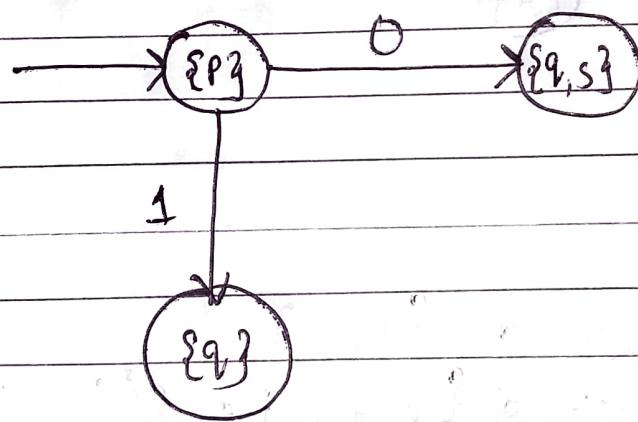


TCS Assignment No. 2.

Q1. Convert the following NFA to DFA

\rightarrow	0	1	
$\{\epsilon_P\}$	$\{\epsilon_q, s\}$	$\{\epsilon_q\}$	
q^*	$\{x\}$	$\{\epsilon_q, r\}$	
x	$\{\epsilon_s\}$	$\{\epsilon_P\}$	
s^*	\emptyset	$\{\epsilon_P\}$	

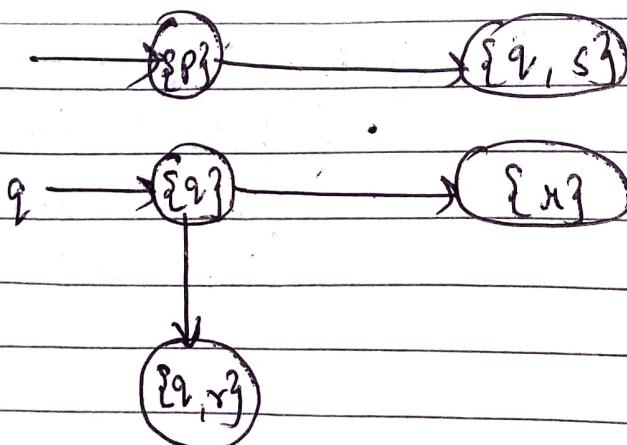
Step 1: Starting state $\{\epsilon_P\}$ is finite state
 0 Successor of $\{\epsilon_P\}$ i.e $\delta(\{\epsilon_P\}, 0) \Rightarrow \{\epsilon_q, s\}$
 1 Successor of $\{\epsilon_P\}$ i.e $\delta(\{\epsilon_P\}, 1) \Rightarrow \{\epsilon_q\}$



Step 2: Successor of $\{\epsilon_q\}$

0: successor of $\{\epsilon_q\}$ i.e $\delta(\{\epsilon_q\}, 0) = \{x\}$

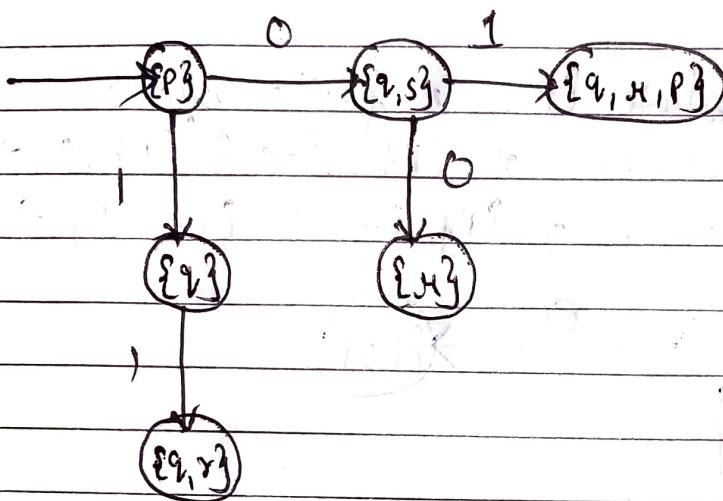
1: successor of $\{\epsilon_q\}$ i.e $\delta(\{\epsilon_q\}, 1) = \{\epsilon_q, r\}$



Step 3 : Successor of $\{q, s\}$

O Successor of $\{q, s\}$ i.e $S(\{q, s\}, 0)$
 $= S(q, 0) \cup S(s, 0)$
 $\therefore = \{x\} \cup \{\emptyset\} = \{\emptyset, x\}$

