# Autonomous Underwater Vehicle:
# A Surveillance Protocol

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# Preface

This project is an attempt at combining all of my major skills into creating a simulation, imaging, perception and control pipeline for Autonomous Underwater Vehicles (AUV). As such, creating this project involves creating a number of pipelines.

The first pipeline is the signal simulation pipeline. The signal simulation pipeline involves sea-floor point-cloud creation and simulating the signals received by the sensor arrays of the AUV. The signals recorded by the sensor-arrays on the AUV contains information from the surrounding environment. The imaging pipeline performs certain operations on the recorded signals to obtain acoustic images of the surrounding environment. To that end, this pipeline involves the topics of signal processing, linear algebra, signals and systems.

As such, the second pipeline is the imaging pipeline. The inputs to the imaging pipeline is the signals recorded by the different sensor-arrays of the AUV, in addition to the parameters of the AUV and its components. This pipeline involves match-filtering, focussing and beamforming operations to create acoustic images of the surrounding environment. Depending on the number of ULAs present, the imaging pipeline is responsible for creating multiple acoustic images in real-time. Thus, this pipeline involves the topics of Digital Signal Processing, Match-Filtering, Estimation and Detection Theory and so on.

The images created by the imaging pipeline are fed to the perception-to-control pipeline. This pipeline takes in the image formed created from the ULA signals, parameters of AUV and its components, and some historical data, it provides instructions regarding the movement of the AUV. The mapping from the inputs to the controls is called policy. Learning policies is a core part of reinforcement learning. Thus, this pipeline mainly involves the topics of reinforcement learning. And since we'll be using convolutional neural nets and transformers for learning the policies, this pipeline involves a significant amount of machine and deep learning.

The final result is an AUV that is primarily trained to map an area of the sea-floor in a constant surveillance mode. The RL-trained policy will also be trained to deal with different kinds of sea-floor terrains: those containing hills, valleys, and path-obstructing features. Due to the resource constrained nature of the marine vessel, we also prioritize efficient policies in the policy-training pipeline.

The project is currently written in C++. And since there is non-trivial amount of training and adaptive features in the pipelines, we'll be using LibTorch (the C++ API of PyTorch) to enable computation graphs, backpropagation and thereby, learning in our AUV pipeline. However, for the sections where a computation graph is not required we will be writing templated STL code.

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# Chapter 1

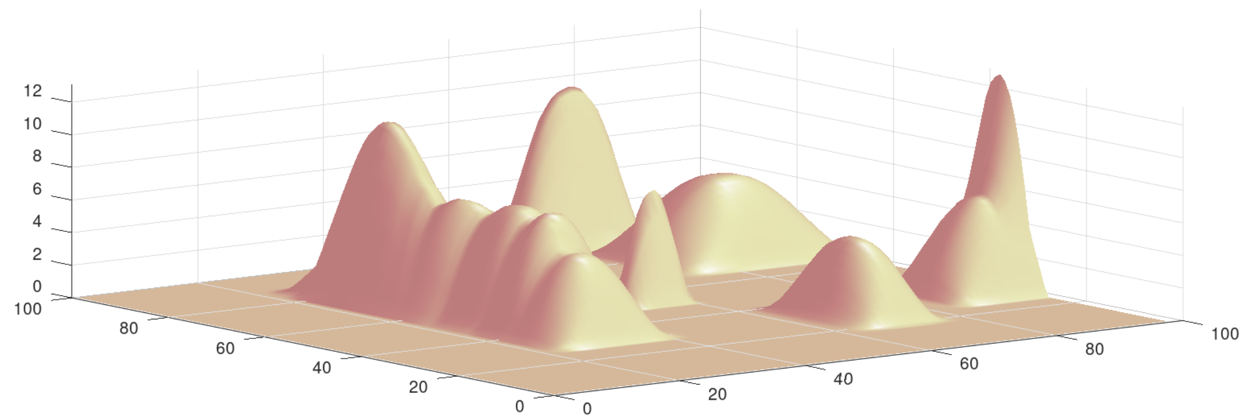
# Underwater Environment

## Overview

All physical matter in this framework is represented using point-clouds. Thus, the sea-floor also is represented using a number of 3D points. In addition to the coordinates, the points also have the additional property of "reflectivity". It is the impulse response of that point.

Sea-floors in real-life are rarely flat. They often contain valleys, mountains, hills and much richer geographical features. Thus, training an agent to function in such environments call for the creation of similar structures in our simulations.

To simplify things, we shall take a more constrained and structured approach. We start by creating different classes of structures and produce instantiations of those structures on the sea-floor. These structures are defined in such a way that the shape and size can be parameterized to enable creation of random sea-floors. The source-code for creation is given in **??**




## 1.1 Underwater Hills

The most basic approach to creating this is to create a flat seafloor, where all the points have the same height. While this is a good place to start, it is good to bring in some realism to the seafloor. To that end, we shall have some rolling hills as the sea-floor. Each "hill " is created using the method outlined in Algorithm 1. The method involves deciding the location of the hills, the dimension of the hills and then designing a hill by combining an exponential function and a cosine function. We're aiming to essentially produce gaussian-looking sea-floor hills. After the creation, this becomes the set of points representing the lowest set of points in the overall seafloor structure.

**Algorithm 1** Hill Creation

1: **Input:** Mean vector $\mathbf{m}$, Dimension vector $\mathbf{d}$, 2D points $\mathbf{P}$
2: **Output:** Updated $\mathbf{P}$ with hill heights
3: num_hills $\leftarrow$ numel($\mathbf{m}_x$)
4: $H \leftarrow$ Zeros tensor of size $(1, \text{numel}(\mathbf{P}_x))$
5: **for** $i = 1$ to num_hills **do**
6: $\quad x_{\text{norm}} \leftarrow \frac{\frac{\pi}{2}(\mathbf{P}_x - \mathbf{m}_x[i])}{\mathbf{d}_x[i]}$
7: $\quad y_{\text{norm}} \leftarrow \frac{\frac{\pi}{2}(\mathbf{P}_y - \mathbf{m}_y[i])}{\mathbf{d}_y[i]}$
8: $\quad h_x \leftarrow \cos(x_{\text{norm}}) \cdot e^{\frac{|x_{\text{norm}}|}{10}}$
9: $\quad h_y \leftarrow \cos(y_{\text{norm}}) \cdot e^{\frac{|y_{\text{norm}}|}{10}}$
10: $\quad h \leftarrow \mathbf{d}_z[i] \cdot h_x \cdot h_y$
11: $\quad$ **Apply boundary conditions:**
12: $\quad$ **if** $x_{\text{norm}} > \frac{\pi}{2}$ or $x_{\text{norm}} < -\frac{\pi}{2}$ or $y_{\text{norm}} > \frac{\pi}{2}$ or $y_{\text{norm}} < -\frac{\pi}{2}$ **then**
13: $\quad\quad h \leftarrow 0$
14: $\quad$ **end if**
15: $\quad H \leftarrow H + h$
16: **end for**
17: $\mathbf{P} \leftarrow \text{concatenate}([\mathbf{P}, H])$

## 1.2 Scatterer Definition

The sea-floor is represented by a single object of the class ScattererClass.

```cpp
/*==============================================================================
Class Declaration
------------------------------------------------------------------------------*/
template <typename T>
class ScattererClass
{
public:
    // members
    std::vector<std::vector<T>> coordinates;
    std::vector<T>              reflectivity;

    // Constructor
    ScattererClass()  {}

    // Constructor
    ScattererClass(std::vector<std::vector<T>> coordinates_arg,
                   std::vector<T>              reflectivity_arg):
                    coordinates(std::move(coordinates_arg)),
                    reflectivity(std::move(reflectivity_arg)) {}

```

```
    // Save to CSV
    void save_to_csv();
};
/*==================================================================================
Saving to CSV-Files
--------------------------------------------------------------------------------*/
template   <typename  T>
void   ScattererClass<T>::save_to_csv()
{
    fWriteMatrix(this->coordinates, "../csv-files/coordinates.csv");
    fWriteVector(this->reflectivity, "../csv-files/reflectivity.csv");
}
```

## 1.3 Sea-Floor Setup Script

Following is the function that will setup the sea-floor script.

```
void fSeaFloorSetup(ScattererClass<double>& scatterers){

    // auto   save_files    {false};
    const  auto    save_files           {false};
    const  auto    hill_creation_flag    {true};

    // sea-floor bounds
    auto    bed_width     {100.00};
    auto    bed_length    {100.00};

    // creating tensors for coordinates and reflectivity
    vector<vector<double>>       box_coordinates;
    vector<double>               box_reflectivity;

    // scatter density
    auto    bed_width_density {static_cast<double>( 10.00)};
    auto    bed_length_density {static_cast<double>( 10.00)};

    // setting up coordinates
    auto    xpoints    {linspace<double>(0.00,
                                        bed_width,
                                        bed_width * bed_width_density)};
    auto    ypoints    {linspace<double>(0.00,
                                        bed_length,
                                        bed_length * bed_length_density)};
    if(save_files) fWriteVector(xpoints, "../csv-files/xpoints.csv");   // verified
    if(save_files) fWriteVector(ypoints, "../csv-files/ypoints.csv");   // verified

    // creating mesh
    auto [xgrid, ygrid] = meshgrid(std::move(xpoints), std::move(ypoints));
    if(save_files) fWriteMatrix(xgrid,  "../csv-files/xgrid.csv");      // verified
    if(save_files) fWriteMatrix(ygrid,  "../csv-files/ygrid.csv");      // verified

    // reshaping
    auto    X      {reshape(xgrid, xgrid.size()*xgrid[0].size())};
    auto    Y      {reshape(ygrid, ygrid.size()*ygrid[0].size())};
    if(save_files) fWriteVector(X,      "../csv-files/X.csv");          // verified
    if(save_files) fWriteVector(Y,      "../csv-files/Y.csv");          // verified

```

```
    // creating heights of scatterers
    if(hill_creation_flag){

        // setting up hill parameters
        auto    num_hills     {10};

        // setting up placement of hills
        auto    points2D            {concatenate<0>(X, Y)};                 // verified
        auto    min2D               {min<1, double>(points2D)};             // verified
        auto    max2D               {max<1, double>(points2D)};             // verified
        auto    hill_2D_center      {min2D + \
                                     rand({2, num_hills}) * (max2D - min2D)}; // verified

        // setup: hill-dimensions
        auto    hill_dimensions_min  {transpose(vector<double>{5, 5, 2})};   // verified
        auto    hill_dimensions_max  {transpose(vector<double>{30, 30, 10})}; // verified
        auto    hill_dimensions      {hill_dimensions_min + \
                                     rand({3, num_hills}) * (hill_dimensions_max -
                                         hill_dimensions_min)};              // verified

        // function-call: hill-creation function
        fCreateHills(hill_2D_center,
                     hill_dimensions,
                     points2D);

        // setting up floor reflectivity
        auto   floorScatter_reflectivity     {std::vector<double>(Y.size(), 1.00)};

        // populating the values of the incoming argument
        scatterers.coordinates    = std::move(points2D);
        scatterers.reflectivity   = std::move(floorScatter_reflectivity);

    }
    else{

        // assigning flat heights
        auto    Z      {std::vector<double>(Y.size(), 0)};

        // setting up floor coordinates
        auto    floorScatter_coordinates     {concatenate<0>(X, Y, Z)};
        auto    floorScatter_reflectivity    {std::vector<double>(Y.size(), 1)};

        // populating the values of the incoming argument
        scatterers.coordinates    = std::move(floorScatter_coordinates);
        scatterers.reflectivity   = std::move(floorScatter_reflectivity);

    }
}
```

# Chapter 2

# Transmitter

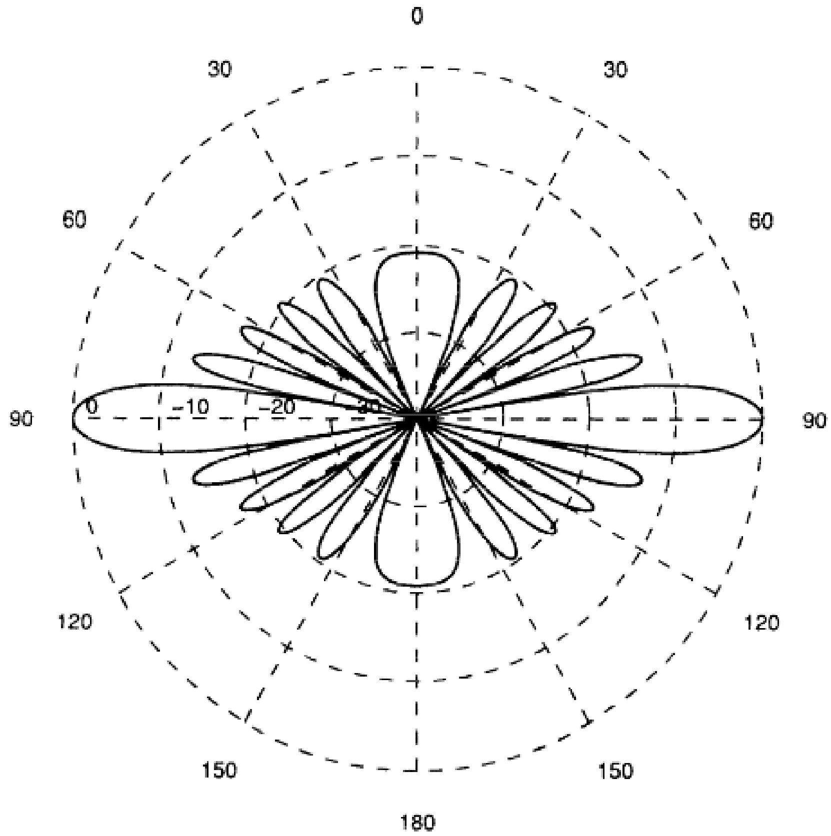



Figure 2.1: Beampattern of a Transmission Uniform Linear Array

## Overview

Probing systems are those systems that send out a signal, listen to the reflection and infer qualitative and quantitative qualities of the environment, matter or object, it was trying to infer information about. The transmitter is one of the most fundamental components of probing systems. As the name suggests, the transmitter is the equipment responsible for sending out the probing signal into the medium.

