

NAVAL PHYSICAL AND OCEANOGRAPHIC
LABORATORY, DRDO

INTERNSHIP REPORT

Beamforming for Uniform Linear Arrays

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NAVAL PHYSICAL AND OCEANOGRAPHIC LABORATORY, DRDO

Abstract

Internship Report

Beamforming for Uniform Linear Arrays

by S.V. RAJENDRAN

Sonar is a class of technologies that involve illuminating an sub-marine environment with an acoustic signal, recording the returns (echoes) and estimating task-specific characteristics of the underwater environment. For oceanographic tasks, this means sea-floor topology, movement of marine life and presence of alien bodies. For defense tasks, this means detection and neutralizing of threats.

Depending on the task, the system-configuration and parameters varies. Some technologies require the hydrophones (sub-marine microphones) arranged in a uniformly spaced linear array, called ULAs. Due to the loss of elevation-information from such configurations, some technologies require the use of uniformly spaced planar array of hydrophones. And over the years, designing application specific hydrophone arrays are not unheard of. However, uniform linear arrays are ubiquitous, and shall be the focus of this report.

However, a uniform linear array is the norm when working with sonar. A towed array sonar is a technology that uses this ULA attached to a moving marine-vessel, and pings the surrounding as it moves. And beamforming is the primary method used to obtain useful information from the received signal.

Beamforming is a linear method of combining recorded signals to obtain spatial information in regards to the immediate environment. This method is used in technologies ranging from simple direction-of-arrival estimation all the way to more advanced synthetic-aperture-sonar implementations. We shall be sticking to topics similar to the former owing to the novice nature of this candidate. The method primarily stems from the fact that signals recorded by the sensors have some inherent delay owing to their delay in arrivals.

In this report, we deal with simulation of reception beamforming under different conditions and environments, with the sole intention of learning the basics and implementing beamforming.

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List of Abbreviations

SONAR	SOund NAvigation And Ranging
RADAR	RAdio Detection And Ranging
SNR	Signal to Noise Ratio
ULA	Uniform Linear Array

Physical Constants

Speed of Light $c_0 = 2.997\,924\,58 \times 10^8 \text{ m s}^{-1}$
Speed of Sound $c = 1500 \text{ m s}^{-1}$

List of Symbols

a	distance	m
P	power	W (J s ⁻¹)
ω	angular frequency	rad

*For Dasher,
Labrador Retriever extraordinaire.*

Introduction

SONAR is a technique that uses sound propagation to navigate, communicate with, or detect objects or or under the surface of the water. Many other methods of detecting the presence of underwater targets in the sea have been investigated such as magnetic, optical signatures, electric field signatures, thermal detection (infrared) , Hydrodynamic changes (pressure) and has had a degree of success. Unfortunately, none of them has surpassed SONAR[1] despite it possessing numerous disadvantages .

There are two types of SONAR - Passive SONAR and Active SONAR .

Passive sonar listens to sound radiated by a target using a hydrophone and detects signals against the background of noise. This noise can be self made noise or ambient noise. Self noise is generated inside the receiver and ambient noise may be a combination of sound generated by waves, turbines, marine life and many more.

Active sonar uses a projector to generate a pulse of sound whose echo is received after it gets reflected by the target. This echo contains both signal and noise, so the signal has to be detected against the background noise. The range of the target is calculated by detecting the power of the received signal and thus determining the transmission loss. The transmission loss is directly related to the distance travelled by the signal. In the case of the active sonar, half of the distance travelled by the signal to get attenuated to undergo a transmission loss detected is the range.

Beamforming is a signal processing technique that originates from the design of spatial filters into pencil shaped beams to strengthen signals in a specified direction and attenuate signals from other directions. Beamforming is applicable to either radiation or reception of energy. Here, we consider the reception part. A beamformer is a processor used along with an array to provide a versatile form of spatial filtering. It essentially performs spatial filtering to separate signals that have overlapping frequency content but generate from different spatial directions .

Background

Despite its major success in air to air detection, RADAR isn't used in sub surface identification and detection. The main reason is the radio wave is an EM wave and the sea water is highly conductive and offers an attenuation of $1400\sqrt{f}$ dB/km. This essentially means that the sea water acts a short circuit to the EM energy[1].

Even if we were to use it, to get tangible results, the target should be in short range and shouldn't be completely submerged. Contemporary submarines are nuclear and has the capacity to go deep underwater for indefinite periods. Hence RADAR is useless in subsurface application and can only used along with SONAR without bringing much to the table. Hence SONAR is the dominating method/technology when it comes to underwater detection and ranging.

Beam forming or spatial filtering is the process by which an array of large number of spatially separated sensors discriminate the signal arriving from a specified direction from a combination of isotropic random noise called ambient noise and other directional signals. In the following simulations, we deal with 32 elements separated by a distance of $\frac{\lambda}{2}$, where λ is the wavelength of the frequency for which the beamformer is designed.

The assumptions under which we perform the simulations are:

1. the noise is isotropic
2. ambient noise at the sensor-to-sensor output is uncorrelated
3. the signal at the sensor-to-sensor outputs are fully correlated .

The sensor spacing in the array is decided based on two important considerations namely, coherence of noise between sensors and formation of grating lobes in the visible region. As far as isotropic noise is concerned, the spatial coherence is zero at spacing in multiples of $\frac{\lambda}{2}$ and small at all points beyond $\frac{\lambda}{2}$. To avoid grating lobe, the spacing between sensors must be less than $\frac{\lambda}{2}$. Hence $\frac{\lambda}{2}$ is chosen as the distance between two elements in the array.

We also assume that the source is far away so as a result, the wave fronts are parallel straight lines. Since the source would be at an angle relative to the axis, the wavefront reaches each element with varying delay. As a result, the output of each element will have a phase delay from each other

Using the above figure, we can calculate the corresponding delay of each element. This will help us in determining to what degree we would have to delay the element

outputs to obtain all the outputs of elements in co phase. Once the outputs are made in-phase, they are added. For an M element array, the co-phase addition increases the signal power N^2 times and the uncorrelated noise power N times. Thus the SNR is enhanced by N times.

By changing the delays of the element's output, we can "steer" the setup to give the gain to signals coming from a certain direction. This is called beam steering and is one of the most attractive features of a beamformer. However, as one 'steers' a beamformer, the width of the main lobe goes on increasing because the effective length of the array decreases.

In our simulation, we create a matrix with the columns corresponding to the output of each element for a source at a certain angle. The noise is then added. This is the output of the elements in the array. This is the basic setup. The array is then manipulated or used as input for other array manipulations to obtain the solution to the problem/objective posed.

Let the source signal be $s(k)$. The output of the elements are time-shifted versions of $s(k)$. So, for an element 'i', the output signal would be

$$y(k) = s[k - \tau_i(\theta)]$$

Using the fourier transform, we get

$$Y_i(\omega, \theta) = e^{-j\omega\tau_i(\theta)} S[\omega]$$

where

- $\tau_i(\theta) = \frac{d_m \cos(\theta)}{c} F_s$
- d_m the distance between the element considered and the element where the wave-front first strikes.
- θ the angle the rays make with the array axis
- c speed of sound in the water
- F_s The sampling frequency of the hydrophones/sensors

Steering Vector

Now, we need to construct a steering vector.

$$d(\omega, \theta) = [1, e^{-j\omega\tau_2(\theta)}, e^{-j\omega\tau_3(\theta)}, e^{-j\omega\tau_4(\theta)}, \dots, e^{-j\omega\tau_M(\theta)}]$$

To obtain the element output, we multiply this matrix with the signal function.

$$\mathbf{Y}(\omega, \theta) = d(\omega, \theta)S[\omega]$$

The output signal is given by

$$\begin{aligned} Z(\omega, \theta) &= \sum_{i=1}^M F_i^*(\omega) Y_i(\omega, \theta) \\ &= F^H(\omega) Y(\omega, \theta) \end{aligned}$$

where F^H is the matrix containing the complex weights

Complex Radiation Field

The complex radiation field produced by a linear array of N passive receivers is given by

$$Z(\omega, \theta) = \frac{1}{M} \sum_{m=1}^M s(\omega) e^{-j(m-1) \frac{\omega d}{c} (\cos(\theta) - \cos(\phi))}$$

Chapter 1

Single-sensor signal simulation

1.1 Aim

Here, a sine wave is created and simulated to replicate the conditions in beamforming and to see the output of a single element in the array. The noise is added by processing the SNR parameter. The changes are then observed by changing SNR. The changes are observed in both time and frequency domain. The change in time domain is observed by plotting amplitude vs time. The change in frequency domain is observed by first taking fourier transform and then plotting absolute value vs frequency.

