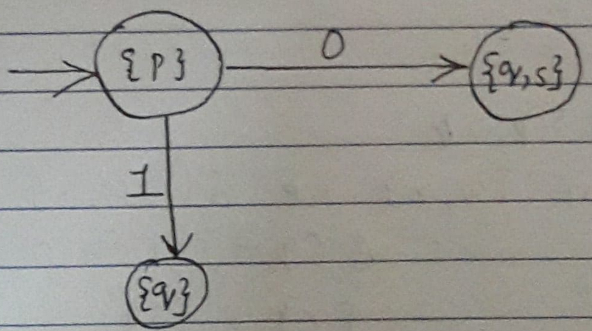
TCS Assignment No. 1

Q1 Convert the following NFA to DFA.

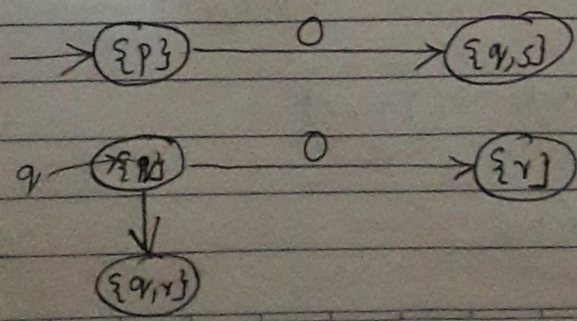
→

	0	1
{p}	{q,s}	{q}
q*	{r}	{q,r}
r	{s}	{p}
s*	ϕ	{p}

Step 1 : Starting state {p} is finite state.
0 successor of {p} i.e δ({p}, 0} ⇒ {q,s}
1 successor of {p} i.e δ({p}, 1) ⇒ {q}



Step 2 : Successor of {q}
0 : successor of {q} i.e δ({q}, 0} = {r}
1 : successor of {q} i.e δ({q}, 1) = {q,r}



