

# Autonomous Underwater Vehicle: A Surveillance Protocol

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# Preface

This project is an attempt at combining all of my major skills into creating a simulation, imaging, perception and control pipeline for Autonomous Underwater Vehicles (AUV). As such, creating this project involves creating a number of pipelines.

The first pipeline is the signal simulation pipeline. The signal simulation pipeline involves sea-floor point-cloud creation and simulating the signals received by the sensor arrays of the AUV. The signals recorded by the sensor-arrays on the AUV contains information from the surrounding environment. The imaging pipeline performs certain operations on the recorded signals to obtain acoustic images of the surrounding environment. To that end, this pipeline involves the topics of signal processing, linear algebra, signals and systems.

As such, the second pipeline is the imaging pipeline. The inputs to the imaging pipeline is the signals recorded by the different sensor-arrays of the AUV, in addition to the parameters of the AUV and its components. This pipeline involves match-filtering, focussing and beamforming operations to create acoustic images of the surrounding environment. Depending on the number of ULAs present, the imaging pipeline is responsible for creating multiple acoustic images in real-time. Thus, this pipeline involves the topics of Digital Signal Processing, Match-Filtering, Estimation and Detection Theory and so on.

The images created by the imaging pipeline are fed to the perception-to-control pipeline. This pipeline takes in the image formed created from the ULA signals, parameters of AUV and its components, and some historical data, it provides instructions regarding the movement of the AUV. The mapping from the inputs to the controls is called policy. Learning policies is a core part of reinforcement learning. Thus, this pipeline mainly involves the topics of reinforcement learning. And since we'll be using convolutional neural nets and transformers for learning the policies, this pipeline involves a significant amount of machine and deep learning.

The final result is an AUV that is primarily trained to map an area of the sea-floor in a constant surveillance mode. The RL-trained policy will also be trained to deal with different kinds of sea-floor terrains: those containing hills, valleys, and path-obstructing features. Due to the resource constrained nature of the marine vessel, we also prioritize efficient policies in the policy-training pipeline.

The project is currently written in C++. And since there is non-trivial amount of training and adaptive features in the pipelines, we'll be using LibTorch (the C++ API of PyTorch) to enable computation graphs, backpropagation and thereby, learning in our AUV pipeline. However, for the sections where a computation graph is not required we will be writing templated STL code.

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# Chapter 1

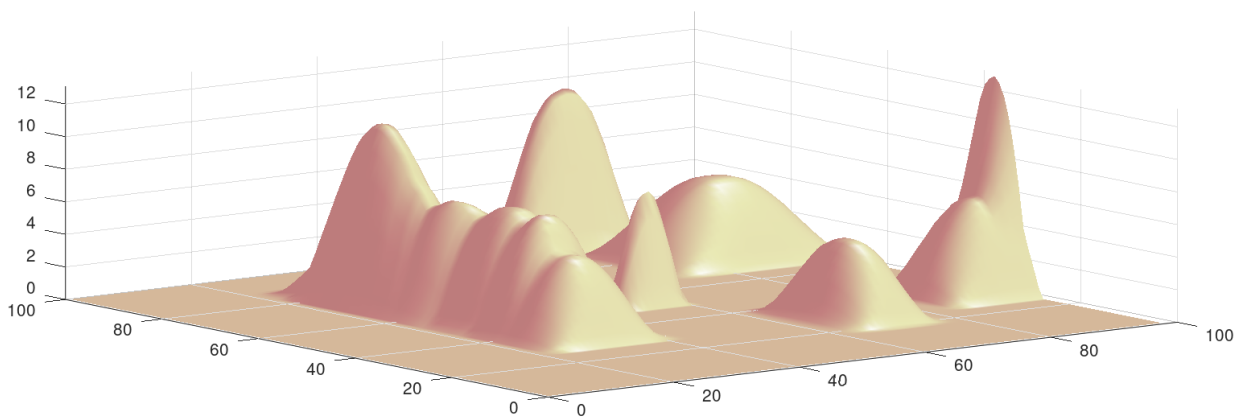
## Underwater Environment

### Overview

All physical matter in this framework is represented using point-clouds. Thus, the sea-floor also is represented using a number of 3D points. In addition to the coordinates, the points also have the additional property of “reflectivity”. It is the impulse response of that point.

Sea-floors in real-life are rarely flat. They often contain valleys, mountains, hills and much richer geographical features. Thus, training an agent to function in such environments call for the creation of similar structures in our simulations.

To simplify things, we shall take a more constrained and structured approach. We start by creating different classes of structures and produce instantiations of those structures on the sea-floor. These structures are defined in such a way that the shape and size can be parameterized to enable creation of random sea-floors. The source-code for creation is given in ??



### 1.1 Underwater Hills

The most basic approach to creating this is to create a flat seafloor, where all the points have the same height. While this is a good place to start, it is good to bring in some realism

to the seafloor. To that end, we shall have some rolling hills as the sea-floor. Each “hill” is created using the method outlined in Algorithm 1. The method involves deciding the location of the hills, the dimension of the hills and then designing a hill by combining an exponential function and a cosine function. We’re aiming to essentially produce gaussian-looking sea-floor hills. After the creation, this becomes the set of points representing the lowest set of points in the overall seafloor structure.

---

**Algorithm 1** Hill Creation
 

---

```

1: Input: Mean vector  $\mathbf{m}$ , Dimension vector  $\mathbf{d}$ , 2D points  $\mathbf{P}$ 
2: Output: Updated  $\mathbf{P}$  with hill heights
3:  $\text{num\_hills} \leftarrow \text{numel}(\mathbf{m}_x)$ 
4:  $H \leftarrow$  Zeros tensor of size  $(1, \text{numel}(\mathbf{P}_x))$ 
5: for  $i = 1$  to  $\text{num\_hills}$  do
6:    $x_{\text{norm}} \leftarrow \frac{\frac{\pi}{2}(\mathbf{P}_x - \mathbf{m}_x[i])}{\mathbf{d}_x[i]}$ 
7:    $y_{\text{norm}} \leftarrow \frac{\frac{\pi}{2}(\mathbf{P}_y - \mathbf{m}_y[i])}{\mathbf{d}_y[i]}$ 
8:    $h_x \leftarrow \cos(x_{\text{norm}}) \cdot e^{\frac{|x_{\text{norm}}|}{10}}$ 
9:    $h_y \leftarrow \cos(y_{\text{norm}}) \cdot e^{\frac{|y_{\text{norm}}|}{10}}$ 
10:   $h \leftarrow \mathbf{d}_z[i] \cdot h_x \cdot h_y$ 
11:  Apply boundary conditions:
12:  if  $x_{\text{norm}} > \frac{\pi}{2}$  or  $x_{\text{norm}} < -\frac{\pi}{2}$  or  $y_{\text{norm}} > \frac{\pi}{2}$  or  $y_{\text{norm}} < -\frac{\pi}{2}$  then
13:     $h \leftarrow 0$ 
14:  end if
15:   $H \leftarrow H + h$ 
16: end for
17:  $\mathbf{P} \leftarrow \text{concatenate}([\mathbf{P}, H])$ 

```

---

## 1.2 Scatterer Definition

The sea-floor is represented by a single object of the class ScattererClass.

---

```

1  /*=====
2  Class Declaration
3  -----*/
4  template <typename T>
5  class ScattererClass
6  {
7  public:
8      // members
9      std::vector<std::vector<T>> coordinates;
10     std::vector<T> reflectivity;
11
12     // Constructor
13     ScattererClass() {}
14
15     // Constructor
16     ScattererClass(std::vector<std::vector<T>> coordinates_arg,
17                    std::vector<T> reflectivity_arg):
18         coordinates(std::move(coordinates_arg)),
19         reflectivity(std::move(reflectivity_arg)) {}
20

```

```

21     // Save to CSV
22     void save_to_csv();
23 };
24 /*=====
25 Saving to CSV-Files
26 -----*/
27 template <typename T>
28 void ScattererClass<T>::save_to_csv()
29 {
30     fWriteMatrix(this->coordinates, "../csv-files/coordinates.csv");
31     fWriteVector(this->reflectivity, "../csv-files/reflectivity.csv");
32 }

```

---

## 1.3 Sea-Floor Setup Script

Following is the function that will setup the sea-floor script.

---

```

1 void fSeaFloorSetup(ScattererClass<double>& scatterers){
2
3     // auto   save_files   {false};
4     const auto   save_files   {false};
5     const auto   hill_creation_flag   {true};
6
7     // sea-floor bounds
8     auto   bed_width   {100.00};
9     auto   bed_length   {100.00};
10
11     // creating tensors for coordinates and reflectivity
12     vector<vector<double>>   box_coordinates;
13     vector<double>           box_reflectivity;
14
15     // scatter density
16     auto   bed_width_density {static_cast<double>( 10.00)};
17     auto   bed_length_density {static_cast<double>( 10.00)};
18
19     // setting up coordinates
20     auto   xpoints   {linspace<double>(0.00,
21                                     bed_width,
22                                     bed_width * bed_width_density)};
23     auto   ypoints   {linspace<double>(0.00,
24                                     bed_length,
25                                     bed_length * bed_length_density)};
26     if(save_files) fWriteVector(xpoints, "../csv-files/xpoints.csv"); // verified
27     if(save_files) fWriteVector(ypoints, "../csv-files/ypoints.csv"); // verified
28
29     // creating mesh
30     auto [xgrid, ygrid] = meshgrid(std::move(xpoints), std::move(ypoints));
31     if(save_files) fWriteMatrix(xgrid, "../csv-files/xgrid.csv"); // verified
32     if(save_files) fWriteMatrix(ygrid, "../csv-files/ygrid.csv"); // verified
33
34     // reshaping
35     auto   X   {reshape(xgrid, xgrid.size()*xgrid[0].size())};
36     auto   Y   {reshape(ygrid, ygrid.size()*ygrid[0].size())};
37     if(save_files) fWriteVector(X, "../csv-files/X.csv"); // verified
38     if(save_files) fWriteVector(Y, "../csv-files/Y.csv"); // verified
39

```

```

40 // creating heights of scatterers
41 if(hill_creation_flag){
42
43     // setting up hill parameters
44     auto num_hills {10};
45
46     // setting up placement of hills
47     auto points2D {concatenate<0>(X, Y)}; // verified
48     auto min2D {min<1, double>(points2D)}; // verified
49     auto max2D {max<1, double>(points2D)}; // verified
50     auto hill_2D_center {min2D + \
51                          rand({2, num_hills}) * (max2D - min2D)}; // verified
52
53     // setup: hill-dimensions
54     auto hill_dimensions_min {transpose(vector<double>{5, 5, 2})}; // verified
55     auto hill_dimensions_max {transpose(vector<double>{30, 30, 10})}; // verified
56     auto hill_dimensions {hill_dimensions_min + \
57                          rand({3, num_hills}) * (hill_dimensions_max -
58                          hill_dimensions_min)}; // verified
59
60     // function-call: hill-creation function
61     fCreateHills(hill_2D_center,
62                  hill_dimensions,
63                  points2D);
64
65     // setting up floor reflectivity
66     auto floorScatter_reflectivity {std::vector<double>(Y.size(), 1.00)};
67
68     // populating the values of the incoming argument
69     scatterers.coordinates = std::move(points2D);
70     scatterers.reflectivity = std::move(floorScatter_reflectivity);
71 }
72 else{
73
74     // assigning flat heights
75     auto Z {std::vector<double>(Y.size(), 0)};
76
77     // setting up floor coordinates
78     auto floorScatter_coordinates {concatenate<0>(X, Y, Z)};
79     auto floorScatter_reflectivity {std::vector<double>(Y.size(), 1)};
80
81     // populating the values of the incoming argument
82     scatterers.coordinates = std::move(floorScatter_coordinates);
83     scatterers.reflectivity = std::move(floorScatter_reflectivity);
84
85 }
86 }

```

---



# Chapter 2

## Transmitter

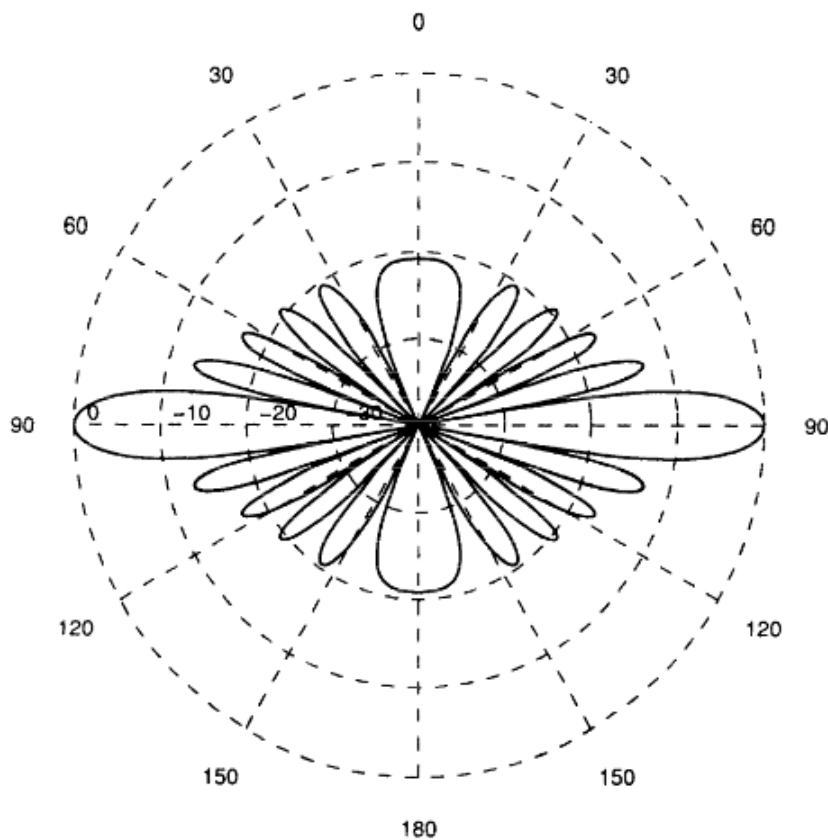


Figure 2.1: Beampattern of a Transmission Uniform Linear Array

### Overview

Probing systems are those systems that send out a signal, listen to the reflection and infer qualitative and quantitative qualities of the environment, matter or object, it was trying to infer information about. The transmitter is one of the most fundamental components of probing systems. As the name suggests, the transmitter is the equipment responsible for sending out the probing signal into the medium.