1.2 Plots

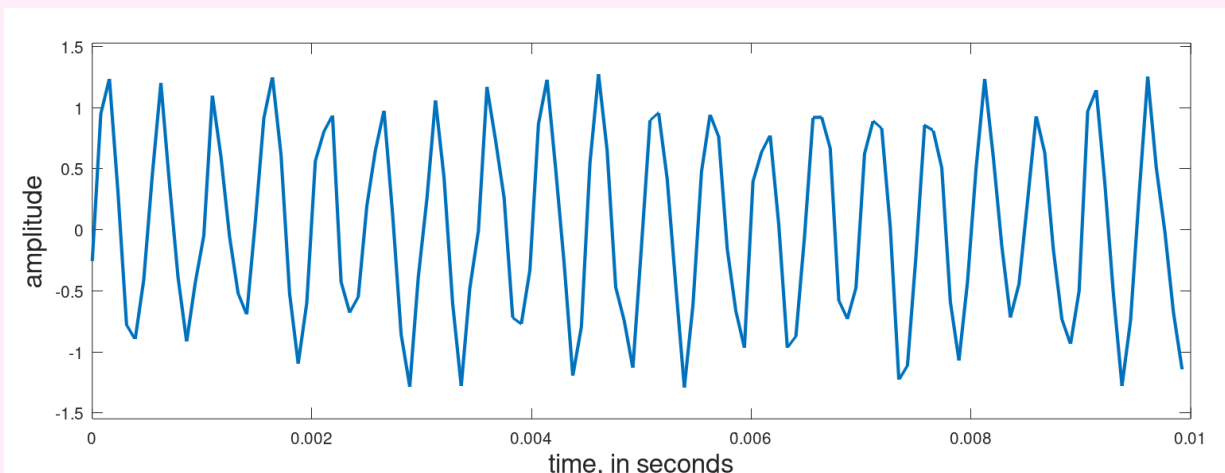


FIGURE 1.1: Time Domain Representation of Signal

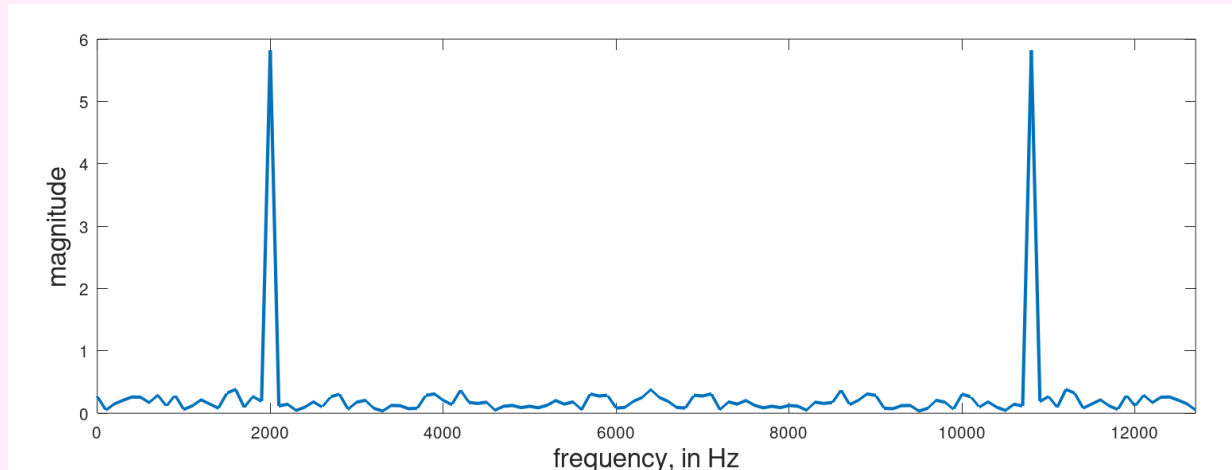


FIGURE 1.2: Magnitude of DFT of input-signal

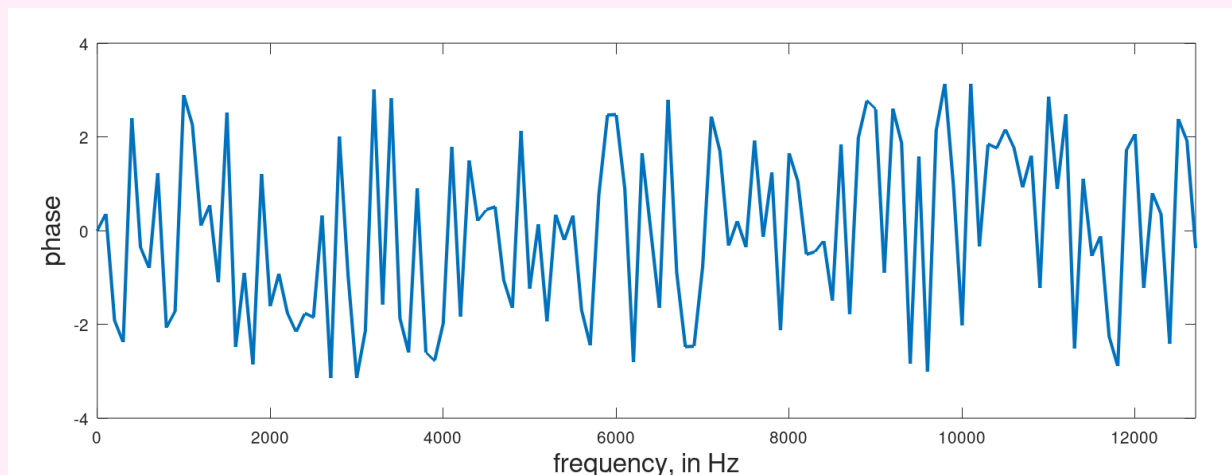


FIGURE 1.3: Phase of DFT of input-signal

1.3 C++ Code

```

1  // =====
2  #include "include/before.hpp"
3  // main-file =====
4  int main(){
5
6      // starting timer
7      auto logfile {string("../csv-files/logfile.csv")};
8      Timer timer(logfile);
9
10     // init-variables
11     auto f {2000}; // frequency of
12     signal // sampling
13     auto Fs {12800}; // corresponding
14     frequency // num-samples
15     auto Ts {1/static_cast<double>(Fs)};
16     time-period
17     auto N {128};

```

```

16     auto snr          {10};                                // signal-to-noise
ratio
17     auto snrweight    {std::pow(10, (-1 * snr * 0.05))};    // corresponding
weight
18
19     // building time-array
20     vector<double> t(N,0); t.reserve(N);
21     t = linspace(0.0, static_cast<double>(N-1), static_cast<size_t>(N)) * Ts;
22     fWriteVector(t, "../csv-files/t-Objective1.csv");
23
24     // creating sine-wave
25     auto y = t;
26     std::transform(t.begin(),
27                   t.end(),
28                   y.begin(),
29                   [&](const auto x){return std::sin( 2 * std::numbers::pi * f *
x);});
30
31     // adding noise to the vector
32     auto newmat        {y + snrweight * rand(-1.0, 1.0, y.size())};
33     auto timeaxis       {linspace(static_cast<double>(0),
34                                   static_cast<double>((N-1)*Ts),
35                                   N)};
36     fWriteVector(timeaxis, "../csv-files/timeaxis-Objective1.csv");
37     fWriteVector(newmat,   "../csv-files/newmat-Objective1.csv");
38
39     // Taking the fourier transform
40     auto nfft           {N};
41     auto fend           {static_cast<double>((nfft-1) * Fs) /
static_cast<double>(nfft)};
42     auto waxis          {linspace(static_cast<double>(0),
43                                   static_cast<double>(fend),
44                                   nfft)};
45     auto Fourier        {fft(newmat)};
46     fWriteVector(waxis,   "../csv-files/waxis-Objective1.csv");
47     fWriteVector(Fourier,  "../csv-files/Fourier-Objective1.csv");
48
49     // return
50     return(0);
51
52 }

```

1.4 Octave Code

```

1  %% Basic Setup
2  clc; clear all; close all;
3  addpath("../include/")
4
5  %% Loading the files
6  timeaxis    = csvread("../csv-files/timeaxis-Objective1.csv");
7  newmat      = csvread("../csv-files/newmat-Objective1.csv");
8  waxis       = csvread("../csv-files/waxis-Objective1.csv");
9  newmatfft   = fReadCSV("../csv-files/Fourier-Objective1.csv");

```

```
10
11 %% Plotting
12 plotwidth  = 1515;
13 plotheight = 500;
14
15 figure(1)
16 set(gcf, 'Position', [0 0 plotwidth plotheight]); % [left bottom width height]
17 plot(timeaxis, newmat, "LineWidth", 2);
18 xlabel("time, in seconds", "fontsize", 16);
19 ylabel("amplitude", "fontsize", 16);
20 ylim([1.2 * min(newmat), 1.2 * max(newmat)]);
21 saveas(gcf, "../Figures/y-objective1.png");
22
23 figure(2);
24 set(gcf, 'Position', [0 0 plotwidth plotheight]); % [left bottom width height]
25 plot(waxis, abs(newmatfft), "LineWidth", 2);
26 xlabel("frequency, in Hz", "fontsize", 16);
27 ylabel("magnitude", "fontsize", 16);
28 xlim([min(waxis), max(waxis)]);
29 saveas(gcf, "../Figures/abs-yfft-objective1.png");
30
31 figure(3);
32 set(gcf, 'Position', [0 0 plotwidth plotheight]); % [left bottom width height]
33 plot(waxis, angle(newmatfft), "LineWidth", 2);
34 xlabel("frequency, in Hz", "fontsize", 16);
35 ylabel("phase", "fontsize", 16);
36 xlim([min(waxis), max(waxis)]);
37 saveas(gcf, "../Figures/phase-yfft-objective1.png");
```


Chapter 2

Simulate the input to a 4 element array.

Here, the outputs of a 4 element array is simulated. The location of the source signal is far and therefore the wave fronts are parallel and arrives at the same angle at each element. As a result there would be a phase difference in the output of each element even for the same source signal due to difference in distance travelled. The sine wave is created as output from each element and corresponding phase difference is brought about. The noise is added by processing the SNR parameter.

2.1 Plots

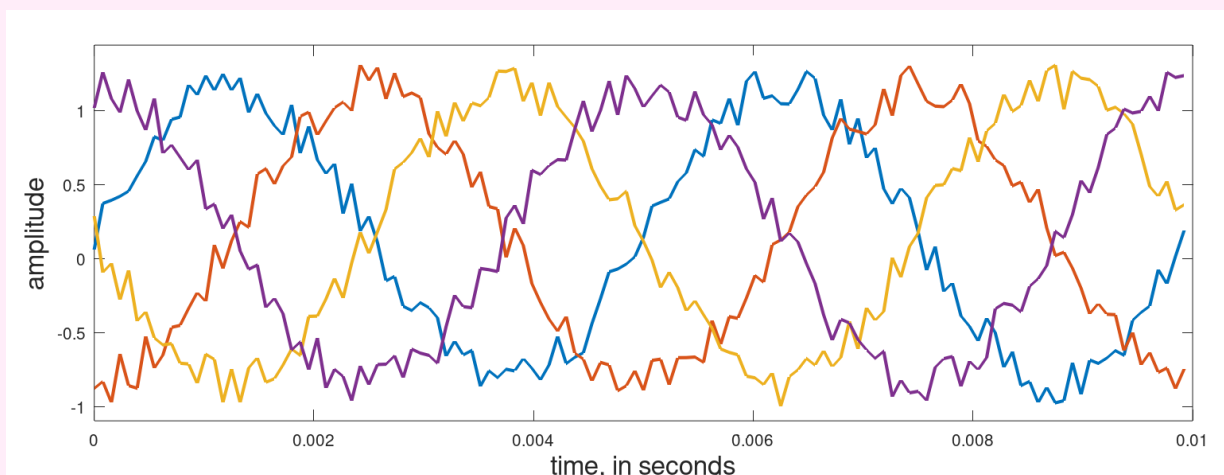


FIGURE 2.1: Time Domain Representation of Signal

2.2 C++ Code

```

1 // =====
2 #include "include/before.hpp"
3 // main-file =====
4 int main(){
5

```

```

6 // starting timer
7 auto logfile {string("../csv-files/logfile.csv")};
8 Timer timer(logfile);
9
10 // init-variables
11 auto f {200}; // frequency
12 // of signal
13 auto Fs {12800}; // sampling
14 // frequency
15 auto Ts {1/static_cast<double>(Fs)}; //
16 // corresponding time-period
17 auto N {128}; //
18 // num-samples
19
20 auto m {4};
21 auto angleofarrival {60};
22 auto speedofsound {1500};
23 auto lambda
24 {static_cast<double>(speedofsound)/static_cast<double>(f)};
25 auto x {lambda/2};
26 auto d {x * std::cos(angleofarrival * std::numbers::pi / 180)
27 / speedofsound};
28
29 auto snr {10}; //
30 // signal-to-noise ratio
31 auto snrweight {std::pow(10, (-1 * snr * 0.05))}; //
32 // corresponding weight
33
34 // building time-array
35 vector<double> t(N,0); t.reserve(N);
36 t = linspace(0.0,
37 static_cast<double>(N-1),
38 static_cast<size_t>(N)) * Ts;
39 fWriteVector(t, "../csv-files/timeaxis-Objective2.csv");
40
41 // building matrix
42 auto matrix = Zeros({m, N});
43
44 // creating sine-wave
45 auto y = sin(2 * std::numbers::pi * f * t);
46 fWriteVector(y, "../csv-files/y-Objective2.csv");
47
48 // building the matrix
49 for(int i = 0; i<m; ++i)
50 matrix[i] = sin(2 * std::numbers::pi * f * (t - i * d));
51 fWriteMatrix(matrix, "../csv-files/matrix-Objective2.csv");
52
53 // Adding noise to the matrix
54 vector<vector<double>> additivenoise = snrweight * rand(0.0, 1.0, {m, N});
55 fWriteMatrix(additivenoise, "../csv-files/additivenoise-Objective2.csv");
56
57 auto newmat {matrix + additivenoise};
58 fWriteMatrix(newmat, "../csv-files/newmat-Objective2.csv");
59
60 // return
61 return(0);

