Autonomous Underwater Vehicle: A Surveillance Protocol

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Preface

This project is an attempt at combining all of my major skills into creating a simulation, imaging, perception and control pipeline for Autonomous Underwater Vehicles (AUV). As such, creating this project involves creating a number of pipelines.

The first pipeline is the signal simulation pipeline. The signal simulation pipeline involves sea-floor point-cloud creation and simulating the signals received by the sensor arrays of the AUV. The signals recorded by the sensor-arrays on the AUV contains information from the surrounding environment. The imaging pipeline performs certain operations on the recorded signals to obtain acoustic images of the surrounding environment. To that end, this pipeline involves the topics of signal processing, linear algebra, signals and systems.

As such, the second pipeline is the imaging pipeline. The inputs to the imaging pipeline is the signals recorded by the different sensor-arrays of the AUV, in addition to the parameters of the AUV and its components. This pipeline involves match-filtering, focusing and beamforming operations to create acoustic images of the surrounding environment. Depending on the number of ULAs present, the imaging pipeline is responsible for creating multiple acoustic images in real-time. Thus, this pipeline involves the topics of Digital Signal Processing, Match-Filtering, Estimation and Detection Theory and so on.

The images created by the imaging pipeline are fed to the perception-to-control pipeline. This pipeline takes in the image formed created from the ULA signals, parameters of AUV and its components, and some historical data, it provides instructions regarding the movement of the AUV. The mapping from the inputs to the controls is called policy. Learning policies is a core part of reinforcement learning. Thus, this pipeline mainly involves the topics of reinforcement learning. And since we'll be using convolutional neural nets and transformers for learning the policies, this pipeline involves a significant amount of machine and deep learning.

The final result is an AUV that is primarily trained to map an area of the sea-floor in a constant surveillance mode. The RL-trained policy will also be trained to deal with different kinds of sea-floor terrains: those containing hills, valleys, and path-obstructing features. Due to the resource constrained nature of the marine vessel, we also prioritize efficient policies in the policy-training pipeline.

The project is currently written in C++. And since there is non-trivial amount of training and adaptive features in the pipelines, we'll be using LibTorch (the C++ API of PyTorch) to enable computation graphs, backpropagation and thereby, learning in our AUV pipeline. However, for the sections where a computation graph is not required we will be writing templated STL code.

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Part I AUV Components & Setup

Chapter 1

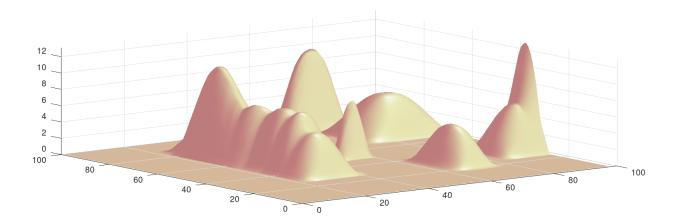
Underwater Environment

Overview

All physical matter in this framework is represented using point-clouds. Thus, the sea-floor also is represented using a number of 3D points. In addition to the coordinates, the points also have the additional property of "reflectivity". It is the impulse response of that point.

Sea-floors in real-life are rarely flat. They often contain valleys, mountains, hills and much richer geographical features. Thus, training an agent to function in such environments call for the creation of similar structures in our simulations.

To simplify things, we shall take a more constrained and structured approach. We start by creating different classes of structures and produce instantiations of those structures on the sea-floor. These structures are defined in such a way that the shape and size can be parameterized to enable creation of random sea-floors.



1.1 Underwater Hills

The most basic approach to creating this is to create a flat seafloor, where all the points have the same height. While this is a good place to start, it is good to bring in some realism to the seafloor. To that end, we shall have some rolling hills as the sea-floor. Each "hill"

is created using the method outlined in Algorithm 1. The method involves deciding the location of the hills, the dimension of the hills and then designing a hill by combining an exponential function and a cosine function. We're aiming to essentially produce gaussian-looking sea-floor hills. After the creation, this becomes the set of points representing the lowest set of points in the overall seafloor structure.

Algorithm 1 Hill Creation

```
1: Input: Mean vector m, Dimension vector d, 2D points P
  2: Output: Updated P with hill heights
  3: num\_hills \leftarrow numel(\mathbf{m}_x)
  4: H \leftarrow \text{Zeros tensor of size } (1, \text{numel}(\mathbf{P}_x))
 5: for i=1 to num_hills do
6: x_{\text{norm}} \leftarrow \frac{\frac{\pi}{2}(\mathbf{P}_x - \mathbf{m}_x[i])}{\mathbf{d}_x[i]}
            y_{\text{norm}} \leftarrow \frac{\frac{\pi}{2}(\mathbf{P}_y - \mathbf{m}_y[i])}{\mathbf{d}_y[i]}
  7:
           h_x \leftarrow \cos(x_{\text{norm}}) \cdot e^{\frac{|x_{\text{norm}}|}{10}}
           h_y \leftarrow \cos(y_{\text{norm}}) \cdot e^{\frac{|y_{\text{norm}}|}{10}}
  9:
            h \leftarrow \mathbf{d}_z[i] \cdot h_x \cdot h_y
10:
            Apply boundary conditions:
11:
            if x_{\text{norm}} > \frac{\pi}{2} or x_{\text{norm}} < -\frac{\pi}{2} or y_{\text{norm}} > \frac{\pi}{2} or y_{\text{norm}} < -\frac{\pi}{2} then
12:
13:
            end if
14:
            H \leftarrow H + h
15:
16: end for
17: \mathbf{P} \leftarrow \text{concatenate}([\mathbf{P}, H])
```

1.2 Scatterer Definition

The sea-floor is represented by a single object of the class ScattererClass.

```
Class Declaration
  template <typename T>
  class ScattererClass
  public:
      // members
      std::vector<std::vector<T>> coordinates;
      std::vector<T> reflectivity;
11
      // Constructor
      ScattererClass() {}
14
      // Constructor
      ScattererClass(std::vector<std::vector<T>> coordinates_arg,
                   std::vector<T>
                                   reflectivity_arg):
                    coordinates(std::move(coordinates_arg)),
18
                    reflectivity(std::move(reflectivity_arg)) {}
19
      // Save to CSV
```

```
void save_to_csv();

;
```

1.3 Sea-Floor Setup Script

Following is the function that will setup the sea-floor script.

```
void fSeaFloorSetup(
      ScattererClass<double>& scatterers
   ){
3
      // auto save_files
                            {false};
                                           {false};
      const auto save_files
      const auto
                   hill_creation_flag
                                          {true};
      // sea-floor bounds
      auto
             bed_width
                            {100.00};
      auto
             bed_length
                            {100.00};
11
      // creating tensors for coordinates and reflectivity
      vector<vector<double>>
                              box_coordinates;
14
      vector<double>
                                   box_reflectivity;
16
      // scatter density
      // auto bed_width_density {static_cast<double>( 10.00)};
18
      // auto
                 bed_length_density {static_cast<double>( 10.00)};
19
      auto
             bed_width_density {static_cast<double>( 5.00)};
20
             bed_length_density {static_cast<double>( 5.00)};
      auto
22
      // setting up coordinates
23
                      {svr::linspace<double>(
             xpoints
      auto
         0.00,
          bed_width,
26
          bed_width * bed_width_density
      )};
29
      auto
             ypoints
                        {svr::linspace<double>(
          0.00,
30
          bed_length,
31
          bed_length * bed_length_density
33
      if(save_files) fWriteVector(xpoints, "../csv-files/xpoints.csv");
                                                                           // verified
34
      if(save_files) fWriteVector(ypoints, "../csv-files/ypoints.csv");
                                                                           // verified
35
      // creating mesh
37
      auto [xgrid, ygrid] = meshgrid(std::move(xpoints), std::move(ypoints));
38
      if(save_files) fWriteMatrix(xgrid, "../csv-files/xgrid.csv"); // verified
39
                                                                           // verified
      if(save_files) fWriteMatrix(ygrid, "../csv-files/ygrid.csv");
41
      // reshaping
                     {reshape(xgrid, xgrid.size()*xgrid[0].size())};
      auto X
            Y
                     {reshape(ygrid, ygrid.size()*ygrid[0].size())};
      auto
                                                                           // verified
      if(save_files) fWriteVector(X, "../csv-files/X.csv");
45
                                          "../csv-files/Y.csv");
                                                                           // verified
      if(save_files) fWriteVector(Y,
46
47
      // creating heights of scatterers
      if(hill_creation_flag){
49
```

```
50
          // setting up hill parameters
51
                  num_hills
          auto
52
          // setting up placement of hills
                                                                                  // verified
          auto
                  points2D
                                         {concatenate<0>(X, Y)};
                  min2D
                                         {min<1, double>(points2D)};
                                                                                  // verified
          auto
56
                  max2D
                                         {max<1, double>(points2D)};
                                                                                  // verified
          auto
                  hill_2D_center
                                         \{min2D + \setminus
          auto
                                          rand({2, num_hills}) * (max2D - min2D)}; // verified
60
          // setup: hill-dimensions
61
                                        {transpose(vector<double>{5, 5, 2})); // verified
                  hill_dimensions_min
          auto
          auto
                  hill_dimensions_max
                                         {transpose(vector<double>{30, 30, 10})}; // verified
63
          auto
                 hill_dimensions
                                         {hill_dimensions_min + \
                                         rand({3, num_hills}) * (hill_dimensions_max -
                                             hill_dimensions_min)};
                                                                                   // verified
66
          // function-call: hill-creation function
67
          fCreateHills(hill_2D_center,
                       hill_dimensions,
                       points2D);
70
          // setting up floor reflectivity
                                                {std::vector<double>(Y.size(), 1.00)};
          auto floorScatter_reflectivity
74
          // populating the values of the incoming argument
75
          scatterers.coordinates = std::move(points2D);
76
          scatterers.reflectivity = std::move(floorScatter_reflectivity);
77
78
       }
79
       else{
80
81
          // assigning flat heights
82
                         {std::vector<double>(Y.size(), 0)};
83
          // setting up floor coordinates
85
          auto
                  floorScatter_coordinates
                                                {concatenate<0>(X, Y, Z)};
86
                  floorScatter_reflectivity
                                                {std::vector<double>(Y.size(), 1)};
          auto
          // populating the values of the incoming argument
89
          scatterers.coordinates = std::move(floorScatter_coordinates);
90
          scatterers.reflectivity = std::move(floorScatter_reflectivity);
91
92
       }
93
94
       // printing status
95
       spdlog::info("Finished Sea-Floor Setup");
96
   }
97
```

Chapter 2

Transmitter

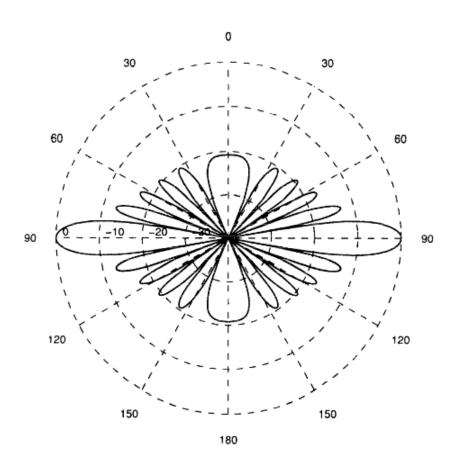


Figure 2.1: Beampattern of a Transmission Uniform Linear Array

Overview

Probing systems are those systems that send out a signal, listen to the reflection and infer qualitative and quantitative qualities of the environment, matter or object, it was trying to infer information about. The transmitter is one of the most fundamental components of probing systems. As the name suggests, the transmitter is the equipment responsible for sending out the probing signal into the medium.

A transmitter is any device or circuit that converts information into a signal and sends it out onto some media like air, cable, water or space. The components of a transmitter are usually as follows

- 1. Input: Information containing signal such as voice, data, video etc
- 2. Process: Encode/modulate the information onto a carrier signal, which can be electromagnetic wave or mechanical wave.
- 3. Transmission: The signal is then transmitted onto the media with electro-mechanical equipment.

Transmitters are of many kinds. But the ones that we will be considering will be directed transmitters, which means that these transmitters have an associated beampattern. To the uninitiated, this means that the power of the transmitted signal is not transmitted in all directions equally. A beampattern is a graphical representation of the power received by an ideal receiver when placed at different angles.

Transmitters made out of a linear-array of individual transmitters use beamforming to "direct" the major power of the transmitter. These kind of systems have well studied beampatterns which can be utilized in our simulations. These kind of studies and inculcating that in our pipelines produce accurate signal simulation pipelines. For now, we stick to a very simple model of a transmitter. We assume that the transmitter sends out the power equally into a particular cone from the AUV position.

2.1 Transmission Signal

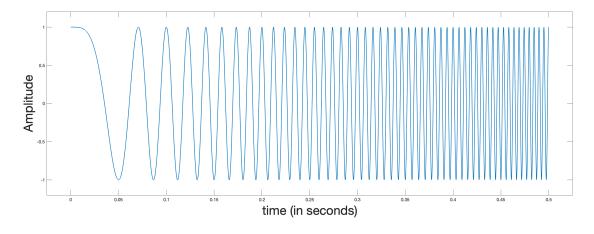


Figure 2.2: Linear Frequency Modulated Wave

The resolution of any probing system is fundamentally tied to the signal bandwidth. A higher bandwidth corresponds to finer resolution $\frac{\text{speed-of-sounds}}{2*\text{bandwidth}}$. Thus, for perfect resolution, an infinite bandwidth is in order. However, infinite bandwidth is impossible for obvious reasons: hardware limitations, spectral regulations, energy limitations and so on.

This is where Linear Frequency Modulation (LFM), also called a "chirp," becomes valuable. An LFM signal linearly sweeps a limited bandwidth over a relatively long duration. This technique spreads the signal's energy in time while retaining the resolution benefits of

the bandwidth. After matched filtering (or pulse compression), we essentially produce pulses corresponding to a base-band LFM of same bandwidth. Overall, LFM is a practical compromise between finite bandwidth and desired performance.

One of the best parts about the resolution depending only on the bandwidth is that it allows us to deploy techniques that would help us improve SNRs without virtually increasing the bandwidth at all. Much of the noise in submarine environments are in and around the baseband region (around frequency, 0). Since resolution depends purely on bandwidth, and LFM can be transmitted at a carrier-frequency, this means that processing the returns after low-pass filtering and basebanding allows us to get rid of the submarine noise, since they do not occupy the same frequency-coefficients. The end-result, thus, is improved SNR compared to use baseband LFM.

Due to all of these advantages, LFM waves are ubiquitous in probing systems, from sonar to radar. Thus, for this project too, the transmitter will be using LFM waves as probing signals, to probe the surrounding submarine environment.

2.2 Transmitter Class Definition

The transmitter is represented by a single object of the class TransmitterClass.

```
template <typename T>
   class TransmitterClass{
   public:
       // A shared pointer to the configuration object
5
       std::shared_ptr<svr::AUVParameters> config_ptr;
6
       // physical/intrinsic properties
8
       std::vector<T>
                         location;
                                               // location tensor
9
                         pointing_direction; // pointing direction
       std::vector<T>
10
11
       // basic parameters
12
       std::vector<T>
                                               // transmitted signal (LFM)
                         signal;
      Т
                         azimuthal_angle;
                                               // transmitter's azimuthal pointing direction
      Т
                         elevation_angle;
                                               // transmitter's elevation pointing direction
      Т
                         azimuthal_beamwidth; // azimuthal beamwidth of transmitter
16
      Т
                         elevation_beamwidth; // elevation beamwidth of transmitter
17
      Τ
                         range;
                                               // a parameter used for spotlight mode.
18
       // transmitted signal attributes
20
      Т
                         f_low;
                                               // lowest frequency of LFM
21
      Т
                                               // highest frequency of LFM
                         f_high;
22
      Т
                                               // center frequency of LFM
23
                                               // bandwidth of LFM
      Т
                         bandwidth;
24
                         speed_of_sound {1500}; // speed of sound
25
       // shadowing properties
27
                         azimuthQuantDensity;
                                                   // quantization of angles along the
       int
28
          azimuth
                                                   // quantization of angles along the
       int
                         elevationQuantDensity;
          elevation
      Т
                                                   // range-cell size when shadowing
                         rangeQuantSize;
30
       Т
                         azimuthShadowThreshold; // azimuth thresholding
31
       Т
                         elevationShadowThreshold; // elevation thresholding
```

```
33
      // shadowing related
                                              // box indicating whether a scatter for a
      std::vector<T>
                         checkbox;
35
          range-angle pair has been found
      std::vector<std::vector<std::vector<T>>> finalScatterBox; // a 3D tensor where the
          third dimension represnets the vector length
      std::vector<T> finalReflectivityBox; // to store the reflectivity
37
       // constructor
      TransmitterClass() = default;
40
41
       // Deleting copy constructors/assignment
42
      TransmitterClass(const TransmitterClass& other)
                                                              = delete;
43
      TransmitterClass& operator=(TransmitterClass& other) = delete;
45
       // Creating move-constructor and move-assignment
      TransmitterClass(TransmitterClass&& other)
                                                               = default;
      TransmitterClass& operator=(TransmitterClass&& other) = default;
48
49
      // member-functions
50
      auto updatePointingAngle(std::vector<T> AUV_pointing_vector);
51
       auto subset_scatterers(const ScattererClass<T>& seafloor,
```

2.3 Transmitter Setup Scripts

The following script shows the setup-script

```
template
       typename
                  Τ,
                     std::enable_if_t<
       typename
                  =
          std::is_same_v<T, double> ||
          std::is_same_v<T, float>
6
   >
   void fTransmitterSetup(
       TransmitterClass<T>& transmitter_fls,
       TransmitterClass<T>& transmitter_portside,
10
       TransmitterClass<T>& transmitter_starboard
11
   ){
12
       // Setting up transmitter
                                     {160e3};
                                                           // sampling frequency
       Τ
              sampling_frequency
14
       Т
                                     {50e3};
                                                           // first frequency of LFM
              f1
       Т
              f2
                                     {70e3};
                                                           // second frequency of LFM
       Τ
                                     \{(f1 + f2)/2.00\};
                                                           // finding center-frequency
17
                                     {std::abs(f2 - f1)}; // bandwidth
       Τ
              bandwidth
18
                                                           // time of recording
19
              pulselength
                                     {5e-2};
       // building LFM
                             {svr::linspace<T>(
       auto
              timearray
          0.00,
          pulselength,
          std::floor(pulselength * sampling_frequency)
       )};
26
                             {f2 - f1/pulselength}; // calculating frequency-slope
       auto
              K
       auto
              Signal
                             {cos(2 * std::numbers::pi * \
                              (f1 + K*timearray) * \
29
```

```
// frequency at each time-step, with f1
                              timearray)};
30
                                  = 0
31
       // Setting up transmitter
32
                                                {std::vector<T>(3, 0)};
       auto
              location
                                                                              // location of
33
          transmitter
              azimuthal_angle_fls
                                                {0};
                                                                              // initial
          pointing direction
              azimuthal_angle_port
                                                {90};
                                                                              // initial
          pointing direction
              azimuthal_angle_starboard
                                                {-90};
                                                                              // initial
36
          pointing direction
37
              elevation_angle
                                                {-60};
                                                                              // initial
38
          pointing direction
30
              azimuthal_beamwidth_fls
                                                {20};
                                                                              // azimuthal
           beamwidth of the signal cone
                                                                              // azimuthal
              azimuthal_beamwidth_port
                                                {20};
41
          beamwidth of the signal cone
       Т
              azimuthal_beamwidth_starboard
                                                {20};
                                                                              // azimuthal
42
          beamwidth of the signal cone
43
                                                                              // elevation
      Т
              elevation_beamwidth_fls
                                                {20};
          beamwidth of the signal cone
              elevation_beamwidth_port
                                                {20};
                                                                              // elevation
45
          beamwidth of the signal cone
              elevation_beamwidth_starboard
                                                                              // elevation
       Τ
                                                {20};
46
          beamwidth of the signal cone
47
              azimuthQuantDensity
                                                {10}; // number of points, a degree is split
       int
48
           into quantization density along azimuth (used for shadowing)
              elevationQuantDensity
                                                {10}; // number of points, a degree is split
          into quantization density along elevation (used for shadowing)
              rangeQuantSize
                                                {10}; // the length of a cell (used for
50
           shadowing)
51
              azimuthShadowThreshold
      Т
                                                {1};
                                                       // azimuth threshold
                                                                                (in degrees)
52
              {\tt elevationShadowThreshold}
                                                {1};
       Т
                                                       // elevation threshold (in degrees)
53
55
       // transmitter-fls
56
       transmitter_fls.location
                                            = location;
                                                                              // Assigning
57
          location
       transmitter_fls.signal
                                            = Signal;
                                                                              // Assigning
58
           signal
       transmitter_fls.azimuthal_angle
                                           = azimuthal_angle_fls;
                                                                             // assigning
           azimuth angle
       transmitter_fls.elevation_angle
                                            = elevation_angle;
                                                                              // assigning
60
           elevation angle
       transmitter_fls.azimuthal_beamwidth = azimuthal_beamwidth_fls;
                                                                              // assigning
61
           azimuth-beamwidth
       transmitter_fls.elevation_beamwidth = elevation_beamwidth_fls;
                                                                              // assigning
62
           elevation-beamwidth
       // updating quantization densities
       transmitter_fls.azimuthQuantDensity
                                               = azimuthQuantDensity;
                                                                           // assigning
64
           azimuth quant density
       transmitter_fls.elevationQuantDensity = elevationQuantDensity;
                                                                           // assigning
65
           elevation quant density
```

```
transmitter_fls.rangeQuantSize
                                              = rangeQuantSize;
                                                                          // assigning
           range-quantization
       transmitter_fls.azimuthShadowThreshold = azimuthShadowThreshold; //
67
           azimuth-threshold in shadowing
       transmitter_fls.elevationShadowThreshold = elevationShadowThreshold; //
           elevation-threshold in shadowing
       // signal related
69
       transmitter_fls.f_low
                                                             // assigning lower frequency
                                              = f1;
70
       transmitter_fls.f_high
                                              = f2;
                                                             // assigning higher frequency
71
72
       transmitter_fls.fc
                                              = fc;
                                                             // assigning center frequency
       transmitter_fls.bandwidth
                                              = bandwidth; // assigning bandwidth
73
74
75
       // transmitter-portside
76
       transmitter_portside.location
                                                 = location;
                                                                               // Assigning
77
           location
                                                  = Signal;
                                                                               // Assigning
       transmitter_portside.signal
           signal
       transmitter_portside.azimuthal_angle
                                                 = azimuthal_angle_port;
                                                                              // assigning
79
           azimuth angle
       transmitter_portside.elevation_angle
                                                 = elevation_angle;
                                                                              // assigning
           elevation angle
       transmitter_portside.azimuthal_beamwidth = azimuthal_beamwidth_port; // assigning
81
           azimuth-beamwidth
       transmitter_portside.elevation_beamwidth = elevation_beamwidth_port; // assigning
           elevation-beamwidth
       // updating quantization densities
83
       transmitter_portside.azimuthQuantDensity = azimuthQuantDensity;
                                                                               // assigning
84
           azimuth quant density
       transmitter_portside.elevationQuantDensity = elevationQuantDensity;
85
                                                                               // assigning
           elevation quant density
       transmitter_portside.rangeQuantSize
                                                  = rangeQuantSize;
                                                                               // assigning
           range-quantization
       transmitter_portside.azimuthShadowThreshold = azimuthShadowThreshold; //
87
           azimuth-threshold in shadowing
       transmitter_portside.elevationShadowThreshold = elevationShadowThreshold; //
           elevation-threshold in shadowing
       // signal related
20
       transmitter_portside.f_low
                                                                                // assigning
                                                  = f1;
           lower frequency
       transmitter_portside.f_high
                                                  = f2;
                                                                               // assigning
91
           higher frequency
       transmitter_portside.fc
                                                  = fc;
                                                                               // assigning
92
           center frequency
       transmitter_portside.bandwidth
                                                  = bandwidth;
                                                                               // assigning
93
           bandwidth
04
95
       // transmitter-starboard
96
       transmitter_starboard.location
                                                                                   //
                                                     = location;
97
           assigning location
       transmitter_starboard.signal
                                                     = Signal;
                                                                                   //
           assigning signal
       transmitter_starboard.azimuthal_angle
                                                     = azimuthal_angle_starboard; //
99
           assigning azimuthal signal
       transmitter_starboard.elevation_angle
                                                      = elevation_angle;
100
       transmitter_starboard.azimuthal_beamwidth
                                                      = azimuthal_beamwidth_starboard;
101
       transmitter_starboard.elevation_beamwidth
                                                      = elevation_beamwidth_starboard;
       // updating quantization densities
```

```
transmitter_starboard.azimuthQuantDensity
                                                       = azimuthQuantDensity;
                                                                                       //
104
            assigning azimuth-quant-density
                                                       = elevationQuantDensity;
       transmitter_starboard.elevationQuantDensity
105
       transmitter_starboard.rangeQuantSize
                                                       = rangeQuantSize;
106
       transmitter_starboard.azimuthShadowThreshold = azimuthShadowThreshold;
107
       transmitter_starboard.elevationShadowThreshold = elevationShadowThreshold;
108
       // signal related
109
       transmitter_starboard.f_low
                                                       = f1;
                                                                                       //
110
           assigning lower frequency
111
       transmitter_starboard.f_high
                                                       = f2;
           assigning higher frequency
       transmitter_starboard.fc
                                                       = fc;
112
           assigning center frequency
       transmitter_starboard.bandwidth
                                                       = bandwidth;
113
            assigning bandwidth
114
    }
```

Chapter 3

Uniform Linear Array

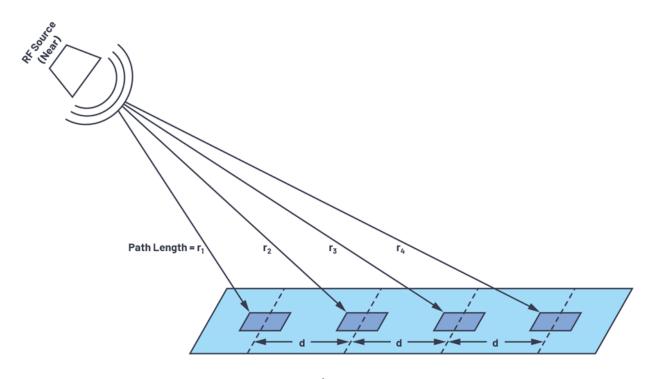


Figure 3.1: Uniform Linear Array

Overview

A Uniform Linear Array (ULA) is a common antenna or sensor configuration in which multiple elements are arranged in a straight line with equal spacing between adjacent elements. This geometry simplifies both the analysis and implementation of array signal processing techniques. In a ULA, each element receives a version of the incoming signal that differs only in phase, depending on the angle of arrival. This phase difference can be exploited to steer the array's beam in a desired direction (beamforming) or to estimate the direction of arrival (DOA) of multiple sources. The equal spacing also leads to a regular phase progression across the elements, which makes the array's response mathematically tractable and allows the use of tools like the discrete Fourier transform (DFT) to analyze spatial frequency content.