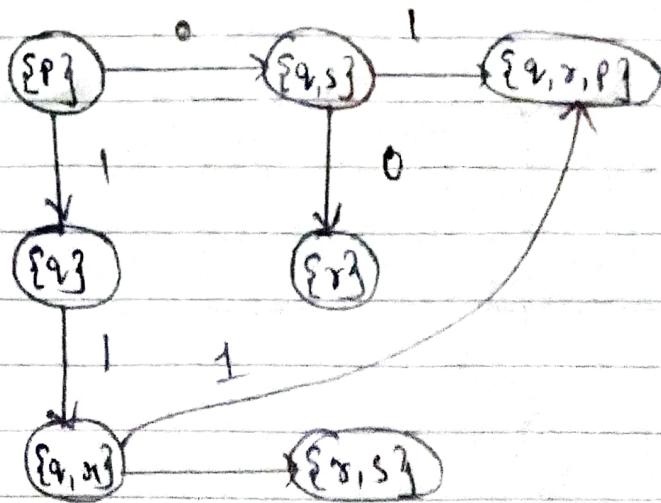
1 Successor of $\{q, s\}$ i.e $S(\{q, s\}, 1)$
 $= S(q, 1) \cup S(s, 1)$
 $= \{q, x\} \cup \{P\}$
 $= \{q, x, P\}$



Step 4 : Successor of $\{q, x\}$

O Successor of $\{q, x\}$ i.e $S(\{q, x\}, 0)$
 $= S(q, 0) \cup S(x, 0)$
 $= \{x\} \cup \{s\}$
 $= \{v, s\}$

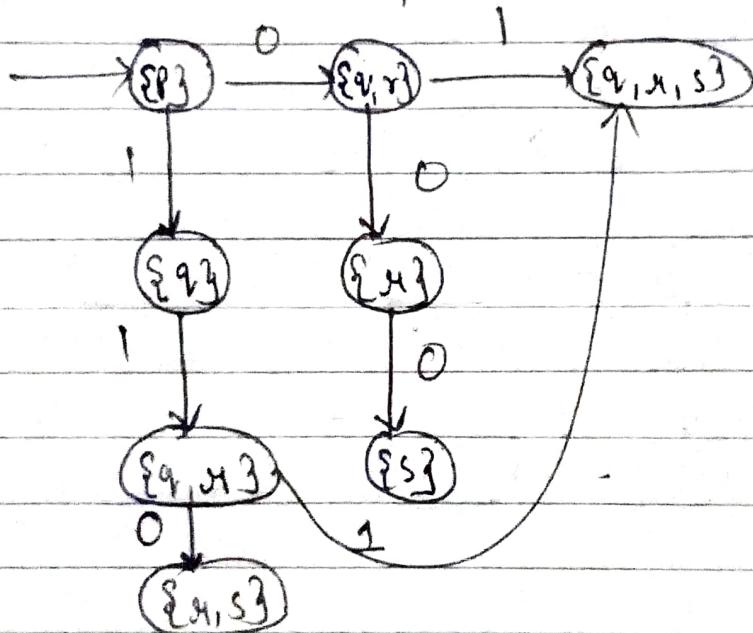
1 Successor of $\{q, x\}$ i.e $S(\{q, x\}, 1)$
 $= S(q, 1) \cup S(x, 1)$
 $= S\{q, x\} \cup \{P\}$
 $= \{P, q, x\}$