```

```
55 }
```

2.3 Octave Code

```
1  %% Basic Setup
2  clc; clear all; close all;
3  addpath("../include/")
4
5  %% Loading the files
6  timeaxis = csvread("../csv-files/timeaxis-Objective2.csv");
7  newmat    = csvread("../csv-files/newmat-Objective2.csv");
8
9  %% Plotting
10 plotwidth = 1515;
11 plotheight = 500;
12
13 figure(1)
14 set(gcf, 'Position', [0 0 plotwidth plotheight]); % [left bottom width height]
15 plot(timeaxis, newmat, "LineWidth", 2);
16 xlabel("time, in seconds", "fontsize", 16);
17 ylabel("amplitude", "fontsize", 16);
18 ylim([1.1 * min(newmat(:)), 1.1 * max(newmat(:))]);
19 saveas(gcf, "../Figures/newmat-Objective2.png");
```


Chapter 3

Narrowband beamformer

3.1 Aim

The outputs of each element in the array differ in phase. They are made in phase again by bringing about an artificial delay corresponding to the array position. In this code, this artificial delay is brought on the property of Fourier transform that

$$x(t - t_0) \Leftrightarrow e^{-j\omega t_0} X(\omega)$$

So the input is Fourier transformed. Then a weight vector is defined for each angle ranging from 0 to 180 degrees and the two matrices are multiplied. Then the absolute value vs angle is plotted.

3.2 Plots

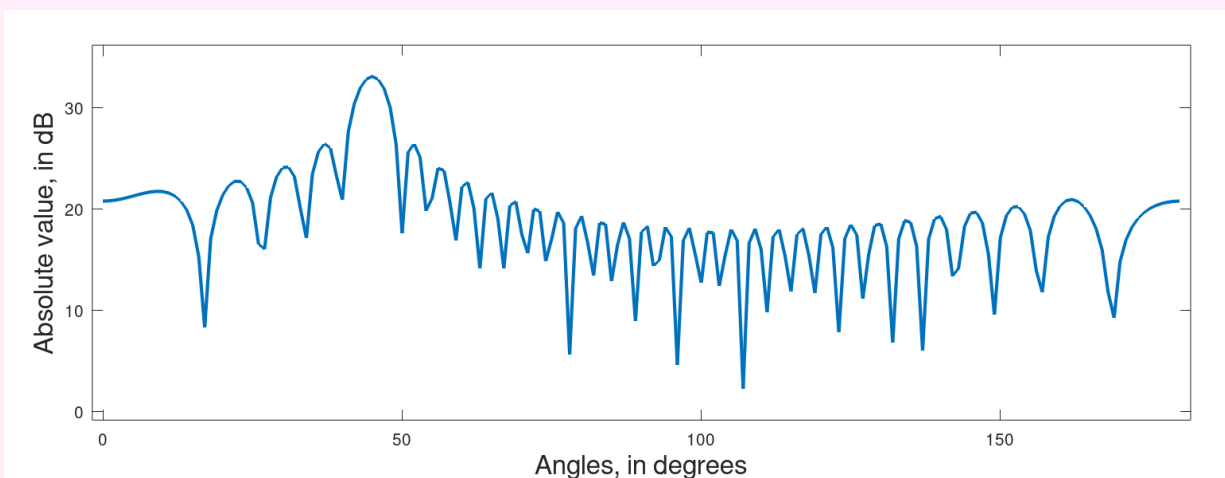


FIGURE 3.1: Beam-pattern for beamformer at angle 45

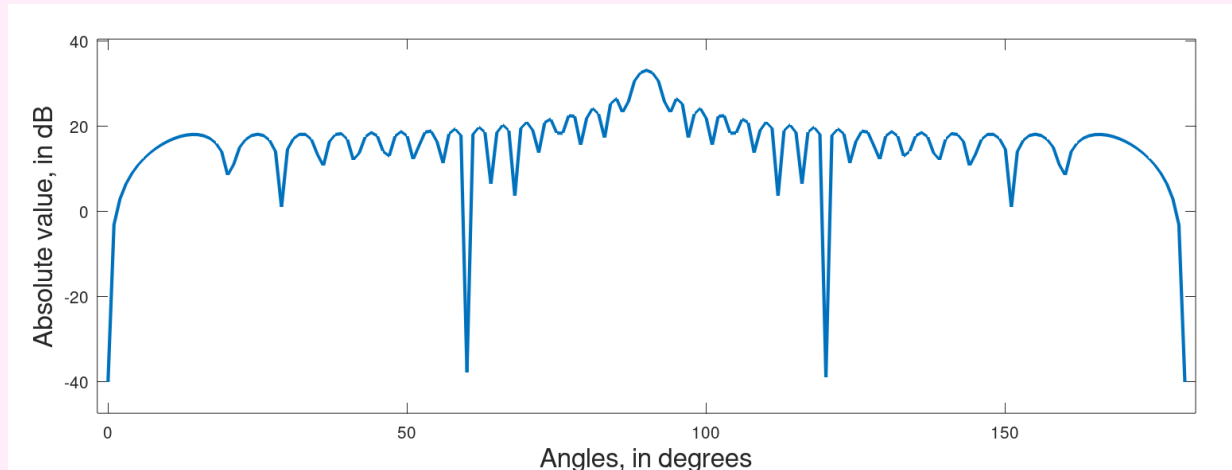


FIGURE 3.2: Beam-pattern for beamformer at angle 90

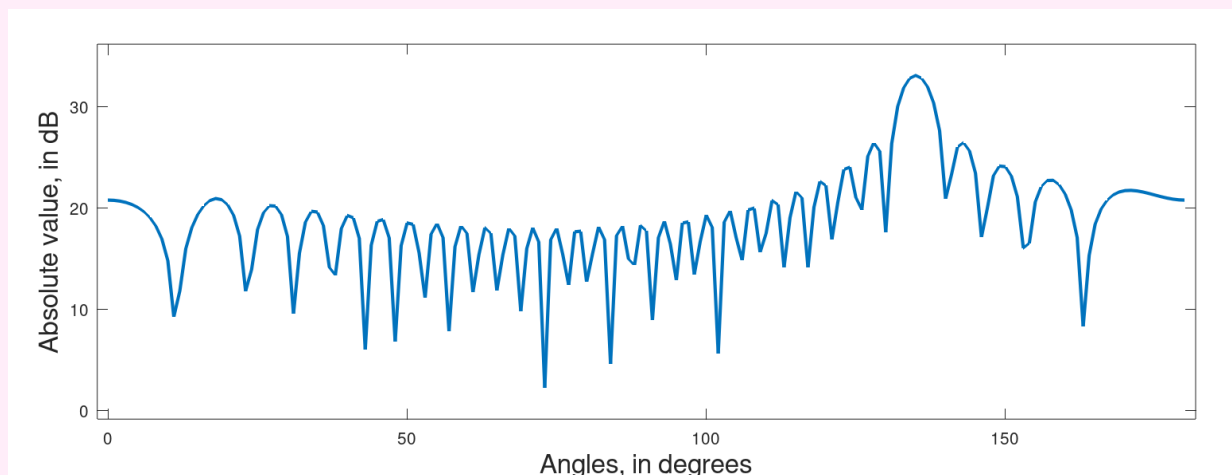


FIGURE 3.3: Beam-pattern for beamformer at angle 135

3.3 Observation

It is seen that for the angle equal to the source angle, a maxima is given and for every other angles, an absolute value much smaller than the peak value is given.

3.4 C++ Code

```

1 // =====
2 #include "include/before.hpp"
3 // main-file =====
4 int main(){
5
6     // starting timer
7     auto logfile {string("../csv-files/logfile-Objective3.csv")};
8     Timer timer(logfile);
9
10    // init-variables

```

```

11  auto f                {2000};                                //
    frequency of signal
12  auto Fs              {12800};                                // sampling
    frequency
13  auto Ts              {1.00/static_cast<double>(Fs)};        //
    corresponding time-period
14  auto N               {128};                                  //
    num-samples

15
16  auto m               {32};
17  auto angleofarrival  {135};
18  auto speedofsound    {1500};
19  auto lambda
    {static_cast<double>(speedofsound)/static_cast<double>(f)};
20  auto x               {lambda/2.00};
21  auto d               {x * std::cos(static_cast<double>(angleofarrival) *
    std::numbers::pi / 180) / speedofsound};

22
23  auto snr             {100};                                  //
    signal-to-noise ratio
24  auto snrweight       {std::pow(10, (-1 * snr * 0.05))};      //
    corresponding weight

25
26
27  // building time-array
28  vector<double>t = linspace(0.0,
    static_cast<double>(N-1) * Ts,
    static_cast<size_t>(N));
29
30  fWriteVector(t, "../csv-files/t-Objective3.csv");
31
32
33  // building matrix
34  auto matrix = Zeros({m, N});
35
36  // creating sine-wave
37  auto y               {sin(2 * std::numbers::pi * f * t)};
38  fWriteVector(y, "../csv-files/y-Objective3.csv");
39
40  // building the matrix
41  for(int sensorindex = 0; sensorindex < m; ++sensorindex)
42      matrix[sensorindex] = sin(2.00 * std::numbers::pi * f * (t - sensorindex *
    d));
43  fWriteMatrix(matrix, "../csv-files/matrix-Objective3.csv");
44
45
46  // Adding noise to the matrix
47  auto newmat           {matrix + snrweight * rand(0.0, 1.0, {m, N})};
48  fWriteMatrix(newmat, "../csv-files/newmat-Objective3.csv");
49
50
51  // Taking the fourier-transform
52  auto nfft             {N};
53  auto fend             {static_cast<double>(nfft - 1) * static_cast<double>(Fs) /
    static_cast<double>(nfft)};
54  auto waxis            {linspace(0, fend, nfft)};
55  auto Fourier          {fft(newmat, nfft)};
56  fWriteMatrix(Fourier, "../csv-files/Fourier-Objective3.csv");
57

```

```

58
59 // choosing the frequency row
60 int index = std::floor(static_cast<double>(f)/
61 (static_cast<double>(Fs)/static_cast<double>(nfft)));
62 auto fmat {slice(Fourier, {-1, -2, index, index})};
63
64 // Bringing the delay in frequency region
65 auto anglematrix {vector<double>(181)};
66 auto delaycolumn {vector<complex<double>>(m)};
67
68 // moving through angle
69 for(int testangle = 0; testangle<181; ++testangle){
70
71     double testd {x * cosd(testangle) / speedofsound};
72
73     for(int currsensor = 0; currsensor < m; ++currsensor){
74         delaycolumn[currsensor] = \
75             std::exp( 1 * std::complex<double>{0, 1} * 2 * std::numbers::pi * f
76             * currsensor * testd);
77     }
78
79     // calculating inner-product
80     auto innerproduct_value {delaycolumn[0] * fmat[0][0]};
81     for(int i = 1; i<delaycolumn.size(); ++i)
82         innerproduct_value += delaycolumn[i] * fmat[i][0];
83
84     // storing to the angle-matrix
85     anglematrix[testangle] = std::abs(innerproduct_value);
86 }
87 fWriteVector(anglematrix, "../csv-files/anglematrix-Objective3.csv");
88
89 // creating angle-axis
90 auto angleaxis {linspace(0, 180, 181)};
91 fWriteVector(angleaxis, "../csv-files/angleaxis-Objective3.csv");
92
93 // return
94 return(0);
95 }