The performance of a ULA depends on the number of elements and their spacing. The spacing is typically chosen to be half the wavelength of the signal to avoid spatial aliasing, also called grating lobes, which can introduce ambiguities in DOA estimation. Increasing the number of elements improves the array's angular resolution and directivity, meaning it can better distinguish closely spaced sources and focus energy more narrowly. ULAs are widely used in radar, sonar, wireless communications, and microphone arrays due to their simplicity, predictable behavior, and compatibility with well-established signal processing algorithms. Their linear structure also makes them easier to implement in hardware compared to more complex array geometries like circular or planar arrays.

3.1 ULA Class Definition

The following is the class used to represent the uniform linear array

```
template <</pre>
      svr::PureFloatingPointType
                                        Τ,
      svr::FFT_SourceDestination_Type sourceType,
      svr::FFT_SourceDestination_Type destinationType
5
   class ULAClass
   {
   public:
8
      // intrinsic parameters
9
                                                                             // number of
      std::size_t
10
                                    num_sensors;
          sensors
                                    inter_element_spacing;
                                                                             // space between
11
          sensors
      std::vector<std::vector<T>> coordinates;
                                                                             // coordinates
12
          of each sensor
                                                                             // sampling
                                    sampling_frequency;
          frequency of the sensors
                                    recording_period;
                                                                             // recording
          period of the ULA
      std::vector<T>
                                                                             // location of
                                    location;
          first coordinate
      // derived
17
      std::vector<T>
                                    sensor_direction;
18
      std::vector<std::vector<T>> signal_matrix;
19
      std::size_t
                                    num_samples;
20
21
      // decimation related
22
      int
                                    decimation_factor;
                                                                                // the new
23
          decimation factor
                                    post_decimation_sampling_frequency;
                                                                                // the new
          sampling frequency
      std::vector<T>
                                    lowpass_filter_coefficients_for_decimation; //
          filter-coefficients for filtering
26
      // imaging related
27
        range_resolution;
                                       // theoretical range-resolution = $\frac{c}{2B}$
28
      T azimuthal_resolution;
                                      // theoretical azimuth-resolution =
          $\frac{\lambda}{(N-1)*inter-element-distance}$
      T range_cell_size;
                                       // the range-cell quanta we're choosing for
          efficiency trade-off
         azimuth_cell_size;
                                       // the azimuth quanta we're choosing
```

```
std::vector<T> azimuth_centers; // tensor containing the azimuth centeres
32
       std::vector<T> range_centers; // tensor containing the range-centers
33
                                        // the frame-size corresponding to a range cell in a
       int frame_size;
34
           decimated signal matrix
35
       std::vector<std::vector<complex<T>>> mulfFTMatrix; // the matrix containing the
          delays for each-element as a slot
       std::vector<complex<T>>
                                            matchFilter; // torch tensor containing the
37
          match-filter
            num_buffer_zeros_per_frame;
                                                           // number of zeros we're adding
38
          per frame to ensure no-rotation
       std::vector<std::vector<T>> beamformedImage;
                                                           // the beamformed image
39
       std::vector<std::vector<T>> cartesianImage;
                                                           // the cartesian version of
40
          beamformed image
41
       // Decimating Related
42
       std::vector<std::complex<T>> basebanding_signal;
43
44
       // Artificial acoustic-image related
45
       std::vector<std::vector<T>> currentArtificialAcousticImage; // acoustic image
           directly produced
47
48
       // Basic Constructor
49
      ULAClass() = default;
51
       // constructor
52
       ULAClass(const int
                             num_sensors_arg,
              const auto
                             inter_element_spacing_arg,
55
               const auto& coordinates_arg,
               const auto& sampling_frequency_arg,
56
               const auto& recording_period_arg,
               const auto&
                             location_arg,
58
                            signalMatrix_arg,
               const auto&
59
               const auto& lowpass_filter_coefficients_for_decimation_arg):
60
                  num_sensors(num_sensors_arg),
                  inter_element_spacing(inter_element_spacing_arg),
62
                  coordinates(std::move(coordinates_arg)),
63
                  sampling_frequency(sampling_frequency_arg),
                  recording_period(recording_period_arg),
                  location(std::move(location_arg)),
66
                  signal_matrix(std::move(signalMatrix_arg)),
67
                  lowpass_filter_coefficients_for_decimation(std::move(lowpass_filter_coefficients_for_decimation)
68
       {
70
          // calculating ULA direction
71
          sensor_direction = std::vector<T>{coordinates[1][0] - coordinates[0][0],
                                            coordinates[1][1] - coordinates[0][1],
                                            coordinates[1][2] - coordinates[0][2]};
74
75
76
          // normalizing
                  norm_value_temp
                                    {std::norm(std::inner_product(sensor_direction.begin(),
                                                                sensor_direction.end(),
78
                                                                 sensor_direction.begin(),
79
                                                                 0.00))};
81
          // dividing
82
          if (norm_value_temp != 0) {sensor_direction = sensor_direction /
83
              norm_value_temp;}
```

3.2 ULA Setup Scripts

The following script shows the setup-script for Uniform Linear Arrays

```
// template <</pre>
        typename
                    Τ,
         typename = std::enable_if_t<</pre>
             std::is_same_v<T, double> ||
  //
             std::is_same_v<T, float>
  //
  //
   // >
   template <
      svr::PureFloatingPointType
10
      svr::FFT_SourceDestination_Type sourceType,
11
      {\tt svr::FFT\_SourceDestination\_Type} \ \ {\tt destinationType}
13
   void fULASetup(
14
      ULAClass<T, sourceType, destinationType>& ula_fls,
16
      ULAClass<T, sourceType, destinationType>& ula_portside,
      ULAClass<T, sourceType, destinationType>& ula_starboard)
17
18
      // setting up ula
19
                                        {static_cast<int>(32)};
                                                                         // number of sensors
20
      auto num_sensors
21
              sampling_frequency
                                        {static_cast<T>(160e3)};
                                                                         // sampling frequency
              inter_element_spacing
                                        {1500/(2*sampling_frequency)};
                                                                         // space between
22
          samples
      Т
                                        {10e-2};
                                                                          // sampling-period
            recording_period
23
      auto num_samples
                                        {static_cast<std::size_t>(
          std::ceil(
25
              sampling_frequency * recording_period
26
      )};
28
29
      // building the direction for the sensors
30
                             {std::vector<T>({-1, 0, 0})};
      auto ULA_direction
            ULA_direction_norm
                                        {norm(ULA_direction)};
32
      if (ULA_direction_norm != 0)
                                        {ULA_direction = ULA_direction/ULA_direction_norm;}
33
      ULA_direction
                                      ULA_direction * inter_element_spacing;
      // building coordinates for sensors
36
            ULA_coordinates
37
          transpose(ULA_direction) * \
          svr::linspace<double>(
              0.00,
40
```

```
num_sensors -1,
41
              num_sensors)
42
      };
43
44
       // coefficients of decimation filter
45
              lowpassfiltercoefficients {std::vector<T>{0.0000, 0.0000, 0.0000, 0.0000,
           0.0000, 0.0000, 0.0001, 0.0003, 0.0006, 0.0015, 0.0030, 0.0057, 0.0100, 0.0163,
           0.0251, 0.0364, 0.0501, 0.0654, 0.0814, 0.0966, 0.1093, 0.1180, 0.1212, 0.1179,
           0.1078, 0.0914, 0.0699, 0.0451, 0.0192, -0.0053, -0.0262, -0.0416, -0.0504,
           -0.0522, -0.0475, -0.0375, -0.0239, -0.0088, 0.0057, 0.0179, 0.0263, 0.0303,
           0.0298, 0.0253, 0.0177, 0.0086, -0.0008, -0.0091, -0.0153, -0.0187, -0.0191,
           -0.0168, -0.0123, -0.0065, -0.0004, 0.0052, 0.0095, 0.0119, 0.0125, 0.0112,
           0.0084, 0.0046, 0.0006, -0.0031, -0.0060, -0.0078, -0.0082, -0.0075, -0.0057,
           -0.0033, -0.0006, 0.0019, 0.0039, 0.0051, 0.0055, 0.0050, 0.0039, 0.0023, 0.0005,
           -0.0012, -0.0025, -0.0034, -0.0036, -0.0034, -0.0026, -0.0016, -0.0004, 0.0007,
           0.0016, 0.0022, 0.0024, 0.0023, 0.0018, 0.0011, 0.0003, -0.0004, -0.0011,
           -0.0015, -0.0016, -0.0015};
47
       // assigning values
48
       ula_fls.num_sensors
49
                                                       = num_sensors;
       ula_fls.inter_element_spacing
                                                       = inter_element_spacing;
       ula_fls.coordinates
                                                      = ULA_coordinates;
51
       ula_fls.sampling_frequency
                                                      = sampling_frequency;
52
      ula_fls.recording_period
                                                       = recording_period;
53
       ula_fls.sensor_direction
                                                       = ULA_direction;
       ula_fls.lowpass_filter_coefficients_for_decimation = lowpassfiltercoefficients;
55
       ula_fls.num_samples
                                                       = num_samples;
56
57
58
       // assigning values
59
       ula_portside.num_sensors
                                                              = num_sensors;
60
       ula_portside.inter_element_spacing
                                                              = inter_element_spacing;
61
       ula_portside.coordinates
                                                              = ULA_coordinates;
62
       ula_portside.sampling_frequency
                                                              = sampling_frequency;
63
       ula_portside.recording_period
                                                              = recording_period;
64
       ula_portside.sensor_direction
                                                              = ULA_direction;
65
       ula_portside.lowpass_filter_coefficients_for_decimation = lowpassfiltercoefficients;
66
       ula_portside.num_samples
                                                                  num_samples;
67
68
70
       // assigning values
       ula_starboard.num_sensors
                                                              = num_sensors;
71
       ula_starboard.inter_element_spacing
                                                              = inter_element_spacing;
72
      ula_starboard.coordinates
                                                              = ULA_coordinates;
73
      ula_starboard.sampling_frequency
                                                              = sampling_frequency;
74
      ula_starboard.recording_period
                                                              = recording_period;
75
      ula_starboard.sensor_direction
                                                              = ULA_direction;
76
      ula_starboard.lowpass_filter_coefficients_for_decimation = lowpassfiltercoefficients;
       ula_starboard.num_samples
                                                               = num_samples;
78
   }
79
```

Chapter 4

Autonomous Underwater Vehicle

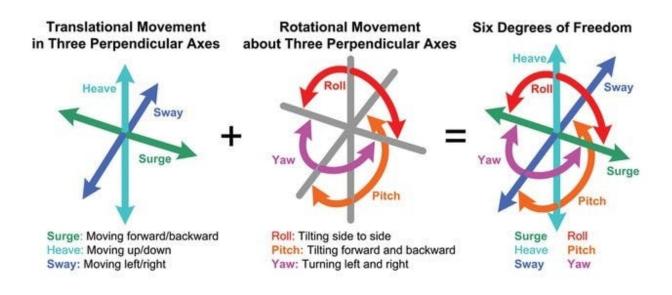


Figure 4.1: AUV degrees of freedom

Overview

Autonomous Underwater Vehicles (AUVs) are robotic systems designed to operate underwater without direct human control. They navigate and perform missions independently using onboard sensors, processors, and preprogrammed instructions. They are widely used in oceanographic research, environmental monitoring, offshore engineering, and military applications. AUVs can vary in size from small, portable vehicles for shallow water surveys to large, torpedo-shaped platforms capable of deep-sea exploration. Their autonomy allows them to access environments that are too dangerous, remote, or impractical for human divers or tethered vehicles.

The navigation and sensing systems of AUVs are critical to their performance. They typically use a combination of inertial measurement units (IMUs), Doppler velocity logs

(DVLs), pressure sensors, magnetometers, and sometimes acoustic positioning systems to estimate their position and orientation underwater. Since GPS signals do not penetrate water, AUVs must rely on these onboard sensors and occasional surfacing for GPS fixes. They are often equipped with sonar systems, cameras, or other scientific instruments to collect data about the seafloor, water column, or underwater structures. Advanced AUVs can also implement adaptive mission planning and obstacle avoidance, enabling them to respond to changes in the environment in real time.

The applications of AUVs are diverse and expanding rapidly. In scientific research, they are used for mapping the seafloor, studying marine life, and monitoring oceanographic parameters such as temperature, salinity, and currents. In the commercial sector, AUVs inspect pipelines, subsea infrastructure, and offshore oil platforms. Military and defense applications include mine countermeasure operations and underwater surveillance. The development of AUVs continues to focus on increasing endurance, improving autonomy, enhancing sensor payloads, and reducing costs, making them a key technology for exploring and understanding the underwater environment efficiently and safely.

4.1 AUV Class Definition

The following is the class used to represent the uniform linear array

```
template <</pre>
      svr::PureFloatingPointType
      svr::FFT_SourceDestination_Type sourceType,
      svr::FFT_SourceDestination_Type destinationType
4
  >
5
  class AUVClass{
  public:
      // Intrinsic attributes
Q
10
      std::vector<T> location;
                                             // location of vessel
      std::vector<T> velocity;
                                             // velocity of the vessel
11
      std::vector<T> acceleration;
                                             // acceleration of vessel
      std::vector<T>
                       pointing_direction; // AUV's pointing direction
      // uniform linear-arrays
      ULAClass<T, sourceType, destinationType>
                                                  ULA_fls;
                                                                       // front-looking
16
          SONAR ULA
      ULAClass<T, sourceType, destinationType>
                                                                       // mounted ULA
                                                  ULA_portside;
17
          [object of class, ULAClass]
      ULAClass<T, sourceType, destinationType>
                                                  ULA_starboard;
                                                                      // mounted ULA
18
          [object of class, ULAClass]
      // transmitters
20
      TransmitterClass<T> transmitter_fls;
                                                    // transmitter for front-looking SONAR
2.1
22
      TransmitterClass<T> transmitter_portside;
                                                    // portside transmitter
      TransmitterClass<T> transmitter_starboard; // starboard transmitter
24
      // derived or dependent attributes
25
      std::vector<std::vector<T>> signalMatrix_1;
                                                             // matrix containing the
          signals obtained from ULA_1
      std::vector<std::vector<T>> largeSignalMatrix_1;
                                                            // matrix holding signal of
27
          synthetic aperture
      std::vector<std::vector<T>> beamformedLargeSignalMatrix; // each column is the
```

```
beamformed signal at each stop-hop
       // plotting mode
30
      bool plottingmode; // to suppress plotting associated with classes
31
      // spotlight mode related
      std::vector<std::vector<T>> absolute_coords_patch_cart; // cartesian coordinates of
          patch
      // Synthetic Aperture Related
36
      std::vector<std::vector<T>> ApertureSensorLocations;
                                                                  // sensor locations of
37
          aperture
38
      // functions
39
      void
              syncComponentAttributes();
40
      void init(
          svr::ThreadPool&
                                                                          thread_pool,
          svr::FFTPlanUniformPoolHandle<T,</pre>
                                                       std::complex<T>>& fph_match_filter,
43
          svr::IFFTPlanUniformPoolHandle<std::complex<T>, T>&
                                                                          ifph_match_filter);
44
      void
              simulate_signal(
                   ScattererClass<T>&
                                                              seafloor,
          svr::ThreadPool&
                                                              thread_pool,
47
          svr::FFTPlanUniformPoolHandle<T, std::complex<T>>& fft_pool_handle,
48
          svr::IFFTPlanUniformPoolHandle<std::complex<T>, T>& ifft_pool_handle);
      void subset_scatterers(
          const ScattererClass<T>&
                                        seafloor,
51
          svr::ThreadPool&
                                        thread_pool,
```

4.2 AUV Setup Scripts

The following script shows the setup-script for Uniform Linear Arrays

```
template <
      svr::PureFloatingPointType
      svr::FFT_SourceDestination_Type sourceType,
      svr::FFT_SourceDestination_Type destinationType
   void fAUVSetup(
      AUVClass<T, sourceType, destinationType>& auv
8
      // building properties for the auv
                               {std::vector<T>{0, 50, 30}}; // starting location
      auto
             location
                                {std::vector<T>{5, 0, 0}}; // starting velocity
      auto
             pointing_direction {std::vector<T>{1, 0, 0}}; // pointing direction
      auto
      // assigning
      auv.location
                            = std::move(location);
                                                             // assigning location
16
      auv.velocity
                            = std::move(velocity);
                                                             // assigning velocity
      auv.pointing_direction = std::move(pointing_direction); // assigning pointing
          direction
      // signaling end
2.0
      spdlog::info("Completed AUV-setup");
21
  }
```

Part II Signal Simulation Pipeline

Chapter 5

Signal Simulation

Overview

The signal simulation pipeline is the pipeline responsible for simulating/modeling the signals sampled by the ULA-sensors under a real sub-marine environment. This chapter, and the subsequent ones, deal with the assumptions, mathematics, physics and code that goes into the design and creation of the pipeline.

A disclaimer that goes without saying is that signal-simulation is a world of its own. There's a reason that comsol, flexcompute and other numerical-simulation based companies exist. To write a signal simulation, from scratch, while these entities exist, and to make any case that this competes with those, would be flirting with delusion.

To that end, we don't write general-purpose signal simulation pipeline. However, the effort in the signal-simulator direction is purely for application-specific reasons. This is something I can talk about. One of the major in-house signal simulation pipelines yours truly developed at Naval Physical and Oceanographic Laboratory did just that. The aim of that pipeline was not to re-invent the wheel. But to create one that existed at the right speed-fidelity trade-off that the institute operated in. The pipeline created during my time there had several toggles corresponding to the different information to consider during simulation. The more information pertaining to the environment, is involved, the higher the compute and time required. Thus, mid-to-high fidelity pipelines often involve writing well-tuned GPU-supported C++ (think, CUDA). And this is important when you have pipelines downstream whose outputs depend on the signal accuracy, and by association, signal simulator fidelity.

To that end, understanding what this pipeline is not, is perhaps just as important as what it is. The core priority of this signal simulator pipeline is to produce signals for navigation. Navigation does not require high-accuracy signals owing to the very simple fact that decisions made from high-accuracy signals and low-accuracy signals tend to be the same as long as environment-topology information is not lost. To grossly oversimplify what I mean by that, the outcome of your driving does't change whether you have high-definition LI-DAR mapping the surrounding environment to the millimeter level or if you're just driving with your eyes. Thus fidelity of simulator is not a priority and I will not be putting in the kind of effort I put in at NPOL, for this reason (also because I don't want OPSEC to be

mad).

To put it simply, the signal simulation pipeline is quite trivial as far as signal simulators are concerned. But it'll work perfectly for our purposes. And thus, we'll be choosing the simplest of systems and one I like to call, "the EE engineer's best friend": the infamous Linear Time Invariant systems.

Part III Imaging Pipeline

Part IV Perception & Control Pipeline

Appendix A

Application Specific Tools

A.1 CSV File-Writes

```
#pragma once
  /*-----
  writing the contents of a vector a csv-file
  template <typename T>
                                        inputvector,
  void fWriteVector(const vector<T>&
                  const string&
                                            filename){
      // opening a file
      std::ofstream fileobj(filename);
      if (!fileobj) {return;}
11
      // writing the real parts in the first column and the imaginary parts int he second
      if constexpr(std::is_same_v<T, std::complex<double>> ||
14
                 std::is_same_v<T, std::complex<float>> ||
                 std::is_same_v<T, std::complex<long double>>){
          for(int i = 0; i<inputvector.size(); ++i){</pre>
17
             // adding entry
             fileobj << inputvector[i].real() << "+" << inputvector[i].imag() << "i";</pre>
             // adding delimiter
             if(i!=inputvector.size()-1) {fileobj << ",";}</pre>
                                      {fileobj << "\n";}
             else
      }
      else{
          for(int i = 0; i<inputvector.size(); ++i){</pre>
             fileobj << inputvector[i];</pre>
             if(i!=inputvector.size()-1) {fileobj << ",";}</pre>
29
                                      {fileobj << "\n";}
30
          }
      // return
34
35
      return;
```

```
writing the contents of a matrix to a csv-file
   template <typename T>
40
   auto fWriteMatrix(const std::vector<std::vector<T>> inputMatrix,
41
                  const string
                                                filename){
42
      // opening a file
44
      std::ofstream fileobj(filename);
      // writing
      if (fileobj){
48
          for(int i = 0; i<inputMatrix.size(); ++i){</pre>
49
             for(int j = 0; j<inputMatrix[0].size(); ++j){</pre>
                 fileobj << inputMatrix[i][j];</pre>
51
                 if (j!=inputMatrix[0].size()-1) {fileobj << ",";}</pre>
                                                {fileobj << "\n";}
                 else
             }
          }
55
      }
56
      else{
          cout << format("File-write to {} failed\n", filename);</pre>
59
60
  }
61
   /*-----
   writing complex-matrix to a csv-file
63
64
  template <>
   auto fWriteMatrix(const std::vector<std::vector<std::complex<double>>> inputMatrix,
67
                  const string
68
      // opening a file
69
      std::ofstream fileobj(filename);
70
71
      // writing
72
      if (fileobj){
          for(int i = 0; i<inputMatrix.size(); ++i){</pre>
             for(int j = 0; j<inputMatrix[0].size(); ++j){</pre>
                 if (j!=inputMatrix[0].size()-1) {fileobj << ",";}</pre>
                                                {fileobj << "\n";}
                 else
78
             }
79
          }
      }
81
      else{
82
          cout << format("File-write to {} failed\n", filename);</pre>
83
84
   }
85
```

A.2 Thread-Pool

```
#pragma once
namespace svr {
class ThreadPool {
public:
```

```
// Members
        boost::asio::thread_pool
                                thread_pool;
                                            // the pool
        std::vector<std::future<void>> future_vector; // futures to wait on
        // Special-Members
        ThreadPool(std::size_t num_threads) : thread_pool(num_threads) {}
        ThreadPool(const ThreadPool& other)
                                        = delete;
11
        ThreadPool& operator=(ThreadPool& other) = delete;
        // Member-functions
        void
                             converge();
        template <typename F> void push_back(F&& func);
                             shutdown();
18
     private:
19
        template<typename F>
20
        std::future<void> _wrap_task(F&& func) {
21
           std::promise<void> p;
22
23
           auto f = p.get_future();
           boost::asio::post(thread_pool,
              [func = std::forward<F>(func), p = std::move(p)]() mutable {
                 func();
                 p.set_value();
              });
30
           return f;
31
        }
     };
33
34
     /*-----
35
     Member-Function: Add new task to the pool
36
     -----*/
37
     template <typename F>
38
     void ThreadPool::push_back(F&& func)
30
     {
        future_vector.push_back(_wrap_task(std::forward<F>(func)));
41
42
     /*-----
     Member-Function: waiting until all the assigned work is done
45
     void ThreadPool::converge()
46
47
        for (auto &fut : future_vector) fut.get();
        future_vector.clear();
49
50
     /*----
     Member-Function: Shutting things down
     void ThreadPool::shutdown()
54
55
        thread_pool.join();
57
58
  }
```

