
     concept FFTPlanClassSourceDestinationType = \
6
        std::is_floating_point_v<T> ||
        (
           std::is_class_v<T> &&
           std::is_floating_point_v<typename T::value_type>
        );
11
     template <
        FFTPlanClassSourceDestinationType sourceType,
        FFTPlanClassSourceDestinationType destinationType
     class FFTPlanClass
16
     {
17
        public:
18
19
           // Members
20
           std::size_t nfft_
                              {std::numeric_limits<std::size_t>::max()};
           fftw_complex* in_
                               {nullptr};
22
           fftw_complex* out_
                              {nullptr};
23
           fftw_plan
                     plan_
                               {nullptr};
           /*-----
           Destructor
           -----*/
           ~FFTPlanClass()
30
              if(plan_ != nullptr) {fftw_destroy_plan( plan_);}
31
              if(in_ != nullptr) {fftw_free(
                                                 in_);}
              if(out_ != nullptr) {fftw_free(
                                                 out_);}
34
           35
           Default Constructor
           -----*/
           FFTPlanClass() = default;
38
           /*-----
30
           Constructor
41
           FFTPlanClass(const std::size_t nfft)
           {
              // allocating nfft
              this->nfft_ = nfft;
46
47
              // allocating input-region
              in_ = reinterpret_cast<fftw_complex*>(
49
                fftw_malloc(nfft_ * sizeof(fftw_complex))
50
              );
              out_ = reinterpret_cast<fftw_complex*>(
                fftw_malloc(nfft_ * sizeof(fftw_complex))
53
              );
54
              if(!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
                 CLASS: FFTPlanClass | REPORT: in-out allocation failed");}
56
              // creating plan
57
              plan_ = fftw_plan_dft_1d(
```

```
static_cast<int>(nfft_),
59
60
                    in_,
                    out_,
61
                    FFTW_FORWARD,
62
                    FFTW_MEASURE
                 if(!plan_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
65
                    CLASS: FFTPlanClass | REPORT: plan-creation failed");}
             }
67
              /*-----
             Copy Constructor
68
69
             FFTPlanClass(const FFTPlanClass& other)
70
71
                 // copying nfft
                 nfft_ = other.nfft_;
                 cout << format("\t\t FFTPlanClass(const FFTPlanClass& other) | nfft_ =</pre>
                    {}\n", nfft_);
75
                 // allocating input-region
76
                 in_ = reinterpret_cast<fftw_complex*>(
                    fftw_malloc(nfft_ * sizeof(fftw_complex))
78
                 );
79
                 out_ = reinterpret_cast<fftw_complex*>(
80
                    fftw_malloc(nfft_ * sizeof(fftw_complex))
82
                 if(!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
83
                    CLASS: FFTPlanClass | REPORT: in-out allocation failed");}
85
                 // copying input-region and output-region
                 std::memcpy(in_, other.in_, nfft_ * sizeof(fftw_complex));
86
                 std::memcpy(out_, other.out_, nfft_ * sizeof(fftw_complex));
                 // creating plan
89
                 plan_ = fftw_plan_dft_1d(
90
                    static_cast<int>(nfft_),
91
                    in_,
92
                    out_,
93
                    FFTW_FORWARD,
                    FFTW_MEASURE
                 );
96
                                 {throw std::runtime_error("FILE: FFTPlanClass.hpp |
                 if(!plan_)
97
                    CLASS: FFTPlanClass | REPORT: plan-creation failed");}
             }
98
             /*----
99
             Copy Assignment
100
101
             FFTPlanClass& operator=(const FFTPlanClass& other)
102
103
                 // handling self-assignment
104
                 if (this == &other) {return *this;}
105
                 // cleaning-up existing resources
107
                 if(plan_ != nullptr) {fftw_destroy_plan(
                                                          plan_);}
108
                 if(in_ != nullptr) {fftw_free(
                                                          in_);}
                 if(out_ != nullptr) {fftw_free(
                                                          out_);}
111
                 // allocating input-region and output-region
                 nfft_ = other.nfft_;
```

```
= reinterpret_cast<fftw_complex*>(
                    fftw_malloc(nfft_ * sizeof(fftw_complex))
                 );
                      = reinterpret_cast<fftw_complex*>(
117
                    fftw_malloc(nfft_ * sizeof(fftw_complex))
118
                 if(!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
120
                    CLASS: FFTPlanClass | FUNCTION: Copy-Assignment | REPORT: in-out
                    allocation failed");}
121
                 // copying contents
122
                 cout << format("\t\t FFTPlanClass& operator=(const FFTPlanClass& other) |</pre>
123
                    nfft_ = {}\n", nfft_);
                 std::memcpy(in_, other.in_, nfft_ * sizeof(fftw_complex));
124
                 std::memcpy(out_, other.out_, nfft_ * sizeof(fftw_complex));
126
                 // creating engine
                 plan_ = fftw_plan_dft_1d(
128
                    static_cast<int>(nfft_),
129
                    in_,
130
131
                    out_,
                    FFTW_FORWARD,
                    FFTW_MEASURE
133
                 );
134
                 if(!plan_) {throw std::runtime_error("FILE: FFTPlanClass.hpp | CLASS:
                    FFTPlanClass | FUNCTION: Copy-Assignment | REPORT: plan-creation
                    failed");}
136
                 // returning
137
                 return *this;
138
             }
139
             140
             Move Constructor
141
142
             FFTPlanClass(FFTPlanClass&& other)
143
                : nfft_( other.nfft_),
                    in_(
                                    other.in_),
145
                    out_(
                                    other.out_),
146
                                     other.plan_)
                    plan_(
148
                 // resetting the others
149
                 other.nfft_ = 0;
150
                 other.in_ = nullptr;
151
                                = nullptr;
                 other.out_
                                = nullptr;
                 other.plan_
153
154
             /*-----
155
             Move Assignment
156
157
             FFTPlanClass& operator=(FFTPlanClass&& other)
158
159
                 // self-assignment check
                 if (this == &other) {return *this;}
161
162
                 // cleaning up existing resources
                 if(plan_ != nullptr) {fftw_destroy_plan(
                                                          plan_);}
164
                 if(in_ != nullptr) {fftw_free(
                                                          in_);}
165
                 if(out_ != nullptr) {fftw_free(
                                                          out_);}
166
```

```
// Copying-values and changing pointers
168
                                      = other.nfft_;
169
                   cout << format("\t\t FFTPlanClass's MOVE assignment | nfft_ = {}\n",</pre>
170
                       nfft_);
                                         other.in_;
171
                   in
                                      =
                                         other.out_;
172
                   out
                   plan_
                                          other.plan_;
173
174
                   // resetting source-members
                   other.nfft_ = 0;
176
                                   = nullptr;
                   other.in_
177
                   other.out_
                                      = nullptr;
178
                   other.plan_
                                    = nullptr;
179
180
                   // returning
181
                   return *this;
182
               }
183
184
185
               Running fft
186
               std::vector<destinationType>
187
               fft(const std::vector<sourceType>& input_vector)
188
189
                   // throwing an error
190
                   if (input_vector.size() > nfft_){
                       cout << format("input_vector.size() = {}, nfft_ = {}\n",</pre>
192
                                     input_vector.size(),
193
194
                                      nfft_);
                       throw std::runtime_error("FILE: FFTPlanClass.hpp | CLASS: FFTPlanClass
195
                           | FUNCTION: fft() | REPORT: input-vector size is greater than
                           NFFT");
                   }
196
197
198
                   // copying inputs
199
                   for(std::size_t index = 0; index < input_vector.size(); ++index)</pre>
201
                       if constexpr(
202
                           std::is_floating_point_v<sourceType>
203
                       ){
204
                           in_[index][0] = input_vector[index];
205
                           in_{in}[index][1] = 0;
206
                       }
207
                       else if constexpr(
                           std::is_same_v<sourceType, std::complex<float>> ||
209
                           std::is_same_v<sourceType, std::complex<double>>
211
                           in_[index][0] =
                                              input_vector[index].real();
                           in_[index][1] = input_vector[index].imag();
213
                       }
214
                   }
215
216
                   // executing fft
217
                   fftw_execute(plan_);
218
219
                   // copying results to output-vector
220
                   std::vector<destinationType> output_vector(nfft_);
221
                   for(std::size_t index = 0; index < nfft_; ++index){</pre>
222
                       if constexpr(
223
```

```
std::is_same_v<destinationType, std::complex<float>> ||
224
                         std::is_same_v<destinationType, std::complex<double>>
225
                     ){
226
                         output_vector[index] = destinationType(
2.2.7
                            out_[index][0],
228
                            out_[index][1]
                         );
230
                     }
231
                     else if constexpr(
                         std::is_floating_point_v<destinationType>
233
                     ){
234
                         output_vector[index] = static_cast<destinationType>(
235
                            std::sqrt(
236
                                std::pow(out_[index][0], 2) + \
237
                                std::pow(out_[index][1], 2)
238
                             )
239
                         );
                     }
241
                  }
242
243
                  // returning output
                  return std::move(output_vector);
245
              }
246
              /*-----
247
              Running fft - balanced
248
              -----*/
249
              std::vector<destinationType>
250
              fft_12_conserved(const std::vector<sourceType>& input_vector)
251
252
                  // throwing an error
253
                  if (input_vector.size() > nfft_)
254
                     throw std::runtime_error("FILE: FFTPlanClass.hpp | CLASS: FFTPlanClass
255
                         | FUNCTION: fft() | REPORT: input-vector size is greater than
                         NFFT");
256
                  // copying inputs
                  for(std::size_t index = 0; index < input_vector.size(); ++index)</pre>
258
                  {
                     if constexpr(
260
261
                         std::is_floating_point_v<sourceType>
262
                         in_[index][0] = input_vector[index];
263
                         in_{in}[index][1] = 0;
264
                     }
                     else if constexpr(
266
                         std::is_same_v<sourceType, std::complex<float>> ||
267
                         std::is_same_v<sourceType, std::complex<double>>
                     ){
269
                         in_[index][0] =
                                           input_vector[index].real();
270
                         in_[index][1] = input_vector[index].imag();
271
                     }
272
273
                  }
274
                  // executing fft
275
                  fftw_execute(plan_);
277
                  // copying results to output-vector
2.78
                  std::vector<destinationType> output_vector(nfft_);
279
                  for(std::size_t index = 0; index < nfft_; ++index)</pre>
280
```

```
{
                       if constexpr(
282
                           std::is_same_v<
                                              destinationType, std::complex<double> >
283
                           std::is_same_v< destinationType, std::complex<float> >
2.84
                       ){
285
                           output_vector[index] = destinationType(
                              out_[index][0] * (1.00 / std::sqrt(nfft_)),
287
                               out_[index][1] * (1.00 / std::sqrt(nfft_))
                           );
                       }
290
                       else if constexpr(
291
                           std::is_floating_point_v<destinationType>
292
                       ){
                           output_vector[index] = destinationType(
294
                               std::sqrt(
295
                                  std::pow(out_[index][0] * (1.00 / std::sqrt(nfft_)), 2) + \
                                  std::pow(out_[index][1] * (1.00 / std::sqrt(nfft_)), 2)
298
                           );
299
                       }
                   }
301
302
                   // returning output
303
                   return std::move(output_vector);
304
               }
       };
306
   }
307
```

A.4 IFFTPlanClass

```
#pragma once
  namespace svr
                   {
      template
                <typename T>
      concept IFFTPlanClassSourceDestinationType = \
         std::is_floating_point_v<T> ||
         (
             std::is_class_v<T> &&
             std::is_floating_point_v<typename T::value_type>
         );
10
      template
         IFFTPlanClassSourceDestinationType sourceType,
12
         IFFTPlanClassSourceDestinationType destinationType
13
14
      class IFFTPlanClass
         public:
            std::size_t
                             nfft_;
18
            fftw_complex*
19
                             in_;
            fftw_complex*
                             out_;
            fftw_plan
                             plan_;
             /*----
            Destructor
25
             ~IFFTPlanClass()
26
```

```
if(plan_ != nullptr) {fftw_destroy_plan(
28
                if(in_ != nullptr) {fftw_free(
                                                          in_);}
29
                if(out_ != nullptr) {fftw_free(
                                                         out_);}
30
31
             }
             /*-----
             Constructor
33
             IFFTPlanClass(const std::size_t nfft): nfft_(nfft)
36
                // allocating space
37
                in_ = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
38
                  sizeof(fftw_complex)));
                out_ = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
39
                    sizeof(fftw_complex)));
                if(!in_ || !out_) {throw std::runtime_error("in_, out_ creation
40
                    failed");}
41
                // creating plan
42
                plan_ = fftw_plan_dft_1d(
43
                   static_cast<int>(nfft_),
                   in_,
45
                   out_,
46
                   FFTW_BACKWARD,
                   FFTW_MEASURE
                );
49
                                  {throw std::runtime_error("File: FFTPlanClass.hpp |
                if(!plan_)
50
                   Class: IFFTPlanClass | report: plan-creation failed");}
             }
51
             /*-----
52
             Copy Constructor
53
             IFFTPlanClass(const IFFTPlanClass& other)
56
                // allocating space
57
                nfft_ = other.nfft_;
                      = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
                   sizeof(fftw_complex)));
                     = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
60
                    sizeof(fftw_complex)));
                if (!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
61
                    Class: IFFTPlanClass | Function: Copy-Constructor | Report: in-out
                    plan creation failed");}
                // copying contents
63
                std::memcpy(in_, other.in_, nfft_ * sizeof(fftw_complex));
                std::memcpy(out_, other.out_, nfft_ * sizeof(fftw_complex));
                // creating a new plan since its more of an engine
67
                plan_ = fftw_plan_dft_1d(
68
                   static_cast<int>(nfft_),
69
                   in_,
                   out_,
71
                   FFTW_BACKWARD,
                   FFTW_MEASURE
                if(!plan_) {throw std::runtime_error("FILE: FFTPlanClass.hpp | Class:
75
                    IFFTPlanClass | Function: Copy-Constructor | Report: plan-creation
                    failed");}
```

```
76
77
             /*----
78
             Copy Assignment
79
             -----*/
80
             IFFTPlanClass& operator=(const IFFTPlanClass& other)
82
                // handling self-assignment
83
                if(this == &other) {return *this;}
                // cleaning up existing resources
86
                if(plan_ != nullptr) {fftw_destroy_plan( plan_);}
87
                if(in_ != nullptr) {fftw_free(
                                                       in_);}
                if(out_ != nullptr) {fftw_free(
                                                       out_);}
89
90
                // allocating space
                nfft_ = other.nfft_;
                in_ = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
93
                   sizeof(fftw_complex)));
                out_ = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
                    sizeof(fftw_complex)));
                if (!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
95
                    Class: IFFTPlanClass | Function: Copy-Constructor | Report: in-out
                    plan creation failed");}
                // copying contents
97
                std::memcpy(in_, other.in_, nfft_ * sizeof(fftw_complex));
98
                std::memcpy(out_, other.out_, nfft_ * sizeof(fftw_complex));
99
100
101
                // creating a new plan since its more of an engine
                plan_ = fftw_plan_dft_1d(
102
                   static_cast<int>(nfft_),
103
104
                   in_,
                   out_,
105
                   FFTW_BACKWARD,
106
                   FFTW_MEASURE
                );
108
                if(!plan_) {throw std::runtime_error("FILE: FFTPlanClass.hpp | Class:
109
                    IFFTPlanClass | Function: Copy-Constructor | Report: plan-creation
                    failed");}
110
                // returning
111
                return *this;
112
113
114
115
             Move Constructor
116
             -----*/
117
             IFFTPlanClass(IFFTPlanClass&& other) noexcept
118
                : nfft_( other.nfft_),
119
120
                   in_( other.in_),
121
                   out_( other.out_),
                   plan_( other.plan_)
             {
123
                // resetting the source object
124
                other.nfft_ = 0;
125
                other.in_
                             = nullptr;
126
                other.out_ = nullptr;
127
                other.plan_ = nullptr;
128
```

```
129
             /*----
130
             Move-Assignment
131
132
             IFFTPlanClass& operator=(IFFTPlanClass&& other) noexcept
133
                // self-assignment check
135
                if(this == &other) {return *this;}
136
138
                // cleaning up existing
                if(plan_ != nullptr) {fftw_destroy_plan( plan_);}
139
                if(in_ != nullptr) {fftw_free(
                                                        in_);}
140
                if(out_ != nullptr) {fftw_free(
                                                         out_);}
141
142
                // Copying values and changing pointers
143
                nfft_ = other.nfft_;
144
                          = other.in_;
                in_
                          = other.out_;
                out_
146
                         = other.plan_;
147
                plan_
148
                // resetting the source-object
                other.nfft_ = 0;
150
                other.in_ = nullptr;
other.out_ = nullptr;
                            = nullptr;
151
                other.plan_ = nullptr;
154
                // returning
                return *this;
156
157
             /*-----
158
             Running
159
             -----*/
160
             std::vector<destinationType>
161
             ifft(const std::vector<sourceType>& input_vector)
162
163
                // throwing an error
                if (input_vector.size() > nfft_)
165
                    throw std::runtime_error("File: FFTPlanClass | Class: IFFTPlanClass |
166
                       Function: ifft() | Report: size of vector > nfft ");
                // copy input into fftw buffer
168
                for(std::size_t index = 0; index < nfft_; ++index)</pre>
169
170
                    if constexpr(
171
                       std::is_same_v< sourceType, std::complex<double> > ||
172
                       std::is_same_v< sourceType, std::complex<float> >
173
174
                       in_[index][0] = input_vector[index].real();
                       in_[index][1] = input_vector[index].imag();
176
                    }
177
                    else if constexpr(
178
                       std::is_floating_point_v< sourceType >
180
                       in_[index][0] = input_vector[index];
181
                       in_{in}[index][1] = 0;
                    }
183
                }
184
185
                // execute ifft
```