```

3.5 Octave Code

```

1 %% Basic Setup
2 clc; clear all;
3
4 %% Loading the files
5 timearray = csvread("../csv-files/t-Objective3.csv");
6 yarray = csvread("../csv-files/y-Objective3.csv");
7 matrix = csvread("../csv-files/matrix-Objective3.csv");
8
9 ULAMatrix = csvread("../csv-files/newmat-Objective3.csv");
10 FourierMatrix = fReadCSV("../csv-files/Fourier-Objective3.csv");

```



```
11
12 angleaxis      = csvread("../csv-files/angleaxis-Objective3.csv");
13 anglematrix    = csvread("../csv-files/anglematrix-Objective3.csv");
14 anglematrixdB  = 10*log10(anglematrix);
15
16 %% Plotting the signals
17 plotwidth      = 1515;
18 plotheight     = 500;
19
20 figure(1);
21 set(gcf, 'Position', [0 0 plotwidth plotheight]); % [left bottom width height]
22 plot(angleaxis, anglematrixdB, "LineWidth", 2);
23 xlabel("Angles, in degrees", "fontsize", 16);
24 ylabel("Absolute value, in dB", "fontsize", 16);
25 ylim([min(anglematrixdB) - 1e-1 * range(anglematrixdB),
26       max(anglematrixdB) + 1e-1 * range(anglematrixdB)]);
27 xlim([min(angleaxis) - 1e-2 * range(angleaxis),
28       max(angleaxis) + 1e-2 * range(angleaxis)]);
29 saveas(gcf, "../Figures/anglematrixdB-Objective3.png");
```


Chapter 4

Simulate beam pattern by shifting theta

4.1 Aim

In this code, we explore the property of the beamformer in hand. The beam former is essentially a spatial filter. That is, it attenuates the signals which doesn't come from a specific direction (which we decide when designing) and provides a gain for signals coming in the direction. We show this by plotting the gain vs angle.

4.2 Plots

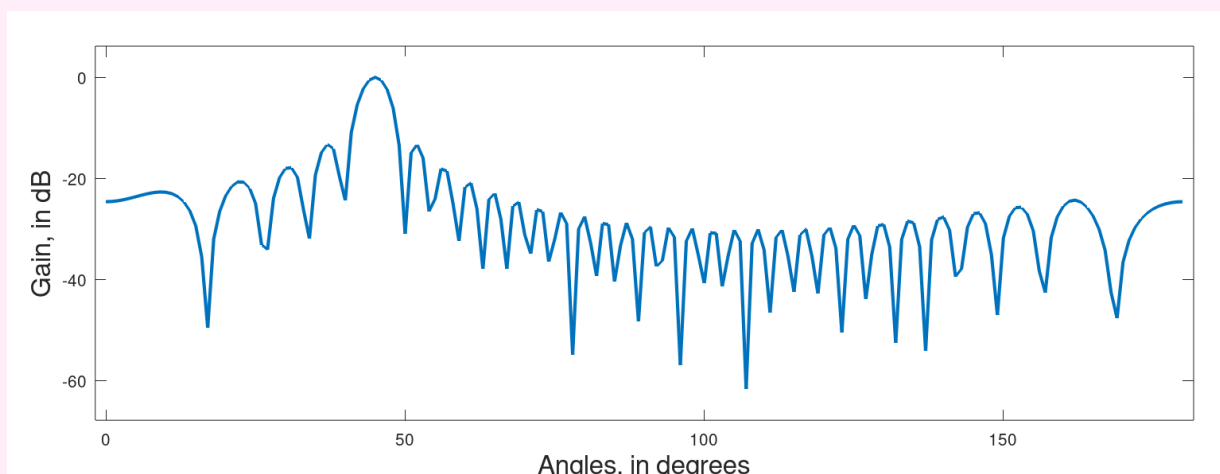


FIGURE 4.1: Beam-pattern for beamformer at angle 45

4.3 Observation

We can see that when we change the designed angle, the gain and attenuation correspondingly move to the designed angle.

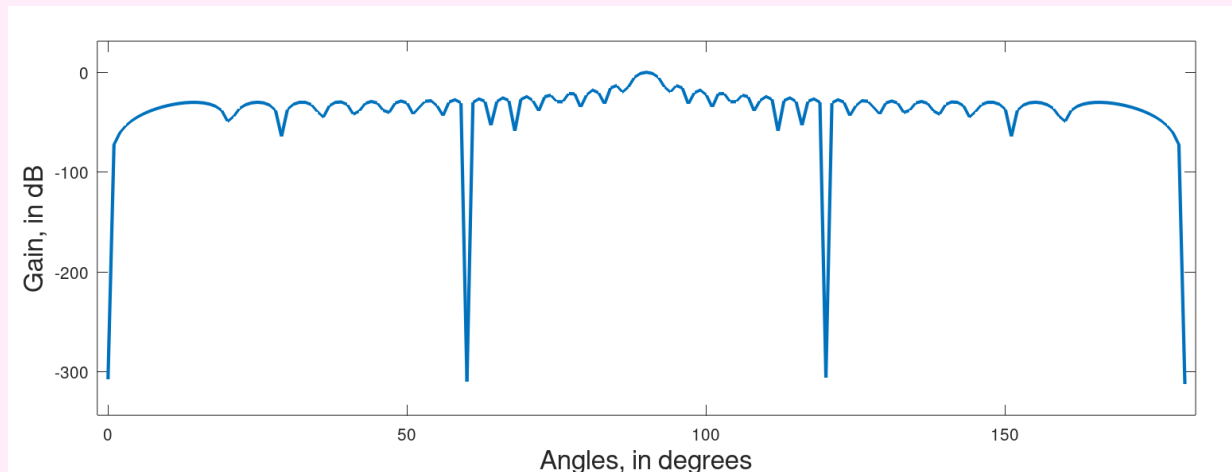


FIGURE 4.2: Beam-pattern for beamformer at angle 90

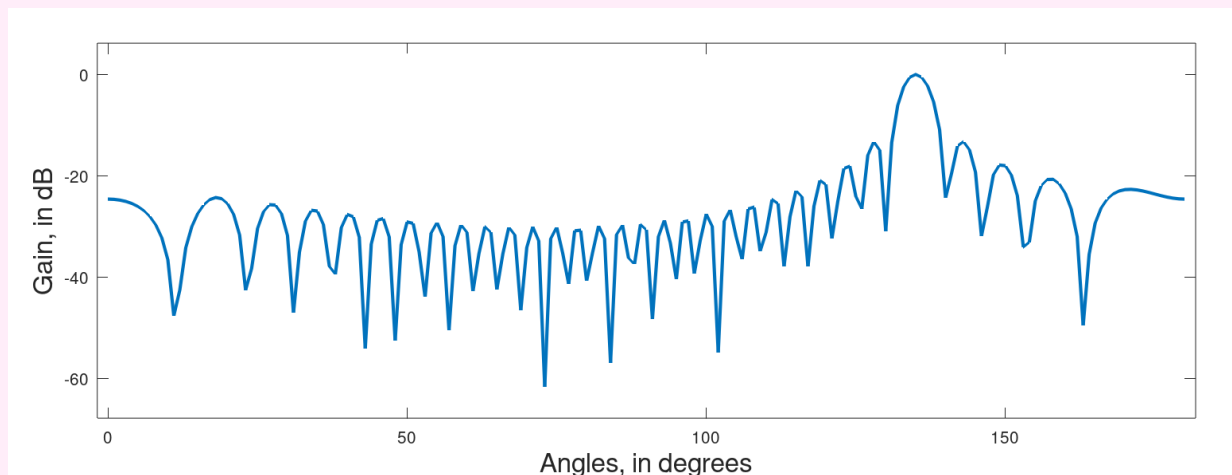


FIGURE 4.3: Beam-pattern for beamformer at angle 135

4.4 C++ Code

```

1 // =====
2 #include "include/before.hpp"
3 // main-file =====
4 int main(){
5
6     // starting timer
7     auto logfile {string("logfile.csv")};
8     Timer timer(logfile);
9
10    // init-variables
11    auto f {2000};
12    auto m {static_cast<double>(32)};
13    auto angle {135};
14    auto c {1500};
15    auto lambda {static_cast<double>(c)/static_cast<double>(f)};
16    auto x {lambda/2};
17    auto d {x * cosd(angle)/c};
18

```

```

19 // bringing about the natural delay
20 auto matrix          {vector<complex<double>>(m, complex<double>(0))};
21 for(auto sensorindex = 0; sensorindex < m; ++sensorindex){
22     matrix[sensorindex] = \
23         (1.00/static_cast<double>(m)) * \
24         std::exp(-1.00 * 1i * 2.00 * \
25             std::numbers::pi * f * (sensorindex) * d);
26 }
27
28 // bringing the delay in frequency region
29 auto delaycolumn      = vector<complex<double>>(m, complex<double>(0));
30 auto anglematrix      = vector<double>(181, 0);
31
32 // calculating
33 for(int testangle = 0; testangle < 181; ++testangle){
34
35     auto testd {x * cosd(testangle)/c};
36
37     for(int sensorindex = 0; sensorindex < m; ++sensorindex)
38         delaycolumn[sensorindex] = \
39             std::exp(1.00 * 1i * 2.00 * \
40                 std::numbers::pi * f * sensorindex * testd);
41
42     // performing inner-product
43     anglematrix[testangle] = \
44         std::abs(std::inner_product(matrix.begin(),
45                                     matrix.end(),
46                                     delaycolumn.begin(),
47                                     complex<double>{0}));
48
49 }
50
51 // producing angle axis
52 auto angleaxis {linspace(0, 180, 181)};
53
54 // saving the tensors
55 fWriteVector(angleaxis,      "../Figures/angleaxis-Objective4.csv");
56 fWriteVector(anglematrix,    "../Figures/anglematrix-Objective4.csv");
57
58 // return
59 return(0);
60
61 }

```

4.5 Octave Code

```

1 %% Basic Setup
2 clc; clear all; close all;
3
4 %% Loading the files
5 angleaxis      = csvread("../Figures/angleaxis-Objective4.csv");
6 anglematrix    = csvread("../Figures/anglematrix-Objective4.csv");
7 anglematrixinDB = 20 * log10(anglematrix);

```

```
8
9  %% Plotting the signals
10 plotwidth  = 1515;
11 plotheight = 500;
12
13 figure(1);
14 set(gcf, 'Position', [0 0 plotwidth plotheight]); % [left bottom width height]
15 plot(angleaxis, anglematrixinDB, "LineWidth", 2);
16 xlabel("Angles, in degrees", "fontsize", 16);
17 ylabel("Gain, in dB", "fontsize", 16);
18 ylim([min(anglematrixinDB) - 1e-1 * range(anglematrixinDB),
19      max(anglematrixinDB) + 1e-1 * range(anglematrixinDB)]);
20 xlim([min(angleaxis) - 1e-2 * range(angleaxis),
21      max(angleaxis) + 1e-2 * range(angleaxis)]);
22 saveas(gcf, "../Figures/anglematrixinDB-Objective4.png");
```