A transmitter is any device or circuit that converts information into a signal and sends it out onto some media like air, cable, water or space. The components of a transmitter are usually as follows

1. Input: Information containing signal such as voice, data, video etc
2. Process: Encode/modulate the information onto a carrier signal, which can be electromagnetic wave or mechanical wave.
3. Transmission: The signal is then transmitted onto the media with electro-mechanical equipment.

Transmitters are of many kinds. But the ones that we will be considering will be directed transmitters, which means that these transmitters have an associated beampattern. To the uninitiated, this means that the power of the transmitted signal is not transmitted in all directions equally. A beampattern is a graphical representation of the power received by an ideal receiver when placed at different angles.

Transmitters made out of a linear-array of individual transmitters use beamforming to “direct” the major power of the transmitter. These kind of systems have well studied beampatterns which can be utilized in our simulations. These kind of studies and inculcating that in our pipelines produce accurate signal simulation pipelines. For now, we stick to a very simple model of a transmitter. We assume that the transmitter sends out the power equally into a particular cone from the AUV position.

## 2.1 Transmission Signal

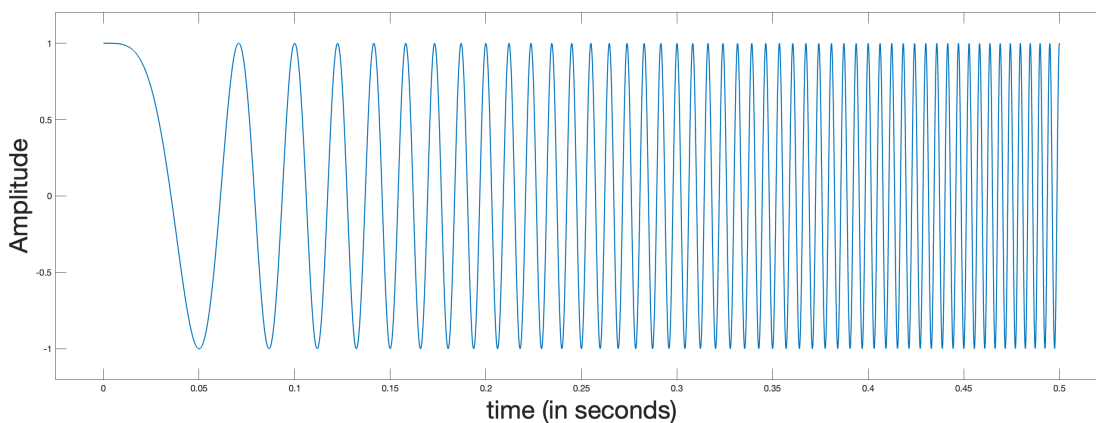


Figure 2.2: Linear Frequency Modulated Wave

The resolution of any probing system is fundamentally tied to the signal bandwidth. A higher bandwidth corresponds to finer resolution  $\frac{\text{speed-of-sounds}}{2 \times \text{bandwidth}}$ . Thus, for perfect resolution, an infinite bandwidth is in order. However, infinite bandwidth is impossible for obvious reasons: hardware limitations, spectral regulations, energy limitations and so on.

This is where Linear Frequency Modulation (LFM), also called a “chirp,” becomes valuable. An LFM signal linearly sweeps a limited bandwidth over a relatively long duration. This technique spreads the signal’s energy in time while retaining the resolution benefits of

the bandwidth. After matched filtering (or pulse compression), we essentially produce pulses corresponding to a base-band LFM of same bandwidth. Overall, LFM is a practical compromise between finite bandwidth and desired performance.

One of the best parts about the resolution depending only on the bandwidth is that it allows us to deploy techniques that would help us improve SNRs without virtually increasing the bandwidth at all. Much of the noise in submarine environments are in and around the baseband region (around frequency, 0). Since resolution depends purely on bandwidth, and LFM can be transmitted at a carrier-frequency, this means that processing the returns after low-pass filtering and basebanding allows us to get rid of the submarine noise, since they do not occupy the same frequency-coefficients. The end-result, thus, is improved SNR compared to use baseband LFM.

Due to all of these advantages, LFM waves are ubiquitous in probing systems, from sonar to radar. Thus, for this project too, the transmitter will be using LFM waves as probing signals, to probe the surrounding submarine environment.

## 2.2 Transmitter Class Definition

The transmitter is represented by a single object of the class TransmitterClass.

---

```

1  template <typename T>
2  class TransmitterClass{
3  public:
4
5      // physical/intrinsic properties
6      std::vector<T>    location;           // location tensor
7      std::vector<T>    pointing_direction; // pointing direction
8
9      // basic parameters
10     std::vector<T>    Signal;             // transmitted signal (LFM)
11     T                  azimuthal_angle;    // transmitter's azimuthal pointing direction
12     T                  elevation_angle;    // transmitter's elevation pointing direction
13     T                  azimuthal_beamwidth; // azimuthal beamwidth of transmitter
14     T                  elevation_beamwidth; // elevation beamwidth of transmitter
15     T                  range;              // a parameter used for spotlight mode.
16
17     // transmitted signal attributes
18     T                  f_low;              // lowest frequency of LFM
19     T                  f_high;             // highest frequency of LFM
20     T                  fc;                 // center frequency of LFM
21     T                  bandwidth;          // bandwidth of LFM
22
23     // shadowing properties
24     int                azimuthQuantDensity; // quantization of angles along the
        azimuth
25     int                elevationQuantDensity; // quantization of angles along the
        elevation
26     T                  rangeQuantSize;      // range-cell size when shadowing
27     T                  azimuthShadowThreshold; // azimuth thresholding
28     T                  elevationShadowThreshold; // elevation thresholding
29
30     // shadowing related
31     std::vector<T>    checkbox;             // box indicating whether a scatter for a
        range-angle pair has been found

```

```

32     std::vector<std::vector<std::vector<T>>> finalScatterBox; // a 3D tensor where the
        third dimension represnets the vector length
33     std::vector<T> finalReflectivityBox; // to store the reflectivity
34
35     // constructor
36     TransmitterClass() = default;
37
38     // Deleting copy constructors/assignment
39     TransmitterClass(const TransmitterClass& other)      = delete;
40     TransmitterClass& operator=(TransmitterClass& other) = delete;
41
42     // Creating move-constructor and move-assignment
43     TransmitterClass(TransmitterClass&& other)          = default;
44     TransmitterClass& operator=(TransmitterClass&& other) = default;
45
46     /*=====
47     Aim: Update pointing angle
48     -----
49     Note:
50         > This function updates pointing angle based on AUV's pointing angle
51         > for now, we're assuming no roll;
52     -----*/
53     auto updatePointingAngle(std::vector<T> AUV_pointing_vector);
54
55 };
56
57
58
59 /*=====
60 Aim: Update pointing angle
61 -----
62 Note:
63     > This function updates pointing angle based on AUV's pointing angle
64     > for now, we're assuming no roll;
65     -----*/
66 template <typename T>
67 auto TransmitterClass<T>::updatePointingAngle(std::vector<T> AUV_pointing_vector)
68 {
69
70     // calculate yaw and pitch
71     auto    AUV_pointing_vector_spherical {svr::cart2sph(AUV_pointing_vector)};
72     auto    yaw                          {AUV_pointing_vector_spherical[0]};
73     auto    pitch                        {AUV_pointing_vector_spherical[1]};
74
75     // calculating azimuth and elevation of transmitter object
76     auto    absolute_azimuth_of_transmitter {yaw + this->azimuthal_angle};
77     auto    absolute_elevation_of_transmitter {pitch + this->elevation_angle};
78
79     // converting back to Cartesian
80     auto    pointing_direction_spherical {std::vector<T>(3, 0)};
81     pointing_direction_spherical[0]      = absolute_azimuth_of_transmitter;
82     pointing_direction_spherical[1]      = absolute_elevation_of_transmitter;
83     pointing_direction_spherical[2]      = 1;
84     this->pointing_direction              = svr::sph2cart(pointing_direction_spherical);
85 }

```

---

## 2.3 Transmitter Setup Scripts

The following script shows the setup-script

---

```

1  template <typename T>
2  void fTransmitterSetup(TransmitterClass<T>& transmitter_fls,
3                          TransmitterClass<T>& transmitter_portside,
4                          TransmitterClass<T>& transmitter_starboard)
5  {
6      // Setting up transmitter
7      T      sampling_frequency    {160e3};           // sampling frequency
8      T      f1                   {50e3};             // first frequency of LFM
9      T      f2                   {70e3};             // second frequency of LFM
10     T      fc                    {(f1 + f2)/2.00};    // finding center-frequency
11     T      bandwidth             {std::abs(f2 - f1)}; // bandwidth
12     T      pulselength           {5e-2};             // time of recording
13
14     // building LFM
15     auto    timearray            {linspace<T>(0.00,
16                                             pulselength,
17                                             std::floor(pulselength * sampling_frequency))};
18     auto    K                    {f2 - f1/pulselength}; // calculating frequency-slope
19     auto    Signal               {cos(2 * std::numbers::pi * \
20                                 (f1 + K*timearray) * \
21                                 timearray)};           // frequency at each time-step, with f1
22                                                         = 0
23
24     // Setting up transmitter
25     auto    location              {std::vector<T>(3, 0)}; // location of
26     // transmitter
27     T      azimuthal_angle_fls    {0};                // initial
28     // pointing direction
29     T      azimuthal_angle_port   {90};               // initial
30     // pointing direction
31     T      azimuthal_angle_starboard {-90};           // initial
32     // pointing direction
33
34     T      elevation_angle        {-60};              // initial
35     // pointing direction
36
37     T      azimuthal_beamwidth_fls {20};              // azimuthal
38     // beamwidth of the signal cone
39     T      azimuthal_beamwidth_port {20};             // azimuthal
40     // beamwidth of the signal cone
41     T      azimuthal_beamwidth_starboard {20};        // azimuthal
42     // beamwidth of the signal cone
43
44     T      elevation_beamwidth_fls  {20};             // elevation
45     // beamwidth of the signal cone
46     T      elevation_beamwidth_port {20};             // elevation
47     // beamwidth of the signal cone
48     T      elevation_beamwidth_starboard {20};        // elevation
49     // beamwidth of the signal cone
50
51     int     azimuthQuantDensity    {10}; // number of points, a degree is split
52     // into quantization density along azimuth (used for shadowing)
53     int     elevationQuantDensity  {10}; // number of points, a degree is split
54     // into quantization density along elevation (used for shadowing)
55     T      rangeQuantSize          {10}; // the length of a cell (used for

