Autonomous Underwater Vehicle: A Surveillance Protocol

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Preface

This project is an attempt at combining all of my major skills into creating a simulation, imaging, perception and control pipeline for Autonomous Underwater Vehicles (AUV). As such, creating this project involves creating a number of pipelines.

The first pipeline is the signal simulation pipeline. The signal simulation pipeline involves sea-floor point-cloud creation and simulating the signals received by the sensor arrays of the AUV. The signals recorded by the sensor-arrays on the AUV contains information from the surrounding environment. The imaging pipeline performs certain operations on the recorded signals to obtain acoustic images of the surrounding environment. To that end, this pipeline involves the topics of signal processing, linear algebra, signals and systems.

As such, the second pipeline is the imaging pipeline. The inputs to the imaging pipeline is the signals recorded by the different sensor-arrays of the AUV, in addition to the parameters of the AUV and its components. This pipeline involves match-filtering, focussing and beamforming operations to create acoustic images of the surrounding environment. Depending on the number of ULAs present, the imaging pipeline is responsible for creating multiple acoustic images in real-time. Thus, this pipeline involves the topics of Digital Signal Processing, Match-Filtering, Estimation and Detection Theory and so on.

The images created by the imaging pipeline are fed to the perception-to-control pipeline. This pipeline takes in the image formed created from the ULA signals, parameters of AUV and its components, and some historical data, it provides instructions regarding the movement of the AUV. The mapping from the inputs to the controls is called policy. Learning policies is a core part of reinforcement learning. Thus, this pipeline mainly involves the topics of reinforcement learning. And since we'll be using convolutional neural nets and transformers for learning the policies, this pipeline involves a significant amount of machine and deep learning.

The final result is an AUV that is primarily trained to map an area of the sea-floor in a constant surveillance mode. The RL-trained policy will also be trained to deal with different kinds of sea-floor terrains: those containing hills, valleys, and path-obstructing features. Due to the resource constrained nature of the marine vessel, we also prioritize efficient policies in the policy-training pipeline.

The project is currently written in C++. And since there is non-trivial amount of training and adaptive features in the pipelines, we'll be using LibTorch (the C++ API of PyTorch) to enable computation graphs, backpropagation and thereby, learning in our AUV pipeline. However, for the sections where a computation graph is not required we will be writing templated STL code.

Contents

Preface				
Ι	AUV	Components & Setup	1	
1	Unde	rwater Environment	2	
	1.1	Underwater Hills	2	
	1.2	Scatterer Definition	3	
	1.3	Sea-Floor Setup Script	4	
2	Trans	mitter	6	
	2.1	Transmission Signal	7	
	2.2	Transmitter Class Definition	8	
	2.3	Transmitter Setup Scripts	9	
3	Unifo	rm Linear Array	13	
	3.1	ULA Class Definition	14	
	3.2	ULA Setup Scripts	16	
4	Autonomous Underwater Vehicle			
	4.1	AUV Class Definition	19	
	4.2	AUV Setup Scripts	20	
II	Sig	nal Simulation Pipeline	21	
5	Signa	l Simulation	22	

CONTENTS iii

III	Imag	ging Pipeline	24
IV	Perce	eption & Control Pipeline	25
A	Applica	tion Specific Tools	26
	A.1	CSV File-Writes	26
	A.2	Thread-Pool	27
	A.3	FFTPlanClass	28
	A.4	IFFTPlanClass	34
	A.5	FFT Plan Pool	39
	A.6	IFFT Plan Pool	40
	A.7	FFT Plan Pool Handle	42
	A.8	IFFT Plan Pool Handle	43
В	General	Purpose Templated Functions	45
	B.1	abs	45
	B.2	Boolean Comparators	46
	B.3	Concatenate Functions	47
	B.4	Conjugate	49
	B.5	Convolution	49
	B.6	Coordinate Change	59
	B.7	Cosine	61
	B.8	Data Structures	62
	B.9	Editing Index Values	62
	B.10	Equality	63
	B.11	Exponentiate	64
	B.12	FFT	64
	B.13	Flipping Containers	68
	B.14	Indexing	68
	B.15	Linspace	70
	B.16	Max	72
	B.17	Meshgrid	
	B.18	Minimum	73
	B.19	Norm	73
	B.20	Division	
	B.21	Addition	77

CONTENTS iv

B.22	Multiplication (Element-wise)
B.23	Subtraction
B.24	Printing Containers
B.25	Random Number Generation
B.26	Reshape
B.27	Summing with containers
B.28	Tangent
B.29	Tiling Operations
B.30	Transpose
B.31	Masking
B.32	Resetting Containers
B.33	Element-wise squaring
B.34	Flooring
B.35	Squeeze
B.36	Tensor Initializations
B.37	Real part
B.38	Imaginary part

Part I AUV Components & Setup

Chapter 1

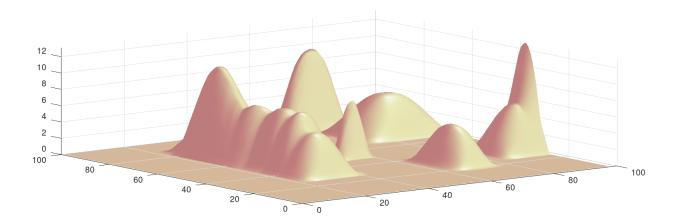
Underwater Environment

Overview

All physical matter in this framework is represented using point-clouds. Thus, the sea-floor also is represented using a number of 3D points. In addition to the coordinates, the points also have the additional property of "reflectivity". It is the impulse response of that point.

Sea-floors in real-life are rarely flat. They often contain valleys, mountains, hills and much richer geographical features. Thus, training an agent to function in such environments call for the creation of similar structures in our simulations.

To simplify things, we shall take a more constrained and structured approach. We start by creating different classes of structures and produce instantiations of those structures on the sea-floor. These structures are defined in such a way that the shape and size can be parameterized to enable creation of random sea-floors.



1.1 Underwater Hills

The most basic approach to creating this is to create a flat seafloor, where all the points have the same height. While this is a good place to start, it is good to bring in some realism to the seafloor. To that end, we shall have some rolling hills as the sea-floor. Each "hill"

is created using the method outlined in Algorithm 1. The method involves deciding the location of the hills, the dimension of the hills and then designing a hill by combining an exponential function and a cosine function. We're aiming to essentially produce gaussian-looking sea-floor hills. After the creation, this becomes the set of points representing the lowest set of points in the overall seafloor structure.

Algorithm 1 Hill Creation

```
1: Input: Mean vector m, Dimension vector d, 2D points P
  2: Output: Updated P with hill heights
  3: num\_hills \leftarrow numel(\mathbf{m}_x)
  4: H \leftarrow \text{Zeros tensor of size } (1, \text{numel}(\mathbf{P}_x))
 5: for i=1 to num_hills do
6: x_{\text{norm}} \leftarrow \frac{\frac{\pi}{2}(\mathbf{P}_x - \mathbf{m}_x[i])}{\mathbf{d}_x[i]}
            y_{\text{norm}} \leftarrow \frac{\frac{\pi}{2}(\mathbf{P}_y - \mathbf{m}_y[i])}{\mathbf{d}_y[i]}
  7:
           h_x \leftarrow \cos(x_{\text{norm}}) \cdot e^{\frac{|x_{\text{norm}}|}{10}}
           h_y \leftarrow \cos(y_{\text{norm}}) \cdot e^{\frac{|y_{\text{norm}}|}{10}}
  9:
            h \leftarrow \mathbf{d}_z[i] \cdot h_x \cdot h_y
10:
            Apply boundary conditions:
11:
            if x_{\text{norm}} > \frac{\pi}{2} or x_{\text{norm}} < -\frac{\pi}{2} or y_{\text{norm}} > \frac{\pi}{2} or y_{\text{norm}} < -\frac{\pi}{2} then
12:
13:
            end if
14:
            H \leftarrow H + h
15:
16: end for
17: \mathbf{P} \leftarrow \text{concatenate}([\mathbf{P}, H])
```

1.2 Scatterer Definition

The sea-floor is represented by a single object of the class ScattererClass.

```
Class Declaration
  template <typename T>
  class ScattererClass
  public:
      // members
      std::vector<std::vector<T>> coordinates;
      std::vector<T> reflectivity;
11
      // Constructor
      ScattererClass() {}
14
      // Constructor
      ScattererClass(std::vector<std::vector<T>> coordinates_arg,
                   std::vector<T>
                                   reflectivity_arg):
                    coordinates(std::move(coordinates_arg)),
18
                    reflectivity(std::move(reflectivity_arg)) {}
19
      // Save to CSV
```

```
void save_to_csv();
};
```

1.3 Sea-Floor Setup Script

Following is the function that will setup the sea-floor script.

```
void fSeaFloorSetup(
      ScattererClass<double>& scatterers
   ){
3
      // auto save_files
                            {false};
      const auto save_files
                                           {false};
      const auto
                   hill_creation_flag
                                           {true};
      // sea-floor bounds
      auto
             bed_width
                            {100.00};
10
      auto
             bed_length
                            {100.00};
11
      // creating tensors for coordinates and reflectivity
      vector<vector<double>>
                              box_coordinates;
14
      vector<double>
                                   box_reflectivity;
16
      // scatter density
      // auto bed_width_density {static_cast<double>( 10.00)};
18
      // auto
                 bed_length_density {static_cast<double>( 10.00)};
19
      auto
              bed_width_density {static_cast<double>( 5.00)};
20
              bed_length_density {static_cast<double>( 5.00)};
      auto
21
22
      // setting up coordinates
23
                         {linspace<double>(0.00,
      auto
              xpoints
                                         bed_width,
                                         bed_width * bed_width_density)};
26
                         {linspace<double>(0.00,
              ypoints
      auto
2.7
                                         bed_length,
                                         bed_length * bed_length_density)};
       if(save_files) fWriteVector(xpoints, "../csv-files/xpoints.csv"); // verified
30
      if(save_files) fWriteVector(ypoints, "../csv-files/ypoints.csv");
                                                                           // verified
31
32
      // creating mesh
33
      auto [xgrid, ygrid] = meshgrid(std::move(xpoints), std::move(ypoints));
34
       if(save_files) fWriteMatrix(xgrid, "../csv-files/xgrid.csv"); // verified
35
       if(save_files) fWriteMatrix(ygrid, "../csv-files/ygrid.csv");
                                                                           // verified
37
      // reshaping
38
             Χ
                     {reshape(xgrid, xgrid.size()*xgrid[0].size())};
39
      auto
      auto
                     {reshape(ygrid, ygrid.size()*ygrid[0].size())};
       if(save_files) fWriteVector(X,
                                          "../csv-files/X.csv");
                                                                           // verified
41
      if(save_files) fWriteVector(Y,
                                           "../csv-files/Y.csv");
                                                                           // verified
      // creating heights of scatterers
       if(hill_creation_flag){
45
46
          // setting up hill parameters
          auto num_hills {10};
49
```

```
// setting up placement of hills
50
                                                                                  // verified
          auto
                  points2D
                                         {concatenate<0>(X, Y)};
51
                  min2D
                                         {min<1, double>(points2D)};
                                                                                  // verified
          auto
52
          auto
                  max2D
                                         {max<1, double>(points2D)};
                                                                                  // verified
54
          auto
                  hill_2D_center
                                         \{\min 2D + \setminus
                                         rand({2, num_hills}) * (max2D - min2D)}; // verified
56
          // setup: hill-dimensions
57
                  hill_dimensions_min
                                         {transpose(vector<double>{5, 5, 2})}; // verified
          auto
          auto
                  hill_dimensions_max
                                         {transpose(vector<double>{30, 30, 10})}; // verified
          auto
                  hill_dimensions
                                         {hill_dimensions_min + \
60
                                         rand({3, num_hills}) * (hill_dimensions_max -
61
                                                                                   // verified
                                             hill_dimensions_min)};
62
          // function-call: hill-creation function
63
          fCreateHills(hill_2D_center,
                       hill_dimensions,
                       points2D);
66
67
          // setting up floor reflectivity
68
          auto floorScatter_reflectivity
                                                {std::vector<double>(Y.size(), 1.00)};
70
          // populating the values of the incoming argument
71
          scatterers.coordinates = std::move(points2D);
72
          scatterers.reflectivity = std::move(floorScatter_reflectivity);
74
75
       else{
76
77
78
          // assigning flat heights
                  Z
                         {std::vector<double>(Y.size(), 0)};
          auto
79
80
          // setting up floor coordinates
81
                  floorScatter_coordinates
                                                {concatenate<0>(X, Y, Z)};
          auto
82
          auto
                  floorScatter_reflectivity
                                                {std::vector<double>(Y.size(), 1)};
83
          // populating the values of the incoming argument
85
          scatterers.coordinates = std::move(floorScatter_coordinates);
86
          scatterers.reflectivity = std::move(floorScatter_reflectivity);
89
       }
90
       // printing status
91
       std::cout << format("> Finished Sea-Floor Setup \n");
92
   }
93
```

Chapter 2

Transmitter

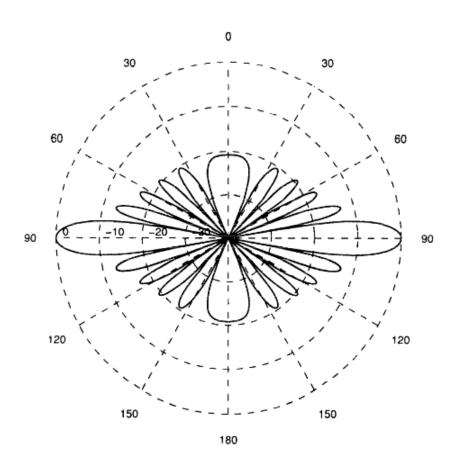


Figure 2.1: Beampattern of a Transmission Uniform Linear Array

Overview

Probing systems are those systems that send out a signal, listen to the reflection and infer qualitative and quantitative qualities of the environment, matter or object, it was trying to infer information about. The transmitter is one of the most fundamental components of probing systems. As the name suggests, the transmitter is the equipment responsible for sending out the probing signal into the medium.

A transmitter is any device or circuit that converts information into a signal and sends it out onto some media like air, cable, water or space. The components of a transmitter are usually as follows

- 1. Input: Information containing signal such as voice, data, video etc
- 2. Process: Encode/modulate the information onto a carrier signal, which can be electromagnetic wave or mechanical wave.
- 3. Transmission: The signal is then transmitted onto the media with electro-mechanical equipment.

Transmitters are of many kinds. But the ones that we will be considering will be directed transmitters, which means that these transmitters have an associated beampattern. To the uninitiated, this means that the power of the transmitted signal is not transmitted in all directions equally. A beampattern is a graphical representation of the power received by an ideal receiver when placed at different angles.

Transmitters made out of a linear-array of individual transmitters use beamforming to "direct" the major power of the transmitter. These kind of systems have well studied beampatterns which can be utilized in our simulations. These kind of studies and inculcating that in our pipelines produce accurate signal simulation pipelines. For now, we stick to a very simple model of a transmitter. We assume that the transmitter sends out the power equally into a particular cone from the AUV position.

2.1 Transmission Signal

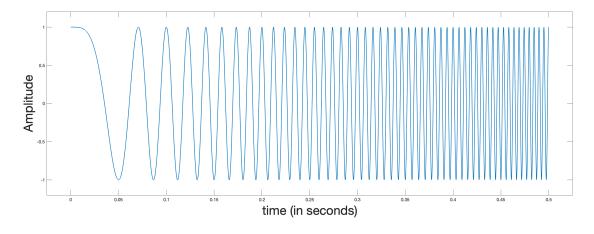


Figure 2.2: Linear Frequency Modulated Wave

The resolution of any probing system is fundamentally tied to the signal bandwidth. A higher bandwidth corresponds to finer resolution $\frac{\text{speed-of-sounds}}{2*\text{bandwidth}}$. Thus, for perfect resolution, an infinite bandwidth is in order. However, infinite bandwidth is impossible for obvious reasons: hardware limitations, spectral regulations, energy limitations and so on.

This is where Linear Frequency Modulation (LFM), also called a "chirp," becomes valuable. An LFM signal linearly sweeps a limited bandwidth over a relatively long duration. This technique spreads the signal's energy in time while retaining the resolution benefits of

the bandwidth. After matched filtering (or pulse compression), we essentially produce pulses corresponding to a base-band LFM of same bandwidth. Overall, LFM is a practical compromise between finite bandwidth and desired performance.

One of the best parts about the resolution depending only on the bandwidth is that it allows us to deploy techniques that would help us improve SNRs without virtually increasing the bandwidth at all. Much of the noise in submarine environments are in and around the baseband region (around frequency, 0). Since resolution depends purely on bandwidth, and LFM can be transmitted at a carrier-frequency, this means that processing the returns after low-pass filtering and basebanding allows us to get rid of the submarine noise, since they do not occupy the same frequency-coefficients. The end-result, thus, is improved SNR compared to use baseband LFM.

Due to all of these advantages, LFM waves are ubiquitous in probing systems, from sonar to radar. Thus, for this project too, the transmitter will be using LFM waves as probing signals, to probe the surrounding submarine environment.

2.2 Transmitter Class Definition

The transmitter is represented by a single object of the class TransmitterClass.

```
template <typename T>
   class TransmitterClass{
   public:
       // A shared pointer to the configuration object
5
       std::shared_ptr<svr::AUVParameters> config_ptr;
6
       // physical/intrinsic properties
8
       std::vector<T>
                         location;
                                               // location tensor
9
                         pointing_direction; // pointing direction
       std::vector<T>
10
11
       // basic parameters
12
       std::vector<T>
                                               // transmitted signal (LFM)
                         signal;
      Τ
                         azimuthal_angle;
                                               // transmitter's azimuthal pointing direction
      Т
                         elevation_angle;
                                               // transmitter's elevation pointing direction
      Т
                         azimuthal_beamwidth; // azimuthal beamwidth of transmitter
16
      Т
                         elevation_beamwidth; // elevation beamwidth of transmitter
17
      Τ
                         range;
                                               // a parameter used for spotlight mode.
18
       // transmitted signal attributes
20
      Т
                         f_low;
                                               // lowest frequency of LFM
21
      Т
                                               // highest frequency of LFM
                         f_high;
22
      Т
                                               // center frequency of LFM
23
                                               // bandwidth of LFM
      Т
                         bandwidth;
24
                         speed_of_sound {1500}; // speed of sound
25
       // shadowing properties
27
                         azimuthQuantDensity;
                                                   // quantization of angles along the
       int
28
          azimuth
                                                   // quantization of angles along the
       int
                         elevationQuantDensity;
          elevation
      Т
                                                   // range-cell size when shadowing
                         rangeQuantSize;
30
       Т
                         azimuthShadowThreshold; // azimuth thresholding
31
       Т
                         elevationShadowThreshold; // elevation thresholding
```

```
33
       // shadowing related
                                              // box indicating whether a scatter for a
      std::vector<T>
                         checkbox;
35
          range-angle pair has been found
      std::vector<std::vector<std::vector<T>>> finalScatterBox; // a 3D tensor where the
          third dimension represnets the vector length
      std::vector<T> finalReflectivityBox; // to store the reflectivity
37
       // constructor
      TransmitterClass() = default;
41
       // Deleting copy constructors/assignment
42
      TransmitterClass(const TransmitterClass& other)
                                                              = delete;
43
      TransmitterClass& operator=(TransmitterClass& other) = delete;
45
       // Creating move-constructor and move-assignment
46
      TransmitterClass(TransmitterClass&& other)
                                                               = default;
      TransmitterClass& operator=(TransmitterClass&& other) = default;
48
49
      // member-functions
50
      auto updatePointingAngle(std::vector<T> AUV_pointing_vector);
51
       auto subset_scatterers(const ScattererClass<T>& seafloor,
52
```

