CHAPTER 30

Vectors

In this chapter you will **learn how to**:

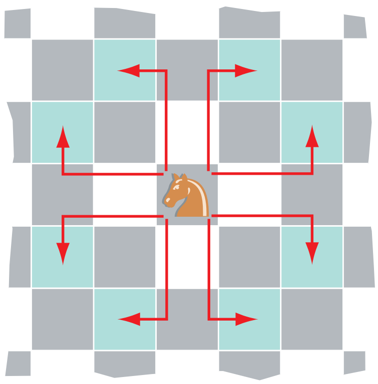
- write vectors as column vectors, and find their magnitudes
- add and subtract vectors, and multiply a vector by a number
- use vectors to prove geometric theorems.

You will also be **challenged to**:

- investigate Queens on a chessboard.

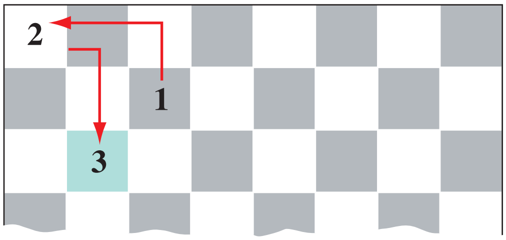
Starter: Knight's tours

When a knight moves on a chessboard, it can move two squares in a straight line and one square at right angles, like this:



A knight can also move one square in a straight line and two squares sideways.

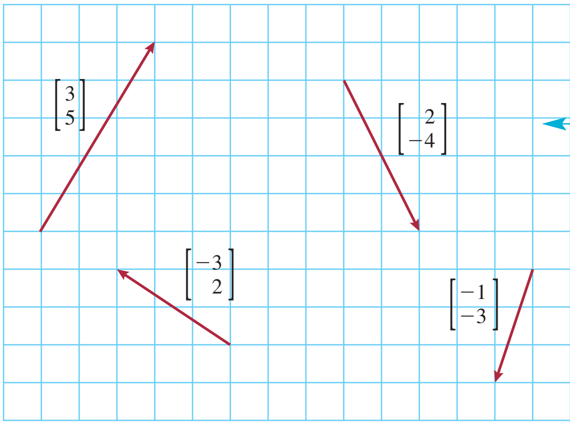
See if you can work out how to move a knight around a chessboard so that it visits all 64 squares. The first three moves have been done to start you off.



If possible, try to find a route so that the 64th square is a knight's hop away from the first square; this will close the tour so that the knight can get back to its starting position.

30.1 Introducing vectors

A **vector** is a quantity that has a magnitude (length) and a direction. Vectors are often described using **column vector** notation such as $\begin{bmatrix}3\\5\end{bmatrix}$. You have already met column vector notation in Chapter 15. Here are some diagrams to remind you how the notation works:



The column vector $\begin{bmatrix}x\\y\end{bmatrix}$ means that the vector can be drawn by going x units to the *right* and y units *up*.
(*Negative* values indicate *left* or *down*.)

The magnitude, or modulus, of a vector is simply its length, regardless of direction. In simple cases the magnitude of a vector may be seen by inspection, but often Pythagoras' theorem is needed.

EXAMPLE

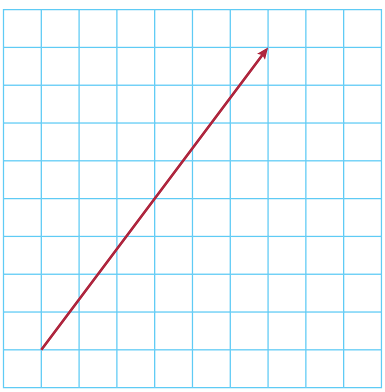
a) Illustrate the vector $\mathbf{a} = \begin{bmatrix}6\\8\end{bmatrix}$ on a square grid.

b) Work out the magnitude of the vector $\mathbf{a}$.

The vector $\mathbf{a} = \begin{bmatrix}6\\8\end{bmatrix}$ means 6 units to the *right* (in the x direction) and 8 units *up* (in the y direction).