Appendix A

C++ Function Definitions

A.1 before.hpp

```
1 // including header-files
2 #include <algorithm>
3 #include <complex>
4 #include <bitset>
5 #include <climits>
6 #include <cstdint>
7 #include <iostream>
8 #include <limits>
9 #include <map>
10 #include <new>
11 #include <stdlib.h>
12 #include <unordered_map>
13 #include <vector>
14 #include <set>
15 #include <numeric>
16 #include <fstream>
17 #include <numbers>
18 #include <cmath>
19 #include <random>
20
21 // custom definitions
22 #include "hashdefines.hpp"
23 #include "usings.hpp"
24 #include "DataStructureDefinitions.hpp"
25 #include "PrintContainers.hpp"
26 #include "TimerClass.hpp"
27 #include "utils.hpp"
```

A.2 hashdefines.hpp

```
1 // hash-deinfes
2 #define PRINTSPACE std::cout << "\n\n\n" << std::endl;
3 #define PRINTLINE std::cout << "=====
    << std::endl;
```

A.3 usings.hpp

```
1  // borrowing from namespace std
2  using std::cout;
3  using std::complex;
4  using std::endl;
5  using std::vector;
6  using std::string;
7  using std::unordered_map;
8  using std::map;
9  using std::format;
10 using std::deque;
11 using std::pair;
12 using std::min;
13 using std::max;
14 using namespace std::complex_literals;
```

A.4 DataStructureDefinitions.hpp

```
1  struct TreeNode {
2      int val;
3      TreeNode *left;
4      TreeNode *right;
5      TreeNode() : val(0), left(nullptr), right(nullptr) {}
6      TreeNode(int x) : val(x), left(nullptr), right(nullptr) {}
7      TreeNode(int x, TreeNode *left, TreeNode *right) : val(x), left(left),
8          right(right) {}
9  };
10
11 struct ListNode {
12     int val;
13     ListNode *next;
14     ListNode() : val(0), next(nullptr) {}
15     ListNode(int x) : val(x), next(nullptr) {}
16     ListNode(int x, ListNode *next) : val(x), next(next) {}
17 };
```

A.5 PrintContainers.hpp

```
1  // vector printing function
2  template<typename T>
3  void fPrintVector(vector<T> input){
```



```
4     for(auto x: input) cout << x << ",";
5     cout << endl;
6 }
7
8 template<typename T>
9 void fpv(vector<T> input){
10     for(auto x: input) cout << x << ",";
11     cout << endl;
12 }
13
14 template<typename T>
15 void fPrintMatrix(vector<T> input){
16     for(auto x: input){
17         for(auto y: x){
18             cout << y << ",";
19         }
20         cout << endl;
21     }
22 }
23
24 template<typename T, typename T1>
25 void fPrintHashmap(unordered_map<T, T1> input){
26     for(auto x: input){
27         cout << format("{}{} | ", x.first, x.second);
28     }
29     cout << endl;
30 }
31
32 void fPrintBinaryTree(TreeNode* root){
33     // sending it back
34     if (root == nullptr) return;
35
36     // printing
37     PRINTLINE
38     cout << "root->val = " << root->val << endl;
39
40     // calling the children
41     fPrintBinaryTree(root->left);
42     fPrintBinaryTree(root->right);
43
44     // returning
45     return;
46 }
47
48
49 void fPrintLinkedList(ListNode* root){
50     if (root == nullptr) return;
51     cout << root->val << " -> ";
52     fPrintLinkedList(root->next);
53     return;
54 }
55
56 template<typename T>
57 void fPrintContainer(T input){
58     for(auto x: input) cout << x << ", ";
59     cout << endl;
60     return;
```

```

61 }
62 // =====
63 template <typename T>
64 auto size(std::vector<std::vector<T>> inputMatrix){
65     cout << format("[{} , {}]\n", inputMatrix.size(), inputMatrix[0].size());
66 }
67
68 template <typename T>
69 auto size(const std::string inputstring, std::vector<std::vector<T>> inputMatrix){
70     cout << format("{} = [{} , {}]\n", inputstring, inputMatrix.size(),
71     inputMatrix[0].size());
72 }

```

A.6 TimerClass.hpp

```

1  struct Timer
2  {
3      std::chrono::time_point<std::chrono::high_resolution_clock> startpoint;
4      std::chrono::time_point<std::chrono::high_resolution_clock> endpoint;
5      std::chrono::duration<long long, std::nano> duration;
6      std::string filename;
7      std::string functionname;
8
9      // constructor
10     Timer() {start();}
11     Timer(std::string logfile_arg): filename(std::move(logfile_arg)) {start();}
12     Timer(std::string logfile_arg,
13           std::string func_arg): filename(std::move(logfile_arg)),
14                                     functionname(std::move(func_arg)) {start();}
15
16     void start() {startpoint = std::chrono::high_resolution_clock::now();}
17     void stop() {endpoint = std::chrono::high_resolution_clock::now();
18     fetchtime();}
19
20     void fetchtime(){
21         duration = std::chrono::duration_cast<std::chrono::nanoseconds>(endpoint -
22         startpoint);
23         cout << format("{} nanoseconds \n", duration.count());
24     }
25     void fetchtime(string stringarg){
26         duration = std::chrono::duration_cast<std::chrono::nanoseconds>(endpoint -
27         startpoint);
28         cout << format("{} took {} nanoseconds \n", stringarg, duration.count());
29     }
30     void measure(){
31         auto temp = std::chrono::high_resolution_clock::now();
32         auto nsduration =
33         std::chrono::duration_cast<std::chrono::nanoseconds>(temp - startpoint);
34         auto msduration =
35         std::chrono::duration_cast<std::chrono::microseconds>(temp - startpoint);
36         auto sduration = std::chrono::duration_cast<std::chrono::seconds>(temp -
37         startpoint);
38         cout << format("{} nanoseconds | {} microseconds | {} seconds \n",

```

```

33         nsduration.count(), msduration.count(), sduration.count());
34     }
35     ~Timer(){
36         auto temp = std::chrono::high_resolution_clock::now();
37         auto nsduration =
38         std::chrono::duration_cast<std::chrono::nanoseconds>(temp - startpoint);
39         auto msduration =
40         std::chrono::duration_cast<std::chrono::microseconds>(temp - startpoint);
41         auto milliduration =
42         std::chrono::duration_cast<std::chrono::milliseconds>(temp - startpoint);
43         auto sduration = std::chrono::duration_cast<std::chrono::seconds>(temp -
44         startpoint);
45         PRINTLINE
46         cout << format("{} nanoseconds | {} microseconds | {} milliseconds | {}
47         seconds \n",
48         nsduration.count(), msduration.count(), milliduration.count(),
49         sduration.count());
50
51         // writing to the file
52         if (!filename.empty()){
53             std::ofstream fileobj(filename, std::ios::app);
54             if (fileobj){
55                 if (functionname.empty()){
56                     fileobj << "main" << "," << nsduration.count() << "," <<
57                     msduration.count() << "," << sduration.count() << "\n";
58                 }
59                 else{
60                     fileobj << functionname << "," << nsduration.count() << "," <<
61                     msduration.count() << "," << sduration.count() << "\n";
62                 }
63             }
64         }
65     }
66 };

```

A.7 utils.hpp

```

1 // =====
2 #include "svr_WriteToCSV.hpp"
3 // =====
4 template <typename F, typename R>
5 constexpr auto fElementWise(F&& func, R& range){
6     std::transform(std::begin(range),
7                   std::end(range),
8                   std::begin(range),
9                   std::forward<F>(func));
10    // return range;
11 }
12 // =====
13 #include "svr_repmat.hpp"
14 // =====
15 auto SineElementWise(auto& input, auto constantvalue){
16     for(auto& x: input) {x = std::sin(constantvalue * x);}

```

```

17 // replace this with std::transform
18 };
19 // =====
20 #include "svr_linspace.hpp"
21 // =====
22 #include "svr_fft.hpp"
23 // =====
24 template <typename T>
25 auto abs(vector<complex<T>> inputvector){
26     vector<T> temp(inputvector.size(), 0);
27     std::transform(temp.begin(),
28                   temp.end(),
29                   temp.begin(),
30                   [](T a){return std::abs(a);});
31     return temp;
32 }
33 // =====
34 #include "svr_rand.hpp"
35 // =====
36 #include "svr_operator_star.hpp"
37 // =====
38 #include "svr_operators.hpp"
39 // =====
40 #include "svr_tensor_inits.hpp"
41 // =====
42 #include "svr_sin.hpp"
43 // =====
44 #include "svr_slice.hpp"
45 // =====
46 #include "svr_matrix_operations.hpp"
47 // =====
48 #include <boost/type_index.hpp>
49 template <typename T>
50 auto type(T inputarg){
51     std::cout <<
52     boost::typeindex::type_id_with_cvr<decltype(inputarg)>().pretty_name()<< "\n";
53 }
54 // =====
55 #include "svr_shape.hpp"
56 // =====
57 // =====
58 // =====
59 // =====
60 // =====
61 // =====
62 // =====
63 // =====
64 // =====
65 // =====
66 // =====
67 // =====
68 // =====
69 // =====
70 // =====
71 // =====
72 // =====

```

```

73 // =====
74 // =====
75 // =====
76 // =====
77 // =====
78 // =====
79 // =====
80 // =====
81 // =====
82 // =====

```

A.8 svr_WriteToCSV.hpp

```

1  // template <typename T>
2  // void fWriteVector(const vector<T>      inputvector,
3  //                  const string      filename){
4
5  //      // opening a file
6  //      std::ofstream fileobj(filename);
7  //      if (fileobj){
8  //          for(int i = 0; i<inputvector.size(); ++i){
9  //              fileobj << inputvector[i];
10 //              if(i!=inputvector.size()-1) {fileobj << ",";}
11 //              else {fileobj << "\n";}
12 //          }
13 //      }
14 // }
15 // =====
16 template <typename T>
17 void fWriteVector(const vector<T>&      inputvector,
18                 const string&          filename){
19
20     // opening a file
21     std::ofstream fileobj(filename);
22     if (!fileobj) {return;}
23
24     // writing the real parts in the first column and the imaginary parts in the
25     // second column
26     if constexpr(std::is_same_v<T, std::complex<double>> ||
27                 std::is_same_v<T, std::complex<float>> ||
28                 std::is_same_v<T, std::complex<long double>>){
29         for(int i = 0; i<inputvector.size(); ++i){
30             // adding entry
31             fileobj << inputvector[i].real() << "+" << inputvector[i].imag() << "i";
32
33             // adding delimiter
34             if(i!=inputvector.size()-1) {fileobj << ",";}
35             else {fileobj << "\n";}
36         }
37     }
38     else{
39         for(int i = 0; i<inputvector.size(); ++i){
40             fileobj << inputvector[i];

