

GCSE Mathematics A Linear Higher

Teaching Notes





Trigonometry

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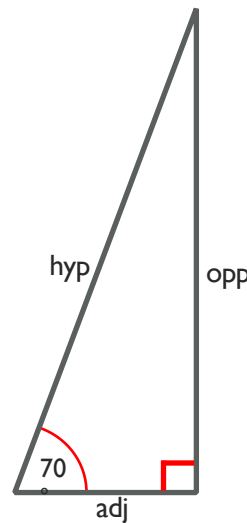
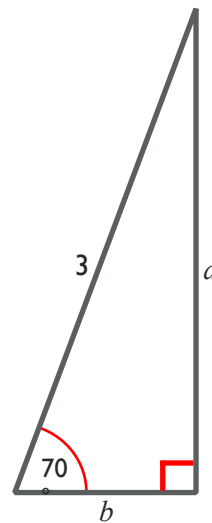
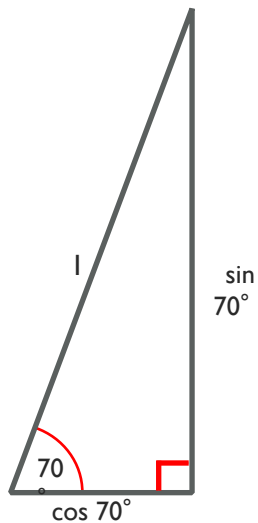
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Trigonometry in right-angled triangles

- The hypotenuse (hyp) of a right-angled triangle is the longest side of the triangle and is opposite the right angle.
 - The other two sides of the triangle are named adjacent and opposite.
 - The side opposite an angle is called the opposite side (opp).
 - The side next to this angle is called the adjacent side (adj).
- Here is a right-angled triangle with its hypotenuse of length 1.
 - The length of the opposite side (opp) in this triangle is known accurately and is called the *sine* of 70° and is written $\sin 70^\circ$.
 - Its value can be found on any scientific calculator.
 - Not all calculators are the same but the key sequence to find $\sin 70^\circ$ applies to many calculators.
- Make sure that the angle mode of your calculator is degrees, usually shown by 'D' on the calculator screen.
 - Press  Key in  Press 
 - The number  should appear on your calculator screen.
 - So correct to four decimal places, $\sin 70^\circ = 0.9397$

Trigonometry in right-angled triangles

- The length of the adjacent side (adj) is called the **cosine** of 70° and is written $\cos 70^\circ$.
 - Using a similar sequence to the one above, but using the **cos** key, correct to four decimal places, $\cos 70^\circ = 0.3420$.
- The terms sine and cosine are called trigonometric ratios, or trig ratios.
- There is another trig ratio called the **tangent** of 70° and written $\tan 70^\circ$.
 - As above, but using the **tan** key, correct to four decimal places, $\tan 70^\circ = 2.7475$.
- You can find the sine, cosine and tangent of any angle.
- Here are three right-angled triangles.



- The second triangle is an enlargement of the first triangle with a scale factor of 3.
- This means that $a = 3 \times 10 \sin 70^\circ$ or $3 \sin 70^\circ$ and $b = 3 \times 10 \cos 70^\circ$ or $3 \cos 70^\circ$.
- The third triangle is an enlargement of the first with a scale factor of hyp.
- This means that $opp = hyp \times \sin 70^\circ$ and $adj = hyp \times \cos 70^\circ$.
- These results can also be written as $\sin 70^\circ = \frac{opp}{hyp}$ and $\cos 70^\circ = \frac{adj}{hyp}$.

Trigonometry in right-angled triangles

- Results like these are true for all right-angled triangles so that

$$\circ \sin x^\circ = \frac{\text{opp}}{\text{hyp}}$$

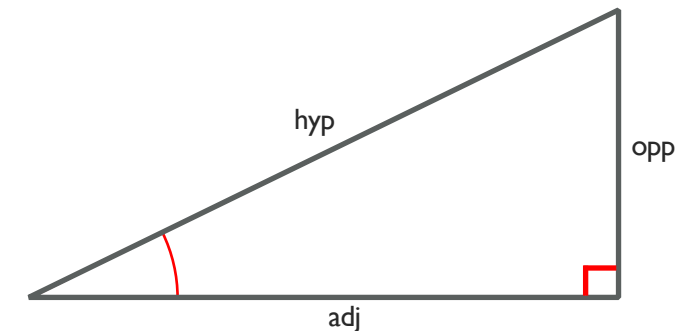
$$\cos x^\circ = \frac{\text{adj}}{\text{hyp}}$$

- When the opposite side and the adjacent side are involved

$$\circ \tan x^\circ = \frac{\text{opp}}{\text{adj}}$$

- SOHCAHTOA might help you remember these results.