2.3 Transmitter Setup Scripts

The following script shows the setup-script

```
template
       typename
                  Τ,
                     std::enable_if_t<
       typename
                  =
          std::is_same_v<T, double> ||
          std::is_same_v<T, float>
6
   >
   void fTransmitterSetup(
       TransmitterClass<T>& transmitter_fls,
       TransmitterClass<T>& transmitter_portside,
10
       TransmitterClass<T>& transmitter_starboard
11
   ){
12
       // Setting up transmitter
                                     {160e3};
                                                           // sampling frequency
       Т
              sampling_frequency
14
       Т
                                     {50e3};
                                                           // first frequency of LFM
              f1
       Т
              f2
                                     {70e3};
                                                           // second frequency of LFM
                                     \{(f1 + f2)/2.00\};
       Τ
              fc
                                                           // finding center-frequency
17
                                     {std::abs(f2 - f1)}; // bandwidth
       Τ
              bandwidth
18
                                                           // time of recording
19
              pulselength
                                     {5e-2};
       // building LFM
21
                             {linspace<T>(0.00,
       auto
              timearray
                                         pulselength,
                                         std::floor(pulselength * sampling_frequency)));
                             {f2 - f1/pulselength}; // calculating frequency-slope
       auto
              Signal
       auto
                             {cos(2 * std::numbers::pi * \
26
                              (f1 + K*timearray) * \
27
                              timearray)};
                                                     // frequency at each time-step, with f1
                                  = 0
```

```
29
       // Setting up transmitter
              location
                                                {std::vector<T>(3, 0)};
                                                                             // location of
       auto
31
          transmitter
                                                                              // initial
              azimuthal_angle_fls
                                                {0};
32
          pointing direction
             azimuthal_angle_port
                                                {90};
                                                                              // initial
33
          pointing direction
              azimuthal_angle_starboard
                                                {-90};
                                                                              // initial
          pointing direction
35
                                                                              // initial
              elevation_angle
                                                {-60};
36
          pointing direction
37
              azimuthal_beamwidth_fls
                                                {20};
                                                                              // azimuthal
38
          beamwidth of the signal cone
              azimuthal_beamwidth_port
                                                {20};
                                                                              // azimuthal
           beamwidth of the signal cone
                                                                              // azimuthal
              azimuthal_beamwidth_starboard
                                                {20};
40
       Т
          beamwidth of the signal cone
41
              elevation_beamwidth_fls
                                                {20};
                                                                              // elevation
42
          beamwidth of the signal cone
                                                                              // elevation
      Т
              elevation_beamwidth_port
                                                {20};
43
          beamwidth of the signal cone
              elevation_beamwidth_starboard
                                                {20};
                                                                              // elevation
44
          beamwidth of the signal cone
45
                                                {10}; // number of points, a degree is split
       int
              azimuthQuantDensity
46
           into quantization density along azimuth (used for shadowing)
              elevationQuantDensity
                                                {10}; // number of points, a degree is split
       int
47
           into quantization density along elevation (used for shadowing)
              rangeQuantSize
                                                {10}; // the length of a cell (used for
48
           shadowing)
49
              azimuthShadowThreshold
                                                {1};
       Т
                                                       // azimuth threshold
                                                                                (in degrees)
50
       Т
              elevationShadowThreshold
                                                {1};
                                                       // elevation threshold (in degrees)
51
52
53
       // transmitter-fls
55
       transmitter_fls.location
                                            = location;
                                                                              // Assigning
          location
       transmitter_fls.signal
                                            = Signal;
                                                                              // Assigning
56
           signal
       transmitter_fls.azimuthal_angle
                                            = azimuthal_angle_fls;
                                                                             // assigning
57
           azimuth angle
       transmitter_fls.elevation_angle
                                          = elevation_angle;
                                                                             // assigning
           elevation angle
       transmitter_fls.azimuthal_beamwidth = azimuthal_beamwidth_fls;
                                                                              // assigning
59
           azimuth-beamwidth
       transmitter_fls.elevation_beamwidth = elevation_beamwidth_fls;
                                                                              // assigning
60
           elevation-beamwidth
       // updating quantization densities
61
       transmitter_fls.azimuthQuantDensity
                                              = azimuthQuantDensity;
                                                                           // assigning
62
           azimuth quant density
       transmitter_fls.elevationQuantDensity = elevationQuantDensity;
                                                                           // assigning
           elevation quant density
       transmitter_fls.rangeQuantSize
                                               = rangeQuantSize;
                                                                           // assigning
          range-quantization
```

```
transmitter_fls.azimuthShadowThreshold = azimuthShadowThreshold; //
           azimuth-threshold in shadowing
       transmitter_fls.elevationShadowThreshold = elevationShadowThreshold; //
66
           elevation-threshold in shadowing
       // signal related
67
                                                             // assigning lower frequency
       transmitter_fls.f_low
                                              = f1;
                                              = f2;
                                                             // assigning higher frequency
       transmitter_fls.f_high
69
                                                            // assigning center frequency
       transmitter_fls.fc
                                              = fc;
70
       transmitter_fls.bandwidth
                                              = bandwidth; // assigning bandwidth
72
73
       // transmitter-portside
74
       transmitter_portside.location
                                                  = location;
                                                                               // Assigning
75
           location
                                                  = Signal;
       transmitter_portside.signal
                                                                               // Assigning
76
           signal
                                                  = azimuthal_angle_port;
                                                                              // assigning
       transmitter_portside.azimuthal_angle
           azimuth angle
       transmitter_portside.elevation_angle = elevation_angle;
                                                                              // assigning
78
           elevation angle
       transmitter_portside.azimuthal_beamwidth = azimuthal_beamwidth_port; // assigning
           azimuth-beamwidth
       transmitter_portside.elevation_beamwidth = elevation_beamwidth_port; // assigning
80
           elevation-beamwidth
       // updating quantization densities
       transmitter_portside.azimuthQuantDensity = azimuthQuantDensity;
                                                                               // assigning
82
           azimuth quant density
       transmitter_portside.elevationQuantDensity = elevationQuantDensity;
                                                                               // assigning
83
           elevation quant density
                                                  = rangeQuantSize;
       transmitter_portside.rangeQuantSize
                                                                               // assigning
84
           range-quantization
       transmitter_portside.azimuthShadowThreshold = azimuthShadowThreshold; //
           azimuth-threshold in shadowing
       transmitter_portside.elevationShadowThreshold = elevationShadowThreshold; //
86
           elevation-threshold in shadowing
       // signal related
87
       transmitter_portside.f_low
                                                  = f1;
                                                                               // assigning
88
           lower frequency
       transmitter_portside.f_high
                                                  = f2;
                                                                               // assigning
           higher frequency
       transmitter_portside.fc
                                                  = fc;
                                                                               // assigning
90
           center frequency
       transmitter_portside.bandwidth
                                                  = bandwidth;
                                                                               // assigning
91
           bandwidth
92
93
       // transmitter-starboard
94
                                                                                   //
       transmitter_starboard.location
                                                     = location;
           assigning location
       transmitter_starboard.signal
                                                     = Signal;
                                                                                   //
           assigning signal
       transmitter_starboard.azimuthal_angle
                                                     = azimuthal_angle_starboard; //
           assigning azimuthal signal
       transmitter_starboard.elevation_angle
                                                     = elevation_angle;
98
       transmitter_starboard.azimuthal_beamwidth
                                                      = azimuthal_beamwidth_starboard;
       transmitter_starboard.elevation_beamwidth
                                                      = elevation_beamwidth_starboard;
100
       // updating quantization densities
101
       transmitter_starboard.azimuthQuantDensity
                                                     = azimuthQuantDensity;
                                                                                    //
           assigning azimuth-quant-density
```

```
transmitter_starboard.elevationQuantDensity
                                                       = elevationQuantDensity;
103
       transmitter_starboard.rangeQuantSize
                                                       = rangeQuantSize;
104
       transmitter_starboard.azimuthShadowThreshold = azimuthShadowThreshold;
105
       transmitter_starboard.elevationShadowThreshold = elevationShadowThreshold;
106
       // signal related
107
       transmitter_starboard.f_low
                                                       = f1;
                                                                                      //
108
           assigning lower frequency
                                                                                      //
       {\tt transmitter\_starboard.f\_high}
                                                       = f2;
109
           assigning higher frequency
       transmitter_starboard.fc
110
                                                       = fc;
           assigning center frequency
       transmitter_starboard.bandwidth
                                                       = bandwidth;
                                                                                      //
111
           assigning bandwidth
112
    }
113
```

Chapter 3

Uniform Linear Array

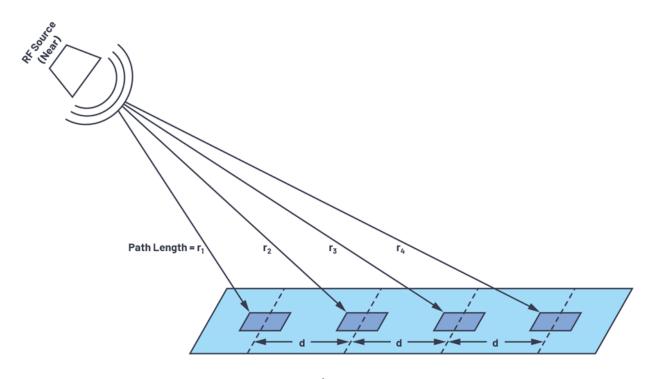


Figure 3.1: Uniform Linear Array

Overview

A Uniform Linear Array (ULA) is a common antenna or sensor configuration in which multiple elements are arranged in a straight line with equal spacing between adjacent elements. This geometry simplifies both the analysis and implementation of array signal processing techniques. In a ULA, each element receives a version of the incoming signal that differs only in phase, depending on the angle of arrival. This phase difference can be exploited to steer the array's beam in a desired direction (beamforming) or to estimate the direction of arrival (DOA) of multiple sources. The equal spacing also leads to a regular phase progression across the elements, which makes the array's response mathematically tractable and allows the use of tools like the discrete Fourier transform (DFT) to analyze spatial frequency content.

The performance of a ULA depends on the number of elements and their spacing. The spacing is typically chosen to be half the wavelength of the signal to avoid spatial aliasing, also called grating lobes, which can introduce ambiguities in DOA estimation. Increasing the number of elements improves the array's angular resolution and directivity, meaning it can better distinguish closely spaced sources and focus energy more narrowly. ULAs are widely used in radar, sonar, wireless communications, and microphone arrays due to their simplicity, predictable behavior, and compatibility with well-established signal processing algorithms. Their linear structure also makes them easier to implement in hardware compared to more complex array geometries like circular or planar arrays.

3.1 ULA Class Definition

The following is the class used to represent the uniform linear array

```
template <typename T>
  class ULAClass
   public:
      // intrinsic parameters
      std::size_t
                                                                                   // number
                                           num_sensors;
          of sensors
                                   inter_element_spacing;
                                                                            // space between
          sensors
      std::vector<std::vector<T>> coordinates;
                                                                            // coordinates
          of each sensor
                                    sampling_frequency;
                                                                            // sampling
9
          frequency of the sensors
                                   recording_period;
                                                                            // recording
10
          period of the ULA
      std::vector<T>
                                                                            // location of
                                    location;
11
          first coordinate
      // derived
      std::vector<T>
                                    sensor_direction;
14
      std::vector<std::vector<T>> signal_matrix;
      // decimation related
17
                                                                    // the new decimation
      int
                        decimation_factor;
18
          factor
                                                                    // the new sampling
                         post_decimation_sampling_frequency;
          frequency
                         lowpass_filter_coefficients_for_decimation; // filter-coefficients
      std::vector<T>
20
          for filtering
21
      // imaging related
22
      T range_resolution;
                                       // theoretical range-resolution = $\frac{c}{2B}$
23
      T azimuthal_resolution;
                                       // theoretical azimuth-resolution =
          $\frac{\lambda}{(N-1)*inter-element-distance}$
      T range_cell_size;
                                       // the range-cell quanta we're choosing for
          efficiency trade-off
         azimuth_cell_size;
                                       // the azimuth quanta we're choosing
26
      std::vector<T> azimuth_centers; // tensor containing the azimuth centeres
      std::vector<T> range_centers; // tensor containing the range-centers
28
                                       // the frame-size corresponding to a range cell in a
      int frame_size;
29
          decimated signal matrix
30
```

```
std::vector<std::vector<complex<T>>> mulFFTMatrix; // the matrix containing the
          delays for each-element as a slot
                                            matchFilter; // torch tensor containing the
      std::vector<complex<T>>
32
          match-filter
      int num_buffer_zeros_per_frame;
                                                          // number of zeros we're adding
33
          per frame to ensure no-rotation
      std::vector<std::vector<T>> beamformedImage;
                                                          // the beamformed image
34
                                                          // the cartesian version of
      std::vector<std::vector<T>> cartesianImage;
35
          beamformed image
36
      // Artificial acoustic-image related
37
      std::vector<std::vector<T>> currentArtificialAcousticImage; // acoustic image
38
          directly produced
39
40
      // Basic Constructor
41
      ULAClass() = default;
42
43
      // constructor
44
      ULAClass(const int
                            num_sensors_arg,
45
               const auto
                           inter_element_spacing_arg,
               const auto& coordinates_arg,
47
               const auto& sampling_frequency_arg,
48
               const auto& recording_period_arg,
               const auto& location_arg,
               const auto& signalMatrix_arg,
51
               const auto& lowpass_filter_coefficients_for_decimation_arg):
52
                 num_sensors(num_sensors_arg),
                  inter_element_spacing(inter_element_spacing_arg),
                  coordinates(std::move(coordinates_arg)),
55
                  sampling_frequency(sampling_frequency_arg),
56
                  recording_period(recording_period_arg),
                  location(std::move(location_arg)),
58
                  signal_matrix(std::move(signalMatrix_arg)),
59
                  lowpass_filter_coefficients_for_decimation(std::move(lowpass_filter_coefficients_for_decimation)
60
      {
61
62
          // calculating ULA direction
63
          sensor_direction = std::vector<T>{coordinates[1][0] - coordinates[0][0],
                                            coordinates[1][1] - coordinates[0][1],
                                            coordinates[1][2] - coordinates[0][2]};
66
67
          // normalizing
68
                                    {std::norm(std::inner_product(sensor_direction.begin(),
          auto norm_value_temp
                                                                sensor_direction.end(),
70
                                                                sensor_direction.begin(),
                                                                0.00));
          // dividing
74
          if (norm_value_temp != 0) {sensor_direction = sensor_direction /
              norm_value_temp;}
      }
77
78
      // // deleting copy constructor/assignment
79
      // ULAClass<T>(const ULAClass<T>& other)
                                                                 = delete;
80
      // ULAClass<T>& operator=(const ULAClass<T>& other)
                                                                 = delete;
81
      ULAClass<T>(ULAClass<T>&& other)
                                                              = delete;
82
      ULAClass<T>& operator=(const ULAClass<T>& other)
                                                              = default;
```

```
84
      // member-functions
85
              buildCoordinatesBasedOnLocation();
      void
86
      void
              buildCoordinatesBasedOnLocation(const std::vector<T>& new_location);
87
      void
              init(const TransmitterClass<T>& transmitterObj);
88
              nfdc_CreateMatchFilter(const TransmitterClass<T>& transmitterObj);
      void
      // void simulate_signals(const ScattererClass<T>& seafloor,
90
      11
                                const std::vector<std::size_t> scatterer_indices,
```

3.2 ULA Setup Scripts

The following script shows the setup-script for Uniform Linear Arrays

```
template
 2
              typename
                                     Τ,
               typename
                                     = std::enable_if_t<
                      std::is_same_v<T, double> ||
                      std::is_same_v<T, float>
 6
      >
      void fULASetup(
 8
              ULAClass<T>&
 9
                                               ula_fls,
              ULAClass<T>&
                                               ula_portside,
10
              ULAClass<T>&
                                               ula_starboard)
      {
              // setting up ula
                             num_sensors
                                                                                    {static_cast<int>(16)};
                                                                                                                                                         // number of sensors
              auto
14
              Т
                              sampling_frequency
                                                                                    {static_cast<T>(160e3)};
                                                                                                                                                         // sampling frequency
                                                                                    {1500/(2*sampling_frequency)};
                                                                                                                                                         // space between
                              inter_element_spacing
16
                       samples
                             recording_period
                                                                                    {10e-2};
                                                                                                                                                         // sampling-period
17
              // building the direction for the sensors
19
                             ULA_direction
                                                                                    {std::vector<T>({-1, 0, 0})};
2.0
                             ULA_direction_norm
                                                                                    {norm(ULA_direction)};
21
22
               if (ULA_direction_norm != 0)
                                                                                    {ULA_direction = ULA_direction/ULA_direction_norm;}
              ULA_direction
                                                                                    ULA_direction * inter_element_spacing;
23
24
              // building coordinates for sensors
25
              auto ULA_coordinates
                                                                                    {transpose(ULA_direction) * \
26
                                                                                      linspace<double>(0.00,
2.7
                                                                                                                      num_sensors -1,
28
                                                                                                                      num_sensors)};
30
               // coefficients of decimation filter
31
                              lowpass filter coefficients \{std::vector < T > \{0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.00000, 0.0000, 0.0000, 0.0000, 0.00000, 0.00000, 0.0000, 0.0000, 0.00000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 
                       0.0000, 0.0000, 0.0001, 0.0003, 0.0006, 0.0015, 0.0030, 0.0057, 0.0100, 0.0163,
                       0.0251, 0.0364, 0.0501, 0.0654, 0.0814, 0.0966, 0.1093, 0.1180, 0.1212, 0.1179,
                       0.1078, 0.0914, 0.0699, 0.0451, 0.0192, -0.0053, -0.0262, -0.0416, -0.0504,
                       -0.0522, -0.0475, -0.0375, -0.0239, -0.0088, 0.0057, 0.0179, 0.0263, 0.0303,
                       0.0298, 0.0253, 0.0177, 0.0086, -0.0008, -0.0091, -0.0153, -0.0187, -0.0191,
                       -0.0168, -0.0123, -0.0065, -0.0004, 0.0052, 0.0095, 0.0119, 0.0125, 0.0112,
                       0.0084, 0.0046, 0.0006, -0.0031, -0.0060, -0.0078, -0.0082, -0.0075, -0.0057,
                      -0.0033, -0.0006, 0.0019, 0.0039, 0.0051, 0.0055, 0.0050, 0.0039, 0.0023, 0.0005,
                       -0.0012, -0.0025, -0.0034, -0.0036, -0.0034, -0.0026, -0.0016, -0.0004, 0.0007,
                       0.0016, 0.0022, 0.0024, 0.0023, 0.0018, 0.0011, 0.0003, -0.0004, -0.0011,
```

```
-0.0015, -0.0016, -0.0015};
33
       // assigning values
34
       ula_fls.num_sensors
                                                        = num_sensors;
                                                                                      11
35
           assigning number of sensors
       ula_fls.inter_element_spacing
                                                        = inter_element_spacing;
           assigning inter-element spacing
       ula_fls.coordinates
                                                        = ULA_coordinates;
                                                                                      //
37
           assigning ULA coordinates
38
       ula_fls.sampling_frequency
                                                        = sampling_frequency;
           assigning sampling frequencys
       ula_fls.recording_period
                                                        = recording_period;
                                                                                      //
39
           assigning recording period
                                                        = ULA_direction;
       ula_fls.sensor_direction
                                                                                      // ULA
40
       ula_fls.lowpass_filter_coefficients_for_decimation = lowpassfiltercoefficients; //
41
           storing coefficients
42
43
       // assigning values
44
       ula_portside.num_sensors
                                                           = num_sensors;
                                                                                          //
45
           assigning number of sensors
       ula_portside.inter_element_spacing
                                                            = inter_element_spacing;
                                                                                          //
46
           assigning inter-element spacing
       ula_portside.coordinates
                                                            = ULA_coordinates;
                                                                                          //
           assigning ULA coordinates
       ula_portside.sampling_frequency
                                                           = sampling_frequency;
                                                                                          //
48
           assigning sampling frequencys
       ula_portside.recording_period
                                                            = recording_period;
                                                                                          //
49
           assigning recording period
      ula_portside.sensor_direction
                                                            = ULA_direction;
                                                                                          //
50
           ULA direction
       ula_portside.lowpass_filter_coefficients_for_decimation = lowpassfiltercoefficients;
           // storing coefficients
52
53
       // assigning values
54
       ula_starboard.num_sensors
                                                                                              //
                                                               = num_sensors;
           assigning number of sensors
       ula_starboard.inter_element_spacing
                                                               = inter_element_spacing;
                                                                                              //
           assigning inter-element spacing
       ula_starboard.coordinates
                                                               = ULA_coordinates;
                                                                                              //
57
           assigning ULA coordinates
       ula_starboard.sampling_frequency
                                                               = sampling_frequency;
                                                                                              //
           assigning sampling frequencys
      ula_starboard.recording_period
                                                               = recording_period;
                                                                                              11
59
           assigning recording period
       ula_starboard.sensor_direction
                                                               = ULA_direction;
                                                                                              //
           ULA direction
       ula_starboard.lowpass_filter_coefficients_for_decimation =
           {\tt lowpassfilter coefficients;}\ //\ {\tt storing}\ {\tt coefficients}
   }
```

Chapter 4

Autonomous Underwater Vehicle

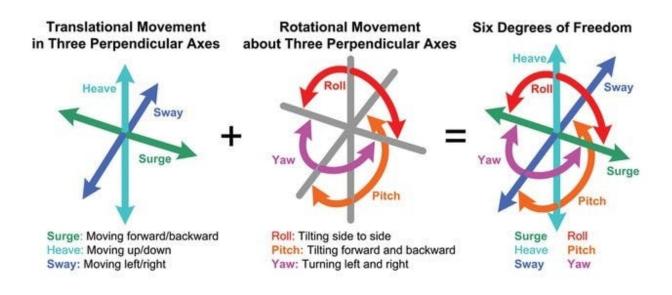


Figure 4.1: AUV degrees of freedom

Overview

Autonomous Underwater Vehicles (AUVs) are robotic systems designed to operate underwater without direct human control. They navigate and perform missions independently using onboard sensors, processors, and preprogrammed instructions. They are widely used in oceanographic research, environmental monitoring, offshore engineering, and military applications. AUVs can vary in size from small, portable vehicles for shallow water surveys to large, torpedo-shaped platforms capable of deep-sea exploration. Their autonomy allows them to access environments that are too dangerous, remote, or impractical for human divers or tethered vehicles.

The navigation and sensing systems of AUVs are critical to their performance. They typically use a combination of inertial measurement units (IMUs), Doppler velocity logs

(DVLs), pressure sensors, magnetometers, and sometimes acoustic positioning systems to estimate their position and orientation underwater. Since GPS signals do not penetrate water, AUVs must rely on these onboard sensors and occasional surfacing for GPS fixes. They are often equipped with sonar systems, cameras, or other scientific instruments to collect data about the seafloor, water column, or underwater structures. Advanced AUVs can also implement adaptive mission planning and obstacle avoidance, enabling them to respond to changes in the environment in real time.

The applications of AUVs are diverse and expanding rapidly. In scientific research, they are used for mapping the seafloor, studying marine life, and monitoring oceanographic parameters such as temperature, salinity, and currents. In the commercial sector, AUVs inspect pipelines, subsea infrastructure, and offshore oil platforms. Military and defense applications include mine countermeasure operations and underwater surveillance. The development of AUVs continues to focus on increasing endurance, improving autonomy, enhancing sensor payloads, and reducing costs, making them a key technology for exploring and understanding the underwater environment efficiently and safely.

4.1 AUV Class Definition

The following is the class used to represent the uniform linear array

```
template <typename T>
2 class AUVClass{
g public:
      // Intrinsic attributes
      std::vector<T> location;
                                             // location of vessel
      std::vector<T>
                       velocity;
                                             // velocity of the vessel
7
      std::vector<T> acceleration;
                                             // acceleration of vessel
8
      std::vector<T> pointing_direction; // AUV's pointing direction
Q
10
      // uniform linear-arrays
11
                                             // front-looking SONAR ULA
      ULAClass<T> ULA_fls;
      ULAClass<T>
                                             // mounted ULA [object of class, ULAClass]
                        ULA_portside;
                                             // mounted ULA [object of class, ULAClass]
      ULAClass<T>
                       ULA_starboard;
      // transmitters
16
      TransmitterClass<T> transmitter_fls;
                                                    // transmitter for front-looking SONAR
17
      TransmitterClass<T> transmitter_portside;
                                                    // portside transmitter
      TransmitterClass<T> transmitter_starboard; // starboard transmitter
19
2.0
      // derived or dependent attributes
21
                                                             // matrix containing the
      std::vector<std::vector<T>> signalMatrix_1;
          signals obtained from ULA_1
      std::vector<std::vector<T>> largeSignalMatrix_1;
                                                             // matrix holding signal of
          synthetic aperture
      std::vector<std::vector<T>> beamformedLargeSignalMatrix; // each column is the
          beamformed signal at each stop-hop
25
      // plotting mode
26
      bool plottingmode; // to suppress plotting associated with classes
27
28
      // spotlight mode related
29
      std::vector<std::vector<T>> absolute_coords_patch_cart; // cartesian coordinates of
```

```
patch
31
       // Synthetic Aperture Related
32
       std::vector<std::vector<T>> ApertureSensorLocations;
                                                                   // sensor locations of
33
           aperture
       // functions
35
       void syncComponentAttributes();
36
       void init(svr::ThreadPool& thread_pool);
       void simulate_signal(
38
          const
                   ScattererClass<T>&
                                                                seafloor,
39
          svr::ThreadPool&
                                                               thread_pool,
40
          svr::FFTPlanUniformPoolHandle<T, std::complex<T>>& fft_pool_handle,
41
          svr::IFFTPlanUniformPoolHandle<std::complex<T>, T>& ifft_pool_handle
42
43
       void subset_scatterers(
          const ScattererClass<T>& seafloor,
          svr::ThreadPool&
                                      thread_pool,
46
          std::vector<std::size_t>& fls_scatterer_indices,
47
          std::vector<std::size_t>& portside_scatterer_indices,
48
          std::vector<std::size_t>& starboard_scatterer_indices
49
50
       void step(T time_step);
51
```

4.2 AUV Setup Scripts

The following script shows the setup-script for Uniform Linear Arrays

```
template <
                 Τ,
      typename
                 =
                     std::enable_if_t<
      typename
          std::is_same_v<T, double> ||
          std::is_same_v<T, float>
8
   void fAUVSetup(AUVClass<T>& auv) {
9
      // building properties for the auv
10
                                {std::vector<T>{0, 50, 30}}; // starting location
      auto location
11
                                {std::vector<T>{5, 0, 0}}; // starting velocity
      auto
              velocity
              pointing_direction {std::vector<T>{1, 0, 0}}; // pointing direction
      auto
      // assigning
      auv.location
                            = std::move(location);
                                                              // assigning location
16
                            = std::move(velocity);
                                                              // assigning velocity
      auv.velocity
      auv.pointing_direction = std::move(pointing_direction); // assigning pointing
          direction
19
      // signaling end
20
      std::cout << format("> Completed AUV-setup\n");
21
22
23
   }
```

Part II Signal Simulation Pipeline

Chapter 5

Signal Simulation

Overview

The signal simulation pipeline is the pipeline responsible for simulating/modeling the signals sampled by the ULA-sensors under a real sub-marine environment. This chapter, and the subsequent ones, deal with the assumptions, mathematics, physics and code that goes into the design and creation of the pipeline.