A transmitter is any device or circuit that converts information into a signal and sends it out onto some media like air, cable, water or space. The components of a transmitter are usually as follows

1. Input: Information containing signal such as voice, data, video etc
2. Process: Encode/modulate the information onto a carrier signal, which can be electromagnetic wave or mechanical wave.
3. Transmission: The signal is then transmitted onto the media with electro-mechanical equipment.

Transmitters are of many kinds. But the ones that we will be considering will be directed transmitters, which means that these transmitters have an associated beampattern. To the uninitiated, this means that the power of the transmitted signal is not transmitted in all directions equally. A beampattern is a graphical representation of the power received by an ideal receiver when placed at different angles.

Transmitters made out of a linear-array of individual transmitters use beamforming to "direct" the major power of the transmitter. These kind of systems have well studied beampatterns which can be utilized in our simulations. These kind of studies and inculcating that in our pipelines produce accurate signal simulation pipelines. For now, we stick to a very simple model of a transmitter. We assume that the transmitter sends out the power equally into a particular cone from the AUV position.

## 2.1 Transmission Signal

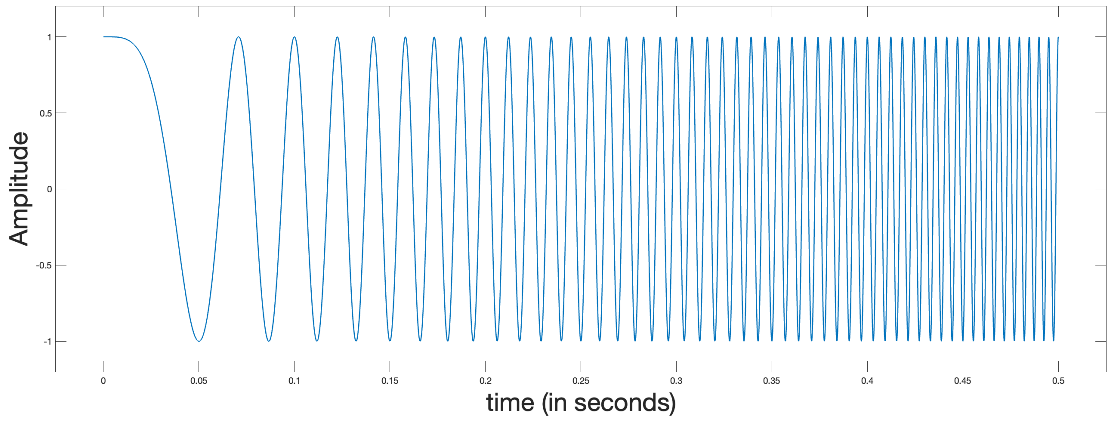



Figure 2.2: Linear Frequency Modulated Wave

The resolution of any probing system is fundamentally tied to the signal bandwidth. A higher bandwidth corresponds to finer resolution $\frac{\text{speed-of-sounds}}{2*\text{bandwidth}}$. Thus, for perfect resolution, an infinite bandwidth is in order. However, infinite bandwidth is impossible for obvious reasons: hardware limitations, spectral regulations, energy limitations and so on.

This is where Linear Frequency Modulation (LFM), also called a "chirp," becomes valuable. An LFM signal linearly sweeps a limited bandwidth over a relatively long duration. This technique spreads the signal's energy in time while retaining the resolution benefits of the bandwidth. After matched filtering (or pulse compression), we essentially produce pulses corresponding to a base-band LFM of same bandwidth. Overall, LFM is a practical compromise between finite bandwidth and desired performance.

One of the best parts about the resolution depending only on the bandwidth is that it allows us to deploy techniques that would help us improve SNRs without virtually increasing the bandwidth at all. Much of the noise in submarine environments are in and around the baseband region (around frequency, 0). Since resolution depends purely on bandwidth, and LFM can be transmitted at a carrier-frequency, this means that processing the returns after low-pass filtering and basebanding allows us to get rid of the submarine noise, since they do not occupy the same frequency-coefficients. The end-result, thus, is improved SNR compared to use baseband LFM.

Due to all of these advantages, LFM waves are ubiquitous in probing systems, from sonar to radar. Thus, for this project too, the transmitter will be using LFM waves as probing signals, to probe the surrounding submarine environment.

## 2.2 Transmitter Class Definition

The transmitter is represented by a single object of the class TransmitterClass.

```
template <typename T>
class TransmitterClass{
public:

    // physical/intrinsic properties
    std::vector<T>    location;             // location tensor
    std::vector<T>    pointing_direction;   // pointing direction

    // basic parameters
    std::vector<T>    Signal;               // transmitted signal (LFM)
    T                 azimuthal_angle;      // transmitter's azimuthal pointing direction
    T                 elevation_angle;      // transmitter's elevation pointing direction
    T                 azimuthal_beamwidth;  // azimuthal beamwidth of transmitter
    T                 elevation_beamwidth;  // elevation beamwidth of transmitter
    T                 range;                // a parameter used for spotlight mode.

    // transmitted signal attributes
    T                 f_low;                // lowest frequency of LFM
    T                 f_high;               // highest frequency of LFM
    T                 fc;                   // center frequency of LFM
    T                 bandwidth;            // bandwidth of LFM

    // shadowing properties
    int               azimuthQuantDensity;      // quantization of angles along the
        azimuth
    int               elevationQuantDensity;    // quantization of angles along the
        elevation
    T                 rangeQuantSize;           // range-cell size when shadowing
    T                 azimuthShadowThreshold;   // azimuth thresholding
    T                 elevationShadowThreshold; // elevation thresholding

    // shadowing related
    std::vector<T>    checkbox;           // box indicating whether a scatter for a
        range-angle pair has been found
```

```
    std::vector<std::vector<std::vector<T>>> finalScatterBox; // a 3D tensor where the
        third dimension represnets the vector length
    std::vector<T> finalReflectivityBox; // to store the reflectivity

    // constructor
    TransmitterClass() = default;

    // Deleting copy constructors/assignment
    TransmitterClass(const TransmitterClass& other)       = delete;
    TransmitterClass& operator=(TransmitterClass& other)  = delete;

    // Creating move-constructor and move-assignment
    TransmitterClass(TransmitterClass&& other)            = default;
    TransmitterClass& operator=(TransmitterClass&& other) =  default;

    /*==========================================================================
    Aim: Update pointing angle
    ----------------------------------------------------------------------------
    Note:
        > This function updates pointing angle based on AUV's pointing angle
        > for now, we're assuming no roll;
    --------------------------------------------------------------------------*/
    auto updatePointingAngle(std::vector<T> AUV_pointing_vector);

};



/*==========================================================================
Aim: Update pointing angle
----------------------------------------------------------------------------
Note:
    > This function updates pointing angle based on AUV's pointing angle
    > for now, we're assuming no roll;
--------------------------------------------------------------------------*/
template <typename T>
auto TransmitterClass<T>::updatePointingAngle(std::vector<T> AUV_pointing_vector)
{

    // calculate yaw and pitch
    auto   AUV_pointing_vector_spherical {svr::cart2sph(AUV_pointing_vector)};
    auto   yaw                           {AUV_pointing_vector_spherical[0]};
    auto   pitch                         {AUV_pointing_vector_spherical[1]};

    // calculating azimuth and elevation of transmitter object
    auto   absolute_azimuth_of_transmitter  {yaw + this->azimuthal_angle};
    auto   absolute_elevation_of_transmitter {pitch + this->elevation_angle};

    // converting back to Cartesian
    auto   pointing_direction_spherical    {std::vector<T>(3, 0)};
    pointing_direction_spherical[0]        = absolute_azimuth_of_transmitter;
    pointing_direction_spherical[1]        = absolute_elevation_of_transmitter;
    pointing_direction_spherical[2]        = 1;
    this->pointing_direction               = svr::sph2cart(pointing_direction_spherical);
}
```