• Sin Opp Hyp Cos Adj Hyp Tan Opp Adj



Example

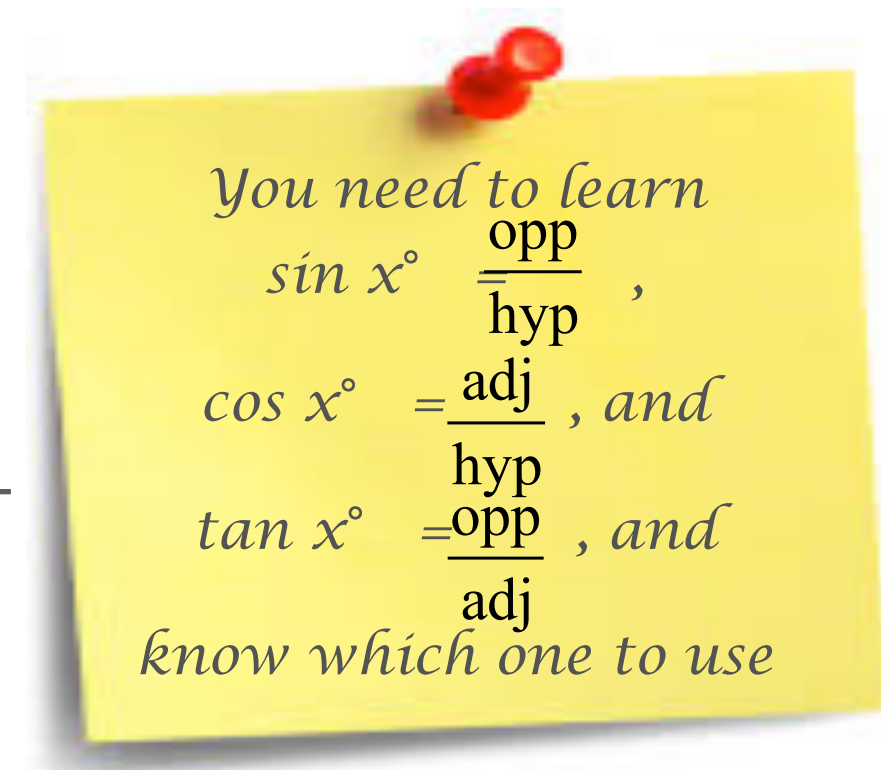
Use a calculator to write down, correct to 4 decimal places, the value of $\cos 74.6^\circ$.



Example

Find the value of x when $\tan x^\circ = 2.7$.

Give your answer correct to 1 decimal place.



Trigonometry in right-angled triangles

Quick Questions

1. Use a calculator to find the value of

$a \sin 20^\circ$	$b \sin 72.6^\circ$	$c \cos 60^\circ$	$d \cos 18.9^\circ$
$e \tan 45^\circ$	$f \tan 86.4^\circ$	$g \cos 137.8^\circ$	$h \tan 4^\circ$
$i \sin 127.2^\circ$	$j \sin 14.7^\circ$	$k \tan 159.5^\circ$	$l \cos 87.3^\circ$