SOLUTION

a)



b) Magnitude of $\mathbf{a}$ is $\sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

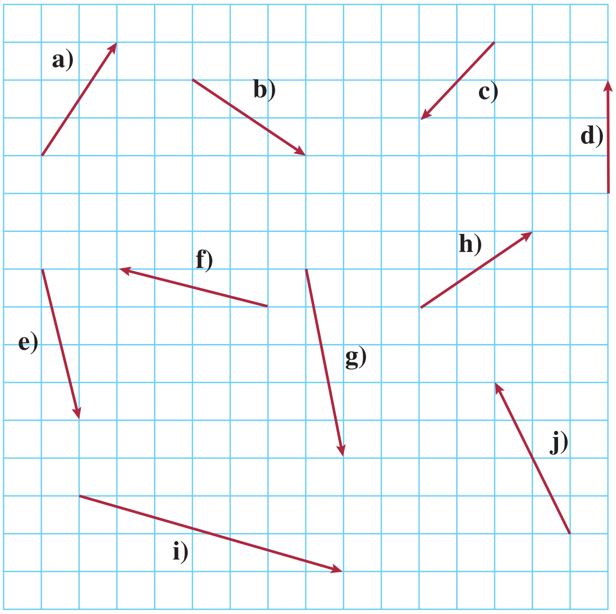
$= \sqrt{100}$

$= 10$

The magnitude of **a** is its length, so we can use Pythagoras' theorem to find it.

EXERCISE 30.1

1 The diagram below shows some vectors drawn on a grid of unit squares.
Write down column vectors to describe each one.



2 Draw a sketch of each of these vectors on squared paper.

a) $\begin{bmatrix} 7 \\ 1 \end{bmatrix}$ **b)** $\begin{bmatrix} -5 \\ 5 \end{bmatrix}$ **c)** $\begin{bmatrix} 4 \\ -6 \end{bmatrix}$ **d)** $\begin{bmatrix} -3 \\ -2 \end{bmatrix}$

3 Calculate the magnitude of each of the following vectors:

a) $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ **b)** $\begin{bmatrix} 5 \\ 12 \end{bmatrix}$ **c)** $\begin{bmatrix} 9 \\ 12 \end{bmatrix}$ **d)** $\begin{bmatrix} 24 \\ 7 \end{bmatrix}$ **e)** $\begin{bmatrix} -4 \\ 7.5 \end{bmatrix}$

30.2 Adding and subtracting vectors

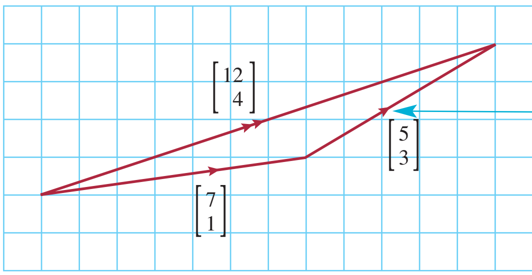
You can add two vectors using simple arithmetic. For example:

$$\begin{bmatrix}7\\1\end{bmatrix}+\begin{bmatrix}5\\3\end{bmatrix}=\begin{bmatrix}7+5\\1+3\end{bmatrix}=\begin{bmatrix}12\\4\end{bmatrix}$$

Subtraction is done in a similar way. For example:

$$\begin{bmatrix}8\\4\end{bmatrix}-\begin{bmatrix}2\\-3\end{bmatrix}=\begin{bmatrix}8-2\\4--3\end{bmatrix}=\begin{bmatrix}6\\7\end{bmatrix}$$

Geometrically, addition corresponds to placing the two vectors head to tail like this:



The sum of the two vectors is called the **resultant**.

Vectors are often named using letter **a**, **b**, **c**, etc. The letters are usually underlined if written by hand, but they are in **bold type** in examination papers and textbooks.

EXAMPLE

The vectors **a**, **b** and **c** are given by $\mathbf{a}=\begin{bmatrix}6\\1\end{bmatrix}$, $\mathbf{b}=\begin{bmatrix}2\\-3\end{bmatrix}$ and $\mathbf{c}=\begin{bmatrix}4\\2\end{bmatrix}$

Work out: **a)** $\mathbf{a}+\mathbf{b}$ **b)** $\mathbf{a}-\mathbf{c}$ **c)** $\mathbf{a}-\mathbf{b}+\mathbf{c}$

SOLUTION

a) $\mathbf{a}+\mathbf{b}=\begin{bmatrix}6\\1\end{bmatrix}+\begin{bmatrix}2\\-3\end{bmatrix}=\begin{bmatrix}6+2\\1+-3\end{bmatrix}=\begin{bmatrix}8\\-2\end{bmatrix}$

b) $\mathbf{a}-\mathbf{c}=\begin{bmatrix}6\\1\end{bmatrix}-\begin{bmatrix}4\\2\end{bmatrix}=\begin{bmatrix}6-4\\1-2\end{bmatrix}=\begin{bmatrix}2\\-1\end{bmatrix}$

c) $\mathbf{a}-\mathbf{b}+\mathbf{c}=\begin{bmatrix}6\\1\end{bmatrix}-\begin{bmatrix}2\\-3\end{bmatrix}+\begin{bmatrix}4\\2\end{bmatrix}=\begin{bmatrix}6-2+4\\1--3+2\end{bmatrix}=\begin{bmatrix}8\\0\end{bmatrix}$

To find the negative of a vector, just reverse the signs of the numbers.

