

Autonomous Underwater Vehicle: A Surveillance Protocol

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Preface

This project is an attempt at combining all of my major skills into creating a simulation, imaging, perception and control pipeline for Autonomous Underwater Vehicles (AUV). As such, creating this project involves creating a number of pipelines.

The first pipeline is the signal simulation pipeline. The signal simulation pipeline involves sea-floor point-cloud creation and simulating the signals received by the sensor arrays of the AUV. The signals recorded by the sensor-arrays on the AUV contains information from the surrounding environment. The imaging pipeline performs certain operations on the recorded signals to obtain acoustic images of the surrounding environment. To that end, this pipeline involves the topics of signal processing, linear algebra, signals and systems.

As such, the second pipeline is the imaging pipeline. The inputs to the imaging pipeline is the signals recorded by the different sensor-arrays of the AUV, in addition to the parameters of the AUV and its components. This pipeline involves match-filtering, focussing and beamforming operations to create acoustic images of the surrounding environment. Depending on the number of ULAs present, the imaging pipeline is responsible for creating multiple acoustic images in real-time. Thus, this pipeline involves the topics of Digital Signal Processing, Match-Filtering, Estimation and Detection Theory and so on.

The images created by the imaging pipeline are fed to the perception-to-control pipeline. This pipeline takes in the image formed created from the ULA signals, parameters of AUV and its components, and some historical data, it provides instructions regarding the movement of the AUV. The mapping from the inputs to the controls is called policy. Learning policies is a core part of reinforcement learning. Thus, this pipeline mainly involves the topics of reinforcement learning. And since we'll be using convolutional neural nets and transformers for learning the policies, this pipeline involves a significant amount of machine and deep learning.

The final result is an AUV that is primarily trained to map an area of the sea-floor in a constant surveillance mode. The RL-trained policy will also be trained to deal with different kinds of sea-floor terrains: those containing hills, valleys, and path-obstructing features. Due to the resource constrained nature of the marine vessel, we also prioritize efficient policies in the policy-training pipeline.

The project is currently written in C++. And since there is non-trivial amount of training and adaptive features in the pipelines, we'll be using LibTorch (the C++ API of PyTorch) to enable computation graphs, backpropagation and thereby, learning in our AUV pipeline. However, for the sections where a computation graph is not required we will be writing templated STL code.

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Part I

AUV Components & Setup

Chapter 1

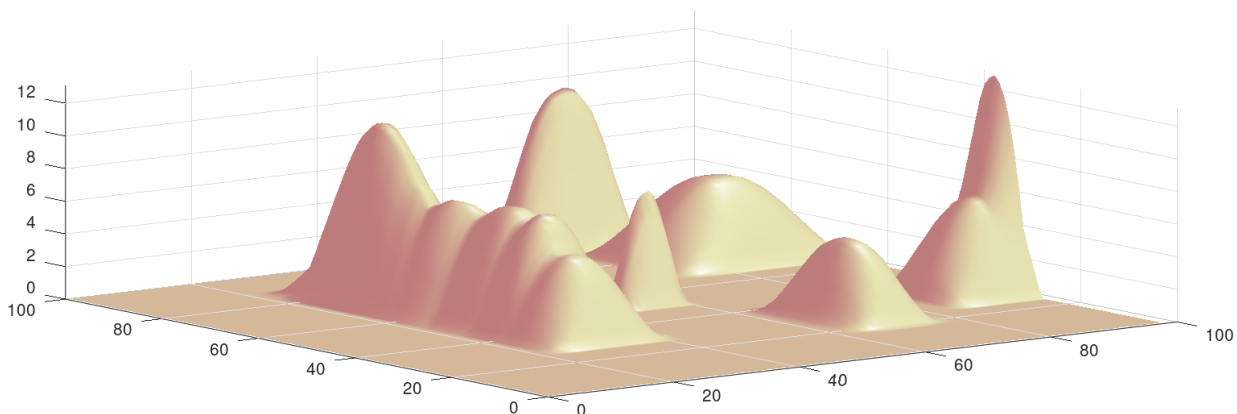
Underwater Environment

Overview

All physical matter in this framework is represented using point-clouds. Thus, the sea-floor also is represented using a number of 3D points. In addition to the coordinates, the points also have the additional property of “reflectivity”. It is the impulse response of that point.

Sea-floors in real-life are rarely flat. They often contain valleys, mountains, hills and much richer geographical features. Thus, training an agent to function in such environments call for the creation of similar structures in our simulations.

To simplify things, we shall take a more constrained and structured approach. We start by creating different classes of structures and produce instantiations of those structures on the sea-floor. These structures are defined in such a way that the shape and size can be parameterized to enable creation of random sea-floors.



1.1 Underwater Hills

The most basic approach to creating this is to create a flat seafloor, where all the points have the same height. While this is a good place to start, it is good to bring in some realism to the seafloor. To that end, we shall have some rolling hills as the sea-floor. Each “hill ”

is created using the method outlined in Algorithm 1. The method involves deciding the location of the hills, the dimension of the hills and then designing a hill by combining an exponential function and a cosine function. We're aiming to essentially produce gaussian-looking sea-floor hills. After the creation, this becomes the set of points representing the lowest set of points in the overall seafloor structure.

Algorithm 1 Hill Creation

```

1: Input: Mean vector  $\mathbf{m}$ , Dimension vector  $\mathbf{d}$ , 2D points  $\mathbf{P}$ 
2: Output: Updated  $\mathbf{P}$  with hill heights
3:  $\text{num\_hills} \leftarrow \text{numel}(\mathbf{m}_x)$ 
4:  $H \leftarrow$  Zeros tensor of size  $(1, \text{numel}(\mathbf{P}_x))$ 
5: for  $i = 1$  to  $\text{num\_hills}$  do
6:    $x_{\text{norm}} \leftarrow \frac{\frac{\pi}{2}(\mathbf{P}_x - \mathbf{m}_x[i])}{\mathbf{d}_x[i]}$ 
7:    $y_{\text{norm}} \leftarrow \frac{\frac{\pi}{2}(\mathbf{P}_y - \mathbf{m}_y[i])}{\mathbf{d}_y[i]}$ 
8:    $h_x \leftarrow \cos(x_{\text{norm}}) \cdot e^{\frac{|x_{\text{norm}}|}{10}}$ 
9:    $h_y \leftarrow \cos(y_{\text{norm}}) \cdot e^{\frac{|y_{\text{norm}}|}{10}}$ 
10:   $h \leftarrow \mathbf{d}_z[i] \cdot h_x \cdot h_y$ 
11:  Apply boundary conditions:
12:  if  $x_{\text{norm}} > \frac{\pi}{2}$  or  $x_{\text{norm}} < -\frac{\pi}{2}$  or  $y_{\text{norm}} > \frac{\pi}{2}$  or  $y_{\text{norm}} < -\frac{\pi}{2}$  then
13:     $h \leftarrow 0$ 
14:  end if
15:   $H \leftarrow H + h$ 
16: end for
17:  $\mathbf{P} \leftarrow \text{concatenate}([\mathbf{P}, H])$ 

```

1.2 Scatterer Definition

The sea-floor is represented by a single object of the class ScattererClass.

```

1  /*=====
2  Class Declaration
3  -----*/
4  template <typename T>
5  class ScattererClass
6  {
7  public:
8      // members
9      std::vector<std::vector<T>> coordinates;
10     std::vector<T> reflectivity;
11
12     // Constructor
13     ScattererClass() {}
14
15     // Constructor
16     ScattererClass(std::vector<std::vector<T>> coordinates_arg,
17                    std::vector<T> reflectivity_arg):
18         coordinates(std::move(coordinates_arg)),
19         reflectivity(std::move(reflectivity_arg)) {}
20
21     // Save to CSV

```



```

22     void save_to_csv();
23 };

```

1.3 Sea-Floor Setup Script

Following is the function that will setup the sea-floor script.

```

1 void fSeaFloorSetup(
2     ScattererClass<double>& scatterers
3 ){
4
5     // auto save_files {false};
6     const auto save_files {false};
7     const auto hill_creation_flag {true};
8
9     // sea-floor bounds
10    auto bed_width {100.00};
11    auto bed_length {100.00};
12
13    // creating tensors for coordinates and reflectivity
14    vector<vector<double>> box_coordinates;
15    vector<double> box_reflectivity;
16
17    // scatter density
18    // auto bed_width_density {static_cast<double>( 10.00)};
19    // auto bed_length_density {static_cast<double>( 10.00)};
20    auto bed_width_density {static_cast<double>( 5.00)};
21    auto bed_length_density {static_cast<double>( 5.00)};
22
23    // setting up coordinates
24    auto xpoints {svr::linspace<double>(
25        0.00,
26        bed_width,
27        bed_width * bed_width_density
28    )};
29    auto ypoints {svr::linspace<double>(
30        0.00,
31        bed_length,
32        bed_length * bed_length_density
33    )};
34    if(save_files) fWriteVector(xpoints, "../csv-files/xpoints.csv"); // verified
35    if(save_files) fWriteVector(ypoints, "../csv-files/ypoints.csv"); // verified
36
37    // creating mesh
38    auto [xgrid, ygrid] = meshgrid(std::move(xpoints), std::move(ypoints));
39    if(save_files) fWriteMatrix(xgrid, "../csv-files/xgrid.csv"); // verified
40    if(save_files) fWriteMatrix(ygrid, "../csv-files/ygrid.csv"); // verified
41
42    // reshaping
43    auto X {reshape(xgrid, xgrid.size()*xgrid[0].size())};
44    auto Y {reshape(ygrid, ygrid.size()*ygrid[0].size())};
45    if(save_files) fWriteVector(X, "../csv-files/X.csv"); // verified
46    if(save_files) fWriteVector(Y, "../csv-files/Y.csv"); // verified
47
48    // creating heights of scatterers
49    if(hill_creation_flag){

```

```

50
51 // setting up hill parameters
52 auto num_hills {10};
53
54 // setting up placement of hills
55 auto points2D {concatenate<0>(X, Y)}; // verified
56 auto min2D {min<1, double>(points2D)}; // verified
57 auto max2D {max<1, double>(points2D)}; // verified
58 auto hill_2D_center {min2D + \
59 rand({2, num_hills}) * (max2D - min2D)}; // verified
60
61 // setup: hill-dimensions
62 auto hill_dimensions_min {transpose(vector<double>{5, 5, 2})}; // verified
63 auto hill_dimensions_max {transpose(vector<double>{30, 30, 10})}; // verified
64 auto hill_dimensions {hill_dimensions_min + \
65 rand({3, num_hills}) * (hill_dimensions_max -
66 hill_dimensions_min)}; // verified
67
68 // function-call: hill-creation function
69 fCreateHills(hill_2D_center,
70 hill_dimensions,
71 points2D);
72
73 // setting up floor reflectivity
74 auto floorScatter_reflectivity {std::vector<double>(Y.size(), 1.00)};
75
76 // populating the values of the incoming argument
77 scatterers.coordinates = std::move(points2D);
78 scatterers.reflectivity = std::move(floorScatter_reflectivity);
79
80 else{
81
82 // assigning flat heights
83 auto Z {std::vector<double>(Y.size(), 0)};
84
85 // setting up floor coordinates
86 auto floorScatter_coordinates {concatenate<0>(X, Y, Z)};
87 auto floorScatter_reflectivity {std::vector<double>(Y.size(), 1)};
88
89 // populating the values of the incoming argument
90 scatterers.coordinates = std::move(floorScatter_coordinates);
91 scatterers.reflectivity = std::move(floorScatter_reflectivity);
92
93 }
94
95 // printing status
96 spdlog::info("Finished Sea-Floor Setup");
97 }

```

Chapter 2

Transmitter

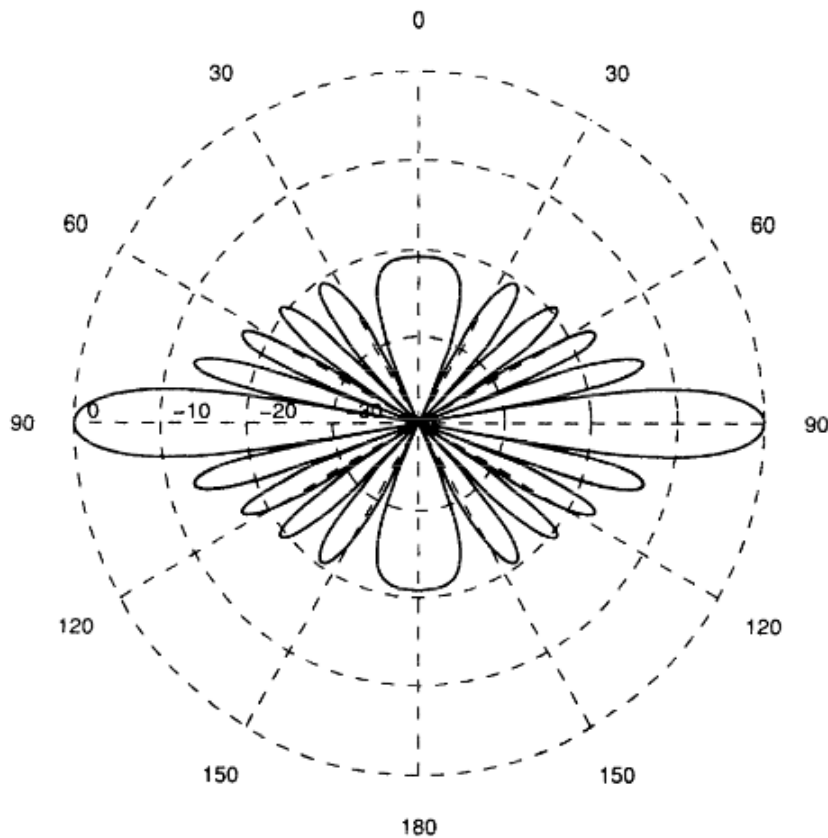


Figure 2.1: Beampattern of a Transmission Uniform Linear Array

Overview

Probing systems are those systems that send out a signal, listen to the reflection and infer qualitative and quantitative qualities of the environment, matter or object, it was trying to infer information about. The transmitter is one of the most fundamental components of probing systems. As the name suggests, the transmitter is the equipment responsible for sending out the probing signal into the medium.

A transmitter is any device or circuit that converts information into a signal and sends it out onto some media like air, cable, water or space. The components of a transmitter are usually as follows

1. Input: Information containing signal such as voice, data, video etc
2. Process: Encode/modulate the information onto a carrier signal, which can be electromagnetic wave or mechanical wave.
3. Transmission: The signal is then transmitted onto the media with electro-mechanical equipment.

Transmitters are of many kinds. But the ones that we will be considering will be directed transmitters, which means that these transmitters have an associated beampattern. To the uninitiated, this means that the power of the transmitted signal is not transmitted in all directions equally. A beampattern is a graphical representation of the power received by an ideal receiver when placed at different angles.

Transmitters made out of a linear-array of individual transmitters use beamforming to “direct” the major power of the transmitter. These kind of systems have well studied beampatterns which can be utilized in our simulations. These kind of studies and inculcating that in our pipelines produce accurate signal simulation pipelines. For now, we stick to a very simple model of a transmitter. We assume that the transmitter sends out the power equally into a particular cone from the AUV position.

2.1 Transmission Signal

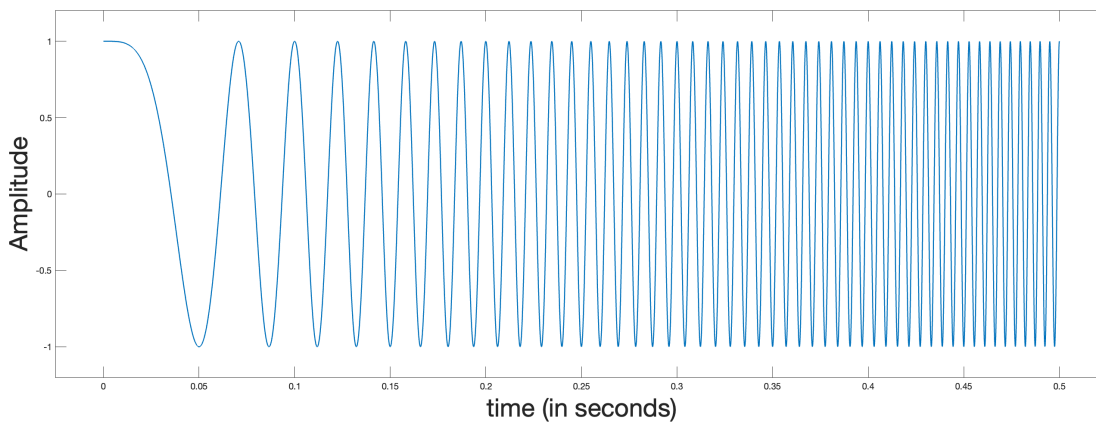


Figure 2.2: Linear Frequency Modulated Wave

The resolution of any probing system is fundamentally tied to the signal bandwidth. A higher bandwidth corresponds to finer resolution $\frac{\text{speed-of-sounds}}{2 \cdot \text{bandwidth}}$. Thus, for perfect resolution, an infinite bandwidth is in order. However, infinite bandwidth is impossible for obvious reasons: hardware limitations, spectral regulations, energy limitations and so on.

This is where Linear Frequency Modulation (LFM), also called a “chirp,” becomes valuable. An LFM signal linearly sweeps a limited bandwidth over a relatively long duration. This technique spreads the signal’s energy in time while retaining the resolution benefits of

the bandwidth. After matched filtering (or pulse compression), we essentially produce pulses corresponding to a base-band LFM of same bandwidth. Overall, LFM is a practical compromise between finite bandwidth and desired performance.

One of the best parts about the resolution depending only on the bandwidth is that it allows us to deploy techniques that would help us improve SNRs without virtually increasing the bandwidth at all. Much of the noise in submarine environments are in and around the baseband region (around frequency, 0). Since resolution depends purely on bandwidth, and LFM can be transmitted at a carrier-frequency, this means that processing the returns after low-pass filtering and basebanding allows us to get rid of the submarine noise, since they do not occupy the same frequency-coefficients. The end-result, thus, is improved SNR compared to use baseband LFM.

Due to all of these advantages, LFM waves are ubiquitous in probing systems, from sonar to radar. Thus, for this project too, the transmitter will be using LFM waves as probing signals, to probe the surrounding submarine environment.

2.2 Transmitter Class Definition

The transmitter is represented by a single object of the class TransmitterClass.

```

1  template <typename T>
2  class TransmitterClass{
3  public:
4
5      // A shared pointer to the configuration object
6      std::shared_ptr<svr::AUVParameters> config_ptr;
7
8      // physical/intrinsic properties
9      std::vector<T>    location;           // location tensor
10     std::vector<T>    pointing_direction; // pointing direction
11
12     // basic parameters
13     std::vector<T>    signal;              // transmitted signal (LFM)
14     T                 azimuthal_angle;     // transmitter's azimuthal pointing direction
15     T                 elevation_angle;     // transmitter's elevation pointing direction
16     T                 azimuthal_beamwidth; // azimuthal beamwidth of transmitter
17     T                 elevation_beamwidth; // elevation beamwidth of transmitter
18     T                 range;              // a parameter used for spotlight mode.
19
20     // transmitted signal attributes
21     T                 f_low;              // lowest frequency of LFM
22     T                 f_high;            // highest frequency of LFM
23     T                 fc;                // center frequency of LFM
24     T                 bandwidth;         // bandwidth of LFM
25     T                 speed_of_sound {1500}; // speed of sound
26
27     // shadowing properties
28     int               azimuthQuantDensity; // quantization of angles along the
29     azimuth           elevationQuantDensity; // quantization of angles along the
30     elevation
31     T                 rangeQuantSize;     // range-cell size when shadowing
32     T                 azimuthShadowThreshold; // azimuth thresholding
33     T                 elevationShadowThreshold; // elevation thresholding

```

```

33
34 // shadowing related
35 std::vector<T> checkbox; // box indicating whether a scatter for a
    range-angle pair has been found
36 std::vector<std::vector<std::vector<T>>> finalScatterBox; // a 3D tensor where the
    third dimension represnets the vector length
37 std::vector<T> finalReflectivityBox; // to store the reflectivity
38
39 // constructor
40 TransmitterClass() = default;
41
42 // Deleting copy constructors/assignment
43 TransmitterClass(const TransmitterClass& other) = delete;
44 TransmitterClass& operator=(TransmitterClass& other) = delete;
45
46 // Creating move-constructor and move-assignment
47 TransmitterClass(TransmitterClass&& other) = default;
48 TransmitterClass& operator=(TransmitterClass&& other) = default;
49
50 // member-functions
51 auto updatePointingAngle(std::vector<T> AUV_pointing_vector);
52 auto subset_scatterers(const ScattererClass<T>& seafloor,
53                       std::vector<std::size_t>& indices,
54                       const T& tilt_angle);
55
56 };
57
58 // Definitions
59 #include "TransmitterClass_subset_scatterers.inl"
60 #include "TransmitterClass_updatePointingAngle.inl"

```

2.3 Transmitter Setup Scripts

The following script shows the setup-script

```

1  template <
2      typename T,
3      typename = std::enable_if_t<
4          std::is_same_v<T, double> ||
5          std::is_same_v<T, float>
6      >
7  >
8  void fTransmitterSetup(
9      TransmitterClass<T>& transmitter_fls,
10     TransmitterClass<T>& transmitter_portside,
11     TransmitterClass<T>& transmitter_starboard
12 ){
13     // Setting up transmitter
14     T sampling_frequency {240e3}; // sampling frequency
15     T f1 {50e3}; // first frequency of LFM
16     T f2 {70e3}; // second frequency of LFM
17     T fc {(f1 + f2)/2.00}; // finding center-frequency
18     T bandwidth {std::abs(f2 - f1)}; // bandwidth
19     T pulselength {5e-2}; // time of recording
20
21     // building LFM

```

```

22  auto    timearray      {svr::linspace<T>(
23      0.00,
24      pulselength,
25      std::floor(pulselength * sampling_frequency)
26  )};
27  auto    K              {static_cast<T>(f2 - f1)/static_cast<T>(pulselength)}; //
      calculating frequency-slope
28  auto    Signal         {cos(2 * std::numbers::pi * \
29      (f1 + 0.5*K*timearray) * \
30      timearray)};      // frequency at each time-step, with f1
                        = 0
31
32  // Setting up transmitter
33  auto    location       {std::vector<T>(3, 0)};      // location of
      transmitter
34  T      azimuthal_angle_fls      {0};      // initial
      pointing direction
35  T      azimuthal_angle_port     {90};      // initial
      pointing direction
36  T      azimuthal_angle_starboard {-90};      // initial
      pointing direction
37
38  T      elevation_angle          {-60};      // initial
      pointing direction
39
40  T      azimuthal_beamwidth_fls   {20};      // azimuthal
      beamwidth of the signal cone
41  T      azimuthal_beamwidth_port  {20};      // azimuthal
      beamwidth of the signal cone
42  T      azimuthal_beamwidth_starboard {20};      // azimuthal
      beamwidth of the signal cone
43
44  T      elevation_beamwidth_fls    {20};      // elevation
      beamwidth of the signal cone
45  T      elevation_beamwidth_port   {20};      // elevation
      beamwidth of the signal cone
46  T      elevation_beamwidth_starboard {20};      // elevation
      beamwidth of the signal cone
47
48  int     azimuthQuantDensity      {10};      // number of points, a degree is split
      into quantization density along azimuth (used for shadowing)
49  int     elevationQuantDensity     {10};      // number of points, a degree is split
      into quantization density along elevation (used for shadowing)
50  T      rangeQuantSize            {10};      // the length of a cell (used for
      shadowing)
51
52  T      azimuthShadowThreshold     {1};      // azimuth threshold   (in degrees)
53  T      elevationShadowThreshold   {1};      // elevation threshold (in degrees)
54
55
56  // transmitter-fls
57  transmitter_fls.location          = location;      // Assigning
      location
58  transmitter_fls.signal           = Signal;      // Assigning
      signal
59  transmitter_fls.azimuthal_angle   = azimuthal_angle_fls;      // assigning
      azimuth angle
60  transmitter_fls.elevation_angle   = elevation_angle;      // assigning
      elevation angle

```

```

61 transmitter_fls.azimuthal_beamwidth = azimuthal_beamwidth_fls; // assigning
    azimuth-beamwidth
62 transmitter_fls.elevation_beamwidth = elevation_beamwidth_fls; // assigning
    elevation-beamwidth
63 // updating quantization densities
64 transmitter_fls.azimuthQuantDensity = azimuthQuantDensity; // assigning
    azimuth quant density
65 transmitter_fls.elevationQuantDensity = elevationQuantDensity; // assigning
    elevation quant density
66 transmitter_fls.rangeQuantSize = rangeQuantSize; // assigning
    range-quantization
67 transmitter_fls.azimuthShadowThreshold = azimuthShadowThreshold; //
    azimuth-threshold in shadowing
68 transmitter_fls.elevationShadowThreshold = elevationShadowThreshold; //
    elevation-threshold in shadowing
69 // signal related
70 transmitter_fls.f_low = f1; // assigning lower frequency
71 transmitter_fls.f_high = f2; // assigning higher frequency
72 transmitter_fls.fc = fc; // assigning center frequency
73 transmitter_fls.bandwidth = bandwidth; // assigning bandwidth
74
75
76 // transmitter-portside
77 transmitter_portside.location = location; // Assigning
    location
78 transmitter_portside.signal = Signal; // Assigning
    signal
79 transmitter_portside.azimuthal_angle = azimuthal_angle_port; // assigning
    azimuth angle
80 transmitter_portside.elevation_angle = elevation_angle; // assigning
    elevation angle
81 transmitter_portside.azimuthal_beamwidth = azimuthal_beamwidth_port; // assigning
    azimuth-beamwidth
82 transmitter_portside.elevation_beamwidth = elevation_beamwidth_port; // assigning
    elevation-beamwidth
83 // updating quantization densities
84 transmitter_portside.azimuthQuantDensity = azimuthQuantDensity; // assigning
    azimuth quant density
85 transmitter_portside.elevationQuantDensity = elevationQuantDensity; // assigning
    elevation quant density
86 transmitter_portside.rangeQuantSize = rangeQuantSize; // assigning
    range-quantization
87 transmitter_portside.azimuthShadowThreshold = azimuthShadowThreshold; //
    azimuth-threshold in shadowing
88 transmitter_portside.elevationShadowThreshold = elevationShadowThreshold; //
    elevation-threshold in shadowing
89 // signal related
90 transmitter_portside.f_low = f1; // assigning
    lower frequency
91 transmitter_portside.f_high = f2; // assigning
    higher frequency
92 transmitter_portside.fc = fc; // assigning
    center frequency
93 transmitter_portside.bandwidth = bandwidth; // assigning
    bandwidth
94
95
96 // transmitter-starboard
97 transmitter_starboard.location = location; //