```
fftw_execute(plan_);
187
188
                  // normalize output
189
                  std::vector<destinationType> output_vector(nfft_);
190
                  for(std::size_t index = 0; index < nfft_; ++index){</pre>
191
                     if constexpr(
                         std::is_floating_point_v< destinationType >
193
                     ){
194
                         output_vector[index] =
                             static_cast<destinationType>(out_[index][0]/nfft_);
196
                     else if constexpr(
197
                                           destinationType, std::complex<double> > ||
                         std::is_same_v<
198
                         std::is_same_v<
                                           destinationType, std::complex<float> >
199
200
                         output_vector[index][0] = destinationType(
201
                            out_[index][0]/nfft_,
                            out_[index][1]/nfft_
203
204
                         );
                     }
205
                  }
206
207
                  // returning
208
                  return std::move(output_vector);
2.09
              }
              /*-----
211
              Running - proper bases change
212
              -----*/
213
              std::vector<destinationType>
214
              ifft_12_conserved(const std::vector<sourceType>& input_vector)
215
216
                  // throwing an error
                  if (input_vector.size() > nfft_)
218
                     throw std::runtime_error("File: FFTPlanClass | Class: IFFTPlanClass |
219
                         Function: ifft() | Report: size of vector > nfft ");
                  // copy input into fftw buffer
221
                  for(std::size_t index = 0; index < nfft_; ++index)</pre>
                     if constexpr(
                         std::is_same_v<
                                          sourceType, std::complex<double> > ||
225
                         std::is_same_v< sourceType, std::complex<float> >
226
                     ){
227
                         in_[index][0] = input_vector[index].real();
                         in_[index][1] = input_vector[index].imag();
229
                     }
230
                     else if constexpr(
231
                         std::is_floating_point_v<sourceType>
233
                         in_[index][0] = input_vector[index];
234
235
                         in_{in}[index][1] = 0;
                     }
                  }
237
238
                  // execute ifft
                  fftw_execute(plan_);
240
241
                  // normalize output
242
                  std::vector<destinationType> output_vector(nfft_);
243
```

```
for(std::size_t index = 0; index < nfft_; ++index)</pre>
245
                       if constexpr(
246
                           std::is_floating_point_v<destinationType>
2.47
                       ){
248
                           output_vector[index] =
249
                               static_cast<destinationType>(out_[index][0] *
                               1.00/std::sqrt(nfft_));
                       }
                       else if constexpr(
251
                           std::is_same_v<
                                               destinationType, std::complex<double> > ||
252
                           std::is_same_v<
                                               destinationType, std::complex<float> >
253
                       ){
254
                           output_vector[index][0] = destinationType(
255
                               out_[index][0] * 1.00/std::sqrt(nfft_),
256
                               out_[index][1] * 1.00/std::sqrt(nfft_)
                       }
259
                   }
260
261
                   // returning
262
                   return std::move(output_vector);
263
               }
264
       };
265
    }
```

A.5 FFT Plan Pool

```
#pragma once
  namespace svr
               {
    template <
       typename sourceType,
       typename destinationType,
       typename = std::enable_if_t<</pre>
          std::is_same_v<sourceType, double> &&
          std::is_same_v<destinationType, std::complex<double>>
10
    class FFTPlanUniformPool {
    public:
       /*-----
       Handle to Plan
16
       struct AccessPairs
          /*-----
          Members
20
2.1
          svr::FFTPlanClass<sourceType, destinationType>& plan;
          std::unique_lock<std::mutex>
          /*-----
          Special Members
          AccessPairs() = delete;
28
```

```
AccessPairs(
               svr::FFTPlanClass<sourceType, destinationType>& plan_arg,
30
               std::mutex&
                                                    plan_mutex
31
              : plan(plan_arg), lock(plan_mutex) {}
32
            AccessPairs(
               svr::FFTPlanClass<sourceType, destinationType>& plan_arg,
               std::unique_lock<std::mutex>&&
35
            ): plan(plan_arg), lock(std::move(lock_arg)) {}
            AccessPairs(const AccessPairs& other)
                                                       = delete;
            AccessPairs& operator=(const AccessPairs& other) =
                                                          delete:
38
            AccessPairs(AccessPairs&& other)
                                                          delete:
39
            AccessPairs& operator=(AccessPairs&& other)
                                                      = delete;
40
        };
41
42
         /*-----
43
        Core Members
         std::vector<svr::FFTPlanClass<sourceType, destinationType>> plans;
46
47
         std::vector<std::mutex>
         /*-----
        Special-Members
50
         -----*/
51
                                                          = default;
        FFTPlanUniformPool()
        FFTPlanUniformPool(const std::size_t num_plans,
                      const std::size_t nfft)
54
55
            // reserving space
            plans.reserve(num_plans);
57
            for(auto i = 0; i < num_plans; ++i){</pre>
58
               plans.emplace_back(nfft);
59
            // creating a vector of mutexes
62
            mutexes = std::move(std::vector<std::mutex>(num_plans));
63
         FFTPlanUniformPool(const FFTPlanUniformPool& other)
                                                         = delete:
         FFTPlanUniformPool& operator=(const FFTPlanUniformPool& other) = delete;
         FFTPlanUniformPool(FFTPlanUniformPool&& other) = default;
         FFTPlanUniformPool& operator=(FFTPlanUniformPool&& other) = default;
69
         /*-----
70
        Function to fetch a plan
71
           > searches for a free-plan
           > if found, locks the plan
73
              > return the handle to the plan
74
                                   -----*/
         AccessPairs fetch_plan() {
            const int num_rounds = 12;
77
            for (int round = 0; round < num_rounds; ++round) {</pre>
               for (int i = 0; i < mutexes.size(); ++i) {</pre>
                  std::unique_lock<std::mutex> curr_lock(
81
                     mutexes[i],
                     std::try_to_lock
                  );
                  if (curr_lock.owns_lock())
85
                     return AccessPairs(plans[i], std::move(curr_lock));
86
```

A.6 IFFT Plan Pool

```
#pragma once
  namespace svr
                 {
9
     template <
        typename
                sourceType,
        typename destinationType,
        typename = std::enable_if_t<</pre>
           std::is_same_v<sourceType, std::complex<double>>&&
           std::is_same_v<destinationType, double>
14
     class IFFTPlanUniformPool
17
        public:
           /*-----
20
           Structure used for interfacing to plans
2.1
           struct AccessPairs
              Core Members
              svr::IFFTPlanClass<sourceType, destinationType>& plan;
28
              std::unique_lock<std::mutex>
29
              /*-----
31
              Special Members
                                                        = delete;
              AccessPairs()
              AccessPairs(
                 svr::IFFTPlanClass<sourceType, destinationType>& plan_arg,
36
                 std::mutex&
                                                        plan_mutex_arg
              ): plan(plan_arg), lock(plan_mutex_arg) {}
              AccessPairs(
                 svr::IFFTPlanClass<sourceType, destinationType>& plan_arg,
                 std::unique_lock<std::mutex>&&
                                                        lock_arg
              ): plan(plan_arg), lock(std::move(lock_arg)) {}
              AccessPairs(const AccessPairs& other)
                                                        = delete;
43
              AccessPairs& operator=(const AccessPairs& other) = delete;
44
```

```
AccessPairs(AccessPairs&& other)
                                                         = delete;
45
               AccessPairs& operator=(AccessPairs&& other)
                                                       = delete;
46
            };
47
48
            /*-----
49
            -----*/
51
            std::vector< svr::IFFTPlanClass<sourceType, destinationType> > plans;
            std::vector< std::mutex</pre>
            /*-----
55
            Special Members
56
            -----*/
57
            IFFTPlanUniformPool()
                                                = default;
58
            IFFTPlanUniformPool(const std::size_t num_plans,
                          const std::size_t nfft)
               // reserving space
62
               plans.reserve(num_plans);
63
               for(auto i = 0; i < num_plans; ++i)</pre>
                  plans.emplace_back(nfft);
               // creating vector of mutexes
               mutexes = std::vector<std::mutex>(num_plans);
            IFFTPlanUniformPool(const IFFTPlanUniformPool& other)
                                                                 = delete;
70
            IFFTPlanUniformPool& operator=(const IFFTPlanUniformPool& other) = delete;
71
            IFFTPlanUniformPool(IFFTPlanUniformPool&& other)
                                                       = default;
            IFFTPlanUniformPool& operator=(IFFTPlanUniformPool&& other)
73
                                                                = default;
74
            /*-----
            Member-Functions
            -----*/
            AccessPairs fetch_plan()
78
79
               // setting the number of rounds to take
               const int num_rounds {12};
81
               // performing rounds
               for(auto round = 0; round < num_rounds; ++round)</pre>
85
                  // going through vector mutexes
86
                  for(auto i =0; i < mutexes.size(); ++i)</pre>
87
                     // trying to lock current mutex
89
                     std::unique_lock<std::mutex> curr_lock(mutexes[i],
90
                        std::try_to_lock);
                     // if our lock contains the mutex, returning the plan and lock
92
                     if (curr_lock.owns_lock())
93
                        return AccessPairs(plans[i], std::move(curr_lock));
                  }
               }
               // throwing error
               throw std::runtime_error("FILE: IFFTPlanPoolClass.hpp | CLASS:
                  IFFTPlanUniformPool | REPORT: COULDN'T FIND ANY AVAILABLE PLANS");
            }
100
      };
```

102 }

A.7 FFT Plan Pool Handle

```
#pragma once
  /*-----
  Dependencies
  -----*/
  #include "FFTPlanPoolClass.hpp"
  namespace svr
9
  {
     template <
       typename sourceType,
       typename
               destinationType,
       typename = std::enable_if_t<</pre>
          std::is_same_v< sourceType, double > &&
          std::is_same_v< destinationType, std::complex<double> >
17
     struct FFTPlanUniformPoolHandle
20
       Core Members
21
       svr::FFTPlanUniformPool<sourceType, destinationType> uniform_pool;
       std::size_t
                                              num_plans;
       std::size_t
                                              nfft;
       /*-----
28
       Special Member-functions
       -----*/
       FFTPlanUniformPoolHandle() = default;
       FFTPlanUniformPoolHandle(const std::size_t num_plans_arg,
                         const std::size_t nfft_arg)
             uniform_pool(num_plans_arg, nfft_arg),
             num_plans(num_plans_arg),
             nfft(nfft_arg)
36
       FFTPlanUniformPoolHandle(const FFTPlanUniformPoolHandle& other) = delete;
37
       FFTPlanUniformPoolHandle& operator=(const FFTPlanUniformPoolHandle& other) =
       FFTPlanUniformPoolHandle(FFTPlanUniformPoolHandle&& other)
       FFTPlanUniformPoolHandle& operator=(FFTPlanUniformPoolHandle&& other) = default;
40
       /*-----
       Member Functions
43
       auto lock()
          return std::unique_lock<std::mutex>(this->mutex);
     };
  }
```

A.8 IFFT Plan Pool Handle

```
#pragma once
  /*-----
4 Dependencies
 -----*/
  #include "IFFTPlanPoolClass.hpp"
  namespace svr
9
  {
10
    template <
11
       typename sourceType,
       typename destinationType,
       typename = std::enable_if_t<</pre>
         std::is_same_v< sourceType,</pre>
                                 std::complex<double> > &&
         std::is_same_v< destinationType, double >
17
18
    struct IFFTPlanUniformPoolHandle
19
       /*-----
21
       -----*/
       IFFTPlanUniformPool< sourceType,</pre>
                      destinationType >
                                       uniform_pool;
25
       std::mutex
                                        mutex;
       std::size_t
                                        num_plans;
       std::size_t
                                        nfft;
       /*-----
       Special Member Functions
       IFFTPlanUniformPoolHandle() = default;
33
       IFFTPlanUniformPoolHandle(const std::size_t num_plans_arg,
                      const std::size_t nfft_arg)
         : uniform_pool( num_plans_arg, nfft_arg),
36
                          num_plans_arg),
            num_plans(
37
            nfft(
                           nfft_arg) {}
       IFFTPlanUniformPoolHandle(const IFFTPlanUniformPoolHandle& other) = delete;
       IFFTPlanUniformPoolHandle& operator=(const IFFTPlanUniformPoolHandle& other) =
       IFFTPlanUniformPoolHandle(IFFTPlanUniformPoolHandle&& other)
       IFFTPlanUniformPoolHandle& operator=(IFFTPlanUniformPoolHandle&& other) = delete;
       /*-----
       Member Functions
       -----*/
       auto
            lock()
         return std::unique_lock<std::mutex>(this->mutex);
       }
51
    };
  }
```

Appendix B

General Purpose Templated Functions

B.1 abs

```
#pragma once
  /*-----
  #include <vector> // for vectors
  #include <algorithm> // for std::transform
  y = abs(vector)
  template <typename T>
  auto abs(const std::vector<T>& input_vector)
      // creating canvas
     auto canvas {input_vector};
     // calculating abs
      std::transform(canvas.begin(),
                  canvas.end(),
                  canvas.begin(),
                  [](auto& argx){return std::abs(argx);});
     // returning
      return std::move(canvas);
  y = abs(matrix)
  template <typename T>
  auto abs(const std::vector<std::vector<T>> input_matrix)
31
      // creating canvas
      auto canvas
                    {input_matrix};
      // applying element-wise abs
      std::transform(input_matrix.begin(),
                  input_matrix.end(),
                  input_matrix.begin(),
```

```
[](auto& argx){return std::abs(argx);});

// returning
return std::move(canvas);

}
```

B.2 Boolean Comparators

```
#pragma once
  template <typename T, typename U>
  auto operator<(const std::vector<T>& input_vector,
             const
                                   scalar)
     // creating canvas
                   {std::vector<bool>(input_vector.size())};
     // transforming
11
     std::transform(input_vector.begin(), input_vector.end(),
                canvas.begin(),
                [&scalar](const auto& argx){
                    return argx < static_cast<T>(scalar);
     // returning
     return std::move(canvas);
19
20 }
  /*-----
  -----*/
  template <typename T, typename U>
  auto operator<=(const std::vector<T>& input_vector,
24
              const
                                   scalar)
25
  {
26
     // creating canvas
          canvas
                    {std::vector<bool>(input_vector.size())};
28
     // transforming
     std::transform(input_vector.begin(), input_vector.end(),
                canvas.begin(),
                 [&scalar](const auto& argx){
                    return argx <= static_cast<T>(scalar);
                });
     // returning
37
     return std::move(canvas);
38
  template <typename T, typename U>
  auto operator>(const std::vector<T>& input_vector,
43
             const
                                   scalar)
44
     // creating canvas
     auto canvas {std::vector<bool>(input_vector.size())};
     // transforming
```

```
std::transform(input_vector.begin(), input_vector.end(),
                   canvas.begin(),
50
                   [&scalar](const auto& argx){
51
                       return argx > static_cast<T>(scalar);
52
                  });
      // returning
55
      return std::move(canvas);
56
  }
57
   /*-----
58
59
  template <typename T, typename U>
60
  auto operator>=(const std::vector<T>& input_vector,
                const U
                                        scalar)
62
63
      // creating canvas
64
                     {std::vector<bool>(input_vector.size())};
      auto canvas
66
      // transforming
67
      std::transform(input_vector.begin(), input_vector.end(),
                  canvas.begin(),
                   [&scalar](const auto& argx){
                       return argx >= static_cast<T>(scalar);
                  });
      // returning
74
      return std::move(canvas);
75
  }
```

B.3 Concatenate Functions

```
#pragma once
  /*----
  input = [vector, vector],
  output = [vector]
                    -----*/
  template <std::size_t axis, typename T>
  auto concatenate(const std::vector<T>& input_vector_A,
               const std::vector<T>&
                                   input_vector_B) -> std::enable_if_t<axis == 1,</pre>
                   std::vector<T> >
  {
9
     // creating canvas vector
10
     auto num_elements {input_vector_A.size() + input_vector_B.size()};
11
     auto canvas
                        {std::vector<T>(num_elements, (T)0) };
     // filling up the canvas
     std::copy(input_vector_A.begin(), input_vector_A.end(),
             canvas.begin());
16
     std::copy(input_vector_B.begin(), input_vector_B.end(),
17
             canvas.begin()+input_vector_A.size());
     // moving it back
20
     return std::move(canvas);
22
  }
23
```

```
input = [vector, vector],
  output = [matrix]
  -----*/
27
28 template <std::size_t axis, typename T>
29 auto concatenate(const std::vector<T>& input_vector_A,
                const std::vector<T>&
                                     input_vector_B) -> std::enable_if_t<axis == 0,</pre>
                    std::vector<std::vector<T>> >
  ₹
31
32
      // throwing error dimensions
33
      if (input_vector_A.size() != input_vector_B.size())
         std::cerr << "concatenate:: incorrect dimensions \n";</pre>
34
35
      // creating canvas
      auto canvas {std::vector<std::vector<T>>(
37
         2, std::vector<T>(input_vector_A.size())
38
      )};
30
      // filling up the dimensions
41
      std::copy(input_vector_A.begin(), input_vector_A.end(), canvas[0].begin());
42
      std::copy(input_vector_B.begin(), input_vector_B.end(), canvas[1].begin());
43
      // moving it back
45
      return std::move(canvas);
46
47
  /*----
49
50 input = [vector, vector, vector],
output = [matrix]
 -----*/
53
  template <std::size_t axis, typename T>
auto concatenate(const std::vector<T>& input_vector_A,
                const std::vector<T>& input_vector_B,
55
                const std::vector<T>& input_vector_C) -> std::enable_if_t<axis == 0,</pre>
                   std::vector<std::vector<T>> >
  ₹
57
      // throwing error dimensions
      if (input_vector_A.size() != input_vector_B.size() ||
59
         input_vector_A.size() != input_vector_C.size())
60
         std::cerr << "concatenate:: incorrect dimensions \n";</pre>
      // creating canvas
63
      auto canvas {std::vector<std::vector<T>>(
64
         3, std::vector<T>(input_vector_A.size())
65
      )};
67
      // filling up the dimensions
68
      std::copy(input_vector_A.begin(), input_vector_A.end(), canvas[0].begin());
69
      std::copy(input_vector_B.begin(), input_vector_B.end(), canvas[1].begin());
70
      std::copy(input_vector_C.begin(), input_vector_C.end(), canvas[2].begin());
71
72
      // moving it back
73
      return std::move(canvas);
75
76 }
  /*-----
  input = [matrix, vector],
78
  output = [matrix]
79
  template <std::size_t axis, typename T>
```

```
auto concatenate(const std::vector<std::vector<T>>& input_matrix,
                   const std::vector<T>
                                                        input_vector) -> std::enable_if_t<axis</pre>
83
                       == 0, std::vector<std::vector<T>> >
   {
84
       // creating canvas
85
       auto canvas
                          {input_matrix};
87
       // adding to the canvas
       canvas.push_back(input_vector);
       // returning
91
       return std::move(canvas);
92
   }
```

B.4 Conjugate