Step 3 : Successor of $\{q, s\}$

0 successor of $\{q, s\}$ i.e $\delta(\{q, s\}, 0)$

$= \delta(q, 0) \cup \delta(s, 0)$

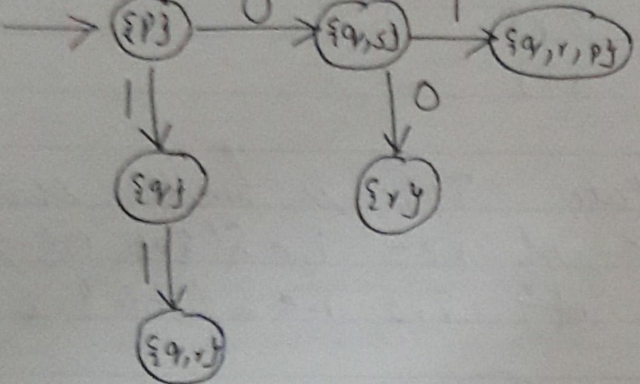
$= \{r\} \cup \{\phi\} = \boxed{\{r\}}$

1 successor of $\{q, s\}$ i.e $\delta(\{q, s\}, 1)$

$= \delta(q, 1) \cup \delta(s, 1)$

$= \{q, r\} \cup \{p\}$

$= \boxed{\{q, r, p\}}$



Step 4 : Successor of $\{q, r\}$

0 successor of $\{q, r\}$ i.e $\delta(\{q, r\}, 0)$

$= \delta(q, 0) \cup (r, 0)$

$= \{r\} \cup \{s\}$

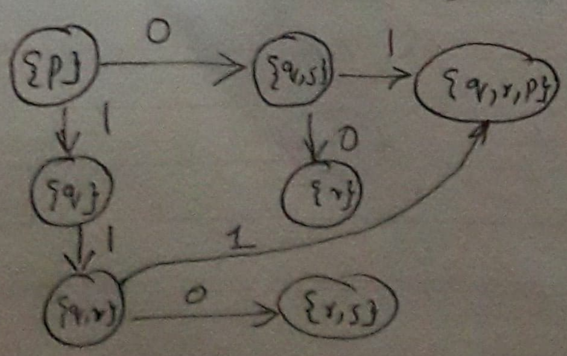
$= \{r, s\}$

1 successor of $\{q, r\}$ i.e $\delta(\{q, r\}, 1)$

$= \delta(q, 1) \cup \delta(r, 1)$

$= \delta \{q, r\} \cup \{p\}$

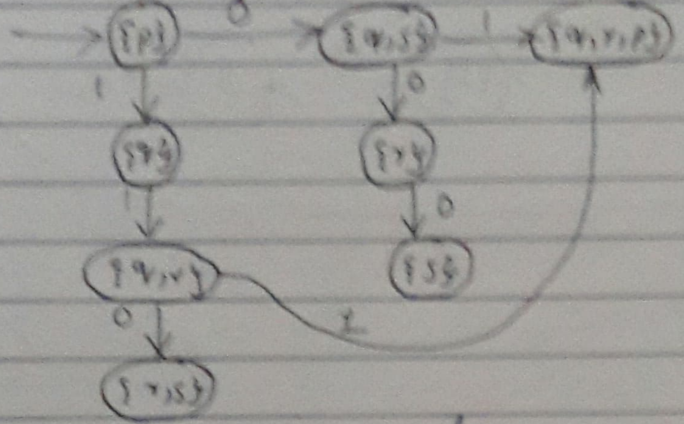
$= \{p, q, r\}$



Step5:- Successor of $\{r\}$

0 successor of $\{r\}$ i.e $\delta(r,0) = \{s\}$

1 successor of $\{r\}$ i.e $\delta(r,1) = \{p\}$