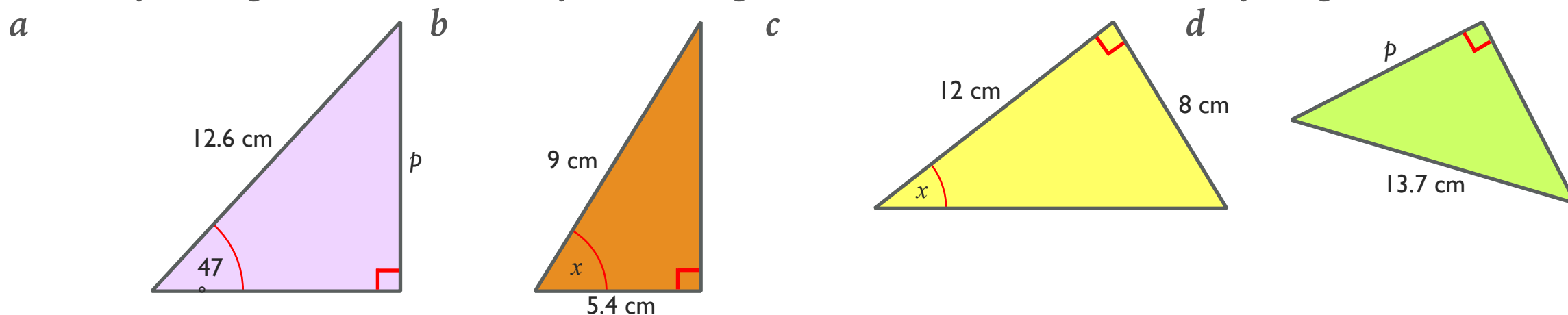
Give your each answer correct to four decimal places, where necessary.

2. Use a calculator to find the value of x when

$a \cos x^\circ = 0.6$	$b \sin x^\circ = 0.43$	$c \cos x^\circ = 0.5$
$d \tan x^\circ = 0.96$	$e \sin x^\circ = 0.8516$	$f \tan x^\circ = 2.03$
$g \sin x^\circ = 0.047$	$h \tan x^\circ = \sqrt{3}$	$i \cos x^\circ = \frac{\sqrt{2}}{2}$

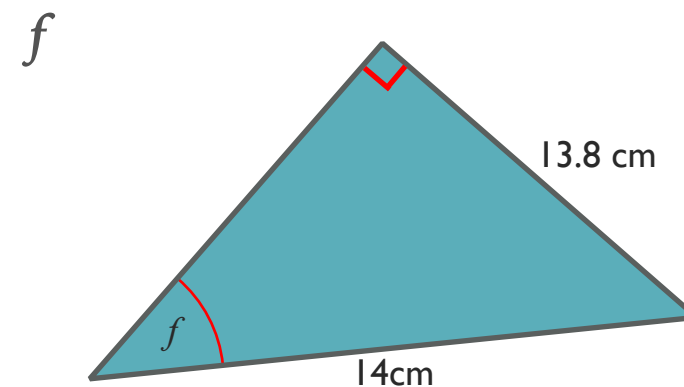
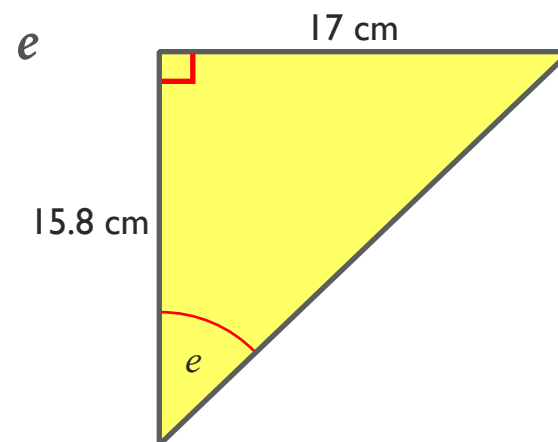
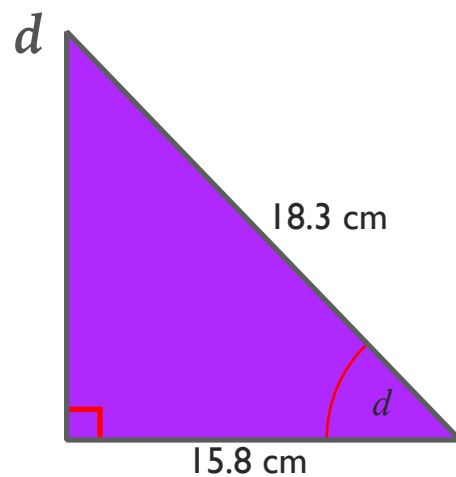
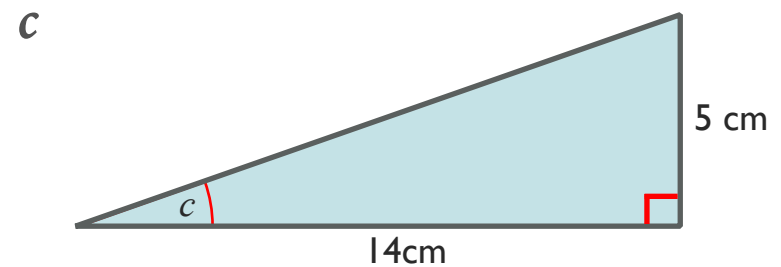
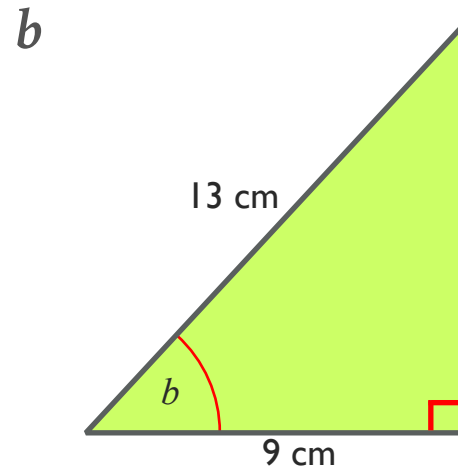
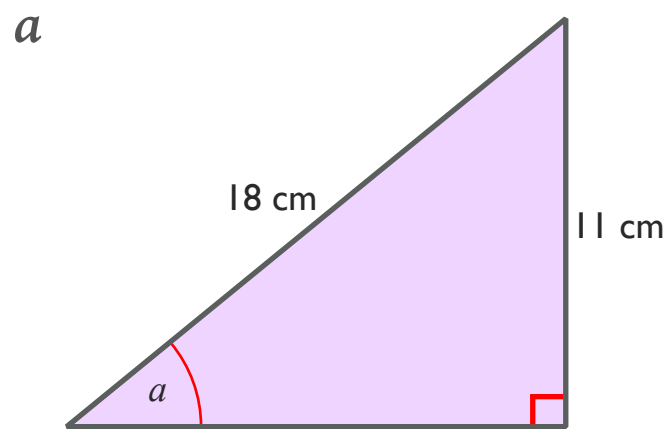
Give each answer correct to 1 decimal place where necessary.