```
#pragma once
  namespace svr {
    /*----
    y = svr::conj(vector);
    -----*/
    template <typename T>
         conj(const std::vector<T>& input_vector)
    auto
       // creating canvas
       auto canvas
                   {std::vector<T>(input_vector.size())};
       // calculating conjugates
       std::for_each(canvas.begin(), canvas.end(),
                [](auto& argx){argx = std::conj(argx);});
       // returning
       return std::move(canvas);
17
    }
18
  }
```

B.5 Convolution

```
17
         // calculating fft of two arrays
18
                fft_A
                          {svr::fft(input_vector_A, canvas_length)};
         auto
19
         auto
                fft_B
                          {svr::fft(input_vector_B, canvas_length)};
2.0
21
         // element-wise multiplying the two matrices
                         {fft_A * fft_B};
         auto
                fft_AB
23
         // finding inverse FFT
         auto convolved_result {ifft(fft_AB)};
26
27
         // returning
28
         return std::move(convolved_result);
29
      }
30
31
      template <>
32
      auto conv1D(const std::vector<double>& input_vector_A,
                   const std::vector<double>& input_vector_B)
34
35
         // creating canvas
36
         auto canvas_length
                                 {input_vector_A.size() + input_vector_B.size() - 1};
38
         // calculating fft of two arrays
39
                         {svr::fft(input_vector_A, canvas_length)};
         auto
                fft_A
                          {svr::fft(input_vector_B, canvas_length)};
         auto
                fft_B
42
         // element-wise multiplying the two matrices
43
         auto fft_AB {fft_A * fft_B};
         // finding inverse FFT
46
         auto convolved_result {ifft(fft_AB)};
47
         // returning
49
         return std::move(convolved_result);
50
      }
51
      /*-----
53
      1D convolution of two vectors
54
      > implemented through fft
55
                           -----*/
57
      template <typename T1, typename T2>
      auto conv1D_fftw(const std::vector<T1>& input_vector_A,
58
                       const std::vector<T2>& input_vector_B)
59
      {
         // resulting type
61
         using T3 = decltype(std::declval<T1>() * std::declval<T2>());
62
         // creating canvas
                                 {input_vector_A.size() + input_vector_B.size() - 1};
         auto canvas_length
65
         // calculating fft of two arrays
67
         auto
                fft_A
                          {svr::fft(input_vector_A, canvas_length)};
         auto
                fft_B
                          {svr::fft(input_vector_B, canvas_length)};
69
70
         // element-wise multiplying the two matrices
         auto fft_AB
                          {fft_A * fft_B};
72
73
         // finding inverse FFT
74
         auto convolved_result {svr::ifft(fft_AB, fft_AB.size())};
```

76

```
// returning
77
          return std::move(convolved_result);
78
79
80
       /*----
81
       Long-signal Conv1D
82
83
       improvements:
          > make an inplace version of this
       -----*/
       template <std::size_t L, typename T>
86
       auto conv1D_long(const std::vector<T>& input_vector_A,
87
                        const std::vector<T>& input_vector_B)
88
89
          // fetching dimensions
90
                                      {std::max(input_vector_A.size(),
          const auto
                       maxlength
                                              input_vector_B.size())};
                                      {std::min(input_vector_A.size(),
          const auto
                        filter_size
93
                                              input_vector_B.size())};
94
          const auto block_size
                                      {L + filter_size - 1};
95
          const auto num_blocks
                                      {2 + static_cast<std::size_t>(
              (maxlength - block_size)/L
97
          )};
98
          // obtaining references
          const auto& large_vector
                                      {input_vector_A.size() >= input_vector_B.size() ? \
101
                                      input_vector_A : input_vector_B};
102
          const auto& small_vector {input_vector_A.size() < input_vector_B.size() ? \</pre>
                                      input_vector_A : input_vector_B};
104
105
          // setup
106
                 starting_index
                                      {static_cast<std::size_t>(0)};
          auto
107
                 ending_index
                                      {static_cast<std::size_t>(0)};
          auto
108
                 length_left_to_fill {ending_index - starting_index};
          auto
109
                                      {std::vector<double>(block_size, 0)};
                 canvas
          auto
          auto
                 finaloutput
                                      {std::vector<double>(maxlength, 0)};
                 block_conv_output_size {L + 2 * filter_size -2};
          auto
                block_conv_output
                                      {std::vector<double>(block_conv_output_size, 0)};
          auto
114
115
          // block-wise processing
          for(auto bid = 0; bid < num_blocks; ++bid)</pre>
116
117
              // estimating indices
118
             starting_index =
                                     L*bid;
119
                                     std::min(starting_index + block_size - 1,maxlength -
              ending_index
120
                 1):
              length_left_to_fill = ending_index - starting_index;
121
              // copying to the common-block
123
              std::copy(large_vector.begin() + starting_index,
124
125
                      large_vector.begin() + ending_index + 1,
126
                      canvas.begin());
127
              // performing convolution
128
              block_conv_output = svr::conv1D_fftw(canvas,
130
                                                 small_vector);
131
              // discarding edges and writing values
              std::copy(block_conv_output.begin() + filter_size-2,
133
```

```
block_conv_output.begin() + filter_size-2 +
134
                           std::min(static_cast<int>(L-1),
                           static_cast<int>(length_left_to_fill)) + 1,
                       finaloutput.begin()+starting_index);
           }
136
137
           // returning
138
          return std::move(finaloutput);
139
140
141
       /*-----
142
       Long-signal Conv1D with FFT Plan
143
       improvements:
144
          > make an inplace version of this
145
                                            -----*/
146
       template <
147
           typename
                     Τ,
           std::enable_if_t<
149
150
              std::is_floating_point_v<T>
151
       >
       auto
              conv1D_long_prototype(
           const std::vector<T>&
                                                  input_vector_A,
154
           const std::vector<T>&
                                                  input_vector_B,
           svr::FFTPlanClass<T, std::complex<T>>& fft_plan,
           svr::IFFTPlanClass<std::complex<T>, T>& ifft_plan
157
       )
158
       {
159
           // Error checks
160
           if (fft_plan.nfft_ != ifft_plan.nfft_)
161
              throw std::runtime_error("fft_plan.nfft_ != ifft_plan.nfft_");
162
163
           // fetching references to large-signal and small-signal
164
           const auto& large_signal_original {
165
              input_vector_A.size() >= input_vector_B.size() ?
166
              input_vector_A
                                   : input_vector_B
           };
168
           const auto& small_signal
                                       {
              input_vector_A.size() < input_vector_B.size() ?
170
              input_vector_A
                                       input_vector_B
          };
172
173
           // copying
174
                  large_signal {std::vector<double>(
175
              input_vector_A.size() + input_vector_B.size() -1
176
           )}:
177
           std::copy(large_signal_original.begin(),
178
                    large_signal_original.end(),
                    large_signal.begin());;
180
181
182
           // calculating parameters
183
           const auto
                         signal_size
                                               {large_signal_original.size()};
           const
                  auto
                         filter_size
                                               {small_signal.size()};
184
                         input_signal_block_size {fft_plan.nfft_ + 1 - filter_size};
           const
                  auto
185
           if (input_signal_block_size <= 0)</pre>
              throw std::runtime_error("input_signal_block_size <= 0 ");</pre>
187
           const auto
                         block_output_length {fft_plan.nfft_};
188
                                               {static_cast<int>(
           const auto
                         num_blocks
189
              1 + std::ceil((signal_size + filter_size - 2)/input_signal_block_size)
190
```

```
)
191
           };
192
                          final_output_size
                                                 {signal_size + filter_size - 1};
           const
                   auto
193
                          useful_sample_length
                                                       {block_output_length - (filter_size -1)
           const
                  auto
194
               - (filter_size -1)};
           // parameters for re-use
196
                   start_index
                                          {static_cast<int>(0)};
197
           auto
           auto
                   end_index
                                          {static_cast<int>(0)};
           auto
                   output_start_index
                                         {static_cast<int>(0)};
199
200
           // calculating fft(filter)
201
                                          {std::vector<double>(block_output_length, 0.0)};
           auto
                   filter_zero_padded
202
           std::copy(small_signal.begin(), small_signal.end(), filter_zero_padded.begin());
203
           auto
                   filter_FFT
                                          {fft_plan.fft(filter_zero_padded)};
204
205
           // allocating space for storing input-blocks
           auto
                   signal_block_zero_padded {std::vector<double>(block_output_length, 0.0)};
207
                                             {std::vector<double>()};
208
           auto
                   fftw_output
           auto
                   conv_output
                                             {std::vector<double>()};
                   finaloutput
                                             {std::vector<double>(final_output_size, 0.0)};
           auto
210
211
           // going through the values
212
           svr::Timer timer("fft-loop");
213
           for(auto i = 0; i<num_blocks; ++i){</pre>
215
               // calculating bounds
               auto
                      analytical_start {
217
                   (i*static_cast<int>(input_signal_block_size)) -
218
                       (static_cast<int>(filter_size) - 1)
               };
219
               auto
                       analytical_end
                                          {(i+1)*input_signal_block_size -1};
               start_index
                             = std::max(
221
                   static_cast<int>(0), static_cast<int>(analytical_start)
222
               );
223
               end_index
                              = std::min(
                   static_cast<int>(signal_size-1),static_cast<int>(analytical_end)
225
               ); // [start-index, end-index)
226
228
               // copying values
               signal_block_zero_padded = std::move(std::vector<double>(block_output_length,
229
                   0.0));
               std::copy(large_signal.begin() + start_index,
230
                         large_signal.begin() + end_index + 1,
231
                         signal_block_zero_padded.begin() + start_index - analytical_start);
232
               // performing ifft(fft(x) * fft(y))
                             = ifft_plan.ifft(
               fftw_output
                   fft_plan.fft(signal_block_zero_padded) * filter_FFT
236
               );
238
239
               // trimming away the first parts (since partial)
               conv_output = std::vector<double>(fftw_output.begin() + filter_size -
240
                   1,fftw_output.end());
               // writing to final-output
242
               std::copy(conv_output.begin(), conv_output.end(), finaloutput.begin() +
243
                   output_start_index);
               output_start_index
                                      += conv_output.size();
```

```
}
245
246
       }
247
248
       /*-----
249
       Long-signal Conv1D with FFT-Plan-Pool
250
251
                 <
252
       template
253
           typename
                      Τ,
           typename = std::enable_if_t<</pre>
254
               std::is_same_v<T, double> ||
255
               std::is_same_v<T, float>
256
257
       >
258
       auto
              conv_per_plan(
           const int
260
           const
                  int&
                                                                input_signal_block_size,
261
           const
                  int&
                                                                filter_size,
262
                  int&
263
           const
                                                                block_output_length,
           const std::vector<T>&
                                                                large_signal,
           std::vector<T>
                                                                signal_block_zero_padded,
265
           svr::FFTPlanUniformPoolHandle<T, std::complex<T>>& fft_pool_handle,
266
           svr::IFFTPlanUniformPoolHandle<std::complex<T>, T>& ifft_pool_handle,
267
                                                                filter_FFT,
           const std::vector<std::complex<T>>&
2.68
           std::vector<T>
                                                                fftw_output,
           std::vector<T>
                                                                conv_output,
270
           std::vector<T>&
                                                                output_vector,
271
           std::mutex&
                                                                output_vector_mutex,
272
           const auto&
                                                                signal_size
273
274
275
           // calculating bounds
277
                  analytical_start {
           auto
278
               (i*static_cast<int>(input_signal_block_size)) -
                   (static_cast<int>(filter_size) - 1)
           };
280
                                     {(i+1)*input_signal_block_size -1};
                  analytical_end
281
           auto start_index =
                                 std::max(
283
               static_cast<int>(0), static_cast<int>(analytical_start)
284
           auto end_index
                                 std::min(
285
               static_cast<int>(signal_size-1),static_cast<int>(analytical_end)
286
           ); // [start-index, end-index)
287
288
           // copying values
2.89
           signal_block_zero_padded = std::move(std::vector<double>(block_output_length,
               0.0));
           std::copy(
291
               large_signal.begin()
2.92
                                                    start_index,
293
               large_signal.begin()
                                                    end_index
                                                                   + 1,
               signal_block_zero_padded.begin() + start_index
                                                                   - analytical_start
           );
295
296
           \ensuremath{//} fetching an fft and IFFT plan
                  fph_lock
                                 {fft_pool_handle.lock()};
298
           auto
                  ifph_lock
                                 {ifft_pool_handle.lock()};
299
                                 {fft_pool_handle.uniform_pool.fetch_plan()};
                  fft_pair
300
           auto
           auto
                  ifft_pair
                                  {ifft_pool_handle.uniform_pool.fetch_plan()};
301
```

```
302
           // performing ifft(fft(x) * filter-FFT)
303
           fftw_output
                           = ifft_pair.plan.ifft_12_conserved(
304
               fft_pair.plan.fft_12_conserved(signal_block_zero_padded) * filter_FFT
305
           );
306
307
           // trimming away the first parts (since partial)
308
           conv_output = std::vector<T>(
309
               fftw_output.begin() + filter_size - 1,
311
               fftw_output.end()
           );
312
313
           // writing to final-output
314
                   output_start_index = i * (block_output_length - (filter_size - 1));
315
           std::lock_guard<std::mutex> output_lock(output_vector_mutex);
           std::copy(
317
               conv_output.begin(), conv_output.end(),
318
               output_vector.begin() + output_start_index
319
320
           );
        }
321
322
323
        template
324
           typename T,
325
           typename
                           std::enable_if_t<
327
               std::is_same_v<T, double> ||
               std::is_same_v<T, float>
328
329
        >
330
        auto
               conv1D_long_FFTPlanPool(
           const std::vector<T>&
                                                                  input_vector_A,
332
                  std::vector<T>&
                                                                  input_vector_B,
333
           svr::FFTPlanUniformPoolHandle<T, std::complex<T>>& fft_pool_handle
334
           svr::IFFTPlanUniformPoolHandle<std::complex<T>, T>& ifft_pool_handle
335
        )
336
        {
337
           // Error checks
338
           if (fft_pool_handle.nfft!=ifft_pool_handle.nfft)
               throw std::runtime_error("FILE: svr_conv.hpp | FUNCTION:
340
                   conv1D_long_FFTPlanPool | Report: the pool-handles are for different
                   nffts");
341
           // fetching references to the large signal and small signal
342
           const auto& large_signal_original {
               input_vector_A.size() >= input_vector_B.size() ?
344
                                          input_vector_B
               input_vector_A
                                      :
345
           };
346
           const auto& small_signal
347
               input_vector_A.size() <</pre>
                                          input_vector_B.size() ?
348
               input_vector_A
                                          input_vector_B
349
                                      :
350
           };
351
           // copying
352
                   large_signal {std::vector<double>(
           auto
353
               input_vector_A.size() + input_vector_B.size() - 1
354
           )};
355
           std::copy(large_signal_original.begin(),
356
                     large_signal_original.end(),
357
                     large_signal.begin());
358
```

```
359
           // calculating some parameters
360
           const
                   auto
                           signal_size
                                           {large_signal_original.size()};
361
                                           {small_signal.size()};
                           filter_size
           const
                   auto
362
                           input_signal_block_size {
           const
                   auto
363
               fft_pool_handle.nfft + 1 - filter_size
364
           };
365
366
           // throwing an error if nfft < filter-size
           if (fft_pool_handle.nfft < filter_size)</pre>
368
               throw std::runtime_error("FILE: svr_conv.hpp | FUNCTION:
369
                    conv1D_long_FFTPlanPool | REPORT: filter is bigger than nfft");
370
           // throwing an error if number of useful samples is less than zero
371
           if (input_signal_block_size <= 0)</pre>
372
               throw std::runtime_error("FILE: svr_conv.hpp | FUNCTION:
                    conv1D_long_FFTPlanPool | REPORT: input_signal_block_size = 0");
374
375
           const
                   auto
                           block_output_length
                                                  {fft_pool_handle.nfft};
376
                           num_blocks
                                                  {static_cast<int>(
377
           const
                   auto
               1 + std::ceil((signal_size + filter_size - 2) / input_signal_block_size)
378
           )};
379
                                                  {signal_size + filter_size - 1};
                           final_output_size
380
           const
                   auto
381
           const
                   auto
                           useful_sample_length {
               block_output_length - (filter_size - 1) - (filter_size -1)
382
           };
383
384
           // parameters for re-use
385
           auto
                   start_index
                                           {static_cast<int>(0)};
386
           auto
                   end_index
                                           {static_cast<int>(0)};
387
                   output_start_index
                                           {static_cast<int>(0)};
           auto
389
           // calculating fft(filter)
390
                                           {std::vector<double>(block_output_length, 0.0)};
                   filter_zero_padded
391
           auto
           std::copy(small_signal.begin(), small_signal.end(), filter_zero_padded.begin());
                                           {fft_pool_handle.lock()};
           auto
                   fph_lock0
393
                                           {fft_pool_handle.uniform_pool.fetch_plan()};
           auto
                   curr_plan_pair
394
                   pool_num_plans
                                           {fft_pool_handle.num_plans};
395
           auto
396
           fph_lock0.unlock();
           auto
                   filter_FFT
397
               curr_plan_pair.plan.fft(
398
                   filter_zero_padded
399
               )
401
           curr_plan_pair.lock.unlock();
402
403
           // allocating space for storing input-blocks
           auto
                   signal_block_zero_padded {std::vector<T>(block_output_length, 0.0)};
405
                                              {std::vector<T>()};
                   fftw_output
406
           auto
407
           auto
                   conv_output
                                              {std::vector<T>()};
           auto
                   output_vector
                                              {std::vector<T>(final_output_size, 0.0)};
           auto
                   output_vector_mutex
                                              {std::mutex()};
409
410
           // creating boost
411
           svr::ThreadPool local_pool(pool_num_plans);
412
413
           // going through the values
414
           for(auto i = 0; i < num_blocks; ++i)</pre>
415
```