A.3 FFTPlanClass

```
#pragma once
  namespace svr
     template <typename T>
     concept FFTPlanClassSourceDestinationType = \
6
        std::is_floating_point_v<T> ||
        (
           std::is_class_v<T> &&
           std::is_floating_point_v<typename T::value_type>
        );
11
     template <
        FFTPlanClassSourceDestinationType sourceType,
        FFTPlanClassSourceDestinationType destinationType
     class FFTPlanClass
16
     {
17
        public:
18
19
           // Members
20
           std::size_t nfft_
                              {std::numeric_limits<std::size_t>::max()};
           fftw_complex* in_
                               {nullptr};
22
           fftw_complex* out_
                              {nullptr};
23
           fftw_plan
                     plan_
                               {nullptr};
           /*-----
           Destructor
           -----*/
           ~FFTPlanClass()
30
              if(plan_ != nullptr) {fftw_destroy_plan( plan_);}
31
              if(in_ != nullptr) {fftw_free(
                                                 in_);}
              if(out_ != nullptr) {fftw_free(
                                                 out_);}
34
           35
           Default Constructor
           -----*/
           FFTPlanClass() = default;
38
           /*-----
30
           Constructor
41
           FFTPlanClass(const std::size_t nfft)
           {
              // allocating nfft
              this->nfft_ = nfft;
46
47
              // allocating input-region
              in_ = reinterpret_cast<fftw_complex*>(
49
                fftw_malloc(nfft_ * sizeof(fftw_complex))
50
              );
              out_ = reinterpret_cast<fftw_complex*>(
                fftw_malloc(nfft_ * sizeof(fftw_complex))
53
              );
54
              if(!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
                 CLASS: FFTPlanClass | REPORT: in-out allocation failed");}
56
              // creating plan
57
              plan_ = fftw_plan_dft_1d(
```

```
static_cast<int>(nfft_),
59
60
                    in_,
                    out_,
61
                    FFTW_FORWARD,
62
                    FFTW_MEASURE
                 if(!plan_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
65
                    CLASS: FFTPlanClass | REPORT: plan-creation failed");}
             }
67
              /*-----
             Copy Constructor
68
69
             FFTPlanClass(const FFTPlanClass& other)
70
71
                 // copying nfft
                 nfft_ = other.nfft_;
                 cout << format("\t\t FFTPlanClass(const FFTPlanClass& other) | nfft_ =</pre>
                    {}\n", nfft_);
75
                 // allocating input-region
76
                 in_ = reinterpret_cast<fftw_complex*>(
                    fftw_malloc(nfft_ * sizeof(fftw_complex))
78
                 );
79
                 out_ = reinterpret_cast<fftw_complex*>(
80
                    fftw_malloc(nfft_ * sizeof(fftw_complex))
82
                 if(!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
83
                    CLASS: FFTPlanClass | REPORT: in-out allocation failed");}
85
                 // copying input-region and output-region
                 std::memcpy(in_, other.in_, nfft_ * sizeof(fftw_complex));
86
                 std::memcpy(out_, other.out_, nfft_ * sizeof(fftw_complex));
                 // creating plan
89
                 plan_ = fftw_plan_dft_1d(
90
                    static_cast<int>(nfft_),
91
                    in_,
92
                    out_,
93
                    FFTW_FORWARD,
                    FFTW_MEASURE
                 );
96
                                 {throw std::runtime_error("FILE: FFTPlanClass.hpp |
                 if(!plan_)
97
                    CLASS: FFTPlanClass | REPORT: plan-creation failed");}
             }
98
             /*----
99
             Copy Assignment
100
101
             FFTPlanClass& operator=(const FFTPlanClass& other)
102
103
                 // handling self-assignment
104
                 if (this == &other) {return *this;}
105
                 // cleaning-up existing resources
107
                 if(plan_ != nullptr) {fftw_destroy_plan(
                                                          plan_);}
108
                 if(in_ != nullptr) {fftw_free(
                                                          in_);}
                 if(out_ != nullptr) {fftw_free(
                                                          out_);}
111
                 // allocating input-region and output-region
                 nfft_ = other.nfft_;
```

```
= reinterpret_cast<fftw_complex*>(
                    fftw_malloc(nfft_ * sizeof(fftw_complex))
                 );
                      = reinterpret_cast<fftw_complex*>(
117
                    fftw_malloc(nfft_ * sizeof(fftw_complex))
118
                 if(!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
120
                    CLASS: FFTPlanClass | FUNCTION: Copy-Assignment | REPORT: in-out
                    allocation failed");}
121
                 // copying contents
122
                 cout << format("\t\t FFTPlanClass& operator=(const FFTPlanClass& other) |</pre>
123
                    nfft_ = {}\n", nfft_);
                 std::memcpy(in_, other.in_, nfft_ * sizeof(fftw_complex));
124
                 std::memcpy(out_, other.out_, nfft_ * sizeof(fftw_complex));
126
                 // creating engine
                 plan_ = fftw_plan_dft_1d(
128
                    static_cast<int>(nfft_),
129
                    in_,
130
131
                    out_,
                    FFTW_FORWARD,
                    FFTW_MEASURE
133
                 );
134
                 if(!plan_) {throw std::runtime_error("FILE: FFTPlanClass.hpp | CLASS:
                    FFTPlanClass | FUNCTION: Copy-Assignment | REPORT: plan-creation
                    failed");}
136
                 // returning
137
                 return *this;
138
             }
139
             140
             Move Constructor
141
142
             FFTPlanClass(FFTPlanClass&& other)
143
                : nfft_( other.nfft_),
                    in_(
                                    other.in_),
145
                    out_(
                                    other.out_),
146
                                     other.plan_)
                    plan_(
148
                 // resetting the others
149
                 other.nfft_ = 0;
150
                 other.in_ = nullptr;
151
                                = nullptr;
                 other.out_
                                = nullptr;
                 other.plan_
153
154
             /*-----
155
             Move Assignment
156
157
             FFTPlanClass& operator=(FFTPlanClass&& other)
158
159
                 // self-assignment check
                 if (this == &other) {return *this;}
161
162
                 // cleaning up existing resources
                 if(plan_ != nullptr) {fftw_destroy_plan(
                                                          plan_);}
164
                 if(in_ != nullptr) {fftw_free(
                                                          in_);}
165
                 if(out_ != nullptr) {fftw_free(
                                                          out_);}
166
```

```
// Copying-values and changing pointers
168
                                      = other.nfft_;
169
                   cout << format("\t\t FFTPlanClass's MOVE assignment | nfft_ = {}\n",</pre>
170
                       nfft_);
                                         other.in_;
171
                   in
                                      =
                                         other.out_;
172
                   out
                   plan_
                                          other.plan_;
173
174
                   // resetting source-members
                   other.nfft_ = 0;
176
                                   = nullptr;
                   other.in_
177
                   other.out_
                                      = nullptr;
178
                   other.plan_
                                    = nullptr;
179
180
                   // returning
181
                   return *this;
182
               }
183
184
185
               Running fft
186
               std::vector<destinationType>
187
               fft(const std::vector<sourceType>& input_vector)
188
189
                   // throwing an error
190
                   if (input_vector.size() > nfft_){
                       cout << format("input_vector.size() = {}, nfft_ = {}\n",</pre>
192
                                     input_vector.size(),
193
194
                                      nfft_);
                       throw std::runtime_error("FILE: FFTPlanClass.hpp | CLASS: FFTPlanClass
195
                           | FUNCTION: fft() | REPORT: input-vector size is greater than
                           NFFT");
                   }
196
197
198
                   // copying inputs
199
                   for(std::size_t index = 0; index < input_vector.size(); ++index)</pre>
201
                       if constexpr(
202
                           std::is_floating_point_v<sourceType>
203
                       ){
204
                           in_[index][0] = input_vector[index];
205
                           in_{in}[index][1] = 0;
206
                       }
207
                       else if constexpr(
                           std::is_same_v<sourceType, std::complex<float>> ||
209
                           std::is_same_v<sourceType, std::complex<double>>
211
                           in_[index][0] =
                                              input_vector[index].real();
                           in_[index][1] = input_vector[index].imag();
213
                       }
214
                   }
215
216
                   // executing fft
217
                   fftw_execute(plan_);
218
219
                   // copying results to output-vector
220
                   std::vector<destinationType> output_vector(nfft_);
221
                   for(std::size_t index = 0; index < nfft_; ++index){</pre>
222
                       if constexpr(
223
```

```
std::is_same_v<destinationType, std::complex<float>> ||
224
                         std::is_same_v<destinationType, std::complex<double>>
225
                     ){
226
                         output_vector[index] = destinationType(
2.2.7
                            out_[index][0],
228
                            out_[index][1]
                         );
230
                     }
231
                     else if constexpr(
                         std::is_floating_point_v<destinationType>
233
                     ){
234
                         output_vector[index] = static_cast<destinationType>(
235
                            std::sqrt(
236
                                std::pow(out_[index][0], 2) + \
237
                                std::pow(out_[index][1], 2)
238
                             )
239
                         );
                     }
241
                  }
242
243
                  // returning output
                  return std::move(output_vector);
245
              }
246
              /*-----
247
              Running fft - balanced
248
              -----*/
249
              std::vector<destinationType>
250
              fft_12_conserved(const std::vector<sourceType>& input_vector)
251
252
                  // throwing an error
253
                  if (input_vector.size() > nfft_)
254
                     throw std::runtime_error("FILE: FFTPlanClass.hpp | CLASS: FFTPlanClass
255
                         | FUNCTION: fft() | REPORT: input-vector size is greater than
                         NFFT");
256
                  // copying inputs
                  for(std::size_t index = 0; index < input_vector.size(); ++index)</pre>
258
                  {
                     if constexpr(
260
261
                         std::is_floating_point_v<sourceType>
262
                         in_[index][0] = input_vector[index];
263
                         in_{in}[index][1] = 0;
264
                     }
                     else if constexpr(
266
                         std::is_same_v<sourceType, std::complex<float>> ||
267
                         std::is_same_v<sourceType, std::complex<double>>
                     ){
269
                         in_[index][0] =
                                           input_vector[index].real();
270
                         in_[index][1] = input_vector[index].imag();
271
                     }
272
273
                  }
274
                  // executing fft
275
                  fftw_execute(plan_);
277
                  // copying results to output-vector
2.78
                  std::vector<destinationType> output_vector(nfft_);
279
                  for(std::size_t index = 0; index < nfft_; ++index)</pre>
280
```

```
{
                       if constexpr(
282
                           std::is_same_v<
                                              destinationType, std::complex<double> >
283
                           std::is_same_v< destinationType, std::complex<float> >
2.84
                       ){
285
                           output_vector[index] = destinationType(
                              out_[index][0] * (1.00 / std::sqrt(nfft_)),
287
                               out_[index][1] * (1.00 / std::sqrt(nfft_))
                           );
                       }
290
                       else if constexpr(
291
                           std::is_floating_point_v<destinationType>
292
                       ){
                           output_vector[index] = destinationType(
294
                               std::sqrt(
295
                                  std::pow(out_[index][0] * (1.00 / std::sqrt(nfft_)), 2) + \
                                  std::pow(out_[index][1] * (1.00 / std::sqrt(nfft_)), 2)
298
                           );
299
                       }
                   }
301
302
                   // returning output
303
                   return std::move(output_vector);
304
               }
       };
306
   }
307
```

A.4 IFFTPlanClass

```
#pragma once
  namespace svr
                   {
      template
                <typename T>
      concept IFFTPlanClassSourceDestinationType = \
         std::is_floating_point_v<T> ||
         (
             std::is_class_v<T> &&
             std::is_floating_point_v<typename T::value_type>
         );
10
      template
         IFFTPlanClassSourceDestinationType sourceType,
12
         IFFTPlanClassSourceDestinationType destinationType
13
14
      class IFFTPlanClass
         public:
            std::size_t
                             nfft_;
18
            fftw_complex*
19
                             in_;
            fftw_complex*
                             out_;
            fftw_plan
                             plan_;
             /*----
            Destructor
25
             ~IFFTPlanClass()
26
```

```
if(plan_ != nullptr) {fftw_destroy_plan(
28
                if(in_ != nullptr) {fftw_free(
                                                          in_);}
29
                if(out_ != nullptr) {fftw_free(
                                                         out_);}
30
31
             }
             /*-----
             Constructor
33
             IFFTPlanClass(const std::size_t nfft): nfft_(nfft)
36
                // allocating space
37
                in_ = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
38
                  sizeof(fftw_complex)));
                out_ = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
39
                    sizeof(fftw_complex)));
                if(!in_ || !out_) {throw std::runtime_error("in_, out_ creation
40
                    failed");}
41
                // creating plan
42
                plan_ = fftw_plan_dft_1d(
43
                   static_cast<int>(nfft_),
                   in_,
45
                   out_,
46
                   FFTW_BACKWARD,
                   FFTW_MEASURE
                );
49
                                  {throw std::runtime_error("File: FFTPlanClass.hpp |
                if(!plan_)
50
                   Class: IFFTPlanClass | report: plan-creation failed");}
             }
51
             /*-----
52
             Copy Constructor
53
             IFFTPlanClass(const IFFTPlanClass& other)
56
                // allocating space
57
                nfft_ = other.nfft_;
                      = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
                   sizeof(fftw_complex)));
                     = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
60
                    sizeof(fftw_complex)));
                if (!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
61
                    Class: IFFTPlanClass | Function: Copy-Constructor | Report: in-out
                    plan creation failed");}
                // copying contents
63
                std::memcpy(in_, other.in_, nfft_ * sizeof(fftw_complex));
                std::memcpy(out_, other.out_, nfft_ * sizeof(fftw_complex));
                // creating a new plan since its more of an engine
67
                plan_ = fftw_plan_dft_1d(
68
                   static_cast<int>(nfft_),
69
                   in_,
                   out_,
71
                   FFTW_BACKWARD,
                   FFTW_MEASURE
                if(!plan_) {throw std::runtime_error("FILE: FFTPlanClass.hpp | Class:
75
                    IFFTPlanClass | Function: Copy-Constructor | Report: plan-creation
                    failed");}
```

```
76
77
             /*----
78
             Copy Assignment
79
             -----*/
80
             IFFTPlanClass& operator=(const IFFTPlanClass& other)
82
                // handling self-assignment
83
                if(this == &other) {return *this;}
                // cleaning up existing resources
86
                if(plan_ != nullptr) {fftw_destroy_plan( plan_);}
87
                if(in_ != nullptr) {fftw_free(
                                                       in_);}
                if(out_ != nullptr) {fftw_free(
                                                       out_);}
89
90
                // allocating space
                nfft_ = other.nfft_;
                in_ = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
93
                   sizeof(fftw_complex)));
                out_ = reinterpret_cast<fftw_complex*>(fftw_malloc(nfft_ *
                    sizeof(fftw_complex)));
                if (!in_ || !out_) {throw std::runtime_error("FILE: FFTPlanClass.hpp |
95
                    Class: IFFTPlanClass | Function: Copy-Constructor | Report: in-out
                    plan creation failed");}
                // copying contents
97
                std::memcpy(in_, other.in_, nfft_ * sizeof(fftw_complex));
98
                std::memcpy(out_, other.out_, nfft_ * sizeof(fftw_complex));
99
100
101
                // creating a new plan since its more of an engine
                plan_ = fftw_plan_dft_1d(
102
                   static_cast<int>(nfft_),
103
104
                   in_,
                   out_,
105
                   FFTW_BACKWARD,
106
                   FFTW_MEASURE
                );
108
                if(!plan_) {throw std::runtime_error("FILE: FFTPlanClass.hpp | Class:
109
                    IFFTPlanClass | Function: Copy-Constructor | Report: plan-creation
                    failed");}
110
                // returning
111
                return *this;
112
113
114
115
             Move Constructor
116
             -----*/
117
             IFFTPlanClass(IFFTPlanClass&& other) noexcept
118
                : nfft_( other.nfft_),
119
120
                   in_( other.in_),
121
                   out_( other.out_),
                   plan_( other.plan_)
             {
123
                // resetting the source object
124
                other.nfft_ = 0;
125
                other.in_
                             = nullptr;
126
                other.out_ = nullptr;
127
                other.plan_ = nullptr;
128
```

```
129
             /*----
130
             Move-Assignment
131
132
             IFFTPlanClass& operator=(IFFTPlanClass&& other) noexcept
133
                // self-assignment check
135
                if(this == &other) {return *this;}
136
138
                // cleaning up existing
                if(plan_ != nullptr) {fftw_destroy_plan( plan_);}
139
                if(in_ != nullptr) {fftw_free(
                                                        in_);}
140
                if(out_ != nullptr) {fftw_free(
                                                         out_);}
141
142
                // Copying values and changing pointers
143
                nfft_ = other.nfft_;
144
                          = other.in_;
                in_
                          = other.out_;
                out_
146
                         = other.plan_;
147
                plan_
148
                // resetting the source-object
                other.nfft_ = 0;
150
                other.in_ = nullptr;
other.out_ = nullptr;
                            = nullptr;
151
                other.plan_ = nullptr;
154
                // returning
                return *this;
156
157
             /*-----
158
             Running
159
             -----*/
160
             std::vector<destinationType>
161
             ifft(const std::vector<sourceType>& input_vector)
162
163
                // throwing an error
                if (input_vector.size() > nfft_)
165
                    throw std::runtime_error("File: FFTPlanClass | Class: IFFTPlanClass |
166
                       Function: ifft() | Report: size of vector > nfft ");
                // copy input into fftw buffer
168
                for(std::size_t index = 0; index < nfft_; ++index)</pre>
169
170
                    if constexpr(
171
                       std::is_same_v< sourceType, std::complex<double> > ||
172
                       std::is_same_v< sourceType, std::complex<float> >
173
174
                       in_[index][0] = input_vector[index].real();
                       in_[index][1] = input_vector[index].imag();
176
                    }
177
                    else if constexpr(
178
                       std::is_floating_point_v< sourceType >
180
                       in_[index][0] = input_vector[index];
181
                       in_{in}[index][1] = 0;
                    }
183
                }
184
185
                // execute ifft
```

```
fftw_execute(plan_);
187
188
                  // normalize output
189
                  std::vector<destinationType> output_vector(nfft_);
190
                  for(std::size_t index = 0; index < nfft_; ++index){</pre>
191
                     if constexpr(
                         std::is_floating_point_v< destinationType >
193
                     ){
194
                         output_vector[index] =
                             static_cast<destinationType>(out_[index][0]/nfft_);
196
                     else if constexpr(
197
                                           destinationType, std::complex<double> > ||
                         std::is_same_v<
198
                         std::is_same_v<
                                           destinationType, std::complex<float> >
199
200
                         output_vector[index][0] = destinationType(
201
                            out_[index][0]/nfft_,
                            out_[index][1]/nfft_
203
204
                         );
                     }
205
                  }
206
207
                  // returning
208
                  return std::move(output_vector);
2.09
              }
              /*-----
211
              Running - proper bases change
212
              -----*/
213
              std::vector<destinationType>
214
              ifft_12_conserved(const std::vector<sourceType>& input_vector)
215
216
                  // throwing an error
                  if (input_vector.size() > nfft_)
218
                     throw std::runtime_error("File: FFTPlanClass | Class: IFFTPlanClass |
219
                         Function: ifft() | Report: size of vector > nfft ");
                  // copy input into fftw buffer
221
                  for(std::size_t index = 0; index < nfft_; ++index)</pre>
                     if constexpr(
                         std::is_same_v<
                                          sourceType, std::complex<double> > ||
225
                         std::is_same_v< sourceType, std::complex<float> >
226
                     ){
227
                         in_[index][0] = input_vector[index].real();
                         in_[index][1] = input_vector[index].imag();
229
                     }
230
                     else if constexpr(
231
                         std::is_floating_point_v<sourceType>
233
                         in_[index][0] = input_vector[index];
234
235
                         in_{in}[index][1] = 0;
                     }
                  }
237
238
                  // execute ifft
                  fftw_execute(plan_);
240
241
                  // normalize output
242
                  std::vector<destinationType> output_vector(nfft_);
243
```

```
for(std::size_t index = 0; index < nfft_; ++index)</pre>
245
                        if constexpr(
246
                           std::is_floating_point_v<destinationType>
2.47
                       ){
248
                           output_vector[index] =
249
                                static_cast<destinationType>(out_[index][0] *
                                1.00/std::sqrt(nfft_));
                       }
                       else if constexpr(
251
                           std::is_same_v<
                                               destinationType, std::complex<double> > ||
252
                           std::is_same_v<
                                               destinationType, std::complex<float> >
253
                       ){
254
                           output_vector[index][0] = destinationType(
255
                               out_[index][0] * 1.00/std::sqrt(nfft_),
256
                               out_[index][1] * 1.00/std::sqrt(nfft_)
257
                       }
259
                   }
260
261
                   // returning
262
                   return std::move(output_vector);
263
               }
264
        };
265
    }
```

A.5 FFT Plan Pool

```
#pragma once
  namespace svr
              {
           <typename T>
    template
    concept FFTPlanUniformPoolSourceDestinationType = \
       std::is_floating_point_v<T> ||
       (
         std::is_class_v<T>
                           &&
         std::is_floating_point_v<typename T::value_type>
       );
    template
10
       FFTPlanUniformPoolSourceDestinationType sourceType,
       FFTPlanUniformPoolSourceDestinationType destinationType
    class FFTPlanUniformPool {
14
    public:
       /*-----
16
       Handle to Plan
       struct AccessPairs
20
         /*-----
2.1
         -----*/
         svr::FFTPlanClass<sourceType, destinationType>& plan;
         std::unique_lock<std::mutex>
                                          lock;
         /*----
         Special Members
28
```

```
-----*/
29
           AccessPairs() = delete;
30
           AccessPairs(
31
              svr::FFTPlanClass<sourceType, destinationType>& plan_arg,
32
              std::mutex&
                                                  plan_mutex
             : plan(plan_arg), lock(plan_mutex) {}
           AccessPairs(
35
              svr::FFTPlanClass<sourceType, destinationType>& plan_arg,
              std::unique_lock<std::mutex>&&
           ): plan(plan_arg), lock(std::move(lock_arg)) {}
38
           AccessPairs(const AccessPairs& other)
                                                    = delete;
39
           AccessPairs& operator=(const AccessPairs& other) = delete;
40
           AccessPairs(AccessPairs&& other)
                                        = delete;
41
           AccessPairs& operator=(AccessPairs&& other) = delete;
42
        };
43
        /*----
        Core Members
46
47
        std::vector<svr::FFTPlanClass<sourceType, destinationType>> plans;
        std::vector<std::mutex>
50
        /*-----
51
        Special-Members
         -----*/
        FFTPlanUniformPool()
                                                       = default;
54
        FFTPlanUniformPool(const std::size_t num_plans,
55
                     const std::size_t nfft)
57
           // reserving space
58
           plans.reserve(num_plans);
           for(auto i = 0; i < num_plans; ++i){</pre>
              plans.emplace_back(nfft);
61
62
63
           // creating a vector of mutexes
           mutexes = std::move(std::vector<std::mutex>(num_plans));
                                                      = delete;
        FFTPlanUniformPool(const FFTPlanUniformPool& other)
        FFTPlanUniformPool& operator=(const FFTPlanUniformPool& other) = delete;
        FFTPlanUniformPool(FFTPlanUniformPool&& other) = default;
69
        FFTPlanUniformPool& operator=(FFTPlanUniformPool&& other) = default;
70
71
        /*-----
        Function to fetch a plan
73
           > searches for a free-plan
           > if found, locks the plan
              > return the handle to the plan
        -----*/
77
        AccessPairs fetch_plan() {
78
           const int num_rounds = 12;
           for (int round = 0; round < num_rounds; ++round) {</pre>
              for (int i = 0; i < mutexes.size(); ++i) {</pre>
81
                 std::unique_lock<std::mutex> curr_lock(
                    mutexes[i],
                    std::try_to_lock
                 );
86
                 if (curr_lock.owns_lock())
```

```
return AccessPairs(plans[i], std::move(curr_lock));

}

throw std::runtime_error(

"FILE: FFTPlanPoolClass.hpp | CLASS: FFTPlanUniformPool | FUNCTION:

fetch_plan() | "

"Report: No plans available despite num_rounds rounds of searching");

}

}

}
```