```

```

    shadowing)
42
43     T      azimuthShadowThreshold      {1};    // azimuth threshold  (in degrees)
44     T      elevationShadowThreshold    {1};    // elevation threshold (in degrees)
45
46
47     // transmitter-fls
48     transmitter_fl.s.location          = location;          // Assigning
49         location
49     transmitter_fl.s.Signal            = Signal;            // Assigning
50         signal
50     transmitter_fl.s.azimuthal_angle   = azimuthal_angle_fl.s;    // assigning
51         azimuth angle
51     transmitter_fl.s.elevation_angle   = elevation_angle;        // assigning
52         elevation angle
52     transmitter_fl.s.azimuthal_beamwidth = azimuthal_beamwidth_fl.s; // assigning
53         azimuth-beamwidth
53     transmitter_fl.s.elevation_beamwidth = elevation_beamwidth_fl.s; // assigning
54         elevation-beamwidth
54     // updating quantization densities
55     transmitter_fl.s.azimuthQuantDensity = azimuthQuantDensity;    // assigning
56         azimuth quant density
56     transmitter_fl.s.elevationQuantDensity = elevationQuantDensity; // assigning
57         elevation quant density
57     transmitter_fl.s.rangeQuantSize      = rangeQuantSize;        // assigning
58         range-quantization
58     transmitter_fl.s.azimuthShadowThreshold = azimuthShadowThreshold; //
59         azimuth-threshold in shadowing
59     transmitter_fl.s.elevationShadowThreshold = elevationShadowThreshold; //
60         elevation-threshold in shadowing
60     // signal related
61     transmitter_fl.s.f_low              = f1;                // assigning lower frequency
62     transmitter_fl.s.f_high             = f2;                // assigning higher frequency
63     transmitter_fl.s.fc                  = fc;                // assigning center frequency
64     transmitter_fl.s.bandwidth           = bandwidth;         // assigning bandwidth
65
66
67     // transmitter-portside
68     transmitter_portside.location        = location;          // Assigning
69         location
69     transmitter_portside.Signal          = Signal;            // Assigning
70         signal
70     transmitter_portside.azimuthal_angle = azimuthal_angle_port; // assigning
71         azimuth angle
71     transmitter_portside.elevation_angle = elevation_angle;    // assigning
72         elevation angle
72     transmitter_portside.azimuthal_beamwidth = azimuthal_beamwidth_port; // assigning
73         azimuth-beamwidth
73     transmitter_portside.elevation_beamwidth = elevation_beamwidth_port; // assigning
74         elevation-beamwidth
74     // updating quantization densities
75     transmitter_portside.azimuthQuantDensity = azimuthQuantDensity; // assigning
76         azimuth quant density
76     transmitter_portside.elevationQuantDensity = elevationQuantDensity; // assigning
77         elevation quant density
77     transmitter_portside.rangeQuantSize      = rangeQuantSize;    // assigning
78         range-quantization
78     transmitter_portside.azimuthShadowThreshold = azimuthShadowThreshold; //
79         azimuth-threshold in shadowing

```

```

79     transmitter_portside.elevationShadowThreshold = elevationShadowThreshold; //
        elevation-threshold in shadowing
80 // signal related
81 transmitter_portside.f_low                = f1;                // assigning
        lower frequency
82 transmitter_portside.f_high              = f2;                // assigning
        higher frequency
83 transmitter_portside.fc                  = fc;                // assigning
        center frequency
84 transmitter_portside.bandwidth            = bandwidth;        // assigning
        bandwidth
85
86
87 // transmitter-starboard
88 transmitter_starboard.location            = location;          //
        assigning location
89 transmitter_starboard.Signal              = Signal;            //
        assigning signal
90 transmitter_starboard.azimuthal_angle     = azimuthal_angle_starboard; //
        assigning azimuthal signal
91 transmitter_starboard.elevation_angle     = elevation_angle;
92 transmitter_starboard.azimuthal_beamwidth = azimuthal_beamwidth_starboard;
93 transmitter_starboard.elevation_beamwidth = elevation_beamwidth_starboard;
94 // updating quantization densities
95 transmitter_starboard.azimuthQuantDensity = azimuthQuantDensity; //
        assigning azimuth-quant-density
96 transmitter_starboard.elevationQuantDensity = elevationQuantDensity;
97 transmitter_starboard.rangeQuantSize       = rangeQuantSize;
98 transmitter_starboard.azimuthShadowThreshold = azimuthShadowThreshold;
99 transmitter_starboard.elevationShadowThreshold = elevationShadowThreshold;
100 // signal related
101 transmitter_starboard.f_low                = f1;                //
        assigning lower frequency
102 transmitter_starboard.f_high              = f2;                //
        assigning higher frequency
103 transmitter_starboard.fc                  = fc;                //
        assigning center frequency
104 transmitter_starboard.bandwidth            = bandwidth;        //
        assigning bandwidth
105
106 }

```

---

# Chapter 3

## Uniform Linear Array

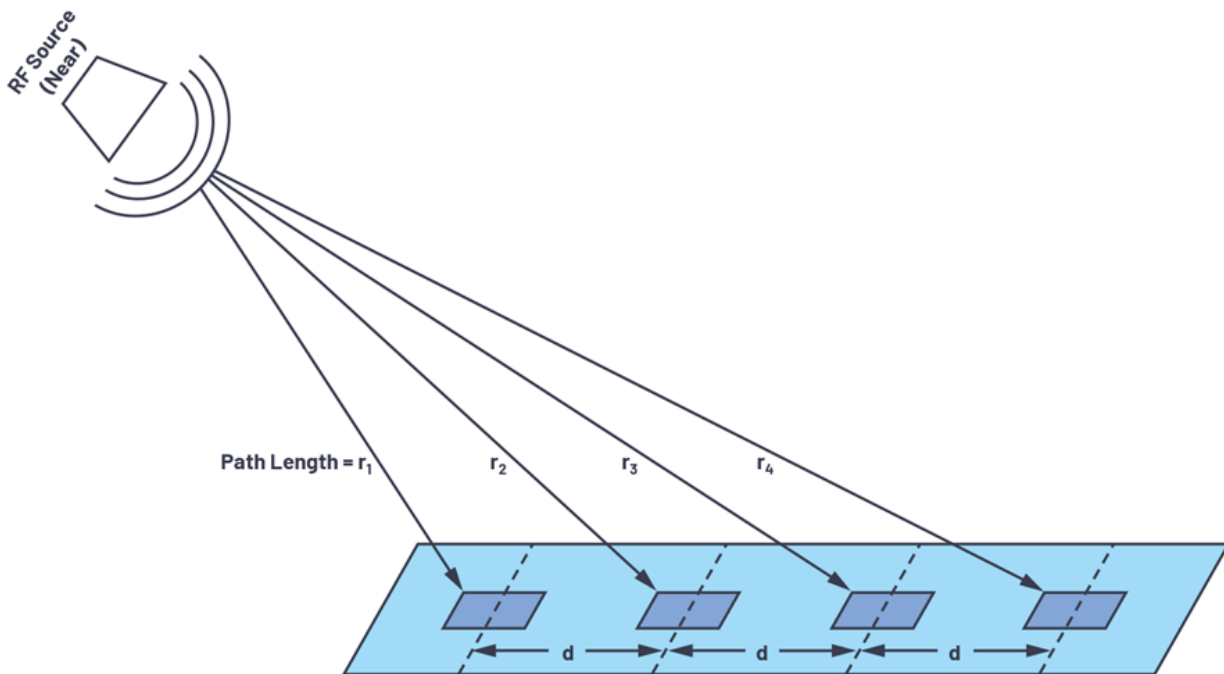


Figure 3.1: Uniform Linear Array

### Overview

A Uniform Linear Array (ULA) is a common antenna or sensor configuration in which multiple elements are arranged in a straight line with equal spacing between adjacent elements. This geometry simplifies both the analysis and implementation of array signal processing techniques. In a ULA, each element receives a version of the incoming signal that differs only in phase, depending on the angle of arrival. This phase difference can be exploited to steer the array's beam in a desired direction (beamforming) or to estimate the direction of arrival (DOA) of multiple sources. The equal spacing also leads to a regular phase progression across the elements, which makes the array's response mathematically tractable and allows the use of tools like the discrete Fourier transform (DFT) to analyze spatial frequency content.



The performance of a ULA depends on the number of elements and their spacing. The spacing is typically chosen to be half the wavelength of the signal to avoid spatial aliasing, also called grating lobes, which can introduce ambiguities in DOA estimation. Increasing the number of elements improves the array's angular resolution and directivity, meaning it can better distinguish closely spaced sources and focus energy more narrowly. ULAs are widely used in radar, sonar, wireless communications, and microphone arrays due to their simplicity, predictable behavior, and compatibility with well-established signal processing algorithms. Their linear structure also makes them easier to implement in hardware compared to more complex array geometries like circular or planar arrays.

### 3.1 ULA Class Definition

The following is the class used to represent the uniform linear array

---

```

1  template <typename T>
2  class ULAClass
3  {
4  public:
5      // intrinsic parameters
6      int num_sensors; // number of
7          sensors
8      T inter_element_spacing; // space between
9          sensors
10     std::vector<std::vector<T>> coordinates; // coordinates
11         of each sensor
12     T sampling_frequency; // sampling
13         frequency of the sensors
14     T recording_period; // recording
15         period of the ULA
16     std::vector<T> location; // location of
17         first coordinate
18
19     // derived
20     std::vector<T> sensor_direction;
21     std::vector<std::vector<T>> signalMatrix;
22
23     // decimation related
24     int decimation_factor; // the new decimation
25         factor
26     T post_decimation_sampling_frequency; // the new sampling
27         frequency
28     std::vector<T> lowpass_filter_coefficients_for_decimation; // filter-coefficients
29         for filtering
30
31     // imaging related
32     T range_resolution; // theoretical range-resolution =  $\frac{c}{2B}$ 
33     T azimuthal_resolution; // theoretical azimuth-resolution =
34          $\frac{\lambda}{(N-1) \cdot \text{inter-element-distance}}$ 
35     T range_cell_size; // the range-cell quanta we're choosing for
36         efficiency trade-off
37     T azimuth_cell_size; // the azimuth quanta we're choosing
38     std::vector<T> azimuth_centers; // tensor containing the azimuth centers
39     std::vector<T> range_centers; // tensor containing the range-centers
40     int frame_size; // the frame-size corresponding to a range cell in a
41         decimated signal matrix

```

```

31     std::vector<std::vector<complex<T>>> mulFFTMatrix; // the matrix containing the
        delays for each-element as a slot
32     std::vector<complex<T>> matchFilter; // torch tensor containing the
        match-filter
33     int num_buffer_zeros_per_frame; // number of zeros we're adding
        per frame to ensure no-rotation
34     std::vector<std::vector<T>> beamformedImage; // the beamformed image
35     std::vector<std::vector<T>> cartesianImage; // the cartesian version of
        beamformed image
36
37     // Artificial acoustic-image related
38     std::vector<std::vector<T>> currentArtificialAcousticImage; // acoustic image
        directly produced
39
40
41     // Basic Constructor
42     ULAClass() = default;
43
44     // constructor
45     ULAClass(const int num_sensors_arg,
46               const auto inter_element_spacing_arg,
47               const auto& coordinates_arg,
48               const auto& sampling_frequency_arg,
49               const auto& recording_period_arg,
50               const auto& location_arg,
51               const auto& signalMatrix_arg,
52               const auto& lowpass_filter_coefficients_for_decimation_arg):
53         num_sensors(num_sensors_arg),
54         inter_element_spacing(inter_element_spacing_arg),
55         coordinates(std::move(coordinates_arg)),
56         sampling_frequency(sampling_frequency_arg),
57         recording_period(recording_period_arg),
58         location(std::move(location_arg)),
59         signalMatrix(std::move(signalMatrix_arg)),
60         lowpass_filter_coefficients_for_decimation(std::move(lowpass_filter_coefficients_for_decima
61     {
62
63         // calculating ULA direction
64         sensor_direction = std::vector<T>{coordinates[1][0] - coordinates[0][0],
65                                           coordinates[1][1] - coordinates[0][1],
66                                           coordinates[1][2] - coordinates[0][2]};
67
68         // normalizing
69         auto norm_value_temp {std::inner_product(sensor_direction.begin(),
70                                                   sensor_direction.end(),
71                                                   sensor_direction.begin(),
72                                                   0.00)};
73
74         // dividing
75         if (norm_value_temp != 0) {sensor_direction = sensor_direction /
76             norm_value_temp;}
77
78     }
79
80     // deleting copy constructor/assignment
81     // ULAClass(const ULAClass& other) = delete;
82     // ULAClass& operator=(const ULAClass& other) = delete;

```

```

83  /* =====
84  Aim: Build Coordinates Based On Location
85  ----- */
86  void    buildCoordinatesBasedOnLocation();
87
88  /* =====
89  Aim: Init
90  ----- */
91  void    init(TransmitterClass<T>&    transmitterObj);
92
93  /* =====
94  Aim: Creating match-filter
95  ----- */
96  void    nfdc_CreateMatchFilter(TransmitterClass<T>& transmitterObj);
97
98  };