```
{
416
              local_pool.push_back(
417
                  418
                     i.
419
                      &input_signal_block_size,
420
                      &filter_size,
421
                     &block_output_length,
422
                     &large_signal,
423
                      signal_block_zero_padded,
425
                     &fft_pool_handle,
                     &ifft_pool_handle,
426
                     &filter_FFT,
427
                     fftw_output,
428
                      conv_output,
429
                      &output_vector,
430
                      &output_vector_mutex,
431
                      &signal_size
                  ]{
433
                      conv_per_plan<T>(
434
435
                         i,
                         std::ref(input_signal_block_size),
436
                         std::ref(filter_size),
437
                         std::ref(block_output_length),
438
                         std::ref(large_signal),
                         signal_block_zero_padded,
                         fft_pool_handle,
441
                         ifft_pool_handle,
442
                         filter_FFT,
443
                         fftw_output,
444
                         conv_output,
445
                         std::ref(output_vector),
446
                         output_vector_mutex,
                         signal_size
448
                     );
449
                  }
450
              );
451
           }
452
           local_pool.converge();
453
           // returning final output
           return std::move(output_vector);
456
           // return output_vector;
457
458
459
       /*-----
460
       Short-conv1D
461
                                      -----*/
462
       template
                  <typename
                                T1,
463
                   typename
464
              conv1D_short(const std::vector<T1>& input_vector_A,
465
       auto
466
                          const std::vector<T2>& input_vector_B)
           // resulting type
468
           using T3 = decltype(std::declval<T1>() * std::declval<T2>());
469
           // creating canvas
471
                                    {input_vector_A.size() + input_vector_B.size() - 1};
           auto
                  canvas_length
472
473
           // calculating fft of two arrays
474
```

```
fft_A
                               {svr::fft(input_vector_A, canvas_length)};
475
            auto
            auto
                   fft_B
                               {svr::fft(input_vector_B, canvas_length)};
476
477
            // element-wise multiplying the two matrices
478
479
                   fft_AB
                               {fft_A * fft_B};
            auto
480
           // finding inverse FFT
481
                   convolved_result {ifft(fft_AB)};
            auto
482
            // returning
484
            // return std::move(convolved_result);
485
           return convolved_result;
486
487
        }
488
489
490
        1D Convolution of a matrix and a vector
492
493
494
        template
                  <typename T>
        auto conv1D(const std::vector<std::vector<T>>& input_matrix,
                      const
                               std::vector<T>&
                                                              input_vector,
496
                      const
                               std::size_t&
                                                              dim)
497
498
            // getting dimensions
            const auto& num_rows_matrix
                                                  {input_matrix.size()};
500
                                                  {input_matrix[0].size()};
            const auto& num_cols_matrix
501
            const auto& num_elements_vector
                                                  {input_vector.size()};
502
503
            // creating canvas
504
            auto
                   canvas
                               {std::vector<std::vector<T>>()};
505
            // creating output based on dim
507
            if (dim == 1)
508
            {
509
               // performing convolutions row by row
               for(auto row = 0; row < num_rows_matrix; ++row)</pre>
511
                {
512
                   cout << format("\t\t row = {}/{}\n", row, num_rows_matrix);
513
514
                   auto bruh {conv1D(input_matrix[row], input_vector)};
                   auto bruh_real {svr::real(std::move(bruh))};
515
                   canvas.push_back(
517
                           svr::real(
518
                               std::move(bruh_real)
519
520
521
                   );
               }
            else{
524
                std::cerr << "svr_conv.hpp | conv1D | yet to be implemented \n";</pre>
525
526
527
            // returning
528
            return std::move(canvas);
530
531
532
    }
533
```

B.6 Coordinate Change

```
#pragma once
  namespace svr {
     /*-----
     y = cart2sph(vector)
      -----*/
     template <typename T>
      auto cart2sph(const std::vector<T>& cartesian_vector){
         // splatting the point onto xy-plane
              xysplat
                        {cartesian_vector};
                   = 0;
         xysplat[2]
         // finding splat lengths
               xysplat_lengths
                               {norm(xysplat)};
         // finding azimuthal and elevation angles
16
               azimuthal_angles {svr::atan2(xysplat[1],
                                        xysplat[0]) \
18
                                * 180.00/std::numbers::pi};
19
         auto
               elevation_angles {svr::atan2(cartesian_vector[2],
                                         xysplat_lengths) \
2.1
                                * 180.00/std::numbers::pi};
         auto
               rho_values
                               {norm(cartesian_vector)};
         // creating tensor to send back
25
               spherical_vector {std::vector<T>{azimuthal_angles,
         auto
                                            elevation_angles,
                                            rho_values}};
29
         // moving it back
30
         return std::move(spherical_vector);
33
     y = cart2sph(vector)
34
      -----*/
      template <typename T>
36
      auto cart2sph_inplace(std::vector<T>& cartesian_vector){
37
         // splatting the point onto xy-plane
         auto xysplat
                        {cartesian_vector};
40
         xysplat[2]
41
         // finding splat lengths
               xysplat_lengths
                               {norm(xysplat)};
         // finding azimuthal and elevation angles
         auto azimuthal_angles {svr::atan2(xysplat[1], xysplat[0]) *
            180.00/std::numbers::pi};
               elevation_angles {svr::atan2(cartesian_vector[2],
48
         auto
                                         xysplat_lengths) * 180.00/std::numbers::pi};
         auto
               rho_values
                               {norm(cartesian_vector)};
50
51
         // creating tesnor
         cartesian_vector[0]
                               azimuthal_angles;
         cartesian_vector[1]
                               elevation_angles;
54
         cartesian_vector[2] = rho_values;
55
     }
```

```
/*-----
57
       y = cart2sph(input_matrix, dim)
58
       -----*/
59
       template <typename T>
60
       auto cart2sph(const std::vector<std::vector<T>>& input_matrix,
61
                  const
                           std::size_t
                                                      axis)
63
          // fetching dimensions
          const auto& num_rows
                                {input_matrix.size()};
          const auto& num_cols {input_matrix[0].size()};
67
          // checking the axis and dimensions
68
          if (axis == 0 && num_rows != 3) {std::cerr << "cart2sph: incorrect num-elements
          if (axis == 1 && num_cols != 3) {std::cerr << "cart2sph: incorrect num-elements
70
             n";}
          // creating canvas
72
          auto canvas {std::vector<std::vector<T>>(
73
             num_rows,
             std::vector<T>(num_cols, 0)
          )};
76
          // if axis = 0, performing operation column-wise
          if(axis == 0)
80
             for(auto col = 0; col < num_cols; ++col)</pre>
81
                 // fetching current column
                 auto curr_column {std::vector<T>({input_matrix[0][col],
84
                                                  input_matrix[1][col],
                                                  input_matrix[2][col]})};
                 // performing inplace transformation
88
                 cart2sph_inplace(curr_column);
20
                 // storing it back
                 canvas[0][col] = curr_column[0];
                 canvas[1][col] = curr_column[1];
                 canvas[2][col] = curr_column[2];
             }
95
96
          // if axis == 1, performing operations row-wise
97
          else if(axis == 0)
99
             std::cerr << "cart2sph: yet to be implemented \n";</pre>
100
          }
101
          else
          {
103
             std::cerr << "cart2sph: yet to be implemented \n";</pre>
104
          }
105
          // returning
107
          return std::move(canvas);
108
       }
110
111
       template <typename T>
```

```
sph2cart(const std::vector<T> spherical_vector){
       auto
114
           // creating cartesian vector
                  cartesian_vector {std::vector<T>(spherical_vector.size(), 0)};
           auto
118
           // populating
           cartesian_vector[0] =
                                     spherical_vector[2] * \
120
                                     cos(spherical_vector[1] * std::numbers::pi / 180.00) * \
                                     cos(spherical_vector[0] * std::numbers::pi / 180.00);
           cartesian_vector[1]
                                     spherical_vector[2] * \
                                     cos(spherical_vector[1] * std::numbers::pi / 180.00) * \
124
                                     sin(spherical_vector[0] * std::numbers::pi / 180.00);
125
           cartesian_vector[2]
                                     spherical_vector[2] * \
                                     sin(spherical_vector[1] * std::numbers::pi / 180.00);
127
128
           // returning
129
           return std::move(cartesian_vector);
130
131
132
```

B.7 Cosine

```
#pragma once
  y = cos(input_vector)
  template <typename T>
  auto cos(const std::vector<T>& input_vector)
      // created canvas
8
                      {input_vector};
9
      auto canvas
10
      // calling the function
11
      std::transform(input_vector.begin(), input_vector.end(),
                  canvas.begin(),
                   [](auto& argx){return std::cos(argx);});
      // returning the output
      return std::move(canvas);
17
  }
18
  y = cosd(input_vector)
  -----*/
  template <typename T>
  auto cosd(const std::vector<T> input_vector)
24
      // created canvas
      auto canvas {input_vector};
26
2.7
      // calling the function
      std::transform(input_vector.begin(),
                  input_vector.end(),
30
                  input_vector.begin(),
31
                   [](const auto& argx){return std::cos(argx * 180.00/std::numbers::pi);});
33
      // returning the output
34
```

```
return std::move(canvas);
36 }
```

B.8 Data Structures

```
struct TreeNode {
      int val;
      TreeNode *left;
      TreeNode *right;
      TreeNode() : val(0), left(nullptr), right(nullptr) {}
      TreeNode(int x) : val(x), left(nullptr), right(nullptr) {}
      TreeNode(int x, TreeNode *left, TreeNode *right) : val(x), left(left), right(right)
          {}
  };
9
  struct ListNode {
11
      int val;
12
13
      ListNode *next;
      ListNode() : val(0), next(nullptr) {}
      ListNode(int x) : val(x), next(nullptr) {}
      ListNode(int x, ListNode *next) : val(x), next(next) {}
 };
```

B.9 Editing Index Values

```
#pragma once
  /*-----
  Matlab's equivalent of A[A < 0.5] = 0
  template <typename T, typename U>
                                   input_vector,
  auto edit(std::vector<T>&
                                 bool_vector,
         const std::vector<bool>&
         const U
                                   scalar)
     // throwing an error
     if (input_vector.size() != bool_vector.size())
11
        std::cerr << "edit: incompatible size\n";</pre>
     // overwriting input-vector
14
     std::transform(input_vector.begin(), input_vector.end(),
                bool_vector.begin(),
                input_vector.begin(),
17
                 [&scalar](auto& argx, auto argy){
                    if(argy == true) {return static_cast<T>(scalar);}
                    else
                                   {return argx;}
                });
21
22
     // no-returns since in-place
23
  /*-----
  accumulate version of edit, instead of just placing values
```

```
THings to add
       - ensuring template only accepts int, std::size_t and similar for T2
30
       - bring in histogram method to ensure SIMD
31
32
   template <typename T1,
              typename T2>
  auto edit_accumulate(std::vector<T1>&
                                                   input_vector,
35
                         const std::vector<T2>& indices_to_edit,
36
                         const std::vector<T1>& new_values)
37
38
       // certain checks
39
       if (indices_to_edit.size() != new_values.size())
40
          std::cerr << "svr::edit | edit_accumulate | size-disparity occured \n";</pre>
41
42
       // going through each and accumulating
43
       for(auto
                 i = 0; i < input_vector.size(); ++i){</pre>
                 auto target_index {static_cast<std::size_t>(indices_to_edit[i])}; //
                                       {new_values[i]};
          const auto new_value
46
          if (target_index < input_vector.size()){</pre>
47
              input_vector[target_index] = input_vector[target_index] + new_value;
          }
          else{
50
              // std::cout << "warning: FILE: svr_edit.hpp | FUNCTION: edit_accumulate |</pre>
                  REPORT: index out of bounds";
          }
53
       // no-return since in-place
55
  }
56
```

B.10 Equality

```
#pragma once
  template <typename T, typename U>
   auto operator==(const std::vector<T>& input_vector,
                 const U&
                                          scalar)
      // setting up canvas
      auto canvas {std::vector<bool>(input_vector.size())};
      // writing to canvas
      std::transform(input_vector.begin(), input_vector.end(),
                    canvas.begin(),
                    [&scalar](const auto& argx){
                         return argx == scalar;
16
17
      // returning
18
      return std::move(canvas);
19
20
  }
```

B.11 Exponentiate

```
#pragma once
 namespace svr
     /*-----
     y = exp(vector)
     template <typename T>
     auto exp(const std::vector<T>& input_vector)
        // creating canvas
        auto canvas
                      {input_vector};
11
        // transforming
        std::transform(canvas.begin(), canvas.end(),
                 canvas.begin(),
                 [](auto& argx){return std::exp(argx);});
        // returning
18
        return std::move(canvas);
19
20
     /*-----
     y = exp(matrix)
22
                  23
     template <
        typename sourceType,
        typename destinationType,
        typename = std::enable_if_t<</pre>
           std::is_arithmetic_v<sourceType>
30
     auto exp(
31
        const std::vector<std::vector<sourceType>> input_matrix
33
34
        // fetching dimensions
35
        const auto& num_rows {input_matrix.size()};
        const auto& num_cols {input_matrix[0].size()};
38
        // creating canvas
        auto canvas {std::vector<std::vector<destinationType>>(
           num_rows,
41
           std::vector<destinationType>(num_cols)
        )};
        // writing to each entry
        for(auto row = 0; row < num_rows; ++row)</pre>
46
           std::transform(
47
              input_matrix[row].begin(), input_matrix[row].end(),
              canvas[row].begin(),
49
              [](const auto& argx){
50
                 return std::exp(argx);
           );
53
        // returning
55
        return std::move(canvas);
57
     /*-----
58
     Aim: Exponentiating complex matrices with general floating types
```

```
-----*/
60
      template <
61
         typename T,
62
         typename = std::enable_if_t<</pre>
63
             std::is_floating_point_v<T>
66
      auto exp(
67
         const std::vector<std::complex<T>>> input_matrix
69
      {
70
         // fetching dimensions
71
         const auto& num_rows
                                {input_matrix.size()};
         const auto& num_cols {input_matrix[0].size()};
73
74
         // creating canvas
75
                        {std::vector<std::vector<std::complex<T>>>(
         auto canvas
             num_rows,
77
             std::vector<std::complex<T>>(num_cols)
78
         )};
         // writing to each entry
81
         for(auto row = 0; row < num_rows; ++row)</pre>
82
             std::transform(
                input_matrix[row].begin(), input_matrix[row].end(),
                canvas[row].begin(),
85
                [](const auto& argx){
86
                    return std::exp(argx);
                }
             );
89
90
         // returning
         return std::move(canvas);
93
94
   }
```

B.12 FFT

```
#pragma once
  namespace svr {
     /*----
    For type-deductions
         -----*/
    template <typename T>
     struct fft_result_type;
     // specializations
9
     template <> struct fft_result_type<double>{
       using type = std::complex<double>;
11
     template <> struct fft_result_type<std::complex<double>>{
       using type = std::complex<double>;
     template <> struct fft_result_type<float>{
16
       using type = std::complex<float>;
17
```

```
18
      template <> struct fft_result_type<std::complex<float>>{
19
         using type = std::complex<float>;
20
21
22
      template <typename T>
      using fft_result_t = typename fft_result_type<T>::type;
24
      /*-----
27
      y = fft(x, nfft)
         > calculating n-point dft where n-value is explicit
28
29
      template<typename T>
30
      auto fft(const std::vector<T>& input_vector,
31
             const size_t
                                     nfft)
32
33
         // throwing an error
         if (nfft < input_vector.size()) {std::cerr << "size-mistmatch\n";}</pre>
35
         if (nfft <= 0)</pre>
                                       {std::cerr << "size-mistmatch\n";}
36
37
         // fetching data-type
         using RType = fft_result_t<T>;
39
         using baseType = std::conditional_t<std::is_same_v<T, std::complex<double>>,
                                           double,
                                           T>;
43
         // canvas instantiation
         std::vector<RType> canvas(nfft);
         auto nfft_sqrt {static_cast<RType>(std::sqrt(nfft))};
                finaloutput {std::vector<RType>(nfft, 0)};
47
         auto
         // calculating index by index
         for(int frequency_index = 0; frequency_index<nfft; ++frequency_index){</pre>
50
             RType accumulate_value;
51
             for(int signal_index = 0; signal_index < input_vector.size(); ++signal_index){</pre>
                accumulate_value += \
                   static_cast<RType>(input_vector[signal_index]) * \
                   static_cast<RType>(std::exp(-1.00 * std::numbers::pi * \
                                           (static_cast<baseType>(frequency_index)/static_cast<baseType>(n)
                                          static_cast<baseType>(signal_index)));
57
58
            finaloutput[frequency_index] = accumulate_value / nfft_sqrt;
         }
61
         // returning
62
         return std::move(finaloutput);
63
65
      /*-----
66
67
      y = fft(std::vector<double> nfft) // specialization
      -----*/
      #include <fftw3.h>
                            // for fft
69
      template <>
70
      auto fft(const std::vector<double>& input_vector,
             const std::size_t
72
      {
73
         if (nfft < input_vector.size())</pre>
74
             throw std::runtime_error("nfft must be >= input_vector.size()");
```

```
if (nfft <= 0)</pre>
              throw std::runtime_error("nfft must be > 0");
78
           // FFTW real-to-complex output
79
           std::vector<std::complex<double>> output(nfft);
           // Allocate input (double) and output (fftw_complex) arrays
82
                             = reinterpret_cast<double*>(
           double* in
              fftw_malloc(sizeof(double) * nfft)
           fftw_complex* out = reinterpret_cast<fftw_complex*>(
86
              fftw_malloc(sizeof(fftw_complex) * (nfft/2 + 1))
87
           );
           // Copy input and zero-pad if needed
90
           for (std::size_t i = 0; i < nfft; ++i) {</pre>
               in[i] = (i < input_vector.size()) ? input_vector[i] : 0.0;</pre>
93
94
           // Create FFTW plan and execute
95
           fftw_plan plan = fftw_plan_dft_r2c_1d(
               static_cast<int>(nfft), in, out, FFTW_ESTIMATE
97
98
           fftw_execute(plan);
           // Copy FFTW output to std::vector<std::complex<double>>
101
           for (std::size_t i = 0; i < nfft/2 + 1; ++i) {</pre>
102
              output[i] = std::complex<double>(out[i][0], out[i][1]);
103
104
           // Optional: fill remaining bins with zeros to match full nfft size
105
           for (std::size_t i = nfft/2 + 1; i < nfft; ++i) {</pre>
106
               output[i] = std::complex<double>(0.0, 0.0);
           }
108
109
           // Cleanup
110
           fftw_destroy_plan(plan);
           fftw_free(in);
           fftw_free(out);
115
           // filling up the other half of the output
           const auto halfpoint {static_cast<std::size_t>(nfft/2)};
116
           std::transform(
117
              output.begin() + 1,
                                        // first half (skip DC)
118
              output.begin() + halfpoint, // end of first half
                                // start writing from last element backward (skip
              output.rbegin(),
120
                  Nyquist)
               [](const auto& x) { return std::conj(x); }
121
           );
123
           // returning
124
           return std::move(output);
125
126
127
128
       /*------
       y = ifft(x, nfft)
130
131
       template<typename T>
132
       auto ifft(const std::vector<T>& input_vector)
133
```