A.6 IFFT Plan Pool

```
#pragma once
  Dependencies
           namespace svr {
    template <typename T>
9
            IFFTPlanUniformPoolSourceDestinationType = \
10
       std::is_floating_point_v<T> ||
11
          std::is_class_v<T> &&
          std::is_floating_point_v<typename T::value_type>
       );
    template
       IFFTPlanUniformPoolSourceDestinationType sourceType,
       IFFTPlanUniformPoolSourceDestinationType destinationType
18
19
    class IFFTPlanUniformPool
       public:
          Structure used for interfacing to plans
          struct AccessPairs
26
27
            Core Members
            -----*/
30
            svr::IFFTPlanClass<sourceType, destinationType>& plan;
            std::unique_lock<std::mutex>
            /*----
            Special Members
            -----*/
            AccessPairs()
                                                 = delete;
            AccessPairs(
               svr::IFFTPlanClass<sourceType, destinationType>& plan_arg,
               std::mutex&
                                                 plan_mutex_arg
            ): plan(plan_arg), lock(plan_mutex_arg) {}
41
            AccessPairs(
42
```

```
svr::IFFTPlanClass<sourceType, destinationType>& plan_arg,
43
                   std::unique_lock<std::mutex>&&
                ): plan(plan_arg), lock(std::move(lock_arg)) {}
45
                AccessPairs(const AccessPairs& other)
                                                             = delete:
46
                AccessPairs& operator=(const AccessPairs& other) = delete;
47
                AccessPairs(AccessPairs&& other)
                                                            = delete;
                AccessPairs& operator=(AccessPairs&& other)
                                                            = delete;
49
            };
50
            /*----
52
            Core Members
53
54
            std::vector< svr::IFFTPlanClass<sourceType, destinationType> > plans;
            std::vector< std::mutex</pre>
56
57
            Special Members
60
                                                   = default;
            IFFTPlanUniformPool()
61
            IFFTPlanUniformPool(const std::size_t num_plans,
                        const std::size_t nfft)
            {
                // reserving space
65
               plans.reserve(num_plans);
                for(auto i = 0; i < num_plans; ++i)</pre>
                   plans.emplace_back(nfft);
68
69
                // creating vector of mutexes
70
                mutexes = std::vector<std::mutex>(num_plans);
71
            IFFTPlanUniformPool(const IFFTPlanUniformPool& other) = delete;
            IFFTPlanUniformPool& operator=(const IFFTPlanUniformPool& other) = delete;
            IFFTPlanUniformPool(IFFTPlanUniformPool&& other)
75
            IFFTPlanUniformPool& operator=(IFFTPlanUniformPool&& other)
76
            Member-Functions
80
            AccessPairs fetch_plan()
                // setting the number of rounds to take
83
                const int num_rounds {12};
84
85
                // performing rounds
                for(auto round = 0; round < num_rounds; ++round)</pre>
87
88
                   // going through vector mutexes
                   for(auto i =0; i < mutexes.size(); ++i)</pre>
91
                      // trying to lock current mutex
92
                      std::unique_lock<std::mutex> curr_lock(mutexes[i],
                          std::try_to_lock);
                      // if our lock contains the mutex, returning the plan and lock
                      if (curr_lock.owns_lock())
                         return AccessPairs(plans[i], std::move(curr_lock));
                   }
98
               }
99
```

A.7 FFT Plan Pool Handle

```
#pragma once
  /*-----
  Dependencies
  #include "FFTPlanPoolClass.hpp"
8
  namespace svr
  {
9
10
     template
              <typename T>
              FFTPlanUniformPoolHandleSourceDestinationType = \
        std::is_floating_point_v<T> ||
           std::is_class_v<T>
                                &&
           std::is_floating_point_v<typename T::value_type>
        );
16
     template
        FFTPlanUniformPoolHandleSourceDestinationType sourceType,
        FFTPlanUniformPoolHandleSourceDestinationType destinationType
20
     struct FFTPlanUniformPoolHandle
2.1
        /*-----
23
        Core Members
24
                    -----*/
        svr::FFTPlanUniformPool<sourceType, destinationType> uniform_pool;
        std::mutex
        std::size_t
                                                   num_plans;
        std::size_t
                                                   nfft;
        /*-----
31
        Special Member-functions
32
        FFTPlanUniformPoolHandle()
                               = default;
        FFTPlanUniformPoolHandle(const std::size_t num_plans_arg,
35
                           const std::size_t nfft_arg)
              uniform_pool(num_plans_arg, nfft_arg),
              num_plans(num_plans_arg),
              nfft(nfft_arg)
                           {}
39
        FFTPlanUniformPoolHandle(const FFTPlanUniformPoolHandle& other) = delete;
40
        FFTPlanUniformPoolHandle& operator=(const FFTPlanUniformPoolHandle& other) =
        FFTPlanUniformPoolHandle(FFTPlanUniformPoolHandle&& other)
                                                                  = default:
42
        FFTPlanUniformPoolHandle& operator=(FFTPlanUniformPoolHandle&& other) = default;
        Member Functions
46
```

```
47 -----*/
48 auto lock()
49 {
50 return std::unique_lock<std::mutex>(this->mutex);
51 }
52 };
53 }
```

A.8 IFFT Plan Pool Handle

```
#pragma once
  /*-----
  -----*/
  #include "IFFTPlanPoolClass.hpp"
9
  namespace svr
10
     template <typename T>
              IFFTPlanUniformPoolHandleSourceDestinationType = \
        std::is_floating_point_v<T> ||
           std::is_class_v<T> &&
           std::is_floating_point_v<typename T::value_type>
16
        );
     template
        IFFTPlanUniformPoolHandleSourceDestinationType sourceType,
19
        {\tt IFFTPlanUniformPoolHandleSourceDestinationType\ destinationType}
2.0
     struct IFFTPlanUniformPoolHandle
23
        IFFTPlanUniformPool< sourceType,</pre>
                         destinationType >
                                              uniform_pool;
        std::mutex
                                              mutex;
        std::size_t
                                              num_plans;
30
        std::size_t
                                              nfft;
31
        /*-----
        Special Member Functions
        IFFTPlanUniformPoolHandle() = default;
        IFFTPlanUniformPoolHandle(const std::size_t num_plans_arg,
                          const std::size_t nfft_arg)
           : uniform_pool(
                              num_plans_arg, nfft_arg),
              num_plans(
                              num_plans_arg),
              nfft(
                               nfft_arg) {}
41
        IFFTPlanUniformPoolHandle(const IFFTPlanUniformPoolHandle& other) = delete;
        IFFTPlanUniformPoolHandle& operator=(const IFFTPlanUniformPoolHandle& other) =
           delete:
        IFFTPlanUniformPoolHandle(IFFTPlanUniformPoolHandle&& other)
        IFFTPlanUniformPoolHandle& operator=(IFFTPlanUniformPoolHandle&& other) = delete;
45
```

Appendix B

General Purpose Templated Functions

B.1 abs

```
#pragma once
  /*-----
  #include <vector> // for vectors
  #include <algorithm> // for std::transform
  y = abs(vector)
  template <typename T>
  auto abs(const std::vector<T>& input_vector)
      // creating canvas
     auto canvas {input_vector};
     // calculating abs
      std::transform(canvas.begin(),
                  canvas.end(),
                  canvas.begin(),
                  [](auto& argx){return std::abs(argx);});
     // returning
      return std::move(canvas);
  y = abs(matrix)
  template <typename T>
  auto abs(const std::vector<std::vector<T>> input_matrix)
31
      // creating canvas
      auto canvas
                    {input_matrix};
      // applying element-wise abs
      std::transform(input_matrix.begin(),
                  input_matrix.end(),
                  input_matrix.begin(),
```

```
[](auto& argx){return std::abs(argx);});

// returning
return std::move(canvas);

}
```

B.2 Boolean Comparators

```
#pragma once
  template <typename T, typename U>
  auto operator<(const std::vector<T>& input_vector,
             const
                                   scalar)
     // creating canvas
                   {std::vector<bool>(input_vector.size())};
     // transforming
11
     std::transform(input_vector.begin(), input_vector.end(),
                canvas.begin(),
                [&scalar](const auto& argx){
                    return argx < static_cast<T>(scalar);
     // returning
     return std::move(canvas);
19
20 }
  /*-----
  -----*/
  template <typename T, typename U>
  auto operator<=(const std::vector<T>& input_vector,
24
              const
                                   scalar)
25
  {
26
     // creating canvas
          canvas
                    {std::vector<bool>(input_vector.size())};
28
     // transforming
     std::transform(input_vector.begin(), input_vector.end(),
                canvas.begin(),
                 [&scalar](const auto& argx){
                    return argx <= static_cast<T>(scalar);
                });
     // returning
37
     return std::move(canvas);
38
  template <typename T, typename U>
  auto operator>(const std::vector<T>& input_vector,
43
             const
                                   scalar)
44
     // creating canvas
     auto canvas {std::vector<bool>(input_vector.size())};
     // transforming
```

```
std::transform(input_vector.begin(), input_vector.end(),
                   canvas.begin(),
50
                   [&scalar](const auto& argx){
51
                       return argx > static_cast<T>(scalar);
52
                  });
      // returning
55
      return std::move(canvas);
56
  }
57
   /*-----
58
59
  template <typename T, typename U>
60
  auto operator>=(const std::vector<T>& input_vector,
                const U
                                        scalar)
62
63
      // creating canvas
64
                     {std::vector<bool>(input_vector.size())};
      auto canvas
66
      // transforming
67
      std::transform(input_vector.begin(), input_vector.end(),
                  canvas.begin(),
                   [&scalar](const auto& argx){
                       return argx >= static_cast<T>(scalar);
                  });
      // returning
74
      return std::move(canvas);
75
  }
```

B.3 Concatenate Functions

```
#pragma once
  /*----
  input = [vector, vector],
  output = [vector]
                    -----*/
  template <std::size_t axis, typename T>
  auto concatenate(const std::vector<T>& input_vector_A,
               const std::vector<T>&
                                   input_vector_B) -> std::enable_if_t<axis == 1,</pre>
                   std::vector<T> >
  {
9
     // creating canvas vector
10
     auto num_elements {input_vector_A.size() + input_vector_B.size()};
11
     auto canvas
                        {std::vector<T>(num_elements, (T)0) };
     // filling up the canvas
     std::copy(input_vector_A.begin(), input_vector_A.end(),
             canvas.begin());
16
     std::copy(input_vector_B.begin(), input_vector_B.end(),
17
             canvas.begin()+input_vector_A.size());
     // moving it back
20
     return std::move(canvas);
22
  }
23
```

```
input = [vector, vector],
  output = [matrix]
  -----*/
27
28 template <std::size_t axis, typename T>
29 auto concatenate(const std::vector<T>& input_vector_A,
                const std::vector<T>&
                                     input_vector_B) -> std::enable_if_t<axis == 0,</pre>
                    std::vector<std::vector<T>> >
  ₹
31
32
      // throwing error dimensions
33
      if (input_vector_A.size() != input_vector_B.size())
         std::cerr << "concatenate:: incorrect dimensions \n";</pre>
34
35
      // creating canvas
      auto canvas {std::vector<std::vector<T>>(
37
         2, std::vector<T>(input_vector_A.size())
38
      )};
30
      // filling up the dimensions
41
      std::copy(input_vector_A.begin(), input_vector_A.end(), canvas[0].begin());
42
      std::copy(input_vector_B.begin(), input_vector_B.end(), canvas[1].begin());
43
      // moving it back
45
      return std::move(canvas);
46
47
  /*----
49
50 input = [vector, vector, vector],
output = [matrix]
 -----*/
53 template <std::size_t axis, typename T>
auto concatenate(const std::vector<T>& input_vector_A,
                const std::vector<T>& input_vector_B,
55
                const std::vector<T>& input_vector_C) -> std::enable_if_t<axis == 0,</pre>
                   std::vector<std::vector<T>> >
  ₹
57
      // throwing error dimensions
      if (input_vector_A.size() != input_vector_B.size() ||
59
         input_vector_A.size() != input_vector_C.size())
60
         std::cerr << "concatenate:: incorrect dimensions \n";</pre>
      // creating canvas
63
      auto canvas {std::vector<std::vector<T>>(
64
         3, std::vector<T>(input_vector_A.size())
65
      )};
67
      // filling up the dimensions
68
      std::copy(input_vector_A.begin(), input_vector_A.end(), canvas[0].begin());
69
      std::copy(input_vector_B.begin(), input_vector_B.end(), canvas[1].begin());
70
      std::copy(input_vector_C.begin(), input_vector_C.end(), canvas[2].begin());
71
72
      // moving it back
73
      return std::move(canvas);
75
76 }
  /*-----
  input = [matrix, vector],
78
  output = [matrix]
79
  template <std::size_t axis, typename T>
```

```
auto concatenate(const std::vector<std::vector<T>>& input_matrix,
                   const std::vector<T>
                                                        input_vector) -> std::enable_if_t<axis</pre>
83
                       == 0, std::vector<std::vector<T>> >
   {
84
       // creating canvas
85
       auto canvas
                          {input_matrix};
87
       // adding to the canvas
       canvas.push_back(input_vector);
       // returning
91
       return std::move(canvas);
92
   }
93
```

B.4 Conjugate

```
#pragma once
  namespace svr {
    /*----
    y = svr::conj(vector);
    -----*/
    template <typename T>
         conj(const std::vector<T>& input_vector)
    auto
       // creating canvas
       auto canvas
                   {std::vector<T>(input_vector.size())};
       // calculating conjugates
       std::for_each(canvas.begin(), canvas.end(),
                [](auto& argx){argx = std::conj(argx);});
       // returning
       return std::move(canvas);
17
    }
18
  }
```