```



```

    assigning location
98 transmitter_starboard.signal = Signal; //
    assigning signal
99 transmitter_starboard.azimuthal_angle = azimuthal_angle_starboard; //
    assigning azimuthal signal
100 transmitter_starboard.elevation_angle = elevation_angle;
101 transmitter_starboard.azimuthal_beamwidth = azimuthal_beamwidth_starboard;
102 transmitter_starboard.elevation_beamwidth = elevation_beamwidth_starboard;
103 // updating quantization densities
104 transmitter_starboard.azimuthQuantDensity = azimuthQuantDensity; //
    assigning azimuth-quant-density
105 transmitter_starboard.elevationQuantDensity = elevationQuantDensity;
106 transmitter_starboard.rangeQuantSize = rangeQuantSize;
107 transmitter_starboard.azimuthShadowThreshold = azimuthShadowThreshold;
108 transmitter_starboard.elevationShadowThreshold = elevationShadowThreshold;
109 // signal related
110 transmitter_starboard.f_low = f1; //
    assigning lower frequency
111 transmitter_starboard.f_high = f2; //
    assigning higher frequency
112 transmitter_starboard.fc = fc; //
    assigning center frequency
113 transmitter_starboard.bandwidth = bandwidth; //
    assigning bandwidth
114
115 }

```

Chapter 3

Uniform Linear Array

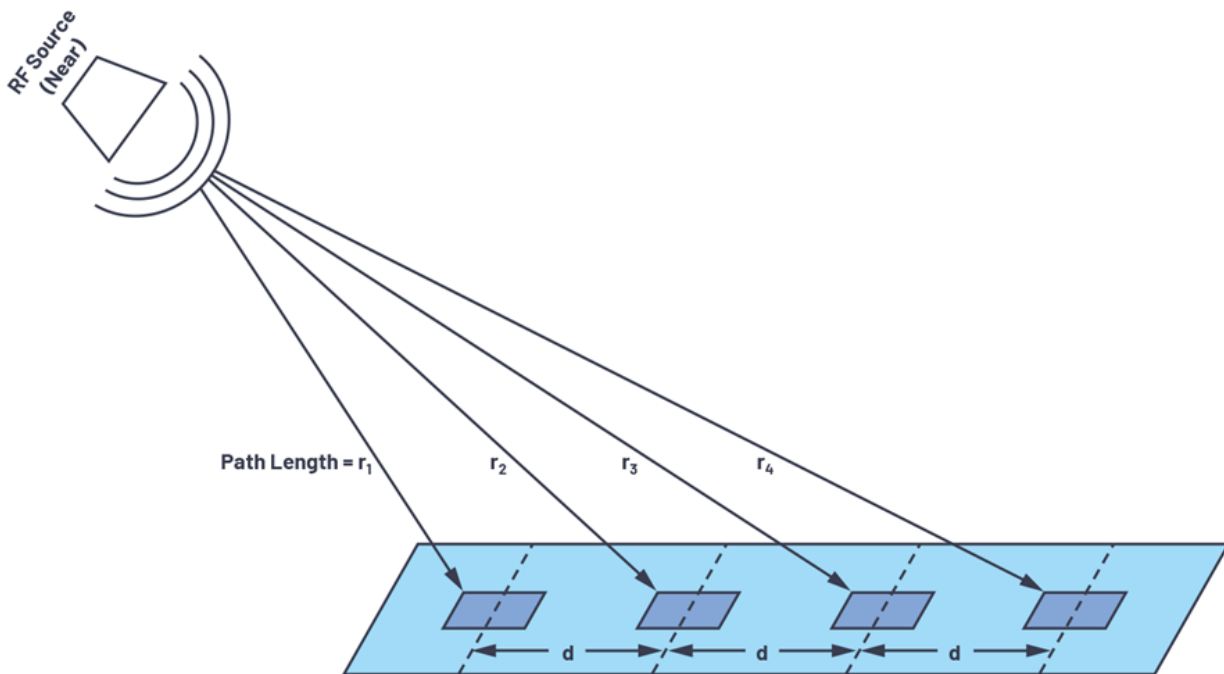


Figure 3.1: Uniform Linear Array

Overview

A Uniform Linear Array (ULA) is a common antenna or sensor configuration in which multiple elements are arranged in a straight line with equal spacing between adjacent elements. This geometry simplifies both the analysis and implementation of array signal processing techniques. In a ULA, each element receives a version of the incoming signal that differs only in phase, depending on the angle of arrival. This phase difference can be exploited to steer the array's beam in a desired direction (beamforming) or to estimate the direction of arrival (DOA) of multiple sources. The equal spacing also leads to a regular phase progression across the elements, which makes the array's response mathematically tractable and allows the use of tools like the discrete Fourier transform (DFT) to analyze spatial frequency content.

The performance of a ULA depends on the number of elements and their spacing. The spacing is typically chosen to be half the wavelength of the signal to avoid spatial aliasing, also called grating lobes, which can introduce ambiguities in DOA estimation. Increasing the number of elements improves the array's angular resolution and directivity, meaning it can better distinguish closely spaced sources and focus energy more narrowly. ULAs are widely used in radar, sonar, wireless communications, and microphone arrays due to their simplicity, predictable behavior, and compatibility with well-established signal processing algorithms. Their linear structure also makes them easier to implement in hardware compared to more complex array geometries like circular or planar arrays.

3.1 ULA Class Definition

The following is the class used to represent the uniform linear array

```

1  template <
2      svr::PureFloatingPointType      T,
3      svr::FFT_SourceDestination_Type sourceType,
4      svr::FFT_SourceDestination_Type destinationType,
5      svr::PureComplexFloatingType    T_PureComplex
6  >
7  class ULAClass
8  {
9  public:
10     // intrinsic parameters
11     std::size_t          num_sensors;                // number of
12         sensors                                                  // space between
13     T                    inter_element_spacing;        // coordinates
14         sensors                                                  // coordinates
15     std::vector<std::vector<T>> coordinates;            // coordinates
16         of each sensor
17     T                    sampling_frequency;           // sampling
18         frequency of the sensors
19     T                    recording_period;             // recording
20         period of the ULA
21     std::vector<T>       location;                    // location of
22         first coordinate
23
24     // derived
25     std::vector<T>       sensor_direction;
26     std::vector<std::vector<T>> signal_matrix;
27     std::vector<std::vector< T_PureComplex >> decimated_signal_matrix;
28     std::size_t          num_samples;
29
30     // decimation related
31     int                  decimation_factor;           // the new
32         decimation factor
33     T                    post_decimation_sampling_frequency; // the new
34         sampling frequency
35     std::vector<T>       lowpass_filter_coefficients_for_decimation; //
36         filter-coefficients for filtering
37
38     // imaging related
39     T range_resolution; // theoretical range-resolution =  $\frac{c}{2B}$ 
40     T azimuthal_resolution; // theoretical azimuth-resolution =
41          $\frac{\lambda}{(N-1)*inter-element-distance}$ 
42     T range_cell_size; // the range-cell quanta we're choosing for

```

```

    efficiency trade-off
33  T azimuth_cell_size;           // the azimuth quanta we're choosing
34  std::vector<T> azimuth_centers; // tensor containing the azimuth centers
35  std::vector<T> range_centers;   // tensor containing the range-centers
36  int frame_size;                // the frame-size corresponding to a range cell in a
    decimated signal matrix

37
38  std::vector<std::vector<complex<T>>> mulFFTMMatrix; // the matrix containing the
    delays for each-element as a slot
39  std::vector<complex<T>> matchFilter; // torch tensor containing the
    match-filter
40  int num_buffer_zeros_per_frame;      // number of zeros we're adding
    per frame to ensure no-rotation
41  std::vector<std::vector<T>> beamformedImage; // the beamformed image
42  std::vector<std::vector<T>> cartesianImage; // the cartesian version of
    beamformed image

43
44  // Decimating Related
45  std::vector<std::complex<T>> basebanding_signal;
46
47  // Artificial acoustic-image related
48  std::vector<std::vector<T>> currentArtificialAcousticImage; // acoustic image
    directly produced

49
50  // Basic Constructor
51  ULAClass() = default;
52
53  // constructor
54  ULAClass(const int num_sensors_arg,
55           const auto inter_element_spacing_arg,
56           const auto& coordinates_arg,
57           const auto& sampling_frequency_arg,
58           const auto& recording_period_arg,
59           const auto& location_arg,
60           const auto& signalMatrix_arg,
61           const auto& lowpass_filter_coefficients_for_decimation_arg):
62      num_sensors(num_sensors_arg),
63      inter_element_spacing(inter_element_spacing_arg),
64      coordinates(std::move(coordinates_arg)),
65      sampling_frequency(sampling_frequency_arg),
66      recording_period(recording_period_arg),
67      location(std::move(location_arg)),
68      signal_matrix(std::move(signalMatrix_arg)),
69      lowpass_filter_coefficients_for_decimation(std::move(lowpass_filter_coefficients_for_decima
70  {
71
72  // calculating ULA direction
73  sensor_direction = std::vector<T>{coordinates[1][0] - coordinates[0][0],
74                                     coordinates[1][1] - coordinates[0][1],
75                                     coordinates[1][2] - coordinates[0][2]};
76
77  // normalizing
78  auto norm_value_temp {std::norm(std::inner_product(sensor_direction.begin(),
79                                                       sensor_direction.end(),
80                                                       sensor_direction.begin(),
81                                                       0.00))};
82
83  // dividing
84  if (norm_value_temp != 0) {sensor_direction = sensor_direction /

```

```

norm_value_temp; }
85
86 }
87
88 // Copy-related Special Functions
89 ULAClass< T, sourceType, destinationType, T_PureComplex >(
90     const ULAClass<T, sourceType, destinationType, T_PureComplex>& other
91 ) = delete;
92 ULAClass< T, sourceType, destinationType, T_PureComplex >& operator=(
93     const ULAClass<T, sourceType, destinationType, T_PureComplex>& other
94 ) = delete;
95
96 // Move-related Special Functions
97 ULAClass< T, sourceType, destinationType, T_PureComplex >(
98     ULAClass<T, sourceType, destinationType, T_PureComplex>&& other
99 ) = default;
100 ULAClass< T, sourceType, destinationType, T_PureComplex >& operator=(
101     ULAClass<T, sourceType, destinationType, T_PureComplex>&& other
102 ) = default;
103
104 // member-functions
105 void buildCoordinatesBasedOnLocation();
106 void buildCoordinatesBasedOnLocation(const std::vector<T>& new_location);
107 void init(
108     const TransmitterClass<T>& transmitterObj,
109     svr::FFTPlanUniformPoolHandle< T_PureComplex, T_PureComplex>& fph_match_filter,
110     svr::IFFTPlanUniformPoolHandle< T_PureComplex, T_PureComplex>& ifph_match_filter);
111 void nfdc_CreateMatchFilter(
112     const TransmitterClass<T>& transmitterObj,
113     svr::FFTPlanUniformPoolHandle< T_PureComplex, T_PureComplex>& fph_match_filter,
114     svr::IFFTPlanUniformPoolHandle< T_PureComplex, T_PureComplex>& ifph_match_filter);
115 void simulate_signals(
116     const ScattererClass<T>& seafloor,
117     const std::vector<std::size_t> scatterer_indices,
118     const TransmitterClass<T>& transmitter,
119     svr::FFTPlanUniformPoolHandle<T, std::complex<T>>& fft_pool_handle,
120     svr::IFFTPlanUniformPoolHandle<std::complex<T>, T>& ifft_pool_handle);
121 void build_sensor_coordinates_from_location();
122 void image(
123     const TransmitterClass<T>& transmitter,
124     svr::FFTPlanUniformPoolHandle< T_PureComplex, T_PureComplex>& fph,
125     svr::IFFTPlanUniformPoolHandle< T_PureComplex, T_PureComplex>& ifph);
126 void decimate_signal(
127     const TransmitterClass<T>& transmitter,
128     svr::FFTPlanUniformPoolHandle< T_PureComplex, T_PureComplex>& fph,
129     svr::IFFTPlanUniformPoolHandle< T_PureComplex, T_PureComplex>& ifph);
130 void beamform(
131     svr::FFTPlanUniformPoolHandle< T_PureComplex, T_PureComplex >& fph,
132     svr::IFFTPlanUniformPoolHandle< T_PureComplex, T_PureComplex >& ifph);
133 void init_beamforming();
134 };
135
136 // Implementation - includes
137 #include "ULAClass_buildCoordinatesBasedOnLocation.inl"
138 #include "ULAClass_init.inl"
139 #include "ULAClass_init_beamforming.inl"

```

```

140 #include "ULAClass_nfdc_CreateMatchFilter.inl"
141 #include "ULAClass_simulate_signals.inl"
142 #include "ULAClass_image.inl"
143 #include "ULAClass_decimate_signal.inl"
144 #include "ULAClass_beamform.inl"

```

3.2 ULA Setup Scripts

The following script shows the setup-script for Uniform Linear Arrays

```

1  template <
2      svr::PureFloatingPointType      T,
3      svr::FFT_SourceDestination_Type sourceType,
4      svr::FFT_SourceDestination_Type destinationType,
5      svr::PureComplexFloatingType    T_PureComplex
6  >
7  void fULASetup(
8      ULAClass<T, sourceType, destinationType, T_PureComplex>& ula_fls,
9      ULAClass<T, sourceType, destinationType, T_PureComplex>& ula_portside,
10     ULAClass<T, sourceType, destinationType, T_PureComplex>& ula_starboard)
11 {
12     // setting up ula
13     auto num_sensors          {static_cast<int>(32)};          // number of sensors
14     T      sampling_frequency  {static_cast<T>(240e3)};        // sampling frequency
15     spdlog::warn("bring in a better method for choosing system-wide sampling-frequency");
16     T      inter_element_spacing {1500/(2*sampling_frequency)}; // space between
17         samples
18     T      recording_period    {10e-2};                        // sampling-period
19     auto num_samples           {static_cast<std::size_t>(
20         std::ceil(
21             sampling_frequency * recording_period
22         )
23     )};
24     // building the direction for the sensors
25     auto ULA_direction          {std::vector<T>({-1, 0, 0})};
26     auto ULA_direction_norm     {norm(ULA_direction)};
27     if (ULA_direction_norm != 0) {ULA_direction = ULA_direction/ULA_direction_norm;}
28     ULA_direction              = ULA_direction * inter_element_spacing;
29
30     // building coordinates for sensors
31     auto ULA_coordinates        {
32         transpose(ULA_direction) * \
33         svr::linspace<double>(
34             0.00,
35             num_sensors -1,
36             num_sensors)
37     };
38
39     // coefficients of decimation filter
40     auto lowpassfiltercoefficients {std::vector<T>{0.0000, 0.0000, 0.0000, 0.0000,
41         0.0000, 0.0000, 0.0001, 0.0003, 0.0006, 0.0015, 0.0030, 0.0057, 0.0100, 0.0163,
42         0.0251, 0.0364, 0.0501, 0.0654, 0.0814, 0.0966, 0.1093, 0.1180, 0.1212, 0.1179,
43         0.1078, 0.0914, 0.0699, 0.0451, 0.0192, -0.0053, -0.0262, -0.0416, -0.0504,
44         -0.0522, -0.0475, -0.0375, -0.0239, -0.0088, 0.0057, 0.0179, 0.0263, 0.0303,
45         0.0298, 0.0253, 0.0177, 0.0086, -0.0008, -0.0091, -0.0153, -0.0187, -0.0191,

```

```

-0.0168, -0.0123, -0.0065, -0.0004, 0.0052, 0.0095, 0.0119, 0.0125, 0.0112,
0.0084, 0.0046, 0.0006, -0.0031, -0.0060, -0.0078, -0.0082, -0.0075, -0.0057,
-0.0033, -0.0006, 0.0019, 0.0039, 0.0051, 0.0055, 0.0050, 0.0039, 0.0023, 0.0005,
-0.0012, -0.0025, -0.0034, -0.0036, -0.0034, -0.0026, -0.0016, -0.0004, 0.0007,
0.0016, 0.0022, 0.0024, 0.0023, 0.0018, 0.0011, 0.0003, -0.0004, -0.0011,
-0.0015, -0.0016, -0.0015}};

```

```

41
42 // assigning values
43 ula_fls.num_sensors           = num_sensors;
44 ula_fls.inter_element_spacing = inter_element_spacing;
45 ula_fls.coordinates           = ULA_coordinates;
46 ula_fls.sampling_frequency    = sampling_frequency;
47 ula_fls.recording_period      = recording_period;
48 ula_fls.sensor_direction      = ULA_direction;
49 ula_fls.lowpass_filter_coefficients_for_decimation = lowpassfiltercoefficients;
50 ula_fls.num_samples           = num_samples;
51
52
53 // assigning values
54 ula_portside.num_sensors      = num_sensors;
55 ula_portside.inter_element_spacing = inter_element_spacing;
56 ula_portside.coordinates      = ULA_coordinates;
57 ula_portside.sampling_frequency = sampling_frequency;
58 ula_portside.recording_period  = recording_period;
59 ula_portside.sensor_direction  = ULA_direction;
60 ula_portside.lowpass_filter_coefficients_for_decimation = lowpassfiltercoefficients;
61 ula_portside.num_samples      = num_samples;
62
63
64 // assigning values
65 ula_starboard.num_sensors     = num_sensors;
66 ula_starboard.inter_element_spacing = inter_element_spacing;
67 ula_starboard.coordinates     = ULA_coordinates;
68 ula_starboard.sampling_frequency = sampling_frequency;
69 ula_starboard.recording_period  = recording_period;
70 ula_starboard.sensor_direction  = ULA_direction;
71 ula_starboard.lowpass_filter_coefficients_for_decimation = lowpassfiltercoefficients;
72 ula_starboard.num_samples     = num_samples;
73 }

```

Chapter 4

Autonomous Underwater Vehicle

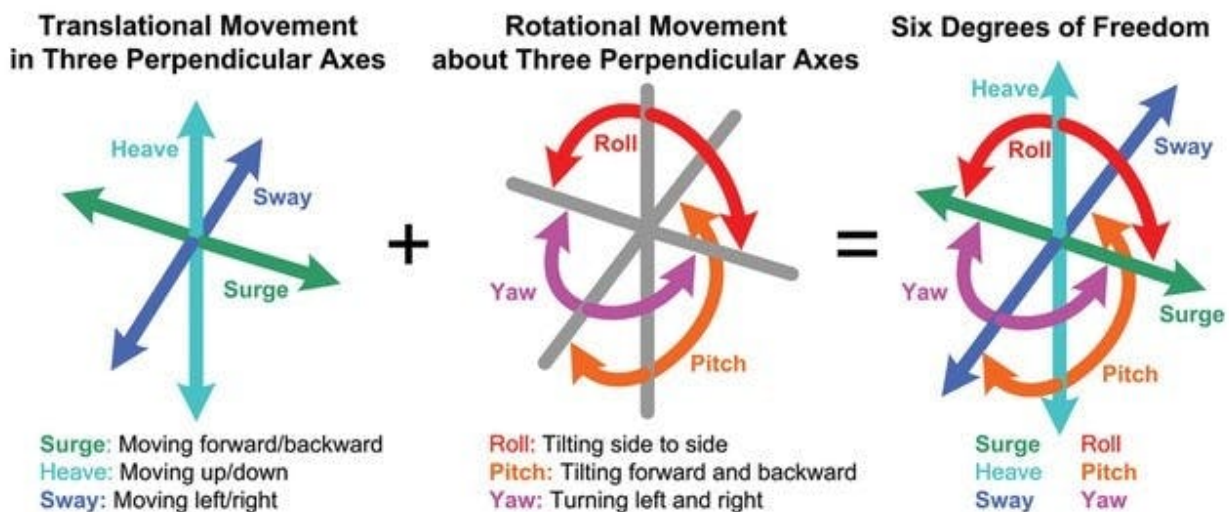


Figure 4.1: AUV degrees of freedom

Overview

Autonomous Underwater Vehicles (AUVs) are robotic systems designed to operate underwater without direct human control. They navigate and perform missions independently using onboard sensors, processors, and preprogrammed instructions. They are widely used in oceanographic research, environmental monitoring, offshore engineering, and military applications. AUVs can vary in size from small, portable vehicles for shallow water surveys to large, torpedo-shaped platforms capable of deep-sea exploration. Their autonomy allows them to access environments that are too dangerous, remote, or impractical for human divers or tethered vehicles.

The navigation and sensing systems of AUVs are critical to their performance. They typically use a combination of inertial measurement units (IMUs), Doppler velocity logs

(DVLs), pressure sensors, magnetometers, and sometimes acoustic positioning systems to estimate their position and orientation underwater. Since GPS signals do not penetrate water, AUVs must rely on these onboard sensors and occasional surfacing for GPS fixes. They are often equipped with sonar systems, cameras, or other scientific instruments to collect data about the seafloor, water column, or underwater structures. Advanced AUVs can also implement adaptive mission planning and obstacle avoidance, enabling them to respond to changes in the environment in real time.

The applications of AUVs are diverse and expanding rapidly. In scientific research, they are used for mapping the seafloor, studying marine life, and monitoring oceanographic parameters such as temperature, salinity, and currents. In the commercial sector, AUVs inspect pipelines, subsea infrastructure, and offshore oil platforms. Military and defense applications include mine countermeasure operations and underwater surveillance. The development of AUVs continues to focus on increasing endurance, improving autonomy, enhancing sensor payloads, and reducing costs, making them a key technology for exploring and understanding the underwater environment efficiently and safely.

4.1 AUV Class Definition

The following is the class used to represent the uniform linear array

```

1  template <
2      svr::PureFloatingPointType      T,
3      svr::FFT_SourceDestination_Type sourceType,
4      svr::FFT_SourceDestination_Type destinationType,
5      svr::PureComplexFloatingType    T_PureComplex
6  >
7  class AUVClass{
8  public:
9
10     // Intrinsic attributes
11     std::vector<T>      location;          // location of vessel
12     std::vector<T>      velocity;          // velocity of the vessel
13     std::vector<T>      acceleration;      // acceleration of vessel
14     std::vector<T>      pointing_direction; // AUV's pointing direction
15
16     // uniform linear-arrays
17     ULAClass<T, sourceType, destinationType, T_PureComplex>  ULA_fls;          //
18     // front-looking SONAR ULA
19     ULAClass<T, sourceType, destinationType, T_PureComplex>  ULA_portside;    //
20     // mounted ULA [object of class, ULAClass]
21     ULAClass<T, sourceType, destinationType, T_PureComplex>  ULA_starboard;    //
22     // mounted ULA [object of class, ULAClass]
23
24     // transmitters
25     TransmitterClass<T>  transmitter_fls;      // transmitter for front-looking SONAR
26     TransmitterClass<T>  transmitter_portside; // portside transmitter
27     TransmitterClass<T>  transmitter_starboard; // starboard transmitter
28
29     // derived or dependent attributes
30     std::vector<std::vector<T>>  signalMatrix_1;          // matrix containing the
31     // signals obtained from ULA_1
32     std::vector<std::vector<T>>  largeSignalMatrix_1;     // matrix holding signal of
33     // synthetic aperture

```

```

29     std::vector<std::vector<T>> beamformedLargeSignalMatrix; // each column is the
        beamformed signal at each stop-hop
30
31     // plotting mode
32     bool plottingmode; // to suppress plotting associated with classes
33
34     // spotlight mode related
35     std::vector<std::vector<T>> absolute_coords_patch_cart; // cartesian coordinates of
        patch
36
37     // Synthetic Aperture Related
38     std::vector<std::vector<T>> ApertureSensorLocations; // sensor locations of
        aperture
39
40     // functions
41     void syncComponentAttributes();
42     void init(
43         svr::ThreadPool& thread_pool,
44         svr::FFTPlanUniformPoolHandle< T_PureComplex, T_PureComplex>& fph_match_filter,
45         svr::IFFTPlanUniformPoolHandle< T_PureComplex, T_PureComplex>& ifph_match_filter);
46     void simulate_signal(
47         const ScattererClass<T>& seafloor,
48         svr::ThreadPool& thread_pool,
49         svr::FFTPlanUniformPoolHandle< sourceType, destinationType >& fft_pool_handle,
50         svr::IFFTPlanUniformPoolHandle< destinationType, sourceType >&
            ifft_pool_handle);
51     void subset_scatterers(
52         const ScattererClass<T>& seafloor,
53         svr::ThreadPool& thread_pool,
54         std::vector<std::size_t>& fls_scatterer_indices,
55         std::vector<std::size_t>& portside_scatterer_indices,
56         std::vector<std::size_t>& starboard_scatterer_indices);
57     void step(
58         T time_step);
59     void image(
60         svr::ThreadPool& thread_pool,
61         svr::FFTPlanUniformPoolHandle< T_PureComplex, T_PureComplex>& fph,
62         svr::IFFTPlanUniformPoolHandle< T_PureComplex, T_PureComplex>& ifph);
63 };
64
65 // Definitions
66 #include "AUVClass_image.inl"
67 #include "AUVClass_step.inl"
68 #include "AUVClass_simulate_signal.inl"
69 #include "AUVClass_subset_scatterers.inl"
70 #include "AUVClass_init.inl"
71 #include "AUVClass_syncComponentAttributes.inl"

```

4.2 AUV Setup Scripts

The following script shows the setup-script for Uniform Linear Arrays

```

1  template <
2      svr::PureFloatingPointType T,
3      svr::FFT_SourceDestination_Type sourceType,
4      svr::FFT_SourceDestination_Type destinationType,

```

```
5     svr::PureComplexFloatingType    T_PureComplex
6 >
7 void fAUVSetup(
8     AUVClass<T, sourceType, destinationType, T_PureComplex>& auv
9 ) {
10
11     // building properties for the auv
12     auto    location      {std::vector<T>{0, 50, 30}}; // starting location
13     auto    velocity      {std::vector<T>{5, 0, 0}};  // starting velocity
14     auto    pointing_direction {std::vector<T>{1, 0, 0}}; // pointing direction
15
16     // assigning
17     auv.location          = std::move(location);        // assigning location
18     auv.velocity          = std::move(velocity);        // assigning velocity
19     auv.pointing_direction = std::move(pointing_direction); // assigning pointing
        direction
20
21     // signaling end
22     spdlog::info("Completed AUV-setup");
23 }
```

Part II

Signal Simulation Pipeline

Chapter 5

Signal Simulation

Overview

The signal simulation pipeline is the pipeline responsible for simulating/modeling the signals sampled by the ULA-sensors under a real sub-marine environment. This chapter, and the subsequent ones, deal with the assumptions, mathematics, physics and code that goes into the design and creation of the pipeline.