For example, if $\mathbf{a} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ then $-\mathbf{a} = \begin{bmatrix} -5 \\ -2 \end{bmatrix}$

When drawn on a grid, the vector $-\mathbf{a}$ will be *parallel* to the vector $\mathbf{a}$, but will point in the *opposite direction*.

EXAMPLE

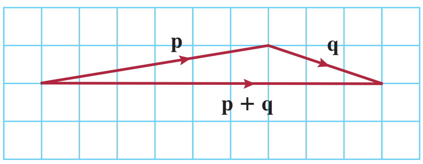
Given that $\mathbf{p} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$ and $\mathbf{q} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, work out:

a) $\mathbf{p} + \mathbf{q}$ **b)** $\mathbf{p} - \mathbf{q}$

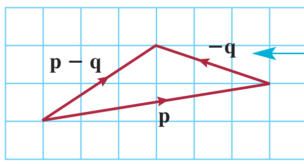
Illustrate your answers graphically.

SOLUTION

a) $\mathbf{p} + \mathbf{q} = \begin{bmatrix} 6 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$



b) $\mathbf{p} - \mathbf{q} = \begin{bmatrix} 6 \\ 1 \end{bmatrix} - \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$



$-\mathbf{q} = -\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

EXERCISE 30.2

The vectors **a**, **b** and **c** are given by $\mathbf{a} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -3 \\ 8 \end{bmatrix}$ and $\mathbf{c} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$.

Work out each of these as a column vector. Illustrate your answer with a diagram.

1 $\mathbf{a} + \mathbf{b}$ **2** $\mathbf{b} - \mathbf{c}$ **3** $\mathbf{a} + \mathbf{c}$ **4** $\mathbf{c} - \mathbf{b}$

5 Work out $\mathbf{a} - \mathbf{c} + \mathbf{b}$ **6** Work out $\mathbf{b} - \mathbf{a} + \mathbf{c}$

6 The vectors **p**, **q** and **r** are given by $\mathbf{p} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} 8 \\ 5 \end{bmatrix}$.

Work out each of these as a column vector. Illustrate your answer with a diagram.

7 $\mathbf{p} - \mathbf{q}$ **8** $\mathbf{q} + \mathbf{r}$ **9** $\mathbf{r} - \mathbf{p}$ **10** $\mathbf{r} - \mathbf{q}$

11 Work out $\mathbf{p} + \mathbf{q} + \mathbf{r}$ **12** Work out $\mathbf{p} - \mathbf{r} + \mathbf{q}$

13 You are given that $\begin{bmatrix} 5 \\ 6 \end{bmatrix} + \begin{bmatrix} 1 \\ x \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$. Find the value of x.

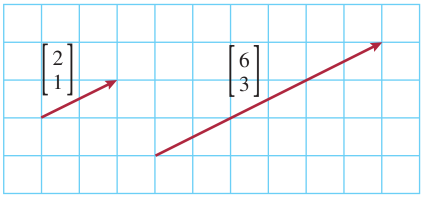
14 You are given that $\begin{bmatrix} x \\ 6 \end{bmatrix} - \begin{bmatrix} 7 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ y \end{bmatrix}$. Find the values of x and y.

15 You are given that $\begin{bmatrix} 5 \\ y \end{bmatrix} + \begin{bmatrix} x \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ 12 \end{bmatrix}$. Find the values of x and y.

30.3 Multiplying a vector by a number (scalar multiplication)

You can **multiply** a vector by an ordinary number, say k. The direction of the vector remains unaltered, but the magnitude is changed by factor k.

For example, $3\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}3\times 2\\3\times 1\end{bmatrix} = \begin{bmatrix}6\\3\end{bmatrix}$



Questions about multiplication are often combined with addition and subtraction.

EXAMPLE

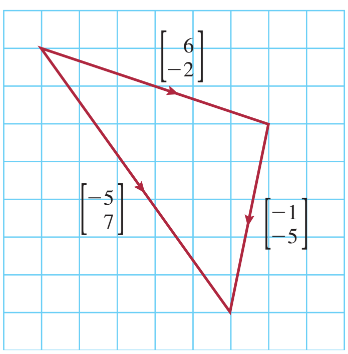
The vectors **a**, **b** and **c** are given by $\mathbf{a} = \begin{bmatrix}3\\-1\end{bmatrix}$, $\mathbf{b} = \begin{bmatrix}4\\2\end{bmatrix}$ and $\mathbf{c} = \begin{bmatrix}1\\5\end{bmatrix}$

a) Work out 3**a**. Give your answer as a column vector.
b) Work out 2**a** − **c**. Give your answer as a column vector. Illustrate with a diagram.
c) Work out 4**a** − 3**b** + 2**c**.

