

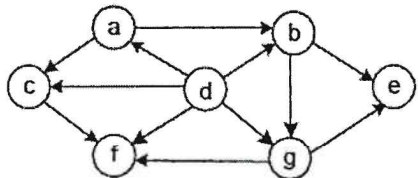
**July 2021: END SEMESTER ASSESSMENT (ESA) B.Tech. IV SEMESTER**

**UE17/18/19CS251: DESIGN AND ANALYSIS OF ALGORITHMS**

Time: 3 Hrs.

## Answer All Questions

Max Marks: 100

1.	a)	Prove that if $t_1(n) \in \Omega(g_1(n))$ and $t_2(n) \in \Omega(g_2(n))$ then $t_1(n) + t_2(n) \in \Omega(\max\{g_1(n), g_2(n)\})$	4
	b)	Solve the following recurrence relations using substitution method i) $f(n) = f(n-1) + n$ for $n > 0, f(0) = 0$ ii) $x(n) = 3x(n-1)$ for $n > 1, x(1) = 4$ iii) $x(n) = x(n/2) + n$ for $n > 1, x(1) = 1, n = 2^k$	6
	c)	$A_1, A_2, A_3, \dots, A_m$ , are $m$ sorted arrays, each having $n$ elements. Give an $O(mn \log m)$ algorithm to combine them into a single sorted array of $mn$ elements. (Assume $n$ is a power of 2)	6
	d)	Prove that $\log n \in O(2^{\sqrt{n/4}})$	4
2.	a)	Design a $\Theta(n)$ algorithm to count the number of substrings that start with an A and end with a B in the given text. (For example, there are 9 such substrings in DAAXBABAGBD)	6
	b)	Assume you are given an algorithm FINDMEDIAN which can determine the median element from a list of $n$ elements with time complexity $O(n)$ . With the help of FINDMEDIAN, design an algorithm with $O(n)$ time complexity to determine $k^{\text{th}}$ ( $1 \leq k \leq n$ ) smallest element in an array of $n$ distinct positive integers (array index starts at 1).	6
	c)	Using FINDMEDIAN algorithm mentioned in Q.2b, design algorithm for modified Quicksort which has $O(n \log n)$ worst case time complexity	8
3.	a)	Derive upper bound for the height of 2-3 tree with $n$ nodes.	4
	b)	Analyze the best-case and worst-case time complexity of Insertion sort.	4
	c)	Explain how to use DFS based topological sorting algorithm to check if a given directed graph is acyclic or not. Apply DFS to list the vertices of given directed graph in topologically sorted ordering  	4+4

3.	d)	State True/False <ul style="list-style-type: none"> <li>i) Johnson Trotter algorithm generates permutations in lexicographic order</li> <li>ii) Root of red black tree should be assigned red color</li> <li>iii) Upper bound on the height of red black tree is <math>2(\log n + 1)</math></li> <li>iv) It is possible to have B Tree of minimum degree one</li> </ul>	4
4.	a)	What are the two tables required to search for a pattern in the given text by Boyer-Moore algorithm. Construct both the tables for the pattern 10101010	6
	b)	Use Prim's algorithm starting at node A to compute the Minimum Spanning Tree (MST) of the given graph. Write down the edges of the MST in the order in which Prim's algorithm adds them to the MST. <div style="text-align: center;"> </div>	6
	c)	Compare the time complexities of the Dijkstra's algorithm and the Floyd's algorithm to determine the minimum weight paths between all pairs of vertices for sparse graphs and dense graphs, and justify which algorithm you would use for each of these two types of graphs	6
	d)	What is time complexity of decrease by constant factor algorithm to solve exponentiation problem( $a^n$ )	2
5.	a)	Define <ul style="list-style-type: none"> <li>i) Np Complete Problem</li> <li>ii) Polynomially Reducible Problem</li> </ul>	4

5.	b)	<p>Draw state space tree to solve Travelling Salesman problem for the given intercity matrix using Branch and Bound Technique</p> <div><div><div>0</div><div>3</div><div>4</div><div>2</div><div>7</div></div><div><div>A</div><div>B</div><div>C</div><div>D</div><div>E</div></div></div>	6															
	c)	<p>What is the key difference between problems that can be solved efficiently by dynamic programming and the ones that can be solved efficiently by divide-and conquer strategy? What is the consequence of this difference for dynamic programming solutions?</p>	2+4															
	d)	<p>Solve following instance of 0/1knapsack problem using Dynamic Programming</p> <table><tr><td>objects</td><td>weights</td><td>Profits</td></tr><tr><td>1</td><td>2</td><td>3</td></tr><tr><td>2</td><td>3</td><td>4</td></tr><tr><td>3</td><td>4</td><td>5</td></tr><tr><td>4</td><td>5</td><td>6</td></tr></table> <p>Capacity of knapsack=5</p>	objects	weights	Profits	1	2	3	2	3	4	3	4	5	4	5	6	4
objects	weights	Profits																
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