```

---

## 3.2 ULA Setup Scripts

The following script shows the setup-script for Uniform Linear Arrays

---

```

1  template <typename T>
2  void fULASetup(ULAClass<T>&    ula_fls,
3                  ULAClass<T>&    ula_portside,
4                  ULAClass<T>&    ula_starboard)
5  {
6      // setting up ula
7      auto    num_sensors          {static_cast<int>(64)};          // number of sensors
8      T        sampling_frequency   {static_cast<T>(160e3)};        // sampling frequency
9      T        inter_element_spacing {1500/(2*sampling_frequency)}; // space between
10     samples
11     T        recording_period      {10e-2};                      // sampling-period
12
13     // building the direction for the sensors
14     auto    ULA_direction          {std::vector<T>({-1, 0, 0})};
15     auto    ULA_direction_norm      {norm(ULA_direction)};
16     if (ULA_direction_norm != 0)    {ULA_direction = ULA_direction/ULA_direction_norm;}
17     ULA_direction                    =    ULA_direction    *    inter_element_spacing;
18
19     // building coordinates for sensors
20     auto    ULA_coordinates          {transpose(ULA_direction) * \
21                                     linspace<double>(0.00,
22                                                         num_sensors -1,
23                                                         num_sensors)};
24
25     // coefficients of decimation filter
26     auto    lowpassfiltercoefficients {std::vector<T>{0.0000, 0.0000, 0.0000, 0.0000,
27     0.0000, 0.0000, 0.0001, 0.0003, 0.0006, 0.0015, 0.0030, 0.0057, 0.0100, 0.0163,
28     0.0251, 0.0364, 0.0501, 0.0654, 0.0814, 0.0966, 0.1093, 0.1180, 0.1212, 0.1179,
29     0.1078, 0.0914, 0.0699, 0.0451, 0.0192, -0.0053, -0.0262, -0.0416, -0.0504,
30     -0.0522, -0.0475, -0.0375, -0.0239, -0.0088, 0.0057, 0.0179, 0.0263, 0.0303,
31     0.0298, 0.0253, 0.0177, 0.0086, -0.0008, -0.0091, -0.0153, -0.0187, -0.0191,
32     -0.0168, -0.0123, -0.0065, -0.0004, 0.0052, 0.0095, 0.0119, 0.0125, 0.0112,
33     0.0084, 0.0046, 0.0006, -0.0031, -0.0060, -0.0078, -0.0082, -0.0075, -0.0057,
34     -0.0033, -0.0006, 0.0019, 0.0039, 0.0051, 0.0055, 0.0050, 0.0039, 0.0023, 0.0005,
35     -0.0012, -0.0025, -0.0034, -0.0036, -0.0034, -0.0026, -0.0016, -0.0004, 0.0007,

```

```
0.0016, 0.0022, 0.0024, 0.0023, 0.0018, 0.0011, 0.0003, -0.0004, -0.0011,
-0.0015, -0.0016, -0.0015}};
```

```
26
27 // assigning values
28 ula_fls.num_sensors = num_sensors; //
    assigning number of sensors
29 ula_fls.inter_element_spacing = inter_element_spacing; //
    assigning inter-element spacing
30 ula_fls.coordinates = ULA_coordinates; //
    assigning ULA coordinates
31 ula_fls.sampling_frequency = sampling_frequency; //
    assigning sampling frequencys
32 ula_fls.recording_period = recording_period; //
    assigning recording period
33 ula_fls.sensor_direction = ULA_direction; // ULA
    direction
34 ula_fls.lowpass_filter_coefficients_for_decimation = lowpassfiltercoefficients; //
    storing coefficients
35
36
37 // assigning values
38 ula_portside.num_sensors = num_sensors; //
    assigning number of sensors
39 ula_portside.inter_element_spacing = inter_element_spacing; //
    assigning inter-element spacing
40 ula_portside.coordinates = ULA_coordinates; //
    assigning ULA coordinates
41 ula_portside.sampling_frequency = sampling_frequency; //
    assigning sampling frequencys
42 ula_portside.recording_period = recording_period; //
    assigning recording period
43 ula_portside.sensor_direction = ULA_direction; //
    ULA direction
44 ula_portside.lowpass_filter_coefficients_for_decimation = lowpassfiltercoefficients;
    // storing coefficients
45
46
47 // assigning values
48 ula_starboard.num_sensors = num_sensors; //
    assigning number of sensors
49 ula_starboard.inter_element_spacing = inter_element_spacing; //
    assigning inter-element spacing
50 ula_starboard.coordinates = ULA_coordinates; //
    assigning ULA coordinates
51 ula_starboard.sampling_frequency = sampling_frequency; //
    assigning sampling frequencys
52 ula_starboard.recording_period = recording_period; //
    assigning recording period
53 ula_starboard.sensor_direction = ULA_direction; //
    ULA direction
54 ula_starboard.lowpass_filter_coefficients_for_decimation =
    lowpassfiltercoefficients; // storing coefficients
55 }
```

---

# Chapter 4

## Autonomous Underwater Vehicle

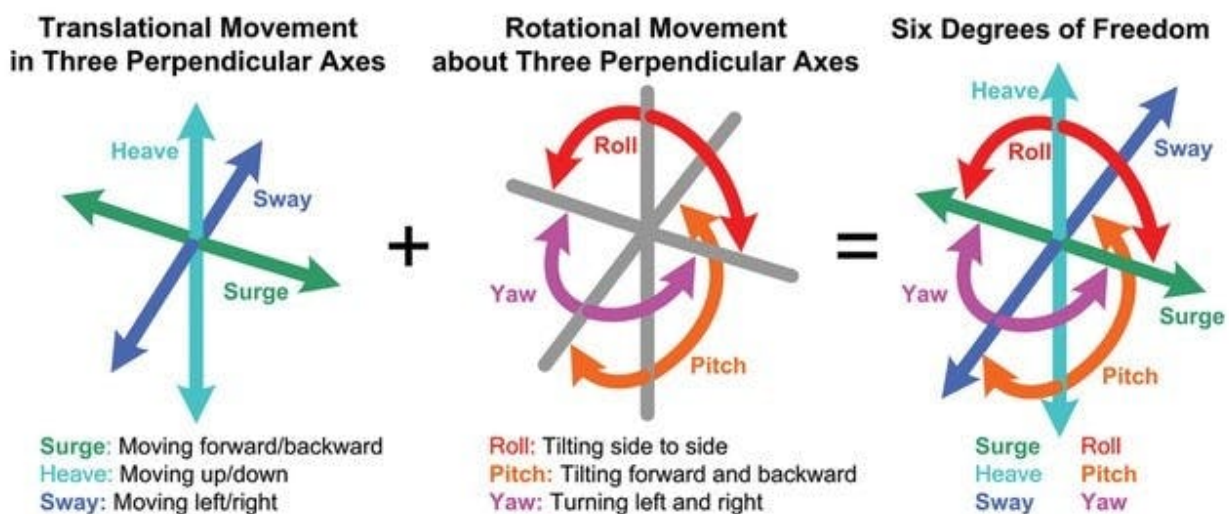


Figure 4.1: AUV degrees of freedom

### Overview

Autonomous Underwater Vehicles (AUVs) are robotic systems designed to operate underwater without direct human control. They navigate and perform missions independently using onboard sensors, processors, and preprogrammed instructions. They are widely used in oceanographic research, environmental monitoring, offshore engineering, and military applications. AUVs can vary in size from small, portable vehicles for shallow water surveys to large, torpedo-shaped platforms capable of deep-sea exploration. Their autonomy allows them to access environments that are too dangerous, remote, or impractical for human divers or tethered vehicles.

The navigation and sensing systems of AUVs are critical to their performance. They typically use a combination of inertial measurement units (IMUs), Doppler velocity logs

(DVLs), pressure sensors, magnetometers, and sometimes acoustic positioning systems to estimate their position and orientation underwater. Since GPS signals do not penetrate water, AUVs must rely on these onboard sensors and occasional surfacing for GPS fixes. They are often equipped with sonar systems, cameras, or other scientific instruments to collect data about the seafloor, water column, or underwater structures. Advanced AUVs can also implement adaptive mission planning and obstacle avoidance, enabling them to respond to changes in the environment in real time.

The applications of AUVs are diverse and expanding rapidly. In scientific research, they are used for mapping the seafloor, studying marine life, and monitoring oceanographic parameters such as temperature, salinity, and currents. In the commercial sector, AUVs inspect pipelines, subsea infrastructure, and offshore oil platforms. Military and defense applications include mine countermeasure operations and underwater surveillance. The development of AUVs continues to focus on increasing endurance, improving autonomy, enhancing sensor payloads, and reducing costs, making them a key technology for exploring and understanding the underwater environment efficiently and safely.

## 4.1 AUV Class Definition

The following is the class used to represent the uniform linear array

---

```

1  template <typename T>
2  class  AUVClass{
3  public:
4
5      // Intrinsic attributes
6      std::vector<T>    location;           // location of vessel
7      std::vector<T>    velocity;          // velocity of the vessel
8      std::vector<T>    acceleration;      // acceleration of vessel
9      std::vector<T>    pointing_direction; // AUV's pointing direction
10
11     // uniform linear-arrays
12     ULAClass<T>        ULA_fls;           // front-looking SONAR ULA
13     ULAClass<T>        ULA_portside;      // mounted ULA [object of class, ULAClass]
14     ULAClass<T>        ULA_starboard;     // mounted ULA [object of class, ULAClass]
15
16     // transmitters
17     TransmitterClass<T> transmitter_fls;   // transmitter for front-looking SONAR
18     TransmitterClass<T> transmitter_portside; // portside transmitter
19     TransmitterClass<T> transmitter_starboard; // starboard transmitter
20
21     // derived or dependent attributes
22     std::vector<std::vector<T>> signalMatrix_1; // matrix containing the
23         signals obtained from ULA_1
24     std::vector<std::vector<T>> largeSignalMatrix_1; // matrix holding signal of
25         synthetic aperture
26     std::vector<std::vector<T>> beamformedLargeSignalMatrix; // each column is the
27         beamformed signal at each stop-hop
28
29     // plotting mode
30     bool plottingmode; // to suppress plotting associated with classes
31
32     // spotlight mode related
33     std::vector<std::vector<T>> absolute_coords_patch_cart; // cartesian coordinates of

```

```

31     patch
32     // Synthetic Aperture Related
33     std::vector<std::vector<T>> ApertureSensorLocations;    // sensor locations of
34     aperture
35     // functions
36     void syncComponentAttributes();
37     void init();
38
39 };

```

---

## 4.2 AUV Setup Scripts

The following script shows the setup-script for Uniform Linear Arrays

---

```

1  template <typename T>
2  void fAUVSetup(AUVClass<T>& auv) {
3
4      // building properties for the auv
5      auto    location          {std::vector<T>{0, 50, 30}};    // starting location
6      auto    velocity          {std::vector<T>{5, 0, 0}};    // starting velocity
7      auto    pointing_direction {std::vector<T>{1, 0, 0}};    // pointing direction
8
9      // assigning
10     auv.location              = std::move(location);          // assigning location
11     auv.velocity              = std::move(velocity);          // assigning velocity
12     auv.pointing_direction    = std::move(pointing_direction); // assigning pointing
13                             direction
14 }

```

---

# Appendix A

## General Purpose Templated Functions

### A.1 CSV File-Writes

```
1 // =====
2 template <typename T>
3 void fWriteVector(const vector<T>&          inputvector,
4                  const string&             filename){
5
6     // opening a file
7     std::ofstream fileobj(filename);
8     if (!fileobj) {return;}
9
10    // writing the real parts in the first column and the imaginary parts in the second
11    // column
12    if constexpr(std::is_same_v<T, std::complex<double>> ||
13                 std::is_same_v<T, std::complex<float>> ||
14                 std::is_same_v<T, std::complex<long double>>){
15        for(int i = 0; i<inputvector.size(); ++i){
16            // adding entry
17            fileobj << inputvector[i].real() << "+" << inputvector[i].imag() << "i";
18
19            // adding delimiter
20            if(i!=inputvector.size()-1) {fileobj << ",";}
21            else {fileobj << "\n";}
22        }
23    }
24    else{
25        for(int i = 0; i<inputvector.size(); ++i){
26            fileobj << inputvector[i];
27            if(i!=inputvector.size()-1) {fileobj << ",";}
28            else {fileobj << "\n";}
29        }
30    }
31
32    // return
33    return;
34 }
35 // Matrix writing =====
36 template <typename T>
37 auto fWriteMatrix(const std::vector<std::vector<T>> inputMatrix,
38                  const string                      filename){
```



```

38
39 // opening a file
40 std::ofstream fileobj(filename);
41
42 // writing
43 if (fileobj){
44     for(int i = 0; i<inputMatrix.size(); ++i){
45         for(int j = 0; j<inputMatrix[0].size(); ++j){
46             fileobj << inputMatrix[i][j];
47             if (j!=inputMatrix[0].size()-1) {fileobj << ",";}
48             else {fileobj << "\n";}
49         }
50     }
51 }
52 else{
53     cout << format("File-write to {} failed\n", filename);
54 }
55
56 }
57
58 template <>
59 auto fWriteMatrix(const std::vector<std::vector<std::complex<double>>> inputMatrix,
60                  const string filename){
61
62     // opening a file
63     std::ofstream fileobj(filename);
64
65     // writing
66     if (fileobj){
67         for(int i = 0; i<inputMatrix.size(); ++i){
68             for(int j = 0; j<inputMatrix[0].size(); ++j){
69                 fileobj << inputMatrix[i][j].real() << "+" << inputMatrix[i][j].imag() <<
70                     "i";
71                 if (j!=inputMatrix[0].size()-1) {fileobj << ",";}
72                 else {fileobj << "\n";}
73             }
74         }
75     }
76     else{
77         cout << format("File-write to {} failed\n", filename);
78     }
79 }