SOLUTION

a) $3\mathbf{a} = 3\begin{bmatrix}3\\-1\end{bmatrix} = \begin{bmatrix}3\times 3\\3\times -1\end{bmatrix} = \begin{bmatrix}9\\-3\end{bmatrix}$

b) $2\mathbf{a} - \mathbf{c} = 2\begin{bmatrix}3\\-1\end{bmatrix} - \begin{bmatrix}1\\5\end{bmatrix} = \begin{bmatrix}6\\-2\end{bmatrix} - \begin{bmatrix}1\\5\end{bmatrix} = \begin{bmatrix}5\\-7\end{bmatrix}$



c) $4\mathbf{a} - 3\mathbf{b} + 2\mathbf{c} = 4\begin{bmatrix}3\\-1\end{bmatrix} - 3\begin{bmatrix}4\\2\end{bmatrix} + 2\begin{bmatrix}1\\5\end{bmatrix} = \begin{bmatrix}12\\-4\end{bmatrix} - \begin{bmatrix}12\\6\end{bmatrix} + \begin{bmatrix}2\\10\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix}$

EXERCISE 30.3

The vectors **a**, **b** and **c** are given by $\mathbf{a} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ and $\mathbf{c} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Work out:

1 $3\mathbf{a}$

2 $2\mathbf{b} + \mathbf{c}$

3 $\mathbf{a} + 3\mathbf{c}$

4 $3\mathbf{c} - 5\mathbf{b}$

5 $4\mathbf{a} + 5\mathbf{b}$

6 $2\mathbf{a} - 4\mathbf{c}$

The vectors **p**, **q** and **r** are given by $\mathbf{p} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

Work out:

7 $5\mathbf{p}$

8 $-3\mathbf{r}$

9 $2\mathbf{r} - 3\mathbf{p}$

10 $4\mathbf{p} + 2\mathbf{q} + \mathbf{r}$

11 $5\mathbf{r} - 3\mathbf{q}$

12 $2\mathbf{p} - 3\mathbf{r} + \mathbf{q}$

13 You are given that $3\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ x \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$. Find the value of x.

14 You are given that $3\begin{bmatrix} 1 \\ 4 \end{bmatrix} - 2\begin{bmatrix} 1 \\ -5 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$. Find the values of x and y.

15 You are given that $4\begin{bmatrix} x \\ 5 \end{bmatrix} + 2\begin{bmatrix} 4 \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ 18 \end{bmatrix}$. Find the values of x and y.

30.4 Using vectors

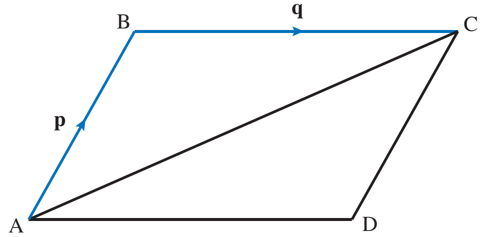
You can use vectors to solve geometric problems, and to prove some theorems about parallel lines. If one vector is a (scalar) multiple of another, then the two vectors must be parallel. The size of the multiple will tell you the scale factor.

In these problems it is often helpful to use $\overrightarrow{AB}$, for example, to represent the vector that would translate you from A to B. You can always rewrite the vector if you need to travel via an intermediate point P:

$$\overrightarrow{AB} = \overrightarrow{AP} + \overrightarrow{PB}$$

EXAMPLE

ABCD is a parallelogram. $\overrightarrow{AB} = \mathbf{p}$, $\overrightarrow{BC} = \mathbf{q}$.



Find, in terms of **p** and **q**, expressions for:

a) $\overrightarrow{BA}$ **b)** $\overrightarrow{AC}$ **c)** $\overrightarrow{BD}$

SOLUTION

a) $\overrightarrow{BA} = -\overrightarrow{AB}$
$= \underline{-\mathbf{p}}$

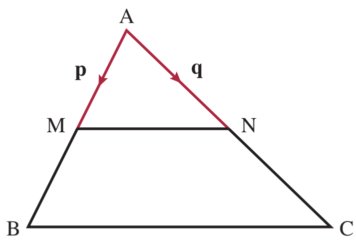
b) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$
$= \underline{\mathbf{p} + \mathbf{q}}$

c) $\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD}$
$= \underline{-\mathbf{p} + \mathbf{q}}$

EXAMPLE

The diagram shows a triangle ABC.
M is the midpoint of AB and
N is the midpoint of AC.

$\overrightarrow{AM} = \mathbf{p}$ and $\overrightarrow{AN} = \mathbf{q}$.



a) Find an expression for $\overrightarrow{MN}$ in terms of **p** and **q**.

b) Find an expression for $\overrightarrow{BC}$ in terms of **p** and **q**.

c) Use your results from **a)** and **b)** to prove that MN is parallel to BC.