## 2.3 Transmitter Setup Scripts

The following script shows the setup-script

```
template <typename T>
void fTransmitterSetup(TransmitterClass<T>& transmitter_fls,
                     TransmitterClass<T>&  transmitter_portside,
                     TransmitterClass<T>&  transmitter_starboard)
{
    // Setting up transmitter
    T       sampling_frequency    {160e3};              // sampling frequency
    T       f1                    {50e3};               // first frequency of LFM
    T       f2                    {70e3};               // second frequency of LFM
    T       fc                    {(f1 + f2)/2.00};     // finding center-frequency
    T       bandwidth             {std::abs(f2 - f1)};  // bandwidth
    T       pulselength           {5e-2};               // time of recording

    // building LFM
    auto    timearray      {linspace<T>(0.00,
                                  pulselength,
                                  std::floor(pulselength * sampling_frequency))};
    auto    K              {f2 - f1/pulselength}; // calculating frequency-slope
    auto    Signal         {cos(2 * std::numbers::pi * \
                            (f1 + K*timearray) * \
                            timearray)};         // frequency at each time-step, with f1 = 0

    // Setting up transmitter
    auto    location                    {std::vector<T>(3, 0)};      // location of transmitter
    T       azimuthal_angle_fls         {0};                         // initial pointing direction
    T       azimuthal_angle_port        {90};                        // initial pointing direction
    T       azimuthal_angle_starboard   {-90};                       // initial pointing direction

    T       elevation_angle             {-60};                       // initial pointing direction

    T       azimuthal_beamwidth_fls     {20};                        // azimuthal beamwidth of the signal cone
    T       azimuthal_beamwidth_port    {20};                        // azimuthal beamwidth of the signal cone
    T       azimuthal_beamwidth_starboard   {20};                    // azimuthal beamwidth of the signal cone

    T       elevation_beamwidth_fls     {20};                        // elevation beamwidth of the signal cone
    T       elevation_beamwidth_port    {20};                        // elevation beamwidth of the signal cone
    T       elevation_beamwidth_starboard   {20};                    // elevation beamwidth of the signal cone

    int     azimuthQuantDensity         {10};   // number of points, a degree is split into quantization density along azimuth (used for shadowing)
    int     elevationQuantDensity       {10};   // number of points, a degree is split into quantization density along elevation (used for shadowing)
    T       rangeQuantSize              {10};   // the length of a cell (used for
```

```
    shadowing)

T      azimuthShadowThreshold          {1};   // azimuth threshold   (in degrees)
T      elevationShadowThreshold        {1};   // elevation threshold (in degrees)


// transmitter-fls
transmitter_fls.location            = location;                     // Assigning location
transmitter_fls.Signal              = Signal;                       // Assigning signal
transmitter_fls.azimuthal_angle     = azimuthal_angle_fls;          // assigning azimuth angle
transmitter_fls.elevation_angle     = elevation_angle;              // assigning elevation angle
transmitter_fls.azimuthal_beamwidth = azimuthal_beamwidth_fls;      // assigning azimuth-beamwidth
transmitter_fls.elevation_beamwidth = elevation_beamwidth_fls;      // assigning elevation-beamwidth
// updating quantization densities
transmitter_fls.azimuthQuantDensity    = azimuthQuantDensity;     // assigning azimuth quant density
transmitter_fls.elevationQuantDensity  = elevationQuantDensity;   // assigning elevation quant density
transmitter_fls.rangeQuantSize         = rangeQuantSize;          // assigning range-quantization
transmitter_fls.azimuthShadowThreshold = azimuthShadowThreshold;  // azimuth-threshold in shadowing
transmitter_fls.elevationShadowThreshold = elevationShadowThreshold; // elevation-threshold in shadowing
// signal related
transmitter_fls.f_low                  = f1;          // assigning lower frequency
transmitter_fls.f_high                 = f2;          // assigning higher frequency
transmitter_fls.fc                     = fc;          // assigning center frequency
transmitter_fls.bandwidth              = bandwidth;   // assigning bandwidth


// transmitter-portside
transmitter_portside.location             = location;                 // Assigning location
transmitter_portside.Signal               = Signal;                   // Assigning signal
transmitter_portside.azimuthal_angle      = azimuthal_angle_port;     // assigning azimuth angle
transmitter_portside.elevation_angle      = elevation_angle;          // assigning elevation angle
transmitter_portside.azimuthal_beamwidth  = azimuthal_beamwidth_port; // assigning azimuth-beamwidth
transmitter_portside.elevation_beamwidth  = elevation_beamwidth_port; // assigning elevation-beamwidth
// updating quantization densities
transmitter_portside.azimuthQuantDensity  = azimuthQuantDensity;      // assigning azimuth quant density
transmitter_portside.elevationQuantDensity = elevationQuantDensity;    // assigning elevation quant density
transmitter_portside.rangeQuantSize       = rangeQuantSize;           // assigning range-quantization
transmitter_portside.azimuthShadowThreshold = azimuthShadowThreshold;  // azimuth-threshold in shadowing
```

```
    transmitter_portside.elevationShadowThreshold = elevationShadowThreshold; //
        elevation-threshold in shadowing
    // signal related
    transmitter_portside.f_low                = f1;                        // assigning
        lower frequency
    transmitter_portside.f_high               = f2;                        // assigning
        higher frequency
    transmitter_portside.fc                   = fc;                        // assigning
        center frequency
    transmitter_portside.bandwidth            = bandwidth;                 // assigning
        bandwidth


    // transmitter-starboard
    transmitter_starboard.location                = location;                  //
        assigning location
    transmitter_starboard.Signal                  = Signal;                    //
        assigning signal
    transmitter_starboard.azimuthal_angle         = azimuthal_angle_starboard; //
        assigning azimuthal signal
    transmitter_starboard.elevation_angle         = elevation_angle;
    transmitter_starboard.azimuthal_beamwidth     = azimuthal_beamwidth_starboard;
    transmitter_starboard.elevation_beamwidth     = elevation_beamwidth_starboard;
    // updating quantization densities
    transmitter_starboard.azimuthQuantDensity     = azimuthQuantDensity;        //
        assigning azimuth-quant-density
    transmitter_starboard.elevationQuantDensity   = elevationQuantDensity;
    transmitter_starboard.rangeQuantSize          = rangeQuantSize;
    transmitter_starboard.azimuthShadowThreshold  = azimuthShadowThreshold;
    transmitter_starboard.elevationShadowThreshold = elevationShadowThreshold;
    // signal related
    transmitter_starboard.f_low                   = f1;                         //
        assigning lower frequency
    transmitter_starboard.f_high                  = f2;                         //
        assigning higher frequency
    transmitter_starboard.fc                      = fc;                         //
        assigning center frequency
    transmitter_starboard.bandwidth               = bandwidth;                  //
        assigning bandwidth

}
```

# Chapter 3

# Uniform Linear Array

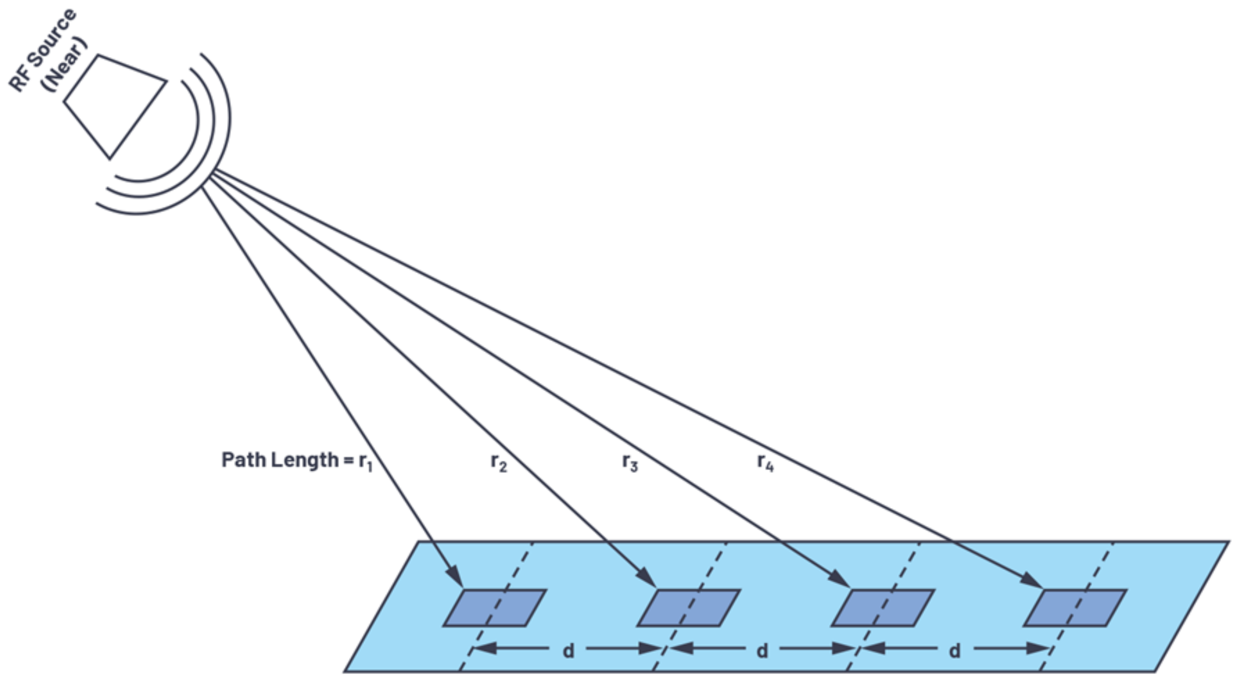



Figure 3.1: Uniform Linear Array

## Overview

A Uniform Linear Array (ULA) is a common antenna or sensor configuration in which multiple elements are arranged in a straight line with equal spacing between adjacent elements. This geometry simplifies both the analysis and implementation of array signal processing techniques. In a ULA, each element receives a version of the incoming signal that differs only in phase, depending on the angle of arrival. This phase difference can be exploited to steer the array's beam in a desired direction (beamforming) or to estimate the direction of arrival (DOA) of multiple sources. The equal spacing also leads to a regular phase progression across the elements, which makes the array's response mathematically tractable and allows the use of tools like the discrete Fourier transform (DFT) to analyze spatial frequency content.

The performance of a ULA depends on the number of elements and their spacing. The spacing is typically chosen to be half the wavelength of the signal to avoid spatial aliasing, also called grating lobes, which can introduce ambiguities in DOA estimation. Increasing the number of elements improves the array's angular resolution and directivity, meaning it can better distinguish closely spaced sources and focus energy more narrowly. ULAs are widely used in radar, sonar, wireless communications, and microphone arrays due to their simplicity, predictable behavior, and compatibility with well-established signal processing algorithms. Their linear structure also makes them easier to implement in hardware compared to more complex array geometries like circular or planar arrays.

## 3.1 ULA Class Definition

The following is the class used to represent the uniform linear array

```
template <typename T>
class ULAClass
{
public:
    // intrinsic parameters
    int                             num_sensors;                            // number of sensors
    T                               inter_element_spacing;                  // space between sensors
    std::vector<std::vector<T>>     coordinates;                            // coordinates of each sensor
    T                               sampling_frequency;                     // sampling frequency of the sensors
    T                               recording_period;                       // recording period of the ULA
    std::vector<T>                  location;                               // location of first coordinate

    // derived
    std::vector<T>                  sensor_direction;
    std::vector<std::vector<T>>     signalMatrix;

    // decimation related
    int                 decimation_factor;                          // the new decimation factor
    T                   post_decimation_sampling_frequency;         // the new sampling frequency
    std::vector<T>      lowpass_filter_coefficients_for_decimation; // filter-coefficients for filtering

    // imaging related
    T   range_resolution;           // theoretical range-resolution = $\frac{c}{2B}$
    T   azimuthal_resolution;       // theoretical azimuth-resolution = $\frac{\lambda}{(N-1)*inter-element-distance}$
    T   range_cell_size;            // the range-cell quanta we're choosing for efficiency trade-off
    T   azimuth_cell_size;          // the azimuth quanta we're choosing
    std::vector<T>  azimuth_centers; // tensor containing the azimuth centeres
    std::vector<T>  range_centers;  // tensor containing the range-centers
    int  frame_size;                // the frame-size corresponding to a range cell in a decimated signal matrix

```

```
    std::vector<std::vector<complex<T>>> mulFFTMatrix; // the matrix containing the delays for each-element as a slot
    std::vector<complex<T>>             matchFilter;  // torch tensor containing the match-filter
    int    num_buffer_zeros_per_frame;                // number of zeros we're adding per frame to ensure no-rotation
    std::vector<std::vector<T>> beamformedImage;      // the beamformed image
    std::vector<std::vector<T>> cartesianImage;       // the cartesian version of beamformed image

    // Artificial acoustic-image related
    std::vector<std::vector<T>> currentArtificialAcousticImage; // acoustic image directly produced


    // Basic Constructor
    ULAClass() = default;

    // constructor
    ULAClass(const int    num_sensors_arg,
             const auto   inter_element_spacing_arg,
             const auto&  coordinates_arg,
             const auto&  sampling_frequency_arg,
             const auto&  recording_period_arg,
             const auto&  location_arg,
             const auto&  signalMatrix_arg,
             const auto&  lowpass_filter_coefficients_for_decimation_arg):
               num_sensors(num_sensors_arg),
               inter_element_spacing(inter_element_spacing_arg),
               coordinates(std::move(coordinates_arg)),
               sampling_frequency(sampling_frequency_arg),
               recording_period(recording_period_arg),
               location(std::move(location_arg)),
               signalMatrix(std::move(signalMatrix_arg)),
               lowpass_filter_coefficients_for_decimation(std::move(lowpass_filter_coefficients_for_decima
    {

        // calculating ULA direction
        sensor_direction = std::vector<T>{coordinates[1][0] - coordinates[0][0],
                                          coordinates[1][1] - coordinates[0][1],
                                          coordinates[1][2] - coordinates[0][2]};

        // normalizing
        auto    norm_value_temp   {std::inner_product(sensor_direction.begin(), sensor_direction.end(),
                                                      sensor_direction.begin(),
                                                      0.00)};

        // dividing
        if (norm_value_temp != 0) {sensor_direction =  sensor_direction / norm_value_temp;}

    }

    // deleting copy constructor/assignment
    // ULAClass(const ULAClass& other)                    = delete;
    // ULAClass& operator=(const   ULAClass&  other)      = delete;


```

```
    /* ===========================================================================
    Aim: Build Coordinates Based On Location
    ------------------------------------------------------------------------ */
    void   buildCoordinatesBasedOnLocation();

    /* ===========================================================================
    Aim: Init
    ------------------------------------------------------------------------ */
    void   init(TransmitterClass<T>&   transmitterObj);

    /* ===========================================================================
    Aim: Creating match-filter
    ------------------------------------------------------------------------ */
    void   nfdc_CreateMatchFilter(TransmitterClass<T>& transmitterObj);

};
```