3. Write down which trigonometric ratio is needed to calculate either the length of the side marked p or the size of the angle marked x in each of these triangles. You do not have to calculate anything.

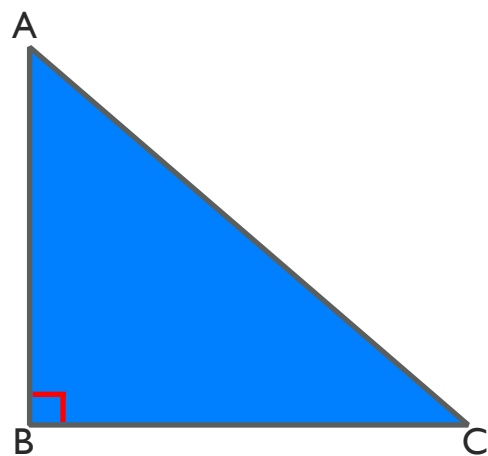


Trigonometry in right-angled triangles

4. Work out the size of each of the lettered angles. Give each answer correct to one decimal place.



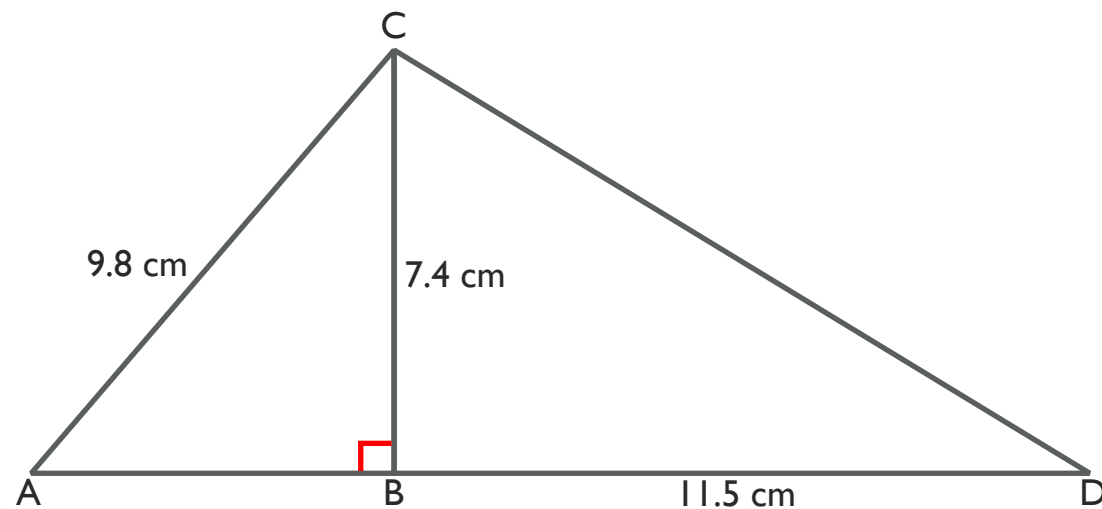
5. Triangle ABC is right-angled at B.