SOLUTION

a) $\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AM}$
$= -\overrightarrow{AM} + \overrightarrow{AN}$
$= (-\mathbf{p}) + \mathbf{q}$
$= \underline{-\mathbf{p} + \mathbf{q}}$

To get $\overrightarrow{MN}$ in terms of **p** and **q**, go from M to N via the point A.

$\overrightarrow{MA}$ has the same length as $\overrightarrow{AM}$ but points in the opposite direction, so $\overrightarrow{MA} = -\overrightarrow{AM}$

b) $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$
$= (-2\mathbf{p}) + 2\mathbf{q}$
$= \underline{-2\mathbf{p} + 2\mathbf{q}}$

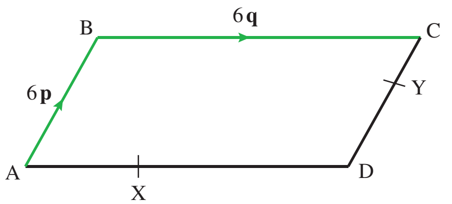
c) $\overrightarrow{BC} = -2\mathbf{p} + 2\mathbf{q}$
$= 2(-\mathbf{p} + \mathbf{q})$
$= 2 \times \overrightarrow{MN}$

Therefore BC is parallel to MN

Some exam questions might refer to a line being divided in a certain *ratio*. For example, you might be told that X is the point on AB for which AX : XB = 2 : 1. This simply means that AX is twice as long as XB, so that X is two-thirds of the way along AB.

EXAMPLE

The diagram shows a parallelogram ABCD. $\overrightarrow{AB} = 6\mathbf{p}$ and $\overrightarrow{BC} = 6\mathbf{q}$.



X is the point on AD for which AX : XD = 1 : 2
Y is the point on DC for which DY : YC = 2 : 1

Find, in terms of **p** and **q**, expressions for:

a) $\overrightarrow{AC}$ **b)** $\overrightarrow{AD}$ **c)** $\overrightarrow{DC}$
d) $\overrightarrow{XD}$ **e)** $\overrightarrow{DY}$ **f)** $\overrightarrow{XY}$

Hence prove that AC is parallel to XY.

SOLUTION

a) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$
$= \underline{6\mathbf{p} + 6\mathbf{q}}$

b) $\overrightarrow{AD} = \overrightarrow{BC}$ since they are opposite sides of the parallelogram
$= \underline{6\mathbf{q}}$

c) $\overrightarrow{DC} = \overrightarrow{AB}$ since they are opposite sides of the parallelogram
$= \underline{6\mathbf{p}}$

d) $\overrightarrow{XD} = \frac{2}{3} \times \overrightarrow{AD}$
$= \frac{2}{3} \times 6\mathbf{q}$
$= \underline{4\mathbf{q}}$

e) $\overrightarrow{DY} = \frac{2}{3} \times \overrightarrow{DC}$
$= \frac{2}{3} \times 6\mathbf{p}$
$= \underline{4\mathbf{p}}$

f) $\overrightarrow{XY} = \overrightarrow{XD} + \overrightarrow{DY}$
$= 4\mathbf{q} + 4\mathbf{p}$
$= \underline{4\mathbf{p} + 4\mathbf{q}}$

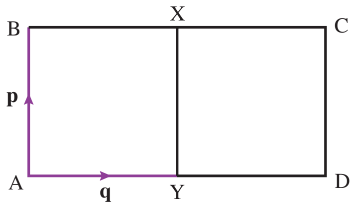
Now $\overrightarrow{AC} = 6\mathbf{p} + 6\mathbf{q} = 6(\mathbf{p} + \mathbf{q})$

and $\overrightarrow{XY} = 4\mathbf{p} + 4\mathbf{q} = 4(\mathbf{p} + \mathbf{q})$

Thus $\overrightarrow{AC} = 1.5 \times \overrightarrow{XY}$, and therefore <u>AC is parallel to XY</u>

EXERCISE 30.4

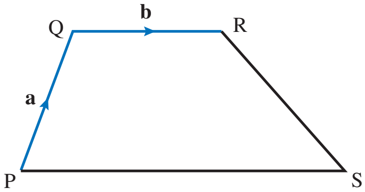
1 The diagram shows two squares ABXY and CDYX.
$\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AY} = \mathbf{q}$.



Find, in terms of **p** and **q**, expressions for:

a) $\overrightarrow{BX}$ **b)** $\overrightarrow{AX}$ **c)** $\overrightarrow{AD}$ **d)** $\overrightarrow{AC}$

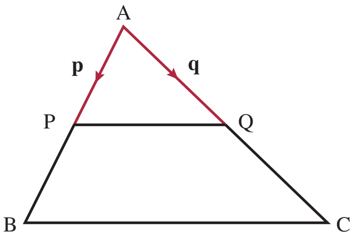
2 The diagram shows a trapezium PQRS.
$\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{QR} = \mathbf{b}$.
PS is twice the length of QR.