Step 5 :- Successor of $\{q, r\}$

0 Successor of $\{q, r\}$ i.e $\delta(q, 0) = \{s\}$

1 Successor of $\{q, r\}$ i.e $\delta(q, 1) = \{p\}$

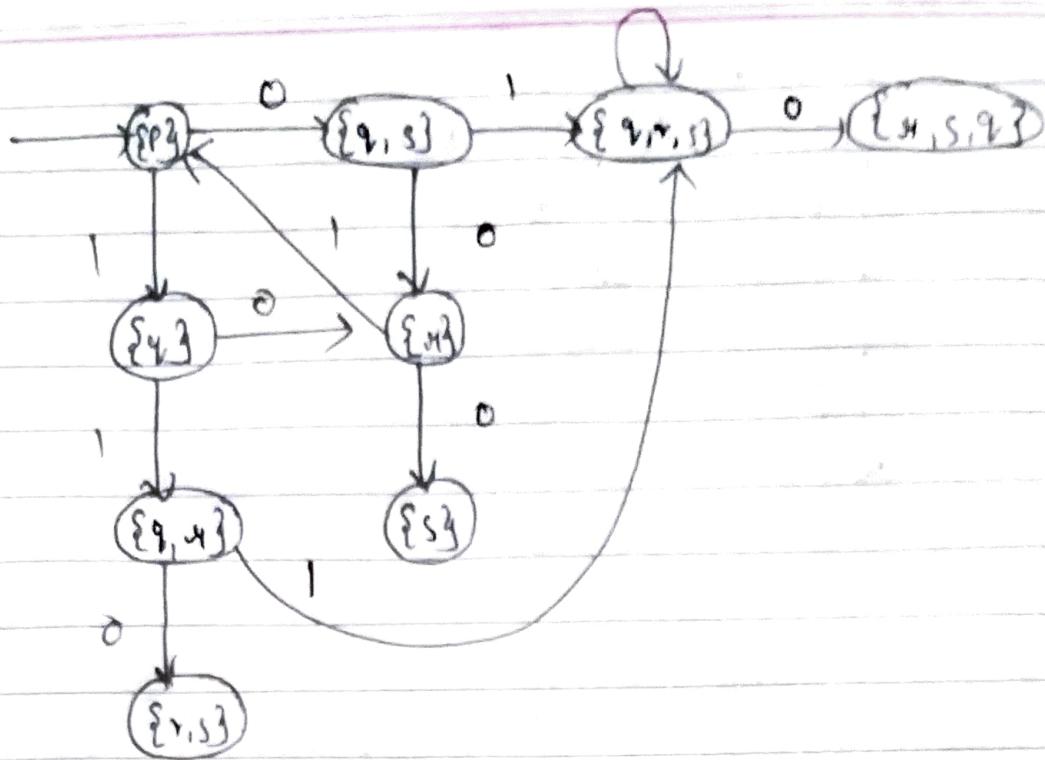


Step 6: Successor of $\{q, r, p\}$

$$\begin{aligned}
 0 \text{ successor of } \{q, r, p\} &\text{ i.e } \delta(\{q, r, p\}) \\
 &= \delta(q, 0) \cup \delta(r, 0) \cup \delta(p, 0) \\
 &= \{s\} \cup \{r\} \cup \{q, s\} \\
 &= \{q, s, r\}
 \end{aligned}$$

1 Successor of $\{q, r, p\}$ i.e $\delta(\{q, r, p\}, 1)$

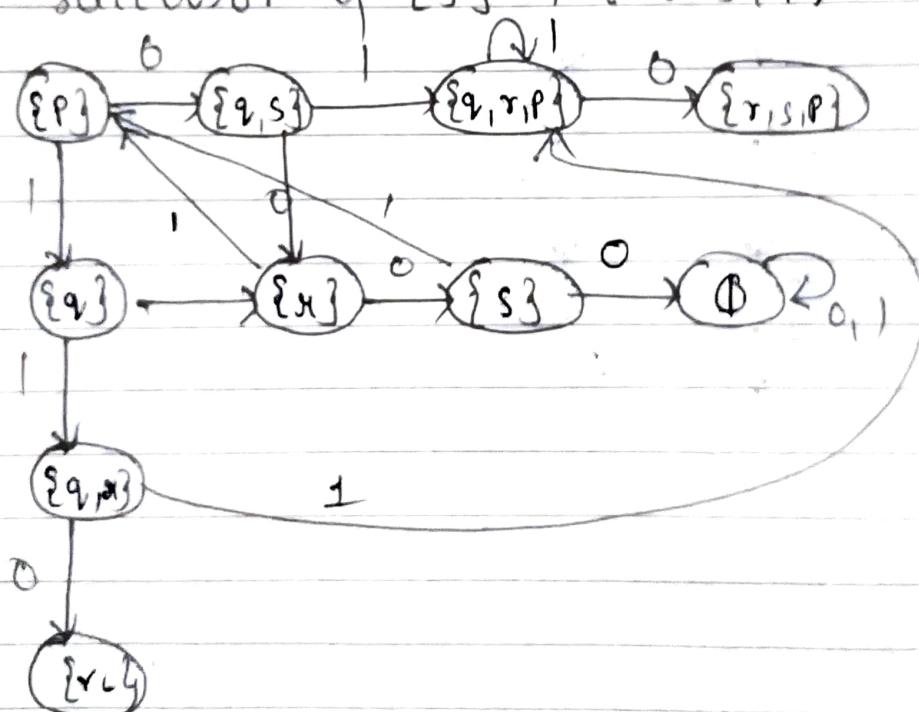
$$\begin{aligned}
 &= \delta(q, 1) \cup \delta(r, 1) \cup \delta(p, 1) \\
 &= \{q, r\} \cup \{p\} \cup \{q\}
 \end{aligned}$$



