Formulario I2 - EYP1113 2024 - 02

Igualdades

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k\, b^{n-k}; \qquad \sum_{k=x}^{\infty} \phi^k = \frac{\phi^x}{1-\phi} \quad \text{si } |\phi| < 1;$$

$$\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = \exp(\lambda); \qquad \sum_{x=0}^{\infty} \binom{x+k-1}{k-1} \phi^x = \frac{1}{(1-\phi)^k} \quad \text{si } 0 < \phi < 1 \text{ y } k \in \mathbb{N}, \qquad \int_{-\infty}^{\infty} e^{-x^2/2}\, dx = \sqrt{2\,\pi}$$

Propiedades función $\Gamma(\cdot)$ y $B(\cdot,\,\cdot)$

$$(1) \quad \Gamma(k) = \int_0^{\infty} u^{k-1}\, e^{-u}\, du = \texttt{gamma}(k); \quad (2) \quad \Gamma(a+1) = a\,\Gamma(a); \quad (3) \quad \Gamma(n+1) = n!, \quad \text{si } n \in \mathbb{N}_0;$$

$$(4) \quad \Gamma(1/2) = \sqrt{\pi}; \quad (5) \qquad B(q,\,r) = \int_0^1 x^{q-1}\,(1-x)^{r-1}\, dx; \quad (6) \quad B(q,\,r) = \frac{\Gamma(q)\,\Gamma(r)}{\Gamma(q+r)} = \texttt{beta}(q,r)$$

Distribución Gamma

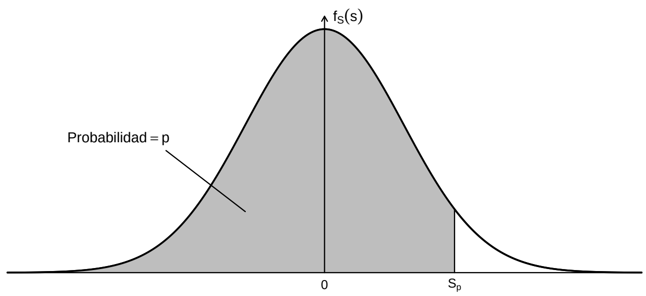
$$(1) \quad \text{Si } T \sim \text{Gamma}(k,\,\nu), \text{ con } k \in \mathbb{N} \longrightarrow F_T(t) = 1 - \sum_{x=0}^{k-1} \frac{(\nu\,t)^x\, e^{-\nu\,t}}{x!}, \quad (2) \quad \text{Gamma}(1,\,\nu) = \text{Exp}(\nu)$$

Medidas descriptivas

$$\mu_X = \mathsf{E}(X), \quad \sigma_X^2 = \mathsf{E}\left[(X-\mu_X)^2\right], \quad \delta_X = \frac{\sigma_X}{\mu_X}, \quad \theta_X = \frac{\mathsf{E}\left[(X-\mu_X)^3\right]}{\sigma_X^3}, \quad K_X = \frac{\mathsf{E}\left[(X-\mu_X)^4\right]}{\sigma_X^4} - 3$$

$$M_X(t) = \mathsf{E}\left(e^{t\,X}\right), \quad \mathsf{E}[g(X)] = \begin{cases} \displaystyle\sum_{x \in \Theta_X} g(x) \cdot p_X(x) \\ \\ \displaystyle\int_{-\infty}^{\infty} g(x) \cdot f_X(x)\, dx \end{cases}, \quad \text{Rango} = \text{máx} - \text{mín}, \quad \text{IQR} = x_{75\,\%} - x_{25\,\%}$$