```

```

40         if(i!=inputvector.size()-1) {fileobj << ",";}
41         else {fileobj << "\n";}
42     }
43 }
44
45 // return
46 return;
47 }
48 // Matrix writing =====
49 template <typename T>
50 auto fWriteMatrix(const std::vector<std::vector<T>> inputMatrix,
51                 const string filename){
52
53     // opening a file
54     std::ofstream fileobj(filename);
55
56     // writing
57     if (fileobj){
58         for(int i = 0; i<inputMatrix.size(); ++i){
59             for(int j = 0; j<inputMatrix[0].size(); ++j){
60                 fileobj << inputMatrix[i][j];
61                 if (j!=inputMatrix[0].size()-1) {fileobj << ",";}
62                 else {fileobj << "\n";}
63             }
64         }
65     }
66     else{
67         cout << format("File-write to {} failed\n", filename);
68     }
69 }
70
71
72 template <>
73 auto fWriteMatrix(const std::vector<std::vector<std::complex<double>>> inputMatrix,
74                 const string filename){
75
76     // opening a file
77     std::ofstream fileobj(filename);
78
79     // writing
80     if (fileobj){
81         for(int i = 0; i<inputMatrix.size(); ++i){
82             for(int j = 0; j<inputMatrix[0].size(); ++j){
83                 fileobj << inputMatrix[i][j].real() << "+" <<
inputMatrix[i][j].imag() << "i";
84                 if (j!=inputMatrix[0].size()-1) {fileobj << ",";}
85                 else {fileobj << "\n";}
86             }
87         }
88     }
89     else{
90         cout << format("File-write to {} failed\n", filename);
91     }
92 }
93
94
95

```

```

96
97
98
99
100
101
102
103
104
105
106
107
108 // AN EXAMPLE OF USING is_same_v<T, std::complex<double>>=====
109
110 // #include <complex>
111 // #include <fstream>
112 // #include <vector>
113 // #include <type_traits>
114
115 // template <typename T>
116 // void fWriteVector(const std::vector<T>& inputvector, const std::string&
    filename) {
117 //     std::ofstream fileobj(filename, std::ios::out | std::ios::trunc);
118 //     if (fileobj) {
119 //         for (size_t i = 0; i < inputvector.size(); ++i) {
120 //             if constexpr (std::is_same_v<T, std::complex<double>> ||
121 //                 std::is_same_v<T, std::complex<float>>) {
122 //                 // write as "real,imag"
123 //                 fileobj << inputvector[i].real() << "," << inputvector[i].imag();
124 //             } else {
125 //                 // normal types (double, int, etc.)
126 //                 fileobj << inputvector[i];
127 //             }
128
129 //             if (i != inputvector.size() - 1)
130 //                 fileobj << ",";
131 //             else
132 //                 fileobj << "\n";
133 //         }
134 //     }
135 // }

```

A.9 svr_repmat.hpp

```

1 template<typename T>
2 constexpr auto repmat(const vector<vector<T>>& input,
3     const vector<int> dimensions){
4
5     // calculating resulting dimensions
6     auto numrows    {static_cast<int>(input.size())    * dimensions[0]};
7     auto numcols    {static_cast<int>(input[0].size()) * dimensions[1]};
8
9     // creating new matrix

```

```

10     vector<vector<T>>    finaloutput;
11     vector<T>           temp;
12     auto sourcerow    {-1};
13     auto sourcecol    {-1};
14     for(int i = 0; i<numrows; ++i){
15         temp.clear();
16         for(int j = 0; j<numcols; ++j){
17             sourcerow = i % static_cast<int>(input.size());
18             sourcecol = j % static_cast<int>(input[0].size());
19             temp.push_back(input[sourcerow][sourcecol]);
20         }
21         finaloutput.push_back(temp);
22     }
23
24     // returning the final output
25     return finaloutput;
26
27 };
28
29 template <typename T>
30 constexpr auto repmat(const vector<T>&          input,
31                      const vector<int>         dimensions){
32
33     // calculating resulting dimensions
34     auto numrows    {static_cast<int>(dimensions[0])};
35     auto numcols    {static_cast<int>(input.size() * dimensions[1])};
36
37     // creating new matrix
38     vector<vector<T>>    finaloutput;
39     vector<T>           temp;
40
41     // filling up the vector
42     auto sourcerow    {-1};
43     auto sourcecol    {-1};
44     for(int i = 0; i<numrows; ++i){
45         temp.clear();
46         for(int j = 0; j<numcols; ++j){
47             sourcerow = i % 1;
48             sourcecol = j % static_cast<int>(input.size());
49             temp.push_back(input[sourcecol]);
50         }
51         finaloutput.push_back(temp);
52     }
53
54     // returning the final output
55     return finaloutput;
56
57 };

```

A.10 svr_linspace.hpp

```

1 // in-place
2 template <typename T>

```



```

3  auto linspace(auto&          input,
4              auto          startvalue,
5              auto          endvalue,
6              auto          numpoints) -> void
7  {
8      auto stepsize = static_cast<T>(endvalue -
9      startvalue)/static_cast<T>(numpoints-1);
10     for(int i = 0; i<input.size(); ++i) {input[i] = startvalue + i*stepsize;}
11 };
12 // in-place
13 template <typename T>
14 auto linspace(vector<complex<T>>&    input,
15              auto          startvalue,
16              auto          endvalue,
17              auto          numpoints) -> void
18 {
19     auto stepsize = static_cast<T>(endvalue -
20     startvalue)/static_cast<T>(numpoints-1);
21     for(int i = 0; i<input.size(); ++i) {
22         input[i] = startvalue + static_cast<T>(i)*stepsize;
23     }
24 };
25 // return-type
26 template <typename T>
27 auto linspace(T          startvalue,
28              T          endvalue,
29              size_t      numpoints)
30 {
31     vector<T> input(numpoints);
32     auto stepsize = static_cast<T>(endvalue -
33     startvalue)/static_cast<T>(numpoints-1);
34
35     for(int i = 0; i<input.size(); ++i) {input[i] = startvalue +
36     static_cast<T>(i)*stepsize;}
37
38     return input;
39 };
40 // return-type
41 template <typename T, typename U>
42 auto linspace(T          startvalue,
43              U          endvalue,
44              size_t      numpoints)
45 {
46     vector<double> input(numpoints);
47     auto stepsize = static_cast<double>(endvalue -
48     startvalue)/static_cast<double>(numpoints-1);
49
50     for(int i = 0; i<input.size(); ++i) {input[i] = startvalue + i*stepsize;}
51
52     return input;
53 };

```



```

55         std::plus<std::complex<double>>>(),
56         [&inputarray](complex<double> a, T b){
57             return a * static_cast<complex<double>>(b) /
static_cast<double>(std::sqrt(inputarray.size()));
58         });
59     }
60
61     // returning finaloutput
62     return finaloutput;
63 }
64 // =====
65 template<typename T>
66 auto fft(vector<T> inputarray, size_t nfft){
67
68     // throwing an error
69     if (nfft < inputarray.size()) {std::cerr << "size-mismatch\n";}
70
71     // building time-only basis
72     vector<complex<T>>
73     basiswithoutfrequency = linspace(static_cast<std::complex<T>>(0),
74                                     static_cast<std::complex<T>>(nfft-1),
75                                     nfft);
76     auto lambda = [&basiswithoutfrequency](const complex<T> arg){
77         return std::exp(-1.00 * complex<double>{0, 1} * 2.00 * \
78             std::numbers::pi * static_cast<complex<T>>(arg) / \
79             static_cast<complex<T>>(basiswithoutfrequency.size()));
80     };
81     std::transform(basiswithoutfrequency.begin(),
82                   basiswithoutfrequency.end(),
83                   basiswithoutfrequency.begin(),
84                   lambda);
85
86
87     // building basis vectors
88     vector<vector<complex<double>>> basisvectors;
89     for(auto i = 0; i < inputarray.size(); ++i){
90
91         // making a copy of the basis-without-frequency
92         vector<complex<double>> temp = basiswithoutfrequency;
93
94         // exponentiating with associated frequency
95         std::transform(temp.begin(),
96                       temp.end(),
97                       temp.begin(),
98                       [i](const auto arg1){return std::pow(arg1, i);});
99
100         // pushing to end of basis vectors
101         basisvectors.push_back(std::move(temp));
102     }
103
104
105     // building the projection
106     vector<complex<double>> finaloutput(inputarray.size(), 0);
107     finaloutput.reserve(finaloutput.size());
108     #pragma omp parallel for
109     for(int i = 0; i<inputarray.size(); ++i){
110         // writing coefficients

```

```

111         finaloutput[i] = \
112             std::inner_product(basisvectors[i].begin(),
113                               basisvectors[i].end(),
114                               inputarray.begin(),
115                               complex<double>(0),
116                               std::plus<std::complex<double>>(),
117                               [&nfft](const complex<double> a,
118                                     const T b){
119                                     return a * static_cast<complex<double>>(b) /
static_cast<double>(std::sqrt(nfft));
120                                     });
121     }
122
123     // returning finaloutput
124     return finaloutput;
125 }
126
127 // =====
128 template <>
129 auto fft(vector<complex<double>> inputarray){
130
131     // aliasing
132     using T = double;
133
134     // building time-only basis-vector
135     vector<complex<T>>
basiswithoutfrequency = linspace(std::complex<T>(0),
136                                std::complex<T>(inputarray.size()-1),
137                                inputarray.size());
138
139     auto lambda = [&basiswithoutfrequency](const complex<T> arg){
140         return std::exp(-1.00 * complex<double>{0, 1} * 2.00 * \
141                     std::numbers::pi * static_cast<complex<T>>(arg) \
142                     / static_cast<complex<T>>(basiswithoutfrequency.size()));
143     };
144     std::transform(basiswithoutfrequency.begin(),
145                   basiswithoutfrequency.end(),
146                   basiswithoutfrequency.begin(),
147                   lambda);
148
149
150     // building basis vectors
151     vector<vector<complex<T>>> basisvectors;
152     for(auto i = 0; i < inputarray.size(); ++i){
153
154         // making a copy of the basis-without-frequency
155         vector<complex<T>> temp = basiswithoutfrequency;
156
157         // adding frequency component
158         std::transform(temp.begin(),
159                       temp.end(),
160                       temp.begin(),
161                       [i](const auto arg1){return std::pow(arg1, i);});
162
163         // pushing to end of basis vectors
164         basisvectors.push_back(std::move(temp));
165     }
166

```

```

167
168 // building the coefficients
169 vector< complex<T> > finaloutput(inputarray.size(), 0);
170 finaloutput.reserve(finaloutput.size());
171 for(int i = 0; i<inputarray.size(); ++i)
172     finaloutput[i] = std::inner_product(basisvectors[i].begin(),
173                                         basisvectors[i].end(),
174                                         inputarray.begin(),
175                                         std::complex<double>(0));
176
177 // scaling down the coefficients
178 std::transform(finaloutput.begin(),
179               finaloutput.end(),
180               finaloutput.begin(),
181               [&inputarray](auto argx){
182                   return argx / static_cast<T>(std::sqrt(inputarray.size()));
183               });
184
185
186 // returning finaloutput
187 return finaloutput;
188 }
189
190 // =====
191 template<typename T>
192 auto fft(std::vector<std::vector<T>> inputMatrix,
193         int nfft){
194
195     // initializing
196     std::vector<std::vector<std::complex<T>>>
197     finaloutput(inputMatrix.size(),
198                 std::vector<std::complex<T>>(inputMatrix[0].size(), 0));
199
200     // checking if we need to pad the rows
201     if (inputMatrix[0].size() > nfft) {std::cerr << "nfft < row-size\n";}
202     else if (inputMatrix[0].size() < nfft) {
203
204         // creating a placeholder
205         std::vector<std::vector<std::complex<T>>>
206         temp(inputMatrix.size(),
207             std::vector<std::complex<T>>(nfft, 0));
208
209         // moving to the finaloutput
210         finaloutput.clear();
211         finaloutput = std::move(temp);
212
213     }
214
215     // filling final-output with the input-values
216     for(int i = 0; i<inputMatrix.size(); ++i)
217         std::copy(inputMatrix[i].begin(),
218                 inputMatrix[i].end(),
219                 finaloutput[i].begin());
220
221     // performing fft
222     #pragma omp parallel for
223     for(auto& row: finaloutput) {row = fft(row);}