## 3.2 ULA Setup Scripts

The following script shows the setup-script for Uniform Linear Arrays

```
template <typename T>
void fULASetup(ULAClass<T>&  ula_fls,
              ULAClass<T>&   ula_portside,
              ULAClass<T>&   ula_starboard)
{
    // setting up ula
    auto    num_sensors             {static_cast<int>(64)};          // number of sensors
    T       sampling_frequency      {static_cast<T>(160e3)};         // sampling frequency
    T       inter_element_spacing   {1500/(2*sampling_frequency)};   // space between
        samples
    T       recording_period        {10e-2};                         // sampling-period

    // building the direction for the sensors
    auto    ULA_direction           {std::vector<T>({-1, 0, 0})};
    auto    ULA_direction_norm      {norm(ULA_direction)};
    if (ULA_direction_norm != 0)    {ULA_direction = ULA_direction/ULA_direction_norm;}
    ULA_direction         =         ULA_direction  *   inter_element_spacing;

    // building coordinates for sensors
    auto    ULA_coordinates         {transpose(ULA_direction) * \
                                     linspace<double>(0.00,
                                                     num_sensors -1,
                                                     num_sensors)};

    // coefficients of decimation filter
    auto    lowpassfiltercoefficients {std::vector<T>{0.0000, 0.0000, 0.0000, 0.0000,
        0.0000, 0.0000, 0.0001, 0.0003, 0.0006, 0.0015, 0.0030, 0.0057, 0.0100, 0.0163,
        0.0251, 0.0364, 0.0501, 0.0654, 0.0814, 0.0966, 0.1093, 0.1180, 0.1212, 0.1179,
        0.1078, 0.0914, 0.0699, 0.0451, 0.0192, -0.0053, -0.0262, -0.0416, -0.0504,
        -0.0522, -0.0475, -0.0375, -0.0239, -0.0088, 0.0057, 0.0179, 0.0263, 0.0303,
        0.0298, 0.0253, 0.0177, 0.0086, -0.0008, -0.0091, -0.0153, -0.0187, -0.0191,
        -0.0168, -0.0123, -0.0065, -0.0004, 0.0052, 0.0095, 0.0119, 0.0125, 0.0112,
        0.0084, 0.0046, 0.0006, -0.0031, -0.0060, -0.0078, -0.0082, -0.0075, -0.0057,
        -0.0033, -0.0006, 0.0019, 0.0039, 0.0051, 0.0055, 0.0050, 0.0039, 0.0023, 0.0005,
        -0.0012, -0.0025, -0.0034, -0.0036, -0.0034, -0.0026, -0.0016, -0.0004, 0.0007,
```

```
    0.0016, 0.0022, 0.0024, 0.0023, 0.0018, 0.0011, 0.0003, -0.0004, -0.0011,
    -0.0015, -0.0016, -0.0015}};

    // assigning values
    ula_fls.num_sensors                              = num_sensors;              // assigning number of sensors
    ula_fls.inter_element_spacing                    = inter_element_spacing;    // assigning inter-element spacing
    ula_fls.coordinates                              = ULA_coordinates;          // assigning ULA coordinates
    ula_fls.sampling_frequency                       = sampling_frequency;       // assigning sampling frequencys
    ula_fls.recording_period                         = recording_period;         // assigning recording period
    ula_fls.sensor_direction                         = ULA_direction;            // ULA direction
    ula_fls.lowpass_filter_coefficients_for_decimation = lowpassfiltercoefficients; // storing coefficients


    // assigning values
    ula_portside.num_sensors                              = num_sensors;              // assigning number of sensors
    ula_portside.inter_element_spacing                    = inter_element_spacing;    // assigning inter-element spacing
    ula_portside.coordinates                              = ULA_coordinates;          // assigning ULA coordinates
    ula_portside.sampling_frequency                       = sampling_frequency;       // assigning sampling frequencys
    ula_portside.recording_period                         = recording_period;         // assigning recording period
    ula_portside.sensor_direction                         = ULA_direction;            // ULA direction
    ula_portside.lowpass_filter_coefficients_for_decimation = lowpassfiltercoefficients; // storing coefficients


    // assigning values
    ula_starboard.num_sensors                              = num_sensors;              // assigning number of sensors
    ula_starboard.inter_element_spacing                    = inter_element_spacing;    // assigning inter-element spacing
    ula_starboard.coordinates                              = ULA_coordinates;          // assigning ULA coordinates
    ula_starboard.sampling_frequency                       = sampling_frequency;       // assigning sampling frequencys
    ula_starboard.recording_period                         = recording_period;         // assigning recording period
    ula_starboard.sensor_direction                         = ULA_direction;            // ULA direction
    ula_starboard.lowpass_filter_coefficients_for_decimation =
        lowpassfiltercoefficients; // storing coefficients
}
```

# Chapter 4

# Autonomous Underwater Vehicle

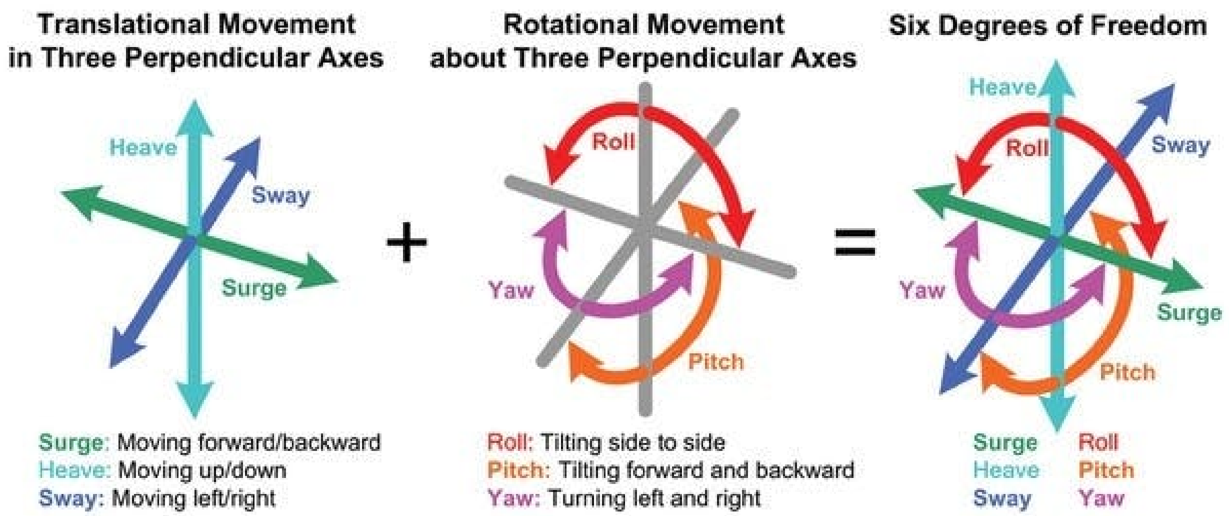



Figure 4.1: AUV degrees of freedom

## Overview

Autonomous Underwater Vehicles (AUVs) are robotic systems designed to operate underwater without direct human control. They navigate and perform missions independently using onboard sensors, processors, and preprogrammed instructions. They are widely used in oceanographic research, environmental monitoring, offshore engineering, and military applications. AUVs can vary in size from small, portable vehicles for shallow water surveys to large, torpedo-shaped platforms capable of deep-sea exploration. Their autonomy allows them to access environments that are too dangerous, remote, or impractical for human divers or tethered vehicles.

The navigation and sensing systems of AUVs are critical to their performance. They typically use a combination of inertial measurement units (IMUs), Doppler velocity logs

(DVLs), pressure sensors, magnetometers, and sometimes acoustic positioning systems to estimate their position and orientation underwater. Since GPS signals do not penetrate water, AUVs must rely on these onboard sensors and occasional surfacing for GPS fixes. They are often equipped with sonar systems, cameras, or other scientific instruments to collect data about the seafloor, water column, or underwater structures. Advanced AUVs can also implement adaptive mission planning and obstacle avoidance, enabling them to respond to changes in the environment in real time.

The applications of AUVs are diverse and expanding rapidly. In scientific research, they are used for mapping the seafloor, studying marine life, and monitoring oceanographic parameters such as temperature, salinity, and currents. In the commercial sector, AUVs inspect pipelines, subsea infrastructure, and offshore oil platforms. Military and defense applications include mine countermeasure operations and underwater surveillance. The development of AUVs continues to focus on increasing endurance, improving autonomy, enhancing sensor payloads, and reducing costs, making them a key technology for exploring and understanding the underwater environment efficiently and safely.

## 4.1 AUV Class Definition

The following is the class used to represent the uniform linear array

```cpp
template <typename T>
class  AUVClass{
public:

    // Intrinsic attributes
    std::vector<T>    location;             // location of vessel
    std::vector<T>    velocity;             // velocity of the vessel
    std::vector<T>    acceleration;         // acceleration of vessel
    std::vector<T>    pointing_direction;   // AUV's pointing direction

    // uniform linear-arrays
    ULAClass<T>       ULA_fls;              // front-looking SONAR ULA
    ULAClass<T>       ULA_portside;         // mounted ULA [object of class, ULAClass]
    ULAClass<T>       ULA_starboard;        // mounted ULA [object of class, ULAClass]

    // transmitters
    TransmitterClass<T>   transmitter_fls;        // transmitter for front-looking SONAR
    TransmitterClass<T>   transmitter_portside;   // portside transmitter
    TransmitterClass<T>   transmitter_starboard;  // starboard transmitter

    // derived or dependent attributes
    std::vector<std::vector<T>>  signalMatrix_1;            // matrix containing the signals obtained from ULA_1
    std::vector<std::vector<T>>  largeSignalMatrix_1;       // matrix holding signal of synthetic aperture
    std::vector<std::vector<T>>  beamformedLargeSignalMatrix; // each column is the beamformed signal at each stop-hop

    // plotting mode
    bool plottingmode;  // to suppress plotting associated with classes

    // spotlight mode related
    std::vector<std::vector<T>>  absolute_coords_patch_cart;  // cartesian coordinates of
```

```
        patch

    // Synthetic Aperture Related
    std::vector<std::vector<T>>  ApertureSensorLocations;     // sensor locations of aperture

    // functions
    void syncComponentAttributes();
    void init();

};
```

## 4.2 AUV Setup Scripts

The following script shows the setup-script for Uniform Linear Arrays

```
template <typename T>
void fAUVSetup(AUVClass<T>& auv) {

    // building properties for the auv
    auto   location             {std::vector<T>{0, 50, 30}};    // starting location
    auto   velocity             {std::vector<T>{5, 0, 0}};      // starting velocity
    auto   pointing_direction   {std::vector<T>{1, 0, 0}};      // pointing direction

    // assigning
    auv.location           = std::move(location);              // assigning location
    auv.velocity           = std::move(velocity);              // assigning velocity
    auv.pointing_direction = std::move(pointing_direction);   // assigning pointing direction

}
```

# Appendix A

# General Purpose Templated Functions

## A.1 CSV File-Writes

```
// ============================================================================
template <typename T>
void fWriteVector(const vector<T>&          inputvector,
                  const string&             filename){

    // opening a file
    std::ofstream fileobj(filename);
    if (!fileobj)  {return;}

    // writing the real parts in the first column and the imaginary parts int he second column
    if constexpr(std::is_same_v<T, std::complex<double>> ||
                 std::is_same_v<T, std::complex<float>> ||
                 std::is_same_v<T, std::complex<long double>>){
        for(int i = 0; i<inputvector.size(); ++i){
            // adding entry
            fileobj << inputvector[i].real() << "+" << inputvector[i].imag() << "i";

            // adding delimiter
            if(i!=inputvector.size()-1) {fileobj << ",";}
            else                        {fileobj << "\n";}
        }
    }
    else{
        for(int i = 0; i<inputvector.size(); ++i){
            fileobj << inputvector[i];
            if(i!=inputvector.size()-1) {fileobj << ",";}
            else                        {fileobj << "\n";}
        }
    }

    // return
    return;
}
// Matrix writing ==============================================================
template <typename T>
auto fWriteMatrix(const std::vector<std::vector<T>> inputMatrix,
                  const string                    filename){
```

```

    // opening a file
    std::ofstream fileobj(filename);

    // writing
    if (fileobj){
        for(int i = 0; i<inputMatrix.size(); ++i){
            for(int j = 0; j<inputMatrix[0].size(); ++j){
                fileobj << inputMatrix[i][j];
                if (j!=inputMatrix[0].size()-1) {fileobj << ",";}
                else                            {fileobj << "\n";}
            }
        }
    }
    else{
        cout << format("File-write to {} failed\n", filename);
    }

}

template <>
auto fWriteMatrix(const std::vector<std::vector<std::complex<double>>> inputMatrix,
                  const string                                         filename){

    // opening a file
    std::ofstream fileobj(filename);

    // writing
    if (fileobj){
        for(int i = 0; i<inputMatrix.size(); ++i){
            for(int j = 0; j<inputMatrix[0].size(); ++j){
                fileobj << inputMatrix[i][j].real() << "+" << inputMatrix[i][j].imag() <<
                    "i";
                if (j!=inputMatrix[0].size()-1) {fileobj << ",";}
                else                            {fileobj << "\n";}
            }
        }
    }
    else{
        cout << format("File-write to {} failed\n", filename);
    }
}
```