```

---

## A.2 abs

```

1 // =====
2 // y = abs(vector)
3 template <typename T>
4 auto abs(const std::vector<T>& input_vector)
5 {
6     // creating canvas
7     auto canvas {input_vector};
8
9     // calculating abs
10    std::transform(canvas.begin(),
11                  canvas.end(),

```



```

31         return argx <= static_cast<T>(scalar);
32     });
33
34     // returning
35     return std::move(canvas);
36 }
37 // =====
38 template <typename T, typename U>
39 auto operator>(const std::vector<T>& input_vector,
40               const U scalar)
41 {
42     // creating canvas
43     auto canvas {std::vector<bool>(input_vector.size())};
44
45     // transforming
46     std::transform(input_vector.begin(), input_vector.end(),
47                   canvas.begin(),
48                   [&scalar](const auto& argx){
49                       return argx > static_cast<T>(scalar);
50                   });
51
52     // returning
53     return std::move(canvas);
54 }
55 // =====
56 template <typename T, typename U>
57 auto operator>=(const std::vector<T>& input_vector,
58                const U scalar)
59 {
60     // creating canvas
61     auto canvas {std::vector<bool>(input_vector.size())};
62
63     // transforming
64     std::transform(input_vector.begin(), input_vector.end(),
65                   canvas.begin(),
66                   [&scalar](const auto& argx){
67                       return argx >= static_cast<T>(scalar);
68                   });
69
70     // returning
71     return std::move(canvas);
72 }

```

---

## A.4 Concatenate Functions

```

1 // input = [vector, vector],
2 // output = [vector]
3 template <std::size_t axis, typename T>
4 auto concatenate(const std::vector<T>& input_vector_A,
5                 const std::vector<T>& input_vector_B) -> std::enable_if_t<axis == 1,
6                 std::vector<T> >
7 {
8     // creating canvas vector
9     auto num_elements {input_vector_A.size() + input_vector_B.size()};
10    auto canvas {std::vector<T>(num_elements, (T)0) };

```

```

11 // filling up the canvas
12 std::copy(input_vector_A.begin(), input_vector_A.end(),
13           canvas.begin());
14 std::copy(input_vector_B.begin(), input_vector_B.end(),
15           canvas.begin()+input_vector_A.size());
16
17 // moving it back
18 return std::move(canvas);
19
20 }
21 // =====
22 // input = [vector, vector],
23 // output = [matrix]
24 template <std::size_t axis, typename T>
25 auto concatenate(const std::vector<T>& input_vector_A,
26                 const std::vector<T>& input_vector_B) -> std::enable_if_t<axis == 0,
27                 std::vector<std::vector<T>>> >
28 {
29     // throwing error dimensions
30     if (input_vector_A.size() != input_vector_B.size())
31         std::cerr << "concatenate:: incorrect dimensions \n";
32
33     // creating canvas
34     auto canvas {std::vector<std::vector<T>>>(
35         2, std::vector<T>(input_vector_A.size())
36     )};
37
38     // filling up the dimensions
39     std::copy(input_vector_A.begin(), input_vector_A.end(), canvas[0].begin());
40     std::copy(input_vector_B.begin(), input_vector_B.end(), canvas[1].begin());
41
42     // moving it back
43     return std::move(canvas);
44 }
45 // =====
46 // input = [vector, vector, vector],
47 // output = [matrix]
48 template <std::size_t axis, typename T>
49 auto concatenate(const std::vector<T>& input_vector_A,
50                 const std::vector<T>& input_vector_B,
51                 const std::vector<T>& input_vector_C) -> std::enable_if_t<axis == 0,
52                 std::vector<std::vector<T>>> >
53 {
54     // throwing error dimensions
55     if (input_vector_A.size() != input_vector_B.size() ||
56         input_vector_A.size() != input_vector_C.size())
57         std::cerr << "concatenate:: incorrect dimensions \n";
58
59     // creating canvas
60     auto canvas {std::vector<std::vector<T>>>(
61         3, std::vector<T>(input_vector_A.size())
62     )};
63
64     // filling up the dimensions
65     std::copy(input_vector_A.begin(), input_vector_A.end(), canvas[0].begin());
66     std::copy(input_vector_B.begin(), input_vector_B.end(), canvas[1].begin());
67     std::copy(input_vector_C.begin(), input_vector_C.end(), canvas[2].begin());

```

```

68     // moving it back
69     return std::move(canvas);
70
71 }
72 // =====
73 // input = [matrix, vector],
74 // output = [matrix]
75 template <std::size_t axis, typename T>
76 auto concatenate(const std::vector<std::vector<T>>& input_matrix,
77                 const std::vector<T> input_vector) -> std::enable_if_t<axis
78                 == 0, std::vector<std::vector<T>>> >
79 {
80     // creating canvas
81     auto canvas {input_matrix};
82
83     // adding to the canvas
84     canvas.push_back(input_vector);
85
86     // returning
87     return std::move(canvas);
88 }

```

---

## A.5 Conjugate

```

1 namespace svr {
2     // =====
3     template <typename T>
4     auto conj(const std::vector<T>& input_vector)
5     {
6         // creating canvas
7         auto canvas {std::vector<T>(input_vector.size())};
8
9         // calculating conjugates
10        std::for_each(canvas.begin(), canvas.end(),
11                      [](auto& argx){argx = std::conj(argx);});
12
13        // returning
14        return std::move(canvas);
15    }
16 }

```

---

## A.6 Convolution

```

1 namespace svr {
2     // =====
3     template <typename T1, typename T2>
4     auto conv1D(const std::vector<T1>& input_vector_A,
5                const std::vector<T2>& input_vector_B)
6     {
7         // resulting type
8         using T3 = decltype(std::declval<T1>() * std::declval<T2>());
9
10        // creating canvas
11        auto canvas_length {input_vector_A.size() + input_vector_B.size() - 1};

```

```

12
13     // calculating fft of two arrays
14     auto    fft_A      {svr::fft(input_vector_A, canvas_length)};
15     auto    fft_B      {svr::fft(input_vector_B, canvas_length)};
16
17     // element-wise multiplying the two matrices
18     auto    fft_AB     {fft_A *  fft_B};
19
20     // finding inverse FFT
21     auto    convolved_result  {ifft(fft_AB)};
22
23     // returning
24     return std::move(convolved_result);
25 }
26
27 }

```

---

## A.7 Coordinate Change

```

1 namespace svr {
2     // =====
3     template <typename T>
4     auto    cart2sph(const    std::vector<T> cartesian_vector){
5
6         // splatting the point onto xy-plane
7         auto    xysplat    {cartesian_vector};
8         xysplat[2]    =    0;
9
10        // finding splat lengths
11        auto    xysplat_lengths    {norm(xysplat)};
12
13        // finding azimuthal and elevation angles
14        auto    azimuthal_angles    {svr::atan2(xysplat[1], xysplat[0]) *
15                                     180.00/std::numbers::pi};
16        auto    elevation_angles    {svr::atan2(cartesian_vector[2], xysplat_lengths) *
17                                     180.00/std::numbers::pi};
18        auto    rho_values          {norm(cartesian_vector)};
19
20        // creating tensor to send back
21        auto    spherical_vector    {std::vector<T>{azimuthal_angles,
22                                                     elevation_angles,
23                                                     rho_values}};
24
25        // moving it back
26        return std::move(spherical_vector);
27    }
28    // =====
29    template <typename T>
30    auto    sph2cart(const    std::vector<T> spherical_vector){
31
32        // creating cartesian vector
33        auto    cartesian_vector    {std::vector<T>(spherical_vector.size(), 0)};
34
35        // populating
36        cartesian_vector[0]    =    spherical_vector[2] * \

```

```

36         cos(spherical_vector[1] * std::numbers::pi / 180.00) * \
37         cos(spherical_vector[0] * std::numbers::pi / 180.00);
38     cartesian_vector[1] = spherical_vector[2] * \
39     cos(spherical_vector[1] * std::numbers::pi / 180.00) * \
40     sin(spherical_vector[0] * std::numbers::pi / 180.00);
41     cartesian_vector[2] = spherical_vector[2] * \
42     sin(spherical_vector[1] * std::numbers::pi / 180.00);
43
44     // returning
45     return std::move(cartesian_vector);
46 }
47 }

```

---

## A.8 Cosine

```

1  // =====
2  // y = cos(input_vector)
3  template <typename T>
4  auto cos(const std::vector<T>& input_vector)
5  {
6      // created canvas
7      auto canvas {input_vector};
8
9      // calling the function
10     std::transform(input_vector.begin(), input_vector.end(),
11                   canvas.begin(),
12                   [](auto& argx){return std::cos(argx);});
13
14     // returning the output
15     return std::move(canvas);
16 }
17 // =====
18 // y = cosd(input_vector)
19 template <typename T>
20 auto cosd(std::vector<T> input_vector)
21 {
22     // created canvas
23     auto canvas {input_vector};
24
25     // calling the function
26     std::transform(input_vector.begin(),
27                   input_vector.end(),
28                   input_vector.begin(),
29                   [](const auto& argx){return std::cos(argx * 180.00/std::numbers::pi);});
30
31     // returning the output
32     return std::move(canvas);
33 }

```

---

## A.9 Data Structures

```

1  struct TreeNode {
2      int val;
3      TreeNode *left;

```





```

13         return argx == scalar;
14     });
15
16     // returning
17     return std::move(canvas);
18 }

```

---

## A.12 Exponentiate

---

```

1 // y = abs(vector)
2 template <typename T>
3 auto exp(const std::vector<T>& input_vector)
4 {
5     // creating canvas
6     auto canvas {input_vector};
7
8     // transforming
9     std::transform(canvas.begin(), canvas.end(),
10                   canvas.begin(),
11                   [](auto& argx){return std::exp(argx);});
12
13     // returning
14     return std::move(canvas);
15 }

```

---

## A.13 FFT

---

```

1 namespace svr {
2     // =====
3     // For type-deductions
4     template <typename T>
5     struct fft_result_type;
6
7     // specializations
8     template <> struct fft_result_type<double>{
9         using type = std::complex<double>;
10    };
11    template <> struct fft_result_type<std::complex<double>>{
12        using type = std::complex<double>;
13    };
14    template <> struct fft_result_type<float>{
15        using type = std::complex<float>;
16    };
17    template <> struct fft_result_type<std::complex<float>>{
18        using type = std::complex<float>;
19    };
20
21    template <typename T>
22    using fft_result_t = typename fft_result_type<T>::type;
23
24    // =====
25    // y = fft(x, nfft)
26    template<typename T>
27    auto fft(const std::vector<T>& input_vector,

```

```

28         const    size_t    nfft)
29 {
30     // throwing an error
31     if (nfft < input_vector.size()) {std::cerr << "size-mismatch\n";}
32     if (nfft <= 0)                  {std::cerr << "size-mismatch\n";}
33
34     // fetching data-type
35     using RType = fft_result_t<T>;
36     using baseType = std::conditional_t<std::is_same_v<T, std::complex<double>>,
37                                         double,
38                                         T>;
39
40     // canvas instantiation
41     std::vector<RType> canvas(nfft);
42     auto nfft_sqrt    {static_cast<RType>(std::sqrt(nfft))};
43     auto finaloutput  {std::vector<RType>(nfft, 0)};
44
45     // calculating index by index
46     for(int frequency_index = 0; frequency_index<nfft; ++frequency_index){
47         RType accumulate_value;
48         for(int signal_index = 0; signal_index < input_vector.size(); ++signal_index){
49             accumulate_value += \
50                 static_cast<RType>(input_vector[signal_index]) * \
51                 static_cast<RType>(std::exp(-1.00 * std::numbers::pi * \
52                                         (static_cast<baseType>(frequency_index)/static_cast<baseType>(nfft) * \
53                                         * \
54                                         static_cast<baseType>(signal_index))));
55             }
56             finaloutput[frequency_index] = accumulate_value / nfft_sqrt;
57         }
58     }
59     // returning
60     return std::move(finaloutput);
61 }
62
63 // =====
64 // y = ifft(x, nfft)
65 template<typename T>
66 auto ifft(const std::vector<T>& input_vector)
67 {
68     // fetching data-type
69     using RType = fft_result_t<T>;
70     using baseType = std::conditional_t<std::is_same_v<T, std::complex<double>>,
71                                         double,
72                                         T>;
73
74     // setup
75     auto nfft    {input_vector.size()};
76
77     // canvas instantiation
78     std::vector<RType> canvas(nfft);
79     auto nfft_sqrt    {static_cast<RType>(std::sqrt(nfft))};
80     auto finaloutput  {std::vector<RType>(nfft, 0)};
81
82     // calculating index by index
83     for(int frequency_index = 0; frequency_index<nfft; ++frequency_index){
84         RType accumulate_value;
85         for(int signal_index = 0; signal_index < input_vector.size(); ++signal_index){
86             accumulate_value += \