A disclaimer that goes without saying is that signal-simulation is a world of its own. There's a reason that comsol, flexcompute and other numerical-simulation based companies exist. To write a signal simulation, from scratch, while these entities exist, and to make any case that this competes with those, would be flirting with delusion.

To that end, we don't write general-purpose signal simulation pipeline. However, the effort in the signal-simulator direction is purely for application-specific reasons. This is something I can talk about. One of the major in-house signal simulation pipelines yours truly developed at Naval Physical and Oceanographic Laboratory did just that. The aim of that pipeline was not to re-invent the wheel. But to create one that existed at the right speed-fidelity trade-off that the institute operated in. The pipeline created during my time there had several toggles corresponding to the different information to consider during simulation. The more information pertaining to the environment, is involved, the higher the compute and time required. Thus, mid-to-high fidelity pipelines often involve writing well-tuned GPU-supported C++ (think, CUDA). And this is important when you have pipelines downstream whose outputs depend on the signal accuracy, and by association, signal simulator fidelity.

To that end, understanding what this pipeline is not, is perhaps just as important as what it is. The core priority of this signal simulator pipeline is to produce signals for navigation. Navigation does not require high-accuracy signals owing to the very simple fact that decisions made from high-accuracy signals and low-accuracy signals tend to be the same as long as environment-topology information is not lost. To grossly oversimplify what I mean by that, the outcome of your driving doesn't change whether you have high-definition LIDAR mapping the surrounding environment to the millimeter level or if you're just driving with your eyes. Thus fidelity of simulator is not a priority and I will not be putting in the kind of effort I put in at NPOL, for this reason (also because I don't want OPSEC to be

mad).

To put it simply, the signal simulation pipeline is quite trivial as far as signal simulators are concerned. But it'll work perfectly for our purposes. And thus, we'll be choosing the simplest of systems and one I like to call, "the EE engineer's best friend": the infamous Linear Time Invariant systems.

Part III

Imaging Pipeline

Part IV

Perception & Control Pipeline

Appendix A

Application Specific Tools

A.1 CSV File-Writes

```
1 #pragma once
2 /*=====
3 writing the contents of a vector a csv-file
4 -----*/
5 template <typename T>
6 void fWriteVector(const vector<T>&          inputvector,
7                  const string&             filename){
8
9     spdlog::warn(
10         "File-write is taking place. Latency will be affected. Ignore if intentional"
11     );
12
13     // opening a file
14     std::ofstream fileobj(filename);
15     if (!fileobj) {return;}
16
17     // writing the real parts in the first column and the imaginary parts int he second
18     // column
19     if constexpr(std::is_same_v<T, std::complex<double>> ||
20                 std::is_same_v<T, std::complex<float>> ||
21                 std::is_same_v<T, std::complex<long double>>){
22         for(int i = 0; i<inputvector.size(); ++i){
23             // adding entry
24             fileobj << inputvector[i].real() << "+" << inputvector[i].imag() << "i";
25
26             // adding delimiter
27             if(i!=inputvector.size()-1) {fileobj << ",";}
28             else {fileobj << "\n";}
29         }
30     }
31     else{
32         for(int i = 0; i<inputvector.size(); ++i){
33             fileobj << inputvector[i];
34             if(i!=inputvector.size()-1) {fileobj << ",";}
35             else {fileobj << "\n";}
36         }
37     }
```

```

38     // return
39     return;
40 }
41 /*=====
42 writing the contents of a matrix to a csv-file
43 -----*/
44 template <typename T>
45 auto fWriteMatrix(const std::vector<std::vector<T>> inputMatrix,
46                  const string filename){
47
48     // opening a file
49     std::ofstream fileobj(filename);
50
51     // writing
52     if (fileobj){
53         for(int i = 0; i<inputMatrix.size(); ++i){
54             for(int j = 0; j<inputMatrix[0].size(); ++j){
55                 fileobj << inputMatrix[i][j];
56                 if (j!=inputMatrix[0].size()-1) {fileobj << ",";}
57                 else {fileobj << "\n";}
58             }
59         }
60     }
61     else{
62         cout << format("File-write to {} failed\n", filename);
63     }
64 }
65
66 /*=====
67 writing complex-matrix to a csv-file
68 -----*/
69 template <>
70 auto fWriteMatrix(const std::vector<std::vector<std::complex<double>>> inputMatrix,
71                  const string filename){
72
73     // opening a file
74     std::ofstream fileobj(filename);
75
76     // writing
77     if (fileobj){
78         for(int i = 0; i<inputMatrix.size(); ++i){
79             for(int j = 0; j<inputMatrix[0].size(); ++j){
80                 fileobj << inputMatrix[i][j].real() << "+" << inputMatrix[i][j].imag() <<
81                     "i";
82                 if (j!=inputMatrix[0].size()-1) {fileobj << ",";}
83                 else {fileobj << "\n";}
84             }
85         }
86     }
87     else{
88         cout << format("File-write to {} failed\n", filename);
89     }
90 }

```

A.2 Thread-Pool

```

1  #pragma once
2  namespace svr {
3      class ThreadPool {
4      public:
5          // Members
6          boost::asio::thread_pool    thread_pool;        // the pool
7          std::vector<std::future<void>> future_vector;    // futures to wait on
8
9          // Special-Members
10         ThreadPool(std::size_t num_threads) : thread_pool(num_threads) {}
11         ThreadPool(const ThreadPool& other)    = delete;
12         ThreadPool& operator=(ThreadPool& other) = delete;
13
14         // Member-functions
15         void                converge();
16         template <typename F> void push_back(F&& func);
17         void                shutdown();
18
19     private:
20         template<typename F>
21         std::future<void> _wrap_task(F&& func) {
22             std::promise<void> p;
23             auto f = p.get_future();
24
25             boost::asio::post(thread_pool,
26                 [func = std::forward<F>(func), p = std::move(p)]() mutable {
27                 func();
28                 p.set_value();
29             });
30
31             return f;
32         }
33     };
34
35     /*=====
36     Member-Function: Add new task to the pool
37     -----*/
38     template <typename F>
39     void ThreadPool::push_back(F&& func)
40     {
41         future_vector.push_back(_wrap_task(std::forward<F>(func)));
42     }
43     /*=====
44     Member-Function: waiting until all the assigned work is done
45     -----*/
46     void ThreadPool::converge()
47     {
48         for (auto &fut : future_vector) fut.get();
49         future_vector.clear();
50     }
51     /*=====
52     Member-Function: Shutting things down
53     -----*/
54     void ThreadPool::shutdown()
55     {
56         thread_pool.join();
57     }
58
59 }

```

A.3 FFTPlanClass

```

1  #pragma once
2
3  /*=====
4  including dependencies
5  -----*/
6  #include "svr_concepts.hpp"
7
8
9  namespace svr {
10
11     template <
12         svr::FFT_SourceDestination_Type sourceType,
13         svr::FFT_SourceDestination_Type destinationType
14     >
15     class FFTPlanClass
16     {
17     public:
18
19         // Members
20         std::size_t    nfft_        {std::numeric_limits<std::size_t>::max()};
21         fftw_complex*  in_          {nullptr};
22         fftw_complex*  out_         {nullptr};
23         fftw_plan      plan_        {nullptr};
24
25         /*=====
26         Destructor
27         -----*/
28         ~FFTPlanClass()
29         {
30             if(plan_ != nullptr) {fftw_destroy_plan(    plan_);}
31             if(in_   != nullptr) {fftw_free(            in_);}
32             if(out_  != nullptr) {fftw_free(            out_);}
33         }
34         /*=====
35         Default Constructor
36         -----*/
37         FFTPlanClass() = default;
38         /*=====
39         Constructor
40         -----*/
41         FFTPlanClass(const std::size_t  nfft)
42         {
43
44             // allocating nfft
45             this->nfft_ = nfft;
46
47             // allocating input-region
48             in_ = reinterpret_cast<fftw_complex*>(
49                 fftw_malloc(nfft_ * sizeof(fftw_complex))
50             );
51             out_ = reinterpret_cast<fftw_complex*>(
52                 fftw_malloc(nfft_ * sizeof(fftw_complex))
53             );
54             if(!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
55                 CLASS: FFTPlanClass | REPORT: in-out allocation failed");}
56
57             // creating plan

```

```

57     plan_ = fftw_plan_dft_1d(
58         static_cast<int>(nfft_),
59         in_,
60         out_,
61         FFTW_FORWARD,
62         FFTW_MEASURE
63     );
64     if(!plan_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
65         CLASS: FFTPlanClass | REPORT: plan-creation failed");}
66 }
67 /*=====
68 Copy Constructor
69 -----*/
70 FFTPlanClass(const FFTPlanClass& other)
71 {
72     // copying nfft
73     nfft_ = other.nfft_;
74     cout << format("\t\t FFTPlanClass(const FFTPlanClass& other) | nfft_ =
75         {}\n", nfft_);
76
77     // allocating input-region
78     in_ = reinterpret_cast<fftw_complex*>(
79         fftw_malloc(nfft_ * sizeof(fftw_complex))
80     );
81     out_ = reinterpret_cast<fftw_complex*>(
82         fftw_malloc(nfft_ * sizeof(fftw_complex))
83     );
84     if(!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
85         CLASS: FFTPlanClass | REPORT: in-out allocation failed");}
86
87     // copying input-region and output-region
88     std::memcpy(in_, other.in_, nfft_ * sizeof(fftw_complex));
89     std::memcpy(out_, other.out_, nfft_ * sizeof(fftw_complex));
90
91     // creating plan
92     plan_ = fftw_plan_dft_1d(
93         static_cast<int>(nfft_),
94         in_,
95         out_,
96         FFTW_FORWARD,
97         FFTW_MEASURE
98     );
99     if(!plan_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
100         CLASS: FFTPlanClass | REPORT: plan-creation failed");}
101 }
102 /*=====
103 Copy Assignment
104 -----*/
105 FFTPlanClass& operator=(const FFTPlanClass& other)
106 {
107     // handling self-assignment
108     if (this == &other) {return *this;}
109
110     // cleaning-up existing resources
111     if(plan_ != nullptr) {fftw_destroy_plan(    plan_);}
112     if(in_ != nullptr) {fftw_free(            in_);}
113     if(out_ != nullptr) {fftw_free(           out_);}
114
115     // allocating input-region and output-region

```

```

112     nfft_ = other.nfft_;
113     in_   = reinterpret_cast<fftw_complex*>(
114         fftw_malloc(nfft_ * sizeof(fftw_complex))
115     );
116     out_  = reinterpret_cast<fftw_complex*>(
117         fftw_malloc(nfft_ * sizeof(fftw_complex))
118     );
119     if(!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
        CLASS: FFTPlanClass | FUNCTION: Copy-Assignment | REPORT: in-out
        allocation failed");}

120
121     // copying contents
122     cout << format("\t\t FFTPlanClass& operator=(const FFTPlanClass& other) |
        nfft_ = {}\n", nfft_);
123     std::memcpy(in_, other.in_, nfft_ * sizeof(fftw_complex));
124     std::memcpy(out_, other.out_, nfft_ * sizeof(fftw_complex));
125
126     // creating engine
127     plan_ = fftw_plan_dft_id(
128         static_cast<int>(nfft_),
129         in_,
130         out_,
131         FFTW_FORWARD,
132         FFTW_MEASURE
133     );
134     if(!plan_) {throw std::runtime_error("FILE: FFTPlanClass.hpp | CLASS:
        FFTPlanClass | FUNCTION: Copy-Assignment | REPORT: plan-creation
        failed");}

135
136     // returning
137     return *this;
138 }
139 /*=====
140 Move Constructor
141 -----*/
142 FFTPlanClass(FFTPlanClass&& other)
143 :   nfft_(      other.nfft_),
144   in_(         other.in_),
145   out_(        other.out_),
146   plan_(       other.plan_)
147 {
148     // resetting the others
149     other.nfft_ = 0;
150     other.in_   = nullptr;
151     other.out_  = nullptr;
152     other.plan_ = nullptr;
153 }
154 /*=====
155 Move Assignment
156 -----*/
157 FFTPlanClass& operator=(FFTPlanClass&& other)
158 {
159     // self-assignment check
160     if (this == &other) {return *this;}
161
162     // cleaning up existing resources
163     if(plan_ != nullptr) {fftw_destroy_plan( plan_);}
164     if(in_   != nullptr) {fftw_free(       in_);}
165     if(out_  != nullptr) {fftw_free(       out_);}

```

```

166
167 // Copying-values and changing pointers
168 nfft_ = other.nfft_;
169 cout << format("\t\t FFTPlanClass's MOVE assignment | nfft_ = {}\n",
    nfft_);
170 in_ = other.in_;
171 out_ = other.out_;
172 plan_ = other.plan_;
173
174 // resetting source-members
175 other.nfft_ = 0;
176 other.in_ = nullptr;
177 other.out_ = nullptr;
178 other.plan_ = nullptr;
179
180 // returning
181 return *this;
182 }
183 /*=====
184 Running fft
185 -----*/
186 std::vector<destinationType>
187 fft(const std::vector<sourceType>& input_vector)
188 {
189 // throwing an error
190 if (input_vector.size() > nfft_){
191     cout << format("input_vector.size() = {}, nfft_ = {}\n",
192         input_vector.size(),
193         nfft_);
194     throw std::runtime_error("FILE: FFTPlanClass.hpp | CLASS: FFTPlanClass
        | FUNCTION: fft() | REPORT: input-vector size is greater than
        NFFT");
195 }
196
197 // copying inputs
198 for(std::size_t index = 0; index < input_vector.size(); ++index)
199 {
200     if constexpr(
201         std::is_floating_point_v<sourceType>
202     ){
203         in_[index][0] = input_vector[index];
204         in_[index][1] = 0;
205     }
206     else if constexpr(
207         std::is_same_v<sourceType, std::complex<float>> ||
208         std::is_same_v<sourceType, std::complex<double>>
209     ){
210         in_[index][0] = input_vector[index].real();
211         in_[index][1] = input_vector[index].imag();
212     }
213 }
214 }
215
216 // executing fft
217 fftw_execute(plan_);
218
219 // copying results to output-vector
220 std::vector<destinationType> output_vector(nfft_);
221 for(std::size_t index = 0; index < nfft_; ++index){

```

```

222         if constexpr(
223             std::is_same_v<destinationType, std::complex<float>> ||
224             std::is_same_v<destinationType, std::complex<double>>
225         ){
226             output_vector[index] = destinationType(
227                 out_[index][0],
228                 out_[index][1]
229             );
230         }
231         else if constexpr(
232             std::is_floating_point_v<destinationType>
233         ){
234             output_vector[index] = static_cast<destinationType>(
235                 std::sqrt(
236                     std::pow(out_[index][0], 2) + \
237                     std::pow(out_[index][1], 2)
238                 )
239             );
240         }
241     }
242
243     // returning output
244     return std::move(output_vector);
245 }
246
247 /*=====
248 Running fft - balanced
249 -----*/
250 std::vector<destinationType>
251 fft_l2_conserved(const std::vector<sourceType>& input_vector)
252 {
253     // throwing an error
254     if (input_vector.size() > nfft_)
255         throw std::runtime_error("FILE: FFTPlanClass.hpp | CLASS: FFTPlanClass
256             | FUNCTION: fft() | REPORT: input-vector size is greater than
257             NFFT");
258
259     // copying inputs
260     for(std::size_t index = 0; index < input_vector.size(); ++index)
261     {
262         if constexpr(
263             std::is_floating_point_v<sourceType>
264         ){
265             in_[index][0] = input_vector[index];
266             in_[index][1] = 0;
267         }
268         else if constexpr(
269             std::is_same_v<sourceType, std::complex<float>> ||
270             std::is_same_v<sourceType, std::complex<double>>
271         ){
272             in_[index][0] = input_vector[index].real();
273             in_[index][1] = input_vector[index].imag();
274         }
275     }
276
277     // executing fft
278     fftw_execute(plan_);
279
280     // copying results to output-vector
281     std::vector<destinationType> output_vector(nfft_);

```



```

279     for(std::size_t index = 0; index < nfft_; ++index)
280     {
281         if constexpr(
282             std::is_same_v< destinationType, std::complex<double> > ||
283             std::is_same_v< destinationType, std::complex<float> >
284         ){
285             output_vector[index] = destinationType(
286                 out_[index][0] * (1.00 / std::sqrt(nfft_)),
287                 out_[index][1] * (1.00 / std::sqrt(nfft_))
288             );
289         }
290         else if constexpr(
291             std::is_floating_point_v<destinationType>
292         ){
293             output_vector[index] = destinationType(
294                 std::sqrt(
295                     std::pow(out_[index][0] * (1.00 / std::sqrt(nfft_)), 2) + \
296                     std::pow(out_[index][1] * (1.00 / std::sqrt(nfft_)), 2)
297                 )
298             );
299         }
300     }
301
302     // returning output
303     return std::move(output_vector);
304 }
305 };
306 }

```

A.4 IFFTPlanClass

```

1  #pragma once
2  namespace svr {
3
4      template <typename T>
5      concept IFFTPlanClassSourceDestinationType = \
6          std::is_floating_point_v<T> ||
7          (
8              std::is_class_v<T> &&
9              std::is_floating_point_v<typename T::value_type>
10         );
11     template <
12         IFFTPlanClassSourceDestinationType sourceType,
13         IFFTPlanClassSourceDestinationType destinationType
14     >
15     class IFFTPlanClass
16     {
17     public:
18         std::size_t      nfft_;
19         fftw_complex*     in_;
20         fftw_complex*     out_;
21         fftw_plan         plan_;
22
23         /*=====
24         Destructor
25         -----*/

```

```

26 ~IFFTPlanClass()
27 {
28     if(plan_ != nullptr) {fftw_destroy_plan(    plan_);}
29     if(in_  != nullptr)  {fftw_free(           in_);}
30     if(out_ != nullptr)  {fftw_free(           out_);}
31 }
32 /*=====
33 Constructor
34 -----*/
35 IFFTPlanClass(const std::size_t nfft): nfft_(nfft)
36 {
37     // allocating space
38     in_  = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
39         sizeof(fftw_complex)));
40     out_ = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
41         sizeof(fftw_complex)));
42     if(!in_ || !out_) {throw std::runtime_error("in_, out_ creation
43         failed");}
44
45     // creating plan
46     plan_ = fftw_plan_dft_1d(
47         static_cast<int>(nfft_),
48         in_,
49         out_,
50         FFTW_BACKWARD,
51         FFTW_MEASURE
52     );
53     if(!plan_) {throw std::runtime_error("File: FFTPlanClass.hpp |
54         Class: IFFTPlanClass | report: plan-creation failed");}
55 }
56 /*=====
57 Copy Constructor
58 -----*/
59 IFFTPlanClass(const IFFTPlanClass& other)
60 {
61     // allocating space
62     nfft_ = other.nfft_;
63     in_  = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
64         sizeof(fftw_complex)));
65     out_ = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
66         sizeof(fftw_complex)));
67     if (!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
68         Class: IFFTPlanClass | Function: Copy-Constructor | Report: in-out
69         plan creation failed");}
70
71     // copying contents
72     std::memcpy(in_, other.in_, nfft_ * sizeof(fftw_complex));
73     std::memcpy(out_, other.out_, nfft_ * sizeof(fftw_complex));
74
75     // creating a new plan since its more of an engine
76     plan_ = fftw_plan_dft_1d(
77         static_cast<int>(nfft_),
78         in_,
79         out_,
80         FFTW_BACKWARD,
81         FFTW_MEASURE
82     );
83     if(!plan_) {throw std::runtime_error("FILE: FFTPlanClass.hpp | Class:
84         IFFTPlanClass | Function: Copy-Constructor | Report: plan-creation

```

```

76         failed");}
77     }
78     /*=====
79     Copy Assignment
80     -----*/
81     IFFTPlanClass& operator=(const IFFTPlanClass& other)
82     {
83         // handling self-assignment
84         if(this == &other) {return *this;}
85
86         // cleaning up existing resources
87         if(plan_ != nullptr) {fftw_destroy_plan(    plan_);}
88         if(in_  != nullptr) {fftw_free(            in_);}
89         if(out_ != nullptr) {fftw_free(            out_);}
90
91         // allocating space
92         nfft_ = other.nfft_;
93         in_   = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
94             sizeof(fftw_complex)));
95         out_  = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
96             sizeof(fftw_complex)));
97         if (!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
98             Class: IFFTPlanClass | Function: Copy-Constructor | Report: in-out
99             plan creation failed");}
100
101         // copying contents
102         std::memcpy(in_, other.in_, nfft_ * sizeof(fftw_complex));
103         std::memcpy(out_, other.out_, nfft_ * sizeof(fftw_complex));
104
105         // creating a new plan since its more of an engine
106         plan_ = fftw_plan_dft_1d(
107             static_cast<int>(nfft_),
108             in_,
109             out_,
110             FFTW_BACKWARD,
111             FFTW_MEASURE
112         );
113         if(!plan_) {throw std::runtime_error("FILE: FFTPlanClass.hpp | Class:
114             IFFTPlanClass | Function: Copy-Constructor | Report: plan-creation
115             failed");}
116
117         // returning
118         return *this;
119     }
120     /*=====
121     Move Constructor
122     -----*/
123     IFFTPlanClass(IFFTPlanClass&& other) noexcept
124     :   nfft_( other.nfft_),
125         in_(   other.in_),
126         out_(  other.out_),
127         plan_( other.plan_)
128     {
129         // resetting the source object
130         other.nfft_ = 0;
131         other.in_   = nullptr;
132         other.out_  = nullptr;

```

```

128     other.plan_    = nullptr;
129 }
130 /*=====
131 Move-Assignment
132 -----*/
133 IFFTPlanClass& operator=(IFFTPlanClass&& other) noexcept
134 {
135     // self-assignment check
136     if(this == &other)    {return *this;}
137
138     // cleaning up existing
139     if(plan_ != nullptr) {fftw_destroy_plan(    plan_);}
140     if(in_   != nullptr) {fftw_free(           in_);}
141     if(out_  != nullptr) {fftw_free(           out_);}
142
143     // Copying values and changing pointers
144     nfft_    = other.nfft_;
145     in_       = other.in_;
146     out_      = other.out_;
147     plan_     = other.plan_;
148
149     // resetting the source-object
150     other.nfft_ = 0;
151     other.in_   = nullptr;
152     other.out_  = nullptr;
153     other.plan_ = nullptr;
154
155     // returning
156     return *this;
157 }
158 /*=====
159 Running
160 -----*/
161 std::vector<destinationType>
162 ifft(const std::vector<sourceType>& input_vector)
163 {
164     // throwing an error
165     if (input_vector.size() > nfft_)
166         throw std::runtime_error("File: FFTPlanClass | Class: IFFTPlanClass |
167                                     Function: ifft() | Report: size of vector > nfft ");
168
169     // copy input into fftw buffer
170     for(std::size_t index = 0; index < nfft_; ++index)
171     {
172         if constexpr(
173             std::is_same_v<    sourceType, std::complex<double> > ||
174             std::is_same_v<    sourceType, std::complex<float> >
175         ){
176             in_[index][0] = input_vector[index].real();
177             in_[index][1] = input_vector[index].imag();
178         }
179         else if constexpr(
180             std::is_floating_point_v< sourceType >
181         ){
182             in_[index][0] = input_vector[index];
183             in_[index][1] = 0;
184         }
185     }

```

```

186 // execute ifft
187 fftw_execute(plan_);
188
189 // normalize output
190 std::vector<destinationType> output_vector(nfft_);
191 for(std::size_t index = 0; index < nfft_; ++index){
192     if constexpr(
193         std::is_floating_point_v< destinationType >
194     ){
195         output_vector[index] =
196             static_cast<destinationType>(out_[index][0]/nfft_);
197     }
198     else if constexpr(
199         std::is_same_v< destinationType, std::complex<double> > ||
200         std::is_same_v< destinationType, std::complex<float> >
201     ){
202         output_vector[index][0] = destinationType(
203             out_[index][0]/nfft_,
204             out_[index][1]/nfft_
205         );
206     }
207 }
208
209 // returning
210 return std::move(output_vector);
211 }
212 /*=====
213 Running - proper bases change
214 -----*/
215 std::vector<destinationType>
216 ifft_l2_conserved(const std::vector<sourceType>& input_vector)
217 {
218     // throwing an error
219     if (input_vector.size() > nfft_)
220         throw std::runtime_error("File: FFTPlanClass | Class: IFFTPlanClass |
221             Function: ifft() | Report: size of vector > nfft ");
222
223     // copy input into fftw buffer
224     for(std::size_t index = 0; index < nfft_; ++index)
225     {
226         if constexpr(
227             std::is_same_v< sourceType, std::complex<double> > ||
228             std::is_same_v< sourceType, std::complex<float> >
229         ){
230             in_[index][0] = input_vector[index].real();
231             in_[index][1] = input_vector[index].imag();
232         }
233         else if constexpr(
234             std::is_floating_point_v<sourceType>
235         ){
236             in_[index][0] = input_vector[index];
237             in_[index][1] = 0;
238         }
239     }
240
241     // execute ifft
242     fftw_execute(plan_);
243
244     // normalize output

