MA080G Cryptography Assignment 1

Viktor Rosvall

March 29, 2019

Question 1

Showing you working, decrypt the shift-encrypted message PM FVB DHUA AV RLLW H ZLJYLA FVB TBZA HSZV RLLW PA MYVT FVBYZLSM.

Answer 1

By looking at the first digram PM, I made the guess that the plaintext is "if". So I start the decryption by shifting the alphabet 7 steps to the left for P to become an "f". After that I begin checking if the remaining ciphertext translate to a plausible English sentence.

By shifting 7 steps to the left the sentence becomes: "if you want to keep a secret you must also keep it from yourself".

Question 2

10.6.1

Write down the cycle notation for permutation which effect the rearrangement.

$$\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 3 & 5 & 7 & 8 & 4 & 6 & 1 & 2 & 9 \end{array}$$

10.6.2

Let σ, τ be the permutations of $\{1, 2, ..., 8\}$ whose effects representations in cycle notation are:

$$\sigma = (1\ 2\ 3)\ (4\ 5\ 6)\ (7\ 8), \quad \tau = (1\ 3\ 5\ 7)\ (2\ 6)\ (4)\ (8)$$

Write down the cycle notations for $\sigma\tau, \tau\sigma, \sigma^2, \sigma^{-1}, \tau^{-1}$.

10.6.4

Show that there are just three members of S_4 which have two cycles of lenght 2 when written in cycle notation.

10.6.5

Let K denote the subset of S_4 which contains the identity permutation i and the three permutations $\alpha_1, \alpha_2, \alpha_3$ described in the previous exercise. Write out the "multiplication table" for K, when multiplication is interpreted as composition of permutations.

Answer 2

By using cycle notation to calculate blabal bla The inverse of a permutation in cycle notation is the number backwards. One-cycles can be discarded as they will not effect the calculations.

10.6.1

Cycle notation: (1 3 7) (2 5 4 8)

10.6.2

$$\begin{aligned}\sigma\tau &= (1\ 2\ 3)\ (4\ 5\ 6)\ (7\ 8) * (1\ 3\ 5\ 7)\ (2\ 6)\\ &= (2\ 4\ 5\ 8\ 7)\ (3\ 6)\end{aligned}$$

$$\begin{aligned}\tau\sigma &= (1\ 3\ 5\ 7)\ (2\ 6) * (1\ 2\ 3)\ (4\ 5\ 6)\ (7\ 8)\\ &= (1\ 6\ 4\ 7\ 8)\ (2\ 5)\end{aligned}$$

$$\begin{aligned}\sigma^2 &= (1\ 2\ 3)\ (4\ 5\ 6)\ (7\ 8) * (1\ 2\ 3)\ (4\ 5\ 6)\ (7\ 8)\\ &= (1\ 3\ 2)\ (4\ 6\ 5)\end{aligned}$$

$$\begin{aligned}\sigma^{-1} &= (3\ 2\ 1)\ (6\ 5\ 4)\ (8\ 7)\\ &= (1\ 3\ 2)\ (4\ 6\ 5)\ (7\ 8)\end{aligned}$$

$$\begin{aligned}\tau^{-1} &= (7\ 5\ 3\ 1)\ (6\ 2)\\ &= (1\ 7\ 5\ 3)\ (2\ 6)\end{aligned}$$

10.6.4

The number of permutations of type $[2^2]$ in S_4 can be calculated as:

$$S_4 \text{ has } \frac{4!}{2^2 2!} = 3 \text{ permutations}$$

Where the base is the loop size, and α is the number of loops.

The loops available are:

$$\begin{aligned}\alpha_1 &= (1\ 2)\ (3\ 4)\\ \alpha_2 &= (1\ 3)\ (2\ 4)\\ \alpha_3 &= (1\ 4)\ (2\ 3)\\ i &= (1)\ (3)\ (2)\ (4)\end{aligned}$$

10.6.5

K is a subset of S_n which contains the permutations $\alpha_1, \alpha_2, \alpha_3, i$ from above.

$\odot$	i	α_1	α_2	α_3
i	i	α_1	α_2	α_3
α_1	α_1	i	α_3	α_2
α_2	α_2	α_3	i	α_1
α_3	α_3	α_2	α_1	i

By looking at the multiplication table of K we see that it's symmetric around the identity because the inverse of α_1 is α_1.