```

```

86         static_cast<RType>(input_vector[signal_index]) * \
87         static_cast<RType>(std::exp(1.00 * std::numbers::pi * \
88             (static_cast<baseType>(frequency_index)/static_cast<baseType>(n)
89             * \
90             static_cast<baseType>(signal_index))));
91     }
92     finaloutput[frequency_index] = accumulate_value / nfft_sqrt;
93 }
94 // returning
95 return std::move(finaloutput);
96 }
97 }

```

---

## A.14 Flipping Containers

```

1 namespace svr {
2     // =====
3     template <typename T>
4     auto fliplr(const std::vector<T>& input_vector)
5     {
6         // creating canvas
7         auto canvas {input_vector};
8
9         // rewriting
10        std::reverse(canvas.begin(), canvas.end());
11
12        // returning
13        return std::move(canvas);
14    }
15 }

```

---

## A.15 Indexing

```

1 namespace svr {
2     // =====
3     template <typename T1, typename T2>
4     auto index(const std::vector<T1>& input_vector,
5               const std::vector<T2>& indices_to_sample)
6     {
7         // creating canvas
8         auto canvas {std::vector<T1>(indices_to_sample.size(), 0)};
9
10        // copying the associated values
11        for(int i = 0; i < indices_to_sample.size(); ++i){
12            auto source_index {indices_to_sample[i]};
13            if(source_index < input_vector.size()){
14                canvas[i] = input_vector[source_index];
15            }
16            else
17                cout << "svr::index | source_index !< input_vector.size()\n";
18        }
19
20        // returning

```

```

21     return std::move(canvas);
22 }
23 }

```

---

## A.16 Linspace

---

```

1  // in-place
2  template <typename T>
3  auto linspace(auto&          input,
4                auto          startvalue,
5                auto          endvalue,
6                auto          numpoints) -> void
7  {
8      auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);
9      for(int i = 0; i<input.size(); ++i) {input[i] = startvalue + i*stepsize;}
10 };
11 // in-place
12 template <typename T>
13 auto linspace(vector<complex<T>>& input,
14               auto          startvalue,
15               auto          endvalue,
16               auto          numpoints) -> void
17 {
18     auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);
19     for(int i = 0; i<input.size(); ++i) {
20         input[i] = startvalue + static_cast<T>(i)*stepsize;
21     }
22 };
23
24 // return-type
25 template <typename T>
26 auto linspace(T          startvalue,
27               T          endvalue,
28               size_t      numpoints)
29 {
30     vector<T> input(numpoints);
31     auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);
32
33     for(int i = 0; i<input.size(); ++i) {input[i] = startvalue +
34         static_cast<T>(i)*stepsize;}
35
36     return input;
37 };
38
39 // return-type
40 template <typename T, typename U>
41 auto linspace(T          startvalue,
42               U          endvalue,
43               size_t      numpoints)
44 {
45     vector<double> input(numpoints);
46     auto stepsize = static_cast<double>(endvalue -
47         startvalue)/static_cast<double>(numpoints-1);
48
49     for(int i = 0; i<input.size(); ++i) {input[i] = startvalue + i*stepsize;}

```

```

49     return input;
50 };

```

---

## A.17 Max

```

1  template <std::size_t axis, typename T>
2  auto    max(const std::vector<std::vector<T>> input_matrix) -> std::enable_if_t<axis ==
      1, std::vector<std::vector<T>> >
3  {
4      // setting up canvas
5      auto    canvas
      {std::vector<std::vector<T>>(input_matrix.size(), std::vector<T>(1))};
6
7      // filling up the canvas
8      for(auto row = 0; row < input_matrix.size(); ++row)
9          canvas[row][0] = *(std::max_element(input_matrix[row].begin(),
      input_matrix[row].end()));
10
11     // returning
12     return std::move(canvas);
13 }

```

---

## A.18 Meshgrid

```

1  // =====
2  template <typename T>
3  auto meshgrid(const std::vector<T>& x,
4                const std::vector<T>& y)
5  {
6
7      // creating and filling x-grid
8      std::vector<std::vector<T>> xcanvas(y.size(), std::vector<T>(x.size(), 0));
9      for(auto row = 0; row < y.size(); ++row)
10         std::copy(x.begin(), x.end(), xcanvas[row].begin());
11
12     // creating and filling y-grid
13     std::vector<std::vector<T>> ycanvas(y.size(), std::vector<T>(x.size(), 0));
14     for(auto col = 0; col < x.size(); ++col)
15         for(auto row = 0; row < y.size(); ++row)
16             ycanvas[row][col] = y[row];
17
18     // returning
19     return std::move(std::pair{xcanvas, ycanvas});
20
21 }
22 // =====
23 template <typename T>
24 auto meshgrid(std::vector<T>&& x,
25              std::vector<T>&& y)
26 {
27
28     // creating and filling x-grid
29     std::vector<std::vector<T>> xcanvas(y.size(), std::vector<T>(x.size(), 0));
30     for(auto row = 0; row < y.size(); ++row)

```

```

31     std::copy(x.begin(), x.end(), xcanvas[row].begin());
32
33     // creating and filling y-grid
34     std::vector<std::vector<T>> ycanvas(y.size(), std::vector<T>(x.size(), 0));
35     for(auto col = 0; col < x.size(); ++col)
36         for(auto row = 0; row < y.size(); ++row)
37             ycanvas[row][col] = y[row];
38
39     // returning
40     return std::move(std::pair{xcanvas, ycanvas});
41
42 }

```

---

## A.19 Minimum

```

1  template <std::size_t axis, typename T>
2  auto min(std::vector<std::vector<T>> input_matrix) -> std::enable_if_t<axis == 1,
   std::vector<std::vector<T>> >
3  {
4      // creating canvas
5      auto canvas
6          {std::vector<std::vector<T>>(input_matrix.size(), std::vector<T>(1))};
7
8      // storing the values
9      for(auto row = 0; row < input_matrix.size(); ++row)
10         canvas[row][0] = *(std::min_element(input_matrix[row].begin(),
11         input_matrix[row].end()));
12
13     // returning the value
14     return std::move(canvas);
15 }

```

---

## A.20 Norm

```

1  // =====
2  template <typename T>
3  auto norm(const std::vector<T>& input_vector)
4  {
5      return std::sqrt(std::inner_product(input_vector.begin(), input_vector.end(),
6      input_vector.begin(),
7      (T)0));
8  }
9
10
11  /*
12  Templates to create
13  - matrix and norm-axis
14  - axis instantiated std::vector<T>
15  */
16

```

---

## A.21 Division

---

```

1 // =====
2 // matrix division with scalars
3 template <typename T>
4 auto operator/(const std::vector<T>& input_vector,
5               const T& input_scalar)
6 {
7     // creating canvas
8     auto canvas {input_vector};
9
10    // filling canvas
11    std::transform(canvas.begin(), canvas.end(),
12                  canvas.begin(),
13                  [&input_scalar](const auto& argx){
14                      return static_cast<double>(argx) /
15                          static_cast<double>(input_scalar);
16                  });
17
18    // returning value
19    return std::move(canvas);
20 }
21 // =====
22 // matrix division with scalars
23 template <typename T>
24 auto operator/=(const std::vector<T>& input_vector,
25                const T& input_scalar)
26 {
27     // creating canvas
28     auto canvas {input_vector};
29
30    // filling canvas
31    std::transform(canvas.begin(), canvas.end(),
32                  canvas.begin(),
33                  [&input_scalar](const auto& argx){
34                      return static_cast<double>(argx) /
35                          static_cast<double>(input_scalar);
36                  });
37
38    // returning value
39    return std::move(canvas);
40 }

```

---

## A.22 Addition

---

```

1 // =====
2 // y = vector + vector
3 template <typename T>
4 std::vector<T> operator+(const std::vector<T>& a,
5                        const std::vector<T>& b)
6 {
7     // Identify which is bigger
8     const auto& big = (a.size() > b.size()) ? a : b;
9     const auto& small = (a.size() > b.size()) ? b : a;
10
11    std::vector<T> result = big; // copy the bigger one

```

```

12
13 // Add elements from the smaller one
14 for (size_t i = 0; i < small.size(); ++i) {
15     result[i] += small[i];
16 }
17
18 return result;
19 }
20 // =====
21 // y = vector + vector
22 template <typename T>
23 std::vector<T>& operator+=(std::vector<T>& a,
24                          const std::vector<T>& b) {
25
26     const auto& small = (a.size() < b.size()) ? a : b;
27     const auto& big = (a.size() < b.size()) ? b : a;
28
29     // If b is bigger, resize 'a' to match
30     if (a.size() < b.size()) {a.resize(b.size());}
31
32     // Add elements
33     for (size_t i = 0; i < small.size(); ++i) {a[i] += b[i];}
34
35     // returning elements
36     return a;
37 }
38 // =====
39 // y = matrix + matrix
40 template <typename T>
41 std::vector<std::vector<T>> operator+(const std::vector<std::vector<T>>& a,
42                                     const std::vector<std::vector<T>>& b)
43 {
44     // fetching dimensions
45     const auto& num_rows_A = {a.size()};
46     const auto& num_cols_A = {a[0].size()};
47     const auto& num_rows_B = {b.size()};
48     const auto& num_cols_B = {b[0].size()};
49
50     // choosing the three different metrics
51     if (num_rows_A != num_rows_B && num_cols_A != num_cols_B){
52         cout << format("a.dimensions = [{},{}, b.shape = [{},{},\n",
53                       num_rows_A, num_cols_A,
54                       num_rows_B, num_cols_B);
55         std::cerr << "dimensions don't match\n";
56     }
57
58     // creating canvas
59     auto canvas = {std::vector<std::vector<T>>(
60         std::max(num_rows_A, num_rows_B),
61         std::vector<T>(std::max(num_cols_A, num_cols_B), (T)0.00)
62     )};
63
64     // performing addition
65     if (num_rows_A == num_rows_B && num_cols_A == num_cols_B){
66         for(auto row = 0; row < num_rows_A; ++row){
67             std::transform(a[row].begin(), a[row].end(),
68                           b[row].begin(),
69                           canvas[row].begin(),
70                           std::plus<T>());

```



```

71     }
72 }
73 else if(num_rows_A == num_rows_B){
74
75     // if number of columns are different, check if one of the cols are one
76     const auto min_num_cols {std::min(num_cols_A, num_cols_B)};
77     if (min_num_cols != 1) {std::cerr<< "Operator+: unable to broadcast\n";}
78     const auto max_num_cols {std::max(num_cols_A, num_cols_B)};
79
80     // using references to tag em differently
81     const auto& big_matrix {num_cols_A > num_cols_B ? a : b};
82     const auto& small_matrix {num_cols_A < num_cols_B ? a : b};
83
84     // Adding to canvas
85     for(auto row = 0; row < canvas.size(); ++row){
86         std::transform(big_matrix[row].begin(), big_matrix[row].end(),
87             canvas[row].begin(),
88             [&small_matrix,
89              &row](const auto& argx){
90                 return argx + small_matrix[row][0];
91             });
92     }
93 }
94 else if(num_cols_A == num_cols_B){
95
96     // check if the smallest column-number is one
97     const auto min_num_rows {std::min(num_rows_A, num_rows_B)};
98     if(min_num_rows != 1) {std::cerr << "Operator+ : unable to broadcast\n";}
99     const auto max_num_rows {std::max(num_rows_A, num_rows_B)};
100
101     // using references to differentiate the two matrices
102     const auto& big_matrix {num_rows_A > num_rows_B ? a : b};
103     const auto& small_matrix {num_rows_A < num_rows_B ? a : b};
104
105     // adding to canvas
106     for(auto row = 0; row < canvas.size(); ++row){
107         std::transform(big_matrix[row].begin(), big_matrix[row].end(),
108             small_matrix[0].begin(),
109             canvas[row].begin(),
110             [](const auto& argx, const auto& argy){
111                 return argx + argy;
112             });
113     }
114 }
115 else {
116     PRINTLINE PRINTLINE PRINTLINE PRINTLINE PRINTLINE
117     cout << format("check this again \n");
118 }
119
120 // returning
121 return std::move(canvas);
122 }
123 // =====
124 // y = vector + scalar
125 template <typename T>
126 auto operator+(const std::vector<T>& input_vector,
127               const T scalar)
128 {
129     // creating canvas

```

```

130     auto    canvas    {input_vector};
131
132     // adding scalar to the canvas
133     std::transform(canvas.begin(), canvas.end(),
134                   canvas.begin(),
135                   [&scalar](auto& argx){return argx + scalar;});
136
137     // returning canvas
138     return std::move(canvas);
139 }
140 // =====
141 // y = scalar + vector
142 template <typename T>
143 auto operator+(const T          scalar,
144               const std::vector<T>& input_vector)
145 {
146     // creating canvas
147     auto    canvas    {input_vector};
148
149     // adding scalar to the canvas
150     std::transform(canvas.begin(), canvas.end(),
151                   canvas.begin(),
152                   [&scalar](auto& argx){return argx + scalar;});
153
154     // returning canvas
155     return std::move(canvas);
156 }