- a* $AB = 8.9 \text{ cm}$ and $BC = 12.1 \text{ cm}$. Calculate the size of angle ACB. Give your answer correct to 0.1° .
- b* $BC = 15.5 \text{ cm}$ and $AC = 24.7 \text{ cm}$. Calculate the size of angle BAC. Give your answer correct to 0.1° .
- c* $AB = 6.3 \text{ cm}$ and $AC = 11.8 \text{ cm}$. Calculate the size of angle ACB. Give your answer correct to 0.1° .

Trigonometry in right-angled triangles

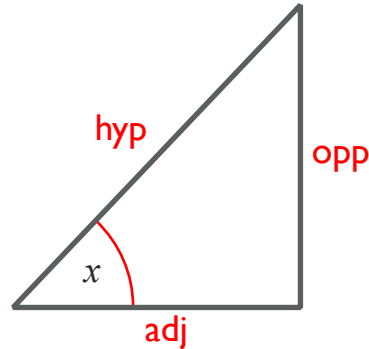
6. In triangle ACD , the point B lies on AD so that CB and AD are perpendicular.
- a* Using triangle ABC , calculate the size of angle ACB .
Give your answer correct to one decimal place.
 - b* Using triangle BCD , calculate the size of angle BCD .
Give your answer correct to one decimal place.
 - c* Hence calculate the size of angle ACD .
Give your answer correct to the nearest degree.



Working out lengths of sides using trigonometry

- The results used in the last section can be written as

- $\text{opp} = \text{hyp} \times \sin x^\circ$
- $\text{adj} = \text{hyp} \times \cos x^\circ$
- $\text{opp} = \text{adj} \times \tan x^\circ$



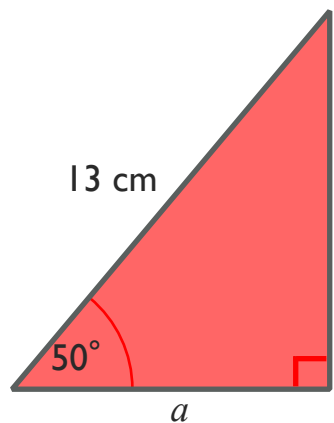
- Trigonometry can be used to solve problems.
 - Sometimes Pythagoras' Theorem is needed as well.
 - Some questions involve bearings and angles of elevation and depression.



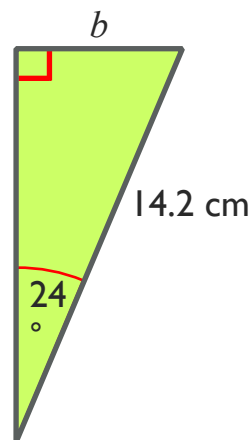
Example

Work out the length of each of the lettered sides.
Give each answer correct to 3 significant figures.