Tabla Percentiles Distribución Normal Estándar



S_p	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998

Distribución	Densidad de Probabilidad	Θ_X	Parámetros	Esperanza y Varianza
Binomial	$\binom{n}{x} p^x (1-p)^{n-x}$	$x = 0, \ldots, n$	$n,\ p$	$\mu_X = n\,p$ $\sigma^2_X = n\,p\,(1-p)$ $M(t) = [p\,e^t + (1-p)]^n, \quad t \in \mathbb{R}$
Geométrica	$p\,(1-p)^{x-1}$	$x = 1, 2, \ldots$	p	$\mu_X = 1/p$ $\sigma^2_X = (1-p)/p^2$ $M(t) = p\,e^t/[1-(1-p)\,e^t], \quad t < -\ln(1-p)$
Binomial-Negativa	$\binom{x-1}{r-1} p^r (1-p)^{x-r}$	$x = r, r+1, \ldots$	$r,\ p$	$\mu_X = r/p$ $\sigma^2_X = r\,(1-p)/p^2$ $M(t) = \left\{p\,e^t/[1-(1-p)\,e^t]\right\}^r, \quad t < -\ln(1-p)$
Poisson	$\dfrac{(\nu\,t)^x\,e^{-\nu\,t}}{x!}$	$x = 0, 1, \ldots$	ν	$\mu_X = \nu\,t$ $\sigma^2_X = \nu\,t$ $M(t) = \exp\left[\lambda\left(e^t - 1\right)\right], \quad t \in \mathbb{R}$
Exponencial	$\nu\,e^{-\nu\,x}$	$x \geq 0$	ν	$\mu_X = 1/\nu$ $\sigma^2_X = 1/\nu^2$ $M(t) = \nu/(\nu - t), \quad t < \nu$
Gamma	$\dfrac{\nu^k}{\Gamma(k)}\,x^{k-1}\,e^{-\nu\,x}$	$x \geq 0$	$k,\ \nu$	$\mu_X = k/\nu$ $\sigma^2_X = k/\nu^2$ $M(t) = [\nu/(\nu - t)]^k, \quad t < \nu$
Normal	$\dfrac{1}{\sqrt{2\,\pi}\,\sigma}\,\exp\left[-\dfrac{1}{2}\left(\dfrac{x-\mu}{\sigma}\right)^2\right]$	$-\infty < x < \infty$	$\mu,\ \sigma$	$\mu_X = \mu$ $\sigma^2_X = \sigma^2$ $M(t) = \exp(\mu\,t + \sigma^2\,t^2/2), \quad t \in \mathbb{R}$
Log-Normal	$\dfrac{1}{\sqrt{2\,\pi}\,(\zeta\,x)}\,\exp\left[-\dfrac{1}{2}\left(\dfrac{\ln\,x - \lambda}{\zeta}\right)^2\right]$	$x \geq 0$	$\lambda,\ \zeta$	$\mu_X = \exp\left(\lambda + \dfrac{1}{2}\,\zeta^2\right)$ $\sigma^2_X = \mu^2_X\left(e^{\zeta^2} - 1\right)$ $E(X^r) = e^{r\,\lambda}\,M_Z(r\,\zeta)$, con $Z \sim$Normal(0,1)
Uniforme	$\dfrac{1}{(b-a)}$	$a \leq x \leq b$	$a,\ b$	$\mu_X = (a+b)/2$ $\sigma^2_X = (b-a)^2/12$ $M(t) = [e^{t\,b} - e^{t\,a}]/[t\,(b-a)], \quad t \in \mathbb{R}$
Beta	$\dfrac{1}{B(q,\ r)}\,\dfrac{(x-a)^{q-1}\,(b-x)^{r-1}}{(b-a)^{q+r-1}}$	$a \leq x \leq b$	$q,\ r$	$\mu_X = a + \frac{q}{q+r}\,(b-a)$ $\sigma^2_X = \frac{q\,r\,(b-a)^2}{(q+r)^2\,(q+r+1)}$
Hipergeométrica	$\dfrac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}}$	$\text{máx}\{0, n+m-N\} \leq x \leq \text{mín}\{n, m\}$	$N,\ m,\ n$	$\mu_X = n\,\frac{m}{N}$ $\sigma^2_X = \left(\frac{N-n}{N-1}\right)\,n\,\frac{m}{N}\,\left(1 - \frac{m}{N}\right)$

Otras distribuciones

- Si $T \sim$ Weibull$(\eta,\, \beta)$, se tiene que

$$F_T(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \quad f_T(t) = \frac{\beta}{\eta}\,\left(\frac{t}{\eta}\right)^{\beta-1}\,\exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right], \quad t > 0$$

  Con $\beta > 0$, es un parámetro de forma y $\eta > 0$, es un parámetro de escala. Si t_p es el percentil $p \times 100\,\%$, entonces

$$\ln(t_p) = \ln(\eta) + \frac{1}{\beta}\cdot\Phi^{-1}_{\text{Weibull}}(p), \quad \Phi^{-1}_{\text{Weibull}}(p) = \ln[-\ln(1-p)]$$