Step6: successor of $\{q,r,p\}$

0 successor of $\{q,r,p\}$ i.e $\delta(\{q,r,p\})$

$= \delta(q,0) \cup \delta(r,0) \cup \delta(p,0)$

$= \{r\} \cup \{s\} \cup \{q,s\}$

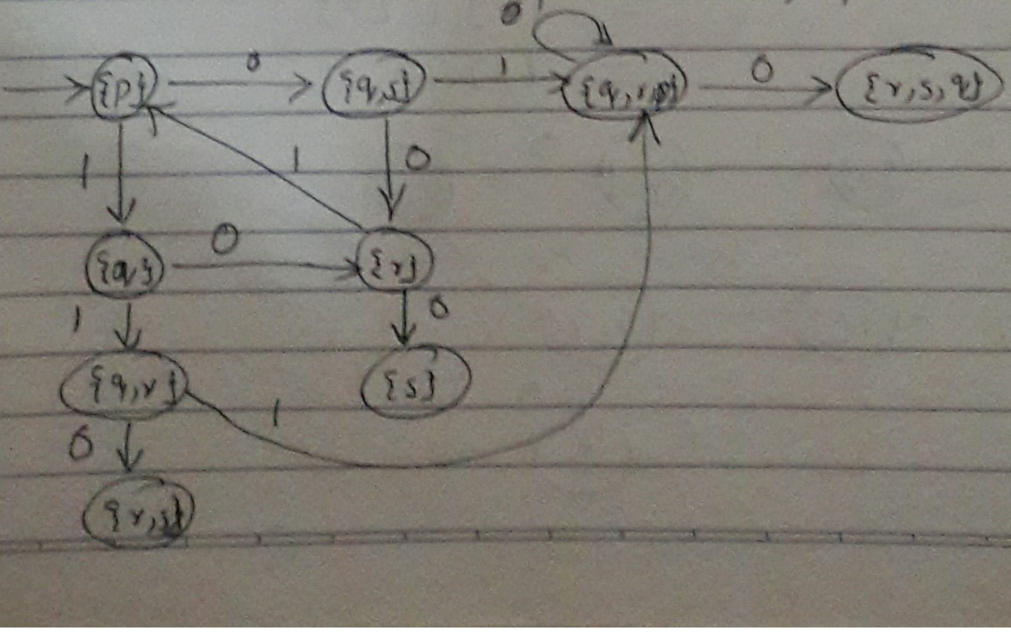
$= \{r,s,q\}$

1 successor of $\{q,r,p\}$ i.e $\delta(\{q,r,p\},1)$

$= \delta(q,1) \cup \delta(r,1) \cup \delta(p,1)$

$= \{q,r\} \cup \{p\} \cup \{q\}$

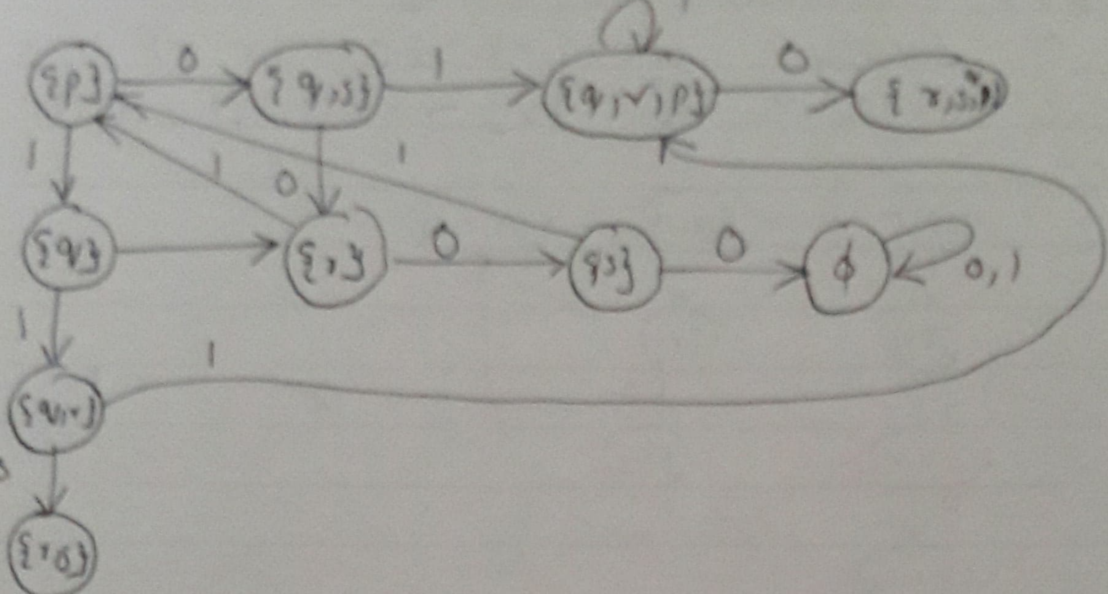
$= \{q,r,p\}$



Step 7 Successor of {s}

0 successor of {s} i.e $\delta(s,0) = \phi$

1 successor of {s} i.e $\delta(s,1) = \{p\}$



Step 8 :- Successor of {r, s}

0 Successor of {r, s} i.e $\delta(\{r,s\},0)$

$= \delta(r,0) \cup \delta(s,0)$

$= \delta \{s\} \cup \{\phi\}$

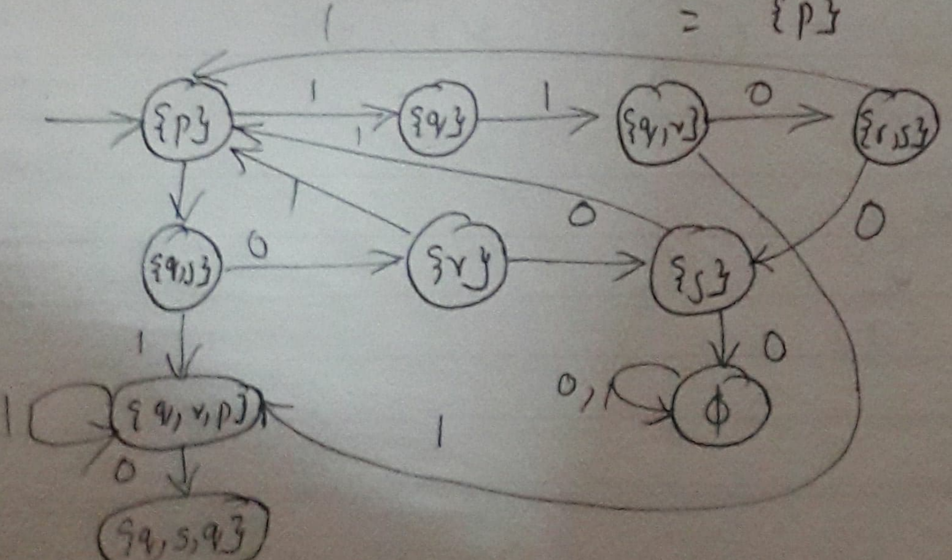
$= \{s\}$

1 Successor of {r, s} i.e $\delta(\{r,s\},1)$

$= \delta(r,1) \cup \delta(s,1)$

$= \{p\} \cup \{p\}$

$= \{p\}$



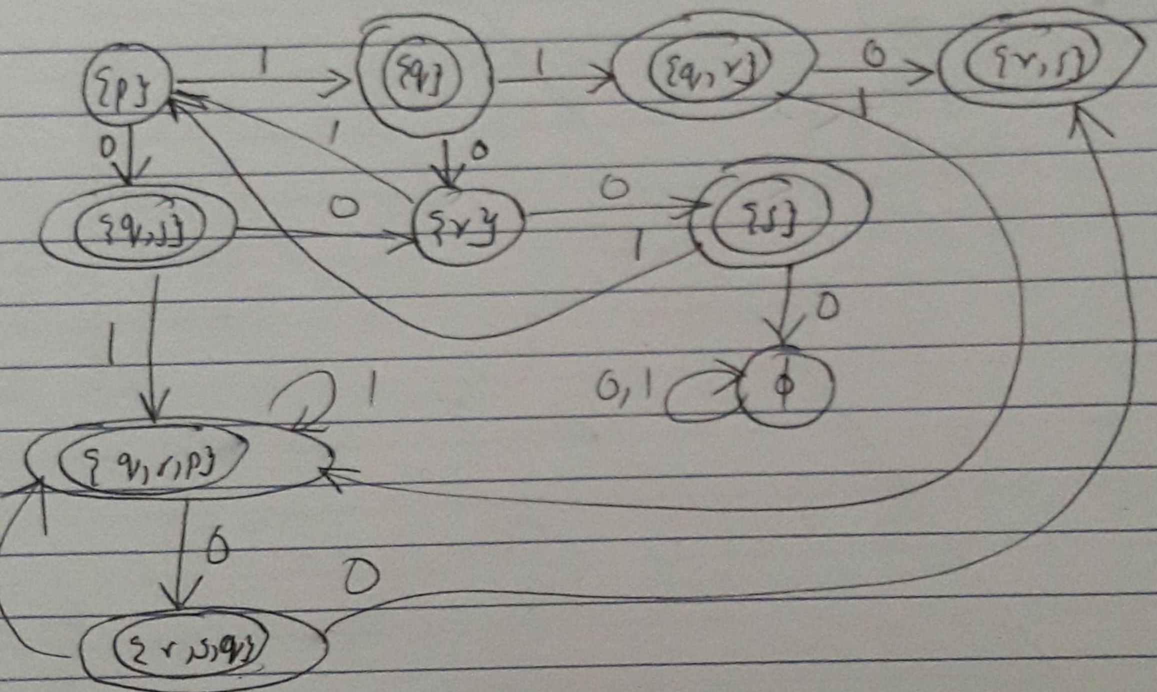