A disclaimer that goes without saying is that signal-simulation is a world of its own. There's a reason that comsol, flexcompute and other numerical-simulation based companies exist. To write a signal simulation, from scratch, while these entities exist, and to make any case that this competes with those, would be flirting with delusion.

To that end, we don't write general-purpose signal simulation pipeline. However, the effort in the signal-simulator direction is purely for application-specific reasons. This is something I can talk about. One of the major in-house signal simulation pipelines yours truly developed at Naval Physical and Oceanographic Laboratory did just that. The aim of that pipeline was not to re-invent the wheel. But to create one that existed at the right speed-fidelity trade-off that the institute operated in. The pipeline created during my time there had several toggles corresponding to the different information to consider during simulation. The more information pertaining to the environment, is involved, the higher the compute and time required. Thus, mid-to-high fidelity pipelines often involve writing well-tuned GPU-supported C++ (think, CUDA). And this is important when you have pipelines downstream whose outputs depend on the signal accuracy, and by association, signal simulator fidelity.

To that end, understanding what this pipeline is not, is perhaps just as important as what it is. The core priority of this signal simulator pipeline is to produce signals for navigation. Navigation does not require high-accuracy signals owing to the very simple fact that decisions made from high-accuracy signals and low-accuracy signals tend to be the same as long as environment-topology information is not lost. To grossly oversimplify what I mean by that, the outcome of your driving does't change whether you have high-definition LI-DAR mapping the surrounding environment to the millimeter level or if you're just driving with your eyes. Thus fidelity of simulator is not a priority and I will not be putting in the kind of effort I put in at NPOL, for this reason (also because I don't want OPSEC to be

mad).

To put it simply, the signal simulation pipeline is quite trivial as far as signal simulators are concerned. But it'll work perfectly for our purposes. And thus, we'll be choosing the simplest of systems and one I like to call, "the EE engineer's best friend": the infamous Linear Time Invariant systems.

Part III Imaging Pipeline

Part IV Perception & Control Pipeline

Appendix A

Application Specific Tools

A.1 CSV File-Writes

```
#pragma once
  /*-----
  writing the contents of a vector a csv-file
  template <typename T>
                                        inputvector,
  void fWriteVector(const vector<T>&
                  const string&
                                            filename){
      // opening a file
      std::ofstream fileobj(filename);
      if (!fileobj) {return;}
11
      // writing the real parts in the first column and the imaginary parts int he second
      if constexpr(std::is_same_v<T, std::complex<double>> ||
14
                 std::is_same_v<T, std::complex<float>> ||
                 std::is_same_v<T, std::complex<long double>>){
          for(int i = 0; i<inputvector.size(); ++i){</pre>
17
             // adding entry
             fileobj << inputvector[i].real() << "+" << inputvector[i].imag() << "i";</pre>
             // adding delimiter
             if(i!=inputvector.size()-1) {fileobj << ",";}</pre>
                                      {fileobj << "\n";}
             else
      }
      else{
          for(int i = 0; i<inputvector.size(); ++i){</pre>
             fileobj << inputvector[i];</pre>
             if(i!=inputvector.size()-1) {fileobj << ",";}</pre>
29
                                      {fileobj << "\n";}
30
          }
      // return
34
35
      return;
```

```
writing the contents of a matrix to a csv-file
   template <typename T>
40
   auto fWriteMatrix(const std::vector<std::vector<T>> inputMatrix,
41
                  const string
                                                filename){
42
      // opening a file
44
      std::ofstream fileobj(filename);
      // writing
      if (fileobj){
48
          for(int i = 0; i<inputMatrix.size(); ++i){</pre>
49
             for(int j = 0; j<inputMatrix[0].size(); ++j){</pre>
                 fileobj << inputMatrix[i][j];</pre>
51
                 if (j!=inputMatrix[0].size()-1) {fileobj << ",";}</pre>
                                                {fileobj << "\n";}
                 else
             }
          }
55
      }
56
      else{
          cout << format("File-write to {} failed\n", filename);</pre>
59
60
  }
61
   /*-----
   writing complex-matrix to a csv-file
63
64
  template <>
   auto fWriteMatrix(const std::vector<std::vector<std::complex<double>>> inputMatrix,
67
                  const string
68
      // opening a file
69
      std::ofstream fileobj(filename);
70
71
      // writing
72
      if (fileobj){
          for(int i = 0; i<inputMatrix.size(); ++i){</pre>
             for(int j = 0; j<inputMatrix[0].size(); ++j){</pre>
                 if (j!=inputMatrix[0].size()-1) {fileobj << ",";}</pre>
                                                {fileobj << "\n";}
                 else
78
             }
79
          }
      }
81
      else{
82
          cout << format("File-write to {} failed\n", filename);</pre>
83
84
   }
85
```

A.2 Thread-Pool

```
#pragma once
namespace svr {
class ThreadPool {
public:
```

```
// Members
        boost::asio::thread_pool
                                thread_pool;
                                            // the pool
        std::vector<std::future<void>> future_vector; // futures to wait on
        // Special-Members
        ThreadPool(std::size_t num_threads) : thread_pool(num_threads) {}
        ThreadPool(const ThreadPool& other)
                                        = delete;
11
        ThreadPool& operator=(ThreadPool& other) = delete;
        // Member-functions
        void
                             converge();
        template <typename F> void push_back(F&& func);
                             shutdown();
18
     private:
19
        template<typename F>
20
        std::future<void> _wrap_task(F&& func) {
21
           std::promise<void> p;
22
23
           auto f = p.get_future();
           boost::asio::post(thread_pool,
              [func = std::forward<F>(func), p = std::move(p)]() mutable {
                 func();
                 p.set_value();
              });
30
           return f;
31
        }
     };
33
34
     /*-----
35
     Member-Function: Add new task to the pool
36
     -----*/
37
     template <typename F>
38
     void ThreadPool::push_back(F&& func)
30
     {
        future_vector.push_back(_wrap_task(std::forward<F>(func)));
41
42
     /*-----
     Member-Function: waiting until all the assigned work is done
45
     void ThreadPool::converge()
46
47
        for (auto &fut : future_vector) fut.get();
        future_vector.clear();
49
50
     /*----
     Member-Function: Shutting things down
     void ThreadPool::shutdown()
54
55
        thread_pool.join();
57
58
  }
```

A.3 FFTPlanClass

```
#pragma once
  namespace svr
     template
              <typename sourceType,
                       destinationType,
               typename
               typename = std::enable_if_t<std::is_same_v<sourceType, double> &&
                                    std::is_same_v<destinationType,</pre>
                                        std::complex<double>>
10
     class FFTPlanClass
11
        public:
14
           // Members
           std::size_t nfft_
                               {std::numeric_limits<std::size_t>::max()};
           fftw_complex* in_
                               {nullptr};
17
           fftw_complex* out_
                               {nullptr};
18
                              {nullptr};
           fftw_plan
19
                      plan_
           /*----
21
           Destructor
22
           -----*/
           ~FFTPlanClass()
              if(plan_ != nullptr) {fftw_destroy_plan(
                                                  plan_);}
26
              if(in_ != nullptr) {fftw_free(
                                                  in_);}
              if(out_ != nullptr) {fftw_free(
                                                  out_);}
29
           /*-----
30
           Default Constructor
31
           -----*/
32
           FFTPlanClass() = default;
33
           /*-----
           {\tt Constructor}
36
           FFTPlanClass(const std::size_t nfft)
37
38
              // allocating nfft
40
              this->nfft_ = nfft;
              // allocating input-region
              in_ = reinterpret_cast<fftw_complex*>(
                 fftw_malloc(nfft_ * sizeof(fftw_complex))
45
              );
46
                  = reinterpret_cast<fftw_complex*>(
              out_
                 fftw_malloc(nfft_ * sizeof(fftw_complex))
48
49
              if(!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
                 CLASS: FFTPlanClass | REPORT: in-out allocation failed");}
51
              // creating plan
52
              plan_ = fftw_plan_dft_1d(
                 static_cast<int>(nfft_),
                 in_,
55
                 out_,
                 FFTW_FORWARD,
```

```
FFTW_MEASURE
                                {throw std::runtime_error("FILE: FFTPlanClass.hpp |
                if(!plan_)
60
                   CLASS: FFTPlanClass | REPORT: plan-creation failed");}
             }
61
             /*-----
             Copy Constructor
63
             FFTPlanClass(const FFTPlanClass& other)
                // copying nfft
67
                nfft_ = other.nfft_;
68
                cout << format("\t\t FFTPlanClass(const FFTPlanClass& other) | nfft_ =</pre>
                   {}\n", nfft_);
70
                // allocating input-region
71
                in_ = reinterpret_cast<fftw_complex*>(
                   fftw_malloc(nfft_ * sizeof(fftw_complex))
73
74
                );
                out_ = reinterpret_cast<fftw_complex*>(
75
                   fftw_malloc(nfft_ * sizeof(fftw_complex))
                if(!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
78
                    CLASS: FFTPlanClass | REPORT: in-out allocation failed");}
                // copying input-region and output-region
80
                std::memcpy(in_, other.in_, nfft_ * sizeof(fftw_complex));
81
                std::memcpy(out_, other.out_, nfft_ * sizeof(fftw_complex));
                // creating plan
84
                plan_ = fftw_plan_dft_1d(
85
                   static_cast<int>(nfft_),
                   in_,
                   out_,
88
                   FFTW_FORWARD,
89
                   FFTW_MEASURE
                );
91
                                {throw std::runtime_error("FILE: FFTPlanClass.hpp |
                if(!plan_)
                    CLASS: FFTPlanClass | REPORT: plan-creation failed");}
             }
             /*----
94
             Copy Assignment
95
             -----*/
96
             FFTPlanClass& operator=(const FFTPlanClass& other)
97
98
                // handling self-assignment
99
                if (this == &other) {return *this;}
100
                // cleaning-up existing resources
102
                if(plan_ != nullptr) {fftw_destroy_plan( plan_);}
103
104
                if(in_ != nullptr) {fftw_free(
                                                       in_);}
                if(out_ != nullptr) {fftw_free(
                                                        out_);}
106
                // allocating input-region and output-region
107
                nfft_ = other.nfft_;
                      = reinterpret_cast<fftw_complex*>(
109
                   fftw_malloc(nfft_ * sizeof(fftw_complex))
                );
111
                out_ = reinterpret_cast<fftw_complex*>(
```

```
fftw_malloc(nfft_ * sizeof(fftw_complex))
113
                );
114
                if(!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
                   CLASS: FFTPlanClass | FUNCTION: Copy-Assignment | REPORT: in-out
                   allocation failed");}
                // copying contents
117
                cout << format("\t\t FFTPlanClass& operator=(const FFTPlanClass& other) |</pre>
118
                   nfft_ = {} \n", nfft_);
119
                std::memcpy(in_, other.in_, nfft_ * sizeof(fftw_complex));
                std::memcpy(out_, other.out_, nfft_ * sizeof(fftw_complex));
120
121
                // creating engine
                plan_ = fftw_plan_dft_1d(
123
                   static_cast<int>(nfft_),
124
                   in_,
                   out_,
126
                   FFTW_FORWARD,
127
                   FFTW_MEASURE
128
                );
129
                if(!plan_) {throw std::runtime_error("FILE: FFTPlanClass.hpp | CLASS:
130
                   FFTPlanClass | FUNCTION: Copy-Assignment | REPORT: plan-creation
                   failed");}
131
                // returning
                return *this;
133
             }
134
             /*-----
135
             Move Constructor
136
             -----*/
137
             FFTPlanClass(FFTPlanClass&& other)
138
                : nfft_(
                                other.nfft_),
139
                   in_(
                                   other.in_),
140
                   \operatorname{out}_{-}(
                                   other.out_),
141
                   plan_(
                                   other.plan_)
142
             {
                // resetting the others
144
                other.nfft_ = 0;
145
                               = nullptr;
                other.in_
146
                                = nullptr;
                other.out_
147
                other.plan_
                                = nullptr;
148
             }
149
             /*-----
150
             Move Assignment
151
             ----*/
152
             FFTPlanClass& operator=(FFTPlanClass&& other)
154
                // self-assignment check
155
                if (this == &other) {return *this;}
156
158
                // cleaning up existing resources
159
                if(plan_ != nullptr) {fftw_destroy_plan(
                                                        plan_);}
                if(in_ != nullptr) {fftw_free(
                                                        in_);}
160
                if(out_ != nullptr) {fftw_free(
                                                        out_);}
161
                // Copying-values and changing pointers
163
                nfft_ = other.nfft_;
164
                cout << format("\t\t FFTPlanClass's MOVE assignment | nfft_ = {}\n",</pre>
165
                   nfft_);
```

```
in_
                                         other.in_;
166
                   \operatorname{out}_-
                                         other.out_;
167
                   plan_
                                         other.plan_;
168
169
                   // resetting source-members
170
                   other.nfft_ = 0;
171
                   other.in_
                                     = nullptr;
172
                                    = nullptr;
                   other.out_
173
                                     = nullptr;
                   other.plan_
175
                   // returning
176
                  return *this;
177
               }
178
               179
               Running fft
180
181
               std::vector<destinationType>
182
               fft(const std::vector<sourceType>& input_vector)
183
184
                   // throwing an error
185
                   if (input_vector.size() > nfft_){
186
                       cout << format("input_vector.size() = {}, nfft_ = {}\n",</pre>
187
                                    input_vector.size(),
188
                                     nfft_);
189
                      throw std::runtime_error("FILE: FFTPlanClass.hpp | CLASS: FFTPlanClass
                          | FUNCTION: fft() | REPORT: input-vector size is greater than
                          NFFT");
                   }
191
192
193
                   // copying inputs
194
                   for(std::size_t index = 0; index < input_vector.size(); ++index)</pre>
195
196
                      if constexpr(
197
                          std::is_same_v< sourceType, double</pre>
198
                      ){
                          in_[index][0] = input_vector[index];
200
                          in_{in}[index][1] = 0;
201
                      }
202
203
                      else if constexpr(
                          std::is_same_v<
                                             sourceType, std::complex<double> >
204
205
                          in_[index][0] =
                                             input_vector[index].real();
206
                          in_[index][1] =
                                             input_vector[index].imag();
                      }
208
                   }
2.09
210
                   // executing fft
211
                   fftw_execute(plan_);
212
213
                   // copying results to output-vector
214
215
                   std::vector<destinationType> output_vector(nfft_);
                   for(std::size_t index = 0; index < nfft_; ++index){</pre>
216
                      if constexpr(
217
                          std::is_same_v< destinationType,std::complex<double> >
                      ){
219
                          output_vector[index] = std::complex<double>(
220
                              out_[index][0],
221
                              out_[index][1]
222
```

```
);
223
                       }
224
                       else if constexpr(
                           std::is_same_v<
                                              destinationType, double
226
                       ){
227
                           output_vector[index] = std::sqrt(
                              std::pow(out_[index][0], 2) + \
229
                               std::pow(out_[index][1], 2)
230
                           );
231
                       }
232
                   }
233
234
                   // returning output
235
                   return std::move(output_vector);
236
               }
               /*===========
238
               Running fft - balanced
239
240
               std::vector<destinationType>
241
               fft_12_conserved(const std::vector<sourceType>& input_vector)
242
243
                   // throwing an error
244
                   if (input_vector.size() > nfft_)
245
                       throw std::runtime_error("FILE: FFTPlanClass.hpp | CLASS: FFTPlanClass
246
                           | FUNCTION: fft() | REPORT: input-vector size is greater than
                           NFFT");
247
                   // copying inputs
248
                   for(std::size_t index = 0; index < input_vector.size(); ++index)</pre>
249
250
                       if constexpr(
251
                                              sourceType, double
                           std::is_same_v<
                       ){
253
                           in_[index][0] =
                                              input_vector[index];
254
                           in_[index][1] =
                                              0;
255
                       }
                       else if constexpr(
257
                           std::is_same_v<
                                              sourceType, std::complex<double> >
258
                       ){
260
                           in_[index][0] =
                                              input_vector[index].real();
                           in_[index][1] =
                                              input_vector[index].imag();
261
                       }
262
                   }
263
                   // executing fft
265
                   fftw_execute(plan_);
266
267
                   // copying results to output-vector
268
                   std::vector<destinationType> output_vector(nfft_);
269
                   for(std::size_t index = 0; index < nfft_; ++index)</pre>
270
271
                   {
272
                       if constexpr(
                           std::is_same_v< destinationType,std::complex<double> >
273
                       ){
274
                           output_vector[index] = std::complex<double>(
                               out_[index][0] * (1.00 / std::sqrt(nfft_)),
276
                               out_[index][1] * (1.00 / std::sqrt(nfft_))
                           );
278
                       }
279
```

```
else if constexpr(
280
                           std::is_same_v<
                                             destinationType, double
281
                       ){
282
                           output_vector[index] = std::sqrt(
2.83
                               std::pow(out_[index][0] * (1.00 / std::sqrt(nfft_)), 2) + \
284
                               std::pow(out_[index][1] * (1.00 / std::sqrt(nfft_)), 2)
                           );
286
                       }
287
                   }
289
                   // returning output
290
                   return std::move(output_vector);
291
               }
292
        };
293
    }
294
```

A.4 IFFTPlanClass

```
#pragma once
   namespace svr
                    {
      template
                <typename sourceType,
                 typename destinationType,
                 typename = std::enable_if_t<std::is_same_v<sourceType,</pre>
                     std::complex<double>> &&
                                           std::is_same_v<destinationType, double>
6
      class IFFTPlanClass
      {
10
         public:
11
             std::size_t
                              nfft_;
             fftw_complex*
                              in_;
             fftw_complex*
                              out_;
14
             fftw_plan
                              plan_;
17
             Destructor
             ~IFFTPlanClass()
20
21
                if(plan_ != nullptr) {fftw_destroy_plan(
                                                          plan_);}
22
                if(in_ != nullptr) {fftw_free(
                                                          in_);}
                if(out_ != nullptr) {fftw_free(
                                                          out_);}
26
             Constructor
             -----*/
28
             IFFTPlanClass(const std::size_t nfft): nfft_(nfft)
29
30
                // allocating space
                     = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
                    sizeof(fftw_complex)));
                      = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
33
                    sizeof(fftw_complex)));
                if(!in_ || !out_)
                                  {throw std::runtime_error("in_, out_ creation
                    failed");}
```

```
35
               // creating plan
36
               plan_ = fftw_plan_dft_1d(
37
                  static_cast<int>(nfft_),
38
39
                  in_,
                  out_,
                  FFTW_BACKWARD,
41
                  FFTW_MEASURE
               );
               if(!plan_)
                                {throw std::runtime_error("File: FFTPlanClass.hpp |
                  Class: IFFTPlanClass | report: plan-creation failed");}
            }
45
            /*----
46
            Copy Constructor
47
                -----*/
48
            IFFTPlanClass(const IFFTPlanClass& other)
49
               // allocating space
51
               nfft_ = other.nfft_;
52
                     = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
                  sizeof(fftw_complex)));
               out_ = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
                  sizeof(fftw_complex)));
               if (!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
                  Class: IFFTPlanClass | Function: Copy-Constructor | Report: in-out
                  plan creation failed");}
56
               // copying contents
               std::memcpy(in_, other.in_, nfft_ * sizeof(fftw_complex));
               std::memcpy(out_, other.out_, nfft_ * sizeof(fftw_complex));
59
60
               // creating a new plan since its more of an engine
               plan_ = fftw_plan_dft_1d(
                  static_cast<int>(nfft_),
63
64
                  in_,
                  out_,
                  FFTW_BACKWARD,
66
                  FFTW_MEASURE
67
               );
               if(!plan_) {throw std::runtime_error("FILE: FFTPlanClass.hpp | Class:
                  IFFTPlanClass | Function: Copy-Constructor | Report: plan-creation
                  failed");}
70
            }
71
            /*-----
72
            Copy Assignment
73
            -----*/
            IFFTPlanClass& operator=(const IFFTPlanClass& other)
76
               // handling self-assignment
77
               if(this == &other) {return *this;}
               // cleaning up existing resources
80
               if(plan_ != nullptr) {fftw_destroy_plan( plan_);}
81
               if(in_ != nullptr) {fftw_free(
                                                     in_);}
               if(out_ != nullptr) {fftw_free(
                                                      out_);}
83
84
               // allocating space
85
               nfft_ = other.nfft_;
```

```
= reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
                in_
                   sizeof(fftw_complex)));
                out_ = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
88
                   sizeof(fftw_complex)));
                if (!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
                   Class: IFFTPlanClass | Function: Copy-Constructor | Report: in-out
                   plan creation failed");}
90
                // copying contents
                std::memcpy(in_, other.in_, nfft_ * sizeof(fftw_complex));
                std::memcpy(out_, other.out_, nfft_ * sizeof(fftw_complex));
93
94
                // creating a new plan since its more of an engine
95
                plan_ = fftw_plan_dft_1d(
96
                   static_cast<int>(nfft_),
97
                   in_,
                   out_,
                   FFTW_BACKWARD,
100
                   FFTW_MEASURE
101
                );
                if(!plan_) {throw std::runtime_error("FILE: FFTPlanClass.hpp | Class:
103
                   IFFTPlanClass | Function: Copy-Constructor | Report: plan-creation
                   failed");}
104
                // returning
                return *this;
106
107
            }
108
             /*-----
109
110
            Move Constructor
             -----*/
111
             IFFTPlanClass(IFFTPlanClass&& other) noexcept
112
                : nfft_( other.nfft_),
113
                   in_( other.in_),
114
                   out_( other.out_),
                   plan_( other.plan_)
             {
117
                // resetting the source object
118
                other.nfft_ = 0;
119
                            = nullptr;
120
                other.in_
121
                other.out_
                            = nullptr;
                other.plan_
                           = nullptr;
122
            }
123
             /*-----
124
            Move-Assignment
125
             -----*/
126
            IFFTPlanClass& operator=(IFFTPlanClass&& other) noexcept
127
128
                // self-assignment check
129
                if(this == &other) {return *this;}
130
131
132
                // cleaning up existing
                if(plan_ != nullptr) {fftw_destroy_plan(
                                                       plan_);}
133
                if(in_ != nullptr) {fftw_free(
                                                       in_);}
134
                if(out_ != nullptr) {fftw_free(
                                                       out_);}
135
136
                // Copying values and changing pointers
                nfft_ = other.nfft_;
138
                in_
                         = other.in_;
139
```

```
out_
                                other.out_;
140
                               other.plan_;
141
                  plan_
142
                  // resetting the source-object
143
                  other.nfft_ = 0;
144
                  other.in_
                               = nullptr;
145
                              = nullptr;
                  other.out_
146
                  other.plan_ = nullptr;
147
                  // returning
149
                  return *this;
150
              }
151
              /*----
153
                         -----*/
154
              std::vector<destinationType>
              ifft(const std::vector<sourceType>& input_vector)
156
157
158
                  // throwing an error
                  if (input_vector.size() > nfft_)
159
                     throw std::runtime_error("File: FFTPlanClass | Class: IFFTPlanClass |
160
                         Function: ifft() | Report: size of vector > nfft ");
161
                  // copy input into fftw buffer
162
                  for(std::size_t index = 0; index < nfft_; ++index)</pre>
164
                     if constexpr(
165
                                           sourceType, std::complex<double> >
                         std::is_same_v<
166
                     ){
167
                                           input_vector[index].real();
                         in_[index][0] =
168
                         in_[index][1] =
                                           input_vector[index].imag();
169
                     }
170
                     else if constexpr(
171
                                           sourceType, double
                         std::is_same_v<
172
                     ){
173
                         in_[index][0] =
                                           input_vector[index];
174
                         in_[index][1] =
175
                     }
176
                  }
177
178
                  // execute ifft
179
                  fftw_execute(plan_);
180
181
                  // normalize output
                  std::vector<destinationType> output_vector(nfft_);
183
                  for(std::size_t index = 0; index < nfft_; ++index){</pre>
184
                     if constexpr(
185
                         std::is_same_v< destinationType, double</pre>
                                                                           >
186
                     ){
187
                         output_vector[index] = out_[index][0]/nfft_;
188
189
190
                     else if constexpr(
                         std::is_same_v<
                                          destinationType, std::complex<double> >
191
                     ){
192
                         output_vector[index][0] = std::complex<double>(
193
                            out_[index][0]/nfft_,
194
                            out_[index][1]/nfft_
195
                         );
196
                     }
197
```

```
}
198
199
                  // returning
200
                  return std::move(output_vector);
2.01
              }
202
               /*----
203
              Running - proper bases change
204
                                            -----*/
205
               std::vector<destinationType>
              ifft_12_conserved(const std::vector<sourceType>& input_vector)
207
208
                  // throwing an error
209
                  if (input_vector.size() > nfft_)
210
                      throw std::runtime_error("File: FFTPlanClass | Class: IFFTPlanClass |
211
                          Function: ifft() | Report: size of vector > nfft ");
212
                  // copy input into fftw buffer
213
                  for(std::size_t index = 0; index < nfft_; ++index)</pre>
214
                  {
215
                      if constexpr(
216
                         std::is_same_v<
                                            sourceType, std::complex<double> >
217
218
                         in_[index][0] = input_vector[index].real();
219
                         in_[index][1] =
                                           input_vector[index].imag();
                      }
                      else if constexpr(
222
                         std::is_same_v<
                                            sourceType, double
223
                      ){
224
                         in_[index][0] =
                                           input_vector[index];
225
                         in_{in}[index][1] = 0;
226
                      }
227
                  }
229
                  // execute ifft
230
                  fftw_execute(plan_);
231
                  // normalize output
233
                  std::vector<destinationType> output_vector(nfft_);
234
                  for(std::size_t index = 0; index < nfft_; ++index)</pre>
235
236
                      if constexpr(
237
                         std::is_same_v<
                                           destinationType, double
238
                      ){
239
                         output_vector[index] = out_[index][0] * 1.00/std::sqrt(nfft_);
241
                      else if constexpr(
2.42
                         std::is_same_v<
                                           destinationType, std::complex<double> >
243
                      ){
                         output_vector[index][0] = std::complex<double>(
245
                             out_[index][0] * 1.00/std::sqrt(nfft_),
246
247
                             out_[index][1] * 1.00/std::sqrt(nfft_)
                         );
                      }
249
                  }
250
251
                  // returning
252
                  return std::move(output_vector);
253
              }
254
       };
```

```
256 }
```