Question 3

In how many ways can you rearrange the letters of the string ABRAABRAKADABRA?

Answer 3

We have 15 letters in total, but they aren't unique. I count 7 As, 3Bs, 3Rs, 1 K and 1 D. If each letter was unique, the number of permutations would be 15!, but since the number of permutations aren't, they can be counted as:

$$\frac{15!}{7!3!3!} = 7207200$$

Question 4

a) (i)

Use Euclid's algorithm to show that $gcd(17, 2018) = 1$

a) (ii)

Find two integers s and t such that:

$$17s + 2018t = 1$$

b)

Showing all your working, find all solutions $[x] \in Z_{2018}$ to the equation

$$[17] \odot [x] = [1]$$

c)

Showing all your working, find the set of all integers x which satisfy the congruence

$$17x \equiv 1 (mod\ 2018)$$

Answer 4

a) (i)

By using Euclid's algorithm step by step we get;

$$\begin{aligned} 2018 &= 17(118) + 12 \rightarrow & 12 &= 2018 - 17(118) \\ 17 &= 12(1) + 5 \rightarrow & 5 &= 17 - 12(1) \\ 12 &= 5(2) + 2 \rightarrow & 2 &= 12 - 5(2) \\ 5 &= 2(2) + \mathbf{1} \rightarrow & 1 &= 5 - 2(2) \\ 2 &= 1(2) + 0 \end{aligned}$$

So $gcd(17, 2018) = 1$.

a) (ii)

Working backwards through the algorithm step by step we get:

$$\begin{aligned} gcd(17, 2018) = 5 &= 5 - 2(2) & &= 5 - [12 - 5(2)](2) \\ &= 5(5) - 12(2) & &= [17 - 12(1)](5) - 12(2) \\ &= 17(5) - 12(7) & &= 17(5) - [2018 - 17(118)](7) \\ &= 17(831) - 2018(7) \end{aligned}$$

So $s = 831$ and $t = 7$.

b)

To solve $[x] \in Z_{2018}$ we need to find the multiplicative inverse of [17]. We know that a multiplicative inverse exists because the $gcd(17, 2018) = 1$.

From Euclid's algorithm we get that $s = 831$ and $t = 7$. The class [831] is thus the multiplicative inverse of [17]

$$[17] \odot [831] = [14127] = [1]$$

c)

To find the set of all integers x in the congruence:

$$17x \equiv 1 (mod\ 2018)$$

is the same as finding all $[x] \in Z_{2018}$ satisfying the equation:

$$[17] \odot [x] = [1]$$

we can calculated the inverse of [17] to be [831], thus the set of all integers is:

$$x \in \{..., -1187, 831, 2849, ...\} \quad \text{or} \quad x \in \{831 + 2018t \mid t \in Z\}$$

Question 5

Using tools to help you do the frequency analysis of cipher text. Decipher the following text and submit the first line of the plaintext together with a description of how you broke the cipher.

Answer 5

The most frequent trigram is RTA, XUO and CUB. I make the assumption that they are the most frequent trigrams: "the", "and", "ing". I focused on trigrams such as CUB and XUO which has letters in common.

By looking at the *digraph frequency* tablet, we can see that R, D, Q and K are frequent digrams (such as "ss" and "oo") with themselves. I thus assume that they are probably the letters "t", "o", "s", "l".

Through much trial and error, by focusing on trigrams mostly, I was able to find all the letter mappings which looks like this:

$$\begin{array}{cccccccccc} A & B & C & D & E & F & G & H & I & J \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ e & g & i & o & c & k & u & x & y & w \\ \\ K & L & M & N & O & P & Q & R & S & T \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ l & m & f & q & d & p & s & t & b & h \\ \\ U & V & W & X & Y & Z \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ n & V & r & a & Y & v \end{array}$$

The ciphertext letters H and N was hard to crack because they only occurred once in the text. It was by process of elimination that I was able to determine them.

The ciphertext letters V and Y has no occurrences which means that the plaintext letters "j" and "z" never occurs too.

The first line of the plaintext is:

"the sun was shining on the sea, shining with all his might"