## A.2 abs

```
// ======================================================================
// y = abs(vector)
template <typename T>
auto abs(const std::vector<T>&   input_vector)
{
    // creating canvas
    auto    canvas     {input_vector};

    // calculating abs
    std::transform(canvas.begin(),
                   canvas.end(),
```

```
                  canvas.begin(),
                  [](auto& argx){return std::abs(argx);});

    // returning
    return std::move(canvas);
}
// ==========================================================================
// y = abs(matrix)
template <typename T>
auto abs(const std::vector<std::vector<T>> input_matrix)
{
    // creating canvas
    auto    canvas     {input_matrix};

    // applying element-wise abs
    std::transform(input_matrix.begin(),
                  input_matrix.end(),
                  input_matrix.begin(),
                   [](auto& argx){return std::abs(argx);});

    // returning
    return std::move(canvas);
}
```

## A.3 Boolean Comparators

```
// ==========================================================================
template <typename T, typename U>
auto operator<(const  std::vector<T>&   input_vector,
              const   U                 scalar)
{
    // creating canvas
    auto    canvas     {std::vector<bool>(input_vector.size())};

    // transforming
    std::transform(input_vector.begin(), input_vector.end(),
                  canvas.begin(),
                  [&scalar](const auto& argx){
                      return argx < static_cast<T>(scalar);
                  });

    // returning
    return std::move(canvas);
}
// ==========================================================================
template <typename T, typename U>
auto operator<=(const  std::vector<T>&   input_vector,
               const   U                 scalar)
{
    // creating canvas
    auto    canvas     {std::vector<bool>(input_vector.size())};

    // transforming
    std::transform(input_vector.begin(), input_vector.end(),
                  canvas.begin(),
                  [&scalar](const auto& argx){
```

```
                    return argx <= static_cast<T>(scalar);
                });

    // returning
    return std::move(canvas);
}
// ===========================================================================
template <typename T, typename U>
auto operator>(const std::vector<T>&   input_vector,
               const   U                 scalar)
{
    // creating canvas
    auto   canvas     {std::vector<bool>(input_vector.size())};

    // transforming
    std::transform(input_vector.begin(), input_vector.end(),
                   canvas.begin(),
                   [&scalar](const auto& argx){
                       return argx > static_cast<T>(scalar);
                   });

    // returning
    return std::move(canvas);
}
// ===========================================================================
template <typename T, typename U>
auto operator>=(const std::vector<T>&   input_vector,
                const   U                 scalar)
{
    // creating canvas
    auto   canvas     {std::vector<bool>(input_vector.size())};

    // transforming
    std::transform(input_vector.begin(), input_vector.end(),
                   canvas.begin(),
                   [&scalar](const auto& argx){
                       return argx >= static_cast<T>(scalar);
                   });

    // returning
    return std::move(canvas);
}
```

## A.4 Concatenate Functions

```
// input = [vector, vector],
// output = [vector]
template <std::size_t axis, typename T>
auto concatenate(const std::vector<T>&  input_vector_A,
                 const std::vector<T>&   input_vector_B) -> std::enable_if_t<axis == 1,
                     std::vector<T> >
{
    // creating canvas vector
    auto   num_elements   {input_vector_A.size() + input_vector_B.size()};
    auto   canvas         {std::vector<T>(num_elements, (T)0) };

```

```
    // filling up the canvas
    std::copy(input_vector_A.begin(), input_vector_A.end(),
              canvas.begin());
    std::copy(input_vector_B.begin(), input_vector_B.end(),
              canvas.begin()+input_vector_A.size());

    // moving it back
    return std::move(canvas);

}
// ==========================================================
// input = [vector, vector],
// output = [matrix]
template <std::size_t axis, typename T>
auto concatenate(const std::vector<T>& input_vector_A,
                 const std::vector<T>&  input_vector_B) -> std::enable_if_t<axis == 0,
                     std::vector<std::vector<T>> >
{
    // throwing error dimensions
    if (input_vector_A.size() != input_vector_B.size())
        std::cerr << "concatenate:: incorrect dimensions \n";

    // creating canvas
    auto   canvas    {std::vector<std::vector<T>>(
        2, std::vector<T>(input_vector_A.size())
    )};

    // filling up the dimensions
    std::copy(input_vector_A.begin(), input_vector_A.end(), canvas[0].begin());
    std::copy(input_vector_B.begin(), input_vector_B.end(), canvas[1].begin());

    // moving it back
    return std::move(canvas);

}
// ==========================================================
// input = [vector, vector, vector],
// output = [matrix]
template <std::size_t axis, typename T>
auto concatenate(const std::vector<T>& input_vector_A,
                 const std::vector<T>&  input_vector_B,
                 const std::vector<T>&  input_vector_C) -> std::enable_if_t<axis == 0,
                     std::vector<std::vector<T>> >
{
    // throwing error dimensions
    if (input_vector_A.size() != input_vector_B.size() ||
        input_vector_A.size() != input_vector_C.size())
        std::cerr << "concatenate:: incorrect dimensions \n";

    // creating canvas
    auto   canvas    {std::vector<std::vector<T>>(
        3, std::vector<T>(input_vector_A.size())
    )};

    // filling up the dimensions
    std::copy(input_vector_A.begin(), input_vector_A.end(), canvas[0].begin());
    std::copy(input_vector_B.begin(), input_vector_B.end(), canvas[1].begin());
    std::copy(input_vector_C.begin(), input_vector_C.end(), canvas[2].begin());

```

```
    // moving it back
    return std::move(canvas);

}
// ==========================================================
// input = [matrix, vector],
// output = [matrix]
template <std::size_t axis, typename T>
auto concatenate(const std::vector<std::vector<T>>& input_matrix,
                const std::vector<T>              input_vector) -> std::enable_if_t<axis
                    == 0, std::vector<std::vector<T>> >
{
    // creating canvas
    auto   canvas     {input_matrix};

    // adding to the canvas
    canvas.push_back(input_vector);

    // returning
    return std::move(canvas);
}
```

## A.5 Conjugate

```
namespace svr {
    // ==========================================================================
    template   <typename T>
    auto   conj(const std::vector<T>&  input_vector)
    {
        // creating canvas
        auto   canvas     {std::vector<T>(input_vector.size())};

        // calculating conjugates
        std::for_each(canvas.begin(), canvas.end(),
                    [](auto& argx){argx = std::conj(argx);});

        // returning
        return std::move(canvas);
    }
}
```

## A.6 Convolution

```
namespace svr {
    // ==========================================================================
    template   <typename T1, typename T2>
    auto   conv1D(const   std::vector<T1>&   input_vector_A,
                  const   std::vector<T2>&   input_vector_B)
    {
        // resulting type
        using   T3  = decltype(std::declval<T1>() * std::declval<T2>());

        // creating canvas
        auto   canvas_length     {input_vector_A.size() + input_vector_B.size() - 1};
```

```

        // calculating fft of two arrays
        auto    fft_A      {svr::fft(input_vector_A, canvas_length)};
        auto    fft_B      {svr::fft(input_vector_B, canvas_length)};

        // element-wise multiplying the two matrices
        auto    fft_AB     {fft_A *   fft_B};

        // finding inverse FFT
        auto    convolved_result  {ifft(fft_AB)};

        // returning
        return std::move(convolved_result);
    }

}
```

## A.7 Coordinate Change

```
namespace svr {
    // ==========================================================================
    template <typename T>
    auto    cart2sph(const    std::vector<T> cartesian_vector){

        // splatting the point onto xy-plane
        auto    xysplat    {cartesian_vector};
        xysplat[2]     =   0;

        // finding splat lengths
        auto    xysplat_lengths   {norm(xysplat)};

        // finding azimuthal and elevation angles
        auto    azimuthal_angles  {svr::atan2(xysplat[1], xysplat[0]) *
            180.00/std::numbers::pi};
        auto    elevation_angles  {svr::atan2(cartesian_vector[2], xysplat_lengths) *
            180.00/std::numbers::pi};
        auto    rho_values        {norm(cartesian_vector)};

        // creating tensor to send back
        auto    spherical_vector  {std::vector<T>{azimuthal_angles,
                                                  elevation_angles,
                                                  rho_values}};

        // moving it back
        return std::move(spherical_vector);

    }
    // ==========================================================================
    template <typename T>
    auto    sph2cart(const std::vector<T> spherical_vector){

        // creating cartesian vector
        auto    cartesian_vector  {std::vector<T>(spherical_vector.size(), 0)};

        // populating
        cartesian_vector[0]    =   spherical_vector[2] * \
```

```
                                cos(spherical_vector[1] * std::numbers::pi / 180.00) * \
                                cos(spherical_vector[0] * std::numbers::pi / 180.00);
        cartesian_vector[1]   =  spherical_vector[2] * \
                                cos(spherical_vector[1] * std::numbers::pi / 180.00) * \
                                sin(spherical_vector[0] * std::numbers::pi / 180.00);
        cartesian_vector[2]   =  spherical_vector[2] * \
                                sin(spherical_vector[1] * std::numbers::pi / 180.00);

        // returning
        return std::move(cartesian_vector);
    }
}
```

## A.8 Cosine

```
// ===========================================================================
// y = cos(input_vector)
template <typename T>
auto cos(const std::vector<T>&  input_vector)
{
    // created canvas
    auto    canvas     {input_vector};

    // calling the function
    std::transform(input_vector.begin(), input_vector.end(),
                   canvas.begin(),
                   [](auto& argx){return std::cos(argx);});

    // returning the output
    return std::move(canvas);
}
// ===========================================================================
// y = cosd(input_vector)
template <typename T>
auto cosd(std::vector<T>  input_vector)
{
    // created canvas
    auto    canvas     {input_vector};

    // calling the function
    std::transform(input_vector.begin(),
                   input_vector.end(),
                   input_vector.begin(),
                   [](const auto& argx){return std::cos(argx * 180.00/std::numbers::pi);});

    // returning the output
    return std::move(canvas);
}
```

## A.9 Data Structures

```
struct TreeNode {
    int val;
    TreeNode *left;
```

```
    TreeNode *right;
    TreeNode() : val(0), left(nullptr), right(nullptr) {}
    TreeNode(int x) : val(x), left(nullptr), right(nullptr) {}
    TreeNode(int x, TreeNode *left, TreeNode *right) : val(x), left(left), right(right)
        {}
};


struct ListNode {
    int val;
    ListNode *next;
    ListNode() : val(0), next(nullptr) {}
    ListNode(int x) : val(x), next(nullptr) {}
    ListNode(int x, ListNode *next) : val(x), next(next) {}
};
```

## A.10 Editing Index Values

```
// ============================================================================
template <typename T, typename BooleanVector, typename U>
auto edit(std::vector<T>&               input_vector,
          BooleanVector                 bool_vector,
          const    U                    scalar)
{
    // throwing an error
    if (input_vector.size() != bool_vector.size())
        std::cerr << "edit: incompatible size\n";

    // overwriting input-vector
    std::transform(input_vector.begin(), input_vector.end(),
                   bool_vector.begin(),
                   input_vector.begin(),
                   [&scalar](auto& argx, auto argy){
                       if(argy == true)  {return static_cast<T>(scalar);}
                       else              {return argx;}
                   });

    // no-returns since in-place
}
```

## A.11 Equality

```
// ============================================================================
template <typename T, typename U>
auto operator==(const std::vector<T>&   input_vector,
                const  U&               scalar)
{
    // setting up canvas
    auto    canvas     {std::vector<bool>(input_vector.size())};

    // writing to canvas
    std::transform(input_vector.begin(), input_vector.end(),
                   canvas.begin(),
                   [&scalar](const auto& argx){
```

```
                    return argx == scalar;
                });

    // returning
    return std::move(canvas);
}
```

## A.12 Exponentiate

```
// y = abs(vector)
template <typename T>
auto exp(const std::vector<T>&  input_vector)
{
    // creating canvas
    auto   canvas    {input_vector};

    // transforming
    std::transform(canvas.begin(), canvas.end(),
                   canvas.begin(),
                   [](auto& argx){return std::exp(argx);});

    // returning
    return std::move(canvas);
}
```