Step 7 Successor of $\{S\}$

0 Successor of $\{S\}$ i.e $\delta(S, 0) = \{\emptyset\}$

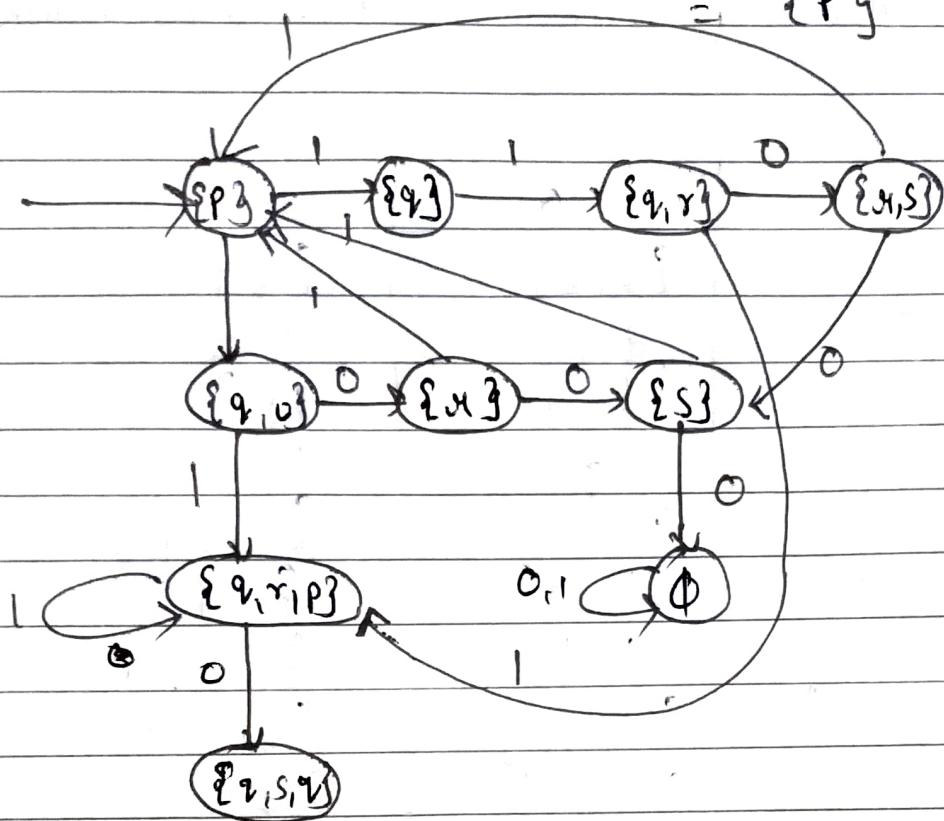
1 Successor of $\{S\}$ i.e $\delta(S, 1) = \{P\}$