Step 9 : Successor of {r, s, q}

① successor of {r, s, q} i.e δ({r, s, q}, 0)

$= \delta(r, 0) \cup \delta(s, 0) \cup \delta(q, 0)$

$= \{p\} \cup \{p\} \cup \{q, r\}$

$= \{p, q, r\}$



Step 10 : Since no new successors are generated the process of subset generation steps. q and s are final state in the NFA. Every subset containg the q and s are final states

DFA

	0	1
A {p}	{q, s}	{q}
B {q}*	{r}	{q, r}
C {r}	{s}	{p}
D {s}*	ϕ	{p}
E {q, r}*	{r, s}	{q, r, p}
F {r, s}*	{s}	{p}
G {q, s}*	{r}	{q, r, p}
H {q, r, p}*	{r, s, q}	{q, r, p}
I {r, s, p}*	{r, s}	{q, r, p}
J ϕ	ϕ	ϕ

	0	1
A	G	B
B	C	E
C	D	A
D	J	A
E	F	H
F	D	A
G	C	H
H	I	H
I	F	H
J	J	J

Q2

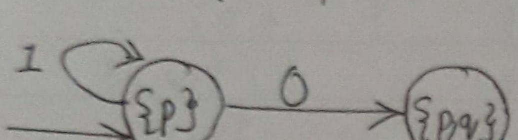
	0	1
$\rightarrow p$	$\{p,q\}$	$\{p\}$
q	$\{r,s\}$	$\{t\}$
r	$\{p,s\}$	$\{t\}$
s^*	$\{\phi$	ϕ
t^*	ϕ	ϕ

Solⁿ:

Step1:- Starting state $\{p\}$ is the first state.