```

```

243     std::vector< destinationType > output_vector(nfft_);
244     for(std::size_t index = 0; index < nfft_; ++index)
245     {
246         if constexpr(
247             std::is_floating_point_v<destinationType>
248         ){
249             output_vector[index] = static_cast<destinationType>(out_[index][0]
250                 * 1.00/std::sqrt(nfft_));
251         }
252         else if constexpr(
253             std::is_same_v< destinationType, std::complex<double> > ||
254             std::is_same_v< destinationType, std::complex<float> >
255         ){
256             output_vector[index] = destinationType(
257                 out_[index][0] * 1.00/std::sqrt(nfft_),
258                 out_[index][1] * 1.00/std::sqrt(nfft_)
259             );
260         }
261     }
262     // returning
263     return std::move(output_vector);
264 }
265 };
266 }

```

A.5 FFT Plan Pool

```

1  #pragma once
2  namespace svr {
3      template <typename T>
4      concept FFTPlanUniformPoolSourceDestinationType = \
5          std::is_floating_point_v<T> ||
6          (
7              std::is_class_v<T> &&
8              std::is_floating_point_v<typename T::value_type>
9          );
10     template <
11         FFTPlanUniformPoolSourceDestinationType sourceType,
12         FFTPlanUniformPoolSourceDestinationType destinationType
13     >
14     class FFTPlanUniformPool {
15     public:
16         /*=====
17         Handle to Plan
18         -----*/
19         struct AccessPairs
20         {
21             /*=====
22             Members
23             -----*/
24             svr::FFTPlanClass<sourceType, destinationType>& plan;
25             std::unique_lock<std::mutex> lock;
26
27             /*=====
28             Special Members

```

```

29 -----*/
30 AccessPairs() = delete;
31 AccessPairs(
32     svr::FFTPlanClass<sourceType, destinationType>& plan_arg,
33     std::mutex& plan_mutex
34 ) : plan(plan_arg), lock(plan_mutex) {}
35 AccessPairs(
36     svr::FFTPlanClass<sourceType, destinationType>& plan_arg,
37     std::unique_lock<std::mutex>&& lock_arg
38 ): plan(plan_arg), lock(std::move(lock_arg)) {}
39 AccessPairs(const AccessPairs& other) = delete;
40 AccessPairs& operator=(const AccessPairs& other) = delete;
41 AccessPairs(AccessPairs&& other) = delete;
42 AccessPairs& operator=(AccessPairs&& other) = delete;
43 };
44
45 /*=====
46 Core Members
47 -----*/
48 std::vector<svr::FFTPlanClass<sourceType, destinationType>> plans;
49 std::vector<std::mutex> mutexes;
50
51 /*=====
52 Special-Members
53 -----*/
54 FFTPlanUniformPool() = default;
55 FFTPlanUniformPool(const std::size_t num_plans,
56                    const std::size_t nfft)
57 {
58     // reserving space
59     plans.reserve(num_plans);
60     for(auto i = 0; i < num_plans; ++i){
61         plans.emplace_back(nfft);
62     }
63
64     // creating a vector of mutexes
65     mutexes = std::move(std::vector<std::mutex>(num_plans));
66 }
67 FFTPlanUniformPool(const FFTPlanUniformPool& other) = delete;
68 FFTPlanUniformPool& operator=(const FFTPlanUniformPool& other) = delete;
69 FFTPlanUniformPool(FFTPlanUniformPool&& other) = default;
70 FFTPlanUniformPool& operator=(FFTPlanUniformPool&& other) = default;
71
72 /*=====
73 Function to fetch a plan
74 > searches for a free-plan
75 > if found, locks the plan
76 > return the handle to the plan
77 -----*/
78 AccessPairs fetch_plan() {
79     const int num_rounds = 12;
80     for (int round = 0; round < num_rounds; ++round) {
81         for (int i = 0; i < mutexes.size(); ++i) {
82
83             std::unique_lock<std::mutex> curr_lock(
84                 mutexes[i],
85                 std::try_to_lock
86             );
87             if (curr_lock.owns_lock())

```

```

88         return AccessPairs(plans[i], std::move(curr_lock));
89     }
90 }
91 }
92 throw std::runtime_error(
93     "FILE: FFTPlanPoolClass.hpp | CLASS: FFTPlanUniformPool | FUNCTION:
94     fetch_plan() | "
95     "Report: No plans available despite num_rounds rounds of searching");
96 };
97 }

```

A.6 IFFT Plan Pool

```

1  #pragma once
2
3  /*=====
4  Dependencies
5  -----*/
6
7  namespace svr {
8
9      template <typename T>
10     concept IFFTPlanUniformPoolSourceDestinationType = \
11         std::is_floating_point_v<T> ||
12         (
13             std::is_class_v<T> &&
14             std::is_floating_point_v<typename T::value_type>
15         );
16     template <
17         IFFTPlanUniformPoolSourceDestinationType sourceType,
18         IFFTPlanUniformPoolSourceDestinationType destinationType
19     >
20     class IFFTPlanUniformPool
21     {
22     public:
23         /*=====
24         Structure used for interfacing to plans
25         -----*/
26         struct AccessPairs
27         {
28             /*=====
29             Core Members
30             -----*/
31             svr::IFFTPlanClass<sourceType, destinationType>& plan;
32             std::unique_lock<std::mutex> lock;
33
34             /*=====
35             Special Members
36             -----*/
37             AccessPairs() = delete;
38             AccessPairs(
39                 svr::IFFTPlanClass<sourceType, destinationType>& plan_arg,
40                 std::mutex& plan_mutex_arg
41             ): plan(plan_arg), lock(plan_mutex_arg) {}
42             AccessPairs(

```



```

43         svr::IFFTPlanClass<sourceType, destinationType>& plan_arg,
44         std::unique_lock<std::mutex>&& lock_arg
45     ): plan(plan_arg), lock(std::move(lock_arg)) {}
46     AccessPairs(const AccessPairs& other) = delete;
47     AccessPairs& operator=(const AccessPairs& other) = delete;
48     AccessPairs(AccessPairs&& other) = delete;
49     AccessPairs& operator=(AccessPairs&& other) = delete;
50 };
51
52 /*=====
53 Core Members
54 -----*/
55 std::vector< svr::IFFTPlanClass<sourceType, destinationType> > plans;
56 std::vector< std::mutex > mutexes;
57
58 /*=====
59 Special Members
60 -----*/
61 IFFTPlanUniformPool() = default;
62 IFFTPlanUniformPool(const std::size_t num_plans,
63                     const std::size_t nfft)
64 {
65     // reserving space
66     plans.reserve(num_plans);
67     for(auto i = 0; i < num_plans; ++i)
68         plans.emplace_back(nfft);
69
70     // creating vector of mutexes
71     mutexes = std::vector<std::mutex>(num_plans);
72 }
73 IFFTPlanUniformPool(const IFFTPlanUniformPool& other) = delete;
74 IFFTPlanUniformPool& operator=(const IFFTPlanUniformPool& other) = delete;
75 IFFTPlanUniformPool(IFFTPlanUniformPool&& other) = default;
76 IFFTPlanUniformPool& operator=(IFFTPlanUniformPool&& other) = default;
77
78 /*=====
79 Member-Functions
80 -----*/
81 AccessPairs fetch_plan()
82 {
83     // setting the number of rounds to take
84     const int num_rounds {12};
85
86     // performing rounds
87     for(auto round = 0; round < num_rounds; ++round)
88     {
89         // going through vector mutexes
90         for(auto i = 0; i < mutexes.size(); ++i)
91         {
92             // trying to lock current mutex
93             std::unique_lock<std::mutex> curr_lock(mutexes[i],
94                                                    std::try_to_lock);
95
96             // if our lock contains the mutex, returning the plan and lock
97             if (curr_lock.owns_lock())
98                 return AccessPairs(plans[i], std::move(curr_lock));
99         }
100     }

```

```

101         // throwing error
102         throw std::runtime_error("FILE: IFFTPlanPoolClass.hpp | CLASS:
103             IFFTPlanUniformPool | REPORT: COULDN'T FIND ANY AVAILABLE PLANS");
104     };
105 }

```

A.7 FFT Plan Pool Handle

```

1  #pragma once
2
3  /*=====
4  Dependencies
5  -----*/
6  #include "FFTPlanPoolClass.hpp"
7
8  namespace svr
9  {
10     template <typename T>
11     concept FFTPlanUniformPoolHandleSourceDestinationType = \
12         std::is_floating_point_v<T> ||
13         (
14             std::is_class_v<T>      &&
15             std::is_floating_point_v<typename T::value_type>
16         );
17     template <
18         FFTPlanUniformPoolHandleSourceDestinationType sourceType,
19         FFTPlanUniformPoolHandleSourceDestinationType destinationType
20     >
21     struct FFTPlanUniformPoolHandle
22     {
23         /*=====
24         Core Members
25         -----*/
26         svr::FFTPlanUniformPool<sourceType, destinationType> uniform_pool;
27         std::mutex                                         mutex;
28         std::size_t                                       num_plans;
29         std::size_t                                       nfft;
30
31         /*=====
32         Special Member-functions
33         -----*/
34         FFTPlanUniformPoolHandle() = default;
35         FFTPlanUniformPoolHandle(const std::size_t num_plans_arg,
36                                 const std::size_t nfft_arg)
37             : uniform_pool(num_plans_arg, nfft_arg),
38               num_plans(num_plans_arg),
39               nfft(nfft_arg) {}
40         FFTPlanUniformPoolHandle(const FFTPlanUniformPoolHandle& other) = delete;
41         FFTPlanUniformPoolHandle& operator=(const FFTPlanUniformPoolHandle& other) =
42             delete;
43         FFTPlanUniformPoolHandle(FFTPlanUniformPoolHandle&& other) = default;
44         FFTPlanUniformPoolHandle& operator=(FFTPlanUniformPoolHandle&& other) = default;
45
46         /*=====
47         Member Functions

```

```

47         -----*/
48     auto    lock()
49     {
50         return std::unique_lock<std::mutex>(this->mutex);
51     }
52 };
53 }

```

A.8 IFFT Plan Pool Handle

```

1  #pragma once
2
3  /*=====
4  Dependencies
5  -----*/
6  #include "IFFTPlanPoolClass.hpp"
7
8
9  namespace svr
10 {
11     template <typename T>
12     concept IFFTPlanUniformPoolHandleSourceDestinationType = \
13         std::is_floating_point_v<T> ||
14         (
15             std::is_class_v<T> &&
16             std::is_floating_point_v<typename T::value_type>
17         );
18     template <
19         IFFTPlanUniformPoolHandleSourceDestinationType sourceType,
20         IFFTPlanUniformPoolHandleSourceDestinationType destinationType
21     >
22     struct IFFTPlanUniformPoolHandle
23     {
24         /*=====
25         Members
26         -----*/
27         IFFTPlanUniformPool< sourceType,
28                             destinationType >    uniform_pool;
29         std::mutex                mutex;
30         std::size_t               num_plans;
31         std::size_t               nfft;
32
33         /*=====
34         Special Member Functions
35         -----*/
36         IFFTPlanUniformPoolHandle() = default;
37         IFFTPlanUniformPoolHandle(const std::size_t num_plans_arg,
38                                   const std::size_t nfft_arg)
39         :    uniform_pool(    num_plans_arg, nfft_arg),
40             num_plans(    num_plans_arg),
41             nfft(    nfft_arg) {}
42         IFFTPlanUniformPoolHandle(const IFFTPlanUniformPoolHandle& other) = delete;
43         IFFTPlanUniformPoolHandle& operator=(const IFFTPlanUniformPoolHandle& other) =
44             delete;
45         IFFTPlanUniformPoolHandle(IFFTPlanUniformPoolHandle&& other) = delete;
46         IFFTPlanUniformPoolHandle& operator=(IFFTPlanUniformPoolHandle&& other) = delete;

```

```
46
47      /*=====
48      Member Functions
49      -----*/
50      auto    lock()
51      {
52          return std::unique_lock<std::mutex>(this->mutex);
53      }
54
55  };
56 }
```

Appendix B

General Purpose Templated Functions

B.1 abs

```
1 #pragma once
2 /*=====
3 Dependencies
4 -----*/
5 #include <vector>    // for vectors
6 #include <algorithm> // for std::transform
7
8 /*=====
9 y = abs(vector)
10 -----*/
11 template <typename T>
12 auto abs(const std::vector<T>& input_vector)
13 {
14     // creating canvas
15     auto canvas {input_vector};
16
17     // calculating abs
18     std::transform(canvas.begin(),
19                   canvas.end(),
20                   canvas.begin(),
21                   [](auto& argx){return std::abs(argx);});
22
23     // returning
24     return std::move(canvas);
25 }
26 /*=====
27 y = abs(matrix)
28 -----*/
29 template <typename T>
30 auto abs(const std::vector<std::vector<T>> input_matrix)
31 {
32     // creating canvas
33     auto canvas {input_matrix};
34
35     // applying element-wise abs
36     std::transform(input_matrix.begin(),
37                   input_matrix.end(),
38                   input_matrix.begin(),
```

```

39         [](auto& argx){return std::abs(argx);});
40
41     // returning
42     return std::move(canvas);
43 }

```

B.2 Boolean Comparators

```

1  #pragma once
2  /*=====
3  -----*/
4  template <typename T, typename U>
5  auto operator<(const std::vector<T>& input_vector,
6                const U scalar)
7  {
8      // creating canvas
9      auto canvas {std::vector<bool>(input_vector.size())};
10
11     // transforming
12     std::transform(input_vector.begin(), input_vector.end(),
13                   canvas.begin(),
14                   [&scalar](const auto& argx){
15                       return argx < static_cast<T>(scalar);
16                   });
17
18     // returning
19     return std::move(canvas);
20 }
21 /*=====
22 -----*/
23 template <typename T, typename U>
24 auto operator<=(const std::vector<T>& input_vector,
25                const U scalar)
26 {
27     // creating canvas
28     auto canvas {std::vector<bool>(input_vector.size())};
29
30     // transforming
31     std::transform(input_vector.begin(), input_vector.end(),
32                   canvas.begin(),
33                   [&scalar](const auto& argx){
34                       return argx <= static_cast<T>(scalar);
35                   });
36
37     // returning
38     return std::move(canvas);
39 }
40 // =====
41 template <typename T, typename U>
42 auto operator>(const std::vector<T>& input_vector,
43               const U scalar)
44 {
45     // creating canvas
46     auto canvas {std::vector<bool>(input_vector.size())};
47
48     // transforming

```

```

49     std::transform(input_vector.begin(), input_vector.end(),
50                   canvas.begin(),
51                   [&scalar](const auto& argx){
52                       return argx > static_cast<T>(scalar);
53                   });
54
55     // returning
56     return std::move(canvas);
57 }
58 /*=====
59 -----*/
60 template <typename T, typename U>
61 auto operator>=(const std::vector<T>& input_vector,
62               const U scalar)
63 {
64     // creating canvas
65     auto canvas {std::vector<bool>(input_vector.size())};
66
67     // transforming
68     std::transform(input_vector.begin(), input_vector.end(),
69                   canvas.begin(),
70                   [&scalar](const auto& argx){
71                       return argx >= static_cast<T>(scalar);
72                   });
73
74     // returning
75     return std::move(canvas);
76 }

```

B.3 Concatenate Functions

```

1  #pragma once
2  /*=====
3  input = [vector, vector],
4  output = [vector]
5  -----*/
6  template <std::size_t axis, typename T>
7  auto concatenate(const std::vector<T>& input_vector_A,
8                 const std::vector<T>& input_vector_B) -> std::enable_if_t<axis == 1,
9                 std::vector<T> >
10 {
11     // creating canvas vector
12     auto num_elements {input_vector_A.size() + input_vector_B.size()};
13     auto canvas {std::vector<T>(num_elements, (T)0) };
14
15     // filling up the canvas
16     std::copy(input_vector_A.begin(), input_vector_A.end(),
17             canvas.begin());
18     std::copy(input_vector_B.begin(), input_vector_B.end(),
19             canvas.begin()+input_vector_A.size());
20
21     // moving it back
22     return std::move(canvas);
23 }
24 /*=====

```

```

25 input = [vector, vector],
26 output = [matrix]
27 -----*/
28 template <std::size_t axis, typename T>
29 auto concatenate(const std::vector<T>& input_vector_A,
30                 const std::vector<T>& input_vector_B) -> std::enable_if_t<axis == 0,
31                 std::vector<std::vector<T>>> >
32 {
33     // throwing error dimensions
34     if (input_vector_A.size() != input_vector_B.size())
35         std::cerr << "concatenate:: incorrect dimensions \n";
36
37     // creating canvas
38     auto canvas {std::vector<std::vector<T>>>(
39         2, std::vector<T>(input_vector_A.size())
40     )};
41
42     // filling up the dimensions
43     std::copy(input_vector_A.begin(), input_vector_A.end(), canvas[0].begin());
44     std::copy(input_vector_B.begin(), input_vector_B.end(), canvas[1].begin());
45
46     // moving it back
47     return std::move(canvas);
48 }
49 /*=====
50 input = [vector, vector, vector],
51 output = [matrix]
52 -----*/
53 template <std::size_t axis, typename T>
54 auto concatenate(const std::vector<T>& input_vector_A,
55                 const std::vector<T>& input_vector_B,
56                 const std::vector<T>& input_vector_C) -> std::enable_if_t<axis == 0,
57                 std::vector<std::vector<T>>> >
58 {
59     // throwing error dimensions
60     if (input_vector_A.size() != input_vector_B.size() ||
61         input_vector_A.size() != input_vector_C.size())
62         std::cerr << "concatenate:: incorrect dimensions \n";
63
64     // creating canvas
65     auto canvas {std::vector<std::vector<T>>>(
66         3, std::vector<T>(input_vector_A.size())
67     )};
68
69     // filling up the dimensions
70     std::copy(input_vector_A.begin(), input_vector_A.end(), canvas[0].begin());
71     std::copy(input_vector_B.begin(), input_vector_B.end(), canvas[1].begin());
72     std::copy(input_vector_C.begin(), input_vector_C.end(), canvas[2].begin());
73
74     // moving it back
75     return std::move(canvas);
76 }
77 /*=====
78 input = [matrix, vector],
79 output = [matrix]
80 -----*/
81 template <std::size_t axis, typename T>

```



```

82 auto concatenate(const std::vector<std::vector<T>>& input_matrix,
83                 const std::vector<T>          input_vector) -> std::enable_if_t<axis
                        == 0, std::vector<std::vector<T>>> >
84 {
85     // creating canvas
86     auto canvas {input_matrix};
87
88     // adding to the canvas
89     canvas.push_back(input_vector);
90
91     // returning
92     return std::move(canvas);
93 }

```

B.4 Conjugate

```

1  #pragma once
2  namespace svr {
3      /*=====
4      y = svr::conj(vector);
5      -----*/
6      template <typename T>
7      auto conj(const std::vector<T>& input_vector)
8      {
9          // creating canvas
10         auto canvas {std::vector<T>(input_vector.size())};
11
12         // calculating conjugates
13         std::for_each(canvas.begin(), canvas.end(),
14             [](auto& argx){argx = std::conj(argx);});
15
16         // returning
17         return std::move(canvas);
18     }
19 }

```

B.5 Convolution

```

1  #pragma once
2  namespace svr {
3
4      /*=====
5      1D convolution of two vectors
6      > implemented through fft
7      -----*/
8      template <typename T1, typename T2>
9      auto conv1D(const std::vector<T1>& input_vector_A,
10                 const std::vector<T2>& input_vector_B)
11      {
12          // resulting type
13          using T3 = decltype(std::declval<T1>() * std::declval<T2>());
14
15          // creating canvas
16          auto canvas_length {input_vector_A.size() + input_vector_B.size() - 1};

```

```

17
18 // calculating fft of two arrays
19 auto fft_A {svr::fft(input_vector_A, canvas_length)};
20 auto fft_B {svr::fft(input_vector_B, canvas_length)};
21
22 // element-wise multiplying the two matrices
23 auto fft_AB {fft_A * fft_B};
24
25 // finding inverse FFT
26 auto convolved_result {ifft(fft_AB)};
27
28 // returning
29 return std::move(convolved_result);
30 }
31
32 template <>
33 auto conv1D(const std::vector<double>& input_vector_A,
34            const std::vector<double>& input_vector_B)
35 {
36 // creating canvas
37 auto canvas_length {input_vector_A.size() + input_vector_B.size() - 1};
38
39 // calculating fft of two arrays
40 auto fft_A {svr::fft(input_vector_A, canvas_length)};
41 auto fft_B {svr::fft(input_vector_B, canvas_length)};
42
43 // element-wise multiplying the two matrices
44 auto fft_AB {fft_A * fft_B};
45
46 // finding inverse FFT
47 auto convolved_result {ifft(fft_AB)};
48
49 // returning
50 return std::move(convolved_result);
51 }
52
53 /*=====
54 1D convolution of two vectors
55 > implemented through fft
56 -----*/
57 template <typename T1, typename T2>
58 auto conv1D_fftw(const std::vector<T1>& input_vector_A,
59                 const std::vector<T2>& input_vector_B)
60 {
61 // resulting type
62 using T3 = decltype(std::declval<T1>() * std::declval<T2>());
63
64 // creating canvas
65 auto canvas_length {input_vector_A.size() + input_vector_B.size() - 1};
66
67 // calculating fft of two arrays
68 auto fft_A {svr::fft(input_vector_A, canvas_length)};
69 auto fft_B {svr::fft(input_vector_B, canvas_length)};
70
71 // element-wise multiplying the two matrices
72 auto fft_AB {fft_A * fft_B};
73
74 // finding inverse FFT
75 auto convolved_result {svr::ifft(fft_AB, fft_AB.size())};

```

```

76
77     // returning
78     return std::move(convolved_result);
79 }
80
81 /*=====
82 Long-signal Conv1D
83 improvements:
84 > make an inplace version of this
85 -----*/
86 template <std::size_t L, typename T>
87 auto conv1D_long(const std::vector<T>& input_vector_A,
88                 const std::vector<T>& input_vector_B)
89 {
90     // fetching dimensions
91     const auto maxlength = {std::max(input_vector_A.size(),
92                                     input_vector_B.size())};
93     const auto filter_size = {std::min(input_vector_A.size(),
94                                     input_vector_B.size())};
95     const auto block_size = {L + filter_size - 1};
96     const auto num_blocks = {2 + static_cast<std::size_t>(
97         (maxlength - block_size)/L
98     )};
99
100    // obtaining references
101    const auto& large_vector = {input_vector_A.size() >= input_vector_B.size() ? \
102                                input_vector_A      : input_vector_B};
103    const auto& small_vector = {input_vector_A.size() < input_vector_B.size() ? \
104                                input_vector_A      : input_vector_B};
105
106    // setup
107    auto starting_index = {static_cast<std::size_t>(0)};
108    auto ending_index = {static_cast<std::size_t>(0)};
109    auto length_left_to_fill = {ending_index - starting_index};
110    auto canvas = {std::vector<double>(block_size, 0)};
111    auto finaloutput = {std::vector<double>(maxlength, 0)};
112    auto block_conv_output_size = {L + 2 * filter_size - 2};
113    auto block_conv_output = {std::vector<double>(block_conv_output_size, 0)};
114
115    // block-wise processing
116    for(auto bid = 0; bid < num_blocks; ++bid)
117    {
118        // estimating indices
119        starting_index = L*bid;
120        ending_index = std::min(starting_index + block_size - 1, maxlength -
121                                1);
122        length_left_to_fill = ending_index - starting_index;
123
124        // copying to the common-block
125        std::copy(large_vector.begin() + starting_index,
126                  large_vector.begin() + ending_index + 1,
127                  canvas.begin());
128
129        // performing convolution
130        block_conv_output = svr::conv1D_fftw(canvas,
131                                              small_vector);
132
133        // discarding edges and writing values
134        std::copy(block_conv_output.begin() + filter_size-2,

```

```

134         block_conv_output.begin() + filter_size-2 +
            std::min(static_cast<int>(L-1),
                    static_cast<int>(length_left_to_fill)) + 1,
135         finaloutput.begin()+starting_index);
136     }
137
138     // returning
139     return std::move(finaloutput);
140
141 }
142 /*=====
143 Long-signal Conv1D with FFT Plan
144 improvements:
145     > make an inplace version of this
146 -----*/
147 template <
148     typename T,
149     std::enable_if_t<
150         std::is_floating_point_v<T>
151     >
152 >
153 auto conv1D_long_prototype(
154     const std::vector<T>& input_vector_A,
155     const std::vector<T>& input_vector_B,
156     svr::FFTPlanClass<T, std::complex<T>>& fft_plan,
157     svr::IFFTPlanClass<std::complex<T>, T>& ifft_plan
158 )
159 {
160     // Error checks
161     if (fft_plan.nfft_ != ifft_plan.nfft_)
162         throw std::runtime_error("fft_plan.nfft_ != ifft_plan.nfft_");
163
164     // fetching references to large-signal and small-signal
165     const auto& large_signal_original {
166         input_vector_A.size() >= input_vector_B.size() ?
167         input_vector_A : input_vector_B
168     };
169     const auto& small_signal {
170         input_vector_A.size() < input_vector_B.size() ?
171         input_vector_A : input_vector_B
172     };
173
174     // copying
175     auto large_signal {std::vector<double>(
176         input_vector_A.size() + input_vector_B.size() - 1
177     )};
178     std::copy(large_signal_original.begin(),
179         large_signal_original.end(),
180         large_signal.begin());;
181
182     // calculating parameters
183     const auto signal_size {large_signal_original.size()};
184     const auto filter_size {small_signal.size()};
185     const auto input_signal_block_size {fft_plan.nfft_ + 1 - filter_size};
186     if (input_signal_block_size <= 0)
187         throw std::runtime_error("input_signal_block_size <= 0 ");
188     const auto block_output_length {fft_plan.nfft_};
189     const auto num_blocks {static_cast<int>(
190         1 + std::ceil((signal_size + filter_size - 2)/input_signal_block_size)