```
134
                        // fetching data-type
135
                        using RType = fft_result_t<T>;
136
                        using baseType = std::conditional_t<std::is_same_v<T, std::complex<double>>,
                                                                                                        double.
138
                                                                                                        T>;
140
                       // setup
141
                                                               {input_vector.size()};
                        auto
                                       nfft
143
                        // canvas instantiation
144
                        std::vector<RType> canvas(nfft);
145
                                                                     {static_cast<RType>(std::sqrt(nfft))};
                                       nfft_sqrt
146
                                                                      {std::vector<RType>(nfft, 0)};
147
                        auto
                                       finaloutput
148
                        // calculating index by index
149
                        for(int frequency_index = 0; frequency_index<nfft; ++frequency_index){</pre>
150
                                RType accumulate_value;
151
                                for(int signal_index = 0; signal_index < input_vector.size(); ++signal_index){</pre>
152
                                       accumulate_value += \
153
                                                static_cast<RType>(input_vector[signal_index]) * \
154
                                                static_cast<RType>(std::exp(1.00 * std::numbers::pi * \
                                                                                                       (static_cast<baseType>(frequency_index)/static_cast<baseType>(note: the content of the cont
156
                                                                                                      static_cast<baseType>(signal_index)));
                                finaloutput[frequency_index] = accumulate_value / nfft_sqrt;
160
161
                       // returning
162
                        return std::move(finaloutput);
163
165
                /*-----
166
                x = ifft(std::vector<std::complex<double>> spectrum, nfft)
167
                #include <fftw3.h>
169
                #include <vector>
170
                #include <complex>
171
172
                #include <stdexcept>
173
                auto ifft(const std::vector<std::complex<double>>& input_vector,
174
                                   const std::size_t
175
176
                        if (nfft <= 0)
177
                                throw std::runtime_error("nfft must be > 0");
178
                        if (input_vector.size() != nfft)
                                throw std::runtime_error("input spectrum must be of size nfft");
181
                        // Output: real-valued time-domain sequence
182
183
                        std::vector<double> output(nfft);
184
                        // Allocate FFTW input/output
185
                        fftw_complex* in = reinterpret_cast<fftw_complex*>(
186
                                fftw_malloc(sizeof(fftw_complex) * (nfft/2 + 1))
188
                        double* out = reinterpret_cast<double*>(
189
                                fftw_malloc(sizeof(double) * nfft)
190
                        );
191
```

```
192
            // Copy *only* the first nfft/2+1 bins (rest are redundant due to symmetry)
193
            for (std::size_t i = 0; i < nfft/2 + 1; ++i) {</pre>
194
                in[i][0] = input_vector[i].real();
195
                in[i][1] = input_vector[i].imag();
            }
198
            // Create inverse FFTW plan
            fftw_plan plan = fftw_plan_dft_c2r_1d(
                static_cast<int>(nfft),
201
                in,
202
                out,
203
                FFTW_ESTIMATE
            );
205
206
            fftw_execute(plan);
207
            // Normalize by nfft (FFTW leaves IFFT unscaled)
209
            for (std::size_t i = 0; i < nfft; ++i) {</pre>
210
                output[i] = out[i] / static_cast<double>(nfft);
212
213
            // Cleanup
214
            fftw_destroy_plan(plan);
215
            fftw_free(in);
            fftw_free(out);
217
218
            return output;
219
220
221
222
223
    }
```

B.13 Flipping Containers

```
#pragma once
   namespace svr {
      Mirror image of a vector
      template <typename T>
             fliplr(const std::vector<T>& input_vector)
          // creating canvas
          auto
                canvas
                            {input_vector};
10
          // rewriting
          std::reverse(canvas.begin(), canvas.end());
          // returning
          return std::move(canvas);
      }
17
   }
```

B.14 Indexing

```
#pragma once
  namespace svr {
     /*-----
     y = index(vector, mask)
     template
        typename T1,
         typename T2,
         typename = std::enable_if_t<</pre>
            (std::is_arithmetic_v<T1>
                                              ш
            std::is_same_v<T1, std::complex<float> > ||
            std::is_same_v<T1, std::complex<double> >) &&
            std::is_integral_v<T2>
        -----*/
16
     template
               <typename T1,
17
               typename T2,
18
                typename = std::enable_if_t<std::is_arithmetic_v<T1>
19
                                     std::is_same_v<T1, std::complex<float> > ||
                                     std::is_same_v<T1, std::complex<double> >
21
            index(const
                        std::vector<T1>&
                                           input_vector,
24
     auto
                const
                        std::vector<T2>&
                                           indices_to_sample)
25
26
         // creating canvas
         auto canvas
                        {std::vector<T1>(indices_to_sample.size(), 0)};
         // copying the associated values
         for(int i = 0; i < indices_to_sample.size(); ++i){</pre>
                source_index {indices_to_sample[i]};
            if(source_index < input_vector.size()){</pre>
33
               canvas[i] = input_vector[source_index];
34
            }
            else{
36
               // cout << "warning: Some chosen samples are out of bounds. svr::index |
                  source_index !< input_vector.size()\n";</pre>
            }
39
        }
40
         // returning
        return std::move(canvas);
43
     }
     /*-----
     y = index(matrix, mask, dim)
46
      -----*/
47
     template <</pre>
48
        typename T1,
         typename T2,
50
         typename =
                     std::enable_if_t<
51
            (std::is_same_v<T1, double> || std::is_same_v<T1, float>) &&
            (std::is_same_v<T2, int> || std::is_same_v<T2, std::size_t>)
54
55
     auto index(const std::vector<std::vector<T1>>& input_matrix,
```

```
std::vector<T2>&
                                                          indices_to_sample,
57
                  const
                  const
                           std::size_t&
                                                          dim)
58
59
           // fetching dimensions
60
           const auto& num_rows_matrix
                                              {input_matrix.size()};
61
           const auto& num_cols_matrix
                                              {input_matrix[0].size()};
           // creating canvas
                   canvas {std::vector<std::vector<T1>>()};
           auto
           // if indices are row-indices
67
           if (dim == 0){
68
               // initializing canvas
70
               canvas = std::vector<std::vector<T1>>(
                   num_rows_matrix,
                   std::vector<T1>(indices_to_sample.size())
               );
74
75
               // filling the canvas
               auto destination_index {0};
               std::for_each(
                   indices_to_sample.begin(), indices_to_sample.end(),
                   [&](const auto& col){
                   for(auto row = 0; row < num_rows_matrix; ++row){</pre>
                       if (col <= input_matrix[0].size()){</pre>
82
                           canvas[row] [destination_index] = input_matrix[row][col];
83
                       }
                   ++destination_index;
86
                   });
           }
           else if (\dim == 1){
               // initializing canvas
90
               canvas = std::vector<std::vector<T1>>(
91
                   indices_to_sample.size(),
                   std::vector<T1>(num_cols_matrix)
               );
               // filling the canvas
               #pragma omp parallel for
97
               for(auto row = 0; row < canvas.size(); ++row){</pre>
98
                           destination_col {0};
99
                   std::for_each(indices_to_sample.begin(), indices_to_sample.end(),
100
                                 [&row,
                                  &input_matrix,
102
                                  &destination_col,
                                  &canvas](const auto& source_col){
104
                                       canvas[row][destination_col++] =
105
                                           input_matrix[row] [source_col];
                                 });
106
107
               }
           }
108
           else {
109
               std::cerr << "svr_index | this dim is not implemented \n";</pre>
111
           // moving it back
           return std::move(canvas);
114
```

```
115 }
116 }
```

B.15 Linspace

```
/*-----
  Dependencies
  -----*/
  #pragma once
  #include <vector>
  #include <complex>
8
  namespace svr {
    /*-----
9
10
    in-place
    template <typename T>
    auto linspace(
       auto\&
                   input,
       const auto
                   startvalue,
       const
             auto
                   endvalue,
       const
              auto
                   numpoints
    ) -> void
19
       auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);
20
       for(int i = 0; i<input.size(); ++i) {input[i] = startvalue + i*stepsize;}</pre>
2.1
    /*-----
24
              -----*/
25
    template <typename T>
    auto linspace(
27
       std::vector<std::complex<T>>& input,
2.8
       const auto
                              startvalue,
       const
              auto
                              endvalue,
                              numpoints
       const
              auto
31
    ) -> void
33
       auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);
       for(int i = 0; i<input.size(); ++i) {</pre>
35
          input[i] = startvalue + static_cast<T>(i)*stepsize;
36
37
    };
38
    /*-----
39
40
    template <
       typename T,
       typename = std::enable_if_t<</pre>
43
          std::is_arithmetic_v<T> ||
          std::is_same_v<T, std::complex<float> > ||
          std::is_same_v<T, std::complex<double> >
46
47
    auto linspace(
       const
                        startvalue,
50
       const
                        endvalue,
51
```

```
const
                 std::size_t
                               numpoints
      )
      {
54
         std::vector<T> input(numpoints);
55
         auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);
         for(int i = 0; i<input.size(); ++i) {input[i] = startvalue +</pre>
             static_cast<T>(i)*stepsize;}
         return std::move(input);
      };
      /*----
61
      template <typename T, typename U>
62
      auto linspace(
         const
                               startvalue,
         const
                               endvalue,
65
         const std::size_t numpoints
68
         std::vector<double> input(numpoints);
69
         auto stepsize = static_cast<double>(endvalue -
             startvalue)/static_cast<double>(numpoints-1);
         for(int i = 0; i<input.size(); ++i) {input[i] = startvalue + i*stepsize;}</pre>
         return std::move(input);
72
      };
73
  }
```

B.16 Max

```
#pragma once
  /*-----
  maximum along dimension 1
  template <std::size_t axis, typename T>
       max(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis</pre>
      == 1, std::vector<std::vector<T>> >
      // setting up canvas
      auto canvas
         {std::vector<std::vector<T>>(input_matrix.size(),std::vector<T>(1))};
      // filling up the canvas
11
      for(auto row = 0; row < input_matrix.size(); ++row)</pre>
         canvas[row][0] = *(std::max_element(input_matrix[row].begin(),
            input_matrix[row].end()));
14
      // returning
      return std::move(canvas);
16
  }
17
```

B.17 Meshgrid

```
#pragma once
   #include <vector> // for std::vector
   #include <utility> // for std::pair
   #include <complex> // for std::complex
  10
11
  mesh-grid when working with l-values
   template <typename T>
13
   auto meshgrid(const std::vector<T>& x,
14
             const std::vector<T>& y)
16
17
      // creating and filling x-grid
18
      std::vector<std::vector<T>> xcanvas(y.size(), std::vector<T>(x.size(), 0));
19
      for(auto row = 0; row < y.size(); ++row)</pre>
          std::copy(x.begin(), x.end(), xcanvas[row].begin());
21
22
23
      // creating and filling y-grid
      std::vector<std::vector<T>> ycanvas(y.size(), std::vector<T>(x.size(), 0));
      for(auto col = 0; col < x.size(); ++col)</pre>
25
          for(auto row = 0; row < y.size(); ++row)</pre>
26
             ycanvas[row][col] = y[row];
      // returning
29
      return std::move(std::pair{xcanvas, ycanvas});
30
31
32 }
  /*-----
33
  meshgrid when working with r-values
   template <typename T>
36
   auto meshgrid(std::vector<T>&& x,
37
               std::vector<T>&& y)
38
39
40
      // creating and filling x-grid
41
      std::vector<std::vector<T>> xcanvas(y.size(), std::vector<T>(x.size(), 0));
43
      for(auto row = 0; row < y.size(); ++row)</pre>
          std::copy(x.begin(), x.end(), xcanvas[row].begin());
44
45
      // creating and filling y-grid
46
      std::vector<std::vector<T>> ycanvas(y.size(), std::vector<T>(x.size(), 0));
      for(auto col = 0; col < x.size(); ++col)</pre>
48
          for(auto row = 0; row < y.size(); ++row)</pre>
49
             ycanvas[row][col] = y[row];
50
      // returning
52
      return std::move(std::pair{xcanvas, ycanvas});
53
54
  }
```

B.18 Minimum

```
/*-----
  minimum along dimension 1
  -----*/
  template <std::size_t axis, typename T>
      min(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis ==</pre>
     1, std::vector<std::vector<T>> >
     // creating canvas
8
     auto canvas
9
        {std::vector<std::vector<T>>(input_matrix.size(),std::vector<T>(1))};
10
     // storing the values
11
     for(auto row = 0; row < input_matrix.size(); ++row)</pre>
        canvas[row][0] = *(std::min_element(input_matrix[row].begin(),
           input_matrix[row].end()));
14
     // returning the value
     return std::move(canvas);
16
  }
17
```

B.19 Norm

```
#pragma once
  /*-----
  calculating norm for vector
5 template <typename T>
6 auto norm(const std::vector<T>& input_vector)
     return std::sqrt(
8
        std::inner_product(
9
           input_vector.begin(), input_vector.end(),
10
           input_vector.begin(),
11
           (T)0
        )
13
     );
14
15 }
  /*-----
  Calculating norm of a complex-vector
17
18
19 template
          <>
20 auto norm(const std::vector<std::complex<double>>& input_vector)
21 {
     return std::sqrt(
        std::inner_product(
23
           input_vector.begin(), input_vector.end(),
           input_vector.begin(),
           static_cast<double>(0),
26
           std::plus<double>(),
2.7
           [](const auto& argx,
              const auto& argy){
              return static_cast<double>(
                 (argx * std::conj(argy)).real()
              );
           }
33
        )
34
```

```
);
35
36
37
  /*-----
38
  -----*/
  template <typename T>
        norm(const std::vector<std::vector<T>>& input_matrix,
41
             const std::size_t
                                              dim)
42
43
      // creating canvas
44
      auto
           canvas
                       {std::vector<std::vector<T>>()};
45
      const auto& num_rows_matrix {input_matrix.size()};
46
      const auto& num_cols_matrix {input_matrix[0].size()};
47
48
      // along dim 0
49
      if(dim == 0)
50
         // allocate canvas
52
         canvas = std::vector<std::vector<T>>(
53
             std::vector<T>(input_matrix[0].size())
         );
         // performing norm
         auto accumulate_vector {std::vector<T>(input_matrix[0].size())};
60
         // going through each row
61
         for(auto row = 0; row < num_rows_matrix; ++row)</pre>
             std::transform(input_matrix[row].begin(), input_matrix[row].end(),
                         accumulate_vector.begin(),
                         accumulate_vector.begin(),
                          [](const auto& argx, auto& argy){
                              return argx*argx + argy;
68
                         });
69
         }
         // calculating element-wise square root
         std::for_each(accumulate_vector.begin(), accumulate_vector.end(),
                     [](auto& argx){
                          argx = std::sqrt(argx);
75
                     });
76
77
         // moving to the canvas
         canvas[0] = std::move(accumulate_vector);
79
80
      else if (dim == 1)
81
         // allocating space in the canvas
83
                   = std::vector<std::vector<T>>(
         canvas
84
             input_matrix[0].size(),
             std::vector<T>(1, 0)
87
         );
         // going through each column
         for(auto row = 0; row < num_cols_matrix; ++row){</pre>
             canvas[row][0] = norm(input_matrix[row]);
91
         }
92
```

```
}
94
        else
95
        {
96
            std::cerr << "norm(matrix, dim): dimension operation not defined \n";</pre>
97
98
        // returning
100
        return std::move(canvas);
101
102
    }
103
104
105
106
    Templates to create
107
          matrix and norm-axis
108
            axis instantiated std::vector<T>
109
    */
```

B.20 Division

```
#pragma once
  element-wise division with scalars
  template <typename T>
  auto operator/(const std::vector<T>& input_vector,
              const
                                    input_scalar)
                     T&
8
9
     // creating canvas
                    {input_vector};
     auto canvas
10
     // filling canvas
12
     std::transform(canvas.begin(), canvas.end(),
                 canvas.begin(),
                 [&input_scalar](const auto& argx){
                     return static_cast<double>(argx) /
16
                        static_cast<double>(input_scalar);
                 });
     // returning value
19
     return std::move(canvas);
20
21 }
 /*----
  element-wise division with scalars
  -----*/
24
  template <typename T>
25
  auto operator/=(const std::vector<T>& input_vector,
26
              const
                                     input_scalar)
27
  {
28
     // creating canvas
29
30
     auto canvas
                    {input_vector};
31
     // filling canvas
     std::transform(canvas.begin(), canvas.end(),
                 canvas.begin(),
34
                 [&input_scalar](const auto& argx){
35
```

```
return static_cast<double>(argx) /
                           static_cast<double>(input_scalar);
                   });
37
38
      // returning value
      return std::move(canvas);
40
  }
41
  /*-----
  element-wise with matrix
   -----*/
  template
            <
45
      typename T,
46
                   std::enable_if_t<
      typename
               =
47
         std::is_floating_point_v<T>
48
49
  >
50
         operator/(const std::vector<std::vector<T>>& input_matrix,
  auto
51
                  const T
                                                  scalar)
52
53
  {
      // fetching matrix-dimensions
54
      const auto& num_rows_matrix
                                     {input_matrix.size()};
55
      const auto& num_cols_matrix
                                     {input_matrix[0].size()};
56
      // creating canvas
                     {std::vector<std::vector<T>>(
      auto
           canvas
         num_rows_matrix,
60
         std::vector<T>(num_cols_matrix)
61
      )};
62
63
      // dividing with values
64
               row = 0; row < num_rows_matrix; ++row){</pre>
      for(auto
65
         std::transform(input_matrix[row].begin(), input_matrix[row].end(),
                      canvas[row].begin(),
                      [&scalar](const auto& argx){
68
                          return argx/scalar;
69
                      });
70
      }
71
72
      // returning values
73
74
      return std::move(canvas);
75
  }
  template
             <
76
                numeratorComplexType,
      typename
77
      typename
                denominatorType,
78
                = std::enable_if_t<
79
         std::is_floating_point_v< numeratorComplexType> &&
80
         std::is_arithmetic_v<
                                 denominatorType>
81
82
83
         operator/(
84
  auto
      const std::vector<std::complex<numeratorComplexType>>>& input_matrix,
85
86
      const denominatorType
                                                                    input_scalar
  )
87
  {
88
      // fetching matrix-dimensions
89
      const auto& num_rows_matrix
                                     {input_matrix.size()};
90
                                     {input_matrix[0].size()};
      const auto& num_cols_matrix
91
92
      // creating canvas
```