Find, in terms of **p** and **q**, expressions for:

a) $\overrightarrow{QP}$ **b)** $\overrightarrow{PR}$ **c)** $\overrightarrow{PS}$ **d)** $\overrightarrow{QS}$

3 The diagram shows a triangle ABC. AP $= \frac{1}{3}$AB, and AQ $= \frac{1}{3}$AC.
$\overrightarrow{AP} = \mathbf{p}$ and $\overrightarrow{AQ} = \mathbf{q}$.



a) Find, in terms of **p** and **q**, expressions for:
(i) $\overrightarrow{PQ}$ **(ii)** $\overrightarrow{AB}$ **(iii)** $\overrightarrow{AC}$ **(iv)** $\overrightarrow{BC}$

b) Use your results from **a)** to prove that PQ is parallel to BC.

4 A quadrilateral ABCD is made by joining points A (1, 1), B (5, 8), C (11, 11) and D (7, 4).

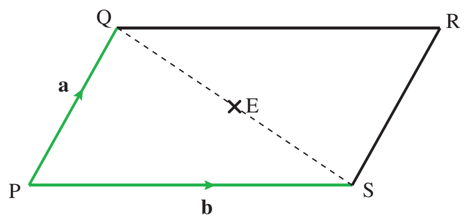
a) Write column vectors for:
(i) $\overrightarrow{AB}$ **(ii)** $\overrightarrow{DC}$

b) What do your answers to part **a)** tell you about AB and DC?

c) Write column vectors for:
(i) $\overrightarrow{BC}$ **(ii)** $\overrightarrow{AD}$

d) What kind of quadrilateral is ABCD?

5 The diagram shows a parallelogram PQRS.
$\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{PS} = \mathbf{b}$.
E is the mid-point of QS.



a) Find, in terms of **a** and **b**:
(i) $\overrightarrow{QS}$ **(ii)** $\overrightarrow{QE}$ **(iii)** $\overrightarrow{PE}$

b) Explain why $\overrightarrow{SR} = \mathbf{a}$.

c) Find $\overrightarrow{PR}$ in terms of **a** and **b**.

d) What can you deduce about the diagonals of a parallelogram?

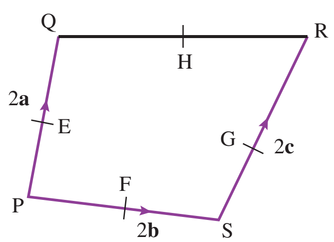
6 A quadrilateral ABCD is made by joining A $(-3, -3)$, B $(9, 3)$, C $(3, 7)$ and D $(-1, 5)$.

a) Write column vectors for:
(i) $\overrightarrow{AB}$ **(ii)** $\overrightarrow{DC}$

b) What do your answers to part **a)** tell you about AB and DC?

c) What kind of quadrilateral is ABCD?

7 The diagram shows a quadrilateral PQRS.
$\overrightarrow{PQ} = 2\mathbf{a}$, $\overrightarrow{PS} = 2\mathbf{b}$ and $\overrightarrow{SR} = 2\mathbf{c}$.
E, F, G and H are the mid-points of PQ, PS, SR and QR respectively.



a) Explain why $\overrightarrow{QR} = -2\mathbf{a} + 2\mathbf{b} + 2\mathbf{c}$.

b) Find $\overrightarrow{EH}$ in terms of **a**, **b** and **c**.

c) Find $\overrightarrow{FG}$ in terms of **a**, **b** and **c**.

d) What can you deduce about the line segments EH and FG?

e) What type of quadrilateral is EFGH?

REVIEW EXERCISE 30

1 Given that $3\begin{bmatrix} x \\ 5 \end{bmatrix} - \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 16 \\ y \end{bmatrix}$, find the values of x and y.

2 P is the point (5, 4) and Q is the point (−1, 12).

a) Write $\overrightarrow{PQ}$ and $\overrightarrow{QP}$ as column vectors.

b) Work out the length of the vector $\overrightarrow{PQ}$.

3 A is the point (2, 3) and B is the point (−2, 0).

a) **(i)** Write $\overrightarrow{AB}$ as a column vector.

(ii) Find the length of the vector $\overrightarrow{AB}$.

D is the point such that $\overrightarrow{BD}$ is parallel to $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and the length of $\overrightarrow{AD}$ = the length of $\overrightarrow{AB}$.

O is the point (0, 0).

b) Find $\overrightarrow{OD}$ as a column vector.