```

```

191     )
192 };
193 const auto final_output_size {signal_size + filter_size - 1};
194 const auto useful_sample_length {block_output_length - (filter_size - 1)
    - (filter_size - 1)};
195
196 // parameters for re-use
197 auto start_index {static_cast<int>(0)};
198 auto end_index {static_cast<int>(0)};
199 auto output_start_index {static_cast<int>(0)};
200
201 // calculating fft(filter)
202 auto filter_zero_padded {std::vector<double>(block_output_length, 0.0)};
203 std::copy(small_signal.begin(), small_signal.end(), filter_zero_padded.begin());
204 auto filter_FFT {fft_plan.fft(filter_zero_padded)};
205
206 // allocating space for storing input-blocks
207 auto signal_block_zero_padded {std::vector<double>(block_output_length, 0.0)};
208 auto fftw_output {std::vector<double>()};
209 auto conv_output {std::vector<double>()};
210 auto finaloutput {std::vector<double>(final_output_size, 0.0)};
211
212 // going through the values
213 svr::Timer timer("fft-loop");
214 for(auto i = 0; i<num_blocks; ++i){
215
216     // calculating bounds
217     auto analytical_start {
218         (i*static_cast<int>(input_signal_block_size)) -
219         (static_cast<int>(filter_size) - 1)
220     };
221     auto analytical_end {(i+1)*input_signal_block_size - 1};
222     start_index = std::max(
223         static_cast<int>(0), static_cast<int>(analytical_start)
224     );
225     end_index = std::min(
226         static_cast<int>(signal_size-1), static_cast<int>(analytical_end)
227     ); // [start-index, end-index)
228
229     // copying values
230     signal_block_zero_padded = std::move(std::vector<double>(block_output_length,
231         0.0));
232     std::copy(large_signal.begin() + start_index,
233         large_signal.begin() + end_index + 1,
234         signal_block_zero_padded.begin() + start_index - analytical_start);
235
236     // performing ifft(fft(x) * fft(y))
237     fftw_output = ifft_plan.ifft(
238         fft_plan.fft(signal_block_zero_padded) * filter_FFT
239     );
240
241     // trimming away the first parts (since partial)
242     conv_output = std::vector<double>(fftw_output.begin() + filter_size -
243         1, fftw_output.end());
244
245     // writing to final-output
246     std::copy(conv_output.begin(), conv_output.end(), finaloutput.begin() +
247         output_start_index);
248     output_start_index += conv_output.size();

```

```

245     }
246
247 }
248
249 /*=====
250 Long-signal Conv1D with FFT-Plan-Pool
251 -----*/
252 // concept for the FFTPlans
253 template <typename T>
254 concept FFT_SourceDestination_DataType = \
255     std::is_floating_point_v<T> || \
256     (
257         std::is_class_v<T> &&
258         std::is_floating_point_v<typename T::value_type>
259     );
260
261 // template <
262 //     typename T,
263 //     typename = std::enable_if_t<
264 //         std::is_floating_point_v<T>
265 //     >
266 // >
267 // auto conv_per_plan(
268 //     const int i,
269 //     const int& input_signal_block_size,
270 //     const int& filter_size,
271 //     const int& block_output_length,
272 //     const std::vector<T>& large_signal,
273 //     std::vector<T> signal_block_zero_padded,
274 //     svr::FFTPlanUniformPoolHandle<T, std::complex<T>>& fft_pool_handle,
275 //     svr::IFFTPlanUniformPoolHandle<std::complex<T>, T>& ifft_pool_handle,
276 //     const std::vector<std::complex<T>>& filter_FFT,
277 //     std::vector<T> fftw_output,
278 //     std::vector<T> conv_output,
279 //     std::vector<T>& output_vector,
280 //     std::mutex& output_vector_mutex,
281 //     const auto& signal_size
282 // )
283 // {
284 //     // calculating bounds
285 //     auto analytical_start {
286 //         (i*static_cast<int>(input_signal_block_size)) -
287 //         (static_cast<int>(filter_size) - 1)
288 //     };
289 //     auto analytical_end {(i+1)*input_signal_block_size - 1};
290 //     auto start_index = std::max(
291 //         static_cast<int>(0), static_cast<int>(analytical_start)
292 //     );
293 //     auto end_index = std::min(
294 //         static_cast<int>(signal_size-1), static_cast<int>(analytical_end)
295 //     ); // [start-index, end-index)
296
297 //     // copying values
298 //     signal_block_zero_padded = std::move(std::vector<double>(block_output_length,
299 //         0.0));
300 //     std::copy(
301 //         large_signal.begin() + start_index,
302 //         large_signal.begin() + end_index + 1,
303 //         signal_block_zero_padded.begin() + start_index - analytical_start

```

```

302     // );
303
304     // // fetching an fft and IFFT plan
305     // auto fph_lock {fft_pool_handle.lock()};
306     // auto ifph_lock {ifft_pool_handle.lock()};
307     // auto fft_pair {fft_pool_handle.uniform_pool.fetch_plan()};
308     // auto ifft_pair {ifft_pool_handle.uniform_pool.fetch_plan()};
309
310     // // performing ifft(fft(x) * filter-FFT)
311     // fftw_output = ifft_pair.plan.ifft_l2_conservd(
312     //     fft_pair.plan.fft_l2_conservd(signal_block_zero_padded) * filter_FFT
313     // );
314
315     // // trimming away the first parts (since partial)
316     // conv_output = std::vector<T>(
317     //     fftw_output.begin() + filter_size - 1,
318     //     fftw_output.end()
319     // );
320
321     // // writing to final-output
322     // auto output_start_index = i * (block_output_length - (filter_size - 1));
323     // std::lock_guard<std::mutex> output_lock(output_vector_mutex);
324     // std::copy(
325     //     conv_output.begin(), conv_output.end(),
326     //     output_vector.begin() + output_start_index
327     // );
328     // }
329
330     template <
331         svr::FFT_SourceDestination_Type sourceType,
332         svr::FFT_SourceDestination_Type destinationType
333     >
334     auto conv_per_plan(
335         const int i,
336         const int& input_signal_block_size,
337         const int& filter_size,
338         const int& block_output_length,
339         const std::vector<sourceType>& large_signal,
340         std::vector<sourceType> signal_block_zero_padded,
341         svr::FFTPlanUniformPoolHandle< sourceType, destinationType>& fft_pool_handle,
342         svr::IFFTPlanUniformPoolHandle< destinationType, sourceType>& ifft_pool_handle,
343         const std::vector<destinationType>& filter_FFT,
344         std::vector<sourceType> fftw_output,
345         std::vector<sourceType> conv_output,
346         std::vector<sourceType>& output_vector,
347         std::mutex& output_vector_mutex,
348         const auto& signal_size
349     )
350     {
351
352         // calculating bounds
353         auto analytical_start {
354             (i*static_cast<int>(input_signal_block_size)) -
355             (static_cast<int>(filter_size) - 1)
356         };

```

```

356     auto analytical_end    {(i+1)*input_signal_block_size -1};
357     auto start_index      = std::max(
358         static_cast<int>(0), static_cast<int>(analytical_start)
359     );
360     auto end_index        = std::min(
361         static_cast<int>(signal_size-1), static_cast<int>(analytical_end)
362     ); // [start-index, end-index)
363
364     // copying values
365     signal_block_zero_padded = std::move(std::vector<sourceType>(block_output_length,
366         0.0));
367     std::copy(
368         large_signal.begin()          + start_index,
369         large_signal.begin()          + end_index    + 1,
370         signal_block_zero_padded.begin() + start_index - analytical_start
371     );
372
373     // fetching an fft and IFFT plan
374     auto fph_lock    {fft_pool_handle.lock()};
375     auto ifph_lock   {ifft_pool_handle.lock()};
376     auto fft_pair    {fft_pool_handle.uniform_pool.fetch_plan()};
377     auto ifft_pair   {ifft_pool_handle.uniform_pool.fetch_plan()};
378
379     // performing ifft(fft(x) * filter-FFT)
380     fftw_output      = ifft_pair.plan.ifft_l2_conserved(
381         fft_pair.plan.fft_l2_conserved(signal_block_zero_padded) * filter_FFT
382     );
383
384     // trimming away the first parts (since partial)
385     conv_output = std::vector<sourceType>(
386         fftw_output.begin() + filter_size - 1,
387         fftw_output.end()
388     );
389
390     // writing to final-output
391     auto output_start_index = i * (block_output_length - (filter_size - 1));
392     std::lock_guard<std::mutex> output_lock(output_vector_mutex);
393     std::copy(
394         conv_output.begin(), conv_output.end(),
395         output_vector.begin() + output_start_index
396     );
397 }
398
399 template <
400     svr::FFT_SourceDestination_Type T,
401     svr::FFT_SourceDestination_Type sourceType,
402     svr::FFT_SourceDestination_Type destinationType
403 >
404 auto conv1D_long_FFTPlanPool(
405     const std::vector<T>& input_vector_A,
406     const std::vector<T>& input_vector_B,
407     svr::FFTPlanUniformPoolHandle< sourceType, destinationType>& fft_pool_handle,
408     svr::IFFTPlanUniformPoolHandle< destinationType, sourceType>& ifft_pool_handle
409 )
410 {
411     // Error checks
412     if (fft_pool_handle.nfft!=ifft_pool_handle.nfft)

```



```

413         throw std::runtime_error("FILE: svr_conv.hpp | FUNCTION:
414             conv1D_long_FFTPlanPool | Report: the pool-handles are for different
415             nffts");
416
417     // fetching references to the large signal and small signal
418     const auto& large_signal_original {
419         input_vector_A.size() >= input_vector_B.size() ?
420         input_vector_A      : input_vector_B
421     };
422     const auto& small_signal {
423         input_vector_A.size() < input_vector_B.size() ?
424         input_vector_A      : input_vector_B
425     };
426
427     // copying
428     auto large_signal {std::vector<T>(
429         input_vector_A.size() + input_vector_B.size() - 1
430     )};
431     std::copy(large_signal_original.begin(),
432         large_signal_original.end(),
433         large_signal.begin());
434
435     // calculating some parameters
436     const auto signal_size {large_signal_original.size()};
437     const auto filter_size {small_signal.size()};
438     const auto input_signal_block_size {
439         fft_pool_handle.nfft + 1 - filter_size
440     };
441
442     // throwing an error if nfft < filter-size
443     if (fft_pool_handle.nfft < filter_size)
444         throw std::runtime_error("FILE: svr_conv.hpp | FUNCTION:
445             conv1D_long_FFTPlanPool | REPORT: filter is bigger than nfft");
446
447     // throwing an error if number of useful samples is less than zero
448     if (input_signal_block_size <= 0)
449         throw std::runtime_error("FILE: svr_conv.hpp | FUNCTION:
450             conv1D_long_FFTPlanPool | REPORT: input_signal_block_size = 0");
451
452     const auto block_output_length {fft_pool_handle.nfft};
453     const auto num_blocks {static_cast<int>(
454         1 + std::ceil((signal_size + filter_size - 2) / input_signal_block_size)
455     )};
456     const auto final_output_size {signal_size + filter_size - 1};
457     const auto useful_sample_length {
458         block_output_length - (filter_size - 1) - (filter_size - 1)
459     };
460
461     // parameters for re-use
462     auto start_index {static_cast<int>(0)};
463     auto end_index {static_cast<int>(0)};
464     auto output_start_index {static_cast<int>(0)};
465
466     // calculating fft(filter)
467     auto filter_zero_padded {std::vector<T>(block_output_length, 0.0)};
468     std::copy( small_signal.begin(),
469         small_signal.end(),
470         filter_zero_padded.begin());

```

```

468     auto    fph_lock0          {fft_pool_handle.lock()};
469     auto    curr_plan_pair     {fft_pool_handle.uniform_pool.fetch_plan()};
470     auto    pool_num_plans     {fft_pool_handle.num_plans};
471     fph_lock0.unlock();
472     auto    filter_FFT         {
473         curr_plan_pair.plan.fft(
474             filter_zero_padded
475         )
476     };
477     curr_plan_pair.lock.unlock();
478
479     // allocating space for storing input-blocks
480     auto    signal_block_zero_padded {std::vector<T>(block_output_length, 0.0)};
481     auto    fftw_output             {std::vector<T>()};
482     auto    conv_output             {std::vector<T>()};
483     auto    output_vector           {std::vector<T>(final_output_size, 0.0)};
484     auto    output_vector_mutex     {std::mutex()};
485
486     // creating boost
487     svr::ThreadPool local_pool(pool_num_plans);
488
489     // going through the values
490     for(auto i = 0; i < num_blocks; ++i)
491     {
492         local_pool.push_back(
493             [
494                 i,
495                 &input_signal_block_size,
496                 &filter_size,
497                 &block_output_length,
498                 &large_signal,
499                 signal_block_zero_padded,
500                 &fft_pool_handle,
501                 &ifft_pool_handle,
502                 &filter_FFT,
503                 fftw_output,
504                 conv_output,
505                 &output_vector,
506                 &output_vector_mutex,
507                 &signal_size
508             ]{
509                 conv_per_plan<T>(
510                     i,
511                     std::ref(input_signal_block_size),
512                     std::ref(filter_size),
513                     std::ref(block_output_length),
514                     std::ref(large_signal),
515                     signal_block_zero_padded,
516                     fft_pool_handle,
517                     ifft_pool_handle,
518                     filter_FFT,
519                     fftw_output,
520                     conv_output,
521                     std::ref(output_vector),
522                     output_vector_mutex,
523                     signal_size
524                 );
525             }
526         );

```

```

527     }
528     local_pool.converge();
529
530     // returning final output
531     return std::move(output_vector);
532     // return output_vector;
533 }
534
535 /*=====
536 Short-conv1D
537 -----*/
538 template    <typename    T1,
539             typename    T2>
540 auto    conv1D_short(const    std::vector<T1>& input_vector_A,
541                     const    std::vector<T2>& input_vector_B)
542 {
543     // resulting type
544     using    T3 = decltype(std::declval<T1>() * std::declval<T2>());
545
546     // creating canvas
547     auto    canvas_length    {input_vector_A.size() + input_vector_B.size() - 1};
548
549     // calculating fft of two arrays
550     auto    fft_A    {svr::fft(input_vector_A, canvas_length)};
551     auto    fft_B    {svr::fft(input_vector_B, canvas_length)};
552
553     // element-wise multiplying the two matrices
554     auto    fft_AB    {fft_A * fft_B};
555
556     // finding inverse FFT
557     auto    convolved_result    {ifft(fft_AB)};
558
559     // returning
560     // return std::move(convolved_result);
561     return convolved_result;
562 }
563
564
565 /*=====
566 1D Convolution of a matrix and a vector
567 -----*/
568 template    <typename T>
569 auto    conv1D(const    std::vector<std::vector<T>>& input_matrix,
570               const    std::vector<T>&            input_vector,
571               const    std::size_t&                dim)
572 {
573     // getting dimensions
574     const    auto&    num_rows_matrix    {input_matrix.size()};
575     const    auto&    num_cols_matrix    {input_matrix[0].size()};
576     const    auto&    num_elements_vector    {input_vector.size()};
577
578     // creating canvas
579     auto    canvas    {std::vector<std::vector<T>>()};
580
581     // creating output based on dim
582     if (dim == 1)
583     {
584         // performing convolutions row by row
585

```

```

586     for(auto row = 0; row < num_rows_matrix; ++row)
587     {
588         cout << format("\t\t row = {}/{}\n", row, num_rows_matrix);
589         auto bruh {conv1D(input_matrix[row], input_vector)};
590         auto bruh_real {svr::real(std::move(bruh))};
591
592         canvas.push_back(
593             svr::real(
594                 std::move(bruh_real)
595             )
596         );
597     }
598 }
599 else{
600     std::cerr << "svr_conv.hpp | conv1D | yet to be implemented \n";
601 }
602
603 // returning
604 return std::move(canvas);
605
606 }
607
608 }

```

B.6 Coordinate Change

```

1  #pragma once
2  namespace svr {
3      /*=====
4      y = cart2sph(vector)
5      -----*/
6      template <typename T>
7      auto cart2sph(const std::vector<T>& cartesian_vector){
8
9          // splatting the point onto xy-plane
10         auto xysplat {cartesian_vector};
11         xysplat[2] = 0;
12
13         // finding splat lengths
14         auto xysplat_lengths {norm(xysplat)};
15
16         // finding azimuthal and elevation angles
17         auto azimuthal_angles {svr::atan2(xysplat[1],
18                                           xysplat[0]) \
19                                * 180.00/std::numbers::pi};
20         auto elevation_angles {svr::atan2(cartesian_vector[2],
21                                           xysplat_lengths) \
22                                * 180.00/std::numbers::pi};
23         auto rho_values {norm(cartesian_vector)};
24
25         // creating tensor to send back
26         auto spherical_vector {std::vector<T>{azimuthal_angles,
27                                               elevation_angles,
28                                               rho_values}};
29
30         // moving it back

```

```

31     return std::move(spherical_vector);
32 }
33 /*=====
34 y = cart2sph(vector)
35 -----*/
36 template <typename T>
37 auto cart2sph_inplace(std::vector<T>& cartesian_vector){
38
39     // splatting the point onto xy-plane
40     auto xysplat {cartesian_vector};
41     xysplat[2] = 0;
42
43     // finding splat lengths
44     auto xysplat_lengths {norm(xysplat)};
45
46     // finding azimuthal and elevation angles
47     auto azimuthal_angles {svr::atan2(xysplat[1], xysplat[0]) *
48         180.00/std::numbers::pi};
49     auto elevation_angles {svr::atan2(cartesian_vector[2],
50         xysplat_lengths) * 180.00/std::numbers::pi};
51     auto rho_values {norm(cartesian_vector)};
52
53     // creating tesnor
54     cartesian_vector[0] = azimuthal_angles;
55     cartesian_vector[1] = elevation_angles;
56     cartesian_vector[2] = rho_values;
57 }
58 /*=====
59 y = cart2sph(input_matrix, dim)
60 -----*/
61 template <typename T>
62 auto cart2sph(const std::vector<std::vector<T>>& input_matrix,
63     const std::size_t axis)
64 {
65     // fetching dimensions
66     const auto& num_rows {input_matrix.size()};
67     const auto& num_cols {input_matrix[0].size()};
68
69     // checking the axis and dimensions
70     if (axis == 0 && num_rows != 3) {std::cerr << "cart2sph: incorrect num-elements
71         \n";}
72     if (axis == 1 && num_cols != 3) {std::cerr << "cart2sph: incorrect num-elements
73         \n";}
74
75     // creating canvas
76     auto canvas {std::vector<std::vector<T>>>(
77         num_rows,
78         std::vector<T>(num_cols, 0)
79     )};
80
81     // if axis = 0, performing operation column-wise
82     if(axis == 0)
83     {
84         for(auto col = 0; col < num_cols; ++col)
85         {
86             // fetching current column
87             auto curr_column {std::vector<T>({input_matrix[0][col],
88                 input_matrix[1][col],
89                 input_matrix[2][col]})};

```

```

87
88     // performing inplace transformation
89     cart2sph_inplace(curr_column);
90
91     // storing it back
92     canvas[0][col] = curr_column[0];
93     canvas[1][col] = curr_column[1];
94     canvas[2][col] = curr_column[2];
95 }
96 }
97 // if axis == 1, performing operations row-wise
98 else if(axis == 0)
99 {
100     std::cerr << "cart2sph: yet to be implemented \n";
101 }
102 else
103 {
104     std::cerr << "cart2sph: yet to be implemented \n";
105 }
106
107 // returning
108 return std::move(canvas);
109
110 }
111
112 // =====
113 template <typename T>
114 auto sph2cart(const std::vector<T> spherical_vector){
115
116     // creating cartesian vector
117     auto cartesian_vector {std::vector<T>(spherical_vector.size(), 0)};
118
119     // populating
120     cartesian_vector[0] = spherical_vector[2] * \
121         cos(spherical_vector[1] * std::numbers::pi / 180.00) * \
122         cos(spherical_vector[0] * std::numbers::pi / 180.00);
123     cartesian_vector[1] = spherical_vector[2] * \
124         cos(spherical_vector[1] * std::numbers::pi / 180.00) * \
125         sin(spherical_vector[0] * std::numbers::pi / 180.00);
126     cartesian_vector[2] = spherical_vector[2] * \
127         sin(spherical_vector[1] * std::numbers::pi / 180.00);
128
129     // returning
130     return std::move(cartesian_vector);
131 }
132 }

```

B.7 Cosine

```

1 #pragma once
2 /*=====
3 y = cos(input_vector)
4 -----*/
5 template <typename T>
6 auto cos(const std::vector<T>& input_vector)
7 {

```

```

8      // created canvas
9      auto canvas {input_vector};
10
11     // calling the function
12     std::transform(input_vector.begin(), input_vector.end(),
13                   canvas.begin(),
14                   [](auto& argx){return std::cos(argx);});
15
16     // returning the output
17     return std::move(canvas);
18 }
19 /*=====
20 y = cosd(input_vector)
21 -----*/
22 template <typename T>
23 auto cosd(const std::vector<T> input_vector)
24 {
25     // created canvas
26     auto canvas {input_vector};
27
28     // calling the function
29     std::transform(input_vector.begin(),
30                   input_vector.end(),
31                   input_vector.begin(),
32                   [](const auto& argx){return std::cos(argx * 180.00/std::numbers::pi);});
33
34     // returning the output
35     return std::move(canvas);
36 }

```

B.8 Data Structures

```

1 struct TreeNode {
2     int val;
3     TreeNode *left;
4     TreeNode *right;
5     TreeNode() : val(0), left(nullptr), right(nullptr) {}
6     TreeNode(int x) : val(x), left(nullptr), right(nullptr) {}
7     TreeNode(int x, TreeNode *left, TreeNode *right) : val(x), left(left), right(right)
8     {}
9 };
10
11 struct ListNode {
12     int val;
13     ListNode *next;
14     ListNode() : val(0), next(nullptr) {}
15     ListNode(int x) : val(x), next(nullptr) {}
16     ListNode(int x, ListNode *next) : val(x), next(next) {}
17 };

```

B.9 Editing Index Values

```

1 #pragma once

```

```

2  /*=====
3  Matlab's equivalent of A[A < 0.5] = 0
4  -----*/
5  template <typename T, typename U>
6  auto edit(std::vector<T>&          input_vector,
7           const std::vector<bool>& bool_vector,
8           const U                scalar)
9  {
10     // throwing an error
11     if (input_vector.size() != bool_vector.size())
12         std::cerr << "edit: incompatible size\n";
13
14     // overwriting input-vector
15     std::transform(input_vector.begin(), input_vector.end(),
16                   bool_vector.begin(),
17                   input_vector.begin(),
18                   [&scalar](auto& argx, auto argy){
19                       if(argy == true) {return static_cast<T>(scalar);}
20                       else             {return argx;}
21                   });
22
23     // no-returns since in-place
24 }
25
26 /*=====
27 accumulate version of edit, instead of just placing values
28
29 Things to add
30     - ensuring template only accepts int, std::size_t and similar for T2
31     - bring in histogram method to ensure SIMD
32 -----*/
33 template <typename T1,
34          typename T2>
35 auto edit_accumulate(std::vector<T1>&          input_vector,
36                     const std::vector<T2>&    indices_to_edit,
37                     const std::vector<T1>&    new_values)
38 {
39     // certain checks
40     if (indices_to_edit.size() != new_values.size())
41         std::cerr << "svr::edit | edit_accumulate | size-disparity occurred \n";
42
43     // going through each and accumulating
44     for(auto i = 0; i < input_vector.size(); ++i){
45         const auto target_index {static_cast<std::size_t>(indices_to_edit[i])}; //
46         const auto new_value    {new_values[i]};
47         if (target_index < input_vector.size()){
48             input_vector[target_index] = input_vector[target_index] + new_value;
49         }
50         else{
51             // std::cout << "warning: FILE: svr_edit.hpp | FUNCTION: edit_accumulate |
52             // REPORT: index out of bounds";
53         }
54     }
55
56     // no-return since in-place
57 }

```

B.10 Equality

```

1  #pragma once
2  /*=====
3  -----*/
4  template <typename T, typename U>
5  auto operator==(const std::vector<T>& input_vector,
6                  const U& scalar)
7  {
8      // setting up canvas
9      auto canvas {std::vector<bool>(input_vector.size())};
10
11     // writing to canvas
12     std::transform(input_vector.begin(), input_vector.end(),
13                   canvas.begin(),
14                   [&scalar](const auto& argx){
15                       return argx == scalar;
16                   });
17
18     // returning
19     return std::move(canvas);
20 }

```

B.11 Exponentiate

```

1  #pragma once
2
3  namespace svr {
4      /*=====
5      y = exp(vector)
6      -----*/
7      template <typename T>
8      auto exp(const std::vector<T>& input_vector)
9      {
10         // creating canvas
11         auto canvas {input_vector};
12
13         // transforming
14         std::transform(canvas.begin(), canvas.end(),
15                       canvas.begin(),
16                       [](auto& argx){return std::exp(argx);});
17
18         // returning
19         return std::move(canvas);
20     }
21     /*=====
22     y = exp(matrix)
23     -----*/
24     template <
25         typename sourceType,
26         typename destinationType,
27         typename = std::enable_if_t<
28             std::is_arithmetic_v<sourceType>
29         >
30     >
31     auto exp(

```

```

32     const std::vector<std::vector<sourceType>> input_matrix
33 )
34 {
35     // fetching dimensions
36     const auto& num_rows {input_matrix.size()};
37     const auto& num_cols {input_matrix[0].size()};
38
39     // creating canvas
40     auto canvas {std::vector<std::vector<destinationType>>(
41         num_rows,
42         std::vector<destinationType>(num_cols)
43     )};
44
45     // writing to each entry
46     for(auto row = 0; row < num_rows; ++row)
47         std::transform(
48             input_matrix[row].begin(), input_matrix[row].end(),
49             canvas[row].begin(),
50             [](const auto& argx){
51                 return std::exp(argx);
52             }
53         );
54
55     // returning
56     return std::move(canvas);
57 }
58 /*=====
59 Aim: Exponentiating complex matrices with general floating types
60 -----*/
61 template <
62     typename T,
63     typename = std::enable_if_t<
64         std::is_floating_point_v<T>
65     >
66 >
67 auto exp(
68     const std::vector<std::vector<std::complex<T>>> input_matrix
69 )
70 {
71     // fetching dimensions
72     const auto& num_rows {input_matrix.size()};
73     const auto& num_cols {input_matrix[0].size()};
74
75     // creating canvas
76     auto canvas {std::vector<std::vector<std::complex<T>>>(
77         num_rows,
78         std::vector<std::complex<T>>(num_cols)
79     )};
80
81     // writing to each entry
82     for(auto row = 0; row < num_rows; ++row)
83         std::transform(
84             input_matrix[row].begin(), input_matrix[row].end(),
85             canvas[row].begin(),
86             [](const auto& argx){
87                 return std::exp(argx);
88             }
89         );
90