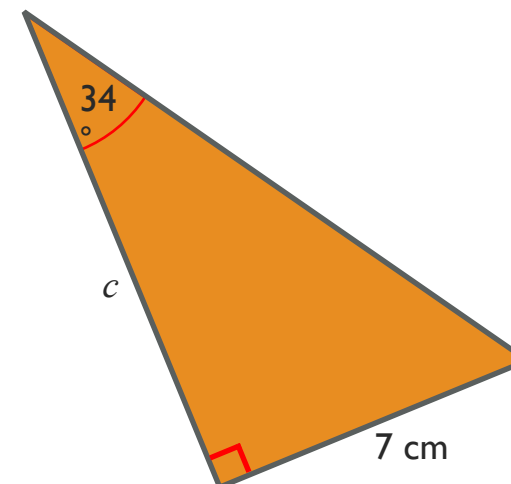
a



b



c



Working out lengths of sides using trigonometry



Example

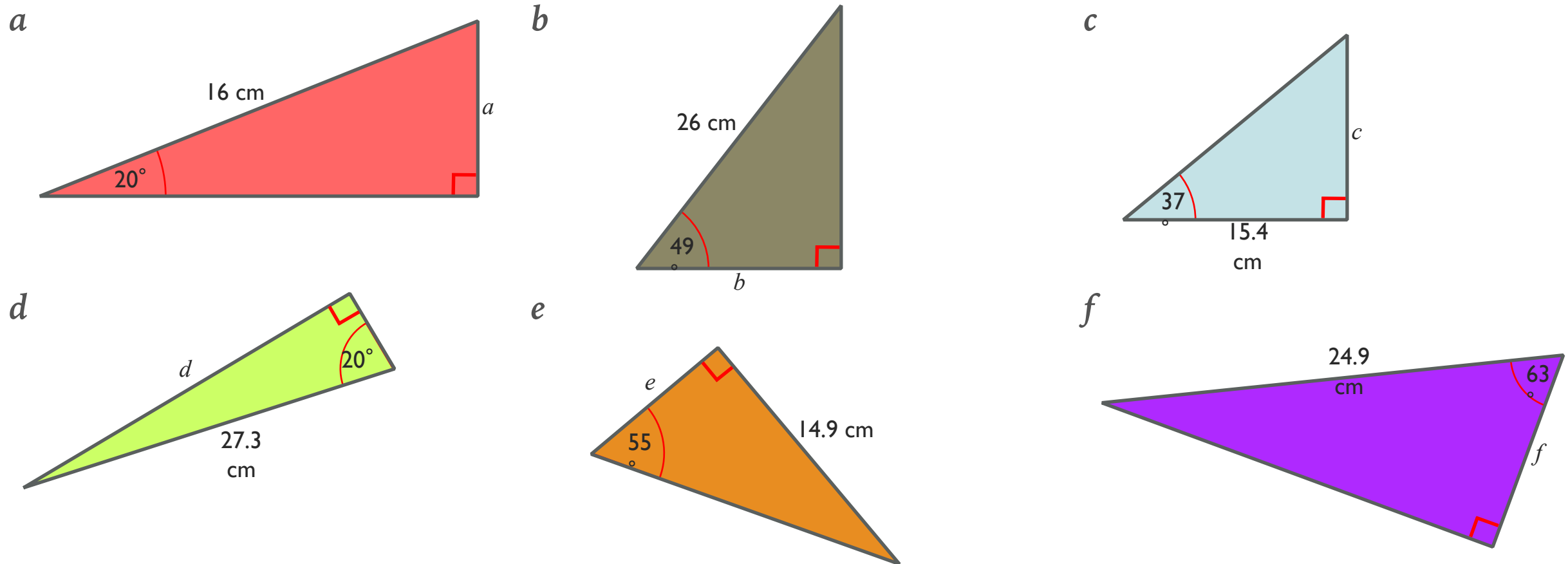
Two towns, Aytown and Beeville, are 40 km apart .
The bearing of Beeville from Aytown is 067° .

- a Calculate how far east and how far north Beeville is from Aytown.
Give your answer to 3 significant figures.
Ceeham is 60 km east of Beeville.
 - b Calculate the distance between Aytown and Ceeham.
Give your answer to the nearest km.
 - c Calculate the bearing of Ceeham from Aytown.
Give your answer to the nearest degree.
-