```

---

## A.23 Multiplication (Element-wise)

```

1 // scalar * vector =====
2 template <typename T>
3 auto operator*(const T          scalar,
4               const std::vector<T>& input_vector)
5 {
6     // creating canvas
7     auto    canvas    {input_vector};
8     // performing operation
9     std::for_each(canvas.begin(), canvas.end(),
10                  [&scalar](auto& argx){argx = argx * scalar;});
11     // returning
12     return std::move(canvas);
13 }
14
15 // scalar * vector =====
16 // template <typename T1, typename T2>
17 template <typename T1, typename T2,
18         typename = std::enable_if_t<!std::is_same_v<std::decay_t<T1>, std::vector<T2>>>>
19 auto operator*(const T1          scalar,
20               const vector<T2>& input_vector)
21 {
22     // fetching final-type
23     using T3 = decltype(std::declval<T1>() * std::declval<T2>());
24     // creating canvas
25     auto    canvas    {std::vector<T3>(input_vector.size())};
26     // multiplying

```

```

27     std::transform(input_vector.begin(), input_vector.end(),
28                    canvas.begin(),
29                    [&scalar](auto& argx){
30                        return static_cast<T3>(scalar) * static_cast<T3>(argx);
31                    });
32     // returning
33     return std::move(canvas);
34 }
35
36 // vector * scalar =====
37 template <typename T>
38 auto operator*(const std::vector<T>& input_vector,
39               const T scalar)
40 {
41     // creating canvas
42     auto canvas {input_vector};
43     // multiplying
44     std::for_each(canvas.begin(), canvas.end(),
45                  [&scalar](auto& argx){
46                      argx = argx * scalar;
47                  });
48     // returning
49     return std::move(canvas);
50 }
51
52 // vector * vector =====
53 template <typename T>
54 auto operator*(const std::vector<T>& input_vector_A,
55               const std::vector<T>& input_vector_B)
56 {
57     // throwing error: size-disparity
58     if (input_vector_A.size() != input_vector_B.size()) {std::cerr << "operator*: size
59         disparity \n";}
60
61     // creating canvas
62     auto canvas {input_vector_A};
63
64     // element-wise multiplying
65     std::transform(input_vector_B.begin(), input_vector_B.end(),
66                   canvas.begin(),
67                   canvas.begin(),
68                   [](const auto& argx, const auto& argy){
69                       return argx * argy;
70                   });
71
72     // moving it back
73     return std::move(canvas);
74 }
75
76 template <typename T1, typename T2>
77 auto operator*(const std::vector<T1>& input_vector_A,
78               const std::vector<T2>& input_vector_B)
79 {
80     // checking size disparity
81     if (input_vector_A.size() != input_vector_B.size())
82         std::cerr << "operator*: error, size-disparity \n";
83
84     // figuring out resulting data type
85     using T3 = decltype(std::declval<T1>() * std::declval<T2>());

```

```

85
86 // creating canvas
87 auto canvas {std::vector<T3>(input_vector_A.size())};
88
89 // performing multiplications
90 std::transform(input_vector_A.begin(), input_vector_A.end(),
91               input_vector_B.begin(),
92               canvas.begin(),
93               [](const auto& argx,
94                 const auto& argy){
95                 return static_cast<T3>(argx) * static_cast<T3>(argy);
96             });
97
98 // returning
99 return std::move(canvas);
100
101 }
102
103 // scalar * matrix =====
104 template <typename T>
105 auto operator*(T scalar,
106               const std::vector<std::vector<T>>& inputMatrix)
107 {
108     std::vector<std::vector<T>> temp {inputMatrix};
109     for(int i = 0; i<inputMatrix.size(); ++i){
110         std::transform(inputMatrix[i].begin(),
111                       inputMatrix[i].end(),
112                       temp[i].begin(),
113                       [&scalar](T x){return scalar * x;});
114     }
115     return temp;
116 }
117 // matrix * matrix =====
118 template <typename T>
119 auto operator*(const std::vector<std::vector<T>>& A,
120               const std::vector<std::vector<T>>& B) -> std::vector<std::vector<T>>
121 {
122     // Case 1: element-wise multiplication
123     if (A.size() == B.size() && A[0].size() == B[0].size()) {
124         std::vector<std::vector<T>> C(A.size(), std::vector<T>(A[0].size()));
125         for (std::size_t row = 0; row < A.size(); ++row) {
126             std::transform(A[row].begin(), A[row].end(),
127                           B[row].begin(),
128                           C[row].begin(),
129                           [](const auto& x, const auto& y){ return x * y; });
130         }
131         return C;
132     }
133
134     // Case 2: broadcast column vector
135     else if (A.size() == B.size() && B[0].size() == 1) {
136         std::vector<std::vector<T>> C(A.size(), std::vector<T>(A[0].size()));
137         for (std::size_t row = 0; row < A.size(); ++row) {
138             std::transform(A[row].begin(), A[row].end(),
139                           C[row].begin(),
140                           [&](const auto& x){ return x * B[row][0]; });
141         }
142         return C;
143     }

```

```

144
145 // case 3: when second matrix contains just one row
146 // case 4: when first matrix is just one column
147 // case 5: when second matrix is just one column
148
149 // Otherwise, invalid
150 else {
151     throw std::runtime_error("operator* dimension mismatch");
152 }
153 }
154 // scalar * matrix =====
155 template <typename T1, typename T2>
156 auto operator*(T1 scalar,
157               const std::vector<std::vector<T2>>& inputMatrix)
158 {
159     std::vector<std::vector<T2>> temp {inputMatrix};
160     for(int i = 0; i<inputMatrix.size(); ++i){
161         std::transform(inputMatrix[i].begin(),
162                        inputMatrix[i].end(),
163                        temp[i].begin(),
164                        [&scalar](T2 x){return static_cast<T2>(scalar) * x;});
165     }
166     return temp;
167 }
168 // matrix-multiplication =====
169 template <typename T1, typename T2>
170 auto matmul(const std::vector<std::vector<T1>>& matA,
171            const std::vector<std::vector<T2>>& matB)
172 {
173
174     // throwing error
175     if (matA[0].size() != matB.size()) {std::cerr << "dimension-mismatch \n";}
176
177     // getting result-type
178     using ResultType = decltype(std::declval<T1>() * std::declval<T2>() + \
179                                std::declval<T1>() * std::declval<T2>());
180
181     // creating aliases
182     auto finalnumrows {matA.size()};
183     auto finalnumcols {matB[0].size()};
184
185     // creating placeholder
186     auto rowcolproduct = [&](auto rowA, auto colB){
187         ResultType temp {0};
188         for(int i = 0; i < matA.size(); ++i) {temp +=
189             static_cast<ResultType>(matA[rowA][i]) +
190             static_cast<ResultType>(matB[i][colB]);}
191         return temp;
192     };
193
194     // producing row-column combinations
195     std::vector<std::vector<ResultType>> finaloutput(finalnumrows,
196                                                    std::vector<ResultType>(finalnumcols));
197     for(int row = 0; row < finalnumrows; ++row){for(int col = 0; col < finalnumcols;
198                                                    ++col){finaloutput[row][col] = rowcolproduct(row, col);}}
199
200     // returning
201     return finaloutput;
202 }

```

```

199 // matrix * vector =====
200 template <typename T>
201 auto operator*(const std::vector<std::vector<T>> input_matrix,
202               const std::vector<T> input_vector)
203 {
204     // fetching dimensions
205     const auto& num_rows_matrix {input_matrix.size()};
206     const auto& num_cols_matrix {input_matrix[0].size()};
207     const auto& num_rows_vector {1};
208     const auto& num_cols_vector {input_vector.size()};
209
210     const auto& max_num_rows {num_rows_matrix > num_rows_vector ?\
211                               num_rows_matrix : num_rows_vector};
212     const auto& max_num_cols {num_cols_matrix > num_cols_vector ?\
213                               num_cols_matrix : num_cols_vector};
214
215     // creating canvas
216     auto canvas {std::vector<std::vector<T>>>(
217         max_num_rows,
218         std::vector<T>(max_num_cols, 0)
219     )};
220
221     //
222     if (num_cols_matrix == 1 && num_rows_vector == 1){
223
224         // writing to canvas
225         for(auto row = 0; row < max_num_rows; ++row)
226             for(auto col = 0; col < max_num_cols; ++col)
227                 canvas[row][col] = input_matrix[row][0] * input_vector[col];
228     }
229     else{
230         std::cerr << "Operator*: [matrix, vector] | not implemented \n";
231     }
232
233     // returning
234     return std::move(canvas);
235 }
236
237 // scalar operators =====
238 auto operator*(const std::complex<double> complexscalar,
239               const double doublescalar){
240     return complexscalar * static_cast<std::complex<double>>(doublescalar);
241 }
242
243 auto operator*(const double doublescalar,
244               const std::complex<double> complexscalar){
245     return complexscalar * static_cast<std::complex<double>>(doublescalar);
246 }
247
248 auto operator*(const std::complex<double> complexscalar,
249               const int scalar){
250     return complexscalar * static_cast<std::complex<double>>(scalar);
251 }
252
253 auto operator*(const int scalar,
254               const std::complex<double> complexscalar){
255     return complexscalar * static_cast<std::complex<double>>(scalar);
256 }

```

---

## A.24 Subtraction

---

```

1 // =====
2 // Aim: subtracting scalar from a vector
3 template <typename T>
4 std::vector<T> operator-(const std::vector<T>& a, const T scalar){
5     std::vector<T> temp(a.size());
6     std::transform(a.begin(),
7                   a.end(),
8                   temp.begin(),
9                   [scalar](T x){return (x - scalar);});
10    return temp;
11 }
12 // =====
13 template <typename T>
14 auto operator-(const std::vector<std::vector<T>>& input_matrix_A,
15               const std::vector<std::vector<T>>& input_matrix_B)
16 {
17     // throwing error in case of dimension differences
18     if (input_matrix_A.size() != input_matrix_B.size() ||
19         input_matrix_A[0].size() != input_matrix_B[0].size())
20         std::cerr << "operator- dimension mismatch\n";
21
22     // setting up canvas
23     auto canvas {std::vector<std::vector<T>>(
24         input_matrix_A.size(),
25         std::vector<T>(input_matrix_A[0].size())
26     )};
27
28     // subtracting values
29     for(auto row = 0; row < input_matrix_B.size(); ++row)
30         std::transform(input_matrix_A[row].begin(), input_matrix_A[row].end(),
31                       input_matrix_B[row].begin(),
32                       canvas[row].begin(),
33                       [](const auto& x, const auto& y){
34                           return x - y;
35                       });
36
37     // returning
38     return std::move(canvas);
39 }
40 
```

---

## A.25 Operator Overloadings

## A.26 Printing Containers

---

```

1 // vector printing function
2 template<typename T>
3 void fPrintVector(vector<T> input){
4     for(auto x: input) cout << x << ", ";
5     cout << endl;
6 }

```

```

6  }
7
8  template<typename T>
9  void fpv(vector<T> input){
10     for(auto x: input) cout << x << ", ";
11     cout << endl;
12 }
13 // =====
14 template<typename T>
15 void fPrintMatrix(const std::vector<std::vector<T>> input_matrix){
16     for(const auto& row: input_matrix)
17         cout << format("{}\n", row);
18 }
19 template <typename T>
20 void fPrintMatrix(const string&                input_string,
21                  const std::vector<std::vector<T>> input_matrix){
22     cout << format("{} = \n", input_string);
23     for(const auto& row: input_matrix)
24         cout << format("{}\n", row);
25 }
26
27
28 template<typename T, typename T1>
29 void fPrintHashmap(unordered_map<T, T1> input){
30     for(auto x: input){
31         cout << format("{}[{}],{}] | ", x.first, x.second);
32     }
33     cout <<endl;
34 }
35
36 void fPrintBinaryTree(TreeNode* root){
37     // sending it back
38     if (root == nullptr) return;
39
40     // printing
41     PRINTLINE
42     cout << "root->val = " << root->val << endl;
43
44     // calling the children
45     fPrintBinaryTree(root->left);
46     fPrintBinaryTree(root->right);
47
48     // returning
49     return;
50 }
51
52
53 void fPrintLinkedList(ListNode* root){
54     if (root == nullptr) return;
55     cout << root->val << " -> ";
56     fPrintLinkedList(root->next);
57     return;
58 }
59
60 template<typename T>
61 void fPrintContainer(T input){
62     for(auto x: input) cout << x << ", ";
63     cout << endl;
64     return;