B.5 Convolution

```
17
         // calculating fft of two arrays
18
                fft_A
                          {svr::fft(input_vector_A, canvas_length)};
         auto
19
         auto
                fft_B
                          {svr::fft(input_vector_B, canvas_length)};
2.0
21
         // element-wise multiplying the two matrices
                        {fft_A * fft_B};
         auto
               fft_AB
23
         // finding inverse FFT
         auto convolved_result {ifft(fft_AB)};
26
27
         // returning
28
         return std::move(convolved_result);
29
      }
30
31
      template <>
32
      auto conv1D(const std::vector<double>& input_vector_A,
                   const std::vector<double>& input_vector_B)
34
35
         // creating canvas
36
         auto canvas_length
                                 {input_vector_A.size() + input_vector_B.size() - 1};
38
         // calculating fft of two arrays
39
                         {svr::fft(input_vector_A, canvas_length)};
         auto
                fft_A
                          {svr::fft(input_vector_B, canvas_length)};
         auto
                fft_B
42
         // element-wise multiplying the two matrices
43
         auto fft_AB {fft_A * fft_B};
         // finding inverse FFT
46
         auto convolved_result {ifft(fft_AB)};
47
         // returning
49
         return std::move(convolved_result);
50
      }
51
      /*-----
53
      1D convolution of two vectors
54
      > implemented through fft
55
                           -----*/
57
      template <typename T1, typename T2>
      auto conv1D_fftw(const std::vector<T1>& input_vector_A,
58
                       const std::vector<T2>& input_vector_B)
59
      {
         // resulting type
61
         using T3 = decltype(std::declval<T1>() * std::declval<T2>());
62
         // creating canvas
                                 {input_vector_A.size() + input_vector_B.size() - 1};
         auto canvas_length
65
         // calculating fft of two arrays
67
         auto
                fft_A
                          {svr::fft(input_vector_A, canvas_length)};
         auto
                fft_B
                          {svr::fft(input_vector_B, canvas_length)};
69
70
         // element-wise multiplying the two matrices
         auto fft_AB
                          {fft_A * fft_B};
72
73
         // finding inverse FFT
74
         auto convolved_result {svr::ifft(fft_AB, fft_AB.size())};
```

```
76
          // returning
77
          return std::move(convolved_result);
78
79
80
       /*----
81
       Long-signal Conv1D
82
83
       improvements:
          > make an inplace version of this
       -----*/
       template <std::size_t L, typename T>
86
       auto conv1D_long(const std::vector<T>& input_vector_A,
87
                        const std::vector<T>& input_vector_B)
88
89
          // fetching dimensions
90
                                      {std::max(input_vector_A.size(),
          const auto
                       maxlength
                                              input_vector_B.size())};
                                      {std::min(input_vector_A.size(),
          const auto
                        filter_size
93
                                              input_vector_B.size())};
94
          const auto block_size
                                      {L + filter_size - 1};
95
          const auto num_blocks
                                      {2 + static_cast<std::size_t>(
              (maxlength - block_size)/L
97
          )};
98
          // obtaining references
          const auto& large_vector
                                      {input_vector_A.size() >= input_vector_B.size() ? \
101
                                      input_vector_A : input_vector_B};
102
          const auto& small_vector {input_vector_A.size() < input_vector_B.size() ? \</pre>
                                      input_vector_A : input_vector_B};
104
105
          // setup
106
                 starting_index
                                      {static_cast<std::size_t>(0)};
          auto
107
                 ending_index
                                      {static_cast<std::size_t>(0)};
          auto
108
                 length_left_to_fill {ending_index - starting_index};
          auto
109
                                      {std::vector<double>(block_size, 0)};
                 canvas
          auto
          auto
                 finaloutput
                                      {std::vector<double>(maxlength, 0)};
                 block_conv_output_size {L + 2 * filter_size -2};
          auto
                block_conv_output
                                      {std::vector<double>(block_conv_output_size, 0)};
          auto
114
115
          // block-wise processing
          for(auto bid = 0; bid < num_blocks; ++bid)</pre>
116
117
              // estimating indices
118
             starting_index =
                                     L*bid;
119
                                     std::min(starting_index + block_size - 1,maxlength -
              ending_index
120
                 1):
              length_left_to_fill = ending_index - starting_index;
121
              // copying to the common-block
123
              std::copy(large_vector.begin() + starting_index,
124
125
                      large_vector.begin() + ending_index + 1,
126
                      canvas.begin());
127
              // performing convolution
128
              block_conv_output = svr::conv1D_fftw(canvas,
130
                                                 small_vector);
131
              // discarding edges and writing values
              std::copy(block_conv_output.begin() + filter_size-2,
133
```

```
block_conv_output.begin() + filter_size-2 +
134
                           std::min(static_cast<int>(L-1),
                           static_cast<int>(length_left_to_fill)) + 1,
                       finaloutput.begin()+starting_index);
           }
136
137
           // returning
138
          return std::move(finaloutput);
139
140
141
       /*-----
142
       Long-signal Conv1D with FFT Plan
143
       improvements:
144
          > make an inplace version of this
145
                                             -----*/
146
       template <
147
           typename
                     Τ,
           std::enable_if_t<
149
150
              std::is_floating_point_v<T>
151
       >
       auto
              conv1D_long_prototype(
           const std::vector<T>&
                                                  input_vector_A,
154
           const std::vector<T>&
                                                  input_vector_B,
           svr::FFTPlanClass<T, std::complex<T>>& fft_plan,
           svr::IFFTPlanClass<std::complex<T>, T>& ifft_plan
157
       )
158
       {
159
           // Error checks
160
           if (fft_plan.nfft_ != ifft_plan.nfft_)
161
              throw std::runtime_error("fft_plan.nfft_ != ifft_plan.nfft_");
162
163
           // fetching references to large-signal and small-signal
164
           const auto& large_signal_original {
165
              input_vector_A.size() >= input_vector_B.size() ?
166
              input_vector_A
                                   : input_vector_B
           };
168
           const auto& small_signal
                                       {
              input_vector_A.size() < input_vector_B.size() ?
170
              input_vector_A
                                       input_vector_B
           };
172
173
           // copying
174
                  large_signal {std::vector<double>(
175
              input_vector_A.size() + input_vector_B.size() -1
176
           )}:
177
           std::copy(large_signal_original.begin(),
178
                    large_signal_original.end(),
                    large_signal.begin());;
180
181
182
           // calculating parameters
183
           const auto
                         signal_size
                                               {large_signal_original.size()};
           const
                  auto
                         filter_size
                                               {small_signal.size()};
184
                         input_signal_block_size {fft_plan.nfft_ + 1 - filter_size};
           const
                  auto
185
           if (input_signal_block_size <= 0)</pre>
              throw std::runtime_error("input_signal_block_size <= 0 ");</pre>
187
           const auto
                         block_output_length {fft_plan.nfft_};
188
                                               {static_cast<int>(
           const auto
                         num_blocks
189
              1 + std::ceil((signal_size + filter_size - 2)/input_signal_block_size)
190
```

```
)
191
           };
192
                          final_output_size
                                                 {signal_size + filter_size - 1};
           const
                   auto
193
                          useful_sample_length
                                                       {block_output_length - (filter_size -1)
           const
                   auto
194
               - (filter_size -1)};
           // parameters for re-use
196
                   start_index
                                          {static_cast<int>(0)};
197
           auto
           auto
                   end_index
                                          {static_cast<int>(0)};
           auto
                   output_start_index
                                          {static_cast<int>(0)};
199
200
           // calculating fft(filter)
201
                                          {std::vector<double>(block_output_length, 0.0)};
           auto
                   filter_zero_padded
202
           std::copy(small_signal.begin(), small_signal.end(), filter_zero_padded.begin());
203
           auto
                   filter_FFT
                                          {fft_plan.fft(filter_zero_padded)};
204
205
           // allocating space for storing input-blocks
           auto
                   signal_block_zero_padded {std::vector<double>(block_output_length, 0.0)};
207
                                             {std::vector<double>()};
208
           auto
                   fftw_output
           auto
                   conv_output
                                             {std::vector<double>()};
                   finaloutput
                                             {std::vector<double>(final_output_size, 0.0)};
           auto
210
211
           // going through the values
212
           svr::Timer timer("fft-loop");
213
           for(auto i = 0; i<num_blocks; ++i){</pre>
215
               // calculating bounds
               auto
                      analytical_start {
217
                   (i*static_cast<int>(input_signal_block_size)) -
218
                       (static_cast<int>(filter_size) - 1)
               };
219
               auto
                       analytical_end
                                          {(i+1)*input_signal_block_size -1};
               start_index
                             = std::max(
221
                   static_cast<int>(0), static_cast<int>(analytical_start)
222
               );
223
               end_index
                              = std::min(
                   static_cast<int>(signal_size-1),static_cast<int>(analytical_end)
225
               ); // [start-index, end-index)
226
228
               // copying values
               signal_block_zero_padded = std::move(std::vector<double>(block_output_length,
229
                   0.0)):
               std::copy(large_signal.begin() + start_index,
230
                         large_signal.begin() + end_index + 1,
231
                         signal_block_zero_padded.begin() + start_index - analytical_start);
232
               // performing ifft(fft(x) * fft(y))
               fftw_output
                             = ifft_plan.ifft(
                   fft_plan.fft(signal_block_zero_padded) * filter_FFT
236
               );
238
239
               // trimming away the first parts (since partial)
               conv_output = std::vector<double>(fftw_output.begin() + filter_size -
240
                   1,fftw_output.end());
               // writing to final-output
242
               std::copy(conv_output.begin(), conv_output.end(), finaloutput.begin() +
243
                   output_start_index);
               output_start_index
                                      += conv_output.size();
```

```
}
245
246
       }
247
248
       /*-----
249
       Long-signal Conv1D with FFT-Plan-Pool
250
       -----*/
251
       // concept for the FFTPlans
252
253
       template <typename T>
       concept FFT_SourceDestination_DataType = \
254
          std::is_floating_point_v<T> || \
255
256
              std::is_class_v<T> &&
257
              std::is_floating_point_v<typename T::value_type>
258
          );
260
       // template <</pre>
261
                                         Τ,
       //
             typename
262
       //
             typename
263
                      = std::enable_if_t<
       //
                std::is_floating_point_v<T>
264
       //
       // >
266
       // auto conv_per_plan(
267
       //
             const int
268
       //
             const
                   int&
                                                              input_signal_block_size,
       //
            const
                    int&
                                                              filter_size,
270
       //
           const int&
                                                              block_output_length,
271
       //
          const std::vector<T>&
272
                                                              large_signal,
       // std::vector<T>
                                                              signal_block_zero_padded,
273
       // svr::FFTPlanUniformPoolHandle<T, std::complex<T>>& fft_pool_handle,
274
       //
          svr::IFFTPlanUniformPoolHandle<std::complex<T>, T>& ifft_pool_handle,
275
       //
            const std::vector<std::complex<T>>&
                                                              filter_FFT,
276
       //
            std::vector<T>
                                                              fftw_output,
277
       //
            std::vector<T>
                                                              conv_output,
278
       11
            std::vector<T>&
                                                              output_vector,
279
       //
             std::mutex&
                                                              output_vector_mutex,
280
             const auto&
                                                              signal_size
281
       // )
282
       // {
283
       //
284
             // calculating bounds
       //
             auto
                   analytical_start {
285
                (i*static_cast<int>(input_signal_block_size)) -
       //
286
           (static_cast<int>(filter_size) - 1)
       //
             };
287
       //
                    analytical_end {(i+1)*input_signal_block_size -1};
288
       //
             auto start_index = std::max(
289
       //
                static_cast<int>(0), static_cast<int>(analytical_start)
290
       //
           );
291
       //
             auto end_index
                               = std::min(
292
                static_cast<int>(signal_size-1),static_cast<int>(analytical_end)
       //
293
       //
             ); // [start-index, end-index)
294
             // copying values
296
             signal_block_zero_padded = std::move(std::vector<double>(block_output_length,
297
          0.0));
       //
             std::copy(
298
                large_signal.begin()
       //
                                                   start_index,
299
       //
                large_signal.begin()
                                                   end_index
                                                                  + 1,
300
       //
                signal_block_zero_padded.begin() + start_index - analytical_start
```

```
//
              );
302
303
        //
              // fetching an fft and IFFT plan
304
        11
                      fph_lock
                                      {fft_pool_handle.lock()};
              auto
305
        11
              auto
                      ifph_lock
                                      {ifft_pool_handle.lock()};
306
        //
              auto
                      fft_pair
                                      {fft_pool_handle.uniform_pool.fetch_plan()};
307
                                      {ifft_pool_handle.uniform_pool.fetch_plan()};
        //
              auto
                      ifft_pair
308
309
        //
              // performing ifft(fft(x) * filter-FFT)
310
311
              fftw_output
                             = ifft_pair.plan.ifft_12_conserved(
        //
                  fft_pair.plan.fft_12_conserved(signal_block_zero_padded) * filter_FFT
312
        //
              );
313
314
        //
              // trimming away the first parts (since partial)
315
        //
              conv_output = std::vector<T>(
316
        //
                  fftw_output.begin() + filter_size - 1,
317
        //
                  fftw_output.end()
318
        //
              );
319
320
        //
              // writing to final-output
321
        //
                      output_start_index = i * (block_output_length - (filter_size - 1));
322
        //
              std::lock_guard<std::mutex> output_lock(output_vector_mutex);
323
        //
              std::copy(
324
        11
                  conv_output.begin(), conv_output.end(),
325
        //
                  output_vector.begin() + output_start_index
        //
              );
327
        // }
328
329
        template
330
           FFT_SourceDestination_DataType sourceType,
           FFT_SourceDestination_DataType destinationType
332
333
               conv_per_plan(
334
        auto
           const
                   int
                                                                                  i,
                  int&
336
           const
                input_signal_block_size,
           const int&
                                                                                  filter_size,
337
           const
                  int&
338
               block_output_length,
339
           const std::vector<sourceType>&
                                                                                  large_signal,
           std::vector<sourceType>
340
               signal_block_zero_padded,
           svr::FFTPlanUniformPoolHandle< sourceType,</pre>
                                                              destinationType>& fft_pool_handle,
341
           svr::IFFTPlanUniformPoolHandle< destinationType, sourceType>&
                                                                                  ifft_pool_handle,
           const std::vector<destinationType>&
                                                                                  filter_FFT,
343
           std::vector<sourceType>
                                                                                  fftw_output,
344
           std::vector<sourceType>
                                                                                  conv_output,
345
           std::vector<sourceType>&
                                                                                  output_vector,
346
           std::mutex&
347
               output_vector_mutex,
348
           const auto&
                                                                                  signal_size
        )
350
351
           // calculating bounds
                   analytical_start {
353
           auto
               (i*static_cast<int>(input_signal_block_size)) -
354
                    (static_cast<int>(filter_size) - 1)
           };
355
```

```
analytical_end
                                       {(i+1)*input_signal_block_size -1};
           auto
356
                                   std::max(
           auto start_index =
357
               static_cast<int>(0), static_cast<int>(analytical_start)
           );
359
                                    std::min(
           auto end_index
360
               static_cast<int>(signal_size-1),static_cast<int>(analytical_end)
361
           ); // [start-index, end-index)
362
363
           // copying values
           signal_block_zero_padded = std::move(std::vector<double>(block_output_length,
365
               0.0));
           std::copy(
366
               large_signal.begin()
                                                      start_index,
367
               large_signal.begin()
                                                      end_index
368
               signal_block_zero_padded.begin() +
                                                      start_index
                                                                      - analytical_start
369
           );
370
371
           // fetching an fft and IFFT plan
373
           auto
                   fph_lock
                                   {fft_pool_handle.lock()};
           {\tt auto}
                   ifph_lock
                                   {ifft_pool_handle.lock()};
374
                   fft_pair
                                   {fft_pool_handle.uniform_pool.fetch_plan()};
           auto
375
           auto
                   ifft_pair
                                   {ifft_pool_handle.uniform_pool.fetch_plan()};
376
377
           // performing ifft(fft(x) * filter-FFT)
           fftw_output
                          = ifft_pair.plan.ifft_l2_conserved(
               fft_pair.plan.fft_12_conserved(signal_block_zero_padded) * filter_FFT
380
           );
381
382
           // trimming away the first parts (since partial)
383
           conv_output = std::vector<sourceType>(
384
               fftw_output.begin() + filter_size - 1,
385
               fftw_output.end()
           );
387
388
           // writing to final-output
389
                   output_start_index = i * (block_output_length - (filter_size - 1));
           std::lock_guard<std::mutex> output_lock(output_vector_mutex);
391
           std::copy(
392
               conv_output.begin(), conv_output.end(),
394
               output_vector.begin() + output_start_index
           );
395
        }
396
397
        template
399
           typename T,
400
           FFT_SourceDestination_DataType sourceType,
401
           FFT_SourceDestination_DataType destinationType,
402
           typename = std::enable_if_t<</pre>
403
               std::is_floating_point_v<T>
404
405
406
        >
        auto
               conv1D_long_FFTPlanPool(
407
           const std::vector<T>&
                                                                                 input_vector_A,
408
           const
                   std::vector<T>&
                                                                                 input_vector_B,
           svr::FFTPlanUniformPoolHandle< sourceType,</pre>
                                                              destinationType>& fft_pool_handle,
410
                                                                                 ifft_pool_handle
           svr::IFFTPlanUniformPoolHandle< destinationType, sourceType>&
411
        )
412
        {
413
```

```
// Error checks
           if (fft_pool_handle.nfft!=ifft_pool_handle.nfft)
415
               throw std::runtime_error("FILE: svr_conv.hpp | FUNCTION:
416
                   conv1D_long_FFTPlanPool | Report: the pool-handles are for different
                   nffts");
           // fetching references to the large signal and small signal
418
           const auto& large_signal_original {
419
               input_vector_A.size() >= input_vector_B.size() ?
421
               input_vector_A
                                      :
                                          input_vector_B
           };
422
           const auto& small_signal
                                          {
423
               input_vector_A.size() <</pre>
                                          input_vector_B.size() ?
424
                                          input_vector_B
425
               input_vector_A
                                      :
           };
426
427
           // copying
                                 {std::vector<double>(
                   large_signal
429
               input_vector_A.size() + input_vector_B.size() - 1
430
           )};
431
           std::copy(large_signal_original.begin(),
432
                     large_signal_original.end(),
433
                     large_signal.begin());
434
435
           // calculating some parameters
           const
                  auto
                           signal_size
                                          {large_signal_original.size()};
437
           const
                   auto
                           filter_size
                                          {small_signal.size()};
438
           const
                   auto
                           input_signal_block_size {
439
               fft_pool_handle.nfft + 1 - filter_size
440
           };
441
442
           // throwing an error if nfft < filter-size
           if (fft_pool_handle.nfft < filter_size)</pre>
               throw std::runtime_error("FILE: svr_conv.hpp | FUNCTION:
445
                   conv1D_long_FFTPlanPool | REPORT: filter is bigger than nfft");
446
           // throwing an error if number of useful samples is less than zero
447
           if (input_signal_block_size <= 0)</pre>
448
               throw std::runtime_error("FILE: svr_conv.hpp | FUNCTION:
449
                   conv1D_long_FFTPlanPool | REPORT: input_signal_block_size = 0");
450
451
                           block_output_length
                                                  {fft_pool_handle.nfft};
452
           const
                   auto
           const
                   auto
                           num_blocks
                                                  {static_cast<int>(
453
               1 + std::ceil((signal_size + filter_size - 2) / input_signal_block_size)
454
           )}:
455
                           final_output_size
                                                  {signal_size + filter_size - 1};
           const
                   auto
                           useful_sample_length {
457
                   auto
               block_output_length - (filter_size - 1) - (filter_size -1)
458
459
           };
460
461
           // parameters for re-use
           auto
                   start_index
                                          {static_cast<int>(0)};
462
                                          {static_cast<int>(0)};
                   end_index
           auto
463
                                          {static_cast<int>(0)};
           auto
                   output_start_index
465
           // calculating fft(filter)
466
                                          {std::vector<double>(block_output_length, 0.0)};
                   filter_zero_padded
467
           std::copy(small_signal.begin(), small_signal.end(), filter_zero_padded.begin());
```

```
fph_lock0
                                            {fft_pool_handle.lock()};
469
            auto
            auto
                    curr_plan_pair
                                            {fft_pool_handle.uniform_pool.fetch_plan()};
470
            auto
                    pool_num_plans
                                            {fft_pool_handle.num_plans};
471
            fph_lock0.unlock();
472
                    filter_FFT
                                            {
            auto
473
                curr_plan_pair.plan.fft(
                    filter_zero_padded
475
                )
476
            };
            curr_plan_pair.lock.unlock();
478
479
            // allocating space for storing input-blocks
480
                    signal_block_zero_padded {std::vector<T>(block_output_length, 0.0)};
481
                    fftw_output
                                                {std::vector<T>()};
482
            auto
            auto
                    conv_output
                                                {std::vector<T>()};
483
                                                {std::vector<T>(final_output_size, 0.0)};
                    output_vector
            auto
            auto
                    output_vector_mutex
                                                {std::mutex()};
486
487
            // creating boost
            svr::ThreadPool local_pool(pool_num_plans);
488
489
            // going through the values
490
            for(auto i = 0; i < num_blocks; ++i)</pre>
491
                local_pool.push_back(
                    Γ
494
                        i,
495
                        &input_signal_block_size,
496
                        &filter_size,
497
                        &block_output_length,
498
                        &large_signal,
                        signal_block_zero_padded,
                        &fft_pool_handle,
501
                        &ifft_pool_handle,
502
                        &filter_FFT,
503
                        fftw_output,
                        conv_output,
505
                        &output_vector,
506
                        &output_vector_mutex,
507
508
                        &signal_size
                    ]{
509
                        conv_per_plan<T>(
511
                            i,
                            std::ref(input_signal_block_size),
                            std::ref(filter_size),
513
                            std::ref(block_output_length),
514
                            std::ref(large_signal),
515
                            signal_block_zero_padded,
516
                            fft_pool_handle,
517
                            ifft_pool_handle,
518
519
                            filter_FFT,
520
                            fftw_output,
                            conv_output,
521
                            std::ref(output_vector),
522
                            output_vector_mutex,
                            signal_size
524
                        );
525
                    }
526
                );
527
```

```
528
          local_pool.converge();
529
530
          // returning final output
531
          return std::move(output_vector);
532
          // return output_vector;
533
534
535
       /*-----
537
       Short-conv1D
538
       template <typename
                              T1.
539
                              T2>
                  typename
540
             conv1D_short(const std::vector<T1>& input_vector_A,
541
                         const std::vector<T2>& input_vector_B)
542
543
          // resulting type
          using T3 = decltype(std::declval<T1>() * std::declval<T2>());
545
546
          // creating canvas
547
                 canvas_length
                                  {input_vector_A.size() + input_vector_B.size() - 1};
549
          // calculating fft of two arrays
550
                           {svr::fft(input_vector_A, canvas_length)};
          auto
                 fft_A
551
          auto
                 fft_B
                           {svr::fft(input_vector_B, canvas_length)};
553
          // element-wise multiplying the two matrices
554
                           {fft_A * fft_B};
                 fft_AB
          auto
555
556
          // finding inverse FFT
557
                 convolved_result {ifft(fft_AB)};
          auto
558
559
          // returning
560
          // return std::move(convolved_result);
561
          return convolved_result;
562
      }
564
565
566
       1D Convolution of a matrix and a vector
568
       -----*/
569
       template
                <typename T>
570
       auto conv1D(const std::vector<std::vector<T>>& input_matrix,
571
                   const std::vector<T>&
572
                                                      input_vector,
                    const std::size_t&
                                                       dim)
573
574
          // getting dimensions
                                            {input_matrix.size()};
          const auto& num_rows_matrix
576
                                            {input_matrix[0].size()};
          const auto& num_cols_matrix
577
          const auto& num_elements_vector {input_vector.size()};
          // creating canvas
580
                           {std::vector<std::vector<T>>()};
          auto
                canvas
581
          // creating output based on dim
583
          if (dim == 1)
584
          {
585
              // performing convolutions row by row
```

```
for(auto
                             row = 0; row < num_rows_matrix; ++row)</pre>
587
588
                    \verb|cout| << format("\t row = {}/{}\n", row, num_rows_matrix); \\
589
                    auto bruh {conv1D(input_matrix[row], input_vector)};
590
                    auto bruh_real {svr::real(std::move(bruh))};
591
                    canvas.push_back(
593
                             svr::real(
                                 std::move(bruh_real)
596
                    );
597
                }
598
            }
            else{
600
                 std::cerr << "svr_conv.hpp | conv1D | yet to be implemented \n";</pre>
601
            }
602
            // returning
604
            return std::move(canvas);
605
        }
607
608
    }
609
```

B.6 Coordinate Change

```
#pragma once
  namespace svr {
     /*----
     y = cart2sph(vector)
                            -----*/
     template <typename T>
           cart2sph(const
                           std::vector<T>& cartesian_vector){
         // splatting the point onto xy-plane
               xysplat
                        {cartesian_vector};
         xysplat[2]
                        0;
         // finding splat lengths
               xysplat_lengths
                               {norm(xysplat)};
14
         // finding azimuthal and elevation angles
               azimuthal_angles {svr::atan2(xysplat[1],
17
                                        xysplat[0]) \
18
                                * 180.00/std::numbers::pi};
19
               elevation_angles {svr::atan2(cartesian_vector[2],
         auto
                                        xysplat_lengths) \
                                * 180.00/std::numbers::pi};
                               {norm(cartesian_vector)};
         auto
               rho_values
23
         // creating tensor to send back
               spherical_vector {std::vector<T>{azimuthal_angles,
                                            elevation_angles,
                                            rho_values}};
         // moving it back
30
```

```
return std::move(spherical_vector);
31
     }
32
     33
     y = cart2sph(vector)
34
      -----*/
35
     template <typename T>
36
     auto cart2sph_inplace(std::vector<T>& cartesian_vector){
37
38
         // splatting the point onto xy-plane
              xysplat {cartesian_vector};
40
         xysplat[2]
41
42
        // finding splat lengths
43
              xysplat_lengths
                              {norm(xysplat)};
45
        // finding azimuthal and elevation angles
               azimuthal_angles {svr::atan2(xysplat[1], xysplat[0]) *
            180.00/std::numbers::pi};
              elevation_angles {svr::atan2(cartesian_vector[2],
48
         auto
                                        xysplat_lengths) * 180.00/std::numbers::pi};
49
        auto rho_values
                               {norm(cartesian_vector)};
51
        // creating tesnor
52
        cartesian_vector[0] = azimuthal_angles;
         cartesian_vector[1] =
                              elevation_angles;
         cartesian_vector[2] = rho_values;
55
56
     /*-----
57
     y = cart2sph(input_matrix, dim)
58
      -----*/
59
     template <typename T>
60
     auto cart2sph(const std::vector<std::vector<T>>& input_matrix,
61
                const
                        std::size_t
63
         // fetching dimensions
64
         const auto& num_rows {input_matrix.size()};
         const auto& num_cols {input_matrix[0].size()};
         // checking the axis and dimensions
         if (axis == 0 && num_rows != 3) {std::cerr << "cart2sph: incorrect num-elements
            \n";}
         if (axis == 1 && num_cols != 3) {std::cerr << "cart2sph: incorrect num-elements
70
            n";}
71
        // creating canvas
72
        auto canvas
                       {std::vector<std::vector<T>>(
73
            num_rows,
            std::vector<T>(num_cols, 0)
        )};
76
77
         // if axis = 0, performing operation column-wise
         if(axis == 0)
         {
80
            for(auto col = 0; col < num_cols; ++col)</pre>
               // fetching current column
83
               auto curr_column {std::vector<T>({input_matrix[0][col],
84
                                             input_matrix[1][col],
85
                                              input_matrix[2][col]})};
```

```
// performing inplace transformation
88
                  cart2sph_inplace(curr_column);
89
90
                  // storing it back
91
                  canvas[0][col] =
                                      curr_column[0];
                  canvas[1][col] =
                                      curr_column[1];
                                      curr_column[2];
                  canvas[2][col] =
              }
           }
96
           // if axis == 1, performing operations row-wise
97
           else if(axis == 0)
98
99
              std::cerr << "cart2sph: yet to be implemented \n";</pre>
100
           }
          else
              std::cerr << "cart2sph: yet to be implemented \n";
104
105
106
           // returning
107
           return std::move(canvas);
108
109
       }
110
       // -----
112
       template <typename T>
              sph2cart(const std::vector<T> spherical_vector){
       auto
114
115
           // creating cartesian vector
           auto
                  cartesian_vector {std::vector<T>(spherical_vector.size(), 0)};
117
           // populating
119
           cartesian_vector[0]
                                    spherical_vector[2] * \
120
                                    cos(spherical_vector[1] * std::numbers::pi / 180.00) * \
                                    cos(spherical_vector[0] * std::numbers::pi / 180.00);
           cartesian_vector[1]
                                    spherical_vector[2] * \
123
                                    cos(spherical_vector[1] * std::numbers::pi / 180.00) * \
124
                                    sin(spherical_vector[0] * std::numbers::pi / 180.00);
126
           cartesian_vector[2]
                                    spherical_vector[2] * \
                                    sin(spherical_vector[1] * std::numbers::pi / 180.00);
127
128
           // returning
129
           return std::move(cartesian_vector);
       }
131
   }
132
```

B.7 Cosine

```
// created canvas
      auto canvas
                      {input_vector};
      // calling the function
11
      std::transform(input_vector.begin(), input_vector.end(),
                  canvas.begin(),
                  [](auto& argx){return std::cos(argx);});
      // returning the output
17
      return std::move(canvas);
  }
18
  /*-----
  y = cosd(input_vector)
  template <typename T>
  auto cosd(const std::vector<T> input_vector)
      // created canvas
25
26
      auto canvas
                    {input_vector};
27
      // calling the function
      std::transform(input_vector.begin(),
                  input_vector.end(),
30
                  input_vector.begin(),
                  [](const auto& argx){return std::cos(argx * 180.00/std::numbers::pi);});
33
      // returning the output
34
     return std::move(canvas);
35
  }
36
```

B.8 Data Structures

```
struct TreeNode {
      int val;
      TreeNode *left;
      TreeNode *right;
      TreeNode() : val(0), left(nullptr), right(nullptr) {}
      TreeNode(int x) : val(x), left(nullptr), right(nullptr) {}
      TreeNode(int x, TreeNode *left, TreeNode *right) : val(x), left(left), right(right)
          {}
   };
8
  struct ListNode {
11
      int val;
      ListNode *next;
      ListNode() : val(0), next(nullptr) {}
      ListNode(int x) : val(x), next(nullptr) {}
      ListNode(int x, ListNode *next) : val(x), next(next) {}
16
  };
```

B.9 Editing Index Values

```
Matlab's equivalent of A[A < 0.5] = 0
  -----*/
5 template <typename T, typename U>
6 auto edit(std::vector<T>&
                                    input_vector,
                                   bool_vector,
          const std::vector<bool>&
                                    scalar)
8
  {
9
      // throwing an error
10
11
      if (input_vector.size() != bool_vector.size())
         std::cerr << "edit: incompatible size\n";</pre>
12
13
      // overwriting input-vector
14
      std::transform(input_vector.begin(), input_vector.end(),
                  bool_vector.begin(),
16
                  input_vector.begin(),
17
                  [&scalar](auto& argx, auto argy){
                      if(argy == true) {return static_cast<T>(scalar);}
19
                     else
                                     {return argx;}
20
                  });
21
      // no-returns since in-place
23
24
25
  /*-----
  accumulate version of edit, instead of just placing values
27
2.8
29 THings to add
     - ensuring template only accepts int, std::size_t and similar for T2
30
31
      - bring in histogram method to ensure SIMD
                        -----*/
32
  template <typename T1,
33
            typename T2>
34
  auto edit_accumulate(std::vector<T1>&
                                           input_vector,
35
                     const std::vector<T2>& indices_to_edit,
36
                     const std::vector<T1>& new_values)
37
38
      // certain checks
39
      if (indices_to_edit.size() != new_values.size())
         std::cerr << "svr::edit | edit_accumulate | size-disparity occured \n";</pre>
41
42
      // going through each and accumulating
43
     for(auto i = 0; i < input_vector.size(); ++i){</pre>
44
         const auto target_index {static_cast<std::size_t>(indices_to_edit[i])}; //
         const auto new_value {new_values[i]};
46
         if (target_index < input_vector.size()){</pre>
47
            input_vector[target_index] = input_vector[target_index] + new_value;
         }
         else{
50
            // std::cout << "warning: FILE: svr_edit.hpp | FUNCTION: edit_accumulate |
               REPORT: index out of bounds";
         }
53
      // no-return since in-place
55
  }
```

B.10 Equality

```
#pragma once
  template <typename T, typename U>
   auto operator==(const std::vector<T>& input_vector,
                 const U&
                                          scalar)
      // setting up canvas
      auto canvas {std::vector<bool>(input_vector.size())};
      // writing to canvas
11
      std::transform(input_vector.begin(), input_vector.end(),
                    canvas.begin(),
                    [&scalar](const auto& argx){
                        return argx == scalar;
                    });
      // returning
18
      return std::move(canvas);
19
  }
```

B.11 Exponentiate

```
#pragma once
 namespace svr {
    /*----
    y = exp(vector)
    -----*/
    template <typename T>
    auto exp(const std::vector<T>& input_vector)
      // creating canvas
      auto canvas
                 {input_vector};
      // transforming
      std::transform(canvas.begin(), canvas.end(),
             canvas.begin(),
             [](auto& argx){return std::exp(argx);});
      // returning
      return std::move(canvas);
20
    /*----
    y = exp(matrix)
    ------*/
23
    template
24
      typename sourceType,
      typename destinationType,
      typename = std::enable_if_t<</pre>
        std::is_arithmetic_v<sourceType>
    auto
       exp(
```

```
const std::vector<std::vector<sourceType>> input_matrix
32
      )
33
34
          // fetching dimensions
35
          const auto& num_rows
                                 {input_matrix.size()};
          const auto& num_cols {input_matrix[0].size()};
38
         // creating canvas
39
                           {std::vector<std::vector<destinationType>>(
          auto canvas
             num_rows,
41
             std::vector<destinationType>(num_cols)
42
          )};
43
          // writing to each entry
45
          for(auto row = 0; row < num_rows; ++row)</pre>
46
             std::transform(
                 input_matrix[row].begin(), input_matrix[row].end(),
                 canvas[row].begin(),
49
                 [](const
                          auto& argx){
50
51
                    return std::exp(argx);
                 }
             );
53
          // returning
          return std::move(canvas);
57
      /*-----
58
59
      Aim: Exponentiating complex matrices with general floating types
60
61
      template
         typename
                   Τ,
62
          typename = std::enable_if_t<</pre>
             std::is_floating_point_v<T>
65
      >
66
      auto
             exp(
          const std::vector<std::complex<T>>> input_matrix
68
      {
70
71
          // fetching dimensions
          const auto& num_rows
                                 {input_matrix.size()};
72
          const auto& num_cols {input_matrix[0].size()};
73
74
          // creating canvas
                           {std::vector<std::complex<T>>>(
          auto canvas
76
             num_rows,
             std::vector<std::complex<T>>(num_cols)
          )};
80
          // writing to each entry
81
          for(auto row = 0; row < num_rows; ++row)</pre>
             std::transform(
                 input_matrix[row].begin(), input_matrix[row].end(),
                 canvas[row].begin(),
                 [](const auto& argx){
                    return std::exp(argx);
                 }
88
             );
89
```

```
// returning
return std::move(canvas);
}

// returning
// return std::move(canvas);
// returning
// return std::move(canvas);
// returning
// return std::move(canvas);
// returning
// return std::move(canvas);
// return std::move(canvas);
// returning
// returni
```