```

```

91     // returning
92     return std::move(canvas);
93 }
94
95 }

```

B.12 FFT

```

1  #pragma once
2  namespace svr {
3      /*=====
4      For type-deductions
5      -----*/
6      template <typename T>
7      struct fft_result_type;
8
9      // specializations
10     template <> struct fft_result_type<double>{
11         using type = std::complex<double>;
12     };
13     template <> struct fft_result_type<std::complex<double>>{
14         using type = std::complex<double>;
15     };
16     template <> struct fft_result_type<float>{
17         using type = std::complex<float>;
18     };
19     template <> struct fft_result_type<std::complex<float>>{
20         using type = std::complex<float>;
21     };
22
23     template <typename T>
24     using fft_result_t = typename fft_result_type<T>::type;
25
26     /*=====
27     y = fft(x, nfft)
28     > calculating n-point dft where n-value is explicit
29     -----*/
30     template<typename T>
31     auto fft(const std::vector<T>& input_vector,
32             const size_t nfft)
33     {
34         // throwing an error
35         if (nfft < input_vector.size()) {std::cerr << "size-mismatch\n";}
36         if (nfft <= 0) {std::cerr << "size-mismatch\n";}
37
38         // fetching data-type
39         using RType = fft_result_t<T>;
40         using baseType = std::conditional_t<std::is_same_v<T, std::complex<double>>,
41                                           double,
42                                           T>;
43
44         // canvas instantiation
45         std::vector<RType> canvas(nfft);
46         auto nfft_sqrt = {static_cast<RType>(std::sqrt(nfft))};
47         auto finaloutput = {std::vector<RType>(nfft, 0)};
48

```

```

49 // calculating index by index
50 for(int frequency_index = 0; frequency_index < nfft; ++frequency_index){
51     RType accumulate_value;
52     for(int signal_index = 0; signal_index < input_vector.size(); ++signal_index){
53         accumulate_value += \
54             static_cast<RType>(input_vector[signal_index]) * \
55             static_cast<RType>(std::exp(-1.00 * std::numbers::pi * \
56                                     (static_cast<baseType>(frequency_index)/static_cast<baseType>(nfft) * \
57                                         * \
58                                         static_cast<baseType>(signal_index))));
59     }
60     finaloutput[frequency_index] = accumulate_value / nfft_sqrt;
61 }
62
63 // returning
64 return std::move(finaloutput);
65 }
66
67 /*=====
68 y = fft(std::vector<double> nfft) // specialization
69 -----*/
70 #include <fftw3.h> // for fft
71 template <>
72 auto fft(const std::vector<double>& input_vector,
73          const std::size_t nfft)
74 {
75     if (nfft < input_vector.size())
76         throw std::runtime_error("nfft must be >= input_vector.size()");
77     if (nfft <= 0)
78         throw std::runtime_error("nfft must be > 0");
79
80     // FFTW real-to-complex output
81     std::vector<std::complex<double>> output(nfft);
82
83     // Allocate input (double) and output (fftw_complex) arrays
84     double* in = reinterpret_cast<double*>(
85         fftw_malloc(sizeof(double) * nfft)
86     );
87     fftw_complex* out = reinterpret_cast<fftw_complex*>(
88         fftw_malloc(sizeof(fftw_complex) * (nfft/2 + 1))
89     );
90
91     // Copy input and zero-pad if needed
92     for (std::size_t i = 0; i < nfft; ++i) {
93         in[i] = (i < input_vector.size()) ? input_vector[i] : 0.0;
94     }
95
96     // Create FFTW plan and execute
97     fftw_plan plan = fftw_plan_dft_r2c_1d(
98         static_cast<int>(nfft), in, out, FFTW_ESTIMATE
99     );
100     fftw_execute(plan);
101
102     // Copy FFTW output to std::vector<std::complex<double>>
103     for (std::size_t i = 0; i < nfft/2 + 1; ++i) {
104         output[i] = std::complex<double>(out[i][0], out[i][1]);
105     }
106     // Optional: fill remaining bins with zeros to match full nfft size
107     for (std::size_t i = nfft/2 + 1; i < nfft; ++i) {

```

```

107     output[i] = std::complex<double>(0.0, 0.0);
108 }
109
110 // Cleanup
111 fftw_destroy_plan(plan);
112 fftw_free(in);
113 fftw_free(out);
114
115 // filling up the other half of the output
116 const auto halfpoint {static_cast<std::size_t>(nfft/2)};
117 std::transform(
118     output.begin() + 1,          // first half (skip DC)
119     output.begin() + halfpoint, // end of first half
120     output.rbegin(),             // start writing from last element backward (skip
        Nyquist)
121     [](const auto& x) { return std::conj(x); }
122 );
123
124 // returning
125 return std::move(output);
126 }
127
128
129 /*=====
130 y = ifft(x, nfft)
131 -----*/
132
133 template<typename T>
134 auto ifft(const std::vector<T>& input_vector)
135 {
136     // fetching data-type
137     using RType = fft_result_t<T>;
138     using baseType = std::conditional_t<std::is_same_v<T, std::complex<double>>,
        double,
139     T>;
140
141     // setup
142     auto nfft {input_vector.size()};
143
144     // canvas instantiation
145     std::vector<RType> canvas(nfft);
146     auto nfft_sqrt {static_cast<RType>(std::sqrt(nfft))};
147     auto finaloutput {std::vector<RType>(nfft, 0)};
148
149     // calculating index by index
150     for(int frequency_index = 0; frequency_index<nfft; ++frequency_index){
151         RType accumulate_value;
152         for(int signal_index = 0; signal_index < input_vector.size(); ++signal_index){
153             accumulate_value += \
154                 static_cast<RType>(input_vector[signal_index]) * \
155                 static_cast<RType>(std::exp(1.00 * std::numbers::pi * \
156                     (static_cast<baseType>(frequency_index)/static_cast<baseType>(nfft) * \
157                     static_cast<baseType>(signal_index))));
158         }
159         finaloutput[frequency_index] = accumulate_value / nfft_sqrt;
160     }
161
162     // returning
163     return std::move(finaloutput);

```

```

164     }
165
166     /*=====
167     x = ifft(std::vector<std::complex<double>> spectrum, nfft)
168     -----*/
169     #include <fftw3.h>
170     #include <vector>
171     #include <complex>
172     #include <stdexcept>
173
174     auto ifft(const std::vector<std::complex<double>>& input_vector,
175              const std::size_t nfft)
176     {
177         if (nfft <= 0)
178             throw std::runtime_error("nfft must be > 0");
179         if (input_vector.size() != nfft)
180             throw std::runtime_error("input spectrum must be of size nfft");
181
182         // Output: real-valued time-domain sequence
183         std::vector<double> output(nfft);
184
185         // Allocate FFTW input/output
186         fftw_complex* in = reinterpret_cast<fftw_complex*>(
187             fftw_malloc(sizeof(fftw_complex) * (nfft/2 + 1))
188         );
189         double* out = reinterpret_cast<double*>(
190             fftw_malloc(sizeof(double) * nfft)
191         );
192
193         // Copy *only* the first nfft/2+1 bins (rest are redundant due to symmetry)
194         for (std::size_t i = 0; i < nfft/2 + 1; ++i) {
195             in[i][0] = input_vector[i].real();
196             in[i][1] = input_vector[i].imag();
197         }
198
199         // Create inverse FFTW plan
200         fftw_plan plan = fftw_plan_dft_c2r_1d(
201             static_cast<int>(nfft),
202             in,
203             out,
204             FFTW_ESTIMATE
205         );
206
207         fftw_execute(plan);
208
209         // Normalize by nfft (FFTW leaves IFFT unscaled)
210         for (std::size_t i = 0; i < nfft; ++i) {
211             output[i] = out[i] / static_cast<double>(nfft);
212         }
213
214         // Cleanup
215         fftw_destroy_plan(plan);
216         fftw_free(in);
217         fftw_free(out);
218
219         return output;
220     }
221
222

```

223
224 }

B.13 Flipping Containers

```

1  #pragma once
2  namespace svr {
3      /*=====
4      Mirror image of a vector
5      -----*/
6      template <typename T>
7      auto fliplr(const std::vector<T>& input_vector)
8      {
9          // creating canvas
10         auto canvas {input_vector};
11
12         // rewriting
13         std::reverse(canvas.begin(), canvas.end());
14
15         // returning
16         return std::move(canvas);
17     }
18 }
```

B.14 Indexing

```

1  #pragma once
2  namespace svr {
3      /*=====
4      y = index(vector, mask)
5
6      template <
7          typename T1,
8          typename T2,
9          typename = std::enable_if_t<
10              (std::is_arithmetic_v<T1> ||
11               std::is_same_v<T1, std::complex<float>> > ||
12               std::is_same_v<T1, std::complex<double>> >) &&
13               std::is_integral_v<T2>
14          >
15      >
16      -----*/
17      template <typename T1,
18              typename T2,
19              typename = std::enable_if_t<std::is_arithmetic_v<T1> ||
20                                          std::is_same_v<T1, std::complex<float>> > ||
21                                          std::is_same_v<T1, std::complex<double>> >
22                                          >
23          >
24      auto index(const std::vector<T1>& input_vector,
25                const std::vector<T2>& indices_to_sample)
26      {
27          // creating canvas
28          auto canvas {std::vector<T1>(indices_to_sample.size(), 0)};
```

```

29
30 // copying the associated values
31 for(int i = 0; i < indices_to_sample.size(); ++i){
32     auto source_index {indices_to_sample[i]};
33     if(source_index < input_vector.size()){
34         canvas[i] = input_vector[source_index];
35     }
36     else{
37         // cout << "warning: Some chosen samples are out of bounds. svr::index |
38         // source_index !< input_vector.size()\n";
39     }
40 }
41
42 // returning
43 return std::move(canvas);
44 }
45 /*=====
46 y = index(matrix, mask, dim)
47 -----*/
48 template <
49     typename T1,
50     typename T2,
51     typename = std::enable_if_t<
52         (std::is_same_v<T1, double> || std::is_same_v<T1, float>) &&
53         (std::is_same_v<T2, int> || std::is_same_v<T2, std::size_t>)
54     >
55 >
56 auto index(const std::vector<std::vector<T1>>& input_matrix,
57             const std::vector<T2>& indices_to_sample,
58             const std::size_t& dim)
59 {
60     // fetching dimensions
61     const auto& num_rows_matrix {input_matrix.size()};
62     const auto& num_cols_matrix {input_matrix[0].size()};
63
64     // creating canvas
65     auto canvas {std::vector<std::vector<T1>>()};
66
67     // if indices are row-indices
68     if (dim == 0){
69
70         // initializing canvas
71         canvas = std::vector<std::vector<T1>>(
72             num_rows_matrix,
73             std::vector<T1>(indices_to_sample.size())
74         );
75
76         // filling the canvas
77         auto destination_index {0};
78         std::for_each(
79             indices_to_sample.begin(), indices_to_sample.end(),
80             [&](const auto& col){
81                 for(auto row = 0; row < num_rows_matrix; ++row){
82                     if (col <= input_matrix[0].size()){
83                         canvas[row][destination_index] = input_matrix[row][col];
84                     }
85                 }
86                 ++destination_index;

```



```

87         });
88     }
89     else if(dim == 1){
90         // initializing canvas
91         canvas = std::vector<std::vector<T1>>>(
92             indices_to_sample.size(),
93             std::vector<T1>(num_cols_matrix)
94         );
95
96         // filling the canvas
97         #pragma omp parallel for
98         for(auto row = 0; row < canvas.size(); ++row){
99             auto destination_col {0};
100             std::for_each(indices_to_sample.begin(), indices_to_sample.end(),
101                 [&row,
102                  &input_matrix,
103                  &destination_col,
104                  &canvas](const auto& source_col){
105                 canvas[row][destination_col++] =
106                     input_matrix[row][source_col];
107             });
108         }
109     }
110     else {
111         std::cerr << "svr_index | this dim is not implemented \n";
112     }
113
114     // moving it back
115     return std::move(canvas);
116 }

```

B.15 Linspace

```

1  /*****
2  Dependencies
3  *****/
4  #pragma once
5  #include <vector>
6  #include <complex>
7
8  namespace svr {
9      /*****
10     in-place
11     *****/
12     template <typename T>
13     auto linspace(
14         auto&          input,
15         const auto     startvalue,
16         const auto     endvalue,
17         const auto     numpoints
18     ) -> void
19     {
20         auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);
21         for(int i = 0; i<input.size(); ++i) {input[i] = startvalue + i*stepsize;}
22     };

```

```

23  /*=====
24  in-place
25  -----*/
26  template <typename T>
27  auto linspace(
28      std::vector<std::complex<T>>& input,
29      const auto startvalue,
30      const auto endvalue,
31      const auto numpoints
32  ) -> void
33  {
34      auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);
35      for(int i = 0; i<input.size(); ++i) {
36          input[i] = startvalue + static_cast<T>(i)*stepsize;
37      }
38  };
39  /*=====
40  -----*/
41  template <
42      typename T,
43      typename = std::enable_if_t<
44          std::is_arithmetic_v<T> ||
45          std::is_same_v<T, std::complex<float>> > ||
46          std::is_same_v<T, std::complex<double>> >
47      >
48  >
49  auto linspace(
50      const T startvalue,
51      const T endvalue,
52      const std::size_t numpoints
53  )
54  {
55      std::vector<T> input(numpoints);
56      auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);
57      for(int i = 0; i<input.size(); ++i) {input[i] = startvalue +
58          static_cast<T>(i)*stepsize;}
59      return std::move(input);
60  };
61  /*=====
62  -----*/
63  template <typename T, typename U>
64  auto linspace(
65      const T startvalue,
66      const U endvalue,
67      const std::size_t numpoints
68  )
69  {
70      std::vector<double> input(numpoints);
71      auto stepsize = static_cast<double>(endvalue -
72          startvalue)/static_cast<double>(numpoints-1);
73      for(int i = 0; i<input.size(); ++i) {input[i] = startvalue + i*stepsize;}
74      return std::move(input);
75  };
76  }

```

B.16 Max

```

1  #pragma once
2  /*=====
3  maximum along dimension 1
4  -----*/
5  template <std::size_t axis, typename T>
6  auto max(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis
    == 1, std::vector<std::vector<T>>> >
7  {
8      // setting up canvas
9      auto canvas
        {std::vector<std::vector<T>>(input_matrix.size(), std::vector<T>(1))};
10
11     // filling up the canvas
12     for(auto row = 0; row < input_matrix.size(); ++row)
13         canvas[row][0] = *(std::max_element(input_matrix[row].begin(),
            input_matrix[row].end()));
14
15     // returning
16     return std::move(canvas);
17 }

```

B.17 Meshgrid

```

1  /*=====
2  Dependencies
3  -----*/
4  #pragma once
5  #include <vector> // for std::vector
6  #include <utility> // for std::pair
7  #include <complex> // for std::complex
8
9
10 /*=====
11 mesh-grid when working with l-values
12 -----*/
13 template <typename T>
14 auto meshgrid(const std::vector<T>& x,
15              const std::vector<T>& y)
16 {
17
18     // creating and filling x-grid
19     std::vector<std::vector<T>> xcanvas(y.size(), std::vector<T>(x.size(), 0));
20     for(auto row = 0; row < y.size(); ++row)
21         std::copy(x.begin(), x.end(), xcanvas[row].begin());
22
23     // creating and filling y-grid
24     std::vector<std::vector<T>> ycanvas(y.size(), std::vector<T>(x.size(), 0));
25     for(auto col = 0; col < x.size(); ++col)
26         for(auto row = 0; row < y.size(); ++row)
27             ycanvas[row][col] = y[row];
28
29     // returning
30     return std::move(std::pair{xcanvas, ycanvas});
31 }
32 }
33 /*=====

```

```

34 meshgrid when working with r-values
35 -----*/
36 template <typename T>
37 auto meshgrid(std::vector<T>&& x,
38               std::vector<T>&& y)
39 {
40
41     // creating and filling x-grid
42     std::vector<std::vector<T>> xcanvas(y.size(), std::vector<T>(x.size(), 0));
43     for(auto row = 0; row < y.size(); ++row)
44         std::copy(x.begin(), x.end(), xcanvas[row].begin());
45
46     // creating and filling y-grid
47     std::vector<std::vector<T>> ycanvas(y.size(), std::vector<T>(x.size(), 0));
48     for(auto col = 0; col < x.size(); ++col)
49         for(auto row = 0; row < y.size(); ++row)
50             ycanvas[row][col] = y[row];
51
52     // returning
53     return std::move(std::pair{xcanvas, ycanvas});
54
55 }

```

B.18 Minimum

```

1 #pragma once
2 /*=====
3 minimum along dimension 1
4 -----*/
5 template <std::size_t axis, typename T>
6 auto min(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis ==
7     1, std::vector<std::vector<T>> >
8 {
9     // creating canvas
10     auto canvas
11         {std::vector<std::vector<T>>(input_matrix.size(), std::vector<T>(1))};
12
13     // storing the values
14     for(auto row = 0; row < input_matrix.size(); ++row)
15         canvas[row][0] = *(std::min_element(input_matrix[row].begin(),
16             input_matrix[row].end()));
17
18     // returning the value
19     return std::move(canvas);
20 }

```

B.19 Norm

```

1 #pragma once
2 /*=====
3 calculating norm for vector
4 -----*/
5 template <typename T>
6 auto norm(const std::vector<T>& input_vector)

```



```

66         accumulate_vector.begin(),
67         [](const auto& argx, auto& argy){
68             return argx*argx + argy;
69         });
70     }
71
72     // calculating element-wise square root
73     std::for_each(accumulate_vector.begin(), accumulate_vector.end(),
74         [](auto& argx){
75             argx = std::sqrt(argx);
76         });
77
78     // moving to the canvas
79     canvas[0] = std::move(accumulate_vector);
80 }
81 else if (dim == 1)
82 {
83     // allocating space in the canvas
84     canvas = std::vector<std::vector<T>>>(
85         input_matrix[0].size(),
86         std::vector<T>(1, 0)
87     );
88
89     // going through each column
90     for(auto row = 0; row < num_cols_matrix; ++row){
91         canvas[row][0] = norm(input_matrix[row]);
92     }
93
94 }
95 else
96 {
97     std::cerr << "norm(matrix, dim): dimension operation not defined \n";
98 }
99
100 // returning
101 return std::move(canvas);
102 }
103
104
105
106 /*
107 Templates to create
108 - matrix and norm-axis
109 - axis instantiated std::vector<T>
110 */

```

B.20 Division

```

1 #pragma once
2 /*=====
3 element-wise division with scalars
4 -----*/
5 template <typename T>
6 auto operator/(const std::vector<T>& input_vector,
7               const T& input_scalar)
8 {

```

```

9      // creating canvas
10     auto canvas {input_vector};
11
12     // filling canvas
13     std::transform(canvas.begin(), canvas.end(),
14                    canvas.begin(),
15                    [&input_scalar](const auto& argx){
16                        return static_cast<double>(argx) /
17                               static_cast<double>(input_scalar);
18                    });
19
20     // returning value
21     return std::move(canvas);
22 }
23
24 /*=====
25 element-wise division with scalars
26 -----*/
27
28 template <typename T>
29 auto operator/=(const std::vector<T>& input_vector,
30                const T& input_scalar)
31 {
32     // creating canvas
33     auto canvas {input_vector};
34
35     // filling canvas
36     std::transform(canvas.begin(), canvas.end(),
37                    canvas.begin(),
38                    [&input_scalar](const auto& argx){
39                        return static_cast<double>(argx) /
40                               static_cast<double>(input_scalar);
41                    });
42
43     // returning value
44     return std::move(canvas);
45 }
46
47 /*=====
48 element-wise with matrix
49 -----*/
50
51 template <
52     typename T,
53     typename = std::enable_if_t<
54         std::is_floating_point_v<T>
55     >
56 >
57 auto operator/(const std::vector<std::vector<T>>& input_matrix,
58               const T scalar)
59 {
60     // fetching matrix-dimensions
61     const auto& num_rows_matrix {input_matrix.size()};
62     const auto& num_cols_matrix {input_matrix[0].size()};
63
64     // creating canvas
65     auto canvas {std::vector<std::vector<T>>(
66         num_rows_matrix,
67         std::vector<T>(num_cols_matrix)
68     )};
69
70     // dividing with values
71     for(auto row = 0; row < num_rows_matrix; ++row){

```



```

124 )
125 {
126     // creating canvas
127     auto canvas {std::vector<std::complex<T>>(input_vector.size())};
128
129     // filling the canvas
130     std::transform(
131         input_vector.begin(), input_vector.end(),
132         canvas.begin(),
133         [&input_scalar](const auto& argx){
134             return argx/static_cast<std::complex<T>>(input_scalar);
135         }
136     );
137
138     // returning
139     return std::move(canvas);
140 }

```

B.21 Addition

```

1  #pragma once
2  /*=====
3  y = vector + vector
4  -----*/
5  template <typename T>
6  std::vector<T> operator+(const std::vector<T>& a,
7                          const std::vector<T>& b)
8  {
9      // Identify which is bigger
10     const auto& big = (a.size() > b.size()) ? a : b;
11     const auto& small = (a.size() > b.size()) ? b : a;
12
13     std::vector<T> result = big; // copy the bigger one
14
15     // Add elements from the smaller one
16     for (size_t i = 0; i < small.size(); ++i) {
17         result[i] += small[i];
18     }
19
20     return result;
21 }
22 /*=====
23 -----*/
24 // y = vector + vector
25 template <typename T>
26 std::vector<T>& operator+=(std::vector<T>& a,
27                           const std::vector<T>& b) {
28
29     const auto& small = (a.size() < b.size()) ? a : b;
30     const auto& big = (a.size() < b.size()) ? b : a;
31
32     // If b is bigger, resize 'a' to match
33     if (a.size() < b.size()) {a.resize(b.size());}
34
35     // Add elements
36     for (size_t i = 0; i < small.size(); ++i) {a[i] += b[i];}

```

```

37
38     // returning elements
39     return a;
40 }
41 // =====
42 // y = matrix + matrix
43 template <typename T>
44 std::vector<std::vector<T>> operator+(const std::vector<std::vector<T>>& a,
45                                     const std::vector<std::vector<T>>& b)
46 {
47     // fetching dimensions
48     const auto& num_rows_A    {a.size()};
49     const auto& num_cols_A    {a[0].size()};
50     const auto& num_rows_B    {b.size()};
51     const auto& num_cols_B    {b[0].size()};
52
53     // choosing the three different metrics
54     if (num_rows_A != num_rows_B && num_cols_A != num_cols_B){
55         cout << format("a.dimensions = [{},{}], b.shape = [{},{}]\n",
56                       num_rows_A, num_cols_A,
57                       num_rows_B, num_cols_B);
58         std::cerr << "dimensions don't match\n";
59     }
60
61     // creating canvas
62     auto canvas {std::vector<std::vector<T>>(
63         std::max(num_rows_A, num_rows_B),
64         std::vector<T>(std::max(num_cols_A, num_cols_B), (T)0.00)
65     )};
66
67     // performing addition
68     if (num_rows_A == num_rows_B && num_cols_A == num_cols_B){
69         for(auto row = 0; row < num_rows_A; ++row){
70             std::transform(a[row].begin(), a[row].end(),
71                           b[row].begin(),
72                           canvas[row].begin(),
73                           std::plus<T>());
74         }
75     }
76     else if(num_rows_A == num_rows_B){
77
78         // if number of columns are different, check if one of the cols are one
79         const auto min_num_cols {std::min(num_cols_A, num_cols_B)};
80         if (min_num_cols != 1) {std::cerr<< "Operator+: unable to broadcast\n";}
81         const auto max_num_cols {std::max(num_cols_A, num_cols_B)};
82
83         // using references to tag em differently
84         const auto& big_matrix    {num_cols_A > num_cols_B ? a : b};
85         const auto& small_matrix  {num_cols_A < num_cols_B ? a : b};
86
87         // Adding to canvas
88         for(auto row = 0; row < canvas.size(); ++row){
89             std::transform(big_matrix[row].begin(), big_matrix[row].end(),
90                           canvas[row].begin(),
91                           [&small_matrix,
92                            &row](const auto& argx){
93                             return argx + small_matrix[row][0];
94                         });
95     }

```

```

96     }
97     else if(num_cols_A == num_cols_B){
98
99         // check if the smallest column-number is one
100         const auto min_num_rows {std::min(num_rows_A, num_rows_B)};
101         if(min_num_rows != 1) {std::cerr << "Operator+ : unable to broadcast\n";}
102         const auto max_num_rows {std::max(num_rows_A, num_rows_B)};
103
104         // using references to differentiate the two matrices
105         const auto& big_matrix {num_rows_A > num_rows_B ? a : b};
106         const auto& small_matrix {num_rows_A < num_rows_B ? a : b};
107
108         // adding to canvas
109         for(auto row = 0; row < canvas.size(); ++row){
110             std::transform(big_matrix[row].begin(), big_matrix[row].end(),
111                             small_matrix[0].begin(),
112                             canvas[row].begin(),
113                             [](const auto& argx, const auto& argy){
114                                 return argx + argy;
115                             });
116         }
117     }
118     else {
119         std::cerr << "operator+: yet to be implemented \n";
120     }
121
122     // returning
123     return std::move(canvas);
124 }
125 /*=====
126 y = vector + scalar
127 -----*/
128 template <typename T>
129 auto operator+(const std::vector<T>& input_vector,
130               const T scalar)
131 {
132     // creating canvas
133     auto canvas {input_vector};
134
135     // adding scalar to the canvas
136     std::transform(canvas.begin(), canvas.end(),
137                     canvas.begin(),
138                     [&scalar](auto& argx){return argx + scalar;});
139
140     // returning canvas
141     return std::move(canvas);
142 }
143 /*=====
144 y = scalar + vector
145 -----*/
146 template <typename T>
147 auto operator+(const T scalar,
148               const std::vector<T>& input_vector)
149 {
150     // creating canvas
151     auto canvas {input_vector};
152
153     // adding scalar to the canvas
154     std::transform(canvas.begin(), canvas.end(),