C is a point such that ABCD is a rhombus. AC is a diagonal of the rhombus.

c) Find the coordinates of C. **[Edexcel]**

4 OPQ is a triangle.
R is the midpoint of OP.
S is the midpoint of PQ.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.

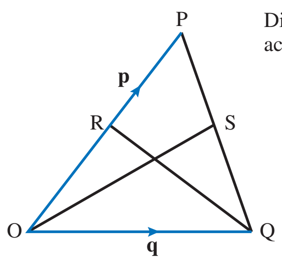


Diagram *not* accurately drawn

a) Find $\overrightarrow{OS}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

b) Show that RS is parallel to OQ. **[Edexcel]**

5 OPQ is a triangle.
T is the point on PQ for which PT : TQ = 2 : 1.
$\overrightarrow{OP} = \mathbf{a}$ and $\overrightarrow{OQ} = \mathbf{b}$.

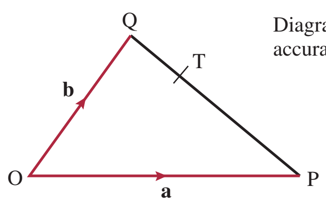
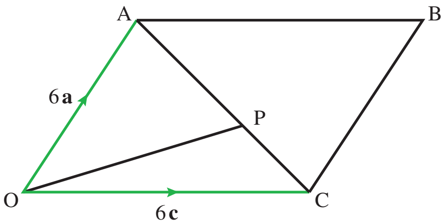


Diagram *not* accurately drawn

a) Write down, in terms of $\mathbf{a}$ and $\mathbf{b}$, an expression for $\overrightarrow{PQ}$.

b) Express $\overrightarrow{OT}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. Give your answer in its simplest form. **[Edexcel]**

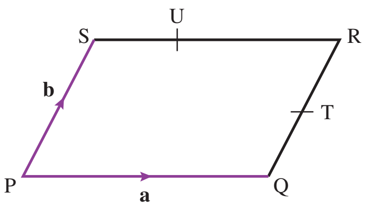
6 OABC is a parallelogram.
P is the point on AC such that $AP = \frac{2}{3}AC$.
$\overrightarrow{OA} = 6\mathbf{a}$ and $\overrightarrow{OC} = 6\mathbf{c}$.



a) Find the vector $\overrightarrow{OP}$. Give your answer in terms of **a** and **c**.

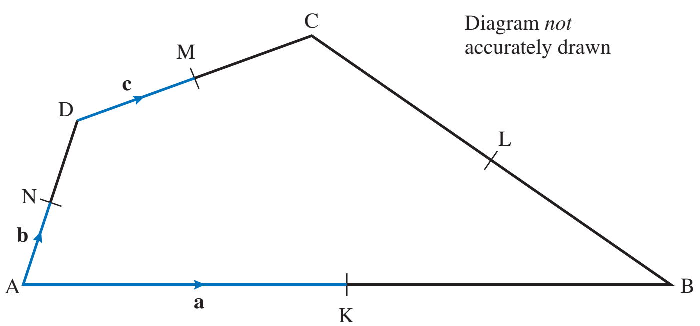
The midpoint of CB is M.
b) Prove that OPM is a straight line. **[Edexcel]**

7 PQRS is a parallelogram.
T is the midpoint of QR.
U is the point on SR for which SU **:** UR = 1 **:** 2.
$\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{PS} = \mathbf{b}$.



Write down, in terms of **a** and **b**, expressions for:
a) $\overrightarrow{PT}$
b) $\overrightarrow{TU}$ **[Edexcel]**

8 ABCD is a quadrilateral.



K is the midpoint of AB. L is the midpoint of BC.
M is the midpoint of CD. N is the midpoint of AD.
$\overrightarrow{AK} = \mathbf{a}$, $\overrightarrow{AN} = \mathbf{b}$ and $\overrightarrow{DM} = \mathbf{c}$

a) Find, in terms of **a**, **b** and **c**, the vectors:
(i) $\overrightarrow{KN}$ **(ii)** $\overrightarrow{AC}$ **(iii)** $\overrightarrow{BC}$ **(iv)** $\overrightarrow{LM}$

b) Write down two geometrical facts about the lines KN and LM which could be deduced from your answers to part **a)**. **[Edexcel]**

9 The diagram shows a regular hexagon ABCDEF with centre O.

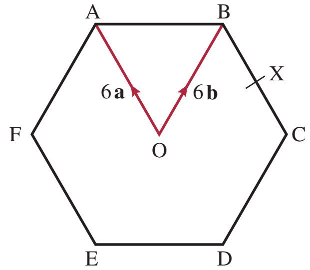


Diagram *not* accurately drawn

$\overrightarrow{OA} = 6\mathbf{a}$ and $\overrightarrow{OB} = 6\mathbf{b}$.

a) Express in terms of **a** and/or **b**.