```
{std::vector<std::complex<numeratorComplexType>>>(
       auto
              canvas
          num_rows_matrix,
95
           std::vector<std::complex<numeratorComplexType>>(num_cols_matrix)
96
       )};
97
98
       // dividing with values
       for(auto row = 0; row < num_rows_matrix; ++row){</pre>
100
           std::transform(
101
              input_matrix[row].begin(), input_matrix[row].end(),
              canvas[row].begin(),
103
              [&input_scalar](const auto& argx){
104
                  return argx /
105
                     static_cast<std::complex<numeratorComplexType>>(input_scalar);
              });
106
108
       // returning values
       return std::move(canvas);
111
    112
   y = std::vector<std::complex<T>> / T
113
114
   template <
115
       typename
                 Τ,
116
117
       typename = std::enable_if_t<</pre>
          std::is_floating_point_v<T>
118
119
120
   >
          operator/(
121
       const std::vector<std::complex<T>>& input_vector,
122
       const
                                           input_scalar
123
   )
124
   {
125
       // creating canvas
126
              canvas {std::vector<std::complex<T>>(input_vector.size())};
127
       // filling the canvas
129
       std::transform(
130
           input_vector.begin(), input_vector.end(),
131
           canvas.begin(),
           [&input_scalar](const auto& argx){
133
              return argx/static_cast<std::complex<T>>(input_scalar);
134
           }
135
       );
136
137
       // returning
138
       return std::move(canvas);
139
   }
140
```

B.21 Addition

```
std::vector<T> operator+(const std::vector<T>& a,
                        const std::vector<T>& b)
  {
8
      // Identify which is bigger
9
      const auto& big = (a.size() > b.size()) ? a : b;
10
      const auto& small = (a.size() > b.size()) ? b : a;
      std::vector<T> result = big; // copy the bigger one
      // Add elements from the smaller one
     for (size_t i = 0; i < small.size(); ++i) {</pre>
16
         result[i] += small[i];
17
19
      return result;
20
21 }
  /*-----
23
  // y = vector + vector
24
25 template <typename T>
std::vector<T>& operator+=(std::vector<T>& a,
                         const std::vector<T>& b) {
2.7
28
      const auto& small = (a.size() < b.size()) ? a : b;</pre>
29
      const auto& big = (a.size() < b.size()) ? b : a;</pre>
31
      // If b is bigger, resize 'a' to match
32
      if (a.size() < b.size())</pre>
                                               {a.resize(b.size());}
     // Add elements
35
      for (size_t i = 0; i < small.size(); ++i) {a[i] += b[i];}</pre>
      // returning elements
      return a:
39
40 }
// y = matrix + matrix
43 template <typename T>
44 std::vector<std::vector<T>> operator+(const std::vector<std::vector<T>>& a,
                                   const std::vector<std::vector<T>>& b)
46
      // fetching dimensions
47
      const auto& num_rows_A {a.size()};
48
      const auto& num_cols_A {a[0].size()};
      const auto& num_rows_B {b.size()};
50
      const auto& num_cols_B {b[0].size()};
51
      // choosing the three different metrics
      if (num_rows_A != num_rows_B && num_cols_A != num_cols_B){
54
         cout << format("a.dimensions = [\{\},\{\}], b.shape = [\{\},\{\}]\n",
55
                       num_rows_A, num_cols_A,
                       num_rows_B, num_cols_B);
         std::cerr << "dimensions don't match\n";</pre>
58
59
      // creating canvas
                    {std::vector<std::vector<T>>(
      auto canvas
62
         std::max(num_rows_A, num_rows_B),
63
         std::vector<T>(std::max(num_cols_A, num_cols_B), (T)0.00)
```

```
)};
65
66
       // performing addition
67
        if (num_rows_A == num_rows_B && num_cols_A == num_cols_B){
68
           for(auto row = 0; row < num_rows_A; ++row){</pre>
69
               std::transform(a[row].begin(), a[row].end(),
                             b[row].begin(),
                             canvas[row].begin(),
                             std::plus<T>());
74
           }
       }
75
       else if(num_rows_A == num_rows_B){
76
77
           // if number of columns are different, check if one of the cols are one
78
                          min_num_cols {std::min(num_cols_A, num_cols_B)};
79
           if (min_num_cols != 1) {std::cerr<< "Operator+: unable to broadcast\n";}</pre>
           const auto
                          max_num_cols {std::max(num_cols_A, num_cols_B)};
           // using references to tag em differently
83
                   auto& big_matrix
                                          {num_cols_A > num_cols_B ? a : b};
           const
                  auto& small_matrix {num_cols_A < num_cols_B ? a : b};</pre>
           // Adding to canvas
           for(auto row = 0; row < canvas.size(); ++row){</pre>
               std::transform(big_matrix[row].begin(), big_matrix[row].end(),
                             canvas[row].begin(),
90
                              [&small_matrix,
91
                              &row](const auto& argx){
92
                                  return argx + small_matrix[row][0];
                             });
94
           }
95
       }
       else if(num_cols_A == num_cols_B){
98
           // check if the smallest column-number is one
99
           const auto min_num_rows {std::min(num_rows_A, num_rows_B)};
           if(min_num_rows != 1)
                                      {std::cerr << "Operator+ : unable to broadcast\n";}
101
                         max_num_rows {std::max(num_rows_A, num_rows_B)};
           const auto
104
           // using references to differentiate the two matrices
                  auto& big_matrix
                                          {num_rows_A > num_rows_B ? a : b};
105
           const
                   auto& small_matrix {num_rows_A < num_rows_B ? a : b};</pre>
106
107
           // adding to canvas
           for(auto row = 0; row < canvas.size(); ++row){</pre>
109
               std::transform(big_matrix[row].begin(), big_matrix[row].end(),
                             small_matrix[0].begin(),
                             canvas[row].begin(),
                              [](const auto& argx, const auto& argy){
113
                              return argx + argy;
114
115
                             });
116
           }
       }
117
       else {
118
           std::cerr << "operator+: yet to be implemented \n";</pre>
119
120
       // returning
       return std::move(canvas);
```

```
/*-----
125
   y = vector + scalar
126
127
   template <typename T>
128
   auto operator+(const std::vector<T>& input_vector,
129
              const
                                  scalar)
130
131
      // creating canvas
133
      auto canvas
                   {input_vector};
134
     // adding scalar to the canvas
135
      std::transform(canvas.begin(), canvas.end(),
136
                 canvas.begin(),
137
                 [&scalar](auto& argx){return argx + scalar;});
138
139
      // returning canvas
      return std::move(canvas);
141
142
   /*-----
   y = scalar + vector
   -----*/
145
   template <typename T>
146
   auto operator+(const T
147
                                   scalar,
             const std::vector<T>& input_vector)
148
149
     // creating canvas
150
      auto canvas
                    {input_vector};
151
152
     // adding scalar to the canvas
153
      std::transform(canvas.begin(), canvas.end(),
154
                 canvas.begin(),
                 [&scalar](auto& argx){return argx + scalar;});
156
157
      // returning canvas
158
      return std::move(canvas);
160
  }
```

B.22 Multiplication (Element-wise)

```
#pragma once
  /*-----
  v = scalar * vector
              .-----*/
  template <typename T>
  auto operator*(
    const T
                       scalar,
         std::vector<T>& input_vector
    const
  )
9
11
    // creating canvas
    auto canvas
                {input_vector};
    // performing operation
    std::for_each(canvas.begin(), canvas.end(),
             [&scalar](auto& argx){argx = argx * scalar;});
    // returning
```

```
return std::move(canvas);
17
  }
18
  20 y = scalar * vector
  _____*/
  template <
     typename T1,
23
     typename T2,
24
     typename = std::enable_if_t<</pre>
26
        !std::is_same_v<std::decay_t<T1>, std::vector<T2> > &&
        std::is_arithmetic_v<T1>
27
28
29 >
30 auto operator*(const T1
                                    scalar,
              const vector<T2>& input_vector)
31
  {
32
     // fetching final-type
33
     using T3 = decltype(std::declval<T1>() * std::declval<T2>());
34
35
     // creating canvas
36
     auto canvas {std::vector<T3>(input_vector.size())};
38
     // multiplying
39
     std::transform(
        input_vector.begin(), input_vector.end(),
        canvas.begin(),
42
         [&scalar](auto& argx){
43
            return static_cast<T3>(scalar) * static_cast<T3>(argx);
        });
46
     // returning
47
     return std::move(canvas);
49 }
  /*-----
50
51 y = scalar * vecotor (subset init)
  template <
53
     typename T,
54
     typename = std::enable_if_t<</pre>
55
56
        std::is_floating_point_v<T>
57
58 >
59 auto operator*(
    const std::complex<T> scalar,
     const std::vector<T>& input_vector
61
62 )
  {
63
     // creating canvas
     auto canvas {std::vector<std::complex<T>>(
65
        input_vector.size()
66
     )};
67
     // copying to canvas
69
     std::transform(
70
        input_vector.begin(), input_vector.end(),
71
         canvas.begin(),
73
         [&scalar](const auto& argx){
            return scalar * static_cast<std::complex<T>>(argx);
74
        }
```

```
);
76
      // moving it back
78
      return std::move(canvas);
79
  }
80
   /*-----
81
   y = vector * scalar
82
   template <typename T>
85
   auto operator*(const std::vector<T>& input_vector,
                 const
                                        scalar)
86
  {
87
      // creating canvas
88
      auto canvas
                     {input_vector};
89
      // multiplying
90
      std::for_each(canvas.begin(), canvas.end(),
                 [&scalar](auto& argx){
                   argx = argx * scalar;
93
                 });
94
      // returning
95
      return std::move(canvas);
  }
97
  /*-----
   y = vector * vector
                     -----*/
   template <typename T>
101
   auto operator*(const std::vector<T>& input_vector_A,
102
              const std::vector<T>& input_vector_B)
103
104
105
      // throwing error: size-desparity
      if (input_vector_A.size() != input_vector_B.size()) {std::cerr << "operator*: size</pre>
106
         disparity \n";}
107
      // creating canvas
108
      auto
                     {input_vector_A};
            canvas
109
      // element-wise multiplying
111
      std::transform(input_vector_B.begin(), input_vector_B.end(),
                  canvas.begin(),
                  canvas.begin(),
                  [](const auto& argx, const auto& argy){
                      return argx * argy;
                  });
117
118
      // moving it back
119
      return std::move(canvas);
120
121
   /*-----
   y = vecotr * vector
123
124
125
   template <
126
      typename T1,
      typename T2,
127
      typename
              = std::enable_if_t<
128
         std::is_arithmetic_v<T1> &&
         std::is_arithmetic_v<T2>
130
131
132 >
         operator*(const
                        std::vector<T1>& input_vector_A,
```

```
std::vector<T2>& input_vector_B)
134
                   const
   {
135
136
       // checking size disparity
       if (input_vector_A.size() != input_vector_B.size())
138
          std::cerr << "operator*: error, size-disparity \n";</pre>
139
140
       // figuring out resulting data type
141
       using T3
                  = decltype(std::declval<T1>() * std::declval<T2>());
143
       // creating canvas
144
            canvas
                        {std::vector<T3>(input_vector_A.size())};
       auto
145
146
       // performing multiplications
147
       std::transform(input_vector_A.begin(), input_vector_A.end(),
148
                    input_vector_B.begin(),
149
                    canvas.begin(),
150
                    [](const
                                          argx,
                               auto&
151
152
                       const
                               auto&
                                          argy){
153
                        return static_cast<T3>(argx) * static_cast<T3>(argy);
                    });
154
       // returning
156
       return std::move(canvas);
157
158
159
   /*-----
160
   y = vector * complex_vector
                 162
   template
163
       typename
                 Τ,
164
       typename = std::enable_if_t<</pre>
165
          std::is_floating_point_v<T>
166
167
168
   >
          operator*(
       const std::vector<T>&
                                          input_vector_A,
170
       const std::vector<std::complex<T>>& input_vector_B
   )
172
173
174
       // checking size issue
       if (input_vector_A.size() != input_vector_B.size())
175
          throw std::runtime_error(
176
              "FILE: svr_operator_star.hpp | FUNCTION: operator* | REPORT: error disparity
177
                  in the two input-vectors"
          );
178
179
       // creating canvas
180
                        {std::vector<std::complex<T>>( input_vector_A.size() )};
       auto canvas
181
182
183
       // filling up the canvas
184
       std::transform(
          input_vector_B.begin(), input_vector_B.end(),
185
          input_vector_A.begin(),
186
          canvas.begin(),
           [](const
                    auto& argx, const auto& argy){
188
              return argx + static_cast<std::complex<T>>(argy);
189
          }
190
       );
191
```

```
192
       // moving it back
       return std::move(canvas);
194
195
   196
   y = complex_vector * vector
197
   -----*/
198
   template <
199
200
       typename
                 Τ,
201
       typename = std::enable_if_t<</pre>
          std::is_floating_point_v<T>
202
203
204
   >
          operator*(
205
       const std::vector<std::complex<T>>& input_vector_A,
206
       const std::vector<T>&
                                          input_vector_B
207
   )
208
   {
209
       // enforcing size
210
       if (input_vector_A.size() != input_vector_B.size())
211
          throw std::runtime_error(
212
              "FILE: svr_operator_star.hpp | FUNCTION: operator* overload"
213
          );
214
215
       // creating canvas
       auto canvas
                        {std::vector<std::complex<T>>(input_vector_A.size())};
217
218
      // filling values
219
       std::transform(
220
          input_vector_A.begin(), input_vector_A.end(),
221
          input_vector_B.begin(),
222
          canvas.begin(),
          [](const auto& argx, const auto& argy){
              return argx * static_cast<std::complex<T>>(argy);
225
          }
226
       );
227
228
       // returning
       return std::move(canvas);
230
231
   }
232
   y = complex-vector * complex-vector
233
234
   template
235
       typename
236
       typename = std::enable_if_t<</pre>
          std::is_floating_point_v<T>
238
239
240
          operator*(
241
   auto
242
       const std::vector<std::complex<T>> input_vector_A,
243
       const std::vector<std::complex<T>> input_vector_B
   )
244
   {
245
       // checking size
246
       if (input_vector_A.size() != input_vector_B.size())
247
          throw std::runtime_error(
248
              "FILE: svr_operator_star.hpp | FUNCTION: operator*(complex-vector,
249
                 complex-vector)"
```

```
);
250
251
       // creating canvas
252
       auto canvas
                        {std::vector<std::complex<T>>(input_vector_A.size())};
253
254
       // filling canvas
255
       std::transform(
256
          input_vector_A.begin(),
                                   input_vector_A.end(),
257
          input_vector_B.begin(),
          canvas.begin(),
259
          [](const auto& argx, const auto& argy){
260
              return argx * argy;
261
          }
262
       );
263
264
       // returning values
265
       return std::move(canvas);
267
   268
   template <typename T>
   auto operator*(const T
270
                const std::vector<std::vector<T>>& inputMatrix)
271
   {
272
       std::vector<std::vector<T>> temp {inputMatrix};
273
       for(int i = 0; i<inputMatrix.size(); ++i){</pre>
          std::transform(inputMatrix[i].begin(),
275
                       inputMatrix[i].end(),
276
                       temp[i].begin(),
277
                        [&scalar](T x){return scalar * x;});
278
279
       return std::move(temp);
280
281
   /*-----
282
   y = matrix * scalar
283
284
   template <typename T>
         operator*(const
                            std::vector<std::vector<T>>& input_matrix,
286
                   const
                                                        scalar)
287
288
289
       // fetching matrix dimensions
       const auto& num_rows_matrix
                                      {input_matrix.size()};
290
                                     {input_matrix[0].size()};
       const auto& num_cols_matrix
291
292
       // creating canvas
293
            canvas
                        {std::vector<std::vector<T>>(
294
          num_rows_matrix,
295
          std::vector<T>(num_cols_matrix)
296
       )};
297
298
       // storing the values
299
       for(auto row = 0; row < num_rows_matrix; ++row)</pre>
300
          std::transform(input_matrix[row].begin(), input_matrix[row].end(),
                       canvas[row].begin(),
302
                        [&scalar](const auto& argx){
303
                            return argx * scalar;
                       });
305
306
       // returning
307
       return std::move(canvas);
```

```
/*-----
310
311
   y = matrix * matrix
312
   template <typename T>
313
   auto operator*(const std::vector<std::vector<T>>& A,
314
                 const std::vector<std::vector<T>>& B) -> std::vector<std::vector<T>>
315
316
317
       // Case 1: element-wise multiplication
       if (A.size() == B.size() && A[0].size() == B[0].size()) {
318
           std::vector<std::vector<T>> C(A.size(), std::vector<T>(A[0].size()));
319
           for (std::size_t row = 0; row < A.size(); ++row) {</pre>
320
               std::transform(A[row].begin(), A[row].end(),
                            B[row].begin(),
322
                             C[row].begin(),
323
                             [](const auto& x, const auto& y){ return x * y; });
324
           }
           return C;
326
327
328
       // Case 2: broadcast column vector
329
       else if (A.size() == B.size() && B[0].size() == 1) {
330
           std::vector<std::vector<T>> C(A.size(), std::vector<T>(A[0].size()));
331
           for (std::size_t row = 0; row < A.size(); ++row) {</pre>
               std::transform(A[row].begin(), A[row].end(),
                            C[row].begin(),
334
                             [&](const auto& x){ return x * B[row][0]; });
335
           }
336
           return C;
337
338
339
       // case 3: when second matrix contains just one row
340
       // case 4: when first matrix is just one column
341
       // case 5: when second matrix is just one column
342
343
       // Otherwise, invalid
       else {
345
           throw std::runtime_error("operator* dimension mismatch");
346
347
348
   }
349
   y = scalar * matrix
350
351
   template <typename T1, typename T2>
   auto operator*(const T1
353
                 const std::vector<std::vector<T2>>& inputMatrix)
354
355
       std::vector<std::vector<T2>> temp {inputMatrix};
356
       for(int i = 0; i<inputMatrix.size(); ++i){</pre>
357
           std::transform(inputMatrix[i].begin(),
358
359
                         inputMatrix[i].end(),
360
                         temp[i].begin(),
                         [&scalar](T2 x){return static_cast<T2>(scalar) * x;});
361
       }
362
       return temp;
364
365
   matrix-multiplication
```