0 successors of $\{p\}$ i.e $\delta(p,0) = \{p,q\}$

1 successor of $\{p\}$ i.e $\delta(p,1) = \{p\}$



Step2:- Successor of $\{p,q\}$

0 successor of $\{p,q\}$ i.e $\delta(\{p,q\},0\}$

$= \delta(p,0) \cup \delta(q,0)$

$= \{p,q\} \cup \{r,s\}$

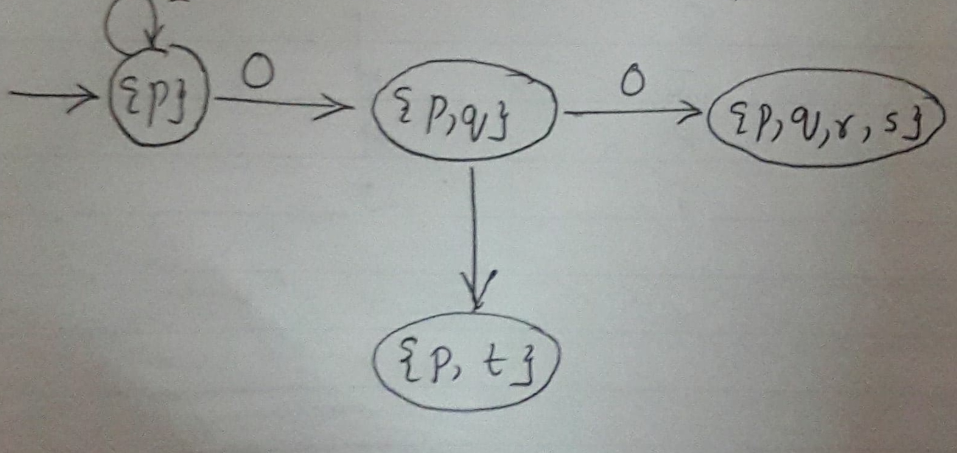
$= \{p,q,r,s\}$

1 successor of $\{p,q\}$ i.e $\delta(\{p,q\},1)$

$= \delta(p,1) \cup \delta(q,1)$

$= \{p\} \cup \{t\}$

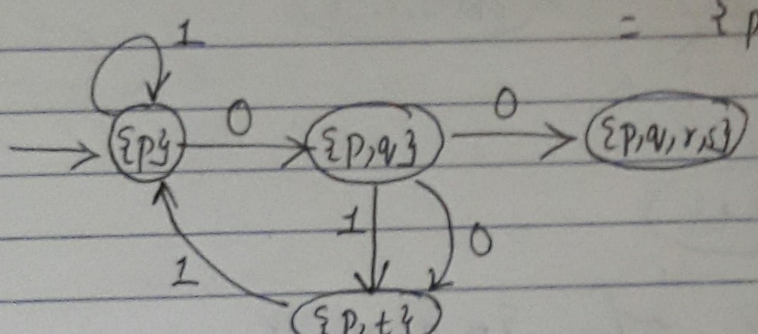
$= \{p,t\}$



Step 3 :- Successor of $\{p, t\}$

0 successors of $\{p, t\}$ i.e $\delta(\{p, t\}, 0)$
$= \delta(p, 0) \cup \delta(t, 0)$
$= \{p, q\} \cup \{\phi\}$
$= \{p, q\}$

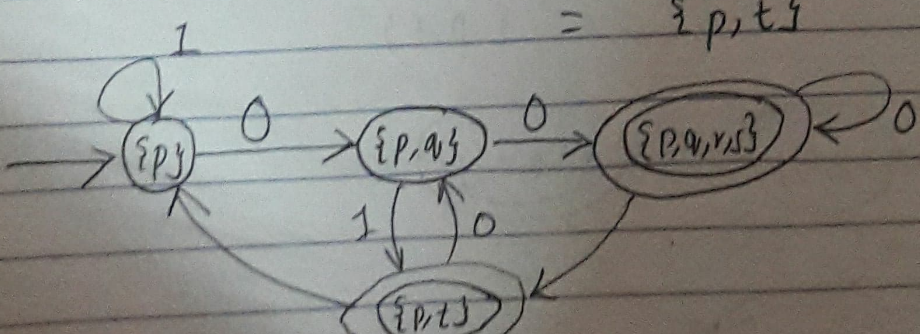
1 successor of $\{p, t\}$ i.e $\delta(\{p, t\}, 1)$
$= \delta(p, 1) \cup (t, 1)$
$= \{p\} \cup \{\phi\}$
$= \{p\}$