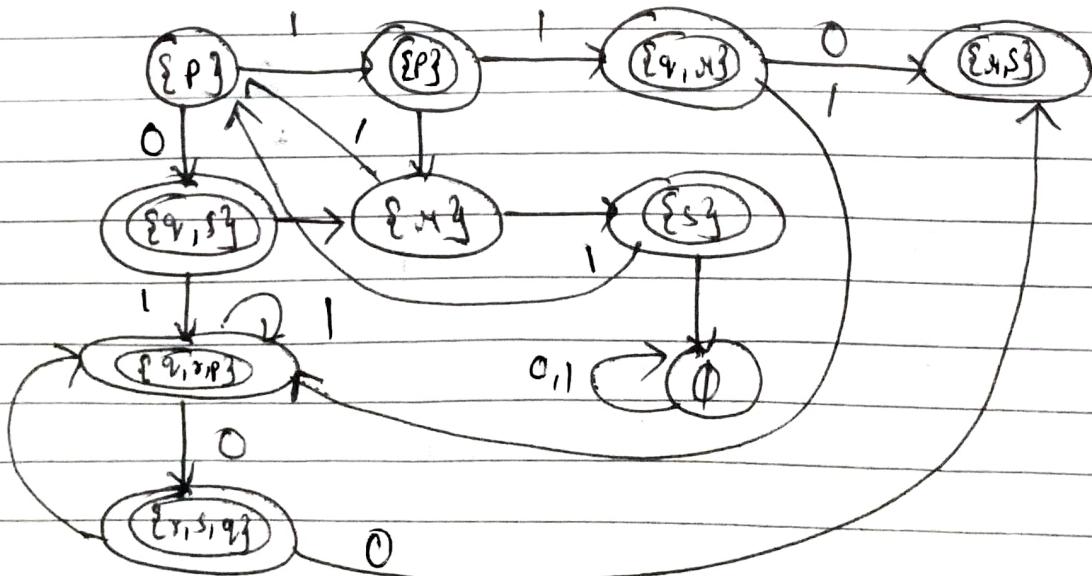
Step 8 :- Successor of $\{R, S\}$

$$\begin{aligned}
 0 \text{ Successor of } \{R, S\} \text{ i.e } & \delta(\{R, S\}, 0) \\
 &= \delta(R, 0) \cup \delta(S, 0) \\
 &= \delta\{\{S\}\} \cup \delta\{\emptyset\} \\
 &= \{\{S\}\}
 \end{aligned}$$

$$\begin{aligned}
 & \exists \text{ successor of } \{r, s\} \text{ i.e } \delta(\{r, s\}, 1) \\
 & = \delta(r, 1) \cup \delta(s, 1) \\
 & = \{P\} \cup \{P\} \\
 & = \{P\}
 \end{aligned}$$



$$\begin{aligned}
 & \text{Step 9 : Successor of } \{r, s, q\} \\
 & \exists \text{ successor of } \{r, s, q\} \text{ i.e } \delta(\{r, s, q\}, 0) \\
 & = \delta(r, 0) \cup \delta(s, 0) \cup \delta(q, 0) \\
 & = \{P\} \cup \{P\} \cup \{q\} \\
 & = \{P, q, m\}
 \end{aligned}$$



Step 10: Since no new successors are generated the process of subset generation step of and s are final state in the NFA. Every subset containing the q and s are final states.

OF A

		0	1
A	{P}	{q, s}	{q}
B	{q}	{x}	{q, x}
C	{x}	{s}	{p}
D	{s}	\emptyset	{p}
E	{q, x}	{x, s}	{q, x, p}
F	{x, s}	{s}	{p}
G	{q, s}	{x}	{q, x, p}
H	{q, x, p}	{x, s, q}	{q, x, p}
I	{x, s, p}	{x, s}	{q, x, p}
J	\emptyset	\emptyset	\emptyset

	0	1
A	G	B
B	C	E
C	D	A
D	J	A
E	F	H
F	D	A
G	C	H
H	I	H
I	F	H
J	J	J

Q2.

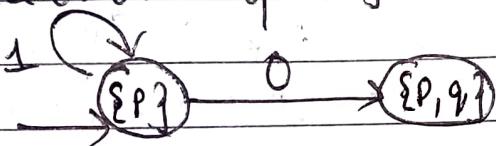
	0	1
$\rightarrow P$	$\{\{P, q\}\}$	$\{\{P\}\}$
q	$\{\{M, s\}\}$	$\{\{t\}\}$
M	$\{\{P, s\}\}$	$\{\{t\}\}$
s*	$\{\emptyset\}$	\emptyset
t*	\emptyset	\emptyset