```
template <typename T1, typename T2>
    auto matmul(const std::vector<std::vector<T1>>& matA,
369
               const std::vector<std::vector<T2>>& matB)
370
   {
371
372
       // throwing error
373
        if (matA[0].size() != matB.size()) {std::cerr << "dimension-mismatch \n";}</pre>
374
375
        // getting result-type
       using ResultType = decltype(std::declval<T1>() * std::declval<T2>() + \
377
                                     std::declval<T1>() * std::declval<T2>() );
378
379
       // creating aliasses
380
       auto finalnumrows {matA.size()};
381
       auto finalnumcols {matB[0].size()};
382
383
        // creating placeholder
        auto rowcolproduct = [&](auto rowA, auto colB){
385
           ResultType temp {0};
386
           for(int i = 0; i < matA.size(); ++i) {temp +=</pre>
387
               static_cast<ResultType>(matA[rowA][i]) +
               static_cast<ResultType>(matB[i][colB]);}
           return temp;
388
       };
389
       // producing row-column combinations
391
       std::vector<std::vector<ResultType>> finaloutput(finalnumrows,
392
            std::vector<ResultType>(finalnumcols));
       for(int row = 0; row < finalnumrows; ++row){for(int col = 0; col < finalnumcols;</pre>
393
            ++col){finaloutput[row][col] = rowcolproduct(row, col);}}
394
       // returning
395
       return finaloutput;
396
397
398
   y = matrix * vector
400
    template <
401
402
       typename T,
403
        typename
                  = std::enable_if_t<
           std::is_arithmetic_v<T>
404
405
406
   auto operator*(const std::vector<std::vector<T>>& input_matrix,
407
                  const
                           std::vector<T>&
                                                           input_vector)
408
   {
409
       // fetching dimensions
410
       const auto& num_rows_matrix
                                          {input_matrix.size()};
411
       const
               auto& num_cols_matrix
                                          {input_matrix[0].size()};
412
       const auto& num_rows_vector
                                          {1};
413
414
       const auto& num_cols_vector
                                          {input_vector.size()};
415
       const auto& max_num_rows
                                          {num_rows_matrix > num_rows_vector ?\
                                           num_rows_matrix : num_rows_vector};
416
                                          {num_cols_matrix > num_cols_vector ?\
       const auto& max_num_cols
417
                                           num_cols_matrix : num_cols_vector};
418
419
       // creating canvas
420
                           {std::vector<std::vector<T>>(
421
       auto
               canvas
           max_num_rows,
```

```
std::vector<T>(max_num_cols, 0)
423
       )};
424
425
       // multiplying column matrix with row matrix
426
       if (num_cols_matrix == 1 && num_rows_vector == 1){
427
          // writing to canvas
429
          for(auto row = 0; row < max_num_rows; ++row)</pre>
430
             for(auto col = 0; col < max_num_cols; ++col)</pre>
431
432
                 canvas[row][col] = input_matrix[row][0] * input_vector[col];
433
       /*-----
434
      Multiplying each row with the input-vector
435
       -----*/
436
       else if(
437
          num_cols_matrix == num_cols_vector &&
438
          num_rows_vector == 1
440
441
          // writing to canvas
442
          for(auto row = 0; row < max_num_rows; ++row)</pre>
             std::transform(
444
                 input_matrix[row].begin(), input_matrix[row].end(),
445
                 input_vector.begin(),
446
                 canvas[row].begin(),
                 [](const auto& argx, const auto& argy){return argx * argy;}
448
             );
449
      }
450
      else
451
       {
452
          std::cerr << "Operator*: [matrix, vector] | not implemented \n";</pre>
453
454
455
       // returning
456
       return std::move(canvas);
457
458
459
460
   complex-matrix * vector
461
462
              -----*/
   template <
463
       typename T,
464
      typename
               = std::enable_if_t<
465
          std::is_floating_point_v<T>
467
   >
468
469
   auto
          operator*(
       const std::vector<std::complex<T>>>& input_matrix,
470
       const std::vector<T>&
                                                   input_vector
471
   )
472
473
   {
       // fetching dimensions
       const auto
                  num_rows_matrix
                                     {input_matrix.size()};
475
                                     {input_matrix[0].size()};
                    num_cols_matrix
       const auto
476
                                     {static_cast<std::size_t>(1)};
       const
             auto
                    num_rows_vector
                    num_cols_vector
                                     {input_vector.size()};
478
       const
             auto
479
       // throwing an error
480
       if (num_cols_matrix != num_cols_vector)
```

```
throw std::runtime_error(
482
               "FILE: svr_operator_star.hpp | FUNCTION: operator*(complex-matrix, vector)"
483
           );
484
485
       // creating canvas
486
              canvas
                           {std::vector<std::vector<std::complex<T>>>(
487
           num_rows_matrix,
488
           std::vector<std::complex<T>>(num_cols_matrix)
489
       )};
491
       // performing operations
492
       for(auto row = 0; row < num_rows_matrix; ++row)</pre>
493
           std::transform(
494
               input_matrix[row].begin(), input_matrix[row].end(),
495
               input_vector.begin(),
496
               canvas[row].begin(),
               [](const
                          auto& argx, const auto& argy){
                   return argx * static_cast<std::complex<T>>(argy);
499
500
501
           );
502
        // returning the final output
503
       return std::move(canvas);
504
505
506
507
   martix * complex-vector
508
509
   template
510
       typename
511
                   = std::enable_if_t<
512
        typename
           std::is_floating_point_v<T>
513
514
515
           operator*(
516
   auto
       const std::vector<std::vector<T>>& input_matrix,
517
              std::vector<std::complex<T>>& input_vector
518
   )
519
   {
520
521
       // fetching dimensions
       const auto
                     num_rows_matrix
                                          {input_matrix.size()};
522
       const
              auto
                      num_cols_matrix
                                          {input_matrix[0].size()};
                                          {static_cast<std::size_t>(1)};
       const auto
                     num_rows_vector
524
       const auto
                      num_cols_vector
                                          {input_vector.size()};
525
526
       // fetching dimension mismatch
527
       if (num_cols_matrix != num_cols_vector){
528
           cout << format("input_matrix.shape = [{}, {}], input_vector.shape = [{}, {}]\n",</pre>
               num_rows_matrix, num_cols_matrix, 1, num_cols_vector);
           throw std::runtime_error(
530
531
               "FILE: svr_operator_star.hpp | FUNCTION: operator*(input-matrix,
                   complex-vector)"
           );
532
       }
533
534
535
       // creating canvas
536
                           {std::vector<std::complex<T>>>(
537
       auto
               canvas
           num_rows_matrix,
```

```
std::vector<std::complex<T>>(
539
             num_cols_matrix
540
541
      )};
542
543
      // filling the values
      for(auto row = 0; row < num_rows_matrix; ++row)</pre>
545
          std::transform(
546
             input_matrix[row].begin(), input_matrix[row].end(),
             input_vector.begin(),
548
             canvas[row].begin(),
549
             [](const auto& argx, const auto& argy){
550
                return static_cast<std::complex<T>>(argx) * argy;
551
             }
552
          );
553
      // returning final-output
      return std::move(canvas);
556
557
   /*-----
558
   scalar operators
   -----*/
560
   auto operator*(const std::complex<double> complexscalar,
561
               const double
                                       doublescalar){
562
563
      return complexscalar * static_cast<std::complex<double>>(doublescalar);
564
   auto operator*(const double
                                        doublescalar,
565
               const std::complex<double> complexscalar){
      return complexscalar * static_cast<std::complex<double>>(doublescalar);
567
568
   auto operator*(const std::complex<double> complexscalar,
569
                const int
                                        scalar){
      return complexscalar * static_cast<std::complex<double>>(scalar);
571
572
   auto operator*(const int
573
                                        scalar,
                const std::complex<double> complexscalar){
574
      return complexscalar * static_cast<std::complex<double>>(scalar);
575
   }
576
```

B.23 Subtraction

```
#pragma once
  /*-----
  y = vector - scalar
                   -----*/
  template <typename T>
  auto operator-(const std::vector<T>& a,
           const T
                           scalar){
    std::vector<T> temp(a.size());
    std::transform(a.begin(),
              a.end(),
              temp.begin(),
11
              [scalar](T x){return (x - scalar);});
    return std::move(temp);
  }
```

```
y = vector - vector
  -----*/
17
  template <typename T>
18
  auto operator-(const std::vector<T>& input_vector_A,
19
               const std::vector<T>& input_vector_B)
20
21
      // throwing error
22
      if (input_vector_A.size() != input_vector_B.size())
23
         std::cerr << "operator-(vector, vector): size disparity\n";</pre>
     // creating canvas
26
      const auto& num_cols {input_vector_A.size()};
2.7
      auto
                            {std::vector<T>()};
                 canvas
28
29
      // peforming operations
30
      std::transform(input_vector_A.begin(), input_vector_A.begin(),
31
                  input_vector_B.begin(),
                  canvas.begin(),
33
                   [](const auto& argx, const auto& argy){
34
                      return argx - argy;
35
                   });
37
      // return
38
      return std::move(canvas);
39
  }
40
  /*----
41
42 y = matrix - matrix
43 -----*/
  template <typename T>
45
  auto operator-(const std::vector<std::vector<T>>& input_matrix_A,
               const std::vector<std::vector<T>>& input_matrix_B)
46
47
      // fetching dimensions
48
      const auto& num_rows_A {input_matrix_A.size()};
49
      const auto& num_cols_A {input_matrix_A[0].size()};
50
      const auto& num_rows_B {input_matrix_B.size()};
51
      const auto& num_cols_B {input_matrix_B[0].size()};
52
      // creating canvas
      auto canvas {std::vector<std::vector<T>>()};
56
      // if both matrices are of equal dimensions
57
      if (num_rows_A == num_rows_B && num_cols_A == num_cols_B)
58
      {
59
         // copying one to the canvas
60
         canvas = input_matrix_A;
61
         // subtracting
         for(auto row = 0; row < num_rows_B; ++row)</pre>
64
            std::transform(canvas[row].begin(), canvas[row].end(),
65
                         input_matrix_B[row].begin(),
                         canvas[row].begin(),
                         [](auto& argx, const auto& argy){
68
                             return argx - argy;
69
                         });
70
71
      // column broadcasting (case 1)
72
      else if(num_rows_A == num_rows_B && num_cols_B == 1)
73
      {
```

```
// copying canvas
          canvas = input_matrix_A;
          // substracting
          for(auto row = 0; row < num_rows_A; ++row){</pre>
              std::transform(canvas[row].begin(), canvas[row].end(),
                            canvas[row].begin(),
                             [&input_matrix_B,
                             &row](auto& argx){
                                 return argx - input_matrix_B[row][0];
                            });
          }
      }
      else{
          std::cerr << "operator-: not implemented for this case \n";</pre>
      // returning
      return std::move(canvas);
93
  }
```

B.24 Printing Containers

```
#pragma once
  /*----
4 template<typename T>
 void fPrintVector(const vector<T> input){
     for(auto x: input) cout << x << ",";</pre>
     cout << endl;</pre>
  }
  template<typename T>
 void fpv(vector<T> input){
     for(auto x: input) cout << x << ",";</pre>
     cout << endl;</pre>
14 }
  /*-----
  -----*/
  template<typename T>
void fPrintMatrix(const std::vector<std::vector<T>> input_matrix){
     for(const auto& row: input_matrix)
       cout << format("{}\n", row);</pre>
20
21 }
  template <typename T>
  void fPrintMatrix(const string&
                                     input_string,
25
              const std::vector<std::vector<T>> input_matrix){
26
     cout << format("{} = \n", input_string);</pre>
     for(const auto& row: input_matrix)
       cout << format("{}\n", row);</pre>
29
30 }
  /*-----
33 template<typename T, typename T1>
```

```
void fPrintHashmap(unordered_map<T, T1> input){
    for(auto x: input){
35
       cout << format("[{},{}] | ", x.first, x.second);</pre>
36
37
    cout <<endl;</pre>
38
 }
 /*-----
40
  void fPrintBinaryTree(TreeNode* root){
43
    // sending it back
    if (root == nullptr) return;
44
45
    // printing
    PRINTLINE
    cout << "root->val = " << root->val << endl;</pre>
48
    // calling the children
    fPrintBinaryTree(root->left);
51
    fPrintBinaryTree(root->right);
52
53
    // returning
    return;
55
56
57 }
  /*-----
60 void fPrintLinkedList(ListNode* root){
    if (root == nullptr) return;
    cout << root->val << " -> ";
63
    fPrintLinkedList(root->next);
    return;
64
65 }
  /*-----
  -----*/
67
68 template<typename T>
 void fPrintContainer(T input){
    for(auto x: input) cout << x << ", ";</pre>
    cout << endl;</pre>
    return;
  }
73
  /*-----
  -----*/
76 template <typename T>
77 auto size(std::vector<std::vector<T>> inputMatrix){
    cout << format("[{}, {}]\n",
78
              inputMatrix.size(),
79
              inputMatrix[0].size());
80
81
  /*-----
82
83
 template <typename T>
  auto size(const std::string&
                                    inputstring,
       const std::vector<std::vector<T>>& inputMatrix){
86
    cout << format("{} = [{}, {}]\n",
87
              inputstring,
              inputMatrix.size(),
              inputMatrix[0].size());
90
  }
```

B.25 Random Number Generation

```
#pragma once
4 template <typename T>
5 auto rand(const T min,
          const T max) {
      static std::random_device rd; // Seed
      static std::mt19937 gen(rd()); // Mersenne Twister generator
      std::uniform_real_distribution<> dist(min, max);
      return dist(gen);
10
11 }
   template <typename T>
   auto rand(const T
                                min,
                  T
16
        const
          const std::size_t numelements)
17
18 {
    static std::random_device rd; // Seed
19
    static std::mt19937 gen(rd()); // Mersenne Twister generator
      std::uniform_real_distribution<> dist(min, max);
21
    // building the fianloutput
      vector<T> finaloutput(numelements);
      for(int i = 0; i<finaloutput.size(); ++i) {finaloutput[i] =</pre>
          static_cast<T>(dist(gen));}
      return finaloutput;
27
28 }
31 template <typename T>
32 auto rand(const T
                                 argmin,
33 const T
                                 argmax,
          const std::vector<int> dimensions)
35 {
36
      // throwing an error if dimension is greater than two
37
      if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}</pre>
      // creating random engine
40
      static std::random_device rd; // Seed
      static std::mt19937 gen(rd()); // Mersenne Twister generator
      std::uniform_real_distribution<> dist(argmin, argmax);
43
      // building the finaloutput
      vector<vector<T>> finaloutput;
46
      for(int i = 0; i<dimensions[0]; ++i){</pre>
47
         vector<T> temp;
48
          for(int j = 0; j<dimensions[1]; ++j) {temp.push_back(dist(gen));}</pre>
         // cout << format("\t\t temp = {}\n", temp);
51
          finaloutput.push_back(temp);
54
      // returning the finaloutput
55
      return finaloutput;
```

```
57
   }
58
60
   auto rand(const std::vector<int> dimensions)
       using ReturnType = double;
63
       // throwing an error if dimension is greater than two
       if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}</pre>
67
       // creating random engine
68
       static std::random_device rd; // Seed
       static std::mt19937 gen(rd()); // Mersenne Twister generator
70
       std::uniform_real_distribution<> dist(0.00, 1.00);
71
       // building the finaloutput
       vector<vector<ReturnType>> finaloutput;
74
       for(int i = 0; i<dimensions[0]; ++i){</pre>
75
           vector<ReturnType> temp;
           for(int j = 0; j<dimensions[1]; ++j) {temp.push_back(dist(gen));}</pre>
           finaloutput.push_back(std::move(temp));
78
79
       // returning the finaloutput
       return std::move(finaloutput);
82
83
84 }
   /*----
85
86
   template <typename T>
   auto rand_complex_double(const T
                                                   argmin,
                          const T
                                                   argmax,
89
                           const std::vector<int>& dimensions)
90
   {
91
       // throwing an error if dimension is greater than two
93
       if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}</pre>
       // creating random engine
97
       static std::random_device rd; // Seed
       static std::mt19937 gen(rd()); // Mersenne Twister generator
98
       std::uniform_real_distribution<> dist(argmin, argmax);
99
100
       // building the finaloutput
101
       vector<vector<complex<double>>> finaloutput;
102
       for(int i = 0; i<dimensions[0]; ++i){</pre>
           vector<complex<double>> temp;
104
           for(int j = 0; j<dimensions[1]; ++j)</pre>
105
               {temp.push_back(static_cast<double>(dist(gen)));}
           finaloutput.push_back(std::move(temp));
106
107
       }
108
       // returning the finaloutput
109
       return finaloutput;
   }
111
```

B.26 Reshape

```
#pragma once
  /*-----
  reshaping a matrix into another matrix
  template <std::size_t M, std::size_t N, typename T>
  auto reshape(const std::vector<std::vector<T>>& input_matrix){
      // verifying size stuff
      if (M*N != input_matrix.size() * input_matrix[0].size())
         std::cerr << "Dimensions are quite different\n";</pre>
      // creating canvas
      auto canvas
                      {std::vector<std::vector<T>>(
         M, std::vector<T>(N, (T)0)
      )};
     // writing to canvas
     size_t tid
                          {0};
19
     size_t target_row
                        {0};
                         {0};
     size_t target_col
21
     for(auto row = 0; row<input_matrix.size(); ++row){</pre>
         for(auto col = 0; col < input_matrix[0].size(); ++col){</pre>
                         = row * input_matrix[0].size() + col;
            target_row
                        = tid/N;
25
                        = tid%N;
            target_col
            canvas[target_row][target_col] = input_matrix[row][col];
      }
29
      // moving it back
      return std::move(canvas);
32
33 }
34 /*=================================
35 reshaping a matrix into a vector
                              -----*/
36
  template<std::size_t M, typename T>
37
  auto reshape(const std::vector<std::vector<T>>& input_matrix){
      // checking element-count validity
40
      if (M != input_matrix.size() * input_matrix[0].size())
41
         std::cerr << "Number of elements differ\n";</pre>
     // creating canvas
     auto canvas {std::vector<T>(M, 0)};
      // filling canvas
      for(auto row = 0; row < input_matrix.size(); ++row)</pre>
48
         for(auto col = 0; col < input_matrix[0].size(); ++col)</pre>
49
            canvas[row * input_matrix.size() + col] = input_matrix[row][col];
51
      // moving it back
52
      return std::move(canvas);
53
55
  Matrix to matrix
56
```