Step 4 :- Successor of $\{p, q, r, s\}$

0 successor $\{p, q, r, s\}$ i.e $\delta(\{p, q, r, s\}, 0\}$
$= \delta(p, 0) \cup \delta(q, p) \cup \delta(r, 0) \cup \delta(s, 0)$
$= \{p, q\} \cup \{r, s\} \cup \{p, r\} \cup \{\phi\}$
$= \{p, q, r, s\}$

1 successor of $\{p, q, r, s\}$ i.e $\delta(\{p, q, r, s\}, 1)$
$= \delta(p, 1) \cup \delta(q, 1) \cup (r, 1) \cup \delta(s, 1)$
$= \delta\ \{p\} \cup \{t\} \cup \{t\} \cup \{\phi\}$
$= \{p, t\}$



Step 3 :- Successor of $\{p,t\}$

0 successor of $\{p,t\}$ i.e $\delta(\{p,t\},0)$

$= \delta(p,0) \cup \delta(t,0)$

$= \delta\{p,q\} \cup \{\phi\}$

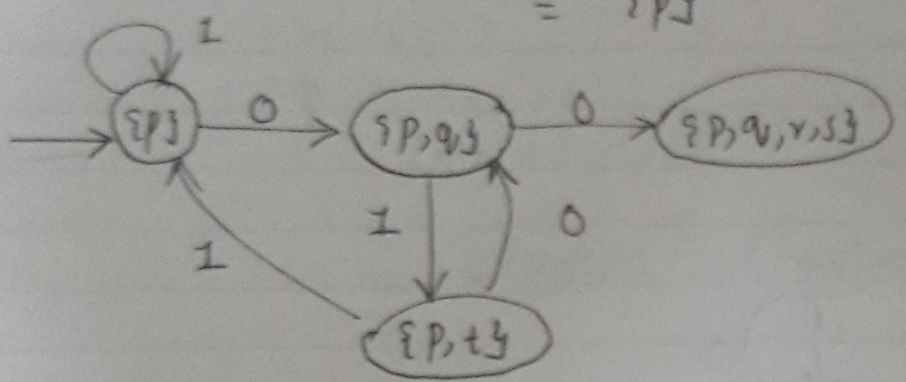
$= \{p,q\}$

1 Successor of $\{p,t\}$ i.e $\delta(\{p,t\},1)$

$= \delta(p,1) \cup \delta(t,1)$

$= \{p\} \cup \{\phi\}$

$= \{p\}$



Step 4 :- Successor of $\{p,q,r,s\}$

0 Successor of $\{p,q,r,s\}$ i.e $\delta(\{p,q,r,s\},0)$

$= \delta(p,0) \cup \delta(q,0) \cup \delta(r,0) \cup \delta(s,0)$

$= \{p,q\} \cup \{r,s\} \cup \{p,r\} ; \phi$

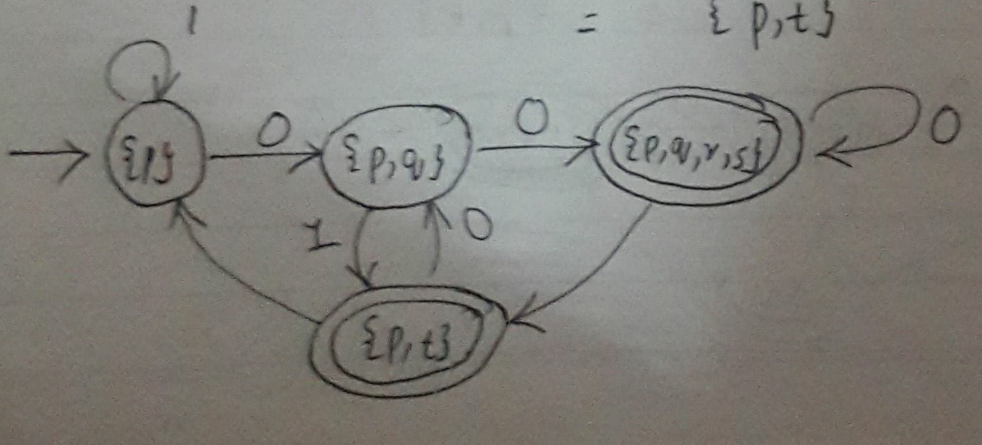
$= \{p,q,r,s\}$

1 successor of $\{p,q,r,s\}$ i.e $\delta(\{p,q,r,s\},1)$

$= \delta(p,1) \cup \delta(q,1) \cup \delta(r,1) \cup \delta(s,1)$

$= \{p\} \cup \{t\} \cup \{t\} \cup \{\phi\}$

$= \{p,t\}$



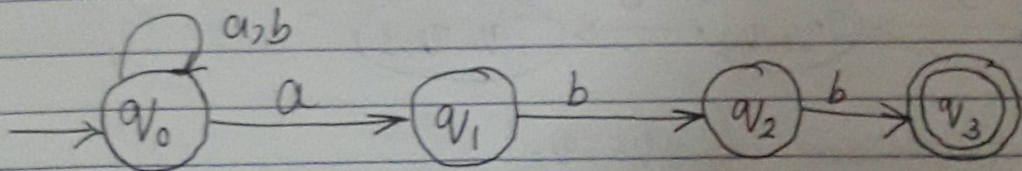
Step5 :- Since, no. of successors are generated, the process of subset generation steps. s and t are final states in the NFA. Every subset containing s and t is the final state

DFA

		0	1
A	{p}	{p, q}	{p}
B	{p, q}	{p, q, r, s}	{p, t}
C	{p, t} *	{p, q}	{p}
D	{p, q, r, s} *	{p, q, r, s}	{p, t}

	0	1
A	B	A
B	D	C
C*	B	A
D*	D	C

Q3



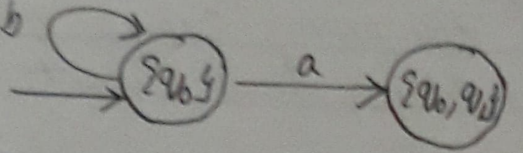
	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	ϕ	$\{q_0\}$
q_2	$\{q_3\}$	ϕ
q_3	ϕ	ϕ