Soln

Step 1 : Starting State $\{\{P\}\}$ is the first state

0 successor of $\{\{P\}\}$ i.e $S(\{P\}, 0) = \{\{P, q\}\}$

1 successor of $\{\{P\}\}$ i.e $S(\{P\}, 1) = \{\{P\}\}$



Step 2 : Successor of $\{\{P, q\}\}$

0 successor of $\{\{P, q\}\}$ i.e $S(\{\{P, q\}\}, 0)$

$$= S(\{P, 0\}) \cup S(q, 0)$$

$$= \{\{P, q\}\} \cup \{\{M, s\}\}$$

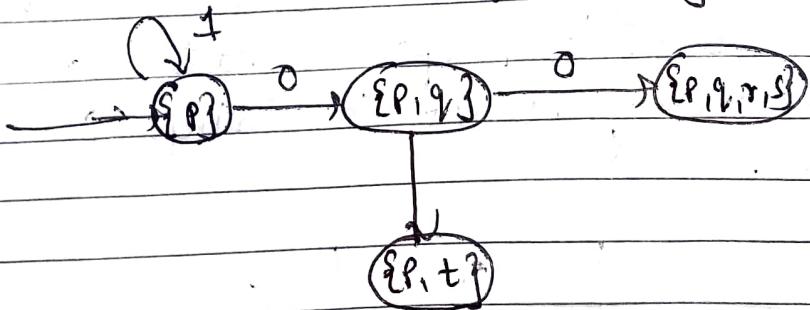
$$= \{\{P, q, M, s\}\}$$

1 successor of $\{\{P, q\}\}$ i.e $S(\{\{P, q\}\}, 1)$

$$= S(P, 1) \cup S(q, 1)$$

$$= \{\{P\}\} \cup \{\{t\}\}$$

$$= \{\{P, t\}\}$$



Step 3 :- Successor of $\{\{P, t\}\}$

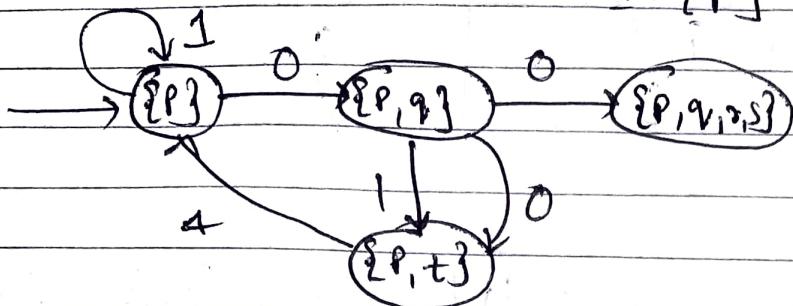
0 successor of $\{\{P, t\}\}$ i.e $S(\{\{P, t\}\}, 0)$

$$= S(P, 0) \cup S(t, 0)$$

$$= \{\{P, q\}\} \cup \{\{\emptyset\}\}$$

$$= \{\{P, q\}\}$$

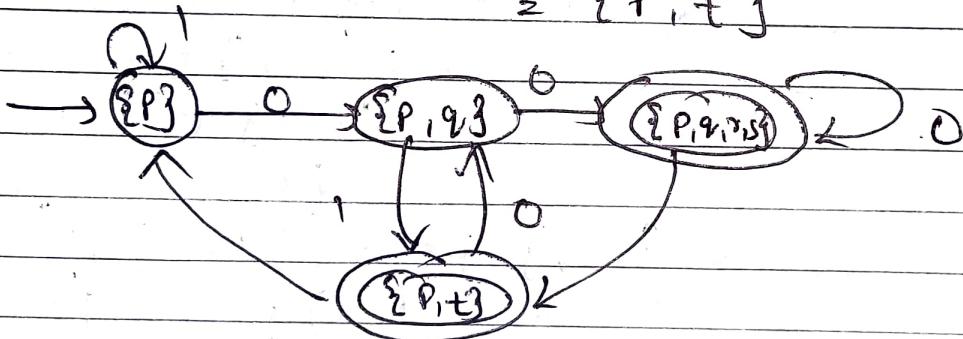
$$\begin{aligned}
 1 \text{ Successor of } \{p, t\} \text{ i.e. } & S(\{p, t\}, 1) \\
 = & S(p, 1) \cup S(t, 1) \\
 = & \{p\} \cup \{t\} \\
 = & \{p\}
 \end{aligned}$$



Step 4 :- Successor of $\{p, q, r, s\}$

$$\begin{aligned}
 0 \text{ Successor } \{p, q, r, s\} \text{ i.e. } & S(\{p, q, r, s\}, 0) \\
 = & S(p, 0) \cup S(q, 0) \cup S(r, 0) \cup S(s, 0)
 \end{aligned}$$

$$\begin{aligned}
 = & \{p, q\} \cup \{r, s\} \cup \{p, r\} \cup \{q, s\} \\
 = & \{p, t\}
 \end{aligned}$$



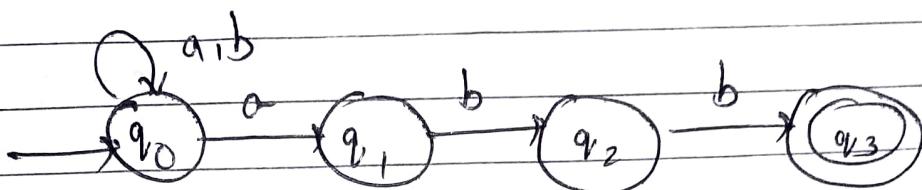
Step 5 :- Since, no. of successors are generated, the process of subset generation stops. s and t are final state in the NFA: every subset containing s and t is the final state.