A.5 FFT Plan Pool

```
#pragma once
  namespace svr
    template <
       typename sourceType,
       typename destinationType,
       typename = std::enable_if_t<</pre>
          std::is_same_v<sourceType, double> &&
          std::is_same_v<destinationType, std::complex<double>>
11
    class FFTPlanUniformPool {
    public:
       Handle to Plan
       -----*/
       struct AccessPairs
          20
          -----*/
21
          svr::FFTPlanClass<sourceType, destinationType>& plan;
          std::unique_lock<std::mutex>
          /*-----
          Special Members
                         -----*/
          AccessPairs() = delete;
          AccessPairs(
            svr::FFTPlanClass<sourceType, destinationType>& plan_arg,
            std::mutex&
                                            plan_mutex
           : plan(plan_arg), lock(plan_mutex) {}
          AccessPairs(
            svr::FFTPlanClass<sourceType, destinationType>& plan_arg,
            std::unique_lock<std::mutex>&&
                                               lock_arg
          ): plan(plan_arg), lock(std::move(lock_arg)) {}
36
          AccessPairs(const AccessPairs& other)
                                                delete;
37
          AccessPairs& operator=(const AccessPairs& other) = delete;
          AccessPairs(AccessPairs&& other)
                                                delete;
          AccessPairs& operator=(AccessPairs&& other)
                                             = delete;
40
       };
41
       /*-----
       std::vector<svr::FFTPlanClass<sourceType, destinationType>> plans;
       std::vector<std::mutex>
       /*-----
       Special-Members
       FFTPlanUniformPool()
                                                 = default;
```

```
FFTPlanUniformPool(const std::size_t
                                                   num_plans,
                            const
                                    std::size_t
                                                   nfft)
          {
55
              // reserving space
56
              plans.reserve(num_plans);
              for(auto i = 0; i < num_plans; ++i){</pre>
                  plans.emplace_back(nfft);
              // creating a vector of mutexes
              mutexes = std::move(std::vector<std::mutex>(num_plans));
63
          FFTPlanUniformPool(const FFTPlanUniformPool& other)
                                                                  = delete;
          FFTPlanUniformPool& operator=(const FFTPlanUniformPool& other) = delete;
          FFTPlanUniformPool(FFTPlanUniformPool&& other)
67
          FFTPlanUniformPool& operator=(FFTPlanUniformPool&& other) = default;
70
71
          Function to fetch a plan
              > searches for a free-plan
              > if found, locks the plan
                 > return the handle to the plan
          AccessPairs fetch_plan() {
              const int num_rounds = 12;
              for (int round = 0; round < num_rounds; ++round) {</pre>
78
                  for (int i = 0; i < mutexes.size(); ++i) {</pre>
79
                      std::unique_lock<std::mutex> curr_lock(
81
                         mutexes[i],
82
                         std::try_to_lock
                     );
                     if (curr_lock.owns_lock())
                         return AccessPairs(plans[i], std::move(curr_lock));
86
                  }
              }
              throw std::runtime_error(
                  "FILE: FFTPlanPoolClass.hpp | CLASS: FFTPlanUniformPool | FUNCTION:
                      fetch_plan() | "
                  "Report: No plans available despite num_rounds rounds of searching");
92
93
          }
      };
94
  }
```

A.6 IFFT Plan Pool

```
typename sourceType,
10
       {\tt typename} \quad {\tt destinationType},
11
       typename = std::enable_if_t<</pre>
          std::is_same_v<sourceType, std::complex<double>>&&
          std::is_same_v<destinationType, double>
16
     class IFFTPlanUniformPool
17
       public:
19
          /*-----
20
          Structure used for interfacing to plans
21
          -----*/
22
          struct AccessPairs
23
24
             25
27
             svr::IFFTPlanClass<sourceType, destinationType>& plan;
28
             std::unique_lock<std::mutex>
                                                   lock;
29
             /*-----
31
             Special Members
32
             -----*/
             AccessPairs()
                                                   = delete;
             AccessPairs(
35
               svr::IFFTPlanClass<sourceType, destinationType>& plan_arg,
36
               std::mutex&
                                                   plan_mutex_arg
37
             ): plan(plan_arg), lock(plan_mutex_arg) {}
             AccessPairs(
39
                svr::IFFTPlanClass<sourceType, destinationType>& plan_arg,
                std::unique_lock<std::mutex>&&
                                                   lock_arg
             ): plan(plan_arg), lock(std::move(lock_arg)) {}
42
             AccessPairs(const AccessPairs& other)
                                                   = delete;
43
             AccessPairs& operator=(const AccessPairs& other) = delete;
             AccessPairs(AccessPairs&& other)
                                                  = delete;
             AccessPairs& operator=(AccessPairs&& other)
                                                  = delete;
          };
          /*----
          Core Members
50
          -----*/
51
          std::vector< svr::IFFTPlanClass<sourceType, destinationType> > plans;
52
          std::vector< std::mutex
                                                      > mutexes;
54
          /*-----
55
          Special Members
          -----*/
                                           = default;
          IFFTPlanUniformPool()
58
          IFFTPlanUniformPool(const std::size_t num_plans,
59
                    const std::size_t nfft)
          {
             // reserving space
62
             plans.reserve(num_plans);
63
             for(auto i = 0; i < num_plans; ++i)</pre>
                plans.emplace_back(nfft);
66
             // creating vector of mutexes
67
             mutexes = std::vector<std::mutex>(num_plans);
```

```
IFFTPlanUniformPool(const IFFTPlanUniformPool& other)
70
             IFFTPlanUniformPool& operator=(const IFFTPlanUniformPool& other) = delete;
71
             IFFTPlanUniformPool(IFFTPlanUniformPool&& other)
                                                                             default;
             IFFTPlanUniformPool& operator=(IFFTPlanUniformPool&& other)
                                                                            default;
             /*-----
             Member-Functions
             -----*/
             AccessPairs fetch_plan()
78
79
                // setting the number of rounds to take
80
                const int num_rounds {12};
                // performing rounds
83
                for(auto round = 0; round < num_rounds; ++round)</pre>
                    // going through vector mutexes
86
                    for(auto i =0; i < mutexes.size(); ++i)</pre>
                    {
                       // trying to lock current mutex
                       std::unique_lock<std::mutex> curr_lock(mutexes[i],
                           std::try_to_lock);
                       // if our lock contains the mutex, returning the plan and lock
                       if (curr_lock.owns_lock())
93
                          return AccessPairs(plans[i], std::move(curr_lock));
94
                    }
                }
97
                // throwing error
                throw std::runtime_error("FILE: IFFTPlanPoolClass.hpp | CLASS:
                    IFFTPlanUniformPool | REPORT: COULDN'T FIND ANY AVAILABLE PLANS");
             }
100
      };
101
   }
```

A.7 FFT Plan Pool Handle

```
#pragma once
  /*-----
  Dependencies
  -----*/
  #include "FFTPlanPoolClass.hpp"
  namespace svr
8
9
  {
    template <
11
      typename
             sourceType,
      typename destinationType,
      typename = std::enable_if_t<</pre>
         std::is_same_v< sourceType, double > &&
         std::is_same_v< destinationType, std::complex<double> >
16
17
```

```
struct FFTPlanUniformPoolHandle
        /*-----
20
       Core Members
2.1
        -----*/
       svr::FFTPlanUniformPool<sourceType, destinationType> uniform_pool;
       std::size_t
                                               num_plans;
       std::size_t
                                               nfft;
        /*-----
28
       Special Member-functions
29
        FFTPlanUniformPoolHandle() = default;
31
       FFTPlanUniformPoolHandle(const std::size_t num_plans_arg,
                         const std::size_t nfft_arg)
             uniform_pool(num_plans_arg, nfft_arg),
             num_plans(num_plans_arg),
35
             nfft(nfft_arg)
36
                         {}
        FFTPlanUniformPoolHandle(const FFTPlanUniformPoolHandle& other) = delete;
        FFTPlanUniformPoolHandle& operator=(const FFTPlanUniformPoolHandle& other) =
          delete;
       FFTPlanUniformPoolHandle(FFTPlanUniformPoolHandle&& other)
       FFTPlanUniformPoolHandle& operator=(FFTPlanUniformPoolHandle&& other) = delete;
        /*-----
       Member Functions
43
       auto
            lock()
46
          return std::unique_lock<std::mutex>(this->mutex);
     };
49
  }
50
```

A.8 IFFT Plan Pool Handle

```
#pragma once
4 Dependencies
   #include "IFFTPlanPoolClass.hpp"
   namespace svr
9
10
      template <
11
          typename sourceType,
          typename destinationType,
          typename = std::enable_if_t<</pre>
             std::is_same_v< sourceType,</pre>
                                                 std::complex<double> > &&
              std::is_same_v< destinationType, double >
18
      struct IFFTPlanUniformPoolHandle
```

```
20
        /*-----
21
        Members
22
23
        IFFTPlanUniformPool< sourceType,</pre>
                          destinationType >
                                             uniform_pool;
        std::mutex
                                              mutex;
26
        std::size_t
                                              num_plans;
        std::size_t
                                              nfft;
        /*-----
30
        Special Member Functions
31
        IFFTPlanUniformPoolHandle() = default;
        IFFTPlanUniformPoolHandle(const std::size_t num_plans_arg,
                           const std::size_t nfft_arg)
              uniform_pool(
                               num_plans_arg, nfft_arg),
                               num_plans_arg),
              num_plans(
37
              nfft(
                               nfft_arg) {}
38
        IFFTPlanUniformPoolHandle(const IFFTPlanUniformPoolHandle& other) = delete;
39
        IFFTPlanUniformPoolHandle& operator=(const IFFTPlanUniformPoolHandle& other) =
        IFFTPlanUniformPoolHandle(IFFTPlanUniformPoolHandle&& other)
41
        IFFTPlanUniformPoolHandle& operator=(IFFTPlanUniformPoolHandle&& other) = delete;
        /*----
        Member Functions
45
        auto lock()
48
           return std::unique_lock<std::mutex>(this->mutex);
        }
50
51
     };
52
  }
53
```

Appendix B

General Purpose Templated Functions

B.1 abs

```
#pragma once
  /*-----
  #include <vector> // for vectors
  #include <algorithm> // for std::transform
  y = abs(vector)
  template <typename T>
  auto abs(const std::vector<T>& input_vector)
      // creating canvas
     auto canvas {input_vector};
     // calculating abs
      std::transform(canvas.begin(),
                  canvas.end(),
                  canvas.begin(),
                  [](auto& argx){return std::abs(argx);});
     // returning
      return std::move(canvas);
  y = abs(matrix)
  template <typename T>
  auto abs(const std::vector<std::vector<T>> input_matrix)
31
      // creating canvas
      auto canvas
                    {input_matrix};
      // applying element-wise abs
      std::transform(input_matrix.begin(),
                  input_matrix.end(),
                  input_matrix.begin(),
```

```
[](auto& argx){return std::abs(argx);});

// returning
return std::move(canvas);

}
```

B.2 Boolean Comparators

```
#pragma once
  template <typename T, typename U>
  auto operator<(const std::vector<T>& input_vector,
             const
                                   scalar)
     // creating canvas
                   {std::vector<bool>(input_vector.size())};
     // transforming
11
     std::transform(input_vector.begin(), input_vector.end(),
                canvas.begin(),
                [&scalar](const auto& argx){
                    return argx < static_cast<T>(scalar);
     // returning
     return std::move(canvas);
19
20 }
  /*-----
  -----*/
  template <typename T, typename U>
  auto operator<=(const std::vector<T>& input_vector,
24
              const
                                   scalar)
25
  {
26
     // creating canvas
          canvas
                    {std::vector<bool>(input_vector.size())};
28
     // transforming
     std::transform(input_vector.begin(), input_vector.end(),
                canvas.begin(),
                 [&scalar](const auto& argx){
                    return argx <= static_cast<T>(scalar);
                });
     // returning
37
     return std::move(canvas);
38
  template <typename T, typename U>
  auto operator>(const std::vector<T>& input_vector,
43
             const
                                   scalar)
44
     // creating canvas
     auto canvas {std::vector<bool>(input_vector.size())};
     // transforming
```

```
std::transform(input_vector.begin(), input_vector.end(),
                   canvas.begin(),
50
                   [&scalar](const auto& argx){
51
                       return argx > static_cast<T>(scalar);
52
                  });
      // returning
55
      return std::move(canvas);
56
  }
57
   /*-----
58
59
  template <typename T, typename U>
60
  auto operator>=(const std::vector<T>& input_vector,
                const U
                                        scalar)
62
63
      // creating canvas
64
                     {std::vector<bool>(input_vector.size())};
      auto canvas
66
      // transforming
67
      std::transform(input_vector.begin(), input_vector.end(),
                  canvas.begin(),
                   [&scalar](const auto& argx){
                       return argx >= static_cast<T>(scalar);
                  });
      // returning
74
      return std::move(canvas);
75
  }
```

B.3 Concatenate Functions

```
#pragma once
  /*----
  input = [vector, vector],
  output = [vector]
                    -----*/
  template <std::size_t axis, typename T>
  auto concatenate(const std::vector<T>& input_vector_A,
               const std::vector<T>&
                                   input_vector_B) -> std::enable_if_t<axis == 1,</pre>
                   std::vector<T> >
  {
9
     // creating canvas vector
10
     auto num_elements {input_vector_A.size() + input_vector_B.size()};
11
     auto canvas
                        {std::vector<T>(num_elements, (T)0) };
     // filling up the canvas
     std::copy(input_vector_A.begin(), input_vector_A.end(),
             canvas.begin());
16
     std::copy(input_vector_B.begin(), input_vector_B.end(),
17
             canvas.begin()+input_vector_A.size());
     // moving it back
20
     return std::move(canvas);
22
  }
23
```

```
input = [vector, vector],
  output = [matrix]
  -----*/
27
28 template <std::size_t axis, typename T>
29 auto concatenate(const std::vector<T>& input_vector_A,
                const std::vector<T>&
                                     input_vector_B) -> std::enable_if_t<axis == 0,</pre>
                    std::vector<std::vector<T>> >
  ₹
31
32
      // throwing error dimensions
33
      if (input_vector_A.size() != input_vector_B.size())
         std::cerr << "concatenate:: incorrect dimensions \n";</pre>
34
35
      // creating canvas
      auto canvas {std::vector<std::vector<T>>(
37
         2, std::vector<T>(input_vector_A.size())
38
      )};
30
      // filling up the dimensions
41
      std::copy(input_vector_A.begin(), input_vector_A.end(), canvas[0].begin());
42
      std::copy(input_vector_B.begin(), input_vector_B.end(), canvas[1].begin());
43
      // moving it back
45
      return std::move(canvas);
46
47
  /*----
49
50 input = [vector, vector, vector],
output = [matrix]
 -----*/
53
  template <std::size_t axis, typename T>
auto concatenate(const std::vector<T>& input_vector_A,
                const std::vector<T>& input_vector_B,
55
                const std::vector<T>& input_vector_C) -> std::enable_if_t<axis == 0,</pre>
                   std::vector<std::vector<T>> >
  ₹
57
      // throwing error dimensions
      if (input_vector_A.size() != input_vector_B.size() ||
59
         input_vector_A.size() != input_vector_C.size())
60
         std::cerr << "concatenate:: incorrect dimensions \n";</pre>
      // creating canvas
63
      auto canvas {std::vector<std::vector<T>>(
64
         3, std::vector<T>(input_vector_A.size())
65
      )};
67
      // filling up the dimensions
68
      std::copy(input_vector_A.begin(), input_vector_A.end(), canvas[0].begin());
69
      std::copy(input_vector_B.begin(), input_vector_B.end(), canvas[1].begin());
70
      std::copy(input_vector_C.begin(), input_vector_C.end(), canvas[2].begin());
71
72
      // moving it back
73
      return std::move(canvas);
75
76 }
  /*-----
  input = [matrix, vector],
78
  output = [matrix]
79
  template <std::size_t axis, typename T>
```

```
auto concatenate(const std::vector<std::vector<T>>& input_matrix,
                   const std::vector<T>
                                                        input_vector) -> std::enable_if_t<axis</pre>
83
                       == 0, std::vector<std::vector<T>> >
   {
84
       // creating canvas
85
       auto canvas
                          {input_matrix};
87
       // adding to the canvas
       canvas.push_back(input_vector);
       // returning
91
       return std::move(canvas);
92
   }
```

B.4 Conjugate

```
#pragma once
  namespace svr {
    /*----
    y = svr::conj(vector);
    -----*/
    template <typename T>
         conj(const std::vector<T>& input_vector)
    auto
       // creating canvas
       auto canvas
                   {std::vector<T>(input_vector.size())};
       // calculating conjugates
       std::for_each(canvas.begin(), canvas.end(),
                [](auto& argx){argx = std::conj(argx);});
       // returning
       return std::move(canvas);
17
    }
18
  }
```

B.5 Convolution

```
17
         // calculating fft of two arrays
18
                fft_A
                          {svr::fft(input_vector_A, canvas_length)};
         auto
19
         auto
                fft_B
                          {svr::fft(input_vector_B, canvas_length)};
2.0
21
         // element-wise multiplying the two matrices
                         {fft_A * fft_B};
         auto
                fft_AB
23
         // finding inverse FFT
         auto convolved_result {ifft(fft_AB)};
26
27
         // returning
28
         return std::move(convolved_result);
29
      }
30
31
      template <>
32
      auto conv1D(const std::vector<double>& input_vector_A,
                   const std::vector<double>& input_vector_B)
34
35
         // creating canvas
36
         auto canvas_length
                                 {input_vector_A.size() + input_vector_B.size() - 1};
38
         // calculating fft of two arrays
39
                         {svr::fft(input_vector_A, canvas_length)};
         auto
                fft_A
                          {svr::fft(input_vector_B, canvas_length)};
         auto
                fft_B
42
         // element-wise multiplying the two matrices
43
         auto fft_AB {fft_A * fft_B};
         // finding inverse FFT
46
         auto convolved_result {ifft(fft_AB)};
47
         // returning
49
         return std::move(convolved_result);
50
      }
51
      /*-----
53
      1D convolution of two vectors
54
      > implemented through fft
55
                           -----*/
57
      template <typename T1, typename T2>
      auto conv1D_fftw(const std::vector<T1>& input_vector_A,
58
                       const std::vector<T2>& input_vector_B)
59
      {
         // resulting type
61
         using T3 = decltype(std::declval<T1>() * std::declval<T2>());
62
         // creating canvas
                                 {input_vector_A.size() + input_vector_B.size() - 1};
         auto canvas_length
65
         // calculating fft of two arrays
67
         auto
                fft_A
                          {svr::fft(input_vector_A, canvas_length)};
         auto
                fft_B
                          {svr::fft(input_vector_B, canvas_length)};
69
70
         // element-wise multiplying the two matrices
         auto fft_AB
                          {fft_A * fft_B};
72
73
         // finding inverse FFT
74
         auto convolved_result {svr::ifft(fft_AB, fft_AB.size())};
```