Working out lengths of sides using trigonometry

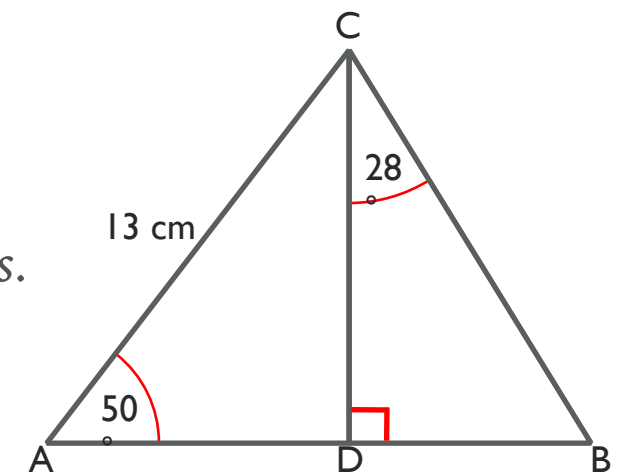
Quick Questions

1. Work out the length of each lettered side.



2. In triangle ABD, the point C lies on AD so that BC and AD are perpendicular.

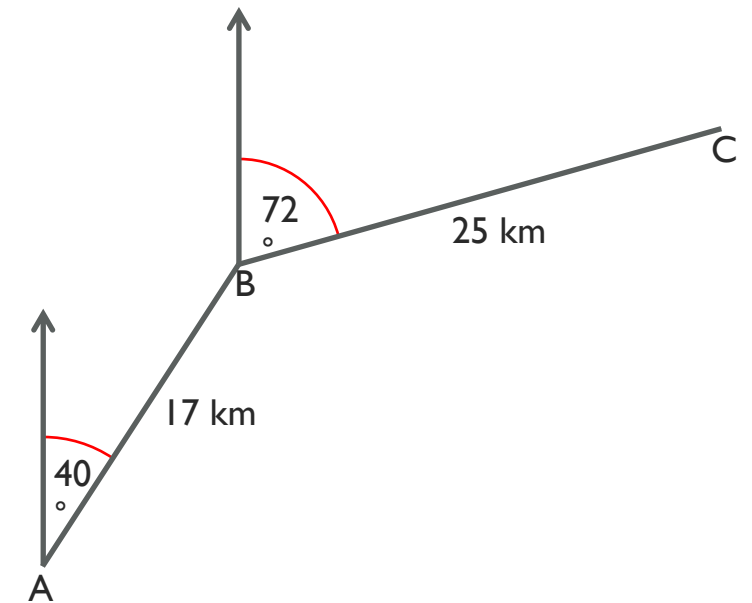
- Using triangle ABC, work out the length of
 i BC
 ii AC.
 Give each answer correct to three significant figures.
- Using triangle BCD, work out the length of CD, correct to three significant figures.
- Hence calculate the area of triangle AD, correct to three significant figures.
- Calculate the area of triangle ABD.
 Give your answer correct to the nearest cm^2 .



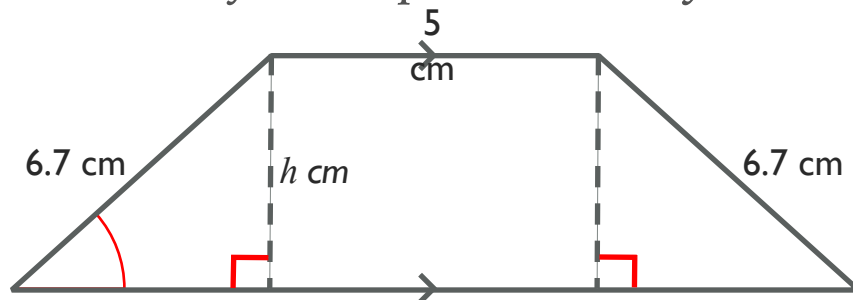
Working out lengths of sides using trigonometry

3. The points P and Q are marked on a horizontal field. The distance from P and Q is 100 m. The bearing of Q from P is 062° . Work out how far:
- Q is north of P
 - Q is east of P .

4. A , B and C are three buoys marking the course of a yacht race.
- Calculate how far B is:
 - north of A
 - east of A .
 - Calculate how far C is:
 - north of B
 - east of B .
 - Hence calculate how far C is:
 - north of A
 - east of A .
 - Calculate the distance and bearing of C from A .



5. The diagram shows an isosceles trapezium.
- Work out the distance, h cm, between the two parallel sides of the trapezium. The length of the shorter parallel side of the trapezium is 5 cm, as shown in the diagram.
 - Work out the length of the longer parallel side of the trapezium.
 - Calculate the area of the trapezium. Give your answer to the nearest cm^2 .



Homework
