Questions on Gravity and Orbits

1. Using the usual symbols write down an equation for

(i) Newton's law of gravitation

....

(ii) Coulomb's law

....

(2)

State one difference and one similarity between gravitational and electric fields.

Difference ....

....

Similarity ....

(2)

A speck of dust has a mass of 1.0×10^{-18} kg and carries a charge equal to that of one electron. Near to the Earth's surface it experiences a uniform downward electric field of strength 100 N C^{-1} and a uniform gravitational field of strength 9.8 N kg^{-1}.

Draw a free-body force diagram for the speck of dust. Label the forces clearly.

Calculate the magnitude and direction of the resultant force on the speck of dust.

....

....

....

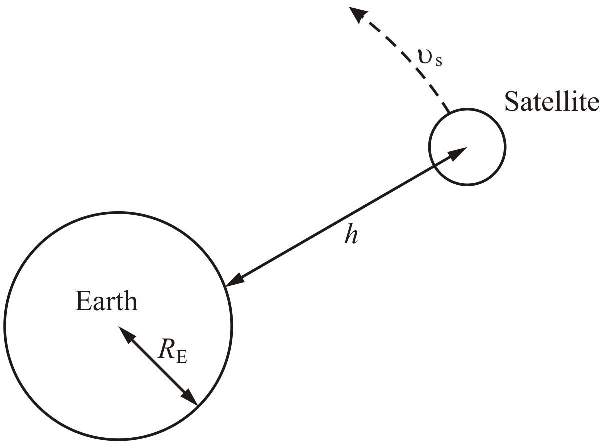
....

....

Force = ...

(6)
(Total 10 marks)

2. The diagram (not to scale) shows a satellite of mass m_s in circular orbit at speed υ_s around the Earth, mass M_E. The satellite is at a height h above the Earth's surface and the radius of the Earth is R_E.



Using the symbols above write down an expression for the centripetal force needed to maintain the satellite in this orbit.

....

....

....

(2)

Write down an expression for the gravitational field strength in the region of the satellite.

....

....

....

State an appropriate unit for this quantity.

....

(3)

Use your two expressions to show that the greater the height of the satellite above the Earth, the smaller will be its orbital speed.

....

....

....

....

....

(3)

Explain why, if a satellite slows down in its orbit, it nevertheless gradually spirals in towards the Earth's surface.

....

....

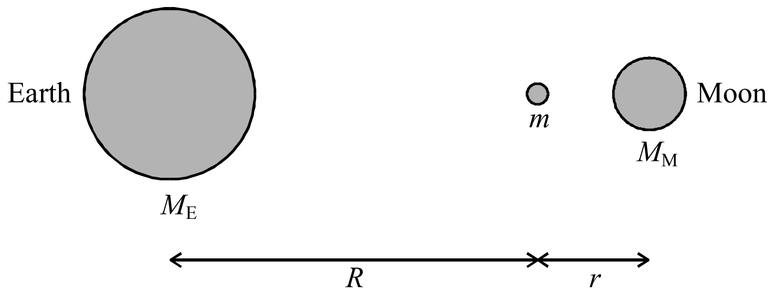
....

....

....

(2)
(Total 10 marks)

3. The diagram shows a body of mass m situated at a point which is a distance R from the centre of the Earth and r from the centre of the Moon.



The masses of the Earth and Moon are M_E and M_M respectively. The gravitational constant is G.

Using the symbols given, write down an expression for

(i) the gravitational force of attraction between the body and the Earth,

....

(ii) the gravitational force of attraction between the body and the Moon.

....

(2)

The resultant gravitational force exerted upon the body at this point is zero. Calculate the distance R of the body from the centre of the Earth given that

$$r = 3.9 \times 10^7 \text{ m and } M_E = 81\, M_M$$

....

....

....

....

....

R = ...

(3)
(Total 5 marks)

4. The orbit of the Moon, which has a mass m, is a circle of approximate radius $60R$, where R is the radius of the Earth. Show that the gravitational attraction between the Earth, mass M, and the Moon is given by

$$\frac{GMm}{3600R^2}$$

……..……..………………………………………………………………………………………..

……..……..………………………………………………………………………………………..