```

```

155         canvas.begin(),
156         [&scalar](auto& argx){return argx + scalar;});
157
158     // returning canvas
159     return std::move(canvas);
160 }

```

B.22 Multiplication (Element-wise)

```

1  #pragma once
2  /*=====
3  y = scalar * vector
4  -----*/
5  template <typename T>
6  auto operator*(
7      const T scalar,
8      const std::vector<T>& input_vector
9  )
10 {
11     // creating canvas
12     auto canvas {input_vector};
13     // performing operation
14     std::for_each(canvas.begin(), canvas.end(),
15         [&scalar](auto& argx){argx = argx * scalar;});
16     // returning
17     return std::move(canvas);
18 }
19 /*=====
20 y = scalar * vector
21 -----*/
22 template <
23     typename T1,
24     typename T2,
25     typename = std::enable_if_t<
26         !std::is_same_v<std::decay_t<T1>, std::vector<T2>> &&
27         std::is_arithmetic_v<T1>
28     >
29 >
30 auto operator*(const T1 scalar,
31     const vector<T2>& input_vector)
32 {
33     // fetching final-type
34     using T3 = decltype(std::declval<T1>() * std::declval<T2>());
35
36     // creating canvas
37     auto canvas {std::vector<T3>(input_vector.size())};
38
39     // multiplying
40     std::transform(
41         input_vector.begin(), input_vector.end(),
42         canvas.begin(),
43         [&scalar](auto& argx){
44             return static_cast<T3>(scalar) * static_cast<T3>(argx);
45         });
46
47     // returning

```

```

48     return std::move(canvas);
49 }
50 /*=====
51 y = scalar * vecotor (subset init)
52 -----*/
53 template <
54     typename T,
55     typename = std::enable_if_t<
56         std::is_floating_point_v<T>
57     >
58 >
59 auto operator*(
60     const std::complex<T> scalar,
61     const std::vector<T>& input_vector
62 )
63 {
64     // creating canvas
65     auto canvas {std::vector<std::complex<T>>(
66         input_vector.size()
67     )};
68
69     // copying to canvas
70     std::transform(
71         input_vector.begin(), input_vector.end(),
72         canvas.begin(),
73         [&scalar](const auto& argx){
74             return scalar * static_cast<std::complex<T>>(argx);
75         }
76     );
77
78     // moving it back
79     return std::move(canvas);
80 }
81 /*=====
82 y = vector * scalar
83 -----*/
84 template <typename T>
85 auto operator*(const std::vector<T>& input_vector,
86               const T scalar)
87 {
88     // creating canvas
89     auto canvas {input_vector};
90     // multiplying
91     std::for_each(canvas.begin(), canvas.end(),
92         [&scalar](auto& argx){
93             argx = argx * scalar;
94         });
95     // returning
96     return std::move(canvas);
97 }
98 /*=====
99 y = vector * vector
100 -----*/
101 template <typename T>
102 auto operator*(const std::vector<T>& input_vector_A,
103               const std::vector<T>& input_vector_B)
104 {
105     // throwing error: size-desparity

```

```

106     if (input_vector_A.size() != input_vector_B.size()) {std::cerr << "operator*: size
107         disparity \n";}
108
109     // creating canvas
110     auto canvas {input_vector_A};
111
112     // element-wise multiplying
113     std::transform(input_vector_B.begin(), input_vector_B.end(),
114                   canvas.begin(),
115                   canvas.begin(),
116                   [](const auto& argx, const auto& argy){
117                       return argx * argy;
118                   });
119
120     // moving it back
121     return std::move(canvas);
122 }
123
124 /*=====
125 y = vecotr * vector
126 -----*/
127
128 template <
129     typename T1,
130     typename T2,
131     typename = std::enable_if_t<
132         std::is_arithmetic_v<T1> &&
133         std::is_arithmetic_v<T2>
134     >
135 >
136 auto operator*(const std::vector<T1>& input_vector_A,
137               const std::vector<T2>& input_vector_B)
138 {
139
140     // checking size disparity
141     if (input_vector_A.size() != input_vector_B.size())
142         std::cerr << "operator*: error, size-disparity \n";
143
144     // figuring out resulting data type
145     using T3 = decltype(std::declval<T1>() * std::declval<T2>());
146
147     // creating canvas
148     auto canvas {std::vector<T3>(input_vector_A.size())};
149
150     // performing multiplications
151     std::transform(input_vector_A.begin(), input_vector_A.end(),
152                   input_vector_B.begin(),
153                   canvas.begin(),
154                   [](const auto& argx,
155                     const auto& argy){
156                       return static_cast<T3>(argx) * static_cast<T3>(argy);
157                   });
158
159     // returning
160     return std::move(canvas);
161 }
162
163 /*=====
164 y = vector * complex_vector
165 -----*/
166
167 template <

```

```

164     typename T,
165     typename = std::enable_if_t<
166         std::is_floating_point_v<T>
167     >
168 >
169 auto operator*(
170     const std::vector<T>& input_vector_A,
171     const std::vector<std::complex<T>>& input_vector_B
172 )
173 {
174     // checking size issue
175     if (input_vector_A.size() != input_vector_B.size())
176         throw std::runtime_error(
177             "FILE: svr_operator_star.hpp | FUNCTION: operator* | REPORT: error disparity
              in the two input-vectors"
178         );
179
180     // creating canvas
181     auto canvas {std::vector<std::complex<T>>( input_vector_A.size() )};
182
183     // filling up the canvas
184     std::transform(
185         input_vector_B.begin(), input_vector_B.end(),
186         input_vector_A.begin(),
187         canvas.begin(),
188         [](const auto& argx, const auto& argy){
189             return argx + static_cast<std::complex<T>>(argy);
190         }
191     );
192
193     // moving it back
194     return std::move(canvas);
195 }
196 /*=====
197 y = complex_vector * vector
198 -----*/
199 template <
200     typename T,
201     typename = std::enable_if_t<
202         std::is_floating_point_v<T>
203     >
204 >
205 auto operator*(
206     const std::vector<std::complex<T>>& input_vector_A,
207     const std::vector<T>& input_vector_B
208 )
209 {
210     // enforcing size
211     if (input_vector_A.size() != input_vector_B.size())
212         throw std::runtime_error(
213             "FILE: svr_operator_star.hpp | FUNCTION: operator* overload"
214         );
215
216     // creating canvas
217     auto canvas {std::vector<std::complex<T>>(input_vector_A.size())};
218
219     // filling values
220     std::transform(
221         input_vector_A.begin(), input_vector_A.end(),

```

```

222     input_vector_B.begin(),
223     canvas.begin(),
224     [](const auto& argx, const auto& argy){
225         return argx * static_cast<std::complex<T>>(argy);
226     }
227 );
228
229 // returning
230 return std::move(canvas);
231 }
232 /*=====
233 y = complex-vector * complex-vector
234 -----*/
235 template <
236     typename T,
237     typename = std::enable_if_t<
238         std::is_floating_point_v<T>
239     >
240 >
241 auto operator*(
242     const std::vector<std::complex<T>> input_vector_A,
243     const std::vector<std::complex<T>> input_vector_B
244 )
245 {
246     // checking size
247     if (input_vector_A.size() != input_vector_B.size())
248         throw std::runtime_error(
249             "FILE: svr_operator_star.hpp | FUNCTION: operator*(complex-vector,
250                 complex-vector)"
251         );
252
253     // creating canvas
254     auto canvas {std::vector<std::complex<T>>(input_vector_A.size())};
255
256     // filling canvas
257     std::transform(
258         input_vector_A.begin(), input_vector_A.end(),
259         input_vector_B.begin(),
260         canvas.begin(),
261         [](const auto& argx, const auto& argy){
262             return argx * argy;
263         }
264     );
265
266     // returning values
267     return std::move(canvas);
268 }
269 // scalar * matrix =====
270 template <typename T>
271 auto operator*(const T scalar,
272     const std::vector<std::vector<T>>& inputMatrix)
273 {
274     std::vector<std::vector<T>> temp {inputMatrix};
275     for(int i = 0; i<inputMatrix.size(); ++i){
276         std::transform(inputMatrix[i].begin(),
277             inputMatrix[i].end(),
278             temp[i].begin(),
279             [&scalar](T x){return scalar * x;});

```



```

280     return std::move(temp);
281 }
282 /*=====
283 y = matrix * scalar
284 -----*/
285 template <typename T>
286 auto operator*(const std::vector<std::vector<T>>& input_matrix,
287                const T scalar)
288 {
289     // fetching matrix dimensions
290     const auto& num_rows_matrix {input_matrix.size()};
291     const auto& num_cols_matrix {input_matrix[0].size()};
292
293     // creating canvas
294     auto canvas {std::vector<std::vector<T>>(
295         num_rows_matrix,
296         std::vector<T>(num_cols_matrix)
297     )};
298
299     // storing the values
300     for(auto row = 0; row < num_rows_matrix; ++row)
301         std::transform(input_matrix[row].begin(), input_matrix[row].end(),
302             canvas[row].begin(),
303             [&scalar](const auto& argx){
304                 return argx * scalar;
305             });
306
307     // returning
308     return std::move(canvas);
309 }
310 /*=====
311 y = matrix * matrix
312 -----*/
313 template <typename T>
314 auto operator*(const std::vector<std::vector<T>>& A,
315                const std::vector<std::vector<T>>& B) -> std::vector<std::vector<T>>
316 {
317     // Case 1: element-wise multiplication
318     if (A.size() == B.size() && A[0].size() == B[0].size()) {
319         std::vector<std::vector<T>> C(A.size(), std::vector<T>(A[0].size()));
320         for (std::size_t row = 0; row < A.size(); ++row) {
321             std::transform(A[row].begin(), A[row].end(),
322                 B[row].begin(),
323                 C[row].begin(),
324                 [](const auto& x, const auto& y){ return x * y; });
325         }
326         return C;
327     }
328
329     // Case 2: broadcast column vector
330     else if (A.size() == B.size() && B[0].size() == 1) {
331         std::vector<std::vector<T>> C(A.size(), std::vector<T>(A[0].size()));
332         for (std::size_t row = 0; row < A.size(); ++row) {
333             std::transform(A[row].begin(), A[row].end(),
334                 C[row].begin(),
335                 [&](const auto& x){ return x * B[row][0]; });
336         }
337         return C;
338     }

```

```

339
340 // case 3: when second matrix contains just one row
341 // case 4: when first matrix is just one column
342 // case 5: when second matrix is just one column
343
344 // Otherwise, invalid
345 else {
346     throw std::runtime_error("operator* dimension mismatch");
347 }
348 }
349 /*=====
350 y = scalar * matrix
351 -----*/
352 template <typename T1, typename T2>
353 auto operator*(const T1 scalar,
354               const std::vector<std::vector<T2>>& inputMatrix)
355 {
356     std::vector<std::vector<T2>> temp {inputMatrix};
357     for(int i = 0; i<inputMatrix.size(); ++i){
358         std::transform(inputMatrix[i].begin(),
359                        inputMatrix[i].end(),
360                        temp[i].begin(),
361                        [&scalar](T2 x){return static_cast<T2>(scalar) * x;});
362     }
363     return temp;
364 }
365 /*=====
366 matrix-multiplication
367 -----*/
368 template <typename T1, typename T2>
369 auto matmul(const std::vector<std::vector<T1>>& matA,
370            const std::vector<std::vector<T2>>& matB)
371 {
372
373     // throwing error
374     if (matA[0].size() != matB.size()) {std::cerr << "dimension-mismatch \n";}
375
376     // getting result-type
377     using ResultType = decltype(std::declval<T1>() * std::declval<T2>() + \
378                                std::declval<T1>() * std::declval<T2>());
379
380     // creating aliases
381     auto finalnumrows {matA.size()};
382     auto finalnumcols {matB[0].size()};
383
384     // creating placeholder
385     auto rowcolproduct = [&](auto rowA, auto colB){
386         ResultType temp {0};
387         for(int i = 0; i < matA.size(); ++i) {temp +=
388             static_cast<ResultType>(matA[rowA][i]) +
389             static_cast<ResultType>(matB[i][colB]);}
390         return temp;
391     };
392
393     // producing row-column combinations
394     std::vector<std::vector<ResultType>> finaloutput(finalnumrows,
395                                                    std::vector<ResultType>(finalnumcols));
396     for(int row = 0; row < finalnumrows; ++row){for(int col = 0; col < finalnumcols;
397         ++col){finaloutput[row][col] = rowcolproduct(row, col);}}

```

```

394
395     // returning
396     return finaloutput;
397 }
398 /*=====
399 y = matrix * vector
400 -----*/
401 template <
402     typename T,
403     typename = std::enable_if_t<
404         std::is_arithmetic_v<T>
405     >
406 >
407 auto operator*(const std::vector<std::vector<T>>& input_matrix,
408               const std::vector<T>& input_vector)
409 {
410     // fetching dimensions
411     const auto& num_rows_matrix {input_matrix.size()};
412     const auto& num_cols_matrix {input_matrix[0].size()};
413     const auto& num_rows_vector {1};
414     const auto& num_cols_vector {input_vector.size()};
415     const auto& max_num_rows {num_rows_matrix > num_rows_vector ?\
416                               num_rows_matrix : num_rows_vector};
417     const auto& max_num_cols {num_cols_matrix > num_cols_vector ?\
418                               num_cols_matrix : num_cols_vector};
419
420     // creating canvas
421     auto canvas {std::vector<std::vector<T>>>(
422         max_num_rows,
423         std::vector<T>(max_num_cols, 0)
424     )};
425
426     // multiplying column matrix with row matrix
427     if (num_cols_matrix == 1 && num_rows_vector == 1){
428
429         // writing to canvas
430         for(auto row = 0; row < max_num_rows; ++row)
431             for(auto col = 0; col < max_num_cols; ++col)
432                 canvas[row][col] = input_matrix[row][0] * input_vector[col];
433     }
434     /*=====
435     Multiplying each row with the input-vector
436     -----*/
437     else if(
438         num_cols_matrix == num_cols_vector &&
439         num_rows_vector == 1
440     )
441     {
442         // writing to canvas
443         for(auto row = 0; row < max_num_rows; ++row)
444             std::transform(
445                 input_matrix[row].begin(), input_matrix[row].end(),
446                 input_vector.begin(),
447                 canvas[row].begin(),
448                 [](const auto& argx, const auto& argy){return argx * argy;}
449             );
450     }
451     else
452     {

```

```

453     std::cerr << "Operator*: [matrix, vector] | not implemented \n";
454 }
455
456 // returning
457 return std::move(canvas);
458
459 }
460 /*=====
461 complex-matrix * vector
462 -----*/
463 template <
464     typename T,
465     typename = std::enable_if_t<
466         std::is_floating_point_v<T>
467     >
468 >
469 auto operator*(
470     const std::vector<std::vector<std::complex<T>>>& input_matrix,
471     const std::vector<T>& input_vector
472 )
473 {
474     // fetching dimensions
475     const auto num_rows_matrix {input_matrix.size()};
476     const auto num_cols_matrix {input_matrix[0].size()};
477     const auto num_rows_vector {static_cast<std::size_t>(1)};
478     const auto num_cols_vector {input_vector.size()};
479
480     // throwing an error
481     if (num_cols_matrix != num_cols_vector)
482         throw std::runtime_error(
483             "FILE: svr_operator_star.hpp | FUNCTION: operator*(complex-matrix, vector)"
484         );
485
486     // creating canvas
487     auto canvas {std::vector<std::vector<std::complex<T>>>(
488         num_rows_matrix,
489         std::vector<std::complex<T>>(num_cols_matrix)
490     )};
491
492     // performing operations
493     for(auto row = 0; row < num_rows_matrix; ++row)
494         std::transform(
495             input_matrix[row].begin(), input_matrix[row].end(),
496             input_vector.begin(),
497             canvas[row].begin(),
498             [](const auto& argx, const auto& argy){
499                 return argx * static_cast<std::complex<T>>(argy);
500             }
501         );
502
503     // returning the final output
504     return std::move(canvas);
505
506 }
507 /*=====
508 martix * complex-vector
509 -----*/
510 template <
511     typename T,

```

```

512     typename    = std::enable_if_t<
513         std::is_floating_point_v<T>
514     >
515 >
516 auto    operator*(
517     const    std::vector<std::vector<T>>& input_matrix,
518     const    std::vector<std::complex<T>>& input_vector
519 )
520 {
521     // fetching dimensions
522     const auto    num_rows_matrix    {input_matrix.size()};
523     const auto    num_cols_matrix    {input_matrix[0].size()};
524     const auto    num_rows_vector    {static_cast<std::size_t>(1)};
525     const auto    num_cols_vector    {input_vector.size()};
526
527     // fetching dimension mismatch
528     if (num_cols_matrix != num_cols_vector)
529         throw std::runtime_error(
530             "FILE: svr_operator_star.hpp | FUNCTION: operator*(input-matrix,
531                 complex-vector)"
532         );
533
534     // creating canvas
535     auto    canvas    {std::vector<std::vector<std::complex<T>>>(
536         num_rows_matrix,
537         std::vector<std::complex<T>>(
538             num_cols_matrix
539         )
540     )};
541
542     // filling the values
543     for(auto row = 0; row < num_rows_matrix; ++row)
544         std::transform(
545             input_matrix[row].begin(), input_matrix[row].end(),
546             input_vector.begin(),
547             canvas[row].begin(),
548             [](const auto& argx, const auto& argy){
549                 return static_cast<std::complex<T>>(argx) * argy;
550             }
551         );
552
553     // returning final-output
554     return std::move(canvas);
555 }
556 //=====
557 scalar operators
558 -----*/
559 auto operator*(const std::complex<double> complexscalar,
560     const double    doublescalar){
561     return complexscalar * static_cast<std::complex<double>>(doublescalar);
562 }
563 auto operator*(const double    doublescalar,
564     const std::complex<double> complexscalar){
565     return complexscalar * static_cast<std::complex<double>>(doublescalar);
566 }
567 auto operator*(const std::complex<double> complexscalar,
568     const int    scalar){
569     return complexscalar * static_cast<std::complex<double>>(scalar);
570 }

```

```

570 auto operator*(const int          scalar,
571               const std::complex<double> complexscalar){
572     return complexscalar * static_cast<std::complex<double>>(scalar);
573 }

```

B.23 Subtraction

```

1  #pragma once
2  /*=====
3  y = vector - scalar
4  -----*/
5  template <typename T>
6  auto operator-(const std::vector<T>& a,
7               const T          scalar){
8      std::vector<T> temp(a.size());
9      std::transform(a.begin(),
10                   a.end(),
11                   temp.begin(),
12                   [scalar](T x){return (x - scalar);});
13     return std::move(temp);
14 }
15 /*=====
16 y = vector - vector
17 -----*/
18 template <typename T>
19 auto operator-(const std::vector<T>& input_vector_A,
20               const std::vector<T>& input_vector_B)
21 {
22     // throwing error
23     if (input_vector_A.size() != input_vector_B.size())
24         std::cerr << "operator-(vector, vector): size disparity\n";
25
26     // creating canvas
27     const auto& num_cols {input_vector_A.size()};
28     auto canvas {std::vector<T>()};
29
30     // performing operations
31     std::transform(input_vector_A.begin(), input_vector_A.begin(),
32                  input_vector_B.begin(),
33                  canvas.begin(),
34                  [](const auto& argx, const auto& argy){
35                      return argx - argy;
36                  });
37
38     // return
39     return std::move(canvas);
40 }
41 /*=====
42 y = matrix - matrix
43 -----*/
44 template <typename T>
45 auto operator-(const std::vector<std::vector<T>>& input_matrix_A,
46               const std::vector<std::vector<T>>& input_matrix_B)
47 {
48     // fetching dimensions
49     const auto& num_rows_A {input_matrix_A.size()};

```

```

50  const auto& num_cols_A {input_matrix_A[0].size()};
51  const auto& num_rows_B {input_matrix_B.size()};
52  const auto& num_cols_B {input_matrix_B[0].size()};
53
54  // creating canvas
55  auto canvas {std::vector<std::vector<T>>()};
56
57  // if both matrices are of equal dimensions
58  if (num_rows_A == num_rows_B && num_cols_A == num_cols_B)
59  {
60      // copying one to the canvas
61      canvas = input_matrix_A;
62
63      // subtracting
64      for(auto row = 0; row < num_rows_B; ++row)
65          std::transform(canvas[row].begin(), canvas[row].end(),
66                          input_matrix_B[row].begin(),
67                          canvas[row].begin(),
68                          [](auto& argx, const auto& argy){
69                              return argx - argy;
70                          });
71  }
72  // column broadcasting (case 1)
73  else if(num_rows_A == num_rows_B && num_cols_B == 1)
74  {
75      // copying canvas
76      canvas = input_matrix_A;
77
78      // subtracting
79      for(auto row = 0; row < num_rows_A; ++row){
80          std::transform(canvas[row].begin(), canvas[row].end(),
81                          canvas[row].begin(),
82                          [&input_matrix_B,
83                           &row](auto& argx){
84                              return argx - input_matrix_B[row][0];
85                          });
86      }
87  }
88  else{
89      std::cerr << "operator-: not implemented for this case \n";
90  }
91
92  // returning
93  return std::move(canvas);
94  }

```

B.24 Printing Containers

```

1  #pragma once
2  /*=====
3  -----*/
4  template<typename T>
5  void fPrintVector(const vector<T> input){
6      for(auto x: input) cout << x << ", ";
7      cout << endl;
8  }

```

```

9
10 template<typename T>
11 void fpv(vector<T> input){
12     for(auto x: input) cout << x << ", ";
13     cout << endl;
14 }
15 /*=====
16 -----*/
17 template<typename T>
18 void fPrintMatrix(const std::vector<std::vector<T>> input_matrix){
19     for(const auto& row: input_matrix)
20         cout << format("{}\n", row);
21 }
22 /*=====
23 -----*/
24 template <typename T>
25 void fPrintMatrix(const string&          input_string,
26                  const std::vector<std::vector<T>> input_matrix){
27     cout << format("{} = \n", input_string);
28     for(const auto& row: input_matrix)
29         cout << format("{}\n", row);
30 }
31 /*=====
32 -----*/
33 template<typename T, typename T1>
34 void fPrintHashmap(unordered_map<T, T1> input){
35     for(auto x: input){
36         cout << format("[{},{}] | ", x.first, x.second);
37     }
38     cout << endl;
39 }
40 /*=====
41 -----*/
42 void fPrintBinaryTree(TreeNode* root){
43     // sending it back
44     if (root == nullptr) return;
45
46     // printing
47     PRINTLINE
48     cout << "root->val = " << root->val << endl;
49
50     // calling the children
51     fPrintBinaryTree(root->left);
52     fPrintBinaryTree(root->right);
53
54     // returning
55     return;
56 }
57 /*=====
58 -----*/
59 void fPrintLinkedList(ListNode* root){
60     if (root == nullptr) return;
61     cout << root->val << " -> ";
62     fPrintLinkedList(root->next);
63     return;
64 }
65 /*=====
66 -----*/
67

```



```

68 template<typename T>
69 void fPrintContainer(T input){
70     for(auto x: input) cout << x << ", ";
71     cout << endl;
72     return;
73 }
74 /*=====
75 -----*/
76 template <typename T>
77 auto size(std::vector<std::vector<T>> inputMatrix){
78     cout << format("[{}, {}]\n",
79                     inputMatrix.size(),
80                     inputMatrix[0].size());
81 }
82 /*=====
83 -----*/
84 template <typename T>
85 auto size(const std::string& inputstring,
86           const std::vector<std::vector<T>>& inputMatrix){
87     cout << format("{} = [{}, {}]\n",
88                     inputstring,
89                     inputMatrix.size(),
90                     inputMatrix[0].size());
91 }

```

B.25 Random Number Generation

```

1  #pragma once
2  /*=====
3  -----*/
4  template <typename T>
5  auto rand(const T min,
6           const T max) {
7      static std::random_device rd; // Seed
8      static std::mt19937 gen(rd()); // Mersenne Twister generator
9      std::uniform_real_distribution<> dist(min, max);
10     return dist(gen);
11 }
12 /*=====
13 -----*/
14 template <typename T>
15 auto rand(const T min,
16           const T max,
17           const std::size_t numelements)
18 {
19     static std::random_device rd; // Seed
20     static std::mt19937 gen(rd()); // Mersenne Twister generator
21     std::uniform_real_distribution<> dist(min, max);
22
23     // building the finaloutput
24     vector<T> finaloutput(numelements);
25     for(int i = 0; i<finaloutput.size(); ++i) {finaloutput[i] =
26         static_cast<T>(dist(gen));}
27
28     return finaloutput;
29 }

```

```

29  /*=====
30  -----*/
31  template <typename T>
32  auto rand(const T          argmin,
33           const T          argmax,
34           const std::vector<int> dimensions)
35  {
36
37      // throwing an error if dimension is greater than two
38      if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}
39
40      // creating random engine
41      static std::random_device rd; // Seed
42      static std::mt19937 gen(rd()); // Mersenne Twister generator
43      std::uniform_real_distribution<> dist(argmin, argmax);
44
45      // building the finaloutput
46      vector<vector<T>> finaloutput;
47      for(int i = 0; i<dimensions[0]; ++i){
48          vector<T> temp;
49          for(int j = 0; j<dimensions[1]; ++j) {temp.push_back(dist(gen));}
50          // cout << format("\t\t temp = {}\n", temp);
51
52          finaloutput.push_back(temp);
53      }
54
55      // returning the finaloutput
56      return finaloutput;
57  }
58  }
59  /*=====
60  -----*/
61  auto rand(const std::vector<int> dimensions)
62  {
63      using ReturnType = double;
64
65      // throwing an error if dimension is greater than two
66      if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}
67
68      // creating random engine
69      static std::random_device rd; // Seed
70      static std::mt19937 gen(rd()); // Mersenne Twister generator
71      std::uniform_real_distribution<> dist(0.00, 1.00);
72
73      // building the finaloutput
74      vector<vector<ReturnType>> finaloutput;
75      for(int i = 0; i<dimensions[0]; ++i){
76          vector<ReturnType> temp;
77          for(int j = 0; j<dimensions[1]; ++j) {temp.push_back(dist(gen));}
78          finaloutput.push_back(std::move(temp));
79      }
80
81      // returning the finaloutput
82      return std::move(finaloutput);
83  }
84  }
85  /*=====
86  -----*/
87  template <typename T>