B.12 FFT

48

```
#pragma once
2 namespace svr {
     /*-----
     For type-deductions
      -----*/
     template <typename T>
      struct fft_result_type;
      // specializations
9
      template <> struct fft_result_type<double>{
11
         using type = std::complex<double>;
      template <> struct fft_result_type<std::complex<double>>{
        using type = std::complex<double>;
      template <> struct fft_result_type<float>{
16
         using type = std::complex<float>;
17
     };
18
      template <> struct fft_result_type<std::complex<float>>{
19
20
         using type = std::complex<float>;
     };
21
22
      template <typename T>
23
      using fft_result_t = typename fft_result_type<T>::type;
24
25
      /*----
26
     y = fft(x, nfft)
         > calculating n-point dft where n-value is explicit
28
29
      template<typename T>
30
      auto fft(const std::vector<T>& input_vector,
31
            const
                    size_t
                                    nfft)
32
33
         // throwing an error
34
         if (nfft < input_vector.size()) {std::cerr << "size-mistmatch\n";}</pre>
35
         if (nfft <= 0)</pre>
                                    {std::cerr << "size-mistmatch\n";}
36
37
         // fetching data-type
         using RType = fft_result_t<T>;
         using baseType = std::conditional_t<std::is_same_v<T, std::complex<double>>,
40
                                         double,
41
                                         T>;
         // canvas instantiation
         std::vector<RType> canvas(nfft);
         auto nfft_sqrt {static_cast<RType>(std::sqrt(nfft))};
               finaloutput {std::vector<RType>(nfft, 0)};
         auto
47
```

```
// calculating index by index
          for(int frequency_index = 0; frequency_index<nfft; ++frequency_index){</pre>
50
              RType accumulate_value;
51
              for(int signal_index = 0; signal_index < input_vector.size(); ++signal_index){</pre>
52
                  accumulate_value += \
                     static_cast<RType>(input_vector[signal_index]) * \
                     static_cast<RType>(std::exp(-1.00 * std::numbers::pi * \
55
                                              (static_cast<baseType>(frequency_index)/static_cast<baseType>(n
                                              static_cast<baseType>(signal_index)));
58
              finaloutput[frequency_index] = accumulate_value / nfft_sqrt;
59
61
          // returning
62
          return std::move(finaloutput);
63
65
       /*-----
66
       y = fft(std::vector<double> nfft) // specialization
67
       -----*/
68
                               // for fft
       #include <fftw3.h>
69
       template <>
70
       auto fft(const std::vector<double>& input_vector,
71
               const std::size_t
                                         nfft.)
73
          if (nfft < input_vector.size())</pre>
74
              throw std::runtime_error("nfft must be >= input_vector.size()");
75
          if (nfft <= 0)
76
              throw std::runtime_error("nfft must be > 0");
77
          // FFTW real-to-complex output
          std::vector<std::complex<double>> output(nfft);
81
          // Allocate input (double) and output (fftw_complex) arrays
82
          double* in
                            = reinterpret_cast<double*>(
              fftw_malloc(sizeof(double) * nfft)
          fftw_complex* out = reinterpret_cast<fftw_complex*>(
              fftw_malloc(sizeof(fftw_complex) * (nfft/2 + 1))
          );
88
89
          // Copy input and zero-pad if needed
90
          for (std::size_t i = 0; i < nfft; ++i) {</pre>
              in[i] = (i < input_vector.size()) ? input_vector[i] : 0.0;</pre>
92
          }
93
          // Create FFTW plan and execute
          fftw_plan plan = fftw_plan_dft_r2c_1d(
96
              static_cast<int>(nfft), in, out, FFTW_ESTIMATE
97
          );
98
          fftw_execute(plan);
100
          // Copy FFTW output to std::vector<std::complex<double>>
101
          for (std::size_t i = 0; i < nfft/2 + 1; ++i) {</pre>
              output[i] = std::complex<double>(out[i][0], out[i][1]);
103
104
          // Optional: fill remaining bins with zeros to match full nfft size
105
          for (std::size_t i = nfft/2 + 1; i < nfft; ++i) {</pre>
106
```

```
output[i] = std::complex<double>(0.0, 0.0);
107
          }
108
109
          // Cleanup
          fftw_destroy_plan(plan);
111
          fftw_free(in);
112
          fftw_free(out);
113
114
          // filling up the other half of the output
          const auto
                      halfpoint {static_cast<std::size_t>(nfft/2)};
          std::transform(
117
             output.begin() + 1,
                                       // first half (skip DC)
118
              output.begin() + halfpoint, // end of first half
119
              output.rbegin(),
                                 // start writing from last element backward (skip
120
                 Nyquist)
              [](const auto& x) { return std::conj(x); }
          );
          // returning
124
          return std::move(output);
125
       }
126
127
128
       129
130
       y = ifft(x, nfft)
        131
       template<typename T>
132
       auto ifft(const
                         std::vector<T>& input_vector)
134
          // fetching data-type
135
          using RType = fft_result_t<T>;
136
          using baseType = std::conditional_t<std::is_same_v<T, std::complex<double>>,
                                              double,
138
                                              T>;
139
140
          // setup
                            {input_vector.size()};
          auto
                 nfft
142
143
          // canvas instantiation
145
          std::vector<RType> canvas(nfft);
          auto
                 nfft_sqrt
                               {static_cast<RType>(std::sqrt(nfft))};
146
          auto
                 finaloutput
                               {std::vector<RType>(nfft, 0)};
147
148
          // calculating index by index
          for(int frequency_index = 0; frequency_index<nfft; ++frequency_index){</pre>
150
              RType accumulate_value;
              for(int signal_index = 0; signal_index < input_vector.size(); ++signal_index){</pre>
                 accumulate_value += \
                     static_cast<RType>(input_vector[signal_index]) * \
                     static_cast<RType>(std::exp(1.00 * std::numbers::pi * \
                                             (static_cast<baseType>(frequency_index)/static_cast<baseType>(n
156
                                             static_cast<baseType>(signal_index)));
157
              }
158
              finaloutput[frequency_index] = accumulate_value / nfft_sqrt;
          }
160
161
          // returning
162
          return std::move(finaloutput);
163
```

```
}
164
165
166
       x = ifft(std::vector<std::complex<double>> spectrum, nfft)
167
       -----*/
168
       #include <fftw3.h>
169
       #include <vector>
170
       #include <complex>
171
       #include <stdexcept>
173
       auto ifft(const std::vector<std::complex<double>>& input_vector,
174
                 const std::size_t
                                                       nfft)
175
176
           if (nfft <= 0)
177
               throw std::runtime_error("nfft must be > 0");
178
           if (input_vector.size() != nfft)
179
               throw std::runtime_error("input spectrum must be of size nfft");
181
           // Output: real-valued time-domain sequence
182
           std::vector<double> output(nfft);
183
184
           // Allocate FFTW input/output
185
           fftw_complex* in = reinterpret_cast<fftw_complex*>(
186
               fftw_malloc(sizeof(fftw_complex) * (nfft/2 + 1))
           double* out = reinterpret_cast<double*>(
189
               fftw_malloc(sizeof(double) * nfft)
190
           );
191
192
           // Copy *only* the first nfft/2+1 bins (rest are redundant due to symmetry)
193
           for (std::size_t i = 0; i < nfft/2 + 1; ++i) {</pre>
194
               in[i][0] = input_vector[i].real();
               in[i][1] = input_vector[i].imag();
196
197
198
           // Create inverse FFTW plan
           fftw_plan plan = fftw_plan_dft_c2r_1d(
200
               static_cast<int>(nfft),
201
               in,
202
203
               out,
               FFTW_ESTIMATE
204
           );
205
206
           fftw_execute(plan);
208
           // Normalize by nfft (FFTW leaves IFFT unscaled)
2.09
           for (std::size_t i = 0; i < nfft; ++i) {</pre>
210
               output[i] = out[i] / static_cast<double>(nfft);
211
212
213
           // Cleanup
214
215
           fftw_destroy_plan(plan);
           fftw_free(in);
216
           fftw_free(out);
217
218
           return output;
219
       }
220
221
```

222

```
223
224 }
```

B.13 Flipping Containers

B.14 Indexing

```
#pragma once
  namespace svr {
      /*-----
      y = index(vector, mask)
      template
         typename T1,
         typename T2,
         typename = std::enable_if_t<</pre>
             (std::is_arithmetic_v<T1>
            std::is_same_v<T1, std::complex<float> > ||
            std::is_same_v<T1, std::complex<double> >) &&
             std::is_integral_v<T2>
14
                <typename T1,
      template
17
                 typename T2,
                 typename = std::enable_if_t<std::is_arithmetic_v<T1>
                                        std::is_same_v<T1, std::complex<float> > ||
                                        std::is_same_v<T1, std::complex<double> >
21
      auto
            index(const
                          std::vector<T1>&
                                              input_vector,
                 const
                          std::vector<T2>&
                                              indices_to_sample)
         // creating canvas
                          {std::vector<T1>(indices_to_sample.size(), 0)};
```

```
29
          // copying the associated values
30
          for(int i = 0; i < indices_to_sample.size(); ++i){</pre>
31
                     source_index {indices_to_sample[i]};
32
              if(source_index < input_vector.size()){</pre>
                  canvas[i] = input_vector[source_index];
              }
35
              else{
                  // cout << "warning: Some chosen samples are out of bounds. svr::index |
                      source_index !< input_vector.size()\n";</pre>
              }
38
39
          }
40
41
          // returning
42
          return std::move(canvas);
43
45
46
       y = index(matrix, mask, dim)
47
                                   -----*/
       template <
48
          typename T1,
49
          typename T2,
50
          typename = std::enable_if_t<</pre>
51
              (std::is_same_v<T1, double> || std::is_same_v<T1, float>) &&
              (std::is_same_v<T2, int> || std::is_same_v<T2, std::size_t>)
53
54
       >
55
       auto index(const std::vector<std::vector<T1>>& input_matrix,
56
57
                 const
                         std::vector<T2>&
                                                       indices_to_sample,
                 const std::size_t&
                                                       dim)
58
59
          // fetching dimensions
60
          const auto& num_rows_matrix
                                            {input_matrix.size()};
61
                                            {input_matrix[0].size()};
          const auto& num_cols_matrix
62
          // creating canvas
                 canvas {std::vector<std::vector<T1>>()};
          // if indices are row-indices
          if (dim == 0){
68
69
              // initializing canvas
70
              canvas = std::vector<std::vector<T1>>(
71
                  num_rows_matrix,
72
                  std::vector<T1>(indices_to_sample.size())
73
              );
              // filling the canvas
76
              auto destination_index {0};
77
78
              std::for_each(
                  indices_to_sample.begin(), indices_to_sample.end(),
                  [&](const auto& col){
80
                  for(auto row = 0; row < num_rows_matrix; ++row){</pre>
                      if (col <= input_matrix[0].size()){</pre>
                         canvas[row][destination_index] = input_matrix[row][col];
                      }
84
                  }
85
                  ++destination_index;
```

```
});
87
            }
88
            else if(dim == 1){
89
                // initializing canvas
90
                canvas = std::vector<std::vector<T1>>(
91
                    indices_to_sample.size(),
                    std::vector<T1>(num_cols_matrix)
                );
                // filling the canvas
96
                #pragma omp parallel for
97
                for(auto row = 0; row < canvas.size(); ++row){</pre>
98
                           destination_col {0};
99
                    std::for_each(indices_to_sample.begin(), indices_to_sample.end(),
100
                                  [&row,
                                   &input_matrix,
                                   &destination_col,
                                   &canvas](const auto& source_col){
104
                                        canvas[row][destination_col++] =
105
                                            input_matrix[row] [source_col];
                                 });
106
                }
107
            }
108
            else {
                std::cerr << "svr_index | this dim is not implemented \n";</pre>
111
112
            // moving it back
113
            return std::move(canvas);
114
115
    }
116
```

B.15 Linspace

```
/*-----
  Dependencies
  #pragma once
  #include <vector>
  #include <complex>
8
  namespace svr {
     /*-----
10
11
     template <typename T>
     auto linspace(
       auto&
                    input,
14
       const
              auto
                    startvalue,
       const
              auto
                    endvalue,
17
       const
              auto
                    numpoints
     ) -> void
18
19
       auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);
       for(int i = 0; i<input.size(); ++i) {input[i] = startvalue + i*stepsize;}</pre>
21
     };
22
```

```
/*-----
23
25
     template <typename T>
26
27
     auto linspace(
        std::vector<std::complex<T>>& input,
         const auto
                                   startvalue,
        const
                auto
                                    endvalue.
         const
                auto
                                   numpoints
     ) -> void
33
         auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);
34
         for(int i = 0; i<input.size(); ++i) {</pre>
            input[i] = startvalue + static_cast<T>(i)*stepsize;
36
37
     };
38
40
41
     template <
        typename T,
         typename = std::enable_if_t<</pre>
43
            std::is_arithmetic_v<T> ||
            std::is_same_v<T, std::complex<float> > ||
45
            std::is_same_v<T, std::complex<double> >
48
     auto linspace(
49
       const T
                             startvalue,
50
        const T
                             endvalue,
51
52
         const std::size_t numpoints
     )
53
         std::vector<T> input(numpoints);
         auto stepsize = static_cast<T>(endvalue - startvalue)/static_cast<T>(numpoints-1);
56
         for(int i = 0; i<input.size(); ++i) {input[i] = startvalue +</pre>
            static_cast<T>(i)*stepsize;}
         return std::move(input);
58
     };
     /*----
60
      -----*/
     template <typename T, typename U>
62
     auto linspace(
63
        const T
                            startvalue,
64
         const U
                            endvalue,
         const std::size_t numpoints
66
     )
67
         std::vector<double> input(numpoints);
         auto stepsize = static_cast<double>(endvalue -
70
            startvalue)/static_cast<double>(numpoints-1);
         for(int i = 0; i<input.size(); ++i) {input[i] = startvalue + i*stepsize;}</pre>
71
72
         return std::move(input);
     };
73
  }
```

B.16 Max

```
#pragma once
  /*-----
3 maximum along dimension 1
  template <std::size_t axis, typename T>
       max(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis</pre>
      == 1, std::vector<std::vector<T>> >
     // setting up canvas
     auto canvas
         {std::vector<std::vector<T>>(input_matrix.size(),std::vector<T>(1))};
     // filling up the canvas
     for(auto row = 0; row < input_matrix.size(); ++row)</pre>
         canvas[row][0] = *(std::max_element(input_matrix[row].begin(),
13
            input_matrix[row].end()));
     // returning
15
     return std::move(canvas);
16
  }
17
```

B.17 Meshgrid

```
/*----
  Dependencies
  #pragma once
  #include <vector> // for std::vector
6 #include <utility> // for std::pair
  #include <complex> // for std::complex
10 /*-----
 mesh-grid when working with 1-values
11
  -----*/
  template <typename T>
13
  auto meshgrid(const std::vector<T>& x,
          const std::vector<T>& y)
17
     // creating and filling x-grid
18
     std::vector<std::vector<T>> xcanvas(y.size(), std::vector<T>(x.size(), 0));
     for(auto row = 0; row < y.size(); ++row)</pre>
       std::copy(x.begin(), x.end(), xcanvas[row].begin());
21
22
     // creating and filling y-grid
23
     std::vector<std::vector<T>> ycanvas(y.size(), std::vector<T>(x.size(), 0));
     for(auto col = 0; col < x.size(); ++col)</pre>
25
       for(auto row = 0; row < y.size(); ++row)</pre>
          ycanvas[row][col] = y[row];
     // returning
29
     return std::move(std::pair{xcanvas, ycanvas});
30
  /*-----
```

```
meshgrid when working with r-values
   template <typename T>
36
   auto meshgrid(std::vector<T>&& x,
37
                std::vector<T>&& y)
38
40
       // creating and filling x-grid
41
       std::vector<std::vector<T>> xcanvas(y.size(), std::vector<T>(x.size(), 0));
       for(auto row = 0; row < y.size(); ++row)</pre>
43
           std::copy(x.begin(), x.end(), xcanvas[row].begin());
44
45
       // creating and filling y-grid
46
       std::vector<std::vector<T>> ycanvas(y.size(), std::vector<T>(x.size(), 0));
47
       for(auto col = 0; col < x.size(); ++col)</pre>
48
           for(auto row = 0; row < y.size(); ++row)</pre>
49
               ycanvas[row][col] = y[row];
51
       // returning
52
       return std::move(std::pair{xcanvas, ycanvas});
53
  }
55
```

B.18 Minimum

```
#pragma once
  minimum along dimension 1
  template <std::size_t axis, typename T>
  auto min(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis ==</pre>
      1, std::vector<std::vector<T>> >
     // creating canvas
8
      auto canvas
         {std::vector<std::vector<T>>(input_matrix.size(),std::vector<T>(1))};
      // storing the values
11
      for(auto row = 0; row < input_matrix.size(); ++row)</pre>
         canvas[row][0] = *(std::min_element(input_matrix[row].begin(),
            input_matrix[row].end()));
14
      // returning the value
     return std::move(canvas);
16
  }
17
```

B.19 Norm

```
{
     return std::sqrt(
        std::inner_product(
9
           input_vector.begin(), input_vector.end(),
10
           input_vector.begin(),
11
           (T)0
     );
14
  }
15
  /*-----
  Calculating norm of a complex-vector
  -----*/
19 template <>
  auto norm(const std::vector<std::complex<double>>& input_vector)
20
  {
21
     return std::sqrt(
22
        std::inner_product(
           input_vector.begin(), input_vector.end(),
24
25
           input_vector.begin(),
           static_cast<double>(0),
           std::plus<double>(),
           [](const auto& argx,
28
             const auto& argy){
              return static_cast<double>(
                 (argx * std::conj(argy)).real()
              );
           }
33
        )
     );
35
36
37 }
  /*-----
  -----*/
  template <typename T>
40
  auto norm(const std::vector<std::vector<T>>& input_matrix,
41
           const std::size_t
                                         dim)
42
43
     // creating canvas
44
                    {std::vector<std::vector<T>>()};
          canvas
     auto
     const auto& num_rows_matrix {input_matrix.size()};
     const auto& num_cols_matrix {input_matrix[0].size()};
47
48
     // along dim 0
49
     if(dim == 0)
50
51
        // allocate canvas
52
        canvas = std::vector<std::vector<T>>(
           std::vector<T>(input_matrix[0].size())
55
        );
56
57
        // performing norm
        auto accumulate_vector
                             {std::vector<T>(input_matrix[0].size())};
59
        // going through each row
        for(auto
                row = 0; row < num_rows_matrix; ++row)</pre>
63
           std::transform(input_matrix[row].begin(), input_matrix[row].end(),
64
                      accumulate_vector.begin(),
```

```
accumulate_vector.begin(),
                              [](const auto& argx, auto& argy){
                                   return argx*argx + argy;
68
                              });
69
           }
70
            // calculating element-wise square root
            std::for_each(accumulate_vector.begin(), accumulate_vector.end(),
                         [](auto& argx){
                               argx = std::sqrt(argx);
                         });
76
77
            // moving to the canvas
78
            canvas[0] = std::move(accumulate_vector);
79
80
       else if (dim == 1)
81
            // allocating space in the canvas
83
                       = std::vector<std::vector<T>>(
84
            canvas
               input_matrix[0].size(),
               std::vector<T>(1, 0)
            );
87
88
            // going through each column
            for(auto row = 0; row < num_cols_matrix; ++row){</pre>
               canvas[row][0] = norm(input_matrix[row]);
91
92
93
       }
95
       else
96
            std::cerr << "norm(matrix, dim): dimension operation not defined \n";</pre>
97
98
99
        // returning
100
        return std::move(canvas);
   }
102
104
105
106
   Templates to create
107
       - matrix and norm-axis
108
           axis instantiated std::vector<T>
110
```

B.20 Division

```
// creating canvas
      auto canvas
                       {input_vector};
10
11
      // filling canvas
      std::transform(canvas.begin(), canvas.end(),
                  canvas.begin(),
                   [&input_scalar](const auto& argx){
                       return static_cast<double>(argx) /
                          static_cast<double>(input_scalar);
                  });
18
      // returning value
19
      return std::move(canvas);
20
21 }
 /*-----
22
  element-wise division with scalars
  template <typename T>
25
  auto operator/=(const std::vector<T>& input_vector,
26
27
                const
                                        input_scalar)
28
      // creating canvas
29
                      {input_vector};
      auto canvas
30
31
      // filling canvas
      std::transform(canvas.begin(), canvas.end(),
33
                  canvas.begin(),
34
                   [&input_scalar](const auto& argx){
35
                       return static_cast<double>(argx) /
                          static_cast<double>(input_scalar);
                  });
37
      // returning value
      return std::move(canvas);
40
  }
41
  element-wise with matrix
44
  template <
45
46
      typename T,
      typename = std::enable_if_t<</pre>
47
         std::is_floating_point_v<T>
48
49
 >
50
         operator/(const std::vector<std::vector<T>>& input_matrix,
51
                                                 scalar)
                 const T
52
53
      // fetching matrix-dimensions
      const auto& num_rows_matrix
                                   {input_matrix.size()};
55
      const auto& num_cols_matrix
                                   {input_matrix[0].size()};
56
57
      // creating canvas
      auto canvas
                      {std::vector<std::vector<T>>(
59
         num_rows_matrix,
60
         std::vector<T>(num_cols_matrix)
      )};
63
      // dividing with values
64
      for(auto row = 0; row < num_rows_matrix; ++row){</pre>
```

```
std::transform(input_matrix[row].begin(), input_matrix[row].end(),
                        canvas[row].begin(),
                        [&scalar](const auto& argx){
68
                            return argx/scalar;
69
                        });
70
       }
72
       // returning values
73
       return std::move(canvas);
74
75
   }
   template
76
                 numeratorComplexType,
       typename
77
       typename
                 denominatorType,
78
                  = std::enable_if_t<
79
       typename
           std::is_floating_point_v< numeratorComplexType> &&
80
           std::is_arithmetic_v<
                                   denominatorType>
81
82
83
          operator/(
84
   auto
       const std::vector<std::complex<numeratorComplexType>>>& input_matrix,
85
       const denominatorType
                                                                        input_scalar
86
   )
87
   {
88
       // fetching matrix-dimensions
89
       const auto& num_rows_matrix
                                       {input_matrix.size()};
       const auto& num_cols_matrix
                                      {input_matrix[0].size()};
91
92
       // creating canvas
93
            canvas
                         {std::vector<std::complex<numeratorComplexType>>>(
95
          num_rows_matrix,
           std::vector<std::complex<numeratorComplexType>>(num_cols_matrix)
96
       )};
97
98
       // dividing with values
99
                 row = 0; row < num_rows_matrix; ++row){</pre>
       for(auto
100
           std::transform(
              input_matrix[row].begin(), input_matrix[row].end(),
102
              canvas[row].begin(),
              [&input_scalar](const auto& argx){
104
105
                  return argx /
                     static_cast<std::complex<numeratorComplexType>>(input_scalar);
              });
106
       }
107
       // returning values
109
       return std::move(canvas);
111
   /*-----
   y = std::vector<std::complex<T>> / T
113
114
115
   template <
116
       typename
                 Τ,
       typename = std::enable_if_t<</pre>
117
          std::is_floating_point_v<T>
118
119
   >
120
          operator/(
121
       const std::vector<std::complex<T>>& input_vector,
       const
              Т
                                           input_scalar
```

```
)
124
    {
125
       // creating canvas
126
       auto canvas {std::vector<std::complex<T>>(input_vector.size())};
127
128
       // filling the canvas
129
       std::transform(
130
           input_vector.begin(), input_vector.end(),
131
           canvas.begin(),
133
           [&input_scalar](const auto& argx){
               return argx/static_cast<std::complex<T>>(input_scalar);
134
135
       );
136
137
       // returning
138
       return std::move(canvas);
139
   }
140
```