```

```

224     // returning the matrix
225     return finaloutput;
226
227 }
228
229 // =====
230 template<>
231 auto fft(std::vector<std::vector<std::complex<double>>> inputMatrix,
232         int nfft){
233
234     // changing types
235     using T = double;
236
237     // initializing
238     std::vector<std::vector<std::complex<T>>>
239     finaloutput(inputMatrix.size(),
240                 std::vector<std::complex<T>>(inputMatrix[0].size(), complex<T>(0)));
241
242     // checking if we need to pad the rows
243     if (inputMatrix[0].size() > nfft) {std::cerr << "nfft < row-size\n";}
244     else if (inputMatrix[0].size() < nfft) {
245
246         // creating a placeholder
247         std::vector<std::vector<std::complex<T>>>
248         temp(inputMatrix.size(),
249             std::vector<std::complex<T>>(nfft, std::complex<T>(0)));
250
251         // moving to the finaloutput
252         finaloutput.clear();
253         finaloutput = std::move(temp);
254     }
255
256     // filling final-output with the input-values
257     for(int i = 0; i<inputMatrix.size(); ++i){
258         std::copy(inputMatrix[i].begin(),
259                 inputMatrix[i].end(),
260                 finaloutput[i].begin());
261     }
262
263     // performing fft
264     for(int i = 0; i<finaloutput.size(); ++i)
265         finaloutput[i] = std::move(fft(finaloutput[i]));
266
267     // returning the matrix
268     return finaloutput;
269
270 }
271 // ifft (vector) =====
272 template <typename T>
273 auto ifft(const std::vector<T> inputvector){
274
275     // setting up data type
276     using T2 = std::conditional_t<std::is_same_v<T, std::complex<float>>,
277                                   std::complex<float>,
278                                   std::complex<double>>;
279     using T3 = typename T2::value_type;
280

```

```

281 // building basis
282 vector<T2>
283 basiswithoutfrequency {linspace(static_cast<T2>(0),
284                                static_cast<T2>(inputvector.size()-1),
285                                inputvector.size())};
286
287 // lambda for building basis without frequency
288 auto lambda = \
289     [&basiswithoutfrequency](const T2 arg){
290         return std::exp(1.00 * T2{0, 1} * 2.00 * \
291                        std::numbers::pi * arg / \
292                        static_cast<T2>(basiswithoutfrequency.size()));
293     };
294
295 // building the basis without frequency
296 std::transform(basiswithoutfrequency.begin(),
297               basiswithoutfrequency.end(),
298               basiswithoutfrequency.begin(),
299               lambda);
300
301 // building basis vectors
302 std::vector<std::vector<T2>> bases;
303 for(int i = 0; i < inputvector.size(); ++i){
304
305     // creating bases with frequency components
306     auto basiswithfrequency = basiswithoutfrequency;
307     std::transform(basiswithfrequency.begin(),
308                   basiswithfrequency.end(),
309                   basiswithfrequency.begin(),
310                   [i](T2 argx){
311                       return static_cast<T2>(std::pow(argx,
312 static_cast<T3>(i)));
313                   });
314
315     // pushing to the basis vectors
316     bases.push_back(std::move(basiswithfrequency));
317 }
318
319 // computing projections
320 std::vector<T2> projection_coefficients(inputvector.size());
321 for(auto i = 0; i < bases.size(); ++i){
322
323     // calculating inner-product
324     auto temp {std::inner_product(bases[i].begin(), bases[i].end(),
325                                   inputvector.begin(),
326                                   static_cast<T2>(0),
327                                   std::plus<T2>(),
328                                   [&inputvector](auto arg_bases, auto
329 arg_inputvector){
330
331                                     return static_cast<T2>(arg_bases) *
332                                     static_cast<T2>(arg_inputvector)
333
334                                     / \
335
336                                     static_cast<T3>(std::sqrt(inputvector.size())));
337                                     }));
338 }

```

```

333         // // calculating inner-product
334         // auto temp {std::inner_product(bases[i].begin(), bases[i].end(),
335         //                                inputvector.begin(),
336         //                                static_cast<T2>(0),
337         //                                std::plus<T2>(),
338         //                                [&inputvector](auto arg_bases, auto
arg_inputvector){
339         //                                return static_cast<T2>(arg_bases) *
        //                                \
340         //                                static_cast<T2>(arg_inputvector) / \
341         //                                static_cast<T3>((inputvector.size()));
342         //                                }));
343
344         // writing to the final output
345         projection_coefficients[i] = std::move(temp);
346     }
347
348     // returning the coefficients
349     return projection_coefficients;
350 }
351

```

A.12 svr_rand.hpp

```

1  // =====
2  template <typename T>
3  auto rand(const T min, const T max) {
4      static std::random_device rd; // Seed
5      static std::mt19937 gen(rd()); // Mersenne Twister generator
6      std::uniform_real_distribution<> dist(min, max);
7      return dist(gen);
8  }
9  // =====
10 template <typename T>
11 auto rand(const T min,
12           const T max,
13           const size_t numelements) {
14     static std::random_device rd; // Seed
15     static std::mt19937 gen(rd()); // Mersenne Twister generator
16     std::uniform_real_distribution<> dist(min, max);
17
18     // building the final output
19     vector<T> finaloutput(numelements);
20     for(int i = 0; i<finaloutput.size(); ++i) {finaloutput[i] =
static_cast<T>(dist(gen));}
21
22     return finaloutput;
23 }
24 // =====
25 template <typename T>
26 auto rand(const T argmin,

```



```

27         const T          argmax,
28         const vector<int> dimensions){
29
30         // throwing an error if dimension is greater than two
31         if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}
32
33         // creating random engine
34         static std::random_device rd;    // Seed
35         static std::mt19937 gen(rd());   // Mersenne Twister generator
36         std::uniform_real_distribution<> dist(argmin, argmax);
37
38         // building the finaloutput
39         vector<vector<T>> finaloutput;
40         for(int i = 0; i<dimensions[0]; ++i){
41             vector<T> temp;
42             for(int j = 0; j<dimensions[1]; ++j)    {temp.push_back(dist(gen));}
43             // cout << format("\t\t temp = {}\n", temp);
44
45             finaloutput.push_back(temp);
46         }
47
48         // returning the finaloutput
49         return finaloutput;
50     }
51 }
52 // =====
53 template <typename T>
54 auto rand_complex_double(const T          argmin,
55                          const T          argmax,
56                          const vector<int>& dimensions){
57
58     // throwing an error if dimension is greater than two
59     if (dimensions.size() > 2) {std::cerr << "dimensions are too high\n";}
60
61     // creating random engine
62     static std::random_device rd;    // Seed
63     static std::mt19937 gen(rd());   // Mersenne Twister generator
64     std::uniform_real_distribution<> dist(argmin, argmax);
65
66     // building the finaloutput
67     vector<vector<complex<double>>> finaloutput;
68     for(int i = 0; i<dimensions[0]; ++i){
69         vector<complex<double>> temp;
70         for(int j = 0; j<dimensions[1]; ++j)
71             {temp.push_back(static_cast<double>(dist(gen)));}
72         finaloutput.push_back(std::move(temp));
73     }
74
75     // returning the finaloutput
76     return finaloutput;
77 }

```

A.13 svr_operator_star.hpp

```

1  // scalar * vector =====
2  template <typename T>
3  auto operator*(T scalar,
4                const vector<T>& inputvector){
5      vector<T> temp(inputvector.size());
6      std::transform(inputvector.begin(),
7                    inputvector.end(),
8                    temp.begin(),
9                    [&scalar](T x){return scalar * x;});
10     return temp;
11 }
12
13 template <typename T1, typename T2>
14 auto operator*(T1 scalar,
15               const vector<T2>& inputvector){
16     using T3 = decltype(std::declval<T1>() * std::declval<T2>());
17     vector<T3> temp(inputvector.size());
18     std::transform(inputvector.begin(),
19                   inputvector.end(),
20                   temp.begin(),
21                   [&scalar](auto x){return static_cast<T3>(scalar) *
22 static_cast<T3>(x);});
23     return temp;
24 }
25
26 // // template <>
27 // auto operator*(double doublescalar,
28 //               std::vector<std::complex<double>> argvector){
29 //     std::vector<std::complex<double>> temp(argvector.size());
30 //     std::transform(argvector.begin(),
31 //                   argvector.end(),
32 //                   temp.begin(),
33 //                   [&doublescalar](complex<double> x){return
34 // static_cast<complex<double>>(doublescalar) * x;});
35 //     return temp;
36 // }
37
38 // vector * scalar =====
39 template <typename T>
40 auto operator*(const vector<T>& inputvector,
41               T scalar){
42     vector<T> temp(inputvector.size());
43     std::transform(inputvector.begin(), inputvector.end(), temp.begin(),
44                   [&scalar](T x){return scalar * x;});
45     return temp;
46 }
47
48 // scalar * matrix =====
49 template <typename T>
50 auto operator*(T scalar,
51               const std::vector<std::vector<T>>& inputMatrix){
52     std::vector<std::vector<T>> temp {inputMatrix};
53     for(int i = 0; i<inputMatrix.size(); ++i){

```

```

52         std::transform(inputMatrix[i].begin(),
53                         inputMatrix[i].end(),
54                         temp[i].begin(),
55                         [&scalar](T x){return scalar * x;});
56     }
57     return temp;
58 }
59 // scalar * matrix =====
60 template <typename T1, typename T2>
61 auto operator*(T1 scalar,
62               const std::vector<std::vector<T2>>& inputMatrix){
63     std::vector<std::vector<T2>> temp {inputMatrix};
64     for(int i = 0; i<inputMatrix.size(); ++i){
65         std::transform(inputMatrix[i].begin(),
66                         inputMatrix[i].end(),
67                         temp[i].begin(),
68                         [&scalar](T2 x){return static_cast<T2>(scalar) * x;});
69     }
70     return temp;
71 }
72
73 // matrix * matrix =====
74 template <typename T>
75 auto operator*(const std::vector<std::vector<T>>& matA,
76               const std::vector<std::vector<T>>& matB)
77 {
78
79     // throwing error
80     if (matA.size() != matB.size() || matA[0].size() != matB[0].size())
81     {std::cerr << "size issues\n";}
82
83     // creating placeholder
84     auto temp {matA};
85
86     // performing multiplication
87     for(int i = 0; i<matA.size(); ++i){
88         for(int j = 0; j<matA[0].size(); ++j){
89             temp[i][j] *= matB[i][j];
90         }
91     }
92
93     // returning
94     return temp;
95 }
96 // matrix-multiplication =====
97 template <typename T1, typename T2>
98 auto matmul(const std::vector<std::vector<T1>>& matA,
99            const std::vector<std::vector<T2>>& matB)
100 {
101
102     // throwing error
103     if (matA[0].size() != matB.size()) {std::cerr << "dimension-mismatch \n";}
104
105     // getting result-type
106     using ResultType = decltype(std::declval<T1>() * std::declval<T2>() + \
107                                std::declval<T1>() * std::declval<T2>() );