76

```
// returning
77
          return std::move(convolved_result);
78
79
80
       /*----
81
       Long-signal Conv1D
82
83
       improvements:
          > make an inplace version of this
       -----*/
       template <std::size_t L, typename T>
86
       auto conv1D_long(const std::vector<T>& input_vector_A,
87
                        const std::vector<T>& input_vector_B)
88
89
          // fetching dimensions
90
                                      {std::max(input_vector_A.size(),
          const auto
                       maxlength
                                              input_vector_B.size())};
                                      {std::min(input_vector_A.size(),
          const auto
                        filter_size
93
                                              input_vector_B.size())};
94
          const auto block_size
                                      {L + filter_size - 1};
95
          const auto num_blocks
                                      {2 + static_cast<std::size_t>(
              (maxlength - block_size)/L
97
          )};
98
          // obtaining references
          const auto& large_vector
                                      {input_vector_A.size() >= input_vector_B.size() ? \
101
                                      input_vector_A : input_vector_B};
102
          const auto& small_vector {input_vector_A.size() < input_vector_B.size() ? \</pre>
                                      input_vector_A : input_vector_B};
104
105
          // setup
106
                 starting_index
                                      {static_cast<std::size_t>(0)};
          auto
107
                 ending_index
                                      {static_cast<std::size_t>(0)};
          auto
108
                 length_left_to_fill {ending_index - starting_index};
          auto
109
                                      {std::vector<double>(block_size, 0)};
                 canvas
          auto
          auto
                 finaloutput
                                      {std::vector<double>(maxlength, 0)};
                 block_conv_output_size {L + 2 * filter_size -2};
          auto
                block_conv_output
                                      {std::vector<double>(block_conv_output_size, 0)};
          auto
114
115
          // block-wise processing
          for(auto bid = 0; bid < num_blocks; ++bid)</pre>
116
117
              // estimating indices
118
             starting_index =
                                     L*bid;
119
                                     std::min(starting_index + block_size - 1,maxlength -
              ending_index
120
                 1):
              length_left_to_fill = ending_index - starting_index;
121
              // copying to the common-block
123
              std::copy(large_vector.begin() + starting_index,
124
125
                      large_vector.begin() + ending_index + 1,
126
                      canvas.begin());
127
              // performing convolution
128
              block_conv_output = svr::conv1D_fftw(canvas,
130
                                                 small_vector);
131
              // discarding edges and writing values
              std::copy(block_conv_output.begin() + filter_size-2,
133
```

```
block_conv_output.begin() + filter_size-2 +
134
                           std::min(static_cast<int>(L-1),
                           static_cast<int>(length_left_to_fill)) + 1,
                       finaloutput.begin()+starting_index);
          }
136
137
          // returning
138
          return std::move(finaloutput);
139
140
141
       /*-----
142
       Long-signal Conv1D with FFT Plan
143
       improvements:
144
          > make an inplace version of this
145
                                -----*/
146
       template
                 <typename T>
147
       auto conv1D_long_prototype(
148
          const std::vector<T>&
                                                  input_vector_A,
149
                 std::vector<T>&
150
                                                  input_vector_B,
          svr::FFTPlanClass<T, std::complex<T>>& fft_plan,
          svr::IFFTPlanClass<std::complex<T>, T>& ifft_plan
153
154
          // Error checks
          if (fft_plan.nfft_ != ifft_plan.nfft_)
              throw std::runtime_error("fft_plan.nfft_ != ifft_plan.nfft_");
157
158
          // fetching references to large-signal and small-signal
159
          const auto& large_signal_original {
160
              input_vector_A.size() >= input_vector_B.size() ?
161
                                : input_vector_B
              input_vector_A
162
          };
163
          const auto& small_signal
164
              input_vector_A.size() < input_vector_B.size() ?</pre>
165
              input_vector_A
                                       input_vector_B
166
                               :
          };
168
          // copying
                 large_signal {std::vector<double>(
170
              input_vector_A.size() + input_vector_B.size() -1
          std::copy(large_signal_original.begin(),
173
                   large_signal_original.end(),
174
                   large_signal.begin());;
175
176
          // calculating parameters
177
                         signal_size
                                              {large_signal_original.size()};
          const auto
178
          const
                 auto
                         filter_size
                                              {small_signal.size()};
179
          const auto
                         input_signal_block_size {fft_plan.nfft_ + 1 - filter_size};
180
          if (input_signal_block_size <= 0)</pre>
181
              throw std::runtime_error("input_signal_block_size <= 0 ");</pre>
182
183
          const auto block_output_length {fft_plan.nfft_};
          const auto
                        num_blocks
                                              {static_cast<int>(
184
              1 + std::ceil((signal_size + filter_size - 2)/input_signal_block_size)
185
          )
          };
187
          const
                 auto
                         final_output_size
                                              {signal_size + filter_size - 1};
188
                         useful_sample_length
                                                    {block_output_length - (filter_size -1)
          const
189
                 auto
              - (filter_size -1)};
```

```
190
           // parameters for re-use
191
                                        {static_cast<int>(0)};
           auto
                  start_index
192
                  end_index
                                        {static_cast<int>(0)};
           auto
193
                  output_start_index
                                        {static_cast<int>(0)};
           auto
194
           // calculating fft(filter)
196
                  filter_zero_padded
                                        {std::vector<double>(block_output_length, 0.0)};
197
           std::copy(small_signal.begin(), small_signal.end(), filter_zero_padded.begin());
           auto
                  filter_FFT
                                        {fft_plan.fft(filter_zero_padded)};
199
200
           // allocating space for storing input-blocks
201
                  signal_block_zero_padded {std::vector<double>(block_output_length, 0.0)};
202
                  fftw_output
                                            {std::vector<double>()};
203
           auto
           auto
                  conv_output
                                            {std::vector<double>()};
204
                                            {std::vector<double>(final_output_size, 0.0)};
           auto
                  finaloutput
205
           // going through the values
207
           svr::Timer timer("fft-loop");
208
           for(auto i = 0; i<num_blocks; ++i){</pre>
209
210
               // calculating bounds
211
                     analytical_start {
212
                  (i*static_cast<int>(input_signal_block_size)) -
213
                      (static_cast<int>(filter_size) - 1)
              };
214
                      analytical_end
                                        {(i+1)*input_signal_block_size -1};
               auto
               start_index
                            = std::max(
216
                  static_cast<int>(0), static_cast<int>(analytical_start)
217
              );
218
               end_index
                             = std::min(
219
                  static_cast<int>(signal_size-1),static_cast<int>(analytical_end)
              ); // [start-index, end-index)
221
222
               // copying values
223
              signal_block_zero_padded = std::move(std::vector<double>(block_output_length,
                  0.0));
               std::copy(large_signal.begin() + start_index,
                        large_signal.begin() + end_index + 1,
                        signal_block_zero_padded.begin() + start_index - analytical_start);
228
               // performing ifft(fft(x) * fft(y))
229
              fftw_output
                            = ifft_plan.ifft(
230
                  fft_plan.fft(signal_block_zero_padded) * filter_FFT
231
              ):
232
               // trimming away the first parts (since partial)
234
               conv_output = std::vector<double>(fftw_output.begin() + filter_size -
                  1,fftw_output.end());
236
237
               // writing to final-output
               std::copy(conv_output.begin(), conv_output.end(), finaloutput.begin() +
                  output_start_index);
               output_start_index
                                    += conv_output.size();
239
           }
241
       }
242
243
```

```
Long-signal Conv1D with FFT-Plan-Pool
245
246
       template
247
                      Т,
           typename
248
                     =
                          std::enable_if_t<
           typename
249
               std::is_same_v<T, double> ||
               std::is_same_v<T, float>
251
252
253
       >
       auto
              conv_per_plan(
254
           const int
                                                                 i,
255
           const int&
                                                                 input_signal_block_size,
256
           const int&
                                                                 filter_size,
257
                                                                 block_output_length,
258
           const int&
           const std::vector<T>&
                                                                 large_signal,
           std::vector<T>
                                                                 signal_block_zero_padded,
260
           svr::FFTPlanUniformPoolHandle<T, std::complex<T>>& fft_pool_handle,
261
           svr::IFFTPlanUniformPoolHandle<std::complex<T>, T>& ifft_pool_handle,
262
           const std::vector<std::complex<T>>&
263
                                                                 filter_FFT,
           std::vector<T>
                                                                 fftw_output,
           std::vector<T>
                                                                 conv_output,
265
           std::vector<T>&
                                                                 output_vector,
266
           std::mutex&
                                                                 output_vector_mutex,
267
           const auto&
                                                                 signal_size
268
269
       )
       {
270
271
           // calculating bounds
272
                   analytical_start {
273
               (i*static_cast<int>(input_signal_block_size)) -
274
                   (static_cast<int>(filter_size) - 1)
           };
275
                   analytical_end
                                      {(i+1)*input_signal_block_size -1};
           auto
276
           auto start_index = std::max(
277
               static_cast<int>(0), static_cast<int>(analytical_start)
278
           );
                               = std::min(
           auto end_index
280
               static_cast<int>(signal_size-1),static_cast<int>(analytical_end)
281
           ); // [start-index, end-index)
282
283
           // copying values
284
           signal_block_zero_padded = std::move(std::vector<double>(block_output_length,
285
               0.0));
           std::copy(
286
               large_signal.begin()
                                                     start_index,
287
                                                                     + 1,
               large_signal.begin()
                                                     end_index
288
               signal_block_zero_padded.begin() + start_index
                                                                    - analytical_start
289
           );
290
291
           // fetching an fft and IFFT plan
2.92
293
           auto
                   fph_lock
                                  {fft_pool_handle.lock()};
           auto
                   ifph_lock
                                  {ifft_pool_handle.lock()};
           auto
                   fft_pair
                                  {fft_pool_handle.uniform_pool.fetch_plan()};
295
                   ifft_pair
                                  {ifft_pool_handle.uniform_pool.fetch_plan()};
           auto
296
           // performing ifft(fft(x) * filter-FFT)
298
           fftw_output = ifft_pair.plan.ifft_12_conserved(
299
               fft_pair.plan.fft_12_conserved(signal_block_zero_padded) * filter_FFT
300
           );
301
```

```
302
           // trimming away the first parts (since partial)
303
           conv_output = std::vector<T>(
304
               fftw_output.begin() + filter_size - 1,
305
               fftw_output.end()
306
           );
307
308
           // writing to final-output
309
                   output_start_index = i * (block_output_length - (filter_size - 1));
311
           std::lock_guard<std::mutex> output_lock(output_vector_mutex);
           std::copy(
               conv_output.begin(), conv_output.end(),
313
               output_vector.begin() + output_start_index
314
           );
315
317
318
       template
319
320
           typename T,
           typename
                           std::enable_if_t<
321
               std::is_same_v<T, double> ||
322
               std::is_same_v<T, float>
323
324
       >
325
       auto
               conv1D_long_FFTPlanPool(
           const std::vector<T>&
                                                                 input_vector_A,
327
                  std::vector<T>&
                                                                 input_vector_B,
           const
328
           svr::FFTPlanUniformPoolHandle<T, std::complex<T>>& fft_pool_handle,
329
           svr::IFFTPlanUniformPoolHandle<std::complex<T>, T>& ifft_pool_handle
330
332
           // Error checks
333
              (fft_pool_handle.nfft!=ifft_pool_handle.nfft)
334
               throw std::runtime_error("FILE: svr_conv.hpp | FUNCTION:
                   conv1D_long_FFTPlanPool | Report: the pool-handles are for different
                   nffts");
336
           // fetching references to the large signal and small signal
337
           const auto& large_signal_original {
338
               input_vector_A.size() >= input_vector_B.size() ?
               input_vector_A
                                      :
                                          input_vector_B
340
           };
341
           const auto& small_signal
                                          {
342
               input_vector_A.size() < input_vector_B.size() ?</pre>
               input_vector_A
                                  :
                                          input_vector_B
344
           };
345
346
           // copying
347
           auto
                   large_signal {std::vector<double>(
348
               input_vector_A.size() + input_vector_B.size() - 1
349
350
           )};
351
           std::copy(large_signal_original.begin(),
                     large_signal_original.end(),
352
                     large_signal.begin());
353
354
           // calculating some parameters
355
           const
                  auto
                           signal_size
                                          {large_signal_original.size()};
356
                                          {small_signal.size()};
                   auto
                           filter_size
357
           const
                   auto
                           input_signal_block_size {
           const
```

```
fft_pool_handle.nfft + 1 - filter_size
359
           };
360
361
           // throwing an error if nfft < filter-size
362
           if (fft_pool_handle.nfft < filter_size)</pre>
363
               throw std::runtime_error("FILE: svr_conv.hpp | FUNCTION:
364
                    conv1D_long_FFTPlanPool | REPORT: filter is bigger than nfft");
365
           // throwing an error if number of useful samples is less than zero
           if (input_signal_block_size <= 0)</pre>
367
               throw std::runtime_error("FILE: svr_conv.hpp | FUNCTION:
368
                    conv1D_long_FFTPlanPool | REPORT: input_signal_block_size = 0");
369
370
           const
                   auto
                           block_output_length
                                                  {fft_pool_handle.nfft};
371
                           num_blocks
                                                   {static_cast<int>(
           const
                   auto
               1 + std::ceil((signal_size + filter_size - 2) / input_signal_block_size)
374
375
           const
                   auto
                           final_output_size
                                                  {signal_size + filter_size - 1};
           const
                   auto
                           useful_sample_length {
376
               block_output_length - (filter_size - 1) - (filter_size -1)
377
378
379
           // parameters for re-use
380
           auto
                   start_index
                                           {static_cast<int>(0)};
           auto
                   end_index
                                           {static_cast<int>(0)};
382
                                           {static_cast<int>(0)};
           auto
                   output_start_index
383
384
           // calculating fft(filter)
385
           auto
                   filter_zero_padded
                                           {std::vector<double>(block_output_length, 0.0)};
386
           std::copy(small_signal.begin(), small_signal.end(), filter_zero_padded.begin());
387
           auto
                   fph_lock0
                                           {fft_pool_handle.lock()};
                   curr_plan_pair
                                           {fft_pool_handle.uniform_pool.fetch_plan()};
           auto
389
                   pool_num_plans
                                           {fft_pool_handle.num_plans};
           auto
390
           fph_lock0.unlock();
391
           auto
                   filter_FFT
               curr_plan_pair.plan.fft(
393
                   filter_zero_padded
394
               )
395
           };
           curr_plan_pair.lock.unlock();
397
398
           // allocating space for storing input-blocks
399
           auto
                   signal_block_zero_padded {std::vector<T>(block_output_length, 0.0)};
                                              {std::vector<T>()};
           auto
                   fftw_output
401
                   conv_output
                                               {std::vector<T>()};
           auto
402
                   output_vector
                                               {std::vector<T>(final_output_size, 0.0)};
           auto
403
                   output_vector_mutex
                                               {std::mutex()};
404
           auto
405
           // creating boost
406
407
           svr::ThreadPool local_pool(pool_num_plans);
           // going through the values
409
           for(auto i = 0; i < num_blocks; ++i)</pre>
410
411
               local_pool.push_back(
412
                   413
414
                       i,
                       &input_signal_block_size,
415
```

```
&filter_size,
416
                     &block_output_length,
417
                     &large_signal,
418
                     signal_block_zero_padded,
419
                     &fft_pool_handle,
420
                     &ifft_pool_handle,
421
                     &filter_FFT,
422
                     fftw_output,
423
                     conv_output,
425
                     &output_vector,
                     &output_vector_mutex,
426
                     &signal_size
427
                 ]{
428
                     conv_per_plan<T>(
429
                         i,
430
                         std::ref(input_signal_block_size),
431
                         std::ref(filter_size),
                         std::ref(block_output_length),
433
                         std::ref(large_signal),
434
435
                         signal_block_zero_padded,
                         fft_pool_handle,
436
                         ifft_pool_handle,
437
                         filter_FFT,
438
                         fftw_output,
439
                         conv_output,
                         std::ref(output_vector),
441
                         output_vector_mutex,
442
                         signal_size
443
                     );
                 }
445
              );
446
          local_pool.converge();
448
449
          // returning final output
450
          // return std::move(output_vector);
451
          return output_vector;
452
453
454
       Short-conv1D
456
        457
       // template <std::size_t shortsize,</pre>
458
459
       //
                     typename
                                  T1,
       //
                                  T2>
                     typename
460
       template
                  <typename
                                T1.
461
                   typename
                                T2>
462
       auto
              conv1D_short(const std::vector<T1>& input_vector_A,
463
                                 std::vector<T2>& input_vector_B)
                          const
464
       {
465
466
          // resulting type
          using T3 = decltype(std::declval<T1>() * std::declval<T2>());
468
          // creating canvas
469
                 canvas_length
                                   {input_vector_A.size() + input_vector_B.size() - 1};
          auto
471
          // calculating fft of two arrays
472
                 fft_A
                            {svr::fft(input_vector_A, canvas_length)};
          auto
473
                  fft_B
          auto
                            {svr::fft(input_vector_B, canvas_length)};
474
```

475

```
// element-wise multiplying the two matrices
476
                  {	t fft}_{	t AB}
                            \{fft_A * fft_B\};
           auto
477
478
           // finding inverse FFT
479
           auto convolved_result {ifft(fft_AB)};
481
           // returning
482
           // return std::move(convolved_result);
484
           return convolved_result;
485
       }
486
487
488
489
       1D Convolution of a matrix and a vector
490
                  <typename T>
       template
492
       auto conv1D(const std::vector<std::vector<T>>& input_matrix,
493
494
                      const
                              std::vector<T>&
                                                             input_vector,
                      const
                              std::size_t&
                                                             dim)
496
           // getting dimensions
497
           const auto& num_rows_matrix
                                                  {input_matrix.size()};
           const auto& num_cols_matrix
                                                  {input_matrix[0].size()};
           const auto& num_elements_vector {input_vector.size()};
500
501
           // creating canvas
502
           auto canvas {std::vector<std::vector<T>>()};
503
504
           // creating output based on dim
505
           if (dim == 1)
507
               // performing convolutions row by row
508
               for(auto row = 0; row < num_rows_matrix; ++row)</pre>
509
                   cout << format("\t\t row = {}/{}\n", row, num_rows_matrix);</pre>
511
                   auto bruh {conv1D(input_matrix[row], input_vector)};
512
                   auto bruh_real {svr::real(std::move(bruh))};
513
                   canvas.push_back(
515
                          svr::real(
                              std::move(bruh_real)
517
                           )
518
                   );
519
               }
520
           }
521
           else{
               std::cerr << "svr_conv.hpp | conv1D | yet to be implemented \n";
524
525
526
           // returning
           return std::move(canvas);
527
528
       }
529
530
531
       1D Convolution of a matrix and a vector (in-place)
532
533
```

```
534
535 }
```

B.6 Coordinate Change

```
#pragma once
  namespace svr {
     y = cart2sph(vector)
     template <typename T>
     auto cart2sph(const std::vector<T>& cartesian_vector){
         // splatting the point onto xy-plane
         auto xysplat
                         {cartesian_vector};
10
         xysplat[2]
         // finding splat lengths
               xysplat_lengths
                               {norm(xysplat)};
         // finding azimuthal and elevation angles
               azimuthal_angles {svr::atan2(xysplat[1],
         auto
                                         xysplat[0]) \
                                * 180.00/std::numbers::pi};
         auto
               elevation_angles {svr::atan2(cartesian_vector[2],
20
                                         xysplat_lengths) \
2.1
                                * 180.00/std::numbers::pi};
         auto
               rho_values
                                {norm(cartesian_vector)};
         // creating tensor to send back
         auto spherical_vector {std::vector<T>{azimuthal_angles,
                                             elevation_angles,
                                             rho_values}};
2.8
         // moving it back
         return std::move(spherical_vector);
31
      /*-----
33
     y = cart2sph(vector)
35
      template <typename T>
36
      auto cart2sph_inplace(std::vector<T>& cartesian_vector){
37
         // splatting the point onto xy-plane
39
         auto xysplat
                        {cartesian_vector};
40
         xysplat[2]
         // finding splat lengths
43
               xysplat_lengths {norm(xysplat)};
         // finding azimuthal and elevation angles
46
              azimuthal_angles {svr::atan2(xysplat[1], xysplat[0]) *
            180.00/std::numbers::pi};
         auto elevation_angles {svr::atan2(cartesian_vector[2],
                                         xysplat_lengths) * 180.00/std::numbers::pi};
         auto
              rho_values
                                {norm(cartesian_vector)};
50
```

```
51
          // creating tesnor
52
          cartesian_vector[0] = azimuthal_angles;
53
          cartesian_vector[1] = elevation_angles;
54
          cartesian_vector[2] = rho_values;
55
       /*-----
57
      y = cart2sph(input_matrix, dim)
58
       -----*/
59
       template <typename T>
60
       auto cart2sph(const std::vector<std::vector<T>>& input_matrix,
61
                           std::size_t
                                                      axis)
                  const
62
63
          // fetching dimensions
64
          const auto& num_rows
                                 {input_matrix.size()};
65
          const auto& num_cols {input_matrix[0].size()};
66
          // checking the axis and dimensions
68
          if (axis == 0 && num_rows != 3) {std::cerr << "cart2sph: incorrect num-elements
69
              n";}
          if (axis == 1 && num_cols != 3) {std::cerr << "cart2sph: incorrect num-elements
              n'';
71
          // creating canvas
          auto canvas {std::vector<std::vector<T>>(
             num_rows,
74
             std::vector<T>(num_cols, 0)
75
          )};
76
77
78
          // if axis = 0, performing operation column-wise
          if(axis == 0)
79
80
              for(auto col = 0; col < num_cols; ++col)</pre>
81
82
                 // fetching current column
83
                 auto curr_column {std::vector<T>({input_matrix[0][col],
                                                  input_matrix[1][col],
85
                                                  input_matrix[2][col]})};
86
                 // performing inplace transformation
                 cart2sph_inplace(curr_column);
89
90
                 // storing it back
91
                 canvas[0][col] = curr_column[0];
                 canvas[1][col] =
                                   curr_column[1];
93
                 canvas[2][col] = curr_column[2];
94
             }
          }
          // if axis == 1, performing operations row-wise
97
          else if(axis == 0)
98
99
          {
              std::cerr << "cart2sph: yet to be implemented \n";
          }
101
          else
102
          {
              std::cerr << "cart2sph: yet to be implemented \n";
104
          }
105
106
          // returning
107
```

```
return std::move(canvas);
108
109
       }
111
       // -----
112
       template <typename T>
113
             sph2cart(const std::vector<T> spherical_vector){
114
          // creating cartesian vector
                 cartesian_vector {std::vector<T>(spherical_vector.size(), 0)};
117
118
          // populating
119
          cartesian_vector[0]
                                  spherical_vector[2] * \
                                  cos(spherical_vector[1] * std::numbers::pi / 180.00) * \
                                  cos(spherical_vector[0] * std::numbers::pi / 180.00);
          cartesian_vector[1]
                                  spherical_vector[2] * \
                                  cos(spherical_vector[1] * std::numbers::pi / 180.00) * \
124
                                  sin(spherical_vector[0] * std::numbers::pi / 180.00);
          cartesian_vector[2]
                                  spherical_vector[2] * \
126
                                  sin(spherical_vector[1] * std::numbers::pi / 180.00);
127
128
          // returning
          return std::move(cartesian_vector);
130
       }
131
   }
```

B.7 Cosine

```
#pragma once
    ______
  y = cos(input_vector)
  -----*/
  template <typename T>
  auto cos(const std::vector<T>& input_vector)
    // created canvas
    auto canvas
                {input_vector};
10
    // calling the function
    std::transform(input_vector.begin(), input_vector.end(),
             canvas.begin(),
             [](auto& argx){return std::cos(argx);});
14
    // returning the output
16
    return std::move(canvas);
18
  /*-----
  y = cosd(input_vector)
20
  -----*/
21
  template <typename T>
  auto cosd(const std::vector<T> input_vector)
24
    // created canvas
25
    auto canvas
                {input_vector};
26
2.7
    // calling the function
28
```

```
std::transform(input_vector.begin(),
input_vector.end(),
input_vector.begin(),
[](const auto& argx){return std::cos(argx * 180.00/std::numbers::pi);});

// returning the output
return std::move(canvas);
}
```