(1)

The mass of the Earth is 6.0×10^{24} kg and its radius is 6.4×10^6 m. Show that the orbital speed of the Moon around the Earth is approximately 1 km s^{-1}.

……..……..………………………………………………………………………………………..

……..……..………………………………………………………………………………………..

……..……..………………………………………………………………………………………..

……..……..………………………………………………………………………………………..

……..……..………………………………………………………………………………………..

……..……..………………………………………………………………………………………..

……..……..………………………………………………………………………………………..

(4)

Hence confirm that it takes the Moon about 30 days to orbit the Earth.

……..……..………………………………………………………………………………………..

……..……..………………………………………………………………………………………..

……..……..………………………………………………………………………………………..

……..……..………………………………………………………………………………………..

……..……..………………………………………………………………………………………..

……..……..………………………………………………………………………………………..

……..……..………………………………………………………………………………………..

(4)

(Total 9 marks)

5. Write a word equation which states Newton's law of gravitation.

....

....

....

(2)

Mars may be assumed to be a spherical planet with the following properties:

Mass m_M of Mars	=	6.42×10^{23} kg
Radius r_M of Mars	=	3.40×10^{6} m

Calculate the force exerted on a body of mass 1.00 kg on the surface of Mars.

....

....

....

Force =..

(3)

For any planet the relationship between g (the free fall acceleration at the surface) the planet's density ρ and its radius R is

$$g = \tfrac{4}{3}\pi\rho GR$$

Has Mars a larger, smaller or similar radius to the Earth?

....

Explain your reasoning.

....

....

....

....

(2)
(Total 7 marks)

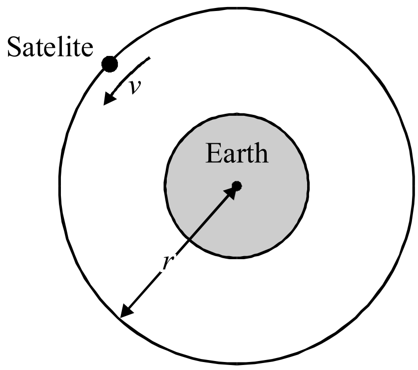
6. The value of G, the gravitational constant, is 6.67×10^{-11} N m^2 kg^{-2}. What are the base units of G?

....

....

(1)

A satellite orbits the Earth, mass M, in a circular path of radius r, with speed v, as shown in the diagram.



It can be shown that the period of orbit T of the satellite is given by

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

Show that this equation is homogeneous with respect to units.

....

....

....

(2)

Personal navigation devices use the Global Positioning System (GPS). GPS satellites are in a non-equatorial orbit at a height of 20 000 km above the Earth. The time to complete one orbit is 12 hours. Given that the radius of the Earth is 6400 km, use the above relationship to find the mass M of the Earth.

....

....

....

....

Mass of the Earth = ...

(3)

(Total 6 marks)

7. In April 2002, the five nearest planets to the Earth lined up in the same part of our sky. As a result, all of them were pulling the Earth in the same general direction.

Some people worried that this would cause disastrous events on Earth, as the following extract from a newspaper article shows.

Doom at the start of the new millennium?
There will be a series of planetary alignments at the start of the new millennium. Will the earth tilt over? Will tidal forces trigger earthquakes? Will the polar ice caps melt?

Write down an expression for the gravitational force between two point masses *M* and *m* a distance *r* apart.

....

....

(1)

Hence derive an expression for the gravitational field strength at a distance *r* from mass *M*.

....

....

....

(2)

The distance between the Sun and the Earth = 1.50×10^{11} m.

Show that the Sun's gravitational field strength at this distance is about 6×10^{-3} N kg^{-1}.

Mass of Sun = 1.99×10^{30} kg.

....

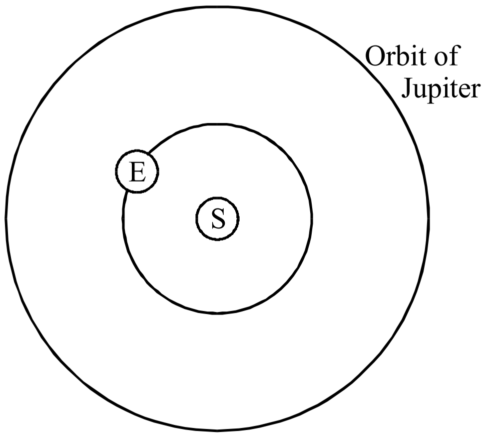
....