B.21 Addition

```
#pragma once
  y = vector + vector
  template <typename T>
6 std::vector<T> operator+(const std::vector<T>& a,
                      const std::vector<T>& b)
      // Identify which is bigger
9
      const auto& big = (a.size() > b.size()) ? a : b;
10
      const auto& small = (a.size() > b.size()) ? b : a;
     std::vector<T> result = big; // copy the bigger one
13
     // Add elements from the smaller one
     for (size_t i = 0; i < small.size(); ++i) {</pre>
         result[i] += small[i];
     return result;
20
21 }
23 -----*/
_{24} // y = vector + vector
25 template <typename T>
std::vector<T>& operator+=(std::vector<T>& a,
                        const std::vector<T>& b) {
28
      const auto& small = (a.size() < b.size()) ? a : b;</pre>
29
     const auto& big = (a.size() < b.size()) ? b : a;</pre>
31
     // If b is bigger, resize 'a' to match
     if (a.size() < b.size())</pre>
                                            {a.resize(b.size());}
     // Add elements
     for (size_t i = 0; i < small.size(); ++i) {a[i] += b[i];}</pre>
```

```
37
      // returning elements
      return a;
39
  }
40
  // -----
41
  // y = matrix + matrix
  template <typename T>
  std::vector<std::vector<T>> operator+(const std::vector<std::vector<T>>% a,
                                     const std::vector<std::vector<T>>& b)
46
      // fetching dimensions
47
      const auto& num_rows_A
                                   {a.size()};
48
      const auto& num_cols_A
                                   {a[0].size()};
49
      const auto& num_rows_B
                                   {b.size()};
50
                                   {b[0].size()};
      const auto& num_cols_B
51
      // choosing the three different metrics
      if (num_rows_A != num_rows_B && num_cols_A != num_cols_B){
54
          cout << format("a.dimensions = [{},{}], b.shape = [{},{}]\n",
55
                        num_rows_A, num_cols_A,
56
                        num_rows_B, num_cols_B);
          std::cerr << "dimensions don't match\n";</pre>
58
59
60
      // creating canvas
      auto
            canvas
                        {std::vector<std::vector<T>>(
62
          std::max(num_rows_A, num_rows_B),
63
          std::vector<T>(std::max(num_cols_A, num_cols_B), (T)0.00)
      )};
66
      // performing addition
67
      if (num_rows_A == num_rows_B && num_cols_A == num_cols_B){
          for(auto row = 0; row < num_rows_A; ++row){</pre>
              std::transform(a[row].begin(), a[row].end(),
70
                           b[row].begin(),
71
                           canvas[row].begin(),
                           std::plus<T>());
73
          }
74
      }
75
76
      else if(num_rows_A == num_rows_B){
77
          // if number of columsn are different, check if one of the cols are one
78
                       min_num_cols {std::min(num_cols_A, num_cols_B)};
          const auto
79
          if (min_num_cols != 1) {std::cerr<< "Operator+: unable to broadcast\n";}</pre>
          const auto max_num_cols {std::max(num_cols_A, num_cols_B)};
81
          // using references to tag em differently
          const auto& big_matrix
                                       {num_cols_A > num_cols_B ? a : b};
          const auto& small_matrix {num_cols_A < num_cols_B ? a : b};</pre>
85
86
          // Adding to canvas
          for(auto row = 0; row < canvas.size(); ++row){</pre>
              std::transform(big_matrix[row].begin(), big_matrix[row].end(),
                           canvas[row].begin(),
                           [&small_matrix,
                            &row](const auto& argx){
                                return argx + small_matrix[row][0];
93
                           });
94
          }
```

```
96
       else if(num_cols_A == num_cols_B){
98
          // check if the smallest column-number is one
99
          const auto min_num_rows {std::min(num_rows_A, num_rows_B)};
100
          if(min_num_rows != 1) {std::cerr << "Operator+ : unable to broadcast\n";}</pre>
                      max_num_rows {std::max(num_rows_A, num_rows_B)};
          const auto
102
103
          // using references to differentiate the two matrices
          const auto& big_matrix {num_rows_A > num_rows_B ? a : b};
105
          const auto& small_matrix {num_rows_A < num_rows_B ? a : b};</pre>
106
107
          // adding to canvas
108
          for(auto row = 0; row < canvas.size(); ++row){</pre>
109
              std::transform(big_matrix[row].begin(), big_matrix[row].end(),
                           small_matrix[0].begin(),
                           canvas[row].begin(),
                           [](const auto& argx, const auto& argy){
                           return argx + argy;
114
                           });
115
          }
116
117
       else {
118
          std::cerr << "operator+: yet to be implemented \n";</pre>
119
121
       // returning
122
       return std::move(canvas);
123
   }
124
   /*----
125
   y = vector + scalar
126
127
   template <typename T>
128
   auto operator+(const std::vector<T>& input_vector,
129
                const
                                         scalar)
130
131
       // creating canvas
       auto canvas
                        {input_vector};
134
       // adding scalar to the canvas
135
       std::transform(canvas.begin(), canvas.end(),
136
                    canvas.begin(),
137
                    [&scalar](auto& argx){return argx + scalar;});
138
139
       // returning canvas
140
       return std::move(canvas);
141
142
   /*-----
143
   y = scalar + vector
144
145
146
   template <typename T>
147
   auto operator+(const T
                                         scalar,
                const
                       std::vector<T>& input_vector)
148
149
       // creating canvas
150
            canvas
                        {input_vector};
151
       // adding scalar to the canvas
       std::transform(canvas.begin(), canvas.end(),
154
```

```
canvas.begin(),
[&scalar](auto& argx){return argx + scalar;});

// returning canvas
return std::move(canvas);

// Particular argx + scalar;});
```

B.22 Multiplication (Element-wise)

```
#pragma once
  y = scalar * vector
  template <typename T>
  auto
        operator*(
      const
              Τ
8
              std::vector<T>& input_vector
  )
9
  {
10
      // creating canvas
      auto canvas
                       {input_vector};
      // performing operation
      std::for_each(canvas.begin(), canvas.end(),
                 [&scalar](auto& argx){argx = argx * scalar;});
      // returning
16
      return std::move(canvas);
17
18 }
  /*-----
  y = scalar * vector
  template <
23
      typename T1,
      typename T2,
24
      typename = std::enable_if_t<</pre>
         !std::is_same_v<std::decay_t<T1>, std::vector<T2> > &&
         std::is_arithmetic_v<T1>
27
28
  >
29
  auto operator*(const T1
                                         scalar,
30
                       vector<T2>&
               const
                                        input_vector)
31
  {
32
      // fetching final-type
33
      using T3 = decltype(std::declval<T1>() * std::declval<T2>());
35
      // creating canvas
36
      auto canvas
                       {std::vector<T3>(input_vector.size())};
      // multiplying
39
      std::transform(
         input_vector.begin(), input_vector.end(),
         canvas.begin(),
          [&scalar](auto& argx){
             return static_cast<T3>(scalar) * static_cast<T3>(argx);
         });
46
      // returning
```

```
return std::move(canvas);
48
  }
  /*-----
50
  y = scalar * vecotor (subset init)
51
  -----*/
  template <
     typename T,
54
     typename = std::enable_if_t<</pre>
55
        std::is_floating_point_v<T>
57
58 >
59 auto operator*(
    const std::complex<T> scalar,
     const std::vector<T>& input_vector
61
62 )
  {
63
     // creating canvas
     auto canvas {std::vector<std::complex<T>>(
65
        input_vector.size()
66
     )};
     // copying to canvas
69
     std::transform(
70
        input_vector.begin(), input_vector.end(),
        canvas.begin(),
        [&scalar](const auto& argx){
73
           return scalar * static_cast<std::complex<T>>(argx);
74
     );
77
     // moving it back
78
     return std::move(canvas);
79
  }
80
  /*-----
81
82
  y = vector * scalar
  -----*/
  template <typename T>
84
  auto operator*(const std::vector<T>& input_vector,
85
               const
                                   scalar)
86
87
     // creating canvas
88
     auto canvas
                  {input_vector};
89
     // multiplying
90
     std::for_each(canvas.begin(), canvas.end(),
               [&scalar](auto& argx){
92
                argx = argx * scalar;
93
               });
     // returning
     return std::move(canvas);
96
  }
97
  /*-----
  y = vector * vector
                  -----*/
100
  template <typename T>
101
  auto operator*(const std::vector<T>& input_vector_A,
102
            const std::vector<T>& input_vector_B)
103
104
     // throwing error: size-desparity
105
```

```
if (input_vector_A.size() != input_vector_B.size()) {std::cerr << "operator*: size</pre>
106
           disparity \n";}
       // creating canvas
108
       auto
             canvas
                        {input_vector_A};
109
110
       // element-wise multiplying
111
       std::transform(input_vector_B.begin(), input_vector_B.end(),
112
                    canvas.begin(),
113
                    canvas.begin(),
114
                    [](const auto& argx, const auto& argy){
115
                        return argx * argy;
116
                    });
117
118
       // moving it back
       return std::move(canvas);
120
121
   y = vecotr * vector
123
   -----*/
124
   template <</pre>
125
       typename T1,
126
       typename T2,
127
       typename
                = std::enable_if_t<
128
          std::is_arithmetic_v<T1> &&
          std::is_arithmetic_v<T2>
130
131
132
   >
          operator*(const
                            std::vector<T1>& input_vector_A,
133
                   const
                            std::vector<T2>& input_vector_B)
134
   {
135
136
       // checking size disparity
137
       if (input_vector_A.size() != input_vector_B.size())
138
          std::cerr << "operator*: error, size-disparity \n";</pre>
139
140
       // figuring out resulting data type
141
       using T3 = decltype(std::declval<T1>() * std::declval<T2>());
142
143
       // creating canvas
             canvas
                        {std::vector<T3>(input_vector_A.size())};
145
146
       // performing multiplications
147
       std::transform(input_vector_A.begin(), input_vector_A.end(),
                    input_vector_B.begin(),
149
                    canvas.begin(),
150
                    [](const
151
                               auto&
                                          argx,
                       const
                               auto&
                                          argy){
                        return static_cast<T3>(argx) * static_cast<T3>(argy);
153
                    });
154
155
156
       // returning
       return std::move(canvas);
157
158
159
   /*-----
160
y = vector * complex_vector
162
   template <
```

```
Τ,
       typename
164
       typename = std::enable_if_t<</pre>
165
           std::is_floating_point_v<T>
166
167
168
   >
          operator*(
169
       const std::vector<T>&
                                            input_vector_A,
170
       const std::vector<std::complex<T>>& input_vector_B
171
172
   )
173
       // checking size issue
174
       if (input_vector_A.size() != input_vector_B.size())
175
           throw std::runtime_error(
176
              "FILE: svr_operator_star.hpp | FUNCTION: operator* | REPORT: error disparity
177
                  in the two input-vectors"
           );
178
       // creating canvas
180
       auto canvas
                         {std::vector<std::complex<T>>( input_vector_A.size() )};
181
182
       // filling up the canvas
183
       std::transform(
184
           input_vector_B.begin(), input_vector_B.end(),
185
           input_vector_A.begin(),
186
           canvas.begin(),
           [](const auto& argx, const auto& argy){
188
              return argx + static_cast<std::complex<T>>(argy);
189
190
           }
       );
191
192
       // moving it back
193
       return std::move(canvas);
194
195
   /*-----
196
   y = complex_vector * vector
197
   template <
199
                 Τ,
       typename
200
       typename = std::enable_if_t<</pre>
201
202
           std::is_floating_point_v<T>
203
204 >
          operator*(
205
   auto
       const std::vector<std::complex<T>>& input_vector_A,
       const std::vector<T>&
                                            input_vector_B
207
   )
208
   {
209
       // enforcing size
210
       if (input_vector_A.size() != input_vector_B.size())
211
           throw std::runtime_error(
212
               "FILE: svr_operator_star.hpp | FUNCTION: operator* overload"
213
214
           );
215
       // creating canvas
216
                        {std::vector<std::complex<T>>(input_vector_A.size())};
217
             canvas
218
       // filling values
219
       std::transform(
220
           input_vector_A.begin(), input_vector_A.end(),
221
```

```
input_vector_B.begin(),
222
          canvas.begin(),
223
           [](const
                    auto& argx, const auto& argy){
224
              return argx * static_cast<std::complex<T>>(argy);
226
       );
227
228
       // returning
229
       return std::move(canvas);
230
231
   }
   /*-----
232
   y = complex-vector * complex-vector
233
234
   template
235
       typename
236
                 = std::enable_if_t<
237
       typename
          std::is_floating_point_v<T>
239
240
   >
   auto
          operator*(
241
       const std::vector<std::complex<T>> input_vector_A,
242
              std::vector<std::complex<T>> input_vector_B
243
   )
244
   {
245
246
       // checking size
       if (input_vector_A.size() != input_vector_B.size())
247
          throw std::runtime_error(
248
              "FILE: svr_operator_star.hpp | FUNCTION: operator*(complex-vector,
249
                  complex-vector)"
          );
250
251
       // creating canvas
252
             canvas
                         {std::vector<std::complex<T>>(input_vector_A.size())};
253
254
       // filling canvas
255
       std::transform(
          input_vector_A.begin(),
                                   input_vector_A.end(),
257
          input_vector_B.begin(),
258
          canvas.begin(),
           [](const
                    auto& argx, const auto& argy){
              return argx * argy;
261
          }
262
       );
263
       // returning values
265
       return std::move(canvas);
266
267
   268
   template <typename T>
269
   auto operator*(const T
270
                                                 scalar.
271
                 const std::vector<std::vector<T>>& inputMatrix)
272
       std::vector<std::vector<T>> temp {inputMatrix};
273
       for(int i = 0; i<inputMatrix.size(); ++i){</pre>
274
          std::transform(inputMatrix[i].begin(),
                        inputMatrix[i].end(),
276
                        temp[i].begin(),
                        [&scalar](T x){return scalar * x;});
278
       }
279
```

```
return std::move(temp);
280
   }
281
   /*-----
282
   y = matrix * scalar
283
   -----*/
284
   template <typename T>
285
         operator*(const
                          std::vector<std::vector<T>>& input_matrix,
286
                                                     scalar)
287
                  const
288
289
      // fetching matrix dimensions
      const auto& num_rows_matrix
                                    {input_matrix.size()};
290
      const auto& num_cols_matrix
                                   {input_matrix[0].size()};
291
292
      // creating canvas
293
      auto
             canvas
                       {std::vector<std::vector<T>>(
294
          num_rows_matrix,
295
          std::vector<T>(num_cols_matrix)
      )};
297
298
      // storing the values
299
      for(auto row = 0; row < num_rows_matrix; ++row)</pre>
300
          std::transform(input_matrix[row].begin(), input_matrix[row].end(),
301
                      canvas[row].begin(),
302
                      [&scalar](const auto& argx){
303
                          return argx * scalar;
                      });
305
306
      // returning
307
      return std::move(canvas);
308
309
   /*-----
310
   y = matrix * matrix
311
   -----*/
312
   template <typename T>
313
   auto operator*(const std::vector<std::vector<T>>& A,
314
               const std::vector<std::vector<T>>& B) -> std::vector<std::vector<T>>
315
316
      // Case 1: element-wise multiplication
317
      if (A.size() == B.size() && A[0].size() == B[0].size()) {
318
319
          std::vector<std::vector<T>> C(A.size(), std::vector<T>(A[0].size()));
          for (std::size_t row = 0; row < A.size(); ++row) {</pre>
             std::transform(A[row].begin(), A[row].end(),
321
                         B[row].begin(),
322
                         C[row].begin(),
323
                          [](const auto& x, const auto& y){ return x * y; });
324
          }
325
          return C;
326
      }
      // Case 2: broadcast column vector
329
      else if (A.size() == B.size() && B[0].size() == 1) {
330
331
          std::vector<std::vector<T>> C(A.size(), std::vector<T>(A[0].size()));
          for (std::size_t row = 0; row < A.size(); ++row) {</pre>
332
             std::transform(A[row].begin(), A[row].end(),
333
                         C[row].begin(),
                          [\&] (const auto& x){ return x * B[row][0]; });
335
          }
336
          return C;
337
      }
```

```
339
       // case 3: when second matrix contains just one row
340
       // case 4: when first matrix is just one column
341
       // case 5: when second matrix is just one column
342
343
       // Otherwise, invalid
       else {
345
          throw std::runtime_error("operator* dimension mismatch");
346
   }
348
   /*-----
349
   y = scalar * matrix
350
   template <typename T1, typename T2>
352
   auto operator*(const T1
353
                 const std::vector<std::vector<T2>>& inputMatrix)
354
355
       std::vector<std::vector<T2>> temp {inputMatrix};
356
       for(int i = 0; i<inputMatrix.size(); ++i){</pre>
357
          std::transform(inputMatrix[i].begin(),
                        inputMatrix[i].end(),
359
                        temp[i].begin(),
360
                        [&scalar](T2 x){return static_cast<T2>(scalar) * x;});
361
       }
362
363
       return temp;
364
365
   matrix-multiplication
   -----*/
367
   template <typename T1, typename T2>
368
   auto matmul(const std::vector<std::vector<T1>>& matA,
369
              const std::vector<std::vector<T2>>& matB)
371
       // throwing error
373
       if (matA[0].size() != matB.size()) {std::cerr << "dimension-mismatch \n";}</pre>
375
       // getting result-type
       using ResultType = decltype(std::declval<T1>() * std::declval<T2>() + \
                                  std::declval<T1>() * std::declval<T2>() );
379
       // creating aliasses
380
       auto finalnumrows {matA.size()};
381
       auto finalnumcols {matB[0].size()};
383
       // creating placeholder
384
       auto rowcolproduct = [&](auto rowA, auto colB){
          ResultType temp {0};
          for(int i = 0; i < matA.size(); ++i) {temp +=</pre>
387
              static_cast<ResultType>(matA[rowA][i]) +
              static_cast<ResultType>(matB[i][colB]);}
          return temp;
       };
389
390
       // producing row-column combinations
       std::vector<std::vector<ResultType>> finaloutput(finalnumrows,
392
           std::vector<ResultType>(finalnumcols));
       for(int row = 0; row < finalnumrows; ++row){for(int col = 0; col < finalnumcols;</pre>
303
           ++col){finaloutput[row][col] = rowcolproduct(row, col);}}
```

```
394
       // returning
395
       return finaloutput;
396
397
   398
   y = matrix * vector
399
400
   template <
401
402
       typename T,
403
       typename
                = std::enable_if_t<
          std::is_arithmetic_v<T>
404
405
   >
406
   auto operator*(const std::vector<std::vector<T>>& input_matrix,
407
                 const
                        std::vector<T>&
                                                      input_vector)
408
409
       // fetching dimensions
410
                                       {input_matrix.size()};
       const auto& num_rows_matrix
411
       const
                                      {input_matrix[0].size()};
412
             auto& num_cols_matrix
413
       const auto& num_rows_vector
                                      {1};
       const auto& num_cols_vector
                                      {input_vector.size()};
414
       const auto& max_num_rows
                                       {num_rows_matrix > num_rows_vector ?\
415
                                       num_rows_matrix : num_rows_vector};
416
       const auto& max_num_cols
                                       {num_cols_matrix > num_cols_vector ?\
417
418
                                       num_cols_matrix : num_cols_vector};
419
       // creating canvas
420
                        {std::vector<std::vector<T>>(
421
       auto
            canvas
422
          max_num_rows,
          std::vector<T>(max_num_cols, 0)
423
       )};
424
425
       // multiplying column matrix with row matrix
426
       if (num_cols_matrix == 1 && num_rows_vector == 1){
427
428
          // writing to canvas
429
          for(auto row = 0; row < max_num_rows; ++row)</pre>
430
                       col = 0; col < max_num_cols; ++col)</pre>
              for(auto
431
                 canvas[row][col] = input_matrix[row][0] *
432
                                                               input_vector[col];
       /*-----
434
       Multiplying each row with the input-vector
435
436
       else if(
437
          num_cols_matrix == num_cols_vector &&
438
          num_rows_vector == 1
439
       )
440
441
          // writing to canvas
442
          for(auto row = 0; row < max_num_rows; ++row)</pre>
443
              std::transform(
                  input_matrix[row].begin(), input_matrix[row].end(),
                 input_vector.begin(),
446
                 canvas[row].begin(),
447
                  [](const auto& argx, const auto& argy){return argx * argy;}
              );
449
       }
450
       else
451
       {
```

```
std::cerr << "Operator*: [matrix, vector] | not implemented \n";</pre>
453
      }
454
455
      // returning
456
      return std::move(canvas);
457
459
   /*-----
460
   complex-matrix * vector
462
   -----*/
   template <</pre>
463
      typename T,
464
      typename = std::enable_if_t<</pre>
465
         std::is_floating_point_v<T>
466
467
   >
468
   auto
         operator*(
      const std::vector<std::complex<T>>>& input_matrix,
470
      const std::vector<T>&
                                                input_vector
471
472
   )
      // fetching dimensions
474
      const auto
                 num_rows_matrix {input_matrix.size()};
475
      const auto
                 num_cols_matrix {input_matrix[0].size()};
476
                                   {static_cast<std::size_t>(1)};
      const
            auto
                  num_rows_vector
      const auto
                 num_cols_vector {input_vector.size()};
478
479
      // throwing an error
480
      if (num_cols_matrix != num_cols_vector)
481
         throw std::runtime_error(
482
             "FILE: svr_operator_star.hpp | FUNCTION: operator*(complex-matrix, vector)"
483
         );
      // creating canvas
486
           canvas
                      {std::vector<std::vector<std::complex<T>>>(
      auto
487
         num_rows_matrix,
         std::vector<std::complex<T>>(num_cols_matrix)
489
      )};
490
491
      // performing operations
      for(auto row = 0; row < num_rows_matrix; ++row)</pre>
493
         std::transform(
494
            input_matrix[row].begin(), input_matrix[row].end(),
495
            input_vector.begin(),
             canvas[row].begin(),
497
             [](const auto& argx, const auto& argy){
498
                return argx * static_cast<std::complex<T>>(argy);
            }
         );
501
502
      // returning the final output
503
      return std::move(canvas);
505
  }
506
   /*-----
   martix * complex-vector
508
                -----*/
509
   template
510
      typename
                Τ,
511
```

```
typename = std::enable_if_t<</pre>
512
           std::is_floating_point_v<T>
513
514
   >
515
          operator*(
   auto
       const std::vector<std::vector<T>>& input_matrix,
517
       const std::vector<std::complex<T>>& input_vector
518
   )
519
   {
520
521
       // fetching dimensions
       const auto
                    num_rows_matrix
                                      {input_matrix.size()};
                                      {input_matrix[0].size()};
       const auto
                    num_cols_matrix
523
                                        {static_cast<std::size_t>(1)};
       const auto
                    num_rows_vector
524
                    num_cols_vector
                                        {input_vector.size()};
525
       const auto
526
       // fetching dimension mismatch
527
       if (num_cols_matrix != num_cols_vector)
           throw std::runtime_error(
529
               "FILE: svr_operator_star.hpp | FUNCTION: operator*(input-matrix,
530
                  complex-vector)"
           );
531
532
       // creating canvas
533
                         {std::vector<std::complex<T>>>(
       auto
             canvas
534
          num_rows_matrix,
           std::vector<std::complex<T>>(
536
              num_cols_matrix
537
           )
538
       )};
539
540
       // filling the values
541
       for(auto row = 0; row < num_rows_matrix; ++row)</pre>
           std::transform(
543
              input_matrix[row].begin(), input_matrix[row].end(),
544
              input_vector.begin(),
545
              canvas[row].begin(),
                        auto& argx, const auto& argy){
              [](const
547
                  return static_cast<std::complex<T>>(argx) * argy;
548
              }
549
          );
551
       // returning final-output
       return std::move(canvas);
553
   }
554
   /*-----
555
   scalar operators
556
    _____
   auto operator*(const std::complex<double> complexscalar,
558
                 const double
                                           doublescalar){
       return complexscalar * static_cast<std::complex<double>>(doublescalar);
560
561
   }
562
   auto operator*(const double
                                           doublescalar,
                 const std::complex<double> complexscalar){
563
       return complexscalar * static_cast<std::complex<double>>(doublescalar);
564
   }
565
   auto operator*(const std::complex<double> complexscalar,
566
                 const int
                                           scalar){
567
       return complexscalar * static_cast<std::complex<double>>(scalar);
568
   }
569
```

```
auto operator*(const int scalar,
const std::complex<double> complexscalar){
return complexscalar * static_cast<std::complex<double>>(scalar);
}
```

B.23 Subtraction

```
#pragma once
  /*-----
  y = vector - scalar
  -----*/
  template <typename T>
  auto operator-(const std::vector<T>& a,
            const T
     std::vector<T> temp(a.size());
     std::transform(a.begin(),
               a.end(),
10
               temp.begin(),
11
                [scalar](T x){return (x - scalar);});
     return std::move(temp);
13
14 }
  /*-----
  y = vector - vector
17
  template <typename T>
18
  auto operator-(const std::vector<T>& input_vector_A,
19
            const std::vector<T>& input_vector_B)
21 {
     // throwing error
22
     if (input_vector_A.size() != input_vector_B.size())
23
        std::cerr << "operator-(vector, vector): size disparity\n";</pre>
     // creating canvas
26
     const auto& num_cols {input_vector_A.size()};
2.7
     auto
               canvas {std::vector<T>()};
     // peforming operations
30
     std::transform(input_vector_A.begin(), input_vector_A.begin(),
               input_vector_B.begin(),
               canvas.begin(),
33
                [](const auto& argx, const auto& argy){
34
                   return argx - argy;
                });
36
37
     // return
38
     return std::move(canvas);
39
40
  /*----
41
  y = matrix - matrix
  ----*/
  template <typename T>
  auto operator-(const std::vector<std::vector<T>>& input_matrix_A,
45
            const std::vector<std::vector<T>>& input_matrix_B)
46
47
     // fetching dimensions
48
     const auto& num_rows_A {input_matrix_A.size()};
49
```

```
auto& num_cols_A {input_matrix_A[0].size()};
       const
              auto& num_rows_B {input_matrix_B.size()};
51
              auto& num_cols_B {input_matrix_B[0].size()};
52
       // creating canvas
              canvas
                          {std::vector<std::vector<T>>()};
       // if both matrices are of equal dimensions
       if (num_rows_A == num_rows_B && num_cols_A == num_cols_B)
          // copying one to the canvas
60
          canvas = input_matrix_A;
61
          // subtracting
          for(auto row = 0; row < num_rows_B; ++row)</pre>
              std::transform(canvas[row].begin(), canvas[row].end(),
                             input_matrix_B[row].begin(),
                             canvas[row].begin(),
                             [](auto& argx, const auto& argy){
68
                                 return argx - argy;
71
      // column broadcasting (case 1)
72
       else if(num_rows_A == num_rows_B && num_cols_B == 1)
          // copying canvas
75
          canvas = input_matrix_A;
76
          // substracting
          for(auto row = 0; row < num_rows_A; ++row){</pre>
              std::transform(canvas[row].begin(), canvas[row].end(),
                             canvas[row].begin(),
                             [&input_matrix_B,
                             &row] (auto& argx) {
83
                                 return argx - input_matrix_B[row][0];
                             });
          }
86
      else{
          std::cerr << "operator-: not implemented for this case \n";</pre>
90
91
       // returning
92
93
      return std::move(canvas);
  }
94
```