## A.13 FFT

```
namespace  svr {
    // ==========================================================================
    // For type-deductions
    template <typename T>
    struct fft_result_type;

    // specializations
    template <> struct fft_result_type<double>{
        using type = std::complex<double>;
    };
    template <> struct fft_result_type<std::complex<double>>{
        using type = std::complex<double>;
    };
    template <> struct fft_result_type<float>{
        using type = std::complex<float>;
    };
    template <> struct fft_result_type<std::complex<float>>{
        using type = std::complex<float>;
    };

    template <typename T>
    using  fft_result_t   = typename fft_result_type<T>::type;

    // ======================================================================
    // y = fft(x, nfft)
    template<typename T>
    auto fft(const    std::vector<T>&   input_vector,
```

```
         const     size_t           nfft)
{
    // throwing an error
    if (nfft < input_vector.size()) {std::cerr << "size-mismatch\n";}
    if (nfft <= 0)                  {std::cerr << "size-mismatch\n";}

    // fetching data-type
    using  RType   = fft_result_t<T>;
    using  baseType   = std::conditional_t<std::is_same_v<T, std::complex<double>>,
                                      double,
                                      T>;

    // canvas instantiation
    std::vector<RType> canvas(nfft);
    auto   nfft_sqrt     {static_cast<RType>(std::sqrt(nfft))};
    auto   finaloutput   {std::vector<RType>(nfft, 0)};

    // calculating index by index
    for(int frequency_index = 0; frequency_index<nfft; ++frequency_index){
        RType  accumulate_value;
        for(int signal_index = 0; signal_index < input_vector.size(); ++signal_index){
            accumulate_value += \
                static_cast<RType>(input_vector[signal_index]) * \
                static_cast<RType>(std::exp(-1.00 * std::numbers::pi * \
                                     (static_cast<baseType>(frequency_index)/static_cast<baseType>(n
                                         * \
                                     static_cast<baseType>(signal_index)));
        }
        finaloutput[frequency_index] = accumulate_value / nfft_sqrt;
    }

    // returning
    return std::move(finaloutput);
}

// =====================================================================
// y = ifft(x, nfft)
template<typename T>
auto ifft(const    std::vector<T>&   input_vector)
{
    // fetching data-type
    using  RType   = fft_result_t<T>;
    using  baseType   = std::conditional_t<std::is_same_v<T, std::complex<double>>,
                                      double,
                                      T>;

    //  setup
    auto   nfft       {input_vector.size()};

    // canvas instantiation
    std::vector<RType> canvas(nfft);
    auto   nfft_sqrt     {static_cast<RType>(std::sqrt(nfft))};
    auto   finaloutput   {std::vector<RType>(nfft, 0)};

    // calculating index by index
    for(int frequency_index = 0; frequency_index<nfft; ++frequency_index){
        RType  accumulate_value;
        for(int signal_index = 0; signal_index < input_vector.size(); ++signal_index){
            accumulate_value += \
```

```
                    static_cast<RType>(input_vector[signal_index]) * \
                    static_cast<RType>(std::exp(1.00 * std::numbers::pi * \
                                            (static_cast<baseType>(frequency_index)/static_cast<baseType>(n
                                                * \
                                            static_cast<baseType>(signal_index)));
            }
            finaloutput[frequency_index] = accumulate_value / nfft_sqrt;
        }

        // returning
        return std::move(finaloutput);
    }
}
```

## A.14 Flipping Containers

```
namespace svr {
    // =======================================================================
    template <typename T>
    auto    fliplr(const   std::vector<T>&    input_vector)
    {
        // creating canvas
        auto    canvas     {input_vector};

        // rewriting
        std::reverse(canvas.begin(), canvas.end());

        // returning
        return std::move(canvas);
    }
}
```

## A.15 Indexing

```
namespace svr {
    // =======================================================================
    template   <typename T1, typename T2>
    auto    index(const    std::vector<T1>&      input_vector,
                  const    std::vector<T2>&      indices_to_sample)
    {
        // creating canvas
        auto    canvas     {std::vector<T1>(indices_to_sample.size(), 0)};

        // copying the associated values
        for(int i = 0; i < indices_to_sample.size(); ++i){
            auto    source_index   {indices_to_sample[i]};
            if(source_index < input_vector.size()){
                canvas[i] = input_vector[source_index];
            }
            else
                cout << "svr::index | source_index !< input_vector.size()\n";
        }

        // returning
```

```
        return std::move(canvas);
    }
}
```

## A.16 Linspace

```
// in-place
template <typename T>
auto linspace(auto&         input,
              auto          startvalue,
              auto          endvalue,
              auto          numpoints) -> void
{
    auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);
    for(int i = 0; i<input.size(); ++i) {input[i] = startvalue + i*stepsize;}
};
// in-place
template <typename T>
auto linspace(vector<complex<T>>& input,
              auto                startvalue,
              auto                endvalue,
              auto                numpoints) -> void
{
    auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);
    for(int i = 0; i<input.size(); ++i) {
        input[i] = startvalue + static_cast<T>(i)*stepsize;
    }
};

// return-type
template <typename T>
auto linspace(T          startvalue,
              T          endvalue,
              size_t     numpoints)
{
    vector<T> input(numpoints);
    auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);

    for(int i = 0; i<input.size(); ++i) {input[i] = startvalue +
        static_cast<T>(i)*stepsize;}

    return input;
};

// return-type
template <typename T, typename U>
auto linspace(T          startvalue,
              U          endvalue,
              size_t     numpoints)
{
    vector<double> input(numpoints);
    auto stepsize = static_cast<double>(endvalue -
        startvalue)/static_cast<double>(numpoints-1);

    for(int i = 0; i<input.size(); ++i) {input[i] = startvalue + i*stepsize;}

```

```
    return input;
};
```

## A.17 Max

```
template <std::size_t axis, typename T>
auto   max(const  std::vector<std::vector<T>>  input_matrix) -> std::enable_if_t<axis ==
    1, std::vector<std::vector<T>> >
{
    // setting up canvas
    auto   canvas
        {std::vector<std::vector<T>>(input_matrix.size(),std::vector<T>(1))};

    // filling up the canvas
    for(auto row = 0; row < input_matrix.size(); ++row)
        canvas[row][0] = *(std::max_element(input_matrix[row].begin(),
            input_matrix[row].end()));

    // returning
    return std::move(canvas);
}
```

## A.18 Meshgrid

```
// ==========================================================================
template <typename T>
auto meshgrid(const   std::vector<T>&   x,
              const    std::vector<T>&   y)
{

    // creating and filling x-grid
    std::vector<std::vector<T>> xcanvas(y.size(), std::vector<T>(x.size(), 0));
    for(auto row = 0; row < y.size(); ++row)
        std::copy(x.begin(), x.end(), xcanvas[row].begin());

    // creating and filling y-grid
    std::vector<std::vector<T>> ycanvas(y.size(), std::vector<T>(x.size(), 0));
    for(auto col = 0; col < x.size(); ++col)
        for(auto row = 0; row < y.size(); ++row)
            ycanvas[row][col] = y[row];

    // returning
    return std::move(std::pair{xcanvas, ycanvas});

}
// ==========================================================================
template <typename T>
auto meshgrid(std::vector<T>&& x,
              std::vector<T>&&  y)
{

    // creating and filling x-grid
    std::vector<std::vector<T>> xcanvas(y.size(), std::vector<T>(x.size(), 0));
    for(auto row = 0; row < y.size(); ++row)
```

```cpp
        std::copy(x.begin(), x.end(), xcanvas[row].begin());

    // creating and filling y-grid
    std::vector<std::vector<T>> ycanvas(y.size(), std::vector<T>(x.size(), 0));
    for(auto col = 0; col < x.size(); ++col)
        for(auto row = 0; row < y.size(); ++row)
            ycanvas[row][col] = y[row];

    // returning
    return std::move(std::pair{xcanvas, ycanvas});

}
```

## A.19 Minimum

```cpp
template <std::size_t axis, typename T>
auto   min(std::vector<std::vector<T>> input_matrix) -> std::enable_if_t<axis == 1,
    std::vector<std::vector<T>> >
{
    // creating canvas
    auto   canvas
        {std::vector<std::vector<T>>(input_matrix.size(),std::vector<T>(1))};

    // storing the values
    for(auto row = 0; row < input_matrix.size(); ++row)
        canvas[row][0]   = *(std::min_element(input_matrix[row].begin(),
            input_matrix[row].end()));

    // returning the value
    return std::move(canvas);
}
```

## A.20 Norm

```cpp
// ============================================================
template <typename T>
auto norm(const   std::vector<T>&   input_vector)
{
    return std::sqrt(std::inner_product(input_vector.begin(), input_vector.end(),
                                        input_vector.begin(),
                                        (T)0));
}



/*
Templates to create
    -   matrix and norm-axis
    -   axis instantiated std::vector<T>
*/
```

## A.21 Division

```
// ==========================================================
// matrix division with scalars
template <typename T>
auto operator/(const std::vector<T>&   input_vector,
              const    T&               input_scalar)
{
    // creating canvas
    auto    canvas     {input_vector};

    // filling canvas
    std::transform(canvas.begin(), canvas.end(),
                   canvas.begin(),
                   [&input_scalar](const auto& argx){
                        return static_cast<double>(argx) /
                            static_cast<double>(input_scalar);
                   });

    // returning value
    return std::move(canvas);
}
// ==========================================================
// matrix division with scalars
template <typename T>
auto operator/=(const  std::vector<T>&   input_vector,
               const    T&               input_scalar)
{
    // creating canvas
    auto    canvas     {input_vector};

    // filling canvas
    std::transform(canvas.begin(), canvas.end(),
                   canvas.begin(),
                   [&input_scalar](const auto& argx){
                        return static_cast<double>(argx) /
                            static_cast<double>(input_scalar);
                   });

    // returning value
    return std::move(canvas);
}
```

## A.22 Addition

```
// ======================================================================
// y = vector + vector
template <typename T>
std::vector<T> operator+(const std::vector<T>& a,
                         const std::vector<T>& b)
{
    // Identify which is bigger
    const auto& big = (a.size() > b.size()) ? a : b;
    const auto& small = (a.size() > b.size()) ? b : a;

    std::vector<T> result = big; // copy the bigger one
```

```

    // Add elements from the smaller one
    for (size_t i = 0; i < small.size(); ++i) {
        result[i] += small[i];
    }

    return result;
}
// ============================================================================
// y = vector + vector
template <typename T>
std::vector<T>& operator+=(std::vector<T>& a,
                           const std::vector<T>& b) {

    const auto& small = (a.size() < b.size()) ? a : b;
    const auto& big = (a.size() < b.size()) ? b : a;

    // If b is bigger, resize 'a' to match
    if (a.size() < b.size())                    {a.resize(b.size());}

    // Add elements
    for (size_t i = 0; i < small.size(); ++i)  {a[i] += b[i];}

    // returning elements
    return a;
}
// ============================================================================
// y = matrix + matrix
template <typename T>
std::vector<std::vector<T>> operator+(const std::vector<std::vector<T>>& a,
                                      const std::vector<std::vector<T>>& b)
{
    // fetching dimensions
    const  auto&  num_rows_A    {a.size()};
    const  auto&  num_cols_A    {a[0].size()};
    const  auto&  num_rows_B    {b.size()};
    const  auto&  num_cols_B    {b[0].size()};

    // choosing the three different metrics
    if (num_rows_A != num_rows_B && num_cols_A != num_cols_B){
        cout << format("a.dimensions = [{},{}], b.shape = [{},{}]\n",
                        num_rows_A, num_cols_A,
                        num_rows_B, num_cols_B);
        std::cerr << "dimensions don't match\n";
    }

    // creating canvas
    auto    canvas     {std::vector<std::vector<T>>(
        std::max(num_rows_A, num_rows_B),
        std::vector<T>(std::max(num_cols_A, num_cols_B), (T)0.00)
    )};

    // performing addition
    if (num_rows_A == num_rows_B && num_cols_A == num_cols_B){
        for(auto row = 0; row < num_rows_A; ++row){
            std::transform(a[row].begin(), a[row].end(),
                           b[row].begin(),
                           canvas[row].begin(),
                           std::plus<T>());
```

```
        }
    }
    else if(num_rows_A == num_rows_B){

        // if number of columsn are different, check if one of the cols are one
        const  auto   min_num_cols   {std::min(num_cols_A, num_cols_B)};
        if (min_num_cols != 1) {std::cerr<< "Operator+: unable to broadcast\n";}
        const  auto   max_num_cols   {std::max(num_cols_A, num_cols_B)};

        // using references to tag em differently
        const  auto&  big_matrix     {num_cols_A > num_cols_B ? a : b};
        const  auto&  small_matrix   {num_cols_A < num_cols_B ? a : b};

        // Adding to canvas
        for(auto row = 0; row < canvas.size(); ++row){
            std::transform(big_matrix[row].begin(), big_matrix[row].end(),
                           canvas[row].begin(),
                           [&small_matrix,
                            &row](const auto& argx){
                                return argx + small_matrix[row][0];
                           });
        }
    }
    else if(num_cols_A == num_cols_B){

        // check if the smallest column-number is one
        const  auto   min_num_rows   {std::min(num_rows_A, num_rows_B)};
        if(min_num_rows != 1)    {std::cerr << "Operator+ : unable to broadcast\n";}
        const  auto   max_num_rows   {std::max(num_rows_A, num_rows_B)};

        // using references to differentiate the two matrices
        const  auto&  big_matrix     {num_rows_A > num_rows_B ? a : b};
        const  auto&  small_matrix   {num_rows_A < num_rows_B ? a : b};

        // adding to canvas
        for(auto row = 0; row < canvas.size(); ++row){
            std::transform(big_matrix[row].begin(), big_matrix[row].end(),
                           small_matrix[0].begin(),
                           canvas[row].begin(),
                           [](const auto& argx, const auto& argy){
                            return argx + argy;
                           });
        }
    }
    else {
        PRINTLINE PRINTLINE PRINTLINE PRINTLINE PRINTLINE
        cout << format("check this again \n");
    }

    // returning
    return std::move(canvas);
}
// =============================================================================
// y = vector + scalar
template <typename T>
auto operator+(const  std::vector<T>&   input_vector,
               const    T                 scalar)
{
    // creating canvas
```

```
    auto    canvas     {input_vector};

    // adding scalar to the canvas
    std::transform(canvas.begin(), canvas.end(),
                   canvas.begin(),
                   [&scalar](auto& argx){return argx + scalar;});

    // returning canvas
    return std::move(canvas);
}
// ============================================================================
// y = scalar + vector
template <typename T>
auto operator+(const T                  scalar,
               const    std::vector<T>&   input_vector)
{
    // creating canvas
    auto    canvas     {input_vector};

    // adding scalar to the canvas
    std::transform(canvas.begin(), canvas.end(),
                   canvas.begin(),
                   [&scalar](auto& argx){return argx + scalar;});

    // returning canvas
    return std::move(canvas);
}
```