```

```

108 // creating aliases
109 auto finalnumrows {matA.size()};
110 auto finalnumcols {matB[0].size()};
111
112 // creating placeholder
113 auto rowcolproduct = [&](auto rowA, auto colB){
114     ResultType temp {0};
115     for(int i = 0; i < matA.size(); ++i) {temp +=
116         static_cast<ResultType>(matA[rowA][i]) +
117         static_cast<ResultType>(matB[i][colB]);}
118     return temp;
119 };
120
121 // producing row-column combinations
122 std::vector<std::vector<ResultType>> finaloutput(finalnumrows,
123 std::vector<ResultType>(finalnumcols));
124 for(int row = 0; row < finalnumrows; ++row){for(int col = 0; col <
125 finalnumcols; ++col){finaloutput[row][col] = rowcolproduct(row, col);}}
126
127 // returning
128 return finaloutput;
129 }
130 // =====

```

A.14 svr_operators.hpp

```

1 template <typename T>
2 std::vector<T> operator+(const std::vector<T>& a, const std::vector<T>& b) {
3     // Identify which is bigger
4     const auto& big = (a.size() > b.size()) ? a : b;
5     const auto& small = (a.size() > b.size()) ? b : a;
6
7     std::vector<T> result = big; // copy the bigger one
8
9     // Add elements from the smaller one
10    for (size_t i = 0; i < small.size(); ++i) {
11        result[i] += small[i];
12    }
13
14    return result;
15 }
16 template <typename T>
17 std::vector<T>& operator+=(std::vector<T>& a, const std::vector<T>& b) {
18
19     const auto& small = (a.size() < b.size()) ? a : b;
20     const auto& big = (a.size() < b.size()) ? b : a;
21
22     // If b is bigger, resize 'a' to match
23     if (a.size() < b.size()) {a.resize(b.size());}
24
25     // Add elements
26     for (size_t i = 0; i < small.size(); ++i) {a[i] += b[i];}
27

```

```

28     // returning elements
29     return a;
30 }
31 template <typename T>
32 std::vector<std::vector<T>> operator+(const std::vector<std::vector<T>>& a,
33                                     const std::vector<std::vector<T>>& b)
34 {
35     // throwing an error if dimension error occurs
36     if ((a.size() != b.size()) || (a[0].size() != b[0].size())) {
37         cout << format("a.dimensions = [{},{}], b.shape = [{},{}]\n",
38             a.size(), a[0].size(), b.size(), b[0].size());
39         std::cerr << "dimensions don't match\n";
40     }
41
42     // performing the addition
43     auto temp {a};
44     for(int i = 0; i<b.size(); ++i){
45         for(int j = 0; j<b[0].size(); ++j){
46             temp[i][j] += b[i][j];
47         }
48     }
49
50     // returning
51     return temp;
52 }
53
54 // =====
55 // Aim: subtracting scalar from a vector
56 template <typename T>
57 std::vector<T> operator-(const std::vector<T>& a, const T scalar){
58     std::vector<T> temp(a.size());
59     std::transform(a.begin(),
60                   a.end(),
61                   temp.begin(),
62                   [scalar](T x){return (x - scalar);});
63     return temp;
64 }
65 // scalar operators =====
66 auto operator*(const std::complex<double>    complexscalar,
67               const double                    doublescalar){
68     return complexscalar * static_cast<std::complex<double>>(doublescalar);
69 }
70 auto operator*(const double                    doublescalar,
71               const std::complex<double>    complexscalar){
72     return complexscalar * static_cast<std::complex<double>>(doublescalar);
73 }
74 auto operator*(const std::complex<double>    complexscalar,
75               const int                      scalar){
76     return complexscalar * static_cast<std::complex<double>>(scalar);
77 }
78 auto operator*(const int                      scalar,
79               const std::complex<double>    complexscalar){
80     return complexscalar * static_cast<std::complex<double>>(scalar);
81 }

```

A.15 svr_tensor_inits.hpp

```

1  std::vector<std::vector<double>> Zeros(vector<int> dimensions){
2
3      // throwing an error if the dimension is more than 2
4      if (dimensions.size() > 2) {std::cerr << "Dimensions are a little too much";}
5
6      // building the vector
7      std::vector<std::vector<double>> temp;
8      for(int i = 0; i<dimensions[0]; ++i){
9          temp.emplace_back(vector<double>(dimensions[1], 0));
10     };
11
12     // returning the output
13     return temp;
14 }
15
16 auto Zeros_complex_double(vector<int> dimensions){
17
18     // throwing an error if the dimension is more than 2
19     if (dimensions.size() > 2) {std::cerr << "Dimensions are a little too much";}
20
21     // building the vector
22     std::vector<std::vector<std::complex<double>>> temp;
23     for(int i = 0; i<dimensions[0]; ++i){
24         temp.emplace_back(std::vector<std::complex<double>>(dimensions[1],
25             std::complex<double>{0,0}));
26     };
27
28     // returning the output
29     return temp;
30 }
31 // =====
32 std::vector<std::vector<double>> Ones(vector<int> dimensions){
33
34     // throwing an error if the dimension is more than 2
35     if (dimensions.size() > 2) {std::cerr << "Dimensions are a little too much";}
36
37     // building the vector
38     std::vector<std::vector<double>> temp;
39     for(int i = 0; i<dimensions[0]; ++i){
40         temp.emplace_back(vector<double>(dimensions[1], 1));
41     };
42
43     // returning the output
44     return temp;
45 }

```

A.16 svr_sin.hpp

```

1  template <typename T>

```

```

2  auto sin(vector<T> input){
3      auto temp    {input};
4      std::transform(input.begin(),
5                      input.end(),
6                      temp.begin(),
7                      [](const T x){return std::sin(x);});
8      return temp;
9  }
10
11 template <typename T>
12 auto sin_inplace(vector<T>& input) -> void
13 {
14     std::transform(input.begin(),
15                     input.end(),
16                     input.begin(),
17                     [](const T x){return std::sin(x);});
18 }
19
20 // =====
21 template <typename T>
22 auto cosd(T input){
23     return std::cos(input * std::numbers::pi / 180);
24 }

```

A.17 svr_slice.hpp

```

1  template<typename T>
2  auto slice(const std::vector<std::vector<T>>&    inputMatrix,
3             vector<int>                          arglist)
4  {
5
6      // updating rows and columns
7      if (arglist[0] == -1)    {arglist[0] = 0;}
8      else if (arglist[0] == -2) {arglist[0] = inputMatrix.size()-1;}
9      if (arglist[1] == -1)    {arglist[1] = 0;}
10     else if (arglist[1] == -2) {arglist[1] = inputMatrix.size()-1;}
11     if (arglist[2] == -1)    {arglist[2] = 0;}
12     else if (arglist[2] == -2) {arglist[2] = inputMatrix[0].size()-1;}
13     if (arglist[3] == -1)    {arglist[3] = 0;}
14     else if (arglist[3] == -2) {arglist[3] = inputMatrix[0].size()-1;}
15
16     // storing dimension values
17     int rowsize    {arglist[1] - arglist[0] + 1};
18     int colsize    {arglist[3] - arglist[2] + 1};
19
20     // building the final-output matrix
21     std::vector<std::vector<T>> finaloutput;
22     for(int row = arglist[0]; row <= arglist[1]; ++row){
23
24         // creating empty sub-row
25         vector<T> temp(colsize, 0);
26
27         // copying corresponding region to subrow

```

```

28         std::copy(inputMatrix[row].begin() + arglist[2],
29                   inputMatrix[row].begin() + arglist[3]+1,
30                   temp.begin());
31
32         // pushing to final output
33         finaloutput.push_back(std::move(temp));
34     }
35
36     // returning the final output
37     return finaloutput;
38 }

```

A.18 svr_matrix_operations.hpp

```

1  // =====
2  template <typename T>
3  auto dotproduct(const vector<T> argx,
4                 const vector<T> argy)
5  {
6      // dimension checks
7      if (argx.size() != argy.size()) {std::cerr << "size disparity\n";}
8
9      // accumulating
10     T temp = 0;
11     for(int i = 0; i<argx.size(); ++i){
12
13         if constexpr(std::is_same_v<T, std::complex<double>> ||
14                     std::is_same_v<T, std::complex<float>> ||
15                     std::is_same_v<T, std::complex<long double>>){
16             temp += std::conj(argx[i]) * argy[i];
17         }
18         else{
19             temp += argx[i] * argy[i];
20         }
21     }
22
23
24     return temp;
25 }

```

A.19 svr_shape.hpp

```

1  // returning shape as vector =====
2  template <typename T>
3  auto shape(const T& inputTensor){
4
5      using U = std::decay_t<T>;
6
7      // type-traits checking

```



```

8     if constexpr (std::is_same_v<U, std::vector<double>>> ||
9                   std::is_same_v<U, std::vector<float>>> ||
10                  std::is_same_v<U, std::vector<int>>> ||
11                  std::is_same_v<U, std::vector<complex<double>>>> ||
12                  std::is_same_v<U, std::vector<complex<float>>>> ||
13                  std::is_same_v<U, std::vector<complex<int>>>> ||
14                  std::is_same_v<U, std::string>){
15         return std::vector<int>{static_cast<int>(inputTensor.size())};
16     }
17     else if constexpr (std::is_same_v<U, std::vector<std::vector<double>>>> ||
18                       std::is_same_v<U, std::vector<std::vector<float>>>> ||
19                       std::is_same_v<U, std::vector<std::vector<int>>>> ||
20                       std::is_same_v<U,
21 std::vector<std::vector<std::complex<double>>>>> ||
22                       std::is_same_v<U,
23 std::vector<std::vector<std::complex<float>>>>> ||
24                       std::is_same_v<U,
25 std::vector<std::vector<std::complex<int>>>>>){
26         return std::vector<int>{static_cast<int>(inputTensor.size()),
                                static_cast<int>(inputTensor[0].size())};
27     }
28 }

```


Appendix B

Octave Function Definitions

B.1 fReadCSV.m

```
1 function finalmatrix = fReadCSV(filename_string)
2     entire_text      = fileread(filename_string);
3     total_num_characters = numel(entire_text);
4     p1               = 1;
5     p2               = 1;
6     currententry     = "";
7     currline         = [];
8     finalmatrix      = [];
9
10    while(p2 <= total_num_characters)
11
12        curr_char = entire_text(p2);
13
14        if(curr_char == ',' || curr_char == "\n")
15            curr_entry = entire_text(p1:p2-1);
16            currline   = [currline, str2num(curr_entry)];
17
18            if (curr_char == "\n")
19                finalmatrix = [finalmatrix; currline];
20                currline    = [];
21            end
22
23            p1 = p2 + 1;
24        end
25        p2 += 1;
26    end
27 end
```