B.24 Printing Containers

```
template<typename T>
void fpv(vector<T> input){
     for(auto x: input) cout << x << ",";</pre>
     cout << endl;</pre>
13
 /*----
  template<typename T>
  void fPrintMatrix(const std::vector<std::vector<T>> input_matrix){
     for(const auto& row: input_matrix)
19
       cout << format("{}\n", row);</pre>
20
21 }
23 -----*/
  template <typename T>
  void fPrintMatrix(const string&
                                      input_string,
              const std::vector<std::vector<T>> input_matrix){
26
     cout << format("{} = \n", input_string);</pre>
27
     for(const auto& row: input_matrix)
       cout << format("{}\n", row);</pre>
30 }
31 /*----
  template<typename T, typename T1>
  void fPrintHashmap(unordered_map<T, T1> input){
34
     for(auto x: input){
35
       cout << format("[{},{}] | ", x.first, x.second);</pre>
38
     cout <<endl;</pre>
39 }
  /*-----
  -----*/
  void fPrintBinaryTree(TreeNode* root){
42
     // sending it back
43
     if (root == nullptr) return;
     // printing
     PRINTLINE
     cout << "root->val = " << root->val << endl;</pre>
     // calling the children
50
     fPrintBinaryTree(root->left);
51
     fPrintBinaryTree(root->right);
    // returning
54
     return;
55
57
void fPrintLinkedList(ListNode* root){
     if (root == nullptr) return;
61
     cout << root->val << " -> ";
62
     fPrintLinkedList(root->next);
65 }
```

```
template<typename T>
  void fPrintContainer(T input){
    for(auto x: input) cout << x << ", ";</pre>
70
    cout << endl;</pre>
71
    return;
73 }
 /*-----
 -----*/
  template <typename T>
  auto size(std::vector<std::vector<T>> inputMatrix){
   cout << format("[{}, {}]\n",</pre>
78
              inputMatrix.size(),
79
              inputMatrix[0].size());
80
81 }
82 /*-----
  template <typename T>
  auto size(const std::string&
                                    inputstring,
85
       const std::vector<std::vector<T>>& inputMatrix){
86
     cout << format("{} = [{}, {}]\n",
87
              inputstring,
              inputMatrix.size(),
89
              inputMatrix[0].size());
90
91 }
```

B.25 Random Number Generation

```
#pragma once
  /*----
  template <typename T>
  auto rand(const T min,
         const
                T max) {
    static std::random_device rd; // Seed
     static std::mt19937 gen(rd()); // Mersenne Twister generator
     std::uniform_real_distribution<> dist(min, max);
     return dist(gen);
10
11 }
  /*-----
13
14 template <typename T>
15 auto rand(const T
                            min,
 const T
                           max,
        const std::size_t numelements)
17
18 €
     static std::random_device rd; // Seed
19
     static std::mt19937 gen(rd()); // Mersenne Twister generator
20
     std::uniform_real_distribution<> dist(min, max);
21
22
     // building the fianloutput
     vector<T> finaloutput(numelements);
     for(int i = 0; i<finaloutput.size(); ++i) {finaloutput[i] =</pre>
        static_cast<T>(dist(gen));}
     return finaloutput;
27
28 }
```

```
template <typename T>
31
   auto rand(const T
                                  argmin,
32
         const T
33
                                 argmax,
           const std::vector<int> dimensions)
  {
35
36
      // throwing an error if dimension is greater than two
      if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}</pre>
38
39
      // creating random engine
40
      static std::random_device rd; // Seed
41
      static std::mt19937 gen(rd()); // Mersenne Twister generator
42
      std::uniform_real_distribution<> dist(argmin, argmax);
43
      // building the finaloutput
      vector<vector<T>> finaloutput;
46
      for(int i = 0; i<dimensions[0]; ++i){</pre>
47
         vector<T> temp;
          for(int j = 0; j<dimensions[1]; ++j) {temp.push_back(dist(gen));}</pre>
          // cout << format("\t\t temp = {}\n", temp);
51
         finaloutput.push_back(temp);
54
      // returning the finaloutput
55
      return finaloutput;
56
57
58 }
  auto rand(const std::vector<int> dimensions)
61
62
      using ReturnType = double;
63
      // throwing an error if dimension is greater than two
65
      if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}</pre>
      // creating random engine
      static std::random_device rd; // Seed
69
      static std::mt19937 gen(rd()); // Mersenne Twister generator
70
      std::uniform_real_distribution<> dist(0.00, 1.00);
71
      // building the finaloutput
73
      vector<vector<ReturnType>> finaloutput;
74
      for(int i = 0; i<dimensions[0]; ++i){</pre>
          vector<ReturnType> temp;
          for(int j = 0; j<dimensions[1]; ++j) {temp.push_back(dist(gen));}</pre>
77
          finaloutput.push_back(std::move(temp));
78
      }
79
      // returning the finaloutput
81
      return std::move(finaloutput);
82
83
84 }
85
  template <typename T>
```

```
auto rand_complex_double(const T
                                                       argmin,
                                                       argmax,
                            const std::vector<int>& dimensions)
90
    {
91
92
        // throwing an error if dimension is greater than two
        if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}</pre>
        // creating random engine
        static std::random_device rd; // Seed
        static std::mt19937 gen(rd()); // Mersenne Twister generator
98
        std::uniform_real_distribution<> dist(argmin, argmax);
99
100
        // building the finaloutput
101
        vector<vector<complex<double>>> finaloutput;
        for(int i = 0; i < dimensions[0]; ++i){</pre>
           vector<complex<double>> temp;
           for(int j = 0; j<dimensions[1]; ++j)</pre>
105
                {temp.push_back(static_cast < double > (dist(gen)));}
           finaloutput.push_back(std::move(temp));
106
        }
107
108
        // returning the finaloutput
109
       return finaloutput;
110
    }
```

B.26 Reshape

```
#pragma once
  /*-----
  reshaping a matrix into another matrix
  template <std::size_t M, std::size_t N, typename T>
  auto reshape(const std::vector<std::vector<T>>& input_matrix){
      // verifying size stuff
      if (M*N != input_matrix.size() * input_matrix[0].size())
10
         std::cerr << "Dimensions are quite different\n";</pre>
      // creating canvas
                       {std::vector<std::vector<T>>(
      auto
            canvas
         M, std::vector<T>(N, (T)0)
16
      // writing to canvas
      size_t tid
                            {0};
      size_t
              target_row
                            {0};
20
              target_col
                            {0};
2.1
      size_t
      for(auto row = 0; row<input_matrix.size(); ++row){</pre>
         for(auto col = 0; col < input_matrix[0].size(); ++col){</pre>
             tid
                          = row * input_matrix[0].size() + col;
                          = tid/N;
             target_row
                          = tid%N;
             target_col
             canvas[target_row][target_col] = input_matrix[row][col];
         }
28
```

```
}
29
      // moving it back
31
      return std::move(canvas);
32
33 }
  /*-----
 reshaping a matrix into a vector
35
  template<std::size_t M, typename T>
  auto reshape(const std::vector<std::vector<T>>& input_matrix){
38
39
      // checking element-count validity
40
      if (M != input_matrix.size() * input_matrix[0].size())
41
         std::cerr << "Number of elements differ\n";</pre>
42
43
      // creating canvas
      auto canvas {std::vector<T>(M, 0)};
46
      // filling canvas
47
      for(auto row = 0; row < input_matrix.size(); ++row)</pre>
         for(auto col = 0; col < input_matrix[0].size(); ++col)</pre>
             canvas[row * input_matrix.size() + col] = input_matrix[row][col];
50
51
      // moving it back
53
      return std::move(canvas);
54 }
56 Matrix to matrix
  -----*/
58
  template<typename T>
  auto reshape(const std::vector<std::vector<T>>& input_matrix,
              const std::size_t
                                          Μ,
                                           N){
              const std::size_t
61
62
      // checking element-count validity
63
      if ( M * N != input_matrix.size() * input_matrix[0].size())
         std::cerr << "Number of elements differ\n";</pre>
      // creating canvas
      auto canvas {std::vector<std::vector<T>>(
69
         M, std::vector<T>(N, (T)0)
      )};
70
71
      // writing to canvas
      size_t tid
                            {0};
73
      size_t target_row
                            {0};
74
      size_t target_col
                           {0};
75
      for(auto row = 0; row<input_matrix.size(); ++row){</pre>
         for(auto col = 0; col < input_matrix[0].size(); ++col){</pre>
77
                          = row * input_matrix[0].size() + col;
             tid
78
                         = tid/N;
             target_row
             target_col
                          = tid%N;
             canvas[target_row][target_col] = input_matrix[row][col];
81
         }
      }
      // moving it back
85
      return std::move(canvas);
86
  }
```

```
/*----
   converting a matrix into a vector
90
   template<typename T>
91
   auto reshape(const std::vector<std::vector<T>>& input_matrix,
              const size_t
      // checking element-count validity
      if (M != input_matrix.size() * input_matrix[0].size())
          std::cerr << "Number of elements differ\n";</pre>
98
      // creating canvas
99
      auto canvas {std::vector<T>(M, 0)};
100
101
      // filling canvas
       for(auto row = 0; row < input_matrix.size(); ++row)</pre>
          for(auto col = 0; col < input_matrix[0].size(); ++col)</pre>
             canvas[row * input_matrix.size() + col] = input_matrix[row][col];
105
106
       // moving it back
      return std::move(canvas);
   }
109
```

B.27 Summing with containers

```
#pragma once
  -----*/
  template <std::size_t axis, typename T>
  auto sum(const std::vector<T>& input_vector) -> std::enable_if_t<axis == 0,</pre>
     std::vector<T>>
     // returning the input as is
     return input_vector;
9
  }
  /*-----
  template <std::size_t axis, typename T>
  auto sum(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis == 0,</pre>
     std::vector<T>>
  {
14
     // creating canvas
     auto canvas {std::vector<T>(input_matrix[0].size(), 0)};
17
     // filling up the canvas
     for(auto row = 0; row < input_matrix.size(); ++row)</pre>
        std::transform(input_matrix[row].begin(), input_matrix[row].end(),
                  canvas.begin(),
                  canvas.begin(),
                  [](auto& argx, auto& argy){return argx + argy;});
     // returning
     return std::move(canvas);
```

```
-----*/
  template <std::size_t axis, typename T>
31
  auto sum(const std::vector<std::vector<T>>& input_matrix) -> std::enable_if_t<axis == 1,</pre>
      std::vector<std::vector<T>>>
33
      // creating canvas
      auto canvas {std::vector<std::vector<T>>(input_matrix.size(),
35
                                         std::vector<T>(1, 0.00))};
36
      // filling up the canvas
38
      for(auto row = 0; row < input_matrix.size(); ++row)</pre>
39
         canvas[row][0] = std::accumulate(input_matrix[row].begin(),
40
                                      input_matrix[row].end(),
41
                                      static_cast<T>(0));
42
43
     // returning
      return std::move(canvas);
46
47
  /*----
48
  -----*/
50 template <std::size_t axis, typename T>
  auto sum(const std::vector<T>& input_vector_A,
51
         const std::vector<T>& input_vector_B) -> std::enable_if_t<axis == 0,</pre>
             std::vector<T> >
53
  {
     // setup
54
     const auto& num_cols_A {input_vector_A.size()};
55
      const auto& num_cols_B {input_vector_B.size()};
57
      // throwing errors
58
      if (num_cols_A != num_cols_B) {std::cerr << "sum: size disparity\n";}</pre>
     // creating canvas
61
     auto canvas {input_vector_A};
      // summing up
      std::transform(input_vector_B.begin(), input_vector_B.end(),
                 canvas.begin(),
                 canvas.begin(),
                 std::plus<T>());
68
69
      // returning
70
      return std::move(canvas);
71
 }
72
```

B.28 Tangent

```
// throw error
         if (input_vector_A.size() != input_vector_B.size())
11
            std::cerr << "atan2: size disparity\n";</pre>
         // create canvas
                          {std::vector<T>(input_vector_A.size(), 0)};
         auto canvas
         // performing element-wise atan2 calculation
         std::transform(input_vector_A.begin(), input_vector_A.end(),
                     input_vector_B.begin(),
19
                     canvas.begin(),
                      [](const auto& arg_a,
                        const auto& arg_b){
                          return std::atan2(arg_a, arg_b);
                     });
26
         // moving things back
         return std::move(canvas);
      /*----
30
      y = tan-inverse(a/b)
31
      template <typename T>
      auto atan2(T scalar_A,
34
                   scalar B)
35
36
         return std::atan2(scalar_A, scalar_B);
38
  }
39
```

B.29 Tiling Operations

```
#pragma once
  namespace svr {
     /*-----
     tiling a vector
     template <typename T>
     auto tile(const std::vector<T>&
                                       input_vector,
             const std::vector<size_t>& mul_dimensions){
        // creating canvas
         const std::size_t& num_rows {1 * mul_dimensions[0]};
         const std::size_t& num_cols {input_vector.size() * mul_dimensions[1]};
         auto
               canvas {std::vector<std::vector<T>>(
           num_rows,
            std::vector<T>(num_cols, 0)
        )};
        // writing
        std::size_t
                   source_row;
        std::size_t source_col;
21
        for(std::size_t row = 0; row < num_rows; ++row){</pre>
```

```
for(std::size_t col = 0; col < num_cols; ++col){</pre>
                source_row = row % 1;
                source_col = col % input_vector.size();
25
                canvas[row][col] = input_vector[source_col];
26
             }
         }
         // returning
30
         return std::move(canvas);
      /*----
33
      tiling a matrix
34
35
      template <typename T>
36
      auto tile(const
                       std::vector<std::vector<T>>& input_matrix,
37
               const
                       std::vector<size_t>&
                                                   mul_dimensions){
38
          // creating canvas
40
          const std::size_t& num_rows
                                        {input_matrix.size() * mul_dimensions[0]};
41
          const std::size_t& num_cols {input_matrix[0].size() * mul_dimensions[1]};
          auto canvas {std::vector<std::vector<T>>(
             num_rows,
             std::vector<T>(num_cols, 0)
45
         )};
         // writing
48
         std::size_t
                      source_row;
49
          std::size_t source_col;
51
         for(std::size_t row = 0; row < num_rows; ++row){</pre>
52
             for(std::size_t col = 0; col < num_cols; ++col){</pre>
                source_row = row % input_matrix.size();
                source_col = col % input_matrix[0].size();
                canvas[row][col] = input_matrix[source_row][source_col];
56
             }
57
         }
         // returning
60
         return std::move(canvas);
61
      }
62
63
  }
```

B.30 Transpose

B.31 Masking

```
#pragma once
  namespace svr {
      /*-----
      y = input_vector[mask == 1]
      template <typename T,
               typename = std::enable_if_t< std::is_arithmetic_v<T>
                                        std::is_same_v<T, std::complex<double>> ||
                                        std::is_same_v<T, std::complex<float>>
10
                                       input_vector,
      auto mask(const
                       std::vector<T>&
                       std::vector<bool>& mask_vector)
14
         // checking dimensionality issues
         if (input_vector.size() != mask_vector.size())
             std::cerr << "mask(vector, mask): incompatible size \n";</pre>
         // creating canas
         auto num_trues {std::count(mask_vector.begin(),
                                     mask_vector.end(),
21
                                     true)}:
                           {std::vector<T>(num_trues)};
         auto
                canvas
         // copying values
         auto destination_index {0};
         for(auto i = 0; i <input_vector.size(); ++i)</pre>
             if (mask_vector[i] == true)
                canvas[destination_index++] = input_vector[i];
29
30
         // returning output
31
         return std::move(canvas);
32
33
34
35
      template <typename T>
      auto mask(const std::vector<std::vector<T>>&
                                                      input_matrix,
37
                     std::vector<bool>
               const
                                                      mask_vector)
38
         // fetching dimensions
         const auto& num_rows_matrix {input_matrix.size()};
         const auto& num_cols_matrix {input_matrix[0].size()};
         const auto& num_cols_vector
                                             {mask_vector.size()};
         // error-checking
45
```

```
if (num_cols_matrix != num_cols_vector)
             std::cerr << "mask(matrix, bool-vector): size disparity";</pre>
48
          // building canvas
49
                 num_trues {std::count(mask_vector.begin(),
                                      mask_vector.end(),
                                      true)};
                            {std::vector<std::vector<T>>(
          auto
                 canvas
             num_rows_matrix,
             std::vector<T>(num_cols_vector, 0)
          )};
56
57
          // writing values
          #pragma omp parallel for
          for(auto row = 0; row < num_rows_matrix; ++row){</pre>
                    destination_index {0};
             for(auto col = 0; col < num_cols_vector; ++col)</pre>
                 if(mask_vector[col] == true)
63
                     canvas[row] [destination_index++] = input_matrix[row][col];
          }
          // returning
          return std::move(canvas);
      /*-----
      Fetch Indices corresponding to mask true's
71
      auto mask_indices(const std::vector<bool>& mask_vector)
          // creating canvas
75
          auto
                 num_trues {std::count(mask_vector.begin(), mask_vector.end(),
                                      true)};
          auto
                 canvas
                            {std::vector<std::size_t>(num_trues)};
          // building canvas
                destination_index {0};
          for(auto i = 0; i < mask_vector.size(); ++i)</pre>
             if (mask_vector[i] == true)
                 canvas[destination_index++] = i;
          // returning
86
          return std::move(canvas);
87
      }
88
  }
```

B.32 Resetting Containers

```
// Recursively reset the remaining vectors
if constexpr (sizeof...(rest_vectors) > 0) {
    reset(rest_vectors...);
}

}
```

B.33 Element-wise squaring

```
#pragma once
  namespace svr {
     /*-----
     Element-wise squaring vector
                         -----*/
     template
             <typename T,
              typename = std::enable_if_t<std::is_arithmetic_v<T>>
     auto
          square(const std::vector<T>& input_vector)
        // creating canvas
        auto
             canvas
                     {std::vector<T>(input_vector.size()));
        // peforming calculations
        std::transform(input_vector.begin(), input_vector.end(),
                  canvas.begin(),
                   [](const auto& argx){
                        return argx * argx;
                   });
19
        // moving it back
        return std::move(canvas);
23
     /*----
24
     Element-wise squaring vector (in-place)
26
     template
             <typename T,
             typename = std::enable_if_t<std::is_arithmetic_v<T>>
          square_inplace(std::vector<T>& input_vector)
     void
30
31
        // performing operations
        std::transform(input_vector.begin(), input_vector.end(),
                  input_vector.begin(),
                  [](auto& argx){
35
                      return argx * argx;
                  });
38
     /*-----
39
     ELement-wise squaring a matrix
41
     -----*/
     template <typename T>
42
          square(const std::vector<std::vector<T>>& input_matrix)
     auto
        // fetching dimensions
45
        const auto& num_rows
                          {input_matrix.size()};
46
```

```
const auto& num_cols {input_matrix[0].size()};
47
48
         // creating canvas
49
              canvas
                     {std::vector<std::vector<T>>(
50
           num_rows,
            std::vector<T>(num_cols, 0)
        )};
         // going through each row
         #pragma omp parallel for
56
         for(auto row = 0; row < num_rows; ++row)</pre>
57
            std::transform(input_matrix[row].begin(), input_matrix[row].end(),
                       canvas[row].begin(),
                        [](const auto& argx){
60
                           return argx * argx;
61
                       });
         // returning
64
        return std::move(canvas);
65
66
     /*-----
     Squaring for scalars
68
                     -----*/
69
     template <typename T>
70
71
     auto square(const T& scalar) {return scalar * scalar;}
72
  }
```

B.34 Flooring

```
namespace svr {
    element-wise flooring of a vector-contents
    -----*/
    template <typename T>
                  std::vector<T>& input_vector)
    auto floor(const
      // creating canvas
           canvas
                  {std::vector<T>(input_vector.size())};
      auto
      // filling the canvas
      std::transform(input_vector.begin(), input_vector.end(),
                canvas.begin(),
                [](const auto& argx){
                  return std::floor(argx);
                }):
      // returning
      return std::move(canvas);
19
20
    /*-----
21
    element-wise flooring of a vector-contents (in-place)
    -----*/
23
    template <typename T>
24
        floor_inplace(std::vector<T>& input_vector)
    auto
26
      // rewriting the contents
```

```
std::transform(input_vector.begin(), input_vector.end(),
                     input_vector.begin(),
                     [](auto& argx){
30
                         return std::floor(argx);
31
                     });
32
      /*-----
34
      element-wise flooring of matrix-contents
35
      -----*/
      template <typename T>
      auto
           floor(const std::vector<std::vector<T>>& input_matrix)
38
39
         // fetching dimensions
40
         const auto& num_rows_matrix
                                      {input_matrix.size()};
41
         const auto& num_cols_matrix {input_matrix[0].size()};
42
         // creating canvas
         auto canvas {std::vector<std::vector<T>>(
45
46
            num_rows_matrix,
            std::vector<T>(num_cols_matrix)
         )};
         // writing contents
50
         for(auto row = 0; row < num_rows_matrix; ++row)</pre>
            std::transform(input_matrix[row].begin(), input_matrix[row].end(),
                        canvas[row].begin(),
                        [](const auto& argx){
54
                            return std::floor(argx);
                        });
57
         // returning contents
58
         return std::move(canvas);
61
      /*-----
62
      element-wise flooring of matrix-contents (in-place)
      template <typename T>
65
      auto
           floor_inplace(std::vector<std::vector<T>>& input_matrix)
         // performing operations
68
         for(auto row = 0; row < input_matrix.size(); ++row)</pre>
69
            std::transform(input_matrix[row].begin(), input_matrix[row].end(),
70
                        input_matrix[row].begin(),
71
                        [](auto& argx){
72
                            return std::floor(argx);
73
                        });
74
      }
75
  }
76
```

B.35 Squeeze

```
namespace svr {
template <typename T>
auto squeeze(const std::vector<std::vector<T>>& input_matrix)
{
```

```
// fetching dimensions
          const auto& num_rows_matrix
                                            {input_matrix.size()};
                                           {input_matrix[0].size()};
          const auto& num_cols_matrix
          // check if any dimension is 1
          if (num_rows_matrix == 0 || num_cols_matrix == 0)
              std::cerr << "at least one dimension should be 1";</pre>
                  final_length {std::max(num_rows_matrix, num_cols_matrix)};
          auto
          // creating canvas
          auto canvas
                                {std::vector<T>(final_length)};
          // building canvas
18
          if (num_rows_matrix == 1)
19
              // filling canvas
              std::copy(input_matrix[0].begin(), input_matrix[0].end(),
22
                       canvas.begin());
23
          }
          else if(num_cols_matrix == 1)
              // filling canvas
              std::transform(input_matrix.begin(), input_matrix.end(),
                            canvas.begin(),
                            [](const auto& argx){
30
                                return argx[0];
31
                            });
          }
34
          // returning
          return std::move(canvas);
37
   }
38
```

B.36 Tensor Initializations

```
namespace svr {
     /*-----
     template <typename T>
     auto zeros(const std::array<std::size_t, 2> input_dimensions)
        // create canvas
        auto canvas
                      {std::vector<std::vector<T>>(
           input_dimensions[0],
           std::vector<T>(input_dimensions[1], 0)
        )};
11
        // returning
        return std::move(canvas);
     }
15
  }
```

B.37 Real part

```
#pragma once
  namespace svr {
     /*-----
     For type-deductions
     template <typename T>
     struct real_result_type;
     template <> struct real_result_type<std::complex<double>>{
        using type = double;
11
     template <> struct real_result_type<std::complex<float>>{
        using type = float;
     template <> struct real_result_type<double> {
16
        using type = double;
17
18
     template <> struct real_result_type<float>{
19
        using type = float;
     };
21
     template
             <typename T>
     using real_result_t = typename real_result_type<T>::type;
24
25
     /*-----
26
     Element-wise real() of a vector
     -----*/
     template <typename T>
29
          real(const std::vector<T>& input_vector)
     auto
30
        // figure out base-type
        using TCanvas = real_result_t<T>;
33
        // creating canvas
                       {std::vector<TCanvas>(
        auto canvas
36
           input_vector.size()
        )};
        // storing values
40
        std::transform(input_vector.begin(), input_vector.end(),
41
                    canvas.begin(),
                     [](const auto&
                                   argx){
                       return std::real(argx);
                    });
        // returning
        return std::move(canvas);
48
     }
49
  }
```

B.38 Imaginary part

```
namespace svr {
     /*-----
     For type-deductions
     -----*/
     template <typename T>
     struct imag_result_type;
     template <> struct imag_result_type<std::complex<double>>{
10
11
        using type = double;
     };
12
     template <> struct imag_result_type<std::complex<float>>{
13
        using type = float;
14
15
     template <> struct imag_result_type<double> {
16
        using type = double;
17
     template <> struct imag_result_type<float>{
19
        using type = float;
20
21
     };
     template <typename T>
23
     using imag_result_t = typename imag_result_type<T>::type;
24
25
     /*-----
27
     template <typename T>
2.8
     auto imag(const std::vector<T>& input_vector)
29
30
        // figure out base-type
31
        using TCanvas = imag_result_t<T>;
        // creating canvas
        auto canvas {std::vector<TCanvas>(
35
           input_vector.size()
36
        )};
        // storing values
        std::transform(input_vector.begin(), input_vector.end(),
41
                     canvas.begin(),
                     [](const auto&
                                    argx){
42
                        return std::imag(argx);
43
                     });
44
        // returning
46
        return std::move(canvas);
47
     }
48
  }
```