B.8 Data Structures

```
struct TreeNode {
      int val;
      TreeNode *left;
      TreeNode *right;
      TreeNode() : val(0), left(nullptr), right(nullptr) {}
      TreeNode(int x) : val(x), left(nullptr), right(nullptr) {}
      TreeNode(int x, TreeNode *left, TreeNode *right) : val(x), left(left), right(right)
          {}
8
   };
  struct ListNode {
11
      int val;
      ListNode *next;
      ListNode() : val(0), next(nullptr) {}
      ListNode(int x) : val(x), next(nullptr) {}
16
      ListNode(int x, ListNode *next) : val(x), next(next) {}
  };
17
```

B.9 Editing Index Values

```
#pragma once
  /*-----
 Matlab's equivalent of A[A < 0.5] = 0
  -----*/
  template <typename T, typename U>
  auto edit(std::vector<T>&
                                  input_vector,
         const std::vector<bool>&
                                  bool_vector,
         const
                                  scalar)
8
  {
9
     // throwing an error
     if (input_vector.size() != bool_vector.size())
11
        std::cerr << "edit: incompatible size\n";</pre>
     // overwriting input-vector
     std::transform(input_vector.begin(), input_vector.end(),
                bool_vector.begin(),
16
                input_vector.begin(),
                [&scalar](auto& argx, auto argy){
                    if(argy == true) {return static_cast<T>(scalar);}
                                  {return argx;}
                });
```

```
// no-returns since in-place
  }
24
25
  /*----
26
  accumulate version of edit, instead of just placing values
  THings to add
29
      - ensuring template only accepts int, std::size_t and similar for T2
30
      - bring in histogram method to ensure SIMD
31
32
  -----*/
  template <typename T1,
33
            typename T2>
34
       edit_accumulate(std::vector<T1>&
                                           input_vector,
35 auto
                     const std::vector<T2>& indices_to_edit,
36
                      const std::vector<T1>& new_values)
37
  {
38
      // certain checks
39
      if (indices_to_edit.size() != new_values.size())
40
         std::cerr << "svr::edit | edit_accumulate | size-disparity occured \n";</pre>
41
42
      // going through each and accumulating
43
      for(auto i = 0; i < input_vector.size(); ++i){</pre>
         const auto target_index {static_cast<std::size_t>(indices_to_edit[i])}; //
45
         const auto new_value
                               {new_values[i]};
         if (target_index < input_vector.size()){</pre>
            input_vector[target_index] = input_vector[target_index] + new_value;
48
         }
49
         else{
            // std::cout << "warning: FILE: svr_edit.hpp | FUNCTION: edit_accumulate |
               REPORT: index out of bounds";
52
      }
53
      // no-return since in-place
55
  }
```

B.10 Equality

```
#pragma once
  /*-----
4 template <typename T, typename U>
  auto operator==(const std::vector<T>& input_vector,
              const U&
  {
     // setting up canvas
8
     auto canvas {std::vector<bool>(input_vector.size())};
10
     // writing to canvas
11
     std::transform(input_vector.begin(), input_vector.end(),
                canvas.begin(),
                 [&scalar](const auto& argx){
                    return argx == scalar;
                });
     // returning
18
```

```
return std::move(canvas);
}
```

B.11 Exponentiate

```
#pragma once
  /*-----
  y = abs(vector)
  template <typename T>
  auto exp(const std::vector<T>& input_vector)
     // creating canvas
     auto canvas
                  {input_vector};
     // transforming
     std::transform(canvas.begin(), canvas.end(),
                canvas.begin(),
                [](auto& argx){return std::exp(argx);});
     // returning
     return std::move(canvas);
17
  }
```

B.12 FFT

```
#pragma once
  namespace svr {
     /*-----
     For type-deductions
                    -----*/
     template <typename T>
     struct fft_result_type;
     // specializations
     template <> struct fft_result_type<double>{
       using type = std::complex<double>;
     template <> struct fft_result_type<std::complex<double>>{
       using type = std::complex<double>;
14
     template <> struct fft_result_type<float>{
       using type = std::complex<float>;
17
     };
     template <> struct fft_result_type<std::complex<float>>{
        using type = std::complex<float>;
     };
21
22
     template <typename T>
     using fft_result_t = typename fft_result_type<T>::type;
     /*-----
     y = fft(x, nfft)
        > calculating n-point dft where n-value is explicit
```

```
-----*/
29
             template<typename T>
30
             auto fft(const std::vector<T>& input_vector,
31
                            const size_t
                                                                                nfft)
32
33
                    // throwing an error
                    if (nfft < input_vector.size()) {std::cerr << "size-mistmatch\n";}</pre>
35
                    if (nfft <= 0)</pre>
                                                                                  {std::cerr << "size-mistmatch\n";}
36
                    // fetching data-type
38
                    using RType = fft_result_t<T>;
39
                    using baseType = std::conditional_t<std::is_same_v<T, std::complex<double>>,
40
                                                                                            double,
41
                                                                                            T>;
42
43
                    // canvas instantiation
                    std::vector<RType> canvas(nfft);
                    auto nfft_sqrt {static_cast<RType>(std::sqrt(nfft))};
46
                                finaloutput {std::vector<RType>(nfft, 0)};
47
                    auto
48
                    // calculating index by index
                    for(int frequency_index = 0; frequency_index<nfft; ++frequency_index){</pre>
50
                           RType accumulate_value;
51
                           for(int signal_index = 0; signal_index < input_vector.size(); ++signal_index){</pre>
                                  accumulate_value += \
                                         static_cast<RType>(input_vector[signal_index]) * \
54
                                         static_cast<RType>(std::exp(-1.00 * std::numbers::pi * \
55
                                                                                           (static_cast<baseType>(frequency_index)/static_cast<baseType>(note: the content of the cont
                                                                                           static_cast<baseType>(signal_index)));
57
58
                           finaloutput[frequency_index] = accumulate_value / nfft_sqrt;
                    }
61
                    // returning
62
                    return std::move(finaloutput);
             }
64
65
             /*-----
             y = fft(std::vector<double> nfft) // specialization
                    _____
68
             #include <fftw3.h> // for fft
69
             template <>
70
             auto fft(const std::vector<double>& input_vector,
71
                            const std::size_t
72
73
                    if (nfft < input_vector.size())</pre>
74
                           throw std::runtime_error("nfft must be >= input_vector.size()");
76
                           throw std::runtime_error("nfft must be > 0");
77
                    // FFTW real-to-complex output
                    std::vector<std::complex<double>> output(nfft);
80
81
                    // Allocate input (double) and output (fftw_complex) arrays
                    double* in = reinterpret_cast<double*>(
83
                           fftw_malloc(sizeof(double) * nfft)
84
85
                    fftw_complex* out = reinterpret_cast<fftw_complex*>(
```

```
fftw_malloc(sizeof(fftw_complex) * (nfft/2 + 1))
          );
88
20
          // Copy input and zero-pad if needed
90
          for (std::size_t i = 0; i < nfft; ++i) {</pre>
91
              in[i] = (i < input_vector.size()) ? input_vector[i] : 0.0;</pre>
93
          // Create FFTW plan and execute
          fftw_plan plan = fftw_plan_dft_r2c_1d(
96
              static_cast<int>(nfft), in, out, FFTW_ESTIMATE
97
98
          fftw_execute(plan);
99
100
          // Copy FFTW output to std::vector<std::complex<double>>
          for (std::size_t i = 0; i < nfft/2 + 1; ++i) {</pre>
              output[i] = std::complex<double>(out[i][0], out[i][1]);
104
          // Optional: fill remaining bins with zeros to match full nfft size
105
          for (std::size_t i = nfft/2 + 1; i < nfft; ++i) {</pre>
              output[i] = std::complex<double>(0.0, 0.0);
107
108
109
          // Cleanup
          fftw_destroy_plan(plan);
          fftw_free(in);
112
          fftw_free(out);
114
          // filling up the other half of the output
115
          const auto
                       halfpoint {static_cast<std::size_t>(nfft/2)};
          std::transform(
117
              output.begin() + 1,
                                        // first half (skip DC)
              output.begin() + halfpoint, // end of first half
119
              output.rbegin(),
                                    // start writing from last element backward (skip
120
                  Nyquist)
              [](const auto& x) { return std::conj(x); }
          );
122
          // returning
125
          return std::move(output);
126
127
128
       /*-----
129
       y = ifft(x, nfft)
130
                                -----*/
131
       template<typename T>
132
       auto ifft(const
                         std::vector<T>& input_vector)
134
          // fetching data-type
135
136
          using RType = fft_result_t<T>;
137
          using baseType
                           = std::conditional_t<std::is_same_v<T, std::complex<double>>,
                                               double,
138
                                               T>;
139
          // setup
141
                            {input_vector.size()};
          auto
                 nfft
142
143
          // canvas instantiation
144
```

```
std::vector<RType> canvas(nfft);
145
                  nfft_sqrt
                                 {static_cast<RType>(std::sqrt(nfft))};
146
                                 {std::vector<RType>(nfft, 0)};
           auto
                  finaloutput
147
148
           // calculating index by index
149
           for(int frequency_index = 0; frequency_index<nfft; ++frequency_index){</pre>
150
              RType accumulate_value;
151
              for(int signal_index = 0; signal_index < input_vector.size(); ++signal_index){</pre>
                  accumulate_value += \
                      static_cast<RType>(input_vector[signal_index]) * \
154
                      static_cast<RType>(std::exp(1.00 * std::numbers::pi * \
                                               (static_cast<baseType>(frequency_index)/static_cast<baseType>(n
156
                                               static_cast<baseType>(signal_index)));
157
              }
158
              finaloutput[frequency_index] = accumulate_value / nfft_sqrt;
           }
161
162
           // returning
           return std::move(finaloutput);
163
164
165
       /*-----
166
       x = ifft(std::vector<std::complex<double>> spectrum, nfft)
167
           ------
       #include <fftw3.h>
169
       #include <vector>
170
       #include <complex>
171
       #include <stdexcept>
172
173
       auto ifft(const std::vector<std::complex<double>>& input_vector,
174
                const std::size_t
                                                     nfft)
176
           if (nfft <= 0)</pre>
              throw std::runtime_error("nfft must be > 0");
178
           if (input_vector.size() != nfft)
              throw std::runtime_error("input spectrum must be of size nfft");
180
181
           // Output: real-valued time-domain sequence
183
           std::vector<double> output(nfft);
184
           // Allocate FFTW input/output
185
           fftw_complex* in = reinterpret_cast<fftw_complex*>(
186
              fftw_malloc(sizeof(fftw_complex) * (nfft/2 + 1))
188
           double* out = reinterpret_cast<double*>(
189
              fftw_malloc(sizeof(double) * nfft)
           );
           // Copy *only* the first nfft/2+1 bins (rest are redundant due to symmetry)
193
194
           for (std::size_t i = 0; i < nfft/2 + 1; ++i) {</pre>
195
              in[i][0] = input_vector[i].real();
              in[i][1] = input_vector[i].imag();
196
           }
197
           // Create inverse FFTW plan
199
           fftw_plan plan = fftw_plan_dft_c2r_1d(
200
              static_cast<int>(nfft),
201
              in,
202
```

```
out,
203
                FFTW_ESTIMATE
204
            );
205
206
            fftw_execute(plan);
207
            // Normalize by nfft (FFTW leaves IFFT unscaled)
209
            for (std::size_t i = 0; i < nfft; ++i) {</pre>
210
                 output[i] = out[i] / static_cast<double>(nfft);
212
213
            // Cleanup
214
            fftw_destroy_plan(plan);
            fftw_free(in);
216
            fftw_free(out);
218
            return output;
220
221
222
    }
224
```

B.13 Flipping Containers

```
#pragma once
  namespace svr {
     /*-----
     Mirror image of a vector
     template <typename T>
          fliplr(const std::vector<T>& input_vector)
        // creating canvas
                      {input_vector};
        auto canvas
        // rewriting
        std::reverse(canvas.begin(), canvas.end());
        // returning
        return std::move(canvas);
16
     }
17
  }
```

B.14 Indexing

```
typename = std::enable_if_t<</pre>
             (std::is_arithmetic_v<T1>
             std::is_same_v<T1, std::complex<float> > ||
11
             std::is_same_v<T1, std::complex<double> >) &&
             std::is_integral_v<T2>
      -----*/
16
      template <typename T1,
                 typename T2,
18
                 typename = std::enable_if_t<std::is_arithmetic_v<T1>
                                                                              11
19
                                         std::is_same_v<T1, std::complex<float> > ||
2.0
                                         std::is_same_v<T1, std::complex<double> >
21
23
             index(const
                                               input_vector,
                          std::vector<T1>&
24
      auto
                 const
                          std::vector<T2>&
                                               indices_to_sample)
26
         // creating canvas
27
                          {std::vector<T1>(indices_to_sample.size(), 0)};
         auto canvas
         // copying the associated values
30
         for(int i = 0; i < indices_to_sample.size(); ++i){</pre>
31
             auto source_index {indices_to_sample[i]};
             if(source_index < input_vector.size()){</pre>
                canvas[i] = input_vector[source_index];
34
             }
35
             else{
                // cout << "warning: Some chosen samples are out of bounds. svr::index |
                    source_index !< input_vector.size()\n";</pre>
             }
38
39
         }
41
         // returning
42
         return std::move(canvas);
45
      y = index(matrix, mask, dim)
         -----*/
      template <
48
         typename T1,
49
         typename T2,
50
         typename = std::enable_if_t<</pre>
             (std::is_same_v<T1, double> || std::is_same_v<T1, float>) &&
52
             (std::is_same_v<T2, int> || std::is_same_v<T2, std::size_t>)
      auto index(const std::vector<std::vector<T1>>& input_matrix,
56
               const std::vector<T2>&
                                                  indices_to_sample,
57
                                                   dim)
58
               const std::size_t&
         // fetching dimensions
60
         const auto& num_rows_matrix {input_matrix.size()};
61
         const auto& num_cols_matrix {input_matrix[0].size()};
         // creating canvas
         auto canvas {std::vector<std::vector<T1>>()};
65
```

```
// if indices are row-indices
           if (dim == 0){
               // initializing canvas
70
               canvas = std::vector<std::vector<T1>>(
                   num_rows_matrix,
                   std::vector<T1>(indices_to_sample.size())
               );
               // filling the canvas
76
               auto destination_index {0};
               std::for_each(
78
                   indices_to_sample.begin(), indices_to_sample.end(),
                   [&](const auto& col){
80
                   for(auto row = 0; row < num_rows_matrix; ++row){</pre>
                       if (col <= input_matrix[0].size()){</pre>
                           canvas[row] [destination_index] = input_matrix[row] [col];
                   }
                   ++destination_index;
                   });
           }
           else if(dim == 1){
               // initializing canvas
               canvas = std::vector<std::vector<T1>>(
                   indices_to_sample.size(),
                   std::vector<T1>(num_cols_matrix)
93
               );
               // filling the canvas
               #pragma omp parallel for
               for(auto row = 0; row < canvas.size(); ++row){</pre>
                           destination_col {0};
                   std::for_each(indices_to_sample.begin(), indices_to_sample.end(),
100
                                 [&row,
101
                                  &input_matrix,
                                  &destination_col,
                                  &canvas](const auto& source_col){
104
                                       canvas[row][destination_col++] =
                                           input_matrix[row] [source_col];
                                 });
106
               }
107
           }
108
           else {
               std::cerr << "svr_index | this dim is not implemented \n";</pre>
110
           }
112
           // moving it back
           return std::move(canvas);
114
        }
116
    }
```

B.15 Linspace

1 /*----

Dependencies

```
-----*/
  #pragma once
  #include <vector>
  #include <complex>
  /*-----
10
  in-place
  -----*/
  template <typename T>
  auto linspace(auto&
                       input,
13
           const auto startvalue,
14
           const auto endvalue,
15
            const auto numpoints) -> void
16
17
     auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);
18
     for(int i = 0; i<input.size(); ++i) {input[i] = startvalue + i*stepsize;}</pre>
20
21
  in-place
  -----*/
  template <typename T>
  auto linspace(std::vector<std::complex<T>>& input,
25
                                   startvalue,
            const
                  auto
            const
                   auto
                                   endvalue,
            const
                  auto
                                   numpoints) -> void
28
 {
29
     auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);
30
     for(int i = 0; i<input.size(); ++i) {</pre>
31
        input[i] = startvalue + static_cast<T>(i)*stepsize;
32
33
  };
  /*-----
  -----*/
36
  template <typename T>
37
  auto linspace(const T
                            startvalue,
           const T
                             endvalue,
39
           const std::size_t numpoints)
40
  {
41
42
     std::vector<T> input(numpoints);
     auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);
43
     for(int i = 0; i<input.size(); ++i) {input[i] = startvalue +</pre>
44
        static_cast<T>(i)*stepsize;}
     return std::move(input);
45
46 }:
  /*-----
47
  template <typename T, typename U>
49
  auto linspace(const T startvalue,
50
           const U
                             endvalue,
51
52
           const std::size_t numpoints)
53
  {
     std::vector<double> input(numpoints);
54
     auto stepsize = static_cast<double>(endvalue -
55
        startvalue)/static_cast<double>(numpoints-1);
     for(int i = 0; i<input.size(); ++i) {input[i] = startvalue + i*stepsize;}</pre>
     return std::move(input);
57
58 };
```

B.16 Max

```
#pragma once
_{\rm 3} maximum along dimension 1
  template <std::size_t axis, typename T>
   auto max(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis</pre>
       == 1, std::vector<std::vector<T>> >
       // setting up canvas
       auto canvas
          {std::vector<std::vector<T>>(input_matrix.size(),std::vector<T>(1))};
11
      // filling up the canvas
      for(auto row = 0; row < input_matrix.size(); ++row)</pre>
          canvas[row][0] = *(std::max_element(input_matrix[row].begin(),
              input_matrix[row].end()));
      // returning
15
      return std::move(canvas);
16
17 }
```

B.17 Meshgrid

```
/*-----
  Dependencies
  -----*/
4 #pragma once
5 #include <vector> // for std::vector
6 #include <utility> // for std::pair
  #include <complex> // for std::complex
  /*----
10
mesh-grid when working with l-values
 -----*/
  template <typename T>
  auto meshgrid(const std::vector<T>& x,
           const std::vector<T>& y)
  {
16
     // creating and filling x-grid
18
     std::vector<std::vector<T>> xcanvas(y.size(), std::vector<T>(x.size(), 0));
     for(auto row = 0; row < y.size(); ++row)</pre>
20
       std::copy(x.begin(), x.end(), xcanvas[row].begin());
     // creating and filling y-grid
     std::vector<std::vector<T>> ycanvas(y.size(), std::vector<T>(x.size(), 0));
24
     for(auto col = 0; col < x.size(); ++col)</pre>
25
       for(auto row = 0; row < y.size(); ++row)</pre>
          ycanvas[row][col] = y[row];
2.8
    // returning
29
     return std::move(std::pair{xcanvas, ycanvas});
31
```

```
}
   /*-----
   meshgrid when working with r-values
34
35
  template <typename T>
36
   auto meshgrid(std::vector<T>&& x,
              std::vector<T>&& y)
38
39
40
41
      // creating and filling x-grid
      std::vector<std::vector<T>> xcanvas(y.size(), std::vector<T>(x.size(), 0));
42
      for(auto row = 0; row < y.size(); ++row)</pre>
43
          std::copy(x.begin(), x.end(), xcanvas[row].begin());
      // creating and filling y-grid
46
      std::vector<std::vector<T>> ycanvas(y.size(), std::vector<T>(x.size(), 0));
      for(auto col = 0; col < x.size(); ++col)</pre>
          for(auto row = 0; row < y.size(); ++row)</pre>
49
             ycanvas[row][col] = y[row];
50
51
      // returning
      return std::move(std::pair{xcanvas, ycanvas});
53
54
  }
55
```

B.18 Minimum

```
#pragma once
  /*----
  minimum along dimension 1
  template <std::size_t axis, typename T>
  auto min(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis ==</pre>
      1, std::vector<std::vector<T>> >
      // creating canvas
      auto canvas
         {std::vector<std::vector<T>>(input_matrix.size(),std::vector<T>(1))};
     // storing the values
11
     for(auto row = 0; row < input_matrix.size(); ++row)</pre>
         canvas[row][0] = *(std::min_element(input_matrix[row].begin(),
            input_matrix[row].end()));
     // returning the value
15
     return std::move(canvas);
16
  }
```

B.19 Norm

```
template <typename T>
  auto norm(const std::vector<T>&
                                  input_vector)
  {
     return std::sqrt(
         std::inner_product(
9
            input_vector.begin(), input_vector.end(),
            input_vector.begin(),
11
            (T)0
         )
      );
14
15 }
16 /*-----
17 Calculating norm of a complex-vector
  -----*/
  template <>
19
  auto norm(const std::vector<std::complex<double>>& input_vector)
20
21
      return std::sqrt(
22
         std::inner_product(
23
            input_vector.begin(), input_vector.end(),
            input_vector.begin(),
            static_cast<double>(0),
26
            std::plus<double>(),
            [](const auto& argx,
               const auto& argy){
                return static_cast<double>(
30
                   (argx * std::conj(argy)).real()
31
                );
            }
         )
34
      );
35
36
37
38
30
  template <typename T>
       norm(const std::vector<std::vector<T>>& input_matrix,
41
             const std::size_t
                                             dim)
42
43
      // creating canvas
                      {std::vector<std::vector<T>>()};
      auto
           canvas
45
      const auto& num_rows_matrix {input_matrix.size()};
46
      const auto& num_cols_matrix {input_matrix[0].size()};
47
      // along dim 0
49
     if(dim == 0)
50
         // allocate canvas
         canvas = std::vector<std::vector<T>>(
            std::vector<T>(input_matrix[0].size())
         );
         // performing norm
         auto accumulate_vector {std::vector<T>(input_matrix[0].size())};
         // going through each row
         for(auto row = 0; row < num_rows_matrix; ++row)</pre>
62
         {
```

```
std::transform(input_matrix[row].begin(), input_matrix[row].end(),
                              accumulate_vector.begin(),
65
                              accumulate_vector.begin(),
66
                              [](const auto& argx, auto& argy){
67
                                   return argx*argx + argy;
                              });
            }
70
            // calculating element-wise square root
            std::for_each(accumulate_vector.begin(), accumulate_vector.end(),
73
                         [](auto& argx){
74
                               argx = std::sqrt(argx);
75
                         });
            // moving to the canvas
78
            canvas[0] = std::move(accumulate_vector);
79
        else if (dim == 1)
81
82
83
            // allocating space in the canvas
                     = std::vector<std::vector<T>>(
               input_matrix[0].size(),
85
               std::vector<T>(1, 0)
86
            );
            // going through each column
89
            for(auto row = 0; row < num_cols_matrix; ++row){</pre>
90
               canvas[row][0] = norm(input_matrix[row]);
91
           }
93
       }
94
       else
95
96
            std::cerr << "norm(matrix, dim): dimension operation not defined \n";</pre>
97
98
99
        // returning
100
        return std::move(canvas);
101
    }
102
103
104
105
106
    Templates to create
       - matrix and norm-axis
108
           axis instantiated std::vector<T>
109
    */
110
```