Step1: q_0 is the starting state

$\because$ q_0 is the final state

a Successor of $\{q_0\}$ i.e $\delta(q_0, a) - \{q_0, q_1\}$

b Successor of $\{q_0\}$ i.e $\delta(q_0, b) - \{q_0\}$



Step2:- Successor of $\{q_0, q_1\}$

a Successor of $\{q_0, q_1\}$ i.e $\delta(\{q_0, q_1\}, a)$

$= \delta(q_0, a) \cup \delta(q_1, a)$

$= \delta\{q_0, q_1\} \cup \{\phi\}$

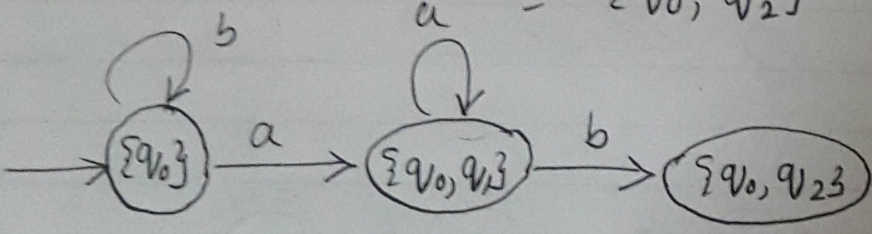
$= \{q_0, q_1\}$

b Successor of $\{q_0, q_1\}$ i.e $\delta(\{q_0, q_1\}, b)$

$= \delta(q_0, b) \cup (q_1, b)$

$= \{q_0\} \cup \{q_2\}$

$= \{q_0, q_2\}$



Step3:- Successor of $\{q_0, q_2\}$

a Successor of $\{q_0, q_1\}$ i.e $\delta(\{q_0, q_2\}, a)$

$= \delta(q_0, a) \cup \delta(q_2, a)$

$= \{q_0, q_1\} \cup \{q_2\}$

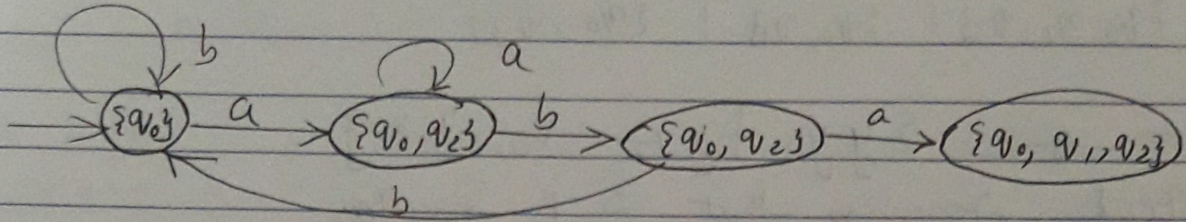
$= \{q_0, q_1, q_3\}$

b Successor of $\{q_0, q_2\}$ i.e $\delta(\{q_0, q_0\}, b)$

$= \delta(q_0, b) \cup \delta(q_2, b)$

$= \{q_0\} \cup \phi$

$= \{q_0\}$



<u>Step4</u> :- Successor of $\{q_0, q_1, q_3\}$

a successor of $\{q_0, q_1, q_3\}$ i.e $(\{q_0, q_1, q_3\}; a)$

$= \delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_3, a)$

$= \{q_0, q_1\} \cup \{\phi\} \cup \{\phi\}$

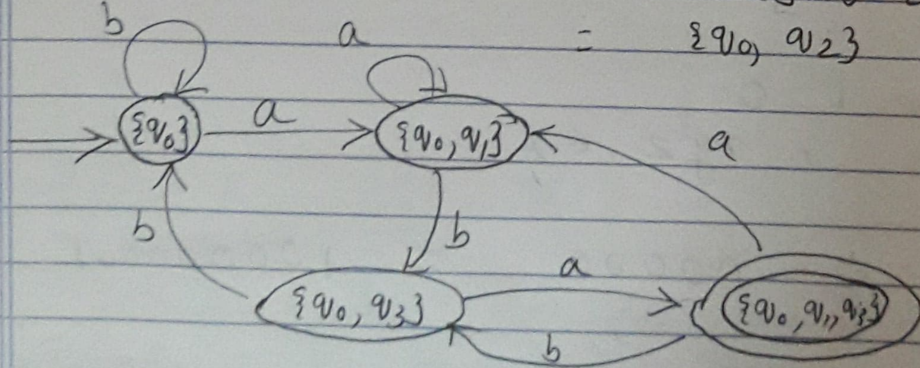
$= \{q_0, q_1\}$

b successor of $\{q_0, q_1, q_3\}$ i.e $\delta(\{q_0, q_1, q_3\}, b)$

$= \delta\{q_0, b\} \cup \delta(q_1, b) \cup \delta(q_3, b)$