(i) $\overrightarrow{AB}$ **(ii)** $\overrightarrow{EF}$

X is the midpoint of BC.

b) Express $\overrightarrow{EX}$ in terms of **a** and/or **b**.

Y is the point on AB extended, such that AB **:** BY = 3 **:** 2.

c) Prove that E, X and Y lie on the same straight line. **[Edexcel]**

10 OPQR is a trapezium. PQ is parallel to OR. $\overrightarrow{OP} = \mathbf{b}$, $\overrightarrow{PQ} = 2\mathbf{a}$, $\overrightarrow{OR} = 6\mathbf{a}$.
M is the midpoint of PQ. N is the midpoint of OR.

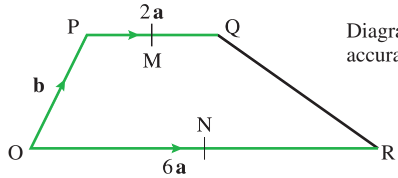


Diagram *not* accurately drawn

a) Find, in terms of **a** and **b**, the vectors:

(i) $\overrightarrow{OM}$ **(ii)** $\overrightarrow{MN}$

X is the midpoint of MN.

b) Find, in terms of **a** and **b**, the vector $\overrightarrow{OX}$.

The lines OX and PQ are extended to meet at the point Y.

c) Find, in terms of **a** and **b**, the vector $\overrightarrow{NY}$. **[Edexcel]**

11 The vector $\mathbf{a} = \begin{bmatrix} 1.5 \\ 2 \end{bmatrix}$ and the vector $\mathbf{b} = 4\mathbf{a}$.

a) Work out the magnitude of vector **a**.

b) Hence write down the magnitude of vector **b**.

12 $\overrightarrow{PQ}$ has magnitude 6 cm, and $\overrightarrow{PR} = 3 \times \overrightarrow{PQ}$.

a) What can you deduce about the directions of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$?

b) Find the magnitude of the vector $\overrightarrow{PR}$.

Key points

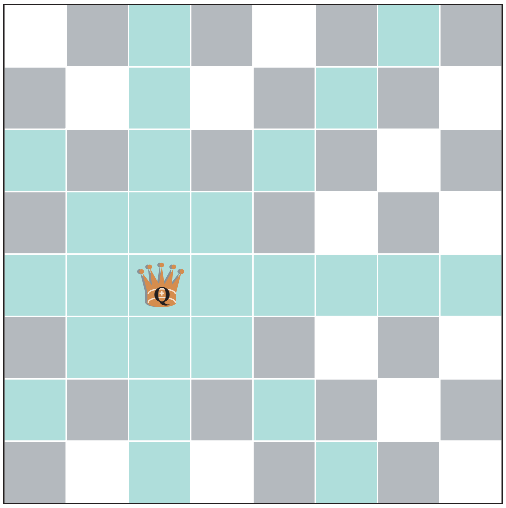
1. A vector has a direction and a length, or magnitude. Vectors are usually written in column form, such as $\begin{bmatrix} 4 \\ 6 \end{bmatrix}$, which represents a translation of 4 units in the x direction and 6 in the y direction.
2. Vectors are often used in examination questions to prove geometric theorems. The method is to use given base vectors **a**, **b**, **c,** etc. and then express other lines in terms of these, for example $2\mathbf{a} + \mathbf{b}$.
3. Two vectors will be parallel if one is a scalar multiple of the other. For example, $6\mathbf{a} + 3\mathbf{b}$ is parallel to $2\mathbf{a} + \mathbf{b}$, since $6\mathbf{a} + 3\mathbf{b} = 3 \times (2\mathbf{a} + \mathbf{b})$

Internet Challenge 30

Queens on a chessboard

Here is another chessboard problem.

The Queen is the most powerful piece on a chessboard. A Queen can attack any squares in a straight line from it, forwards, backwards, left, right or diagonal. The diagram below shows this in green for one position of the Queen:



Place eight Queens on a chessboard so that no two Queens attack each other.
You may want to use squared paper to record your attempts. This problem does have more than one solution. Once you have solved it, you might want to use the internet to help answer the following questions.

1 How many different distinct solutions does this problem have?

2 How many solutions are there in which no three Queens lie on an oblique line?

3 What is a Latin square? Is this a Latin square problem?

4 How many knights can be placed on a chessboard so that no knight attacks any other?

5 How about bishops?

Obviously it is not possible to place nine Queens on a board without at least two Queens attacking each other. (Why not?) There is, however, a 'nine Queens' problem:

Place nine Queens and one pawn on a chessboard so that no two Queens attack each other.

6 Try to solve the nine Queens problem. Use the internet if you get stuck.