B.20 Division

```
T&
                                         input_scalar)
                const
  {
9
      // creating canvas
      auto canvas
                       {input_vector};
11
      // filling canvas
      std::transform(canvas.begin(), canvas.end(),
                   canvas.begin(),
                    [&input_scalar](const auto& argx){
                        return static_cast<double>(argx) /
                           static_cast<double>(input_scalar);
                   });
17
      // returning value
      return std::move(canvas);
20
  }
21
  element-wise division with scalars
24
25
  template <typename T>
  auto operator/=(const std::vector<T>& input_vector,
                const
                                          input_scalar)
27
28
      // creating canvas
29
      auto canvas {input_vector};
31
      // filling canvas
32
      std::transform(canvas.begin(), canvas.end(),
                   canvas.begin(),
                    [&input_scalar](const auto& argx){
35
                        return static_cast<double>(argx) /
                           static_cast<double>(input_scalar);
                   });
38
      // returning value
30
      return std::move(canvas);
 }
41
  element-wise with matrix
   -----*/
  template <typename T>
45
  auto operator/(const std::vector<std::vector<T>>& input_matrix,
46
                  const T
                                                   scalar)
47
48
      // fetching matrix-dimensions
49
      const auto& num_rows_matrix {input_matrix.size()};
50
      const auto& num_cols_matrix {input_matrix[0].size()};
51
      // creating canvas
53
                    {std::vector<std::vector<T>>(
      auto
           canvas
54
55
         num_rows_matrix,
          std::vector<T>(num_cols_matrix)
57
      )};
58
      // dividing with values
      for(auto
               row = 0; row < num_rows_matrix; ++row){</pre>
          std::transform(input_matrix[row].begin(), input_matrix[row].end(),
61
                       canvas[row].begin(),
62
                       [&scalar](const auto& argx){
```

B.21 Addition

```
#pragma once
  /*----
  y = vector + vector
  template <typename T>
  std::vector<T> operator+(const std::vector<T>& a,
                      const std::vector<T>& b)
8
     // Identify which is bigger
     const auto& big = (a.size() > b.size()) ? a : b;
     const auto& small = (a.size() > b.size()) ? b : a;
     std::vector<T> result = big; // copy the bigger one
     // Add elements from the smaller one
     for (size_t i = 0; i < small.size(); ++i) {</pre>
        result[i] += small[i];
19
     return result;
20
  }
21
  /*-----
23
  // y = vector + vector
24
  template <typename T>
  std::vector<T>& operator+=(std::vector<T>& a,
26
                       const std::vector<T>& b) {
28
     const auto& small = (a.size() < b.size()) ? a : b;</pre>
     const auto& big = (a.size() < b.size()) ? b : a;</pre>
30
31
     // If b is bigger, resize 'a' to match
     if (a.size() < b.size())</pre>
                                           {a.resize(b.size());}
     // Add elements
35
     for (size_t i = 0; i < small.size(); ++i) {a[i] += b[i];}</pre>
     // returning elements
38
     return a;
39
40 }
  // y = matrix + matrix
43 template <typename T>
44 std::vector<std::vector<T>> operator+(const std::vector<std::vector<T>>& a,
                                const std::vector<std::vector<T>>& b)
  {
46
```

```
// fetching dimensions
47
       const
              auto& num_rows_A
                                      {a.size()};
48
       const
               auto&
                      num\_cols\_A
                                      {a[0].size()};
49
               auto& num_rows_B
                                      {b.size()};
       const
50
              auto& num_cols_B
                                      {b[0].size()};
       const
51
       // choosing the three different metrics
       if (num_rows_A != num_rows_B && num_cols_A != num_cols_B){
           cout << format("a.dimensions = [{},{}], b.shape = [{},{}]\n",
                          num_rows_A, num_cols_A,
                          num_rows_B, num_cols_B);
57
           std::cerr << "dimensions don't match\n";</pre>
58
       }
59
60
       // creating canvas
61
                          {std::vector<std::vector<T>>(
       auto
               canvas
62
           std::max(num_rows_A, num_rows_B),
           std::vector<T>(std::max(num_cols_A, num_cols_B), (T)0.00)
64
       )};
65
66
       // performing addition
       if (num_rows_A == num_rows_B && num_cols_A == num_cols_B){
68
           for(auto row = 0; row < num_rows_A; ++row){</pre>
69
               std::transform(a[row].begin(), a[row].end(),
70
                             b[row].begin(),
                             canvas[row].begin(),
72
                             std::plus<T>());
73
           }
74
       }
75
76
       else if(num_rows_A == num_rows_B){
           // if number of columsn are different, check if one of the cols are one
                          min_num_cols {std::min(num_cols_A, num_cols_B)};
           if (min_num_cols != 1) {std::cerr<< "Operator+: unable to broadcast\n";}</pre>
80
                          max_num_cols {std::max(num_cols_A, num_cols_B)};
                  auto
81
           // using references to tag em differently
           const
                  auto& big_matrix
                                         {num_cols_A > num_cols_B ? a : b};
                  auto& small_matrix {num_cols_A < num_cols_B ? a : b};</pre>
           const
           // Adding to canvas
87
           for(auto row = 0; row < canvas.size(); ++row){</pre>
88
               std::transform(big_matrix[row].begin(), big_matrix[row].end(),
89
                             canvas[row].begin(),
                             [&small_matrix,
91
                              &row](const auto& argx){
92
                                  return argx + small_matrix[row][0];
93
                             });
           }
95
96
97
       else if(num_cols_A == num_cols_B){
           // check if the smallest column-number is one
99
                         min_num_rows {std::min(num_rows_A, num_rows_B)};
           const auto
100
                                     {std::cerr << "Operator+ : unable to broadcast\n";}
           if(min_num_rows != 1)
                          max_num_rows {std::max(num_rows_A, num_rows_B)};
102
           // using references to differentiate the two matrices
104
           const auto& big_matrix {num_rows_A > num_rows_B ? a : b};
105
```

```
const auto& small_matrix {num_rows_A < num_rows_B ? a : b};</pre>
106
107
          // adding to canvas
108
          for(auto row = 0; row < canvas.size(); ++row){</pre>
109
              std::transform(big_matrix[row].begin(), big_matrix[row].end(),
                           small_matrix[0].begin(),
                           canvas[row].begin(),
112
                           [](const auto& argx, const auto& argy){
113
                            return argx + argy;
                           });
          }
       }
117
      else {
118
          std::cerr << "operator+: yet to be implemented \n";</pre>
119
120
       // returning
       return std::move(canvas);
124
   /*-----
125
   y = vector + scalar
126
   -----*/
127
   template <typename T>
128
   auto operator+(const std::vector<T>& input_vector,
129
130
                const
                        Т
                                         scalar)
   {
131
       // creating canvas
132
            canvas
                        {input_vector};
133
       auto
134
       // adding scalar to the canvas
135
       std::transform(canvas.begin(), canvas.end(),
136
                    canvas.begin(),
137
                    [&scalar](auto& argx){return argx + scalar;});
138
139
       // returning canvas
140
       return std::move(canvas);
141
   }
142
143
   y = scalar + vector
   template <typename T>
146
   auto operator+(const T
                                         scalar,
147
                const
                        std::vector<T>& input_vector)
148
149
       // creating canvas
150
       auto
            canvas
                        {input_vector};
       // adding scalar to the canvas
       std::transform(canvas.begin(), canvas.end(),
154
                    canvas.begin(),
                    [&scalar](auto& argx){return argx + scalar;});
156
157
       // returning canvas
158
       return std::move(canvas);
159
   }
160
```

B.22 Multiplication (Element-wise)

```
#pragma once
  /*-----
  y = scalar * vector
      -----*/
  template <typename T>
  auto operator*(const T
                                   scalar,
              const
                    std::vector<T>& input_vector)
     // creating canvas
     auto canvas {input_vector};
     // performing operation
     std::for_each(canvas.begin(), canvas.end(),
              [&scalar](auto& argx){argx = argx * scalar;});
     // returning
     return std::move(canvas);
  }
16
  /*-----
  y = scalar * vector
  template <typename T1, typename T2,
        typename = std::enable_if_t<!std::is_same_v<std::decay_t<T1>, std::vector<T2>>>>
21
  auto operator*(const T1
                               scalar,
            const vector<T2>& input_vector)
23
24
     // fetching final-type
25
     using T3 = decltype(std::declval<T1>() * std::declval<T2>());
26
     // creating canvas
     auto
         canvas
                  {std::vector<T3>(input_vector.size())};
    // multiplying
     std::transform(input_vector.begin(), input_vector.end(),
               canvas.begin(),
               [&scalar](auto& argx){
32
                return static_cast<T3>(scalar) * static_cast<T3>(argx);
33
               });
34
     // returning
     return std::move(canvas);
36
37 }
  /*-----
  y = vector * scalar
  ----*/
40
  template <typename T>
41
  auto operator*(const std::vector<T>& input_vector,
              const
                                   scalar)
43
44
    // creating canvas
45
         canvas
                 {input_vector};
    auto
     // multiplying
     std::for_each(canvas.begin(), canvas.end(),
48
              [&scalar](auto& argx){
49
                argx = argx * scalar;
51
     // returning
52
     return std::move(canvas);
53
55
  y = vector * vector
                -----*/
```

```
template <typename T>
   auto operator*(const std::vector<T>& input_vector_A,
               const std::vector<T>& input_vector_B)
60
   {
61
      // throwing error: size-desparity
62
      if (input_vector_A.size() != input_vector_B.size()) {std::cerr << "operator*: size</pre>
         disparity \n";}
      // creating canvas
      auto
           canvas
                    {input_vector_A};
66
67
      // element-wise multiplying
68
      std::transform(input_vector_B.begin(), input_vector_B.end(),
69
                  canvas.begin(),
70
                  canvas.begin(),
71
                  [](const auto& argx, const auto& argy){
                      return argx * argy;
                  });
74
75
      // moving it back
76
      return std::move(canvas);
77
  }
78
   /*-----
79
   -----*/
   template <typename T1, typename T2>
81
   auto operator*(const std::vector<T1>& input_vector_A,
82
                 const std::vector<T2>& input_vector_B)
83
84
  {
85
86
      // checking size disparity
      if (input_vector_A.size() != input_vector_B.size())
87
         std::cerr << "operator*: error, size-disparity \n";</pre>
      // figuring out resulting data type
90
                = decltype(std::declval<T1>() * std::declval<T2>());
      using T3
91
      // creating canvas
93
      auto canvas
                     {std::vector<T3>(input_vector_A.size())};
      // performing multiplications
      std::transform(input_vector_A.begin(), input_vector_A.end(),
97
                  input_vector_B.begin(),
98
                  canvas.begin(),
99
                  [](const auto&
                                    argx,
100
                    const
                           auto&
                                    argy){
101
                      return static_cast<T3>(argx) * static_cast<T3>(argy);
102
                  });
104
      // returning
105
      return std::move(canvas);
106
107
108
   /*-----
109
   template <typename T>
112
   auto operator*(const T
                                           scalar.
113
              const std::vector<std::vector<T>>& inputMatrix)
114
   {
115
```

```
std::vector<std::vector<T>> temp {inputMatrix};
116
       for(int i = 0; i<inputMatrix.size(); ++i){</pre>
117
           std::transform(inputMatrix[i].begin(),
118
                         inputMatrix[i].end(),
119
                         temp[i].begin(),
120
                         [&scalar](T x){return scalar * x;});
121
122
       return std::move(temp);
123
124
   }
125
   /*-----
   y = matrix * scalar
126
127
   template <typename T>
128
           operator*(const
                              std::vector<std::vector<T>>& input_matrix,
129
                    const
                                                           scalar)
130
131
       // fetching matrix dimensions
       const auto& num_rows_matrix
                                         {input_matrix.size()};
133
       const auto& num_cols_matrix
                                         {input_matrix[0].size()};
134
135
       // creating canvas
136
                          {std::vector<std::vector<T>>(
       auto
             canvas
137
           num_rows_matrix,
138
           std::vector<T>(num_cols_matrix)
139
       )};
141
       // storing the values
142
       for(auto row = 0; row < num_rows_matrix; ++row)</pre>
143
           std::transform(input_matrix[row].begin(), input_matrix[row].end(),
144
                         canvas[row].begin(),
145
                         [&scalar](const auto& argx){
146
                              return argx * scalar;
147
                         });
148
149
       // returning
150
       return std::move(canvas);
   }
152
154
   y = matrix * matrix
   template <typename T>
156
   auto operator*(const std::vector<std::vector<T>>& A,
157
                 const std::vector<std::vector<T>>& B) -> std::vector<std::vector<T>>
158
   {
159
       // Case 1: element-wise multiplication
160
       if (A.size() == B.size() && A[0].size() == B[0].size()) {
161
           std::vector<std::vector<T>> C(A.size(), std::vector<T>(A[0].size()));
           for (std::size_t row = 0; row < A.size(); ++row) {</pre>
163
               std::transform(A[row].begin(), A[row].end(),
                             B[row].begin(),
165
166
                             C[row].begin(),
167
                             [](const auto& x, const auto& y){ return x * y; });
           }
168
           return C;
169
170
171
       // Case 2: broadcast column vector
       else if (A.size() == B.size() && B[0].size() == 1) {
173
           std::vector<std::vector<T>> C(A.size(), std::vector<T>(A[0].size()));
174
```

```
for (std::size_t row = 0; row < A.size(); ++row) {</pre>
175
              std::transform(A[row].begin(), A[row].end(),
176
                            C[row].begin(),
                            [&](const auto& x){ return x * B[row][0]; });
178
           }
179
           return C;
180
181
182
       // case 3: when second matrix contains just one row
       // case 4: when first matrix is just one column
184
       // case 5: when second matrix is just one column
185
186
       // Otherwise, invalid
187
       else {
188
           throw std::runtime_error("operator* dimension mismatch");
189
190
191
192
193
   y = scalar * matrix
194
   template <typename T1, typename T2>
195
   auto operator*(const T1
                                                  scalar,
196
                 const std::vector<std::vector<T2>>& inputMatrix)
197
198
199
       std::vector<std::vector<T2>> temp {inputMatrix};
       for(int i = 0; i<inputMatrix.size(); ++i){</pre>
200
           std::transform(inputMatrix[i].begin(),
2.01
                        inputMatrix[i].end(),
202
                        temp[i].begin(),
203
                        [&scalar](T2 x){return static_cast<T2>(scalar) * x;});
204
       }
205
       return temp;
206
207
   /*-----
208
   matrix-multiplication
209
   -----*/
   template <typename T1, typename T2>
211
   auto matmul(const std::vector<std::vector<T1>>& matA,
              const std::vector<std::vector<T2>>& matB)
213
214
215
       // throwing error
216
       if (matA[0].size() != matB.size()) {std::cerr << "dimension-mismatch \n";}</pre>
217
218
       // getting result-type
219
       using ResultType = decltype(std::declval<T1>() * std::declval<T2>() + \
220
                                   std::declval<T1>() * std::declval<T2>() );
221
       // creating aliasses
223
       auto finalnumrows {matA.size()};
224
225
       auto finalnumcols {matB[0].size()};
226
       // creating placeholder
227
       auto rowcolproduct = [&](auto rowA, auto colB){
228
           ResultType temp {0};
229
           for(int i = 0; i < matA.size(); ++i) {temp +=</pre>
230
              static_cast<ResultType>(matA[rowA][i]) +
              static_cast<ResultType>(matB[i][colB]);}
           return temp;
231
```

```
};
232
233
      // producing row-column combinations
234
      std::vector<std::vector<ResultType>> finaloutput(finalnumrows,
          std::vector<ResultType>(finalnumcols));
      for(int row = 0; row < finalnumrows; ++row){for(int col = 0; col < finalnumcols;</pre>
          ++col){finaloutput[row][col] = rowcolproduct(row, col);}}
237
      // returning
      return finaloutput;
239
   }
240
   /*-----
241
   y = matrix * vector
   -----*/
243
   template <typename T>
244
   auto operator*(const std::vector<std::vector<T>> input_matrix,
245
               const
                       std::vector<T>
                                                 input_vector)
246
247
      // fetching dimensions
248
      const auto& num_rows_matrix {input_matrix.size()};
249
      const auto& num_cols_matrix {input_matrix[0].size()};
250
                                    {1};
      const auto& num_rows_vector
251
      const auto& num_cols_vector
                                    {input_vector.size()};
252
253
      const auto& max_num_rows
                                    {num_rows_matrix > num_rows_vector ?\
                                    num_rows_matrix : num_rows_vector};
255
                                    {num_cols_matrix > num_cols_vector ?\
      const auto& max_num_cols
256
                                    num_cols_matrix : num_cols_vector};
257
258
      // creating canvas
259
      auto
           canvas
                       {std::vector<std::vector<T>>(
260
         max_num_rows,
261
          std::vector<T>(max_num_cols, 0)
262
      )};
263
264
      if (num_cols_matrix == 1 && num_rows_vector == 1){
266
267
          // writing to canvas
          for(auto row = 0; row < max_num_rows; ++row)</pre>
             for(auto col = 0; col < max_num_cols; ++col)</pre>
270
                canvas[row][col] = input_matrix[row][0] * input_vector[col];
271
      }
272
      else{
          std::cerr << "Operator*: [matrix, vector] | not implemented \n";</pre>
274
      // returning
      return std::move(canvas);
278
280
   /*-----
281
   scalar operators
282
                -----*/
283
   auto operator*(const std::complex<double> complexscalar,
284
               const double
                                       doublescalar){
285
      return complexscalar * static_cast<std::complex<double>>(doublescalar);
286
   }
287
   auto operator*(const double
                                       doublescalar,
```

```
const std::complex<double> complexscalar){
289
       return complexscalar * static_cast<std::complex<double>>(doublescalar);
290
   }
291
   auto operator*(const std::complex<double> complexscalar,
2.92
                  const int
                                             scalar){
293
       return complexscalar * static_cast<std::complex<double>>(scalar);
   }
295
   auto operator*(const int
296
                                             scalar,
                 const std::complex<double> complexscalar){
298
       return complexscalar * static_cast<std::complex<double>>(scalar);
   }
299
```

B.23 Subtraction

```
#pragma once
  /*=========
                   _____
  y = vector - scalar
  template <typename T>
  auto operator-(const std::vector<T>& a,
            const T
                               scalar){
     std::vector<T> temp(a.size());
     std::transform(a.begin(),
10
                a.end(),
                temp.begin(),
11
                [scalar](T x){return (x - scalar);});
12
     return std::move(temp);
13
14 }
  /*----
15
  y = vector - vector
     -----*/
  template <typename T>
18
  auto operator-(const std::vector<T>& input_vector_A,
19
            const std::vector<T>& input_vector_B)
2.0
21
     // throwing error
22
     if (input_vector_A.size() != input_vector_B.size())
        std::cerr << "operator-(vector, vector): size disparity\n";</pre>
     // creating canvas
26
     const auto& num_cols {input_vector_A.size()};
27
     auto
                canvas
                         {std::vector<T>()};
28
     // peforming operations
30
     std::transform(input_vector_A.begin(), input_vector_A.begin(),
31
                input_vector_B.begin(),
                canvas.begin(),
                [](const auto& argx, const auto& argy){
34
                   return argx - argy;
35
                });
     // return
38
     return std::move(canvas);
39
  }
  /*-----
 y = matrix - matrix
```

```
-----*/
   template <typename T>
   auto operator-(const std::vector<std::vector<T>>& input_matrix_A,
45
                         std::vector<std::vector<T>>& input_matrix_B)
                 const
46
   {
47
       // fetching dimensions
       const auto& num_rows_A {input_matrix_A.size()};
49
       const auto& num_cols_A {input_matrix_A[0].size()};
50
       const auto& num_rows_B {input_matrix_B.size()};
       const auto& num_cols_B {input_matrix_B[0].size()};
52
       // creating canvas
54
                         {std::vector<std::vector<T>>()};
       auto canvas
       // if both matrices are of equal dimensions
57
       if (num_rows_A == num_rows_B && num_cols_A == num_cols_B)
          // copying one to the canvas
60
          canvas = input_matrix_A;
61
          // subtracting
          for(auto row = 0; row < num_rows_B; ++row)</pre>
              std::transform(canvas[row].begin(), canvas[row].end(),
                            input_matrix_B[row].begin(),
                            canvas[row].begin(),
                            [](auto& argx, const auto& argy){
68
                                 return argx - argy;
69
                            });
71
72
       // column broadcasting (case 1)
       else if(num_rows_A == num_rows_B && num_cols_B == 1)
73
          // copying canvas
          canvas = input_matrix_A;
76
          // substracting
          for(auto row = 0; row < num_rows_A; ++row){</pre>
              std::transform(canvas[row].begin(), canvas[row].end(),
                            canvas[row].begin(),
                            [&input_matrix_B,
                             &row](auto& argx){
83
                                 return argx - input_matrix_B[row][0];
84
                            });
          }
       }
87
       else{
88
          std::cerr << "operator-: not implemented for this case \n";</pre>
91
       // returning
92
      return std::move(canvas);
93
  }
```

B.24 Printing Containers

```
/*----
  template<typename T>
  void fPrintVector(const vector<T> input){
     for(auto x: input) cout << x << ",";</pre>
     cout << endl;</pre>
  }
8
  template<typename T>
  void fpv(vector<T> input){
11
     for(auto x: input) cout << x << ",";</pre>
     cout << endl;</pre>
13
14 }
15 /*-----
17 template<typename T>
  void fPrintMatrix(const std::vector<std::vector<T>> input_matrix){
     for(const auto& row: input_matrix)
19
       cout << format("{}\n", row);</pre>
20
21 }
  /*-----
24 template <typename T>
 void fPrintMatrix(const string&
                                     input_string,
              const std::vector<std::vector<T>> input_matrix){
    cout << format("{} = \n", input_string);</pre>
   for(const auto& row: input_matrix)
2.8
      cout << format("{}\n", row);</pre>
29
30 }
31 /*-----
  -----*/
  template<typename T, typename T1>
  void fPrintHashmap(unordered_map<T, T1> input){
     for(auto x: input){
35
       cout << format("[{},{}] | ", x.first, x.second);</pre>
36
     cout <<endl;</pre>
38
39 }
  /*-----
    void fPrintBinaryTree(TreeNode* root){
42
   // sending it back
43
    if (root == nullptr) return;
44
    // printing
46
    PRINTLINE
47
     cout << "root->val = " << root->val << endl;</pre>
    // calling the children
50
     fPrintBinaryTree(root->left);
51
     fPrintBinaryTree(root->right);
     // returning
     return;
55
60 void fPrintLinkedList(ListNode* root){
```

```
if (root == nullptr) return;
    cout << root->val << " -> ";
    fPrintLinkedList(root->next);
64
    return:
65 }
 /*-----
 -----*/
68 template<typename T>
 void fPrintContainer(T input){
    for(auto x: input) cout << x << ", ";</pre>
    cout << endl;</pre>
71
    return;
72
73 }
74 /*-----
 -----*/
76 template <typename T>
 auto size(std::vector<std::vector<T>> inputMatrix){
    cout << format("[{}, {}]\n",</pre>
78
            inputMatrix.size(),
79
            inputMatrix[0].size());
80
81 }
 /*-----
82
 -----*/
84 template <typename T>
 auto size(const std::string&
                              inputstring,
  const std::vector<std::vector<T>>& inputMatrix){
86
   cout << format("{} = [{}, {}]\n",
87
           inputstring,
            inputMatrix.size(),
90
            inputMatrix[0].size());
91 }
```

