

Final Exam Study Guide

Exam Date & Time: **December 13, 10:15am-12:15pm**

Location: **EN 2110**

- (1) What is a vector space?
 - (a) What is a subspace of a vector space?
 - (b) Use the vector space axioms to determine properties of vector spaces.
 - (i) Can you multiply elements of a vector space together?
 - (ii) Show the zero vector is unique.
 - (iii) If V is a vector space and \mathbf{v} is a vector in V , is the additive inverse of \mathbf{v} just the scalar -1 times \mathbf{v} ?
 - (c) Give examples of vector spaces.
- (2) What is a linear combination of vectors?
- (3) What does it mean for a set of vectors to span a vector space?
- (4) What does it mean for a set of vectors to generate a vector space?
- (5) What does it mean for a set of vectors to be linearly independent?
- (6) What is a basis for a vector space?
 - (a) Can a vector space have more than one basis?
 - (b) Is a generating set necessarily a basis?
 - (c) Is the number of elements in a basis for a given vector space always the same?
 - (d) In what sense is a basis a *maximally* linearly independent set?
 - (e) In what sense is a basis a *minimal* spanning set?
- (7) Given a basis for a vector space, show that every element in the vector space has a unique representation with respect to the given basis.
- (8) Given a basis \mathcal{B} for a vector space V , what are the coordinates of a vector \mathbf{v} in V with respect to \mathcal{B} ?
- (9) What is a linear transformation?
 - (a) Give examples of linear transformations.
 - (b) Show that differentiation is linear.
 - (c) Determine if a given linear transformation is one-to-one and onto.
 - (d) Construct a one-to-one linear transformation.
 - (e) Construct an onto linear transformation.
- (10) Given a basis \mathcal{B} for a vector space V , what is the coordinate mapping for V with respect to \mathcal{B} ?
 - (a) Is the coordinate mapping linear?
 - (b) Is the coordinate mapping one-to-one?
 - (c) Is the coordinate mapping onto?
- (11) Given a basis \mathcal{B} for a vector space V and a vector \mathbf{v} in V , find the coordinate vector of \mathbf{v} with respect to \mathcal{B} .
- (12) The coordinate mapping tells us that, structurally speaking, every vector space looks like what?
- (13) Explain how one can interpret vectors geometrically.

- (14) Perform matrix operations.
 - (a) When can you multiply matrices?
 - (b) When can you add matrices?
 - (c) What does it mean to multiply a matrix by a vector?
- (15) Solve a system of linear equations.
 - (a) Determine when solutions exist.
 - (b) Determine when there is a unique solution.
 - (c) What is the relationship between matrices and systems of linear equations? Where do solutions fit into this relationship?
- (16) Determine if a matrix is invertible.
- (17) Determine if a set of vectors is linearly independent.
- (18) Determine if a given vector is in the span of some set of vectors.
- (19) Show the statements in the invertible matrix theorem are equivalent.
- (20) Find the inverse of an invertible matrix.
- (21) Find the determinant of a matrix.
 - (a) Given $n \times n$ matrices A and B , what is the determinant of AB ?
 - (b) Given an $n \times n$ invertible matrix A , what is the determinant of A^{-1} ?
- (22) What is the relationship between matrices and linear transformations?
- (23) Find the standard matrix of a linear transformation.
- (24) What is a change of basis matrix?
- (25) Given a vector space V and two bases for V , \mathcal{B}_1 and \mathcal{B}_2 , find the change of basis matrix from \mathcal{B}_1 to \mathcal{B}_2 .
- (26) Let V be a vector space, let T be a linear transformation from V to itself, and let \mathcal{B} be a basis for V . Find the matrix for T with respect to \mathcal{B} .
- (27) What is the rank-nullity theorem?
 - (a) Find the null space of a linear transformation.
 - (b) Determine if a vector is in the range of a given linear transformation.
 - (c) Show that the null space of a linear transformation is a vector space.
- (28) What are eigenvalues and eigenvectors?
 - (a) Find the eigenvalues and corresponding eigenvectors for a given matrix.
 - (b) What is the characteristic polynomial of a matrix?
 - (c) What is the relationship between eigenvectors which correspond to distinct eigenvalues?
 - (d) What is the eigenspace corresponding to a given eigenvalue?
 - (i) What is the algebraic multiplicity of an eigenvalue?
 - (ii) What is the geometric multiplicity of an eigenvalue?
 - (iii) Does algebraic multiplicity always equal geometric?
- (29) What does it mean for matrices to be similar?
- (30) What does it mean for a matrix to be diagonalizable?
 - (a) How do you determine if a matrix is diagonalizable?
 - (b) Diagonalize a matrix.
- (31) Do similar matrices have the same eigenvalues?