....

(2)

Jupiter is the most massive planet in our Solar System. Its maximum gravitational field strength at the Earth is 3.2×10^{-7} N kg^{-1}.

The diagram shows the orbits of the Earth and Jupiter around the Sun.



On the diagram

(i) mark the position of Jupiter at which it has maximum gravitational effect on the Earth,

(ii) draw labelled arrows on E to show the directions of the gravitational fields of Jupiter and the Sun.

(2)

Assume that the main gravitational field acting, on the Earth is that of the Sun.

Calculate the maximum percentage change in this gravitational field strength which Jupiter could make.

....

....

(1)

Venus (our nearest neighbour in space) Is about 400 times less massive than Jupiter, but, at its nearest, is 15 times closer. Calculate the maximum value of the ratio

$$\frac{\text{gravitational field strength at the Earth due to Venus}}{\text{gravitational field strength at the Earth due to Jupiter}}$$

....

....

....

....

....

(2)

With reference to your calculations, comment on whether an alignment of the five planets is likely to cause disastrous events on Earth.

....

....

....

(1)
(Total 11 marks)

8. Astronomers who are interested in finding life elsewhere in the Universe, are searching for planets orbiting other stars. A planet will be too small to see, but still can cause a star to appear to wobble as the planet orbits it.

One such wobbling star is 47 Ursae Majoris, which has a mass of 2.2×10^{30} kg. The period of its wobble is 9.2×10^{7} s.

From the wobble, the astronomers deduced that there is a planet of mass 4.5×10^{27} kg orbiting 47 Ursae Majoris at a distance of 3.1×10^{11} m.

Show that the force on the star due to this planet is about 7×10^{24} N.

....

....

....

....

(2)

Hence calculate the acceleration of the star towards the planet.

....

....

Acceleration = ...

(1)

Explain with the aid of a diagram why the star wobbles.

....

....

....

....

(3)

The star is orbiting about the centre of mass of the star/planet system.

Show that the speed of the star in its circular orbit is about 50 m s^{-1}.

....

....

....

....

....

....

....

(3)

The astronomers found out about this planet by measuring changes in the light emitted by 47 Ursae Majoris. Calculate the maximum change in wavelength which they can observe for light from the hydrogen spectrum of wavelength 656 nm.

....

....

....

....

(2)
(Total 11 marks)

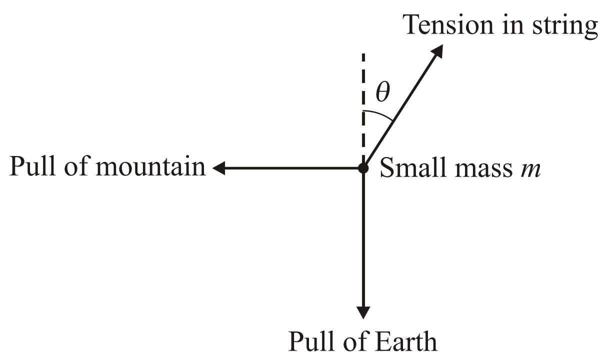
9. Write down a formula which relates the force between two objects to their masses M and m and their distance r apart.

....

....

(1)

Nevil Maskelyne developed a suggestion of Newton to use a small mass m on the end of a string to detect the gravitational effect of a nearby large mass. Maskelyne used a mountain in Scotland as the nearby large mass. The diagram below illustrates the effect of the mountain on the small mass m on the end of the string.



By using a vector diagram or by considering the components of the tension in the string in the vertical and horizontal directions, show that

$$\tan \theta = \frac{MR^2}{M_e r^2}$$

where M is the mass of the mountain,
M_e is the mass of the Earth,
R is the radius of the Earth,
r is the distance between the small mass and the centre of mass of the mountain.

....

....

....

....

....

....

....

(4)

From this experiment Maskelyne determined a value of 4.5×10^3 kg m^{-3} for the average density of the Earth. Use this result to calculate a value for the gravitational constant.

Radius of Earth = 6400 km
$g = 9.8$ m s^{-2}

....

....

....

....

....

....

....

....