B.25 Random Number Generation

```
#pragma once
  template <typename T>
  auto rand(const T min,
      const T max) {
    static std::random_device rd; // Seed
    static std::mt19937 gen(rd()); // Mersenne Twister generator
    std::uniform_real_distribution<> dist(min, max);
    return dist(gen);
10
11 }
-----*/
  template <typename T>
14
15 auto rand(const T
                         min,
    const T
                         max,
17
       const std::size_t numelements)
18 {
    static std::random_device rd; // Seed
19
    static std::mt19937 gen(rd()); // Mersenne Twister generator
    std::uniform_real_distribution<> dist(min, max);
21
22
```

```
// building the fianloutput
      vector<T> finaloutput(numelements);
      for(int i = 0; i<finaloutput.size(); ++i) {finaloutput[i] =</pre>
          static_cast<T>(dist(gen));}
26
      return finaloutput;
28 }
  /*----
29
   -----*/
  template <typename T>
31
32 auto rand(const T
                                  argmin,
     const T
                                  argmax,
33
          const std::vector<int> dimensions)
34
35 {
36
      // throwing an error if dimension is greater than two
37
      if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}</pre>
39
      // creating random engine
40
      static std::random_device rd; // Seed
41
      static std::mt19937 gen(rd()); // Mersenne Twister generator
42
      std::uniform_real_distribution<> dist(argmin, argmax);
43
      // building the finaloutput
      vector<vector<T>> finaloutput;
      for(int i = 0; i<dimensions[0]; ++i){</pre>
47
         vector<T> temp;
48
         for(int j = 0; j<dimensions[1]; ++j) {temp.push_back(dist(gen));}</pre>
         // cout << format("\t\t temp = {}\n", temp);
51
          finaloutput.push_back(temp);
      }
53
      // returning the finaloutput
55
      return finaloutput;
56
58 }
61
   auto rand(const std::vector<int> dimensions)
62
      using ReturnType = double;
63
64
      // throwing an error if dimension is greater than two
      if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}</pre>
66
67
      // creating random engine
      static std::random_device rd; // Seed
69
      static std::mt19937 gen(rd()); // Mersenne Twister generator
70
      std::uniform_real_distribution<> dist(0.00, 1.00);
71
      // building the finaloutput
      vector<vector<ReturnType>> finaloutput;
      for(int i = 0; i<dimensions[0]; ++i){</pre>
          vector<ReturnType> temp;
          for(int j = 0; j<dimensions[1]; ++j) {temp.push_back(dist(gen));}</pre>
          finaloutput.push_back(std::move(temp));
78
      }
79
```

```
// returning the finaloutput
      return std::move(finaloutput);
83
84
   /*-----
     template <typename T>
87
   auto rand_complex_double(const T
88
                                             argmin,
                       const T
                                             argmax,
                       const std::vector<int>& dimensions)
90
   {
91
92
      // throwing an error if dimension is greater than two
93
      if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}</pre>
      // creating random engine
      static std::random_device rd; // Seed
      static std::mt19937 gen(rd()); // Mersenne Twister generator
98
      std::uniform_real_distribution<> dist(argmin, argmax);
99
100
      // building the finaloutput
101
      vector<vector<complex<double>>> finaloutput;
      for(int i = 0; i < dimensions[0]; ++i){</pre>
103
          vector<complex<double>> temp;
          for(int j = 0; j<dimensions[1]; ++j)</pre>
             {temp.push_back(static_cast<double>(dist(gen)));}
          finaloutput.push_back(std::move(temp));
106
107
108
      // returning the finaloutput
109
      return finaloutput;
110
   }
111
```

B.26 Reshape

```
#pragma once
  reshaping a matrix into another matrix
  template <std::size_t M, std::size_t N, typename T>
  auto reshape(const std::vector<std::vector<T>>& input_matrix){
     // verifying size stuff
     if (M*N != input_matrix.size() * input_matrix[0].size())
        std::cerr << "Dimensions are quite different\n";</pre>
     // creating canvas
          canvas
                   {std::vector<std::vector<T>>(
     auto
        M, std::vector<T>(N, (T)0)
     // writing to canvas
                         {0};
     size_t tid
     size_t
            target_row
                         {0};
20
     size_t target_col
                         {0};
21
```

```
for(auto row = 0; row<input_matrix.size(); ++row){</pre>
22
         for(auto col = 0; col < input_matrix[0].size(); ++col){</pre>
23
                         = row * input_matrix[0].size() + col;
            tid
24
            target_row
                         = tid/N;
25
            target_col
                         = tid%N;
            canvas[target_row][target_col] = input_matrix[row][col];
2.8
      }
29
31
      // moving it back
      return std::move(canvas);
32
33 }
34 /*============================
35 reshaping a matrix into a vector
  -----*/
36
  template<std::size_t M, typename T>
37
  auto reshape(const std::vector<std::vector<T>>& input_matrix){
39
      // checking element-count validity
40
      if (M != input_matrix.size() * input_matrix[0].size())
41
         std::cerr << "Number of elements differ\n";</pre>
43
      // creating canvas
      auto canvas
                      {std::vector<T>(M, 0)};
      // filling canvas
47
      for(auto row = 0; row < input_matrix.size(); ++row)</pre>
48
         for(auto col = 0; col < input_matrix[0].size(); ++col)</pre>
49
            canvas[row * input_matrix.size() + col] = input_matrix[row][col];
51
      // moving it back
52
      return std::move(canvas);
53
  }
54
  /*-----
55
  Matrix to matrix
                   -----*/
  template<typename T>
58
  auto reshape(const std::vector<std::vector<T>>& input_matrix,
             const std::size_t
60
                                          Μ,
                                          N){
             const std::size_t
62
      // checking element-count validity
63
      if ( M * N != input_matrix.size() * input_matrix[0].size())
64
         std::cerr << "Number of elements differ\n";</pre>
66
      // creating canvas
67
      auto
           canvas
                    {std::vector<std::vector<T>>(
         M, std::vector<T>(N, (T)0)
      )};
70
71
      // writing to canvas
72
      size_t tid
                           {0};
      size_t
              target_row
                           {0};
74
                           {0};
      size_t
              target_col
75
      for(auto row = 0; row<input_matrix.size(); ++row){</pre>
         for(auto col = 0; col < input_matrix[0].size(); ++col){</pre>
                         = row * input_matrix[0].size() + col;
            tid
78
                         = tid/N;
            target_row
79
            target_col
                       = tid%N;
```

```
canvas[target_row][target_col] = input_matrix[row][col];
83
84
      // moving it back
      return std::move(canvas);
87
  /*-----
   converting a matrix into a vector
   -----*/
   template<typename T>
91
   auto reshape(const std::vector<std::vector<T>>& input_matrix,
             const size_t
      // checking element-count validity
95
      if (M != input_matrix.size() * input_matrix[0].size())
         std::cerr << "Number of elements differ\n";</pre>
98
      // creating canvas
      auto canvas {std::vector<T>(M, 0)};
      // filling canvas
      for(auto row = 0; row < input_matrix.size(); ++row)</pre>
103
         for(auto col = 0; col < input_matrix[0].size(); ++col)</pre>
            canvas[row * input_matrix.size() + col] = input_matrix[row][col];
106
      // moving it back
107
      return std::move(canvas);
108
  }
109
```

B.27 Summing with containers

```
#pragma once
  /*-----
  template <std::size_t axis, typename T>
  auto sum(const std::vector<T>& input_vector) -> std::enable_if_t<axis == 0,</pre>
     std::vector<T>>
     // returning the input as is
     return input_vector;
8
  }
9
 /*-----
  template <std::size_t axis, typename T>
  auto sum(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis == 0,</pre>
     std::vector<T>>
14
     // creating canvas
     auto canvas {std::vector<T>(input_matrix[0].size(), 0)};
     // filling up the canvas
     for(auto row = 0; row < input_matrix.size(); ++row)</pre>
        std::transform(input_matrix[row].begin(), input_matrix[row].end(),
                    canvas.begin(),
21
                    canvas.begin(),
```

```
[](auto& argx, auto& argy){return argx + argy;});
23
      // returning
25
      return std::move(canvas);
26
2.7
  /*----
29
      -----*/
  template <std::size_t axis, typename T>
  auto sum(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis == 1,</pre>
      std::vector<std::vector<T>>>
  {
33
      // creating canvas
34
      auto canvas {std::vector<std::vector<T>>(input_matrix.size(),
35
                                          std::vector<T>(1, 0.00))};
36
37
      // filling up the canvas
      for(auto row = 0; row < input_matrix.size(); ++row)</pre>
39
         canvas[row][0] = std::accumulate(input_matrix[row].begin(),
40
                                       input_matrix[row].end(),
41
                                       static_cast<T>(0));
43
      // returning
44
      return std::move(canvas);
47
  /*-----
48
 template <std::size_t axis, typename T>
51
  auto sum(const std::vector<T>& input_vector_A,
         const std::vector<T>& input_vector_B) -> std::enable_if_t<axis == 0,</pre>
             std::vector<T> >
53
      // setup
54
                               {input_vector_A.size()};
      const auto& num_cols_A
55
      const auto& num_cols_B
                               {input_vector_B.size()};
57
      // throwing errors
      if (num_cols_A != num_cols_B) {std::cerr << "sum: size disparity\n";}</pre>
      // creating canvas
61
      auto canvas {input_vector_A};
62
63
      // summing up
      std::transform(input_vector_B.begin(), input_vector_B.end(),
65
                  canvas.begin(),
66
                  canvas.begin(),
                  std::plus<T>());
69
      // returning
70
      return std::move(canvas);
71
  }
```

B.28 Tangent

```
namespace svr {
     /*-----
     y = tan-inverse(input_vector_A/input_vector_B)
     template <typename T>
     auto atan2(const std::vector<T>
                                  input_vector_A,
              const std::vector<T>
                                  input_vector_B)
        // throw error
        if (input_vector_A.size() != input_vector_B.size())
11
            std::cerr << "atan2: size disparity\n";</pre>
        // create canvas
        auto canvas
                        {std::vector<T>(input_vector_A.size(), 0)};
        // performing element-wise atan2 calculation
        std::transform(input_vector_A.begin(), input_vector_A.end(),
                    input_vector_B.begin(),
                    canvas.begin(),
20
                    [](const auto& arg_a,
                       const auto& arg_b){
                        return std::atan2(arg_a, arg_b);
                    });
        // moving things back
        return std::move(canvas);
2.8
     /*-----
     y = tan-inverse(a/b)
31
     template <typename T>
33
     auto atan2(T scalar_A,
                scalar_B)
35
     {
36
        return std::atan2(scalar_A, scalar_B);
37
38
  }
```

B.29 Tiling Operations

```
)};
         // writing
18
         std::size_t
                      source_row;
19
         std::size_t source_col;
         for(std::size_t row = 0; row < num_rows; ++row){</pre>
             for(std::size_t col = 0; col < num_cols; ++col){</pre>
                source_row = row % 1;
                 source_col = col % input_vector.size();
                 canvas[row][col] = input_vector[source_col];
26
             }
2.7
         }
         // returning
30
         return std::move(canvas);
31
33
34
      tiling a matrix
      -----*/
35
      template <typename T>
      auto tile(const std::vector<std::vector<T>>& input_matrix,
37
               const std::vector<size_t>&
                                                  mul_dimensions){
38
39
         // creating canvas
          const std::size_t& num_rows {input_matrix.size() * mul_dimensions[0]};
41
          const std::size_t& num_cols {input_matrix[0].size() * mul_dimensions[1]};
42
                canvas {std::vector<std::vector<T>>(
          auto
            num_rows,
             std::vector<T>(num_cols, 0)
45
         )};
         // writing
         std::size_t
                       source_row;
49
         std::size_t source_col;
50
         for(std::size_t row = 0; row < num_rows; ++row){</pre>
             for(std::size_t col = 0; col < num_cols; ++col){</pre>
                 source_row = row % input_matrix.size();
                 source_col = col % input_matrix[0].size();
                 canvas[row][col] = input_matrix[source_row][source_col];
56
             }
57
         }
58
         // returning
60
         return std::move(canvas);
61
62
  }
```

B.30 Transpose

```
// creating canvas
auto canvas {std::vector<std::vector<T>>{
    input_vector.size(),
    std::vector<T>(1)
};

// filling canvas
for(auto i = 0; i < input_vector.size(); ++i){
    canvas[i][0] = input_vector[i];
}

// moving it back
return std::move(canvas);
}</pre>
```

B.31 Masking

```
#pragma once
  namespace svr {
      /*-----
      y = input_vector[mask == 1]
      template <typename T,
              typename = std::enable_if_t< std::is_arithmetic_v<T>
                                       std::is_same_v<T, std::complex<double>> ||
                                       std::is_same_v<T, std::complex<float>>
11
      auto mask(const std::vector<T>& input_vector,
             const std::vector<bool>& mask_vector)
         // checking dimensionality issues
         if (input_vector.size() != mask_vector.size())
             std::cerr << "mask(vector, mask): incompatible size \n";</pre>
         // creating canas
         auto num_trues {std::count(mask_vector.begin(),
                                    mask_vector.end(),
                                    true)};
         auto
                canvas
                          {std::vector<T>(num_trues)};
         // copying values
         auto destination_index {0};
         for(auto i = 0; i <input_vector.size(); ++i)</pre>
             if (mask_vector[i] == true)
                canvas[destination_index++] = input_vector[i];
30
         // returning output
31
         return std::move(canvas);
      template <typename T>
      auto mask(const std::vector<std::vector<T>>&
                                                     input_matrix,
                     std::vector<bool>
                                                     mask_vector)
38
```

```
// fetching dimensions
                auto& num_rows_matrix
          const
                                          {input_matrix.size()};
41
                auto& num_cols_matrix
                                         {input_matrix[0].size()};
          const
          const auto& num_cols_vector
                                              {mask_vector.size()};
          // error-checking
          if (num_cols_matrix != num_cols_vector)
             std::cerr << "mask(matrix, bool-vector): size disparity";</pre>
          // building canvas
49
                 num_trues {std::count(mask_vector.begin(),
                                      mask_vector.end(),
                                      true)};
          auto
                 canvas
                            {std::vector<std::vector<T>>(
             num_rows_matrix,
             std::vector<T>(num_cols_vector, 0)
          )};
56
          // writing values
          #pragma omp parallel for
          for(auto row = 0; row < num_rows_matrix; ++row){</pre>
                    destination_index {0};
             for(auto col = 0; col < num_cols_vector; ++col)</pre>
                 if(mask_vector[col] == true)
                     canvas[row] [destination_index++] = input_matrix[row][col];
          }
          // returning
          return std::move(canvas);
68
69
      /*-----
70
      Fetch Indices corresponding to mask true's
71
72
      auto mask_indices(const std::vector<bool>& mask_vector)
73
          // creating canvas
                num_trues {std::count(mask_vector.begin(), mask_vector.end(),
                                      true)};
          auto
                 canvas
                            {std::vector<std::size_t>(num_trues)};
          // building canvas
80
                 destination_index {0};
81
          for(auto i = 0; i < mask_vector.size(); ++i)</pre>
             if (mask_vector[i] == true)
83
                 canvas[destination_index++] = i;
          // returning
          return std::move(canvas);
87
      }
88
  }
```

B.32 Resetting Containers

```
#pragma once
namespace svr {
```

B.33 Element-wise squaring

```
#pragma once
  namespace svr {
     /*-----
     Element-wise squaring vector
     template <typename T,
               typename = std::enable_if_t<std::is_arithmetic_v<T>>
            square(const std::vector<T>& input_vector)
     auto
        // creating canvas
         auto
              canvas
                        {std::vector<T>(input_vector.size())};
         // peforming calculations
         std::transform(input_vector.begin(), input_vector.end(),
                    canvas.begin(),
                     [](const auto& argx){
                           return argx * argx;
                     });
         // moving it back
         return std::move(canvas);
23
     Element-wise squaring vector (in-place)
                        ----*/
     template <typename T,
               typename = std::enable_if_t<std::is_arithmetic_v<T>>
            square_inplace(std::vector<T>& input_vector)
     void
         // performing operations
         std::transform(input_vector.begin(), input_vector.end(),
                    input_vector.begin(),
                    [](auto& argx){
                        return argx * argx;
                    });
```

```
ELement-wise squaring a matrix
40
            ______*/
------*/
41
     template <typename T>
42
           square(const std::vector<std::vector<T>>& input_matrix)
     auto
43
44
     {
        // fetching dimensions
        const auto& num_rows {input_matrix.size()};
46
        const auto& num_cols {input_matrix[0].size()};
        // creating canvas
49
        auto
             canvas {std::vector<std::vector<T>>(
50
           num_rows,
51
           std::vector<T>(num_cols, 0)
        )};
53
        // going through each row
        #pragma omp parallel for
        for(auto row = 0; row < num_rows; ++row)</pre>
57
           std::transform(input_matrix[row].begin(), input_matrix[row].end(),
                      canvas[row].begin(),
                       [](const auto& argx){
                          return argx * argx;
61
                      });
62
        // returning
        return std::move(canvas);
65
66
     /*----
67
     Squaring for scalars
68
     -----*/
69
     template <typename T>
70
     auto square(const T& scalar) {return scalar * scalar;}
71
  }
```

B.34 Flooring

```
namespace svr {
     /*-----
     element-wise flooring of a vector-contents
     template <typename T>
     auto floor(const
                       std::vector<T>& input_vector)
        // creating canvas
        auto canvas
                      {std::vector<T>(input_vector.size())};
10
        // filling the canvas
11
        std::transform(input_vector.begin(), input_vector.end(),
                    canvas.begin(),
                    [](const auto& argx){
                       return std::floor(argx);
                    });
        // returning
        return std::move(canvas);
19
20
```

```
/*-----
21
     element-wise flooring of a vector-contents (in-place)
22
     -----*/
23
     template <typename T>
24
     auto floor_inplace(std::vector<T>&
                                      input_vector)
25
        // rewriting the contents
        std::transform(input_vector.begin(), input_vector.end(),
                   input_vector.begin(),
                   [](auto& argx){
30
                       return std::floor(argx);
31
                   });
32
     }
33
     34
     element-wise flooring of matrix-contents
35
36
     template <typename T>
          floor(const std::vector<std::vector<T>>& input_matrix)
38
39
        // fetching dimensions
40
        const auto& num_rows_matrix
                                   {input_matrix.size()};
41
        const auto& num_cols_matrix {input_matrix[0].size()};
43
        // creating canvas
                      {std::vector<std::vector<T>>(
        auto canvas
           num_rows_matrix,
46
           std::vector<T>(num_cols_matrix)
47
        )};
48
49
        // writing contents
50
        for(auto row = 0; row < num_rows_matrix; ++row)</pre>
           std::transform(input_matrix[row].begin(), input_matrix[row].end(),
                      canvas[row].begin(),
53
                       [](const auto& argx){
54
                          return std::floor(argx);
55
                      });
        // returning contents
        return std::move(canvas);
61
     /*----
62
     element-wise flooring of matrix-contents (in-place)
63
     -----*/
     template <typename T>
65
          floor_inplace(std::vector<std::vector<T>>& input_matrix)
     auto
66
        // performing operations
        for(auto row = 0; row < input_matrix.size(); ++row)</pre>
           std::transform(input_matrix[row].begin(), input_matrix[row].end(),
70
71
                      input_matrix[row].begin(),
                      [](auto& argx){
                          return std::floor(argx);
73
                      });
     }
75
  }
```

B.35 Squeeze

```
namespace svr {
      template
                 <typename T>
              squeeze(const std::vector<std::vector<T>>& input_matrix)
          // fetching dimensions
          const auto& num_rows_matrix {input_matrix.size()};
          const auto& num_cols_matrix {input_matrix[0].size()};
          // check if any dimension is 1
          if (num_rows_matrix == 0 || num_cols_matrix == 0)
10
              std::cerr << "at least one dimension should be 1";</pre>
          auto
                  final_length {std::max(num_rows_matrix, num_cols_matrix)};
          // creating canvas
                 canvas
          auto
                                {std::vector<T>(final_length)};
          // building canvas
18
          if (num_rows_matrix == 1)
19
              // filling canvas
              std::copy(input_matrix[0].begin(), input_matrix[0].end(),
                       canvas.begin());
          else if(num_cols_matrix == 1)
26
              // filling canvas
              std::transform(input_matrix.begin(), input_matrix.end(),
                            canvas.begin(),
                            [](const auto& argx){
30
                                return argx[0];
                            });
          }
33
34
          // returning
35
          return std::move(canvas);
      }
37
  }
38
```

B.36 Tensor Initializations

```
return std::move(canvas);
15 }
16 }
```

B.37 Real part

```
#pragma once
  namespace svr {
     /*----
     For type-deductions
     template <typename T>
     struct real_result_type;
     template <> struct real_result_type<std::complex<double>>{
11
        using type = double;
     template <> struct real_result_type<std::complex<float>>{
        using type = float;
     template <> struct real_result_type<double> {
        using type = double;
18
     template <> struct real_result_type<float>{
19
        using type = float;
2.0
     };
21
     template <typename T>
23
     using real_result_t = typename real_result_type<T>::type;
24
     /*-----
26
     Element-wise real() of a vector
2.7
     -----*/
2.8
     template <typename T>
          real(const std::vector<T>& input_vector)
30
31
        // figure out base-type
        using TCanvas = real_result_t<T>;
        // creating canvas
35
                       {std::vector<TCanvas>(
              canvas
        auto
           input_vector.size()
        )};
        // storing values
        std::transform(input_vector.begin(), input_vector.end(),
                    canvas.begin(),
                     [](const auto&
                                    argx){
43
                       return std::real(argx);
                    });
        // returning
        return std::move(canvas);
49
  }
50
```

B.38 Imaginary part

```
#pragma once
  namespace svr {
      /*-----
     For type-deductions
      -----*/
     template <typename T>
     struct imag_result_type;
      template <> struct imag_result_type<std::complex<double>>{
10
         using type = double;
11
      template <> struct imag_result_type<std::complex<float>>{
        using type = float;
      template <> struct imag_result_type<double> {
         using type = double;
18
      template <> struct imag_result_type<float>{
19
         using type = float;
     };
21
      template <typename T>
23
      using imag_result_t = typename imag_result_type<T>::type;
25
26
27
              <typename T>
           imag(const std::vector<T>& input_vector)
         // figure out base-type
         using TCanvas = imag_result_t<T>;
33
         // creating canvas
         auto canvas
                        {std::vector<TCanvas>(
            input_vector.size()
         )};
37
         // storing values
         std::transform(input_vector.begin(), input_vector.end(),
                     canvas.begin(),
41
                      [](const auto&
                                      argx){
                         return std::imag(argx);
         // returning
         return std::move(canvas);
     }
48
  }
49
```