D.F.A

	0	1
A	$\{p\}$	$\{p\}$
B	$\{p, q\}$	$\{p, t\}$
C	$\{p, t\}$ *	$\{p\}$
D	$\{p, q, r, s\}$	$\{p, t\}$

	0	1
A	B	A
B	D	C
C	B	A
D	D	C

Q3



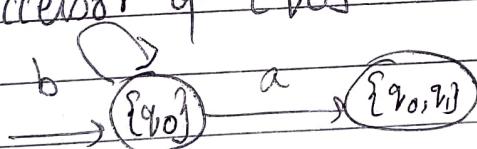
	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	\emptyset	$\{q_0\}$
q_2	$\{q_3\}$	\emptyset
q_3	\emptyset	\emptyset

Step 1 : q_0 is the starting state

$\therefore q_0$ is the final state

a successor of $\{q_0\}$ i.e $\delta(q_0, a) = \{q_0, q_1\}$

b successor of $\{q_0\}$ i.e $\delta(q_0, b) = \{q_0\}$



Step 2 : successor of $\{q_0, q_1\}$

a successor of $\{q_0, q_1\}$ i.e $\delta(\{q_0, q_1\}, a)$

$$= \delta(q_0, a) \cup \delta(q_1, a)$$

$$= \{q_0, q_1\} \cup \{\emptyset\}$$

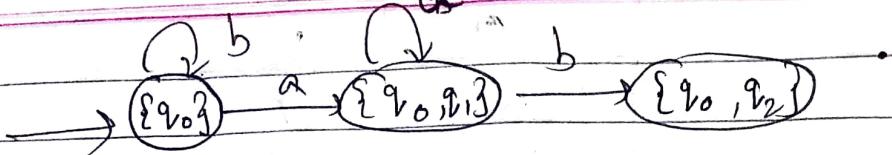
$$= \{q_0, q_1\}$$

b successor of $\{q_0, q_1\}$ i.e $\delta(\{q_0, q_1\}, b)$

$$= \delta(q_0, b) \cup \delta(q_1, b)$$

$$= \{q_0\} \cup \{q_2\}$$

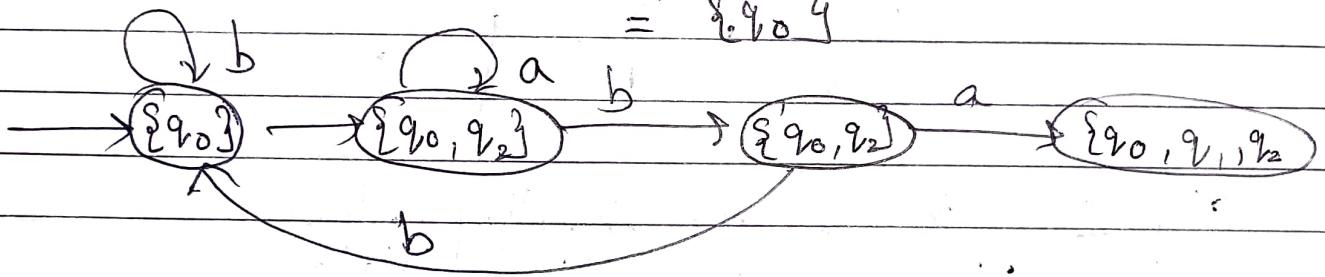
$$= \{q_0, q_2\}$$



Step 3 :- Successor of $\{q_0, q_2\}$

$$\begin{aligned}
 & \text{a successor of } \{q_0, q_2\} \text{ i.e } \delta(\{q_0, q_2\}, a) \\
 &= \delta(q_0, a) \cup \delta(q_2, a) \\
 &= \{q_0, q_1\} \cup \{q_2\} \\
 &= \{q_0, q_1, q_2\}
 \end{aligned}$$