  Mientras que su m-ésimo momento está dado por

$$E(T^m) = \eta^m\,\Gamma(1 + m/\beta)$$

$$\mu_T = \eta\,\Gamma\left(1 + \frac{1}{\beta}\right), \quad \sigma^2_T = \eta^2\,\left[\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right)\right]$$

- Si $Y \sim$ Logística$(\mu,\, \sigma)$, se tiene que

$$F_Y(y) = \Phi_{\text{Logística}}\left(\frac{y-\mu}{\sigma}\right); \qquad f_Y(y) = \frac{1}{\sigma}\,\phi_{\text{Logística}}\left(\frac{y-\mu}{\sigma}\right), \quad -\infty < y < \infty$$

  donde

$$\Phi_{\text{Logística}}(z) = \frac{\exp(z)}{[1+\exp(z)]} \quad \text{y} \quad \phi_{\text{Logística}}(z) = \frac{\exp(z)}{[1+\exp(z)]^2}$$

  son la función de probabilidad y de densidad de una Logística Estándar. $\mu \in \mathbb{R}$, es un parámetro de localización y $\sigma > 0$, es un parámetro de escala. Si y_p es el percentil $p \times 100\,\%$, entonces

$$y_p = \mu + \sigma\,\Phi^{-1}_{\text{Logística}}(p) \quad \text{con} \quad \Phi^{-1}_{\text{Logística}}(p) = \log\left(\frac{p}{1-p}\right)$$

  Su esperanza y varianza están dadas por: $\mu_Y = \mu$ y $\sigma^2_Y = \frac{\sigma^2\,\pi^2}{3}$.

- Si $T \sim$ Log-Logística$(\mu,\, \sigma)$, se tiene que

$$F_T(t) = \Phi_{\text{Logística}}\left(\frac{\ln(t)-\mu}{\sigma}\right); \quad f_T(t) = \frac{1}{\sigma\,t}\,\phi_{\text{Logística}}\left(\frac{\ln(t)-\mu}{\sigma}\right) \quad t > 0$$

  Donde $\exp(\mu)$, es un parámetro de escala y $\sigma > 0$, es un parámetro de forma. Si t_p es el percentil $p\times 100\,\%$, entonces

$$\ln(t_p) = \mu + \sigma\,\Phi^{-1}_{\text{Logística}}(p)$$

  Para un entero $m > 0$ se tiene que

$$E(T^m) = \exp(m\,\mu)\,\Gamma(1 + m\,\sigma)\,\Gamma(1 - m\,\sigma)$$

  El m-ésimo momento no es finito si $m\,\sigma \geq 1$.

  Para $\sigma < 1$: $\mu_T = \exp(\mu)\,\Gamma(1+\sigma)\,\Gamma(1-\sigma)$

  y para $\sigma < 1/2$: $\sigma^2_T = \exp(2\,\mu)\,\left[\Gamma(1 + 2\,\sigma)\,\Gamma(1 - 2\,\sigma) - \Gamma^2(1+\sigma)\,\Gamma^2(1-\sigma)\right]$

- Un variable aleatoria T tiene distribución t-student(ν) si su función de densidad está dada por:

$$f_T(t) = \frac{\Gamma[(\nu+1)/2]}{\sqrt{\pi\,\nu}\,\Gamma(\nu/2)}\,\left(1 + \frac{t^2}{\nu}\right)^{-(\nu+1)/2}, \quad -\infty < t < \infty$$

  - $\mu_T = 0$, para $\nu > 1$.
  - $\sigma^2_T = \frac{\nu}{\nu-2}$, para $\mu > 2$.

- Si $T \sim$ Fisher$(\eta,\, \nu)$, se tiene que

$$f_T(t) = \frac{\Gamma(\frac{\eta+\nu}{2})}{\Gamma(\eta/2)\Gamma(\nu/2)}\,\left(\frac{\eta}{\nu}\right)^{\frac{\eta}{2}}\,\frac{t^{\frac{\eta}{2}-1}}{\left(\frac{\eta}{\nu}\,t + 1\right)^{\frac{\eta+\nu}{2}}}, \quad t > 0$$

  - $\mu_T = \frac{\nu}{\nu-2}$, para $\nu > 2$.
  - $\sigma^2_T = \frac{2\,\nu^2\,(\eta+\nu-2)}{\eta\,(\nu-2)^2\,(\nu-4)}$, para $\nu > 4$.