G = ...

(3)

This value differs from the accepted value by about 20%. Give one reason for this inaccuracy.

....

....

(1)

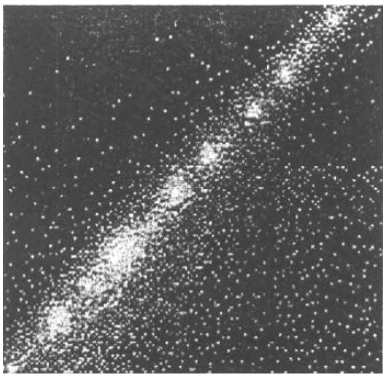
Maskelyne calculated that the average density of the Earth was approximately twice that of the mountain. What does this suggest about the Earth's core?

....

....

(1)
(Total 10 marks)

10. In 1994 the comet Shoemaker-Levy approached the planet Jupiter on a collision course. Just before the collision, the comet consisted of at least 17 separate fragments as shown in the picture.



All the fragments had originally been combined in a single comet with a diameter of 9 km. Comets like this are made of ice with trapped dust and gas and have a typical density of 500 kg m^{-3}.

Show that the minimum mass for the original comet which formed Shoemaker-Levy was about 2 × 10^{14} kg.

....

....

....

....

(3)

As a single comet, Shoemaker-Levy had previously approached Jupiter in 1992. At its closest, the near side of the comet was 96 000 km from the centre of Jupiter. At this near side of the comet, Jupiter's gravitational field strength was 13.7458 N kg^{-1}.

Calculate Jupiter's gravitational field strength at the far side of the comet.

(Mass of Jupiter = 1.8987 × 10^{27} kg.
G, gravitational constant = 6.6720 × 10^{-11} N m^2 kg^{-2}).

....

....

....

....

....

....

(3)

Explain why the comet broke apart as it passed Jupiter in 1992.

....

....

....

....

(2)

State one difference and one similarity between gravitational fields and electric fields.

Difference ....

....

....

Similarity ....

....

....

(2)
(Total 10 marks)

11. A student says that once you know about electric fields, you can apply the same laws to gravitational fields.

Write a set of revision notes defining each of these two types of field and stating the similarities and differences between them.

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

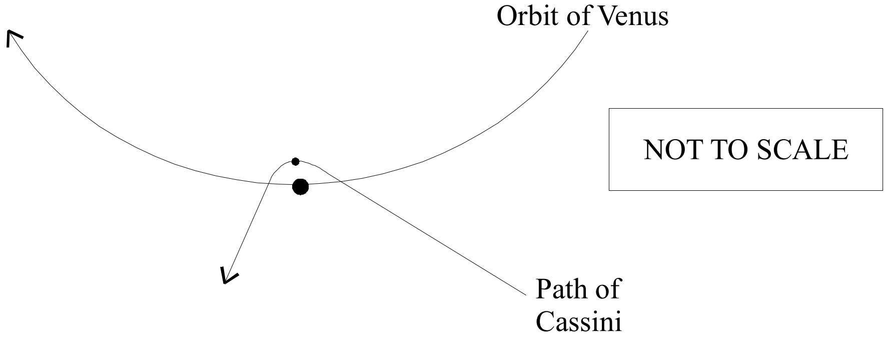
....

....

....

(Total 6 marks)

12. The Cassini spacecraft was launched from Earth in 1997 and is expected to reach Titan, one of Saturn's moons, in 2004. Cassini's energy at launch was insufficient for it to reach Titan without assistance from the gravitational pull of planets like Venus and Jupiter. The diagram shows the path of Cassini past Venus. Cassini is at its **closest** approach, just 284 km from the surface of Venus.



Write an expression for the magnitude of the gravitational force on Cassini at a distance *r* from the centre of Venus.

....

....

(1)

Hence derive an expression for the gravitational field strength at a distance *r* from the centre of Venus.

....

....

....

(2)

Calculate the maximum acceleration of Cassini caused by Venus.

Mass of Venus = 4.87×10^{24} kg

Radius of Venus = 6100 km

....

....

....

Maximum acceleration = ...

(3)

State and explain the effect of this acceleration on the velocity of Cassini at its closest distance to Venus.

....

....

....

....

(2)

(Total 8 marks)