$= \{q_0\} \cup \{q_2\} \cup \{\phi\}$

$= \{q_0, q_2\}$



<u>Step5</u>: Since, now new successor is generated
∴ subset generation stops
q_3 is the final state in NFA.
Every subset containing q_3 is final state

DFA

	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_3\}$
$\{q_0, q_3\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_3\}$

Q4 :– $A = \{yy \mid y \in \{0,1\}^*\}$

→ **Step1** Assuming that A is regular.
pumping length p.

$$s = 0^p 1 0^p 1$$

x y z

Let's assume $p = 7$

$$s = \underbrace{00}_{x}\ \underbrace{00000}_{y}\ \underbrace{100000001}_{z} \qquad |xy| \le p$$

Step2 :– (i) for $i = 0$

$s = xyz$, $xy^iz \in A$

Consider $i = 0$

$x = 00$ $\quad y = 00000$ $\quad z = 100000001$

$$= xy^0z \Rightarrow xz$$
$$= \underbrace{00}_{x}\ \underbrace{100000001}_{z}$$

Hence, $xy^iz \in A$

(2) For $i=1$

$S = xyz$, $xy^i z \in A$

Consider $i=1$,

$x = 00$ $y = 00000$ $z = 100000001$

$xy^i z = xy^1 z = xyz$

$= \underbrace{00}_{x} \underbrace{00000}_{y} \underbrace{100000001}_{z}$

$|xy| \le P$

$\therefore$ It satisfies $xy^i z \in A$ for $i=1$

(3) For $i=2$

$S = xyz$, $xy^i z \in A$

Consider $i=2$.

$x = 00$, $y = 000000$, $z = 1000\ 00001$

$xy^i z = xy^\circ z$

$= \underbrace{00}_{x} \underbrace{00000\ 00000}_{y^2} \underbrace{1000\ 000001}_{z}$

Hence, $xy^i z \notin A$

Hence, all the conditions are not satisfied.
Therefore, s cannot be pumped.