## A.23 Multiplication (Element-wise)

```
// scalar * vector ============================================================
template <typename T>
auto    operator*(const    T                   scalar,
                  const    std::vector<T>&    input_vector)
{
    // creating canvas
    auto    canvas     {input_vector};
    // performing operation
    std::for_each(canvas.begin(), canvas.end(),
                 [&scalar](auto& argx){argx = argx * scalar;});
    // returning
    return std::move(canvas);
}

// scalar * vector ============================================================
// template <typename T1, typename T2>
template <typename T1, typename T2,
          typename = std::enable_if_t<!std::is_same_v<std::decay_t<T1>, std::vector<T2>>>>
auto operator*(const T1                 scalar,
               const    vector<T2>&       input_vector)
{
    // fetching final-type
    using T3 = decltype(std::declval<T1>() * std::declval<T2>());
    // creating canvas
    auto    canvas     {std::vector<T3>(input_vector.size())};
    // multiplying
```

```
    std::transform(input_vector.begin(), input_vector.end(),
                   canvas.begin(),
                   [&scalar](auto& argx){
                    return static_cast<T3>(scalar) * static_cast<T3>(argx);
                   });
    // returning
    return std::move(canvas);
}

// vector * scalar ==============================================================
template <typename T>
auto    operator*(const   std::vector<T>&   input_vector,
                  const    T                 scalar)
{
    // creating canvas
    auto    canvas     {input_vector};
    // multiplying
    std::for_each(canvas.begin(), canvas.end(),
                  [&scalar](auto& argx){
                    argx = argx * scalar;
                  });
    // returning
    return std::move(canvas);
}

// vector * vector ==============================================================
template <typename T>
auto operator*(const std::vector<T>& input_vector_A,
               const std::vector<T>&  input_vector_B)
{
    // throwing error: size-desparity
    if (input_vector_A.size() != input_vector_B.size()) {std::cerr << "operator*: size
        disparity \n";}

    // creating canvas
    auto    canvas     {input_vector_A};

    // element-wise multiplying
    std::transform(input_vector_B.begin(), input_vector_B.end(),
                   canvas.begin(),
                   canvas.begin(),
                   [](const auto& argx, const auto& argy){
                       return argx * argy;
                   });

    // moving it back
    return std::move(canvas);
}
template <typename T1, typename T2>
auto    operator*(const   std::vector<T1>&  input_vector_A,
                  const    std::vector<T2>&  input_vector_B)
{

    // checking size disparity
    if (input_vector_A.size() != input_vector_B.size())
        std::cerr << "operator*: error, size-disparity \n";

    // figuring out resulting data type
    using   T3    = decltype(std::declval<T1>() * std::declval<T2>());
```

```

    // creating canvas
    auto    canvas     {std::vector<T3>(input_vector_A.size())};

    // performing multiplications
    std::transform(input_vector_A.begin(), input_vector_A.end(),
                   input_vector_B.begin(),
                   canvas.begin(),
                   [](const    auto&      argx,
                      const    auto&      argy){
                        return static_cast<T3>(argx) *  static_cast<T3>(argy);
                   });

    // returning
    return std::move(canvas);

}

// scalar * matrix =============================================================
template <typename T>
auto operator*(T                                 scalar,
               const std::vector<std::vector<T>>& inputMatrix)
{
    std::vector<std::vector<T>> temp {inputMatrix};
    for(int i = 0; i<inputMatrix.size(); ++i){
        std::transform(inputMatrix[i].begin(),
                       inputMatrix[i].end(),
                       temp[i].begin(),
                       [&scalar](T x){return scalar * x;});
    }
    return temp;
}
// matrix * matrix =============================================================
template <typename T>
auto operator*(const std::vector<std::vector<T>>& A,
               const std::vector<std::vector<T>>& B) -> std::vector<std::vector<T>>
{
    // Case 1: element-wise multiplication
    if (A.size() == B.size() && A[0].size() == B[0].size()) {
        std::vector<std::vector<T>> C(A.size(), std::vector<T>(A[0].size()));
        for (std::size_t row = 0; row < A.size(); ++row) {
            std::transform(A[row].begin(), A[row].end(),
                           B[row].begin(),
                           C[row].begin(),
                           [](const auto& x, const auto& y){ return x * y; });
        }
        return C;
    }

    // Case 2: broadcast column vector
    else if (A.size() == B.size() && B[0].size() == 1) {
        std::vector<std::vector<T>> C(A.size(), std::vector<T>(A[0].size()));
        for (std::size_t row = 0; row < A.size(); ++row) {
            std::transform(A[row].begin(), A[row].end(),
                           C[row].begin(),
                           [&](const auto& x){ return x * B[row][0]; });
        }
        return C;
    }
```

```

    // case 3: when second matrix contains just one row
    // case 4: when first matrix is just one column
    // case 5: when second matrix is just one column

    // Otherwise, invalid
    else {
        throw std::runtime_error("operator* dimension mismatch");
    }
}
// scalar * matrix =========================================================
template <typename T1, typename T2>
auto operator*(T1 scalar,
               const std::vector<std::vector<T2>>& inputMatrix)
{
    std::vector<std::vector<T2>> temp {inputMatrix};
    for(int i = 0; i<inputMatrix.size(); ++i){
        std::transform(inputMatrix[i].begin(),
                       inputMatrix[i].end(),
                       temp[i].begin(),
                       [&scalar](T2 x){return static_cast<T2>(scalar) * x;});
    }
    return temp;
}
// matrix-multiplication ===================================================
template <typename T1, typename T2>
auto matmul(const std::vector<std::vector<T1>>& matA,
            const std::vector<std::vector<T2>>& matB)
{

    // throwing error
    if (matA[0].size() != matB.size())  {std::cerr << "dimension-mismatch \n";}

    // getting result-type
    using ResultType  = decltype(std::declval<T1>() * std::declval<T2>() + \
                                 std::declval<T1>() * std::declval<T2>() );

    // creating aliasses
    auto finalnumrows {matA.size()};
    auto finalnumcols {matB[0].size()};

    // creating placeholder
    auto rowcolproduct = [&](auto rowA, auto colB){
        ResultType temp {0};
        for(int i = 0; i < matA.size(); ++i) {temp +=
            static_cast<ResultType>(matA[rowA][i]) +
            static_cast<ResultType>(matB[i][colB]);}
        return temp;
    };

    // producing row-column combinations
    std::vector<std::vector<ResultType>> finaloutput(finalnumrows,
        std::vector<ResultType>(finalnumcols));
    for(int row = 0; row < finalnumrows; ++row){for(int col = 0; col < finalnumcols;
        ++col){finaloutput[row][col] = rowcolproduct(row, col);}}

    // returning
    return finaloutput;
}
```

```
// matrix * vector ============================================================
template   <typename T>
auto operator*(const std::vector<std::vector<T>>  input_matrix,
               const   std::vector<T>              input_vector)
{
    // fetching dimensions
    const  auto&  num_rows_matrix   {input_matrix.size()};
    const  auto&  num_cols_matrix   {input_matrix[0].size()};
    const  auto&  num_rows_vector   {1};
    const  auto&  num_cols_vector   {input_vector.size()};

    const  auto&  max_num_rows      {num_rows_matrix > num_rows_vector ?\
                                     num_rows_matrix : num_rows_vector};
    const  auto&  max_num_cols      {num_cols_matrix > num_cols_vector ?\
                                     num_cols_matrix : num_cols_vector};

    // creating canvas
    auto   canvas    {std::vector<std::vector<T>>(
        max_num_rows,
        std::vector<T>(max_num_cols, 0)
    )};

    //
    if (num_cols_matrix == 1 && num_rows_vector == 1){

        // writing to canvas
        for(auto   row = 0; row < max_num_rows; ++row)
            for(auto   col = 0; col < max_num_cols; ++col)
                canvas[row][col]  =   input_matrix[row][0]  *   input_vector[col];
    }
    else{
        std::cerr << "Operator*: [matrix, vector] | not implemented \n";
    }

    // returning
    return std::move(canvas);

}

// scalar operators ============================================================
auto operator*(const std::complex<double> complexscalar,
               const double               doublescalar){
    return complexscalar * static_cast<std::complex<double>>(doublescalar);
}
auto operator*(const double               doublescalar,
               const std::complex<double> complexscalar){
    return complexscalar * static_cast<std::complex<double>>(doublescalar);
}
auto operator*(const std::complex<double> complexscalar,
               const int                  scalar){
    return complexscalar * static_cast<std::complex<double>>(scalar);
}
auto operator*(const int                  scalar,
               const std::complex<double> complexscalar){
    return complexscalar * static_cast<std::complex<double>>(scalar);
}
```

## A.24 Subtraction

```
// ============================================================================
// Aim: substracting scalar from a vector
template <typename T>
std::vector<T> operator-(const std::vector<T>& a, const T scalar){
    std::vector<T> temp(a.size());
    std::transform(a.begin(),
                   a.end(),
                   temp.begin(),
                   [scalar](T x){return (x - scalar);});
    return temp;
}
// ============================================================================
template <typename T>
auto operator-(const  std::vector<std::vector<T>>&  input_matrix_A,
               const  std::vector<std::vector<T>>&  input_matrix_B)
{
    // throwing error in case of dimension differences
    if (input_matrix_A.size() != input_matrix_B.size() ||
        input_matrix_A[0].size() != input_matrix_B[0].size())
        std::cerr << "operator- dimension mismatch\n";

    // setting up canvas
    auto   canvas    {std::vector<std::vector<T>>(
        input_matrix_A.size(),
        std::vector<T>(input_matrix_A[0].size())
    )};

    // subtracting values
    for(auto row = 0; row < input_matrix_B.size(); ++row)
        std::transform(input_matrix_A[row].begin(), input_matrix_A[row].end(),
                       input_matrix_B[row].begin(),
                       canvas[row].begin(),
                       [](const auto& x, const auto& y){
                           return x - y;
                       });

    // returning
    return std::move(canvas);

}
```

## A.25 Operator Overloadings

## A.26 Printing Containers

```
// vector printing function
template<typename T>
void fPrintVector(vector<T> input){
    for(auto x: input) cout << x << ",";
    cout << endl;
```

```cpp
}

template<typename T>
void fpv(vector<T> input){
    for(auto x: input) cout << x << ",";
    cout << endl;
}
// ==========================================================
template<typename T>
void fPrintMatrix(const std::vector<std::vector<T>> input_matrix){
    for(const auto& row: input_matrix)
        cout << format("{}\n", row);
}
template <typename T>
void fPrintMatrix(const string&                input_string,
                  const std::vector<std::vector<T>> input_matrix){
    cout << format("{} = \n", input_string);
    for(const auto& row: input_matrix)
        cout << format("{}\n", row);
}


template<typename T, typename T1>
void fPrintHashmap(unordered_map<T, T1> input){
    for(auto x: input){
        cout << format("[{},{}] | ", x.first, x.second);
    }
    cout <<endl;
}

void fPrintBinaryTree(TreeNode* root){
    // sending it back
    if (root == nullptr) return;

    // printing
    PRINTLINE
    cout << "root->val = " << root->val << endl;

    // calling the children
    fPrintBinaryTree(root->left);
    fPrintBinaryTree(root->right);

    // returning
    return;

}

void fPrintLinkedList(ListNode* root){
    if (root == nullptr) return;
    cout << root->val << " -> ";
    fPrintLinkedList(root->next);
    return;
}

template<typename T>
void fPrintContainer(T input){
    for(auto x: input) cout << x << ", ";
    cout << endl;
    return;
```

```
}
// ============================================================================
template <typename T>
auto size(std::vector<std::vector<T>> inputMatrix){
    cout << format("[{}, {}]\n", inputMatrix.size(), inputMatrix[0].size());
}

template <typename T>
auto size(const std::string inputstring, std::vector<std::vector<T>> inputMatrix){
    cout << format("{} = [{}, {}]\n", inputstring, inputMatrix.size(),
        inputMatrix[0].size());
}
```