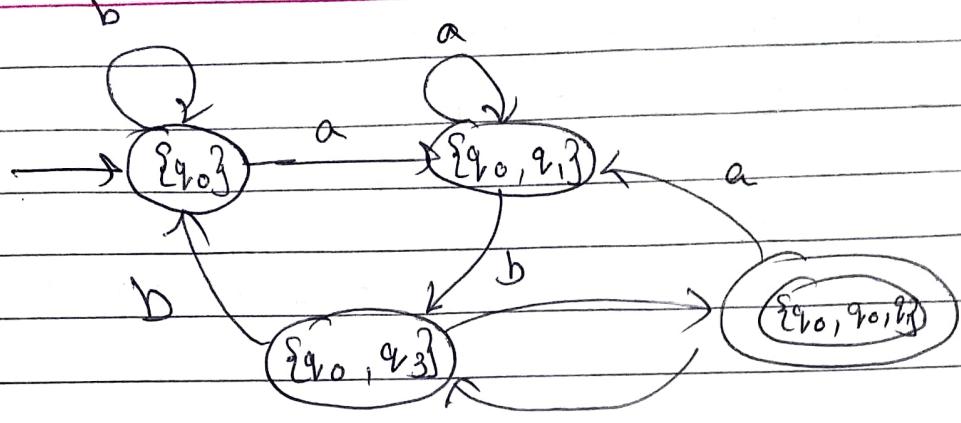
$$\begin{aligned}
 & \text{b successor of } \{q_0, q_2\} \text{ i.e } \delta(\{q_0, q_2\}, b) \\
 &= \delta(q_0, b) \cup \delta(q_2, b) \\
 &= \{q_0\} \cup \emptyset \\
 &= \{q_0\}
 \end{aligned}$$



Step 4 :- Successor of $\{q_0, q_1, q_2\}$

$$\begin{aligned}
 & \text{a successor of } \{q_0, q_1, q_2\} \text{ i.e } (\{q_0, q_1, q_2\}; a) \\
 &= \delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a) \\
 &= \{q_0, q_1\} \cup \{\emptyset\} \cup \{\emptyset\} \\
 &= \{q_0, q_1\}
 \end{aligned}$$

$$\begin{aligned}
 & \text{b successor of } \{q_0, q_1, q_2\} \text{ i.e } \delta(\{q_0, q_1, q_2\}, b) \\
 &= \delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_2, b) \\
 &= \{q_0\} \cup \{q_2\} \cup \{\emptyset\} \\
 &= \{q_0, q_2\}
 \end{aligned}$$



Step 5 : since, now new successor is generated
 : subset generation stops
 q_3 is the final state in NFA
 Every subset containing q_3 is final state

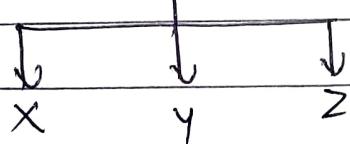
DFA

	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_3\}$
$\{q_0, q_3\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_3\}$

$$Q4. \quad A = \{yy \mid y \in \{0, 1\}^*\}$$

→ Step 1 Assuming that A is regular.
 pumping length p

$$S = 0^p 0^p 1 0^p,$$



let's assume $p = 7$

$$S = \underbrace{00}_x \quad \underbrace{0000}_y \quad \underbrace{1000000}_z \quad |xy| \leq p$$

Step 2:- ① for for $i = 0$

$$S = xyz, xy^i z \in A$$

Consider $i = 0$

$$x = 00 \quad y = 000000 \quad z = 1000000001$$

$$= xy^0 z \Rightarrow xz$$

$$= \underbrace{00}_x \quad \underbrace{100000000}_z$$

Hence, $xy^i z \in A$

② For $i = 1$

$$S = xyz, xy^i z \in A$$

Consider $i = 1$,

$$x = 00 \quad y = 000000 \quad z = 1000000001$$

$$xy^i z = xy^1 z = xyz$$

$$= \underbrace{00}_x \quad \underbrace{00000}_y \quad \underbrace{1000000001}_z$$

$$|xyz| \leq p$$

\therefore It satisfies $xy^i z \in A$ for $i = 1$

③ For $i = 2$

$$S = xyz, xy^i z \in A$$

Consider $i = 2$

$$x = 00, y = 000000, z = 1000000001$$

$$xy^i z = xy^2 z$$

$$= \underbrace{00}_x \quad \underbrace{000\ 00\ 0000}_y \quad \underbrace{1000\ 00001}_z$$

Hence, $xy^i z \in A$

Hence, all the conditions are not satisfied Therefore, S cannot be pumped.