```
template<typename T>
   auto reshape(const std::vector<std::vector<T>>& input_matrix,
               const std::size_t
60
               const std::size_t
                                               N){
61
62
       // checking element-count validity
       if ( M * N != input_matrix.size() * input_matrix[0].size())
           std::cerr << "Number of elements differ\n";</pre>
       // creating canvas
       auto
             canvas
                         {std::vector<std::vector<T>>(
68
          M, std::vector<T>(N, (T)0)
       )};
71
       // writing to canvas
       size_t
                              {0};
                tid
       size_t
                target_row
                              {0};
                              {0};
       size_t
                target_col
75
       for(auto row = 0; row<input_matrix.size(); ++row){</pre>
           for(auto col = 0; col < input_matrix[0].size(); ++col){</pre>
                            = row * input_matrix[0].size() + col;
              target_row
                            = tid/N;
              target_col
                            = tid%N;
              canvas[target_row][target_col] = input_matrix[row][col];
           }
83
       // moving it back
       return std::move(canvas);
86
87
   /*-----
   converting a matrix into a vector
   template<typename T>
91
   auto reshape(const std::vector<std::vector<T>>& input_matrix,
               const size_t
                                              M){
       // checking element-count validity
       if (M != input_matrix.size() * input_matrix[0].size())
           std::cerr << "Number of elements differ\n";</pre>
98
       // creating canvas
99
       auto
            canvas
                         {std::vector<T>(M, 0)};
100
101
       // filling canvas
       for(auto row = 0; row < input_matrix.size(); ++row)</pre>
           for(auto col = 0; col < input_matrix[0].size(); ++col)</pre>
104
              canvas[row * input_matrix.size() + col] = input_matrix[row][col];
106
       // moving it back
107
       return std::move(canvas);
108
```

B.27 Summing with containers

```
template <std::size_t axis, typename T>
  auto sum(const std::vector<T>& input_vector) -> std::enable_if_t<axis == 0,</pre>
      std::vector<T>>
  {
     // returning the input as is
     return input_vector;
8
  }
9
  /*-----
11
  template <std::size_t axis, typename T>
  auto sum(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis == 0,</pre>
     std::vector<T>>
14
     // creating canvas
     auto canvas {std::vector<T>(input_matrix[0].size(), 0)};
17
     // filling up the canvas
18
     for(auto row = 0; row < input_matrix.size(); ++row)</pre>
        std::transform(input_matrix[row].begin(), input_matrix[row].end(),
                    canvas.begin(),
                    canvas.begin(),
                    [](auto& argx, auto& argy){return argx + argy;});
     // returning
25
     return std::move(canvas);
26
27
28 }
29 /*-----
  -----*/
  template <std::size_t axis, typename T>
  auto sum(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis == 1,</pre>
      std::vector<std::vector<T>>>
33
     // creating canvas
                 {std::vector<std::vector<T>>(input_matrix.size(),
35
                                       std::vector<T>(1, 0.00))};
36
     // filling up the canvas
     for(auto row = 0; row < input_matrix.size(); ++row)</pre>
39
        canvas[row][0] = std::accumulate(input_matrix[row].begin(),
40
                                    input_matrix[row].end(),
41
                                    static_cast<T>(0));
43
     // returning
     return std::move(canvas);
45
47
  /*-----
48
49
  template <std::size_t axis, typename T>
  auto sum(const std::vector<T>& input_vector_A,
51
         const std::vector<T>& input_vector_B) -> std::enable_if_t<axis == 0,</pre>
            std::vector<T> >
53
     // setup
54
                             {input_vector_A.size()};
     const auto& num_cols_A
55
                             {input_vector_B.size()};
     const auto& num_cols_B
```

```
57
       // throwing errors
58
       if (num_cols_A != num_cols_B) {std::cerr << "sum: size disparity\n";}</pre>
59
60
       // creating canvas
61
       auto canvas {input_vector_A};
63
       // summing up
       std::transform(input_vector_B.begin(), input_vector_B.end(),
                      canvas.begin(),
66
                      canvas.begin(),
67
                      std::plus<T>());
68
69
       // returning
70
       return std::move(canvas);
71
   }
72
```

B.28 Tangent

```
#pragma once
  namespace svr {
     y = tan-inverse(input_vector_A/input_vector_B)
     template <typename T>
     auto atan2(const std::vector<T>
                                    input_vector_A,
             const std::vector<T> input_vector_B)
        // throw error
10
        if (input_vector_A.size() != input_vector_B.size())
11
            std::cerr << "atan2: size disparity\n";</pre>
        // create canvas
        auto canvas
                        {std::vector<T>(input_vector_A.size(), 0)};
        // performing element-wise atan2 calculation
        std::transform(input_vector_A.begin(), input_vector_A.end(),
                    input_vector_B.begin(),
                    canvas.begin(),
                    [](const auto& arg_a,
21
                       const
                              auto& arg_b){
22
                        return std::atan2(arg_a, arg_b);
                    });
26
        // moving things back
        return std::move(canvas);
28
29
     /*-----
30
31
     y = tan-inverse(a/b)
32
     -----*/
     template <typename T>
33
     auto atan2(T scalar_A,
                scalar_B)
              Т
36
        return std::atan2(scalar_A, scalar_B);
```

```
38 }
39 }
```

B.29 Tiling Operations

```
#pragma once
  namespace svr {
     /*----
     tiling a vector
     -----*/
     template <typename T>
     auto tile(const std::vector<T>& input_vector,
             const std::vector<size_t>& mul_dimensions){
        // creating canvas
10
        const std::size_t& num_rows {1 * mul_dimensions[0]};
        const std::size_t& num_cols {input_vector.size() * mul_dimensions[1]};
        auto canvas {std::vector<std::vector<T>>(
13
           num_rows,
           std::vector<T>(num_cols, 0)
        )};
        // writing
        std::size_t
                   source_row;
        std::size_t source_col;
20
2.1
        for(std::size_t row = 0; row < num_rows; ++row){</pre>
           for(std::size_t col = 0; col < num_cols; ++col){</pre>
              source_row = row % 1;
              source_col = col % input_vector.size();
              canvas[row][col] = input_vector[source_col];
2.8
        // returning
        return std::move(canvas);
31
32
     33
     tiling a matrix
35
     template <typename T>
36
     auto tile(const std::vector<std::vector<T>>& input_matrix,
37
            const std::vector<size_t>& mul_dimensions){
39
        // creating canvas
40
        const std::size_t& num_rows {input_matrix.size() * mul_dimensions[0]};
        const std::size_t& num_cols {input_matrix[0].size() * mul_dimensions[1]};
        auto canvas {std::vector<std::vector<T>>(
43
           num_rows,
           std::vector<T>(num_cols, 0)
        )};
        // writing
        std::size_t
                    source_row;
        std::size_t
50
                   source_col;
51
```

```
for(std::size_t row = 0; row < num_rows; ++row){
    for(std::size_t col = 0; col < num_cols; ++col){
        source_row = row % input_matrix.size();
        source_col = col % input_matrix[0].size();
        canvas[row][col] = input_matrix[source_row][source_col];
}

// returning
return std::move(canvas);
}
</pre>
```

B.30 Transpose

```
template <typename T>
   auto transpose(const std::vector<T>& input_vector){
      // creating canvas
      auto canvas
                         {std::vector<std::vector<T>>{
          input_vector.size(),
          std::vector<T>(1)
      }};
11
      // filling canvas
      for(auto i = 0; i < input_vector.size(); ++i){</pre>
          canvas[i][0] = input_vector[i];
      // moving it back
18
      return std::move(canvas);
19
   }
```

B.31 Masking

```
std::cerr << "mask(vector, mask): incompatible size \n";</pre>
17
18
          // creating canas
19
                 num_trues {std::count(mask_vector.begin(),
          auto
2.0
                                      mask_vector.end(),
                                      true)};
          auto
                 canvas
                           {std::vector<T>(num_trues)};
          // copying values
                destination_index {0};
26
          for(auto i = 0; i <input_vector.size(); ++i)</pre>
27
             if (mask_vector[i] == true)
2.8
                 canvas[destination_index++] = input_vector[i];
30
          // returning output
31
          return std::move(canvas);
34
35
      template <typename T>
36
      auto mask(const std::vector<std::vector<T>>&
                                                       input_matrix,
               const
                       std::vector<bool>
                                                       mask_vector)
38
39
          // fetching dimensions
          const auto& num_rows_matrix {input_matrix.size()};
          const auto& num_cols_matrix {input_matrix[0].size()};
42.
          const auto& num_cols_vector
                                             {mask_vector.size()};
43
          // error-checking
45
          if (num_cols_matrix != num_cols_vector)
46
             std::cerr << "mask(matrix, bool-vector): size disparity";</pre>
          // building canvas
          auto
                 num_trues {std::count(mask_vector.begin(),
50
                                      mask_vector.end(),
51
                                      true)};
                           {std::vector<std::vector<T>>(
          auto
                 canvas
             num_rows_matrix,
             std::vector<T>(num_cols_vector, 0)
          )};
57
          // writing values
58
          #pragma omp parallel for
59
          for(auto row = 0; row < num_rows_matrix; ++row){</pre>
                    destination_index {0};
61
             for(auto col = 0; col < num_cols_vector; ++col)</pre>
                 if(mask_vector[col] == true)
                    canvas[row] [destination_index++] = input_matrix[row] [col];
          }
65
66
67
          // returning
          return std::move(canvas);
69
      /*-----
70
      Fetch Indices corresponding to mask true's
      -----*/
72
      auto mask_indices(const std::vector<bool>& mask_vector)
73
74
          // creating canvas
```

```
num_trues {std::count(mask_vector.begin(), mask_vector.end(),
          auto
          auto
                  canvas
                              {std::vector<std::size_t>(num_trues)};
78
          // building canvas
                 destination_index {0};
          for(auto i = 0; i < mask_vector.size(); ++i)</pre>
              if (mask_vector[i] == true)
                  canvas[destination_index++] = i;
          // returning
86
          return std::move(canvas);
87
   }
89
```

B.32 Resetting Containers

B.33 Element-wise squaring

```
#pragma once
  namespace svr {
     /*-----
     Element-wise squaring vector
     template
              <typename T,
               typename = std::enable_if_t<std::is_arithmetic_v<T>>
           square(const std::vector<T>& input_vector)
     auto
        // creating canvas
                        {std::vector<T>(input_vector.size())};
        auto
              canvas
        // peforming calculations
        std::transform(input_vector.begin(), input_vector.end(),
                    canvas.begin(),
                     [](const auto& argx){
                           return argx * argx;
18
```

```
});
19
20
         // moving it back
21
         return std::move(canvas);
22
23
      /*----
      Element-wise squaring vector (in-place)
25
26
      template <typename T,
                typename = std::enable_if_t<std::is_arithmetic_v<T>>
28
29
            square_inplace(std::vector<T>& input_vector)
      void
30
      {
31
         // performing operations
32
         std::transform(input_vector.begin(), input_vector.end(),
33
                     input_vector.begin(),
                     [](auto& argx){
                         return argx * argx;
36
37
                     });
38
      /*-----
      ELement-wise squaring a matrix
40
41
      template <typename T>
      auto
            square(const std::vector<std::vector<T>>& input_matrix)
44
         // fetching dimensions
45
         const auto& num_rows
                              {input_matrix.size()};
46
         const auto& num_cols {input_matrix[0].size()};
47
48
         // creating canvas
49
                        {std::vector<std::vector<T>>(
         auto
              canvas
            num_rows,
51
            std::vector<T>(num_cols, 0)
         )};
53
         // going through each row
55
         #pragma omp parallel for
         for(auto row = 0; row < num_rows; ++row)</pre>
            std::transform(input_matrix[row].begin(), input_matrix[row].end(),
                        canvas[row].begin(),
59
                        [](const auto& argx){
60
                            return argx * argx;
61
                        });
63
         // returning
64
         return std::move(canvas);
      /*-----
67
68
      Squaring for scalars
69
      template <typename T>
      auto square(const T& scalar) {return scalar * scalar;}
71
  }
72
```

B.34 Flooring

```
namespace svr {
      /*-----
      element-wise flooring of a vector-contents
      template <typename T>
      auto floor(const std::vector<T>& input_vector)
         // creating canvas
         auto canvas {std::vector<T>(input_vector.size())};
         // filling the canvas
         std::transform(input_vector.begin(), input_vector.end(),
                    canvas.begin(),
                    [](const auto& argx){
                        return std::floor(argx);
                    });
16
         // returning
18
        return std::move(canvas);
19
20
      /*-----
21
      element-wise flooring of a vector-contents (in-place)
22
23
24
      template <typename T>
          floor_inplace(std::vector<T>& input_vector)
26
         // rewriting the contents
2.7
         std::transform(input_vector.begin(), input_vector.end(),
                    input_vector.begin(),
                    [](auto& argx){
30
                        return std::floor(argx);
31
32
                    });
33
      /*-----
34
      element-wise flooring of matrix-contents
35
                           -----*/
      template <typename T>
37
      auto
          floor(const std::vector<std::vector<T>>& input_matrix)
38
30
         // fetching dimensions
         const auto& num_rows_matrix {input_matrix.size()};
41
         const auto& num_cols_matrix {input_matrix[0].size()};
         // creating canvas
         auto canvas {std::vector<std::vector<T>>(
            num_rows_matrix,
46
            std::vector<T>(num_cols_matrix)
47
         )};
49
         // writing contents
50
         for(auto row = 0; row < num_rows_matrix; ++row)</pre>
            std::transform(input_matrix[row].begin(), input_matrix[row].end(),
                       canvas[row].begin(),
53
                        [](const auto& argx){
54
                           return std::floor(argx);
55
                       });
57
         // returning contents
         return std::move(canvas);
```

```
60
61
      /*-----
62
      element-wise flooring of matrix-contents (in-place)
63
      template <typename T>
      auto
           floor_inplace(std::vector<std::vector<T>>& input_matrix)
66
         // performing operations
         for(auto row = 0; row < input_matrix.size(); ++row)</pre>
69
            std::transform(input_matrix[row].begin(), input_matrix[row].end(),
70
                        input_matrix[row].begin(),
71
                        [](auto& argx){
                             return std::floor(argx);
73
                        });
74
     }
75
  }
```

B.35 Squeeze

```
namespace svr {
      template
                 <typename T>
              squeeze(const std::vector<std::vector<T>>& input_matrix)
          // fetching dimensions
          const auto& num_rows_matrix {input_matrix.size()};
          const auto& num_cols_matrix {input_matrix[0].size()};
          // check if any dimension is 1
          if (num_rows_matrix == 0 || num_cols_matrix == 0)
10
              std::cerr << "at least one dimension should be 1";</pre>
          auto
                 final_length {std::max(num_rows_matrix, num_cols_matrix)};
          // creating canvas
                 canvas
                                {std::vector<T>(final_length)};
          // building canvas
          if (num_rows_matrix == 1)
20
              // filling canvas
21
              std::copy(input_matrix[0].begin(), input_matrix[0].end(),
                       canvas.begin());
          else if(num_cols_matrix == 1)
              // filling canvas
              std::transform(input_matrix.begin(), input_matrix.end(),
28
                            canvas.begin(),
29
                            [](const auto& argx){
                                return argx[0];
                            });
          }
          // returning
          return std::move(canvas);
36
```

```
37 }
38 }
```

B.36 Tensor Initializations

B.37 Real part

```
#pragma once
  namespace svr {
     /*-----
     For type-deductions
     template <typename T>
     struct real_result_type;
     template <> struct real_result_type<std::complex<double>>{
        using type = double;
     template <> struct real_result_type<std::complex<float>>{
        using type = float;
     template <> struct real_result_type<double> {
16
        using type = double;
17
     template <> struct real_result_type<float>{
19
        using type = float;
2.0
     };
     template <typename T>
23
     using real_result_t = typename real_result_type<T>::type;
24
25
     /*-----
     Element-wise real() of a vector
27
28
     template <typename T>
     auto real(const std::vector<T>& input_vector)
```

```
// figure out base-type
          using TCanvas = real_result_t<T>;
33
34
          // creating canvas
          auto canvas
                            {std::vector<TCanvas>(
              input_vector.size()
          )};
          // storing values
          std::transform(input_vector.begin(), input_vector.end(),
41
                         canvas.begin(),
                         [](const auto&
                                             argx){
                             return std::real(argx);
45
          // returning
          return std::move(canvas);
48
49
50
  }
```

B.38 Imaginary part

```
#pragma once
  namespace svr {
     /*-----
    For type-deductions
                    -----*/
    template <typename T>
    struct imag_result_type;
    template <> struct imag_result_type<std::complex<double>>{
11
       using type = double;
    template <> struct imag_result_type<std::complex<float>>{
       using type = float;
    template <> struct imag_result_type<double> {
       using type = double;
17
18
    template <> struct imag_result_type<float>{
19
       using type = float;
20
    };
21
22
    template <typename T>
    using imag_result_t = typename imag_result_type<T>::type;
25
    /*-----
26
    -----*/
    template <typename T>
    auto
        imag(const std::vector<T>& input_vector)
29
       // figure out base-type
       using TCanvas = imag_result_t<T>;
32
33
```

```
// creating canvas
          auto canvas {std::vector<TCanvas>(
35
             input_vector.size()
36
          )};
37
38
          // storing values
          std::transform(input_vector.begin(), input_vector.end(),
40
                         canvas.begin(),
                         [](const auto&
                                          argx){
                            return std::imag(argx);
43
                         });
44
45
          // returning
46
          return std::move(canvas);
47
48
49 }
```