## A.27 Random Number Generation

```
// ============================================================================
template <typename T>
auto rand(const T min, const T max) {
    static std::random_device rd; // Seed
    static std::mt19937 gen(rd()); // Mersenne Twister generator
    std::uniform_real_distribution<> dist(min, max);
    return dist(gen);
}
// ============================================================================
template <typename T>
auto rand(const T      min,
          const T      max,
          const size_t numelements)
{
    static std::random_device rd; // Seed
    static std::mt19937 gen(rd()); // Mersenne Twister generator
    std::uniform_real_distribution<> dist(min, max);

    // building the fianloutput
    vector<T> finaloutput(numelements);
    for(int i = 0; i<finaloutput.size(); ++i) {finaloutput[i] =
        static_cast<T>(dist(gen));}

    return finaloutput;
}
// ============================================================================
template <typename T>
auto rand(const T           argmin,
          const T           argmax,
          const vector<int>   dimensions)
{

    // throwing an error if dimension is greater than two
    if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}

    // creating random engine
    static std::random_device rd; // Seed
    static std::mt19937 gen(rd()); // Mersenne Twister generator
    std::uniform_real_distribution<> dist(argmin, argmax);

    // building the finaloutput
```

```
    vector<vector<T>> finaloutput;
    for(int i = 0; i<dimensions[0]; ++i){
        vector<T> temp;
        for(int j = 0; j<dimensions[1]; ++j) {temp.push_back(dist(gen));}
        // cout << format("\t\t temp = {}\n", temp);

        finaloutput.push_back(temp);
    }

    // returning the finaloutput
    return finaloutput;

}
// ===========================================================================
auto rand(const vector<int>  dimensions)
{

    using ReturnType = double;

    // throwing an error if dimension is greater than two
    if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}

    // creating random engine
    static std::random_device rd; // Seed
    static std::mt19937 gen(rd()); // Mersenne Twister generator
    std::uniform_real_distribution<> dist(0.00, 1.00);

    // building the finaloutput
    vector<vector<ReturnType>> finaloutput;
    for(int i = 0; i<dimensions[0]; ++i){
        vector<ReturnType> temp;
        for(int j = 0; j<dimensions[1]; ++j) {temp.push_back(dist(gen));}
        finaloutput.push_back(std::move(temp));
    }

    // returning the finaloutput
    return std::move(finaloutput);

}
// ===========================================================================
template <typename T>
auto rand_complex_double(const T            argmin,
                         const T            argmax,
                         const vector<int>& dimensions)
{

    // throwing an error if dimension is greater than two
    if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}

    // creating random engine
    static std::random_device rd; // Seed
    static std::mt19937 gen(rd()); // Mersenne Twister generator
    std::uniform_real_distribution<> dist(argmin, argmax);

    // building the finaloutput
    vector<vector<complex<double>>> finaloutput;
    for(int i = 0; i<dimensions[0]; ++i){
        vector<complex<double>> temp;
```

```
        for(int j = 0; j<dimensions[1]; ++j)
            {temp.push_back(static_cast<double>(dist(gen)));}
        finaloutput.push_back(std::move(temp));
    }

    // returning the finaloutput
    return finaloutput;

}
```

## A.28 Reshape

```
// ===========================================================================
// reshaping a matrix into another matrix
template <std::size_t M, std::size_t N, typename T>
auto reshape(const std::vector<std::vector<T>>& input_matrix){

    // verifying size stuff
    if (M*N != input_matrix.size() * input_matrix[0].size())
        std::cerr << "Dimensions are quite different\n";

    // creating canvas
    auto    canvas     {std::vector<std::vector<T>>(
        M, std::vector<T>(N, (T)0)
    )};

    // writing to canvas
    size_t   tid            {0};
    size_t   target_row     {0};
    size_t   target_col     {0};
    for(auto row = 0; row<input_matrix.size(); ++row){
        for(auto col = 0; col < input_matrix[0].size(); ++col){
            tid             =   row * input_matrix[0].size() + col;
            target_row      =   tid/N;
            target_col      =   tid%N;
            canvas[target_row][target_col]   =   input_matrix[row][col];
        }
    }

    // moving it back
    return std::move(canvas);
}
// ===========================================================================
// reshaping a matrix into a vector
template<std::size_t M, typename T>
auto reshape(const std::vector<std::vector<T>>& input_matrix){

    // checking element-count validity
    if (M != input_matrix.size() * input_matrix[0].size())
        std::cerr << "Number of elements differ\n";

    // creating canvas
    auto    canvas     {std::vector<T>(M, 0)};

    // filling canvas
    for(auto row = 0; row < input_matrix.size(); ++row)
```

```
        for(auto col = 0; col < input_matrix[0].size(); ++col)
            canvas[row * input_matrix.size() + col] = input_matrix[row][col];

    // moving it back
    return std::move(canvas);
}

// ==========================================================
// Matrix to matrix
// ==========================================================
template<typename T>
auto reshape(const std::vector<std::vector<T>>& input_matrix,
             const std::size_t              M,
             const std::size_t              N){

    // checking element-count validity
    if ( M * N != input_matrix.size() * input_matrix[0].size())
        std::cerr << "Number of elements differ\n";

    // creating canvas
    auto   canvas     {std::vector<std::vector<T>>(
        M, std::vector<T>(N, (T)0)
    )};

    // writing to canvas
    size_t   tid           {0};
    size_t   target_row    {0};
    size_t   target_col    {0};
    for(auto row = 0; row<input_matrix.size(); ++row){
        for(auto col = 0; col < input_matrix[0].size(); ++col){
            tid           =   row * input_matrix[0].size() + col;
            target_row    =   tid/N;
            target_col    =   tid%N;
            canvas[target_row][target_col]   =   input_matrix[row][col];
        }
    }

    // moving it back
    return std::move(canvas);
}

// ==========================================================
// converting a matrix into a vector
// ==========================================================
template<typename T>
auto reshape(const std::vector<std::vector<T>>& input_matrix,
             const size_t                   M){

    // checking element-count validity
    if (M != input_matrix.size() * input_matrix[0].size())
        std::cerr << "Number of elements differ\n";

    // creating canvas
    auto   canvas     {std::vector<T>(M, 0)};

    // filling canvas
    for(auto row = 0; row < input_matrix.size(); ++row)
        for(auto col = 0; col < input_matrix[0].size(); ++col)
            canvas[row * input_matrix.size() + col] = input_matrix[row][col];
```

```

    // moving it back
    return std::move(canvas);
}
```

## A.29 Summing with containers

```
// ===========================================================================
template <std::size_t axis, typename T>
auto sum(const std::vector<T>& input_vector) -> std::enable_if_t<axis == 0, std::vector<T>>
{
    // returning the input as is
    return input_vector;
}
// ===========================================================================
template <std::size_t axis, typename T>
auto sum(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis == 0, std::vector<T>>
{
    // creating canvas
    auto    canvas     {std::vector<T>(input_matrix[0].size(), 0)};

    // filling up the canvas
    for(auto row = 0; row < input_matrix.size(); ++row)
        std::transform(input_matrix[row].begin(), input_matrix[row].end(),
                       canvas.begin(),
                       canvas.begin(),
                       [](auto& argx, auto& argy){return argx + argy;});

    // returning
    return std::move(canvas);

}
// ===========================================================================
template <std::size_t axis, typename T>
auto sum(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis == 1, std::vector<std::vector<T>>>
{
    // creating canvas
    auto canvas    {std::vector<std::vector<T>>(input_matrix.size(),
                                                std::vector<T>(1, 0.00))};

    // filling up the canvas
    for(auto row = 0; row < input_matrix.size(); ++row)
        canvas[row][0]    = std::accumulate(input_matrix[row].begin(),
                                            input_matrix[row].end(),
                                            static_cast<T>(0));

    // returning
    return std::move(canvas);

}
// ===========================================================================
template <std::size_t axis, typename T>
auto sum(const std::vector<T>&   input_vector_A,
```

```
        const std::vector<T>&   input_vector_B) -> std::enable_if_t<axis == 0,
            std::vector<T> >
{
    // setup
    const  auto&  num_cols_A    {input_vector_A.size()};
    const  auto&  num_cols_B    {input_vector_B.size()};

    // throwing errors
    if (num_cols_A != num_cols_B) {std::cerr << "sum: size disparity\n";}

    // creating canvas
    auto   canvas {input_vector_A};

    // summing up
    std::transform(input_vector_B.begin(), input_vector_B.end(),
                   canvas.begin(),
                   canvas.begin(),
                   std::plus<T>());

    // returning
    return std::move(canvas);
}
```

## A.30 Tangent

```
namespace svr {
    // =====================================================
    template <typename T>
    auto atan2(const  std::vector<T>    input_vector_A,
               const  std::vector<T>    input_vector_B)
    {
        // throw error
        if (input_vector_A.size() != input_vector_B.size())
            std::cerr << "atan2: size disparity\n";

        // create canvas
        auto    canvas     {std::vector<T>(input_vector_A.size(), 0)};

        // performing element-wise atan2 calculation
        std::transform(input_vector_A.begin(), input_vector_A.end(),
                       input_vector_B.begin(),
                       canvas.begin(),
                       [](const    auto&   arg_a,
                          const    auto&   arg_b){

                            return std::atan2(arg_a, arg_b);
                       });

        // moving things back
        return std::move(canvas);
    }
    // =====================================================
    template <typename T>
    auto atan2(T   scalar_A,
               T   scalar_B)
    {
```

```
        return std::atan2(scalar_A, scalar_B);
    }
}
```

## A.31 Tiling Operations

```
namespace svr {
    // =======================================================
    template <typename T>
    auto tile(const    std::vector<T>&       input_vector,
              const    std::vector<size_t>   mul_dimensions){

        // creating canvas
        const   std::size_t&  num_rows    {1 * mul_dimensions[0]};
        const   std::size_t&  num_cols    {input_vector.size() * mul_dimensions[1]};
        auto    canvas {std::vector<std::vector<T>>(
            num_rows,
            std::vector<T>(num_cols, 0)
        )};

        // writing
        std::size_t    source_row;
        std::size_t    source_col;

        for(std::size_t row = 0; row < num_rows; ++row){
            for(std::size_t col = 0; col < num_cols; ++col){
                source_row =   row % 1;
                source_col =   col % input_vector.size();
                canvas[row][col] = input_vector[source_col];
            }
        }

        // returning
        return std::move(canvas);
    }
    // =======================================================
    template <typename T>
    auto tile(const    std::vector<std::vector<T>>& input_matrix,
              const    std::vector<size_t>          mul_dimensions){

        // creating canvas
        const   std::size_t&  num_rows    {input_matrix.size()    * mul_dimensions[0]};
        const   std::size_t&  num_cols    {input_matrix[0].size() * mul_dimensions[1]};
        auto    canvas {std::vector<std::vector<T>>(
            num_rows,
            std::vector<T>(num_cols, 0)
        )};

        // writing
        std::size_t    source_row;
        std::size_t    source_col;

        for(std::size_t row = 0; row < num_rows; ++row){
            for(std::size_t col = 0; col < num_cols; ++col){
                source_row =   row % input_matrix.size();
                source_col =   col % input_matrix[0].size();
```

```
                canvas[row][col] = input_matrix[source_row][source_col];
            }
        }

        // returning
        return std::move(canvas);
    }
}
```

## A.32 Transpose

```
template <typename T>
auto transpose(const std::vector<T> input_vector){

    // creating canvas
    auto    canvas     {std::vector<std::vector<T>>{
        input_vector.size(),
        std::vector<T>(1)
    }};

    // filling canvas
    for(auto i = 0; i < input_vector.size(); ++i){
        canvas[i][0]   = input_vector[i];
    }

    // moving it back
    return std::move(canvas);
}
```