```



```

65 }
66 // =====
67 template <typename T>
68 auto size(std::vector<std::vector<T>> inputMatrix){
69     cout << format("{} , {} \n", inputMatrix.size(), inputMatrix[0].size());
70 }
71
72 template <typename T>
73 auto size(const std::string inputstring, std::vector<std::vector<T>> inputMatrix){
74     cout << format("{} = {} , {} \n", inputstring, inputMatrix.size(),
75         inputMatrix[0].size());
76 }

```

---

## A.27 Random Number Generation

```

1 // =====
2 template <typename T>
3 auto rand(const T min, const T max) {
4     static std::random_device rd; // Seed
5     static std::mt19937 gen(rd()); // Mersenne Twister generator
6     std::uniform_real_distribution<> dist(min, max);
7     return dist(gen);
8 }
9 // =====
10 template <typename T>
11 auto rand(const T min,
12           const T max,
13           const size_t numelements)
14 {
15     static std::random_device rd; // Seed
16     static std::mt19937 gen(rd()); // Mersenne Twister generator
17     std::uniform_real_distribution<> dist(min, max);
18
19     // building the finaloutput
20     vector<T> finaloutput(numelements);
21     for(int i = 0; i<finaloutput.size(); ++i) {finaloutput[i] =
22         static_cast<T>(dist(gen));}
23
24     return finaloutput;
25 }
26 // =====
27 template <typename T>
28 auto rand(const T argmin,
29           const T argmax,
30           const vector<int> dimensions)
31 {
32     // throwing an error if dimension is greater than two
33     if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}
34
35     // creating random engine
36     static std::random_device rd; // Seed
37     static std::mt19937 gen(rd()); // Mersenne Twister generator
38     std::uniform_real_distribution<> dist(argmin, argmax);
39
40     // building the finaloutput

```

```

41     vector<vector<T>> finaloutput;
42     for(int i = 0; i<dimensions[0]; ++i){
43         vector<T> temp;
44         for(int j = 0; j<dimensions[1]; ++j) {temp.push_back(dist(gen));}
45         // cout << format("\t\t temp = {}\n", temp);
46
47         finaloutput.push_back(temp);
48     }
49
50     // returning the finaloutput
51     return finaloutput;
52
53 }
54 // =====
55 auto rand(const vector<int> dimensions)
56 {
57
58     using ReturnType = double;
59
60     // throwing an error if dimension is greater than two
61     if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}
62
63     // creating random engine
64     static std::random_device rd; // Seed
65     static std::mt19937 gen(rd()); // Mersenne Twister generator
66     std::uniform_real_distribution<> dist(0.00, 1.00);
67
68     // building the finaloutput
69     vector<vector<ReturnType>> finaloutput;
70     for(int i = 0; i<dimensions[0]; ++i){
71         vector<ReturnType> temp;
72         for(int j = 0; j<dimensions[1]; ++j) {temp.push_back(dist(gen));}
73         finaloutput.push_back(std::move(temp));
74     }
75
76     // returning the finaloutput
77     return std::move(finaloutput);
78
79 }
80 // =====
81 template <typename T>
82 auto rand_complex_double(const T argmin,
83                          const T argmax,
84                          const vector<int>& dimensions)
85 {
86
87     // throwing an error if dimension is greater than two
88     if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}
89
90     // creating random engine
91     static std::random_device rd; // Seed
92     static std::mt19937 gen(rd()); // Mersenne Twister generator
93     std::uniform_real_distribution<> dist(argmin, argmax);
94
95     // building the finaloutput
96     vector<vector<complex<double>>> finaloutput;
97     for(int i = 0; i<dimensions[0]; ++i){
98         vector<complex<double>> temp;

```

```

99     for(int j = 0; j<dimensions[1]; ++j)
        {temp.push_back(static_cast<double>(dist(gen)));}
100     finaloutput.push_back(std::move(temp));
101 }
102
103 // returning the finaloutput
104 return finaloutput;
105
106 }

```

---

## A.28 Reshape

```

1 // =====
2 // reshaping a matrix into another matrix
3 template <std::size_t M, std::size_t N, typename T>
4 auto reshape(const std::vector<std::vector<T>>& input_matrix){
5
6     // verifying size stuff
7     if (M*N != input_matrix.size() * input_matrix[0].size())
8         std::cerr << "Dimensions are quite different\n";
9
10    // creating canvas
11    auto canvas {std::vector<std::vector<T>>(
12        M, std::vector<T>(N, (T)0)
13    )};
14
15    // writing to canvas
16    size_t tid {0};
17    size_t target_row {0};
18    size_t target_col {0};
19    for(auto row = 0; row<input_matrix.size(); ++row){
20        for(auto col = 0; col < input_matrix[0].size(); ++col){
21            tid = row * input_matrix[0].size() + col;
22            target_row = tid/N;
23            target_col = tid%N;
24            canvas[target_row][target_col] = input_matrix[row][col];
25        }
26    }
27
28    // moving it back
29    return std::move(canvas);
30 }
31 // =====
32 // reshaping a matrix into a vector
33 template<std::size_t M, typename T>
34 auto reshape(const std::vector<std::vector<T>>& input_matrix){
35
36    // checking element-count validity
37    if (M != input_matrix.size() * input_matrix[0].size())
38        std::cerr << "Number of elements differ\n";
39
40    // creating canvas
41    auto canvas {std::vector<T>(M, 0)};
42
43    // filling canvas
44    for(auto row = 0; row < input_matrix.size(); ++row)

```

```

45     for(auto col = 0; col < input_matrix[0].size(); ++col)
46         canvas[row * input_matrix.size() + col] = input_matrix[row][col];
47
48     // moving it back
49     return std::move(canvas);
50 }
51
52 // =====
53 // Matrix to matrix
54 // =====
55 template<typename T>
56 auto reshape(const std::vector<std::vector<T>>& input_matrix,
57             const std::size_t M,
58             const std::size_t N){
59
60     // checking element-count validity
61     if ( M * N != input_matrix.size() * input_matrix[0].size())
62         std::cerr << "Number of elements differ\n";
63
64     // creating canvas
65     auto canvas {std::vector<std::vector<T>>(
66         M, std::vector<T>(N, (T)0)
67     )};
68
69     // writing to canvas
70     size_t tid {0};
71     size_t target_row {0};
72     size_t target_col {0};
73     for(auto row = 0; row < input_matrix.size(); ++row){
74         for(auto col = 0; col < input_matrix[0].size(); ++col){
75             tid = row * input_matrix[0].size() + col;
76             target_row = tid/N;
77             target_col = tid%N;
78             canvas[target_row][target_col] = input_matrix[row][col];
79         }
80     }
81
82     // moving it back
83     return std::move(canvas);
84 }
85
86 // =====
87 // converting a matrix into a vector
88 // =====
89 template<typename T>
90 auto reshape(const std::vector<std::vector<T>>& input_matrix,
91             const size_t M){
92
93     // checking element-count validity
94     if (M != input_matrix.size() * input_matrix[0].size())
95         std::cerr << "Number of elements differ\n";
96
97     // creating canvas
98     auto canvas {std::vector<T>(M, 0)};
99
100    // filling canvas
101    for(auto row = 0; row < input_matrix.size(); ++row)
102        for(auto col = 0; col < input_matrix[0].size(); ++col)
103            canvas[row * input_matrix.size() + col] = input_matrix[row][col];

```

```

104
105     // moving it back
106     return std::move(canvas);
107 }

```

---

## A.29 Summing with containers

```

1  // =====
2  template <std::size_t axis, typename T>
3  auto sum(const std::vector<T>& input_vector) -> std::enable_if_t<axis == 0,
4      std::vector<T>>
5  {
6      // returning the input as is
7      return input_vector;
8  }
9  // =====
10 template <std::size_t axis, typename T>
11 auto sum(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis == 0,
12     std::vector<T>>
13 {
14     // creating canvas
15     auto canvas {std::vector<T>(input_matrix[0].size(), 0)};
16
17     // filling up the canvas
18     for(auto row = 0; row < input_matrix.size(); ++row)
19         std::transform(input_matrix[row].begin(), input_matrix[row].end(),
20             canvas.begin(),
21             canvas.begin(),
22             [](auto& argx, auto& argy){return argx + argy;});
23
24     // returning
25     return std::move(canvas);
26 }
27 // =====
28 template <std::size_t axis, typename T>
29 auto sum(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis == 1,
30     std::vector<std::vector<T>>>
31 {
32     // creating canvas
33     auto canvas {std::vector<std::vector<T>>(input_matrix.size(),
34         std::vector<T>(1, 0.00))};
35
36     // filling up the canvas
37     for(auto row = 0; row < input_matrix.size(); ++row)
38         canvas[row][0] = std::accumulate(input_matrix[row].begin(),
39             input_matrix[row].end(),
40             static_cast<T>(0));
41
42     // returning
43     return std::move(canvas);
44 }
45 // =====
46 template <std::size_t axis, typename T>
47 auto sum(const std::vector<T>& input_vector_A,

```

```

47         const std::vector<T>&    input_vector_B) -> std::enable_if_t<axis == 0,
           std::vector<T> >
48     {
49         // setup
50         const auto&    num_cols_A    {input_vector_A.size()};
51         const auto&    num_cols_B    {input_vector_B.size()};
52
53         // throwing errors
54         if (num_cols_A != num_cols_B) {std::cerr << "sum: size disparity\n";}
55
56         // creating canvas
57         auto    canvas {input_vector_A};
58
59         // summing up
60         std::transform(input_vector_B.begin(), input_vector_B.end(),
61                        canvas.begin(),
62                        canvas.begin(),
63                        std::plus<T>());
64
65         // returning
66         return std::move(canvas);
67     }

```

---

## A.30 Tangent

```

1  namespace svr {
2      // =====
3      template <typename T>
4      auto atan2(const std::vector<T>    input_vector_A,
5                 const std::vector<T>    input_vector_B)
6      {
7          // throw error
8          if (input_vector_A.size() != input_vector_B.size())
9              std::cerr << "atan2: size disparity\n";
10
11          // create canvas
12          auto    canvas    {std::vector<T>(input_vector_A.size(), 0)};
13
14          // performing element-wise atan2 calculation
15          std::transform(input_vector_A.begin(), input_vector_A.end(),
16                        input_vector_B.begin(),
17                        canvas.begin(),
18                        [](const auto& arg_a,
19                          const auto& arg_b){
20
21                          return std::atan2(arg_a, arg_b);
22                      });
23
24          // moving things back
25          return std::move(canvas);
26      }
27      // =====
28      template <typename T>
29      auto atan2(T    scalar_A,
30                 T    scalar_B)
31      {

```

```

32     return std::atan2(scalar_A, scalar_B);
33 }
34 }

```

---

## A.31 Tiling Operations

---

```

1  namespace svr {
2      // =====
3  template <typename T>
4  auto tile(const std::vector<T>&      input_vector,
5            const std::vector<size_t> mul_dimensions){
6
7      // creating canvas
8      const std::size_t& num_rows  {1 * mul_dimensions[0]};
9      const std::size_t& num_cols  {input_vector.size() * mul_dimensions[1]};
10     auto canvas {std::vector<std::vector<T>>>(
11         num_rows,
12         std::vector<T>(num_cols, 0)
13     )};
14
15     // writing
16     std::size_t source_row;
17     std::size_t source_col;
18
19     for(std::size_t row = 0; row < num_rows; ++row){
20         for(std::size_t col = 0; col < num_cols; ++col){
21             source_row = row % 1;
22             source_col = col % input_vector.size();
23             canvas[row][col] = input_vector[source_col];
24         }
25     }
26
27     // returning
28     return std::move(canvas);
29 }
30 // =====
31 template <typename T>
32 auto tile(const std::vector<std::vector<T>>& input_matrix,
33           const std::vector<size_t>      mul_dimensions){
34
35     // creating canvas
36     const std::size_t& num_rows  {input_matrix.size() * mul_dimensions[0]};
37     const std::size_t& num_cols  {input_matrix[0].size() * mul_dimensions[1]};
38     auto canvas {std::vector<std::vector<T>>>(
39         num_rows,
40         std::vector<T>(num_cols, 0)
41     )};
42
43     // writing
44     std::size_t source_row;
45     std::size_t source_col;
46
47     for(std::size_t row = 0; row < num_rows; ++row){
48         for(std::size_t col = 0; col < num_cols; ++col){
49             source_row = row % input_matrix.size();
50             source_col = col % input_matrix[0].size();

```

```
51         canvas[row][col] = input_matrix[source_row][source_col];
52     }
53 }
54
55 // returning
56 return std::move(canvas);
57 }
58 }
```

---

## A.32 Transpose

---

```
1  template <typename T>
2  auto transpose(const std::vector<T> input_vector){
3
4      // creating canvas
5      auto canvas {std::vector<std::vector<T>>{
6          input_vector.size(),
7          std::vector<T>(1)
8      }};
9
10     // filling canvas
11     for(auto i = 0; i < input_vector.size(); ++i){
12         canvas[i][0] = input_vector[i];
13     }
14
15     // moving it back
16     return std::move(canvas);
17 }
```

---