```

```

88 auto rand_complex_double(const T          argmin,
89                          const T          argmax,
90                          const std::vector<int>& dimensions)
91 {
92
93     // throwing an error if dimension is greater than two
94     if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}
95
96     // creating random engine
97     static std::random_device rd; // Seed
98     static std::mt19937 gen(rd()); // Mersenne Twister generator
99     std::uniform_real_distribution<> dist(argmin, argmax);
100
101     // building the finaloutput
102     vector<vector<complex<double>>> finaloutput;
103     for(int i = 0; i<dimensions[0]; ++i){
104         vector<complex<double>> temp;
105         for(int j = 0; j<dimensions[1]; ++j)
106             {temp.push_back(static_cast<double>(dist(gen)));}
107         finaloutput.push_back(std::move(temp));
108     }
109
110     // returning the finaloutput
111     return finaloutput;
112 }

```

B.26 Reshape

```

1  #pragma once
2
3  /*=====
4  reshaping a matrix into another matrix
5  -----*/
6  template <std::size_t M, std::size_t N, typename T>
7  auto reshape(const std::vector<std::vector<T>>& input_matrix){
8
9      // verifying size stuff
10     if (M*N != input_matrix.size() * input_matrix[0].size())
11         std::cerr << "Dimensions are quite different\n";
12
13     // creating canvas
14     auto canvas {std::vector<std::vector<T>>(
15         M, std::vector<T>(N, (T)0)
16     )};
17
18     // writing to canvas
19     size_t tid {0};
20     size_t target_row {0};
21     size_t target_col {0};
22     for(auto row = 0; row<input_matrix.size(); ++row){
23         for(auto col = 0; col < input_matrix[0].size(); ++col){
24             tid = row * input_matrix[0].size() + col;
25             target_row = tid/N;
26             target_col = tid%N;
27             canvas[target_row][target_col] = input_matrix[row][col];
28         }
29     }
30 }

```

```

29     }
30
31     // moving it back
32     return std::move(canvas);
33 }
34 /*=====
35 reshaping a matrix into a vector
36 -----*/
37 template<std::size_t M, typename T>
38 auto reshape(const std::vector<std::vector<T>>& input_matrix){
39
40     // checking element-count validity
41     if (M != input_matrix.size() * input_matrix[0].size())
42         std::cerr << "Number of elements differ\n";
43
44     // creating canvas
45     auto canvas {std::vector<T>(M, 0)};
46
47     // filling canvas
48     for(auto row = 0; row < input_matrix.size(); ++row)
49         for(auto col = 0; col < input_matrix[0].size(); ++col)
50             canvas[row * input_matrix.size() + col] = input_matrix[row][col];
51
52     // moving it back
53     return std::move(canvas);
54 }
55 /*=====
56 Matrix to matrix
57 -----*/
58 template<typename T>
59 auto reshape(const std::vector<std::vector<T>>& input_matrix,
60             const std::size_t M,
61             const std::size_t N){
62
63     // checking element-count validity
64     if (M * N != input_matrix.size() * input_matrix[0].size())
65         std::cerr << "Number of elements differ\n";
66
67     // creating canvas
68     auto canvas {std::vector<std::vector<T>>(
69         M, std::vector<T>(N, (T)0)
70     )};
71
72     // writing to canvas
73     size_t tid {0};
74     size_t target_row {0};
75     size_t target_col {0};
76     for(auto row = 0; row < input_matrix.size(); ++row){
77         for(auto col = 0; col < input_matrix[0].size(); ++col){
78             tid = row * input_matrix[0].size() + col;
79             target_row = tid/N;
80             target_col = tid%N;
81             canvas[target_row][target_col] = input_matrix[row][col];
82         }
83     }
84
85     // moving it back
86     return std::move(canvas);
87 }

```

```

88  /*=====
89  converting a matrix into a vector
90  -----*/
91  template<typename T>
92  auto reshape(const std::vector<std::vector<T>>& input_matrix,
93              const size_t M){
94
95      // checking element-count validity
96      if (M != input_matrix.size() * input_matrix[0].size())
97          std::cerr << "Number of elements differ\n";
98
99      // creating canvas
100     auto canvas {std::vector<T>(M, 0)};
101
102     // filling canvas
103     for(auto row = 0; row < input_matrix.size(); ++row)
104         for(auto col = 0; col < input_matrix[0].size(); ++col)
105             canvas[row * input_matrix.size() + col] = input_matrix[row][col];
106
107     // moving it back
108     return std::move(canvas);
109 }

```

B.27 Summing with containers

```

1  #pragma once
2  /*=====
3  -----*/
4  template <std::size_t axis, typename T>
5  auto sum(const std::vector<T>& input_vector) -> std::enable_if_t<axis == 0,
6          std::vector<T>>
7  {
8      // returning the input as is
9      return input_vector;
10 }
11 /*=====
12 -----*/
13 template <std::size_t axis, typename T>
14 auto sum(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis == 0,
15         std::vector<T>>
16 {
17     // creating canvas
18     auto canvas {std::vector<T>(input_matrix[0].size(), 0)};
19
20     // filling up the canvas
21     for(auto row = 0; row < input_matrix.size(); ++row)
22         std::transform(input_matrix[row].begin(), input_matrix[row].end(),
23             canvas.begin(),
24             canvas.begin(),
25             [](auto& argx, auto& argy){return argx + argy;});
26
27     // returning
28     return std::move(canvas);
29 }
30 /*=====

```

```

30 -----*/
31 template <std::size_t axis, typename T>
32 auto sum(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis == 1,
    std::vector<std::vector<T>>>
33 {
34     // creating canvas
35     auto canvas {std::vector<std::vector<T>>(input_matrix.size(),
36                                             std::vector<T>(1, 0.00))};
37
38     // filling up the canvas
39     for(auto row = 0; row < input_matrix.size(); ++row)
40         canvas[row][0] = std::accumulate(input_matrix[row].begin(),
41                                           input_matrix[row].end(),
42                                           static_cast<T>(0));
43
44     // returning
45     return std::move(canvas);
46
47 }
48 /*=====
49 -----*/
50 template <std::size_t axis, typename T>
51 auto sum(const std::vector<T>& input_vector_A,
52         const std::vector<T>& input_vector_B) -> std::enable_if_t<axis == 0,
    std::vector<T> >
53 {
54     // setup
55     const auto& num_cols_A {input_vector_A.size()};
56     const auto& num_cols_B {input_vector_B.size()};
57
58     // throwing errors
59     if (num_cols_A != num_cols_B) {std::cerr << "sum: size disparity\n";}
60
61     // creating canvas
62     auto canvas {input_vector_A};
63
64     // summing up
65     std::transform(input_vector_B.begin(), input_vector_B.end(),
66                   canvas.begin(),
67                   canvas.begin(),
68                   std::plus<T>());
69
70     // returning
71     return std::move(canvas);
72 }

```

B.28 Tangent

```

1 #pragma once
2 namespace svr {
3     /*=====
4     y = tan-inverse(input_vector_A/input_vector_B)
5     -----*/
6     template <typename T>
7     auto atan2(const std::vector<T> input_vector_A,
8               const std::vector<T> input_vector_B)

```

```

9      {
10         // throw error
11         if (input_vector_A.size() != input_vector_B.size())
12             std::cerr << "atan2: size disparity\n";
13
14         // create canvas
15         auto canvas = {std::vector<T>(input_vector_A.size(), 0)};
16
17         // performing element-wise atan2 calculation
18         std::transform(input_vector_A.begin(), input_vector_A.end(),
19                        input_vector_B.begin(),
20                        canvas.begin(),
21                        [](const auto& arg_a,
22                          const auto& arg_b){
23
24                             return std::atan2(arg_a, arg_b);
25                         });
26
27         // moving things back
28         return std::move(canvas);
29     }
30     /*=====
31     y = tan-inverse(a/b)
32     -----*/
33     template <typename T>
34     auto atan2(T scalar_A,
35               T scalar_B)
36     {
37         return std::atan2(scalar_A, scalar_B);
38     }
39 }

```

B.29 Tiling Operations

```

1  #pragma once
2  namespace svr {
3      /*=====
4      tiling a vector
5      -----*/
6      template <typename T>
7      auto tile(const std::vector<T>& input_vector,
8               const std::vector<size_t>& mul_dimensions){
9
10         // creating canvas
11         const std::size_t& num_rows = {1 * mul_dimensions[0]};
12         const std::size_t& num_cols = {input_vector.size() * mul_dimensions[1]};
13         auto canvas {std::vector<std::vector<T>>(
14             num_rows,
15             std::vector<T>(num_cols, 0)
16         )};
17
18         // writing
19         std::size_t source_row;
20         std::size_t source_col;
21
22         for(std::size_t row = 0; row < num_rows; ++row){

```

```

23         for(std::size_t col = 0; col < num_cols; ++col){
24             source_row = row % 1;
25             source_col = col % input_vector.size();
26             canvas[row][col] = input_vector[source_col];
27         }
28     }
29
30     // returning
31     return std::move(canvas);
32 }
33 /*=====
34 tiling a matrix
35 -----*/
36 template <typename T>
37 auto tile(const std::vector<std::vector<T>>& input_matrix,
38           const std::vector<size_t>& mul_dimensions){
39
40     // creating canvas
41     const std::size_t& num_rows {input_matrix.size() * mul_dimensions[0]};
42     const std::size_t& num_cols {input_matrix[0].size() * mul_dimensions[1]};
43     auto canvas {std::vector<std::vector<T>>>(
44         num_rows,
45         std::vector<T>(num_cols, 0)
46     )};
47
48     // writing
49     std::size_t source_row;
50     std::size_t source_col;
51
52     for(std::size_t row = 0; row < num_rows; ++row){
53         for(std::size_t col = 0; col < num_cols; ++col){
54             source_row = row % input_matrix.size();
55             source_col = col % input_matrix[0].size();
56             canvas[row][col] = input_matrix[source_row][source_col];
57         }
58     }
59
60     // returning
61     return std::move(canvas);
62 }
63 }

```

B.30 Transpose

```

1  #pragma once
2  /*=====
3  -----*/
4  template <typename T>
5  auto transpose(const std::vector<T>& input_vector){
6
7      // creating canvas
8      auto canvas {std::vector<std::vector<T>>>{
9          input_vector.size(),
10         std::vector<T>(1)
11     }};
12

```



```

13 // filling canvas
14 for(auto i = 0; i < input_vector.size(); ++i){
15     canvas[i][0] = input_vector[i];
16 }
17
18 // moving it back
19 return std::move(canvas);
20 }

```

B.31 Masking

```

1 #pragma once
2 namespace svr {
3     /*=====
4     y = input_vector[mask == 1]
5     -----*/
6     template <typename T,
7             typename = std::enable_if_t< std::is_arithmetic_v<T>          ||
8                                     std::is_same_v<T, std::complex<double>> ||
9                                     std::is_same_v<T, std::complex<float>>>
10                                     >
11             >
12     auto mask(const std::vector<T>& input_vector,
13             const std::vector<bool>& mask_vector)
14     {
15         // checking dimensionality issues
16         if (input_vector.size() != mask_vector.size())
17             std::cerr << "mask(vector, mask): incompatible size \n";
18
19         // creating canvas
20         auto num_trues {std::count(mask_vector.begin(),
21                                   mask_vector.end(),
22                                   true)};
23         auto canvas {std::vector<T>(num_trues)};
24
25         // copying values
26         auto destination_index {0};
27         for(auto i = 0; i < input_vector.size(); ++i)
28             if (mask_vector[i] == true)
29                 canvas[destination_index++] = input_vector[i];
30
31         // returning output
32         return std::move(canvas);
33     }
34     /*=====
35     -----*/
36     template <typename T>
37     auto mask(const std::vector<std::vector<T>>& input_matrix,
38             const std::vector<bool> mask_vector)
39     {
40         // fetching dimensions
41         const auto& num_rows_matrix {input_matrix.size()};
42         const auto& num_cols_matrix {input_matrix[0].size()};
43         const auto& num_cols_vector {mask_vector.size()};
44
45         // error-checking

```

```

46     if (num_cols_matrix != num_cols_vector)
47         std::cerr << "mask(matrix, bool-vector): size disparity";
48
49     // building canvas
50     auto num_trues {std::count(mask_vector.begin(),
51                               mask_vector.end(),
52                               true)};
53     auto canvas {std::vector<std::vector<T>>>(
54         num_rows_matrix,
55         std::vector<T>(num_cols_vector, 0)
56     )};
57
58     // writing values
59     #pragma omp parallel for
60     for(auto row = 0; row < num_rows_matrix; ++row){
61         auto destination_index {0};
62         for(auto col = 0; col < num_cols_vector; ++col)
63             if(mask_vector[col] == true)
64                 canvas[row][destination_index++] = input_matrix[row][col];
65     }
66
67     // returning
68     return std::move(canvas);
69 }
70 /*=====
71 Fetch Indices corresponding to mask true's
72 -----*/
73 auto mask_indices(const std::vector<bool>& mask_vector)
74 {
75     // creating canvas
76     auto num_trues {std::count(mask_vector.begin(), mask_vector.end(),
77                               true)};
78     auto canvas {std::vector<std::size_t>(num_trues)};
79
80     // building canvas
81     auto destination_index {0};
82     for(auto i = 0; i < mask_vector.size(); ++i)
83         if (mask_vector[i] == true)
84             canvas[destination_index++] = i;
85
86     // returning
87     return std::move(canvas);
88 }
89 }

```

B.32 Resetting Containers

```

1  #pragma once
2  namespace svr {
3      /*=====
4      Variadic version of resetting
5      -----*/
6      template <typename T, typename... Rest>
7      void reset(std::vector<T>& first_vector, Rest&... rest_vectors) {
8          // Reset the first vector
9          std::vector<T>().swap(first_vector);

```

```

10
11     // Recursively reset the remaining vectors
12     if constexpr (sizeof...(rest_vectors) > 0) {
13         reset(rest_vectors...);
14     }
15 }
16 }

```

B.33 Element-wise squaring

```

1  #pragma once
2  namespace svr {
3      /*=====
4      Element-wise squaring vector
5      -----*/
6      template <typename T,
7              typename = std::enable_if_t<std::is_arithmetic_v<T>>
8              >
9      auto square(const std::vector<T>& input_vector)
10     {
11         // creating canvas
12         auto canvas {std::vector<T>(input_vector.size())};
13
14         // performing calculations
15         std::transform(input_vector.begin(), input_vector.end(),
16                        canvas.begin(),
17                        [](const auto& argx){
18                            return argx * argx;
19                        });
20
21         // moving it back
22         return std::move(canvas);
23     }
24     /*=====
25     Element-wise squaring vector (in-place)
26     -----*/
27     template <typename T,
28             typename = std::enable_if_t<std::is_arithmetic_v<T>>
29             >
30     void square_inplace(std::vector<T>& input_vector)
31     {
32         // performing operations
33         std::transform(input_vector.begin(), input_vector.end(),
34                        input_vector.begin(),
35                        [](auto& argx){
36                            return argx * argx;
37                        });
38     }
39     /*=====
40     Element-wise squaring a matrix
41     -----*/
42     template <typename T>
43     auto square(const std::vector<std::vector<T>>& input_matrix)
44     {
45         // fetching dimensions
46         const auto& num_rows {input_matrix.size()};

```

```

47     const auto& num_cols {input_matrix[0].size()};
48
49     // creating canvas
50     auto canvas {std::vector<std::vector<T>>(
51         num_rows,
52         std::vector<T>(num_cols, 0)
53     )};
54
55     // going through each row
56     #pragma omp parallel for
57     for(auto row = 0; row < num_rows; ++row)
58         std::transform(input_matrix[row].begin(), input_matrix[row].end(),
59             canvas[row].begin(),
60             [](const auto& argx){
61                 return argx * argx;
62             });
63
64     // returning
65     return std::move(canvas);
66 }
67 /*=====
68 Squaring for scalars
69 -----*/
70 template <typename T>
71 auto square(const T& scalar) {return scalar * scalar;}
72 }

```

B.34 Flooring

```

1 namespace svr {
2     /*=====
3     element-wise flooring of a vector-contents
4     -----*/
5     template <typename T>
6     auto floor(const std::vector<T>& input_vector)
7     {
8         // creating canvas
9         auto canvas {std::vector<T>(input_vector.size())};
10
11         // filling the canvas
12         std::transform(input_vector.begin(), input_vector.end(),
13             canvas.begin(),
14             [](const auto& argx){
15                 return std::floor(argx);
16             });
17
18         // returning
19         return std::move(canvas);
20     }
21     /*=====
22     element-wise flooring of a vector-contents (in-place)
23     -----*/
24     template <typename T>
25     auto floor_inplace(std::vector<T>& input_vector)
26     {
27         // rewriting the contents

```

```

28     std::transform(input_vector.begin(), input_vector.end(),
29                    input_vector.begin(),
30                    [](auto& argx){
31                        return std::floor(argx);
32                    });
33 }
34 /*=====
35 element-wise flooring of matrix-contents
36 -----*/
37 template <typename T>
38 auto floor(const std::vector<std::vector<T>>& input_matrix)
39 {
40     // fetching dimensions
41     const auto& num_rows_matrix {input_matrix.size()};
42     const auto& num_cols_matrix {input_matrix[0].size()};
43
44     // creating canvas
45     auto canvas {std::vector<std::vector<T>>(
46         num_rows_matrix,
47         std::vector<T>(num_cols_matrix)
48     )};
49
50     // writing contents
51     for(auto row = 0; row < num_rows_matrix; ++row)
52         std::transform(input_matrix[row].begin(), input_matrix[row].end(),
53                        canvas[row].begin(),
54                        [](const auto& argx){
55                            return std::floor(argx);
56                        });
57
58     // returning contents
59     return std::move(canvas);
60
61 }
62 /*=====
63 element-wise flooring of matrix-contents (in-place)
64 -----*/
65 template <typename T>
66 auto floor_inplace(std::vector<std::vector<T>>& input_matrix)
67 {
68     // performing operations
69     for(auto row = 0; row < input_matrix.size(); ++row)
70         std::transform(input_matrix[row].begin(), input_matrix[row].end(),
71                        input_matrix[row].begin(),
72                        [](auto& argx){
73                            return std::floor(argx);
74                        });
75 }
76 }

```

B.35 Squeeze

```

1 namespace svr {
2     template <typename T>
3     auto squeeze(const std::vector<std::vector<T>>& input_matrix)
4     {

```

```

5      // fetching dimensions
6      const auto& num_rows_matrix {input_matrix.size()};
7      const auto& num_cols_matrix {input_matrix[0].size()};
8
9      // check if any dimension is 1
10     if (num_rows_matrix == 0 || num_cols_matrix == 0)
11         std::cerr << "at least one dimension should be 1";
12
13     auto final_length {std::max(num_rows_matrix, num_cols_matrix)};
14
15     // creating canvas
16     auto canvas {std::vector<T>(final_length)};
17
18     // building canvas
19     if (num_rows_matrix == 1)
20     {
21         // filling canvas
22         std::copy(input_matrix[0].begin(), input_matrix[0].end(),
23                 canvas.begin());
24     }
25     else if (num_cols_matrix == 1)
26     {
27         // filling canvas
28         std::transform(input_matrix.begin(), input_matrix.end(),
29                 canvas.begin(),
30                 [](const auto& argx){
31                     return argx[0];
32                 });
33     }
34
35     // returning
36     return std::move(canvas);
37 }
38 }

```

B.36 Tensor Initializations

```

1  namespace svr {
2      /*=====
3      -----*/
4      template <typename T>
5      auto zeros(const std::array<std::size_t, 2> input_dimensions)
6      {
7          // create canvas
8          auto canvas {std::vector<std::vector<T>>(
9              input_dimensions[0],
10             std::vector<T>(input_dimensions[1], 0)
11         )};
12
13         // returning
14         return std::move(canvas);
15     }
16 }

```

B.37 Real part

```

1  #pragma once
2  namespace svr {
3
4      /*=====
5      For type-deductions
6      -----*/
7      template <typename T>
8      struct real_result_type;
9
10     template <> struct real_result_type<std::complex<double>>{
11         using type = double;
12     };
13     template <> struct real_result_type<std::complex<float>>{
14         using type = float;
15     };
16     template <> struct real_result_type<double> {
17         using type = double;
18     };
19     template <> struct real_result_type<float>{
20         using type = float;
21     };
22
23     template <typename T>
24     using real_result_t = typename real_result_type<T>::type;
25
26     /*=====
27     Element-wise real() of a vector
28     -----*/
29     template <typename T>
30     auto real(const std::vector<T>& input_vector)
31     {
32         // figure out base-type
33         using TCanvas = real_result_t<T>;
34
35         // creating canvas
36         auto canvas = std::vector<TCanvas>(
37             input_vector.size()
38         );
39
40         // storing values
41         std::transform(input_vector.begin(), input_vector.end(),
42             canvas.begin(),
43             [](const auto& argx){
44                 return std::real(argx);
45             });
46
47         // returning
48         return std::move(canvas);
49     }
50 }

```

B.38 Imaginary part

```

1  #pragma once

```

```

2 namespace svr {
3
4     /*=====
5     For type-deductions
6     -----*/
7     template <typename T>
8     struct imag_result_type;
9
10    template <> struct imag_result_type<std::complex<double>>{
11        using type = double;
12    };
13    template <> struct imag_result_type<std::complex<float>>{
14        using type = float;
15    };
16    template <> struct imag_result_type<double> {
17        using type = double;
18    };
19    template <> struct imag_result_type<float>{
20        using type = float;
21    };
22
23    template <typename T>
24    using imag_result_t = typename imag_result_type<T>::type;
25
26    /*=====
27    -----*/
28    template <typename T>
29    auto imag(const std::vector<T>& input_vector)
30    {
31        // figure out base-type
32        using TCanvas = imag_result_t<T>;
33
34        // creating canvas
35        auto canvas {std::vector<TCanvas>(
36            input_vector.size()
37        )};
38
39        // storing values
40        std::transform(input_vector.begin(), input_vector.end(),
41            canvas.begin(),
42            [](const auto& argx){
43                return std::imag(argx);
44            });
45
46        // returning
47        return std::move(canvas);
48    }
49 }

```

B.39 Down-sample

```

1 #pragma once
2
3 /*=====
4 Includes
5 -----*/

```

```

6  #include "svr_concepts.hpp"
7
8  namespace svr {
9
10     /*=====
11     -----*/
12     template <
13         svr::PureOrComplexFloatingType T0,
14         typename T1,
15         typename = std::enable_if_t<
16             std::is_integral_v<T1>
17         >
18     >
19     auto sample(
20         const std::vector<T0>& input_vector,
21         const T1 starting_point,
22         const T1 constant_difference)
23     {
24         // trivial-cases
25         if (input_vector.size() == 0) {return input_vector;}
26
27         // error-checks
28         if (starting_point >= input_vector.size()) {
29             throw std::runtime_error(
30                 "FILE: svr_sample.hpp | FUNCTION: sample | REPORT: starting-point >=
31                 input-vector-size"
32             );
33         }
34         if (constant_difference < 0){
35             throw std::runtime_error(
36                 "FILE: svr_sample.hpp | FUNCTION: sample | REPORT: constant-difference < 0"
37             );
38         }
39
40         // calculating finaloutput-dimensions
41         auto canvas_size {1 + std::floor(
42             (input_vector.size() -
43              static_cast<std::size_t>(starting_point))/constant_difference
44         )};
45         auto canvas {std::vector<T0>(canvas_size)};
46
47         // writing values
48         for(auto i = 0; i < canvas_size; ++i){
49             const auto source_index {starting_point + i *
50                 constant_difference};
51             const auto& destination_index {i};
52             canvas[destination_index] = input_vector[source_index];
53         }
54
55         // returning the output
56         return std::move(canvas);
57     }
58     /*=====
59     -----*/
60     Matrix-version
61     -----*/
62     template <
63         svr::PureOrComplexFloatingType T0,
64         typename T1,
65         typename = std::enable_if_t<

```

```

62         std::is_integral_v<T1>
63     >
64 >
65     auto sample(
66         const std::vector<std::vector<T0>>& input_matrix,
67         const T1 starting_index,
68         const T1 constant_difference,
69         const T1 dim
70     )
71     {
72         // trivial-cases
73         // error-checks
74         if (dim != 0) {throw std::runtime_error(
75             "FILE: svr_sample.hpp | FUNCTION: sample matrix-version | REPORT:
76             dim-argument is not implemented"
77         );}
78
79         // setup
80         auto canvas {std::vector<std::vector<T0>>()};
81
82         if (dim == 0){
83             // calculating finaloutput-dimensions
84             auto canvas_num_cols {1 + std::floor(
85                 (
86                     input_matrix[0].size() - \
87                     static_cast<std::size_t>(starting_index)
88                 )/constant_difference
89             )};
90             canvas = std::vector<std::vector<T0>>(
91                 input_matrix.size(),
92                 std::vector<T0>(canvas_num_cols)
93             );
94
95             // writing values
96             for(auto row = 0; row < input_matrix.size(); ++row){
97                 for(auto i = 0; i < canvas_num_cols; ++i){
98                     const auto source_index {starting_index + i *
99                     constant_difference};
100                     const auto& destination_index {i};
101                     canvas[row][destination_index] = input_matrix[row][source_index];
102                 }
103             }
104
105             // returning
106             return std::move(canvas);
107         }
108     }
